Calculation of $K \to \pi l \nu$ form factor in $N_f = 2+1$ QCD at physical point on $(10\text{fm})^3$ volume

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(PACS Collaboration)

Introduction

CKM matrix unitarity check (up quark row)

$$\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$$

If CKM matrix is unitary,

$$\Delta_u = 0$$

In recent result of CKM matrix element (PDG 2018)

M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).

$$|V_{ud}| = 0.97420(21)$$

from superallowed nuclear β decay

Hardy and Towner PoS CKM2016, 028 (2016).

$$|V_{us}| = 0.2231(8)$$

from combination experiment and semileptonic decay form factor by N_f=2+1 lattice calculations

$$|V_{us}| = 0.2253(7)$$

from combination experiment and the ratio of decay constants f_K/f_π by N_f=2+1 lattice calculations

$$|V_{ub}| = 3.94(36) \times 10^{-3}$$

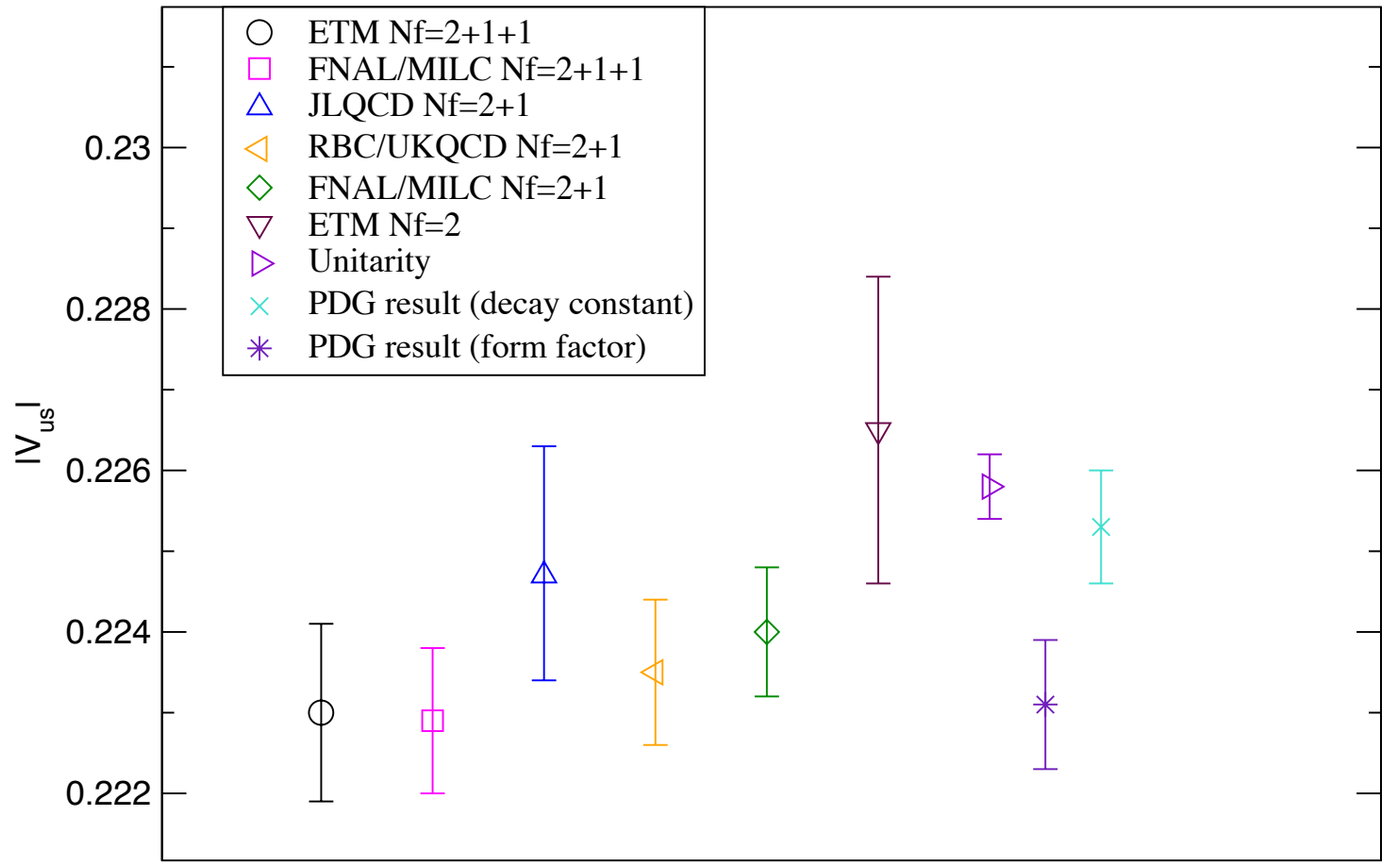
from combination of inclusive and exclusive decay exclude $B \to \tau\ \nu$

Introduction

the result of “unitarity”
is assumed $\Delta_u = 0$
and ignoring V_{ub}

right panel : lattice calculations of Vus from semileptonic form factor and result assuming unitarity

* FLAG, arXiv:1607.00299v1 [hep-lat]
[Eur Phys J C Part Fields. 2017;77(2):112.]



there is difference between “unitarity” and PDG (FF)’s by 2σ

uncertainty from lattice calculation

chiral extrapolation, interpolation (extrapolation) to the zero momentum transfer, finite size effect etc…

Purpose

calculation the semileptonic ($K \to \pi l \nu$) form factor at physical point on the large volume

