Measuring Elliptic Apertures using Rotating Coils

Pierre Schnizer[1], Bernhard Schnizer[2], Pavel Akishin[3], Egbert Fischer[1]

[1]Gesellschaft für Schwerionenforschung
Plankstraße 1, D-64291 Darmstadt
[2]Institut für Theoretische Physik
Technische Universität Graz
Petersgasse 16, A-8010 Graz
[3]Joint Institut for Nuclear Research
Dubna, Moscow Region, Russia

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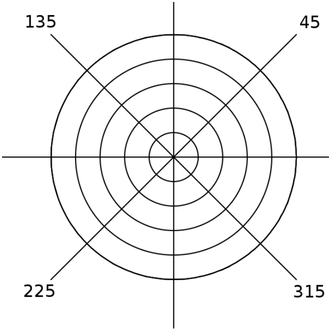
Outline

1. Theory
   - Circular Multipoles vs. Elliptic Multipoles
   - Converting Elliptic to Cyclic Multipoles
2. Test on Field Calculations for SIS 100
   - 8 Turn Single Layer Dipole
   - 5 Turn L Quadrupole
3. Field Reconstruction for Measurement
   - measurement description
   - Connecting measurements

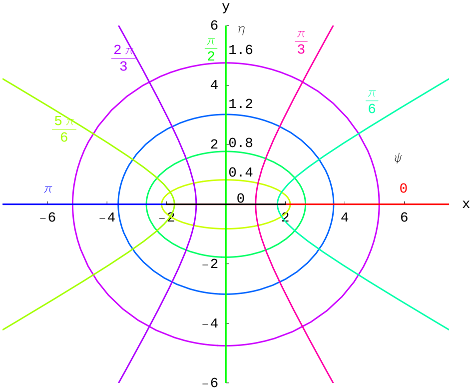
Motivation

- FAIR @ GSI $\longrightarrow$
  - with elliptic beam pipe: SIS 100, NESR, RESR
  - consice description of the field required
  - valid within the whole aperture
    - describe / compare the quality of the magnet(s)
    - managable data for users: e.g. beam dynamics

Coordiante systems I/II



circles and lines



ellipse and hyperbola

Coordiante systems II/II

Plane polar coordinates

$$0 \leq r < \infty \qquad -\pi \leq \phi \leq \pi,$$

$$x = r\cos\phi \qquad y = r\sin\phi,$$

Elliptic coordiantes:

$$\begin{aligned} x &= e\cosh\eta\ \cos\psi \\ y &= e\sinh\eta\ \sin\psi \end{aligned}$$

$$0 \leq \eta < \infty,$$

$$-\pi \leq \psi \leq \pi.$$

$$\varepsilon := e/a = \sqrt{1 - b^2/a^2}$$

Reference ellipse:

$$a = e\cosh\eta_0, \qquad b = e\sinh\eta_0.$$

Field description for potential equation $\Delta\Phi = 0$

Set of functions

$r^m e^{\mathrm{i}m\phi}$		$\sinh(n\eta)\, e^{\mathrm{i}n\psi}$	$\cosh(n\eta)\, e^{\mathrm{i}n\psi}$	for $n \neq 0$
$r^m e^{-\mathrm{i}m\phi}$	$-\infty < m < \infty$	1	η	for $n = 0$

General solution

circular:

$$\Phi(r,\phi) = \sum_{m=-\infty}^{\infty} C_m \;(r/R_{Ref})^{|m|}\, e^{\mathrm{i}m\phi}$$

elliptic:

$$\Psi_g(\eta,\psi) = A_0 + B_0\eta + {\sum_{n=-\infty}^{n=\infty}}' \left[B_n \sinh(|n|\eta) + A_n \cosh(|n|\eta)\right] e^{in\psi}$$

The primed sum does not contain the term $n = 0$.

Elliptic Multipoles

Required solution with real functions

$$\Psi(\eta,\psi) = A_0 \underbrace{\frac{1}{2}}_{:=ce_0} + \sum_{n=1}^{\infty} \Big[A_n \underbrace{\cos(n\psi)\frac{\cosh(n\eta)}{\cosh(n\eta_0)}}_{:=ce_n} + B_n \underbrace{\sin(n\psi)\frac{\sinh(n\eta)}{\sinh(n\eta_0)}}_{:=se_n} \Big]. \quad (1)$$

$\Psi(\eta,\psi)$ can represent a scalar or e. g. $\mathbf{B} = B_y + i\, B_x$.
η_0 corresponds to R_{Ref}.

Calculating A_n, B_n

Calculation of A_m and B_m using $\Psi(\eta_0, \psi)$ (Euler Formulas) or a FFT.

$$\begin{aligned} A_0 &= 2D_0; \\ A_n &= \mathrm{Re}(D_n + D_{-n}) + i\,\mathrm{Im}(D_n + D_{-n}), \\ B_n &= i\,\mathrm{Re}(D_n - D_{-n}) - \mathrm{Im}(D_n - D_{-n}); \\ n &= 1, 2, 3, \ldots. \end{aligned} \tag{2}$$

$$D_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Psi(\eta_0, \psi)\, e^{(-in\,\psi)} \mathrm{d}\psi, \qquad -\infty < n < \infty; \tag{3}$$

The last formulas still simplify if $\Psi(\eta_0, \psi)$ is real, since then $D_n^* = D_{-n}$.

From Elliptic to Cyclic Multipoles

- Circular multipoles widespread (e.g beam dynamics)
- calculation of the multipoles:
  - elliptic multipoles $A_n, \ B_n$
  - the inverse transformation matrix $\hat{S}$ elliptic $\rightarrow$ circular
- calculating $\hat{S}$
  - insert elliptic coordinates (7) and (8) into $(z/R_0)^m$ and $(z^*/R_0)^m$; $m \geq 0$.
  - expand resulting expression in harmonics of hyperbolic and trignometric functions $\rightarrow$ $ce_n(\eta, \psi)$, $se_n(\eta, \psi)$
  - invert the matrix $\hat{T} \rightarrow \hat{S}$

Theory Summary

- Circular Multipoles: Fourier Series on Circles
- Elliptic Multipoles: Fourier Series on Ellipse
- harmonics for both can be calculated using FFT
- but
    - only the circular ones correspond to the Carthesian ones
    - for the elliptic: only if $B_y + \mathrm{i} B_x$ is imposed on the ellipse ones gets a straightforward transform to circular ones
    - SSW measurement on ellipse measures B_η!

Test on SIS 100 magnet options

dipole 8 turn single layer dipole 0.13 T at 873A

quadrupole 5 turn quadrupole providing 35 T/m at 6600A

high current in the cable $\longrightarrow$ field quality

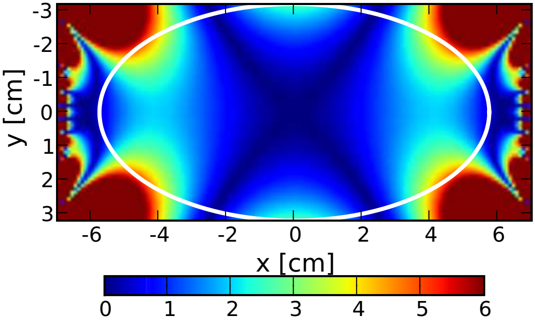
Field quality in units

$$\mathbf{\Delta B(z)} = \left[\mathbf{B(z)} - \mathbf{C_m}\left(\frac{z}{R_{Ref}}\right)^{m-1}\right]\frac{10^4}{\mathbf{C_m}}$$

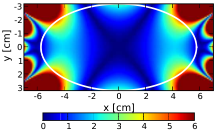
with $\mathbf{C_m}$ the strength of the main multipole

Dipole: Original Data

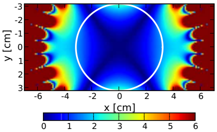
B_y:



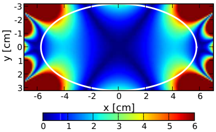
Dipole: Interpolation



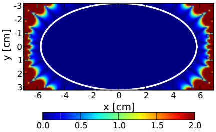
elliptic $\mathcal{C}_e$



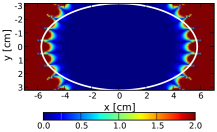
cyclic $\mathcal{C}$



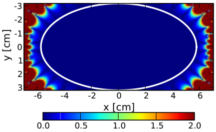
ell. $\longrightarrow$ cyclic in $\mathcal{C}_e$



Δ elliptic $\mathcal{C}_e$



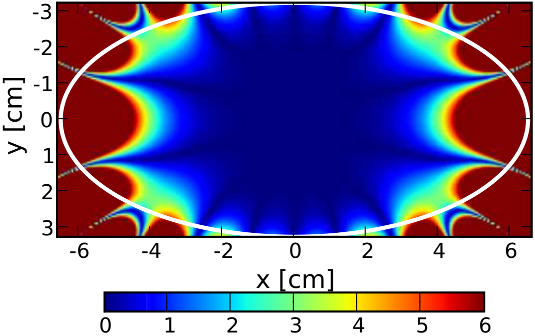
Δ cyclic $\mathcal{C}$



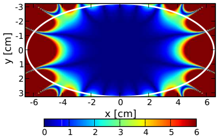
Δ ell. $\longrightarrow$ cyclic in $\mathcal{C}_e$

Quadrupole: Original Data

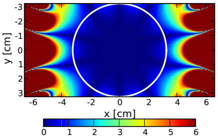
B_y:



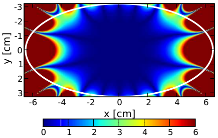
Quadrupole: Interpolation



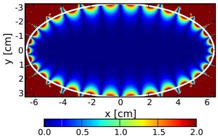
elliptic $\mathcal{C}_e$



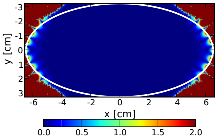
cyclic $\mathcal{C}$



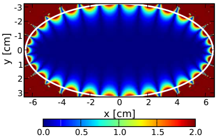
ell. $\rightarrow$ cyclic in $\mathcal{C}_e$