Construction of form factors

Semileptonic form factor

vector form factor $\langle \pi(\vec{p_\pi}) | V_\mu | K(\vec{p_K}) \rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$

scalar form factor $f_0(q^2) = f_+(q^2) + \frac{-q^2}{(m_K^2 - m_\pi^2)} f_-(q^2) = f_+(q^2) \left(1 + \frac{-q^2}{(m_K^2 - m_\pi^2)} \xi(q^2)\right)$

$$\left(\xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}\right)$$

At q^2=0, $f_+(0) = f_0(0)$

to construct form factors, we calculate
vector current 3point functions and 2point functions

$$C_\mu^{XY}(\vec{P}', \vec{P}) = \frac{Z_Y(\vec{P}') Z_X(\vec{P})}{4 E_Y(\vec{P}') E_X(\vec{P})} \frac{1}{Z_v} \times \langle Q(\vec{P}') | V_\mu | X(\vec{P}) \rangle exp(-E_Y(\vec{P}')(t_f - t)) exp(-E_X(\vec{P}) t)$$

$$(0 << t << t_f, X, Y = \pi, K)$$

and construct ratios by 3(or 2)point function

the ratios (combination of 2point and 3point functions)

JLQCD, arXiv:1705.00884v1 [hep-lat]

$$d_1 = \frac{C_4^{\pi K}(\vec{0},\vec{0})C_4^{K\pi}(\vec{0},\vec{0})}{C_4^{KK}(\vec{0},\vec{0})C_4^{\pi\pi}(\vec{0},\vec{0})} \to \frac{(m_K+m_\pi)^2}{4m_K m_\pi}(\boxed{f_0(q^2_{min})})^2$$

$$(0 << t << t_f, q^2_{min} = -(m_K - m_\pi)^2)$$

$$d_2 = \frac{C_4^{\pi K}(\vec{0},\vec{p})C^{\pi}(\vec{0})}{C_4^{\pi K}(\vec{0},\vec{0})C^{\pi}(\vec{p})} \to \left(\frac{E_\pi(\vec{p})+m_K}{m_\pi+m_K} + \frac{E_\pi(\vec{p})-m_K}{m_\pi+m_K}\xi(q^2)\right)\frac{\boxed{f_+(q^2)}}{f_0(q^2_{min})}$$

$$(0 << t << t_f)$$

$$d_3 = \frac{C_i^{\pi K}(\vec{0},\vec{p})C_4^{KK}(\vec{0},\vec{p})}{C_i^{KK}(\vec{0},\vec{p})C_4^{\pi K}(\vec{0},\vec{p})} \to \left(\frac{E_K(\vec{p})+m_K}{p_i}\frac{p_i - p_i\boxed{\xi(q^2)}}{E_\pi(p)+m_K+(m_K-E_\pi(p))\boxed{\xi(q^2)}}\right)$$

$$(0 << t << t_f, i = 1-3)$$

construct the form factors from the ratios

$$\boxed{d_1 \to f_0(q^2_{min})}$$

$$\boxed{d_3 \to \xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}}$$

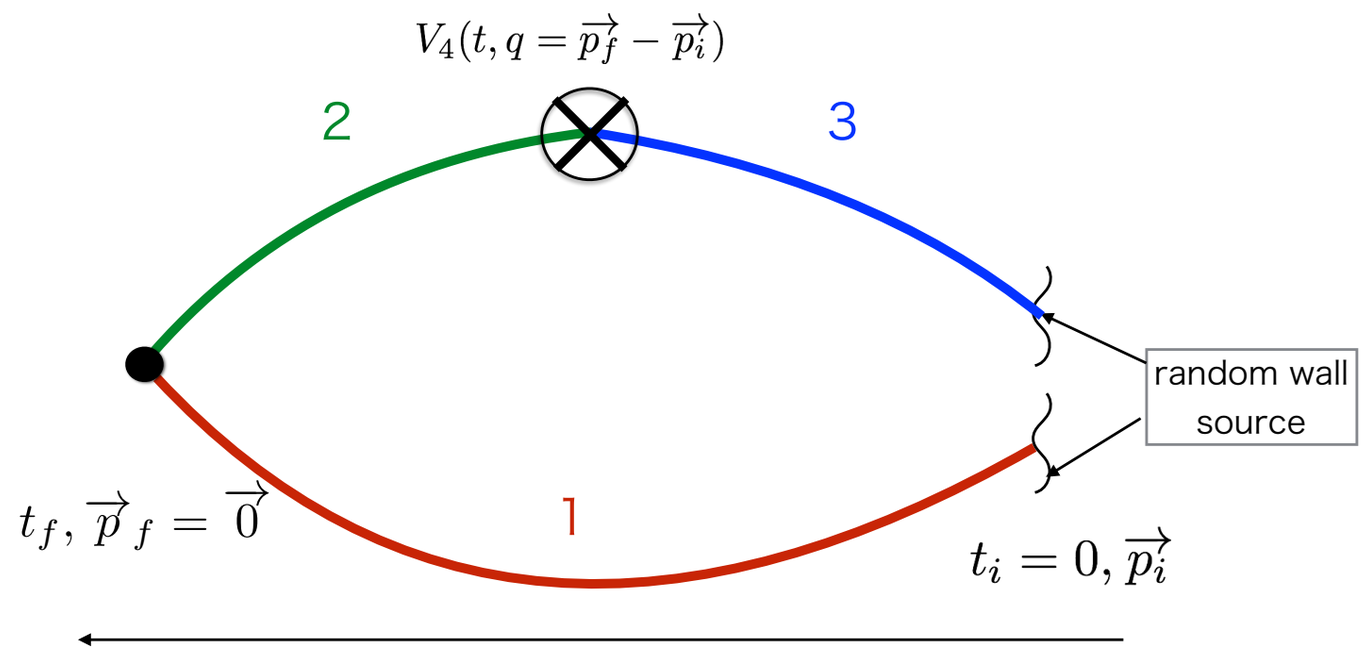
$$\to \boxed{d_2 \to f_+(q^2)} \to f_0(q^2) = f_+(q^2)\left(1 + \frac{-q^2}{(m_K^2 - m_\pi^2)}\xi(q^2)\right)$$

Calculation 3-point function

- connected 3-point function is consist of 3 quark propagators

$$C_\mu^{XY}(\vec{P'},\vec{P}) = Z_V \left\langle 0 \middle| O_Y(t_f,\vec{P'}) V_\mu(t,\vec{q}) O_X^\dagger(t_i,\vec{P}) \middle| 0 \right\rangle$$

- 1 random wall source $\vec{p}_i = \vec{0}$
- 2 sequential source $\vec{p}_f = \vec{0}$
- 3 random wall source $\vec{p}_i \neq \vec{0}$



random wall source(A,B : color&spinor index)

$$\eta_B(\vec{y},t_i) = \left\{\frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}}\right\} \in \mathbb{Z}(2)\otimes\mathbb{Z}(2)$$

$$\lim_{N\to\infty}\frac{1}{N}\sum_{j=0}^{N}\eta_A^j(\vec{x},t_i)\eta_B^{\dagger j}(\vec{y},t_i) = \delta(\vec{x}-\vec{y})\delta_{AB}$$

calculation cost is reduced by random wall source

RBC&UKQCD:JHEP(0807 (2008)112)

disconnected term is vanished by charge symmetry

Simulation details

All the results are preliminary

PACS10 configuration (HPCI System Research project)

arXiv:1807.06237v1 [hep-lat]

$L^3 \times T = 128^3 \times 128 \quad m_\pi L \approx 7.5 \quad (\kappa_{ud}, \kappa_s) = (0.126117, 0.124902)$

$\beta = 1.82, n_{stout} = 6, \rho = 0.1, c_{sw} = 1.11 \quad a^{-1} = 2.333(\mathrm{GeV}) \to a = 0.084(\mathrm{fm})$

$N_f = 2+1$ Iwasaki gauge + stout smeared link Wilson clover action

measurement parameter

20 configurations (every 10 traj.)

8 time slices × 4 directions (x, y, z, t)

×1 random source = 32 meas. per config.

(anti) periodic boundary condition

for temporal direction

bin size:10 traj. $t_f - t_i = 36$

resources : Oakforest-PACS (JCAHPC)

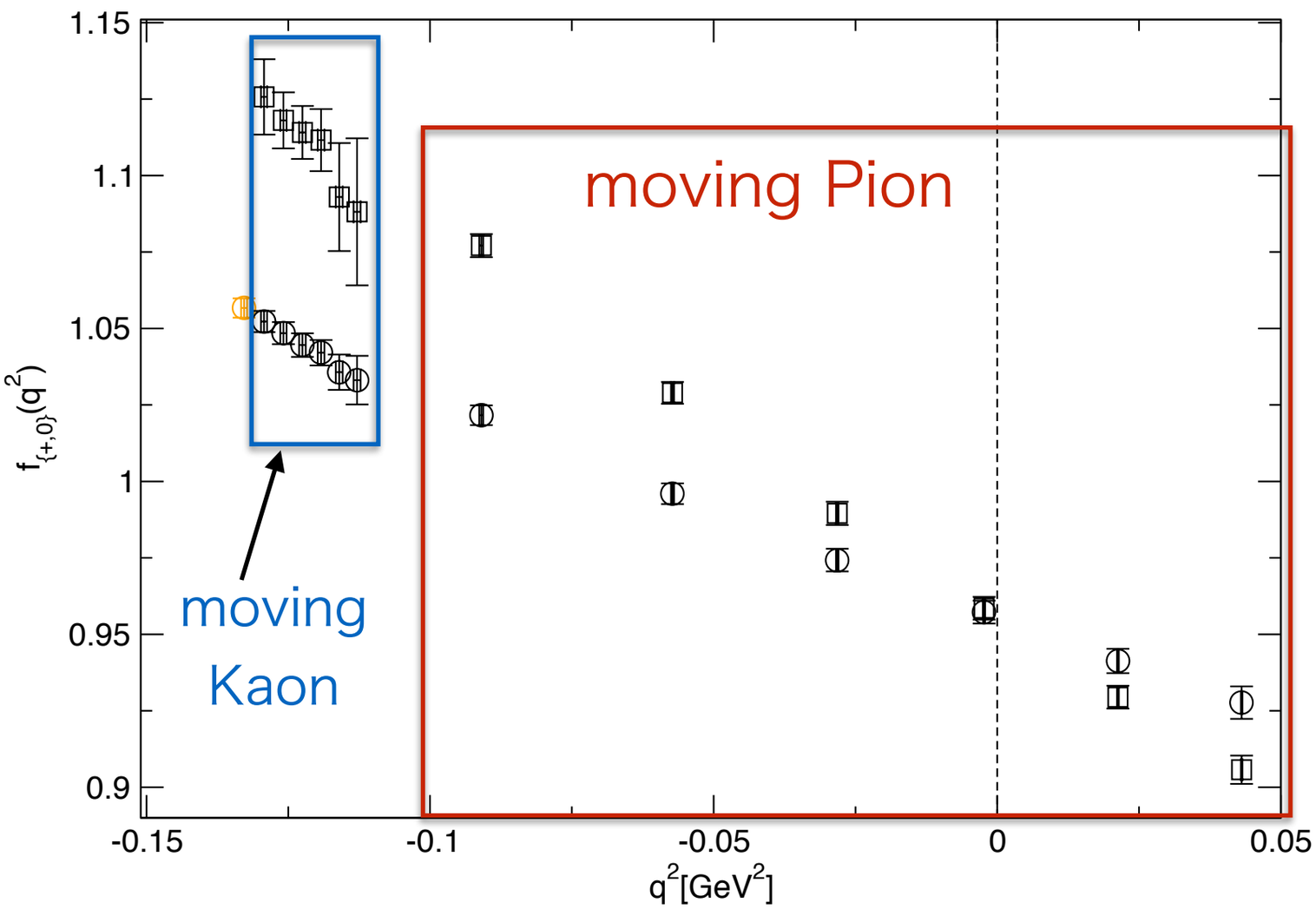
directions of source momenta $\vec{n}$ $\quad \vec{p} = \frac{2\pi}{L}\vec{n}$