Δ elliptic $\mathcal{C}_e$



Δ cyclic $\mathcal{C}$

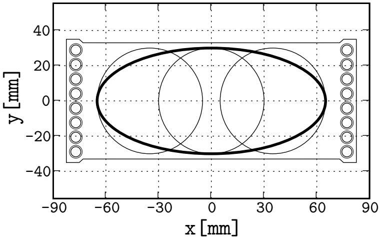


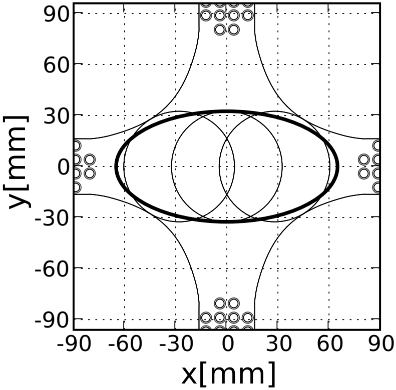
Δ ell. $\rightarrow$ cyclic in $\mathcal{C}_e$

Motiviation for rotating coils measurements

- rectangular apertures → search coils
  - require a precise reference surface → pol
  - do not provide angle of the magnet
- SIS 100 → superconducting → anticryostat (circular, moveable)
  - rotating coils → circular multipoles
  - not covering whole area of interest
  - thus measurements at different locations
  - how many locations are enough?
  - how to combine the measurements?

Postion of measurement





Not the whole ellipse is covered!

Connecting measurement data I/II

- Interpolation between the circles

$$\mathbf{B_i}(\mathbf{z}) = (1-\lambda)\sum_{n=1}^{N}\mathbf{C_n^{c}}\left(\frac{\mathbf{z}}{R_{Ref}}\right)^{(n-1)} + \lambda\sum_{n=1}^{N}\mathbf{C_n^{l,r}}\left(\frac{\mathbf{z}\pm a}{R_{Ref}}\right)^{(n-1)}$$

- How calculate lambda?
- validity of extrapolation: eg. distance from centre

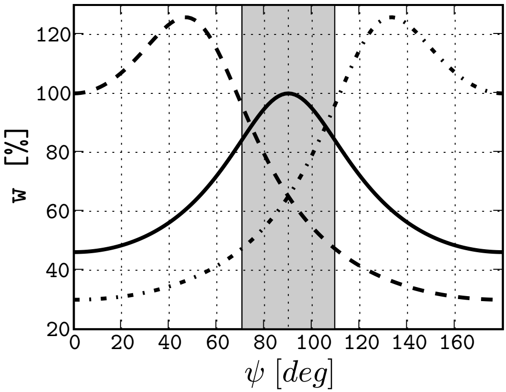
$$w^{l} = \frac{R_{Ref}}{|\mathbf{z}-a|} \qquad w^{c} = \frac{R_{Ref}}{|\mathbf{z}|} \qquad w^{r} = \frac{R_{Ref}}{|\mathbf{z}+a|}$$

$$\lambda^{cl} = w^{c}/\left(w^{c}+w^{l}\right) \qquad \lambda^{cr} = w^{c}/\left(w^{c}+w^{r}\right)$$

Connecting measurement data II/II

$\lambda(p0) = 0$ $\lambda(p1) = 1$
$\lambda'(p_0) = 0$ $\lambda'(p_0)$
$\lambda(p) = 3p^2 - 2p^3$

p_0 intersection ellipse, measurment at left/right, p_1 at $\psi = 90(270)$ (centre)

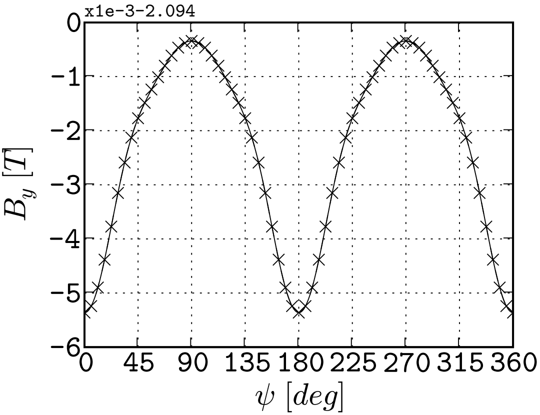


Field on the ellipse

Field on the ellipse $\mathbf{B}(\mathbf{z}) = \begin{cases} \sum_{n=1}^{N} \mathbf{C}_\mathbf{n}^{\mathrm{l,r}} \left(\frac{\mathbf{z} \pm a}{R_{Ref}}\right)^{(n\cdot} \\ \mathbf{B}_\mathbf{i}(\mathbf{z}) \end{cases}$

$\psi = cosh^{-1}(z/e)$

Multipoles on the interpolation



Archieved precision

- for dipole: difference reconstructed $\leftrightarrow$ field 1 ppm
- for quadrupole 1 per mille $\rightarrow$ improvement required

Conclusion

- elliptic multipoles solution of $\Delta\Phi = 0$
- describe field within reference ellipse
- elliptic multipole $\longrightarrow$ cicular multipoles
- field reconstruction from measurement
    - demonstrated for dipole
    - refinement required for quadrupole