$\lvert\vec{n}^2\rvert = n$	# dir. of $\vec{n}$	perm. of $\vec{n} =$
0	1	$(0,0,0)$
1	6	$(1,0,0)$
2	12	$(1,1,0)$
3	8	$(1,1,1)$
4	6	$(2,0,0)$
5	9	$(2,1,0)$
6	9	$(2,1,1)$

sink momentum : 0

Note that our calculation without twisted boundary condition in spatial directions

Preliminary result : momentum transfer dependance of semileptonic decay form factors



constant fit
from all the ratios
in $t = 15 - 21$

there is target momentum (q=0) between mom. of n=4 and n=5

$$n = 0 \rightarrow q^2_{min} = -(m_K - m_\pi)^2 \approx -0.132(\mathrm{GeV}^2)$$

square symbols: $f_+(q^2)$
circle symbols: $f_0(q^2)$

orange circle: $f_0(q^2_{min})$

the form factors of Kaon with mom. are noisier than Pion's with mom. and too far to extrapolate to the target

the data of Kaon with mom. is NOT included in the next analysis

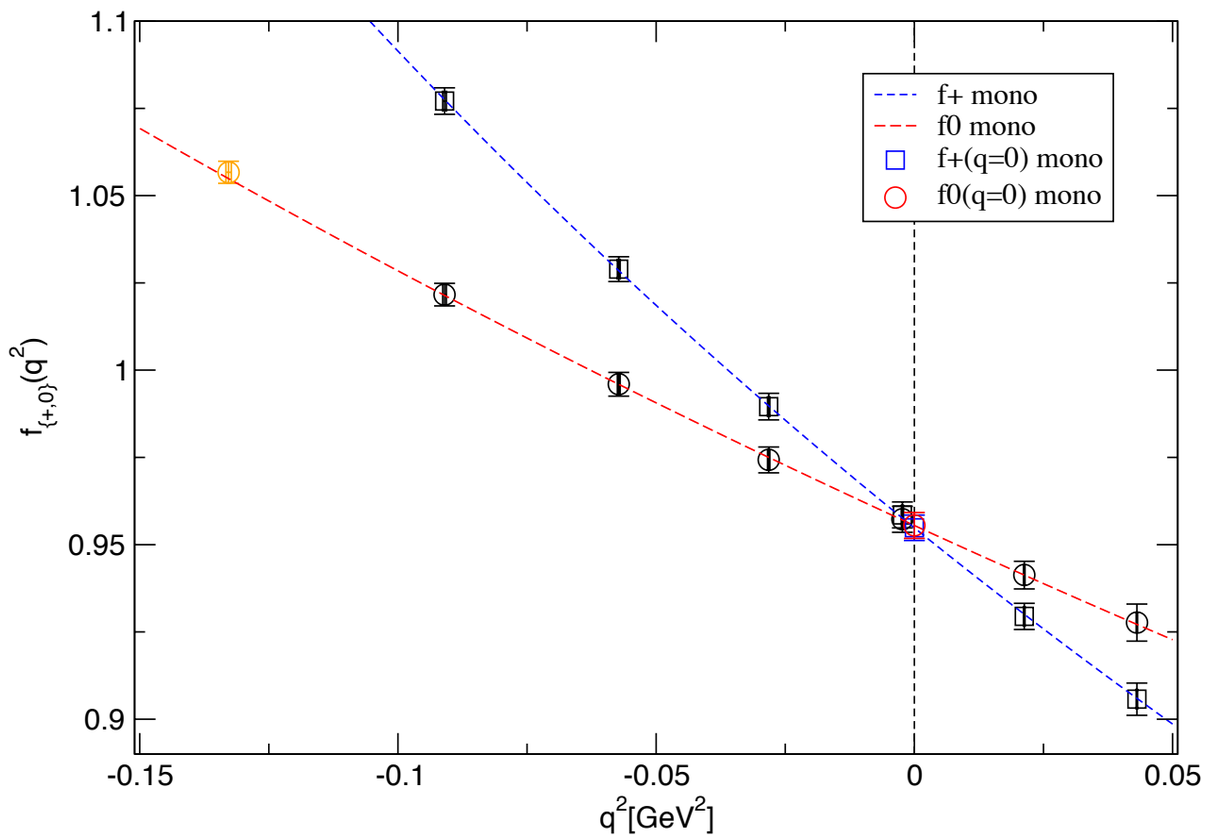
preliminary result : interpolation to zero momentum transfer (monopole fit)

blue dashed line : $$f_+(q^2) = \frac{f_+(0)}{1 + q^2/M^2_{monoV}}$$

red dashed line : $$f_0(q^2) = \frac{f_0(0)}{1 + q^2/M^2_{monoS}}$$

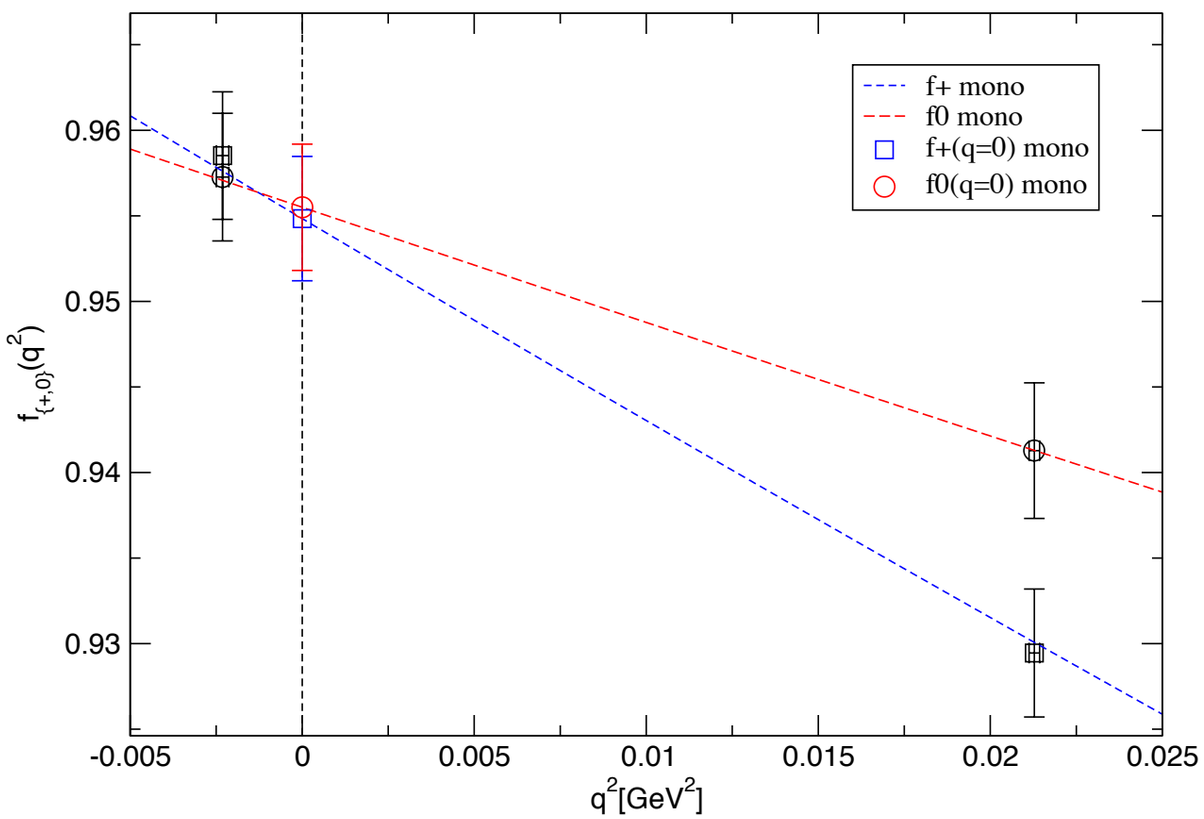
2 free parameters :

$$f_{+,0}(0), M^2_{monoV,S}$$



the fit with the momenta from n=1 to n=6

note that the result of the fit with n=0(qmin) is the almost same



near q=0 region,
between n=4 and n=5 mom.

$$f_+(0) = 0.9548(36)$$

$$f_0(0) = 0.9554(37)$$

preliminary result : interpolation to zero momentum transfer (NLO SU(3) ChPT fit)

$$f_+(q^2) = F_+(s)\left(1 + C_+(s)x + \frac{m_K^2}{(4\pi f_0)^2}\left(-\frac{3}{4}x\log x - xT_1^+(s) - T_2^+(s)\right)\right)$$

$$f_0(q^2) = F_0(s)\left(1 + C_0(s)x + \frac{m_K^2}{(4\pi f_0)^2}\left(-\frac{3}{4}x\log x + xT_1^0(s) - T_2^0(s)\right)\right)$$

ETMC,
arXiv:1012.3573 [hep-lat]

$$s = -\frac{q^2}{m_K^2}, x = \frac{m_\pi^2}{m_K^2}$$

$$T_1^+(s) = [(1-s)\log(1-s) + s(1-s/2)]3(1+s)/4s^2$$

$$T_2^+(s) = [(1-s)\log(1-s) + s(1-s/2)](1-s)^2/4s^2$$

$$T_1^0(s) = [\log(1-s) + s(1+s/2)](9+7s^2)/4s^2$$

$$T_2^0(s) = [(1-s)\log(1-s) + s(1-s/2)](1-s)(3+5s)/4s^2$$

3 free parameters :

$$F_{+,0}(s) = F_{+,0}$$

$$C_{+,0}(s) = C_{+,0}^0 + C_{+,0}^1 s$$

the decay constant : $f_0 = 0.10508(\mathrm{GeV})$

* combine $f = 0.12925(\mathrm{GeV})\ (m_u, m_d \to 0)$ and $\frac{f}{f_0} = 1.229$

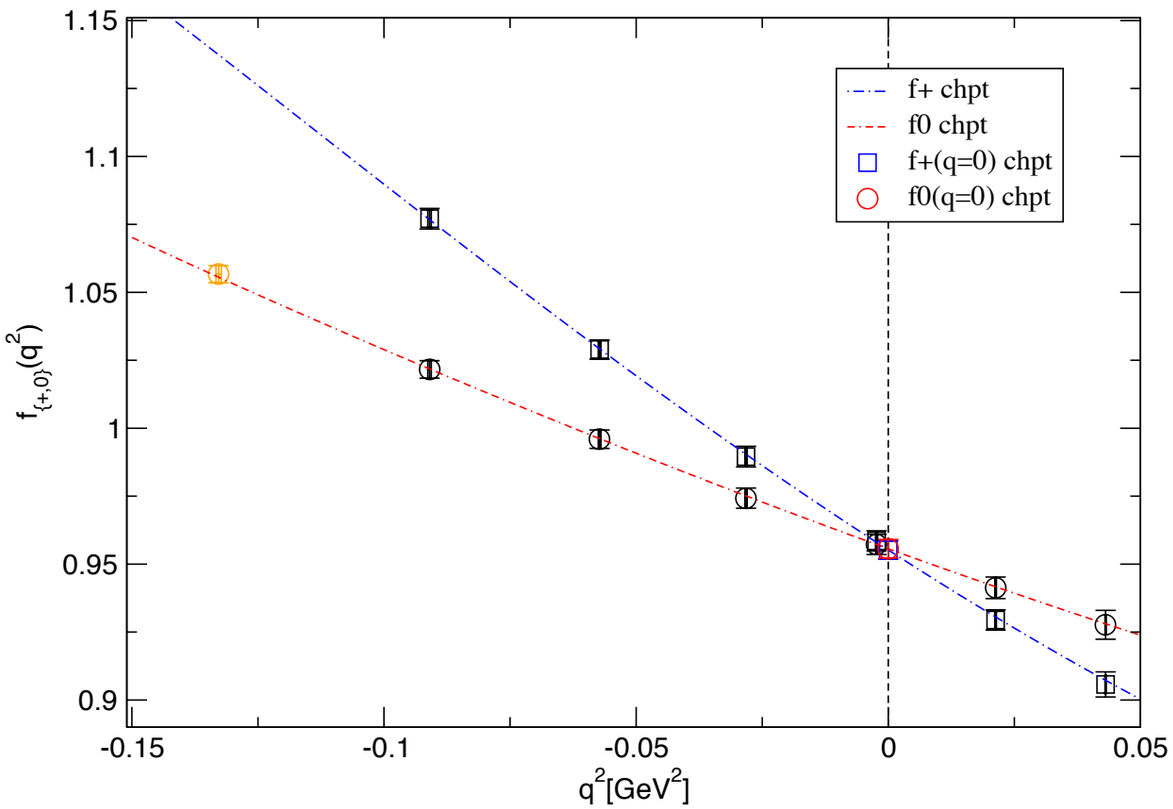
the ratio from arXiv:0804.0473 [hep-lat]

the fit with the momenta from n=1 to n=6

preliminary result : interpolation to zero momentum transfer (NLO ChPT fit)

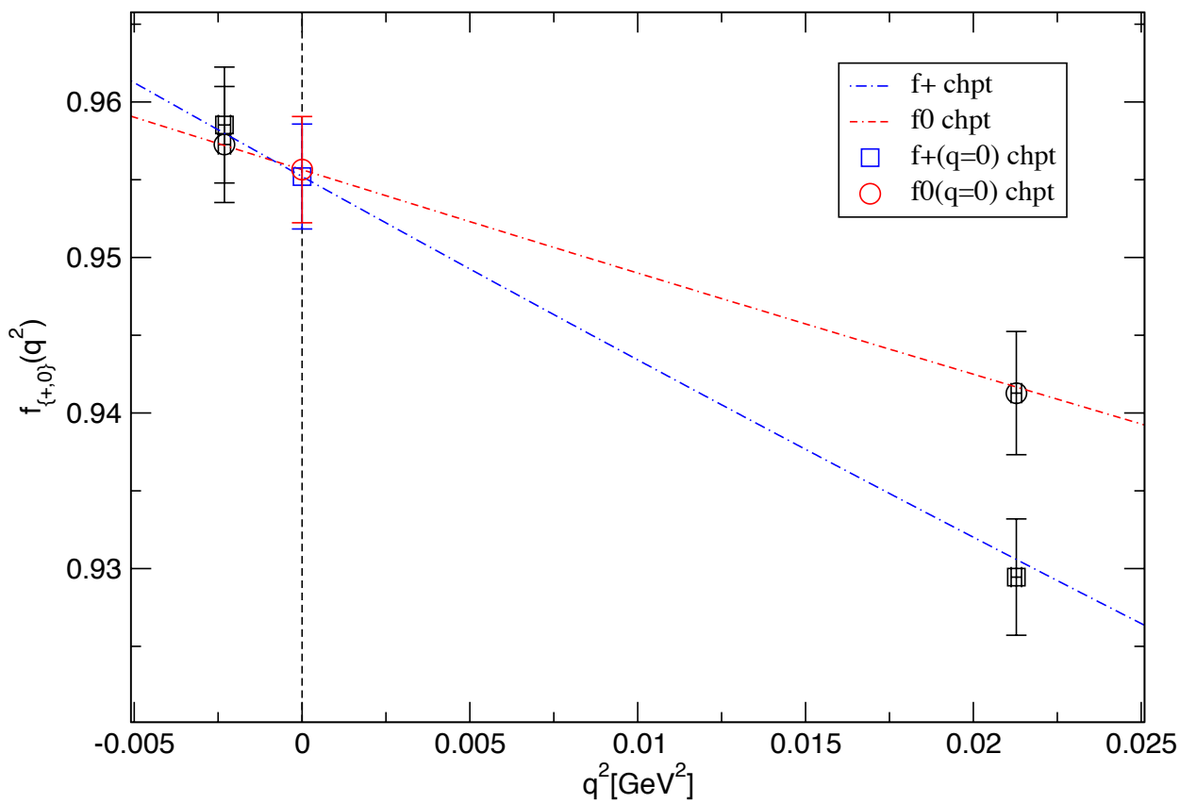
blue dashed line : $f_+(q^2) = F_+\left(1 + (C_+^0 + C_+^1 s)x + \frac{m_K^2}{(4\pi f_0)^2}\left(-\frac{3}{4}x\log x - xT_1^+(s) - T_2^+(s)\right)\right)$

red dashed line : $f_0(q^2) = F_0\left(1 + (C_0^0 + C_0^1 s)x + \frac{m_K^2}{(4\pi f_0)^2}\left(-\frac{3}{4}x\log x + xT_1^0(s) - T_2^0(s)\right)\right)$



the fit with the momenta from n=1 to n=6

note that the result of the fit with n=0(qmin) is also the almost same



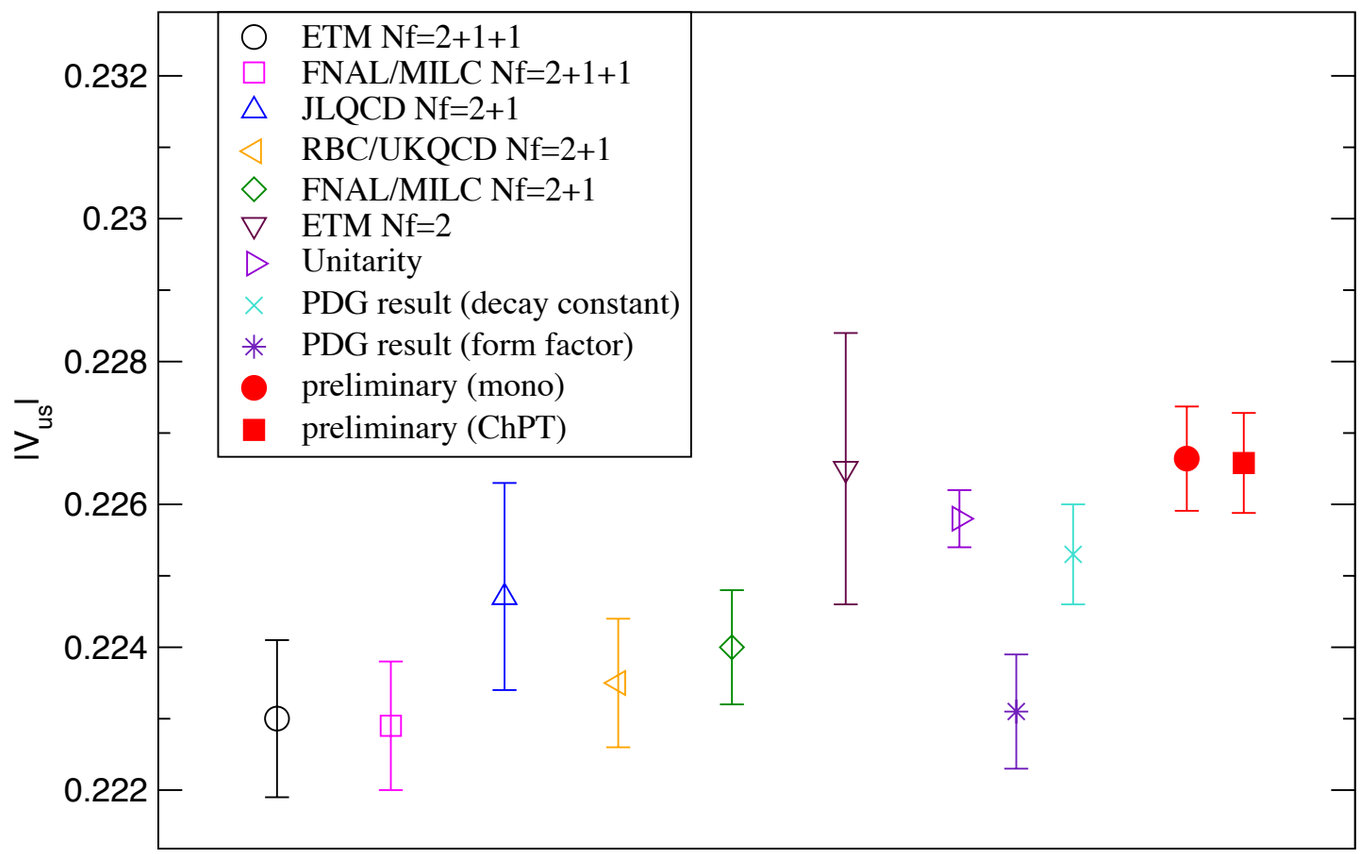
near q=0 region,
between n=4 and n=5 mom.

$f_+(0) = 0.9553(33)$ $\quad$ $f_0(0) = 0.9556(34)$

preliminary result : CKM matrix element

In the experimental result $|V_{us}|f_+(q^2=0) = 0.2165(4)$

our results of CKM (average the results from f+ and f0)



comparison with our preliminary result(red filled symbols) and other calculation

$$|V_{us}|^{mono} = 0.22664(61)(8)(41)$$

$$|V_{us}|^{chpt} = 0.22658(56)(8)(41)$$

1st error : statistical

2nd error : difference of central value between f+ and f0

3rd error : from experiment

our preliminary results are consist with assuming unitarity

preliminary result : slope of form factor

slope of form factor $\lambda'_{(+,0)} = \frac{m^2_{\pi_\pm \mathrm{phys}}}{f_{(+,0)}(0)} \frac{f_{(+,0)}(t)}{dt}\Big|_{t=0}$

can be used for the consistency check with experiment

experiment M. Moulson, PoS CKM2016, 033 [arXiv:1704.04104 [hep-ex]].
JLQCD collaboration, arXiv:1705.00884v1 [hep-lat]

$$\lambda'_{(+)} = 2.58(7) \times 10^{-2} \quad \lambda'_{(0)} = 1.36(7) \times 10^{-2}$$

our result

monopole fit

$$\lambda'_{(+)} = 2.55(6) \times 10^{-2} \quad \lambda'_{(0)} = 1.44(5) \times 10^{-2}$$

ChPT fit

$$\lambda'_{(+)} = 2.44(6) \times 10^{-2} \quad \lambda'_{(0)} = 1.36(7) \times 10^{-2}$$

our results are consistent with experiment

Summary

We calculated semileptonic form factor

in N_f=2+1 Lattice QCD

- calculation on (10 fm)^3
- physical light quark and strange quark mass
- monopole and ChPT to interpolate to zero momentum transfer

our preliminary result of semileptonic form factors

$f_+(0) = 0.9548(36), f_0(0) = 0.9554(37)$: monopole fit

$f_+(0) = 0.9553(33), f_0(0) = 0.9556(34)$: ChPT fit

our preliminary results are consistent with assuming unitarity

and higher than PDG

$|V_{us}|^{mono} = 0.22664(61)(8)(41)$ $|V_{us}|^{chpt} = 0.22658(56)(8)(41)$

Next step

- excited state contribution
- estimate error from finite lattice spacing
- model independent analysis (z expansion)

back up

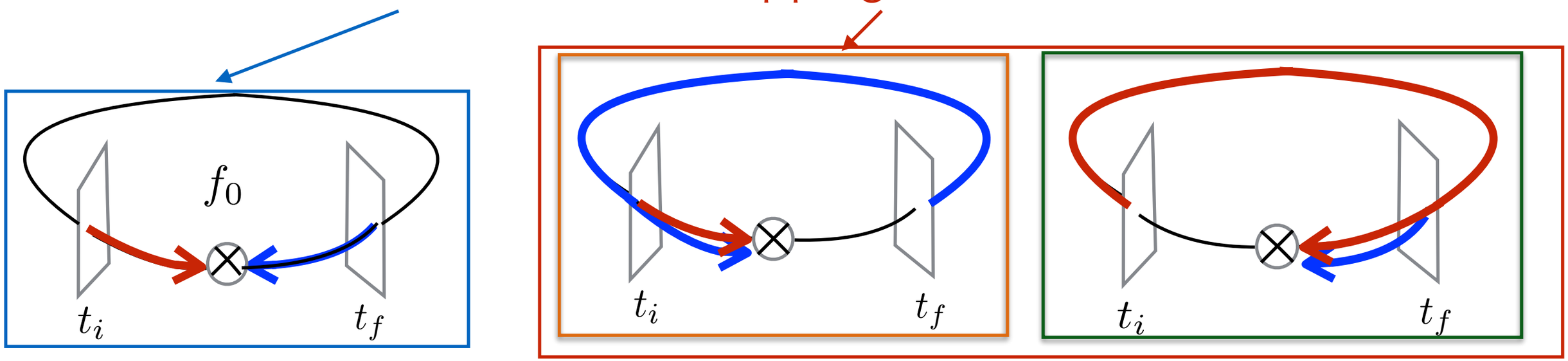
Extraction of (Pion) form factor

light pion mass＋ periodic boundary condition in temporal direction

→<u>current time dependence appears in R</u>

we should consider wrapping around effect

form factor＋wrapping around effect



$$f_0 + f_1 \times \frac{e^{-E_{2\pi}\times t} e^{-E_{\pi_f}\times(T-t_f)} - e^{-E_{2\pi}\times(t_f-t)} e^{-E_{\pi_i}\times(T-t_f)}}{e^{-E_{\pi_i}\times t} e^{-E_{\pi_f}\times(t_f-t)}}$$

f_1 is related to $\langle 0|V_\mu|\pi\pi\rangle$

$E_{2\pi} = E_i + E_f$:sum of single pion energy

Calculation 3-point function

- connected 3-point function is consist of 3 quark propagators

$$C_{\pi V \pi} = Z_V \left\langle 0 \middle| O_\pi(t_f, \vec{p}_f) V_4(t, \vec{q}) O_\pi^\dagger(t_i = 0, \vec{p}_i) \middle| 0 \right\rangle$$

- 1 random wall source $\vec{p}_i = \vec{0}$
- 2 sequential source $\vec{p}_f = \vec{0}$
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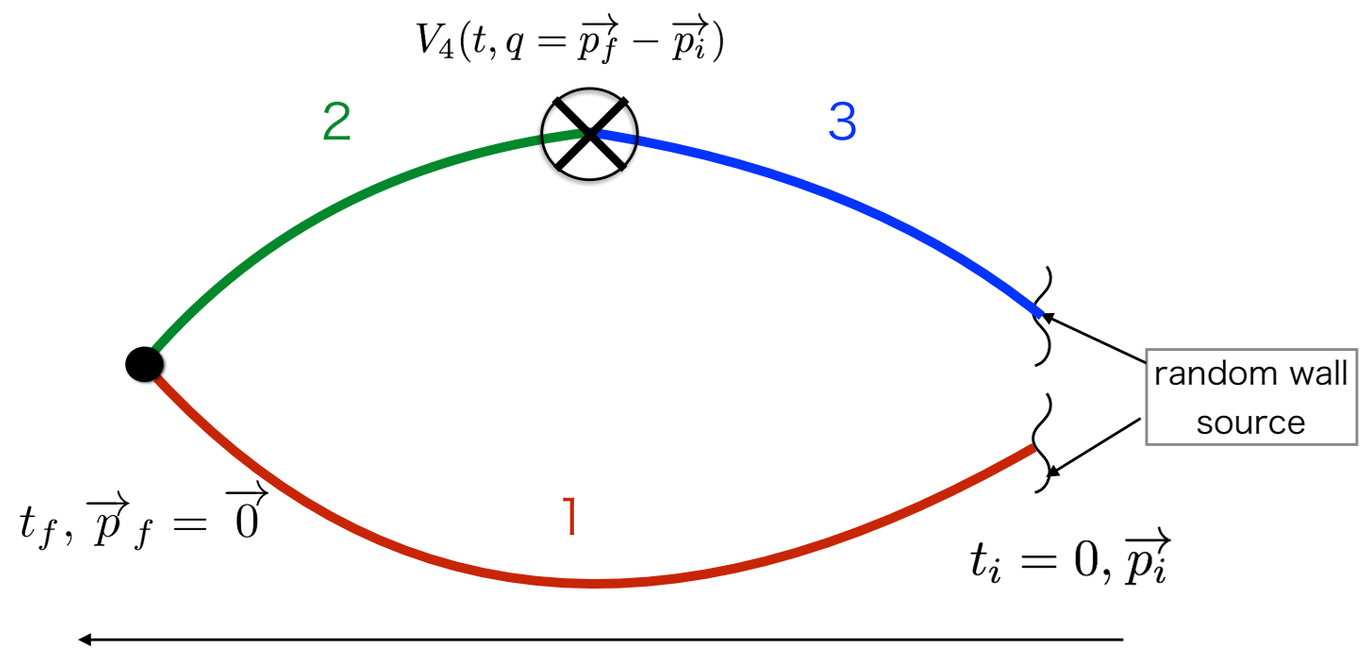

the data of all quark propagator (1,2,3) calculation with temporal periodic b.c.

+

#1 (red) quark propagator calculation with temporal <u>anti</u> periodic b.c.

suppress wrapping around effect in temporal direction