TOWARDS THE P-WAVE $N\pi$ SCATTERING AMPLITUDE IN THE Δ (1232)

Interpolating fields and spectra

July 27, 2018 | Giorgio Silvi | Forschungszentrum Jülich

LATTICE 18 East Lansing, MI, USA

JÜLICH Forschungszentrum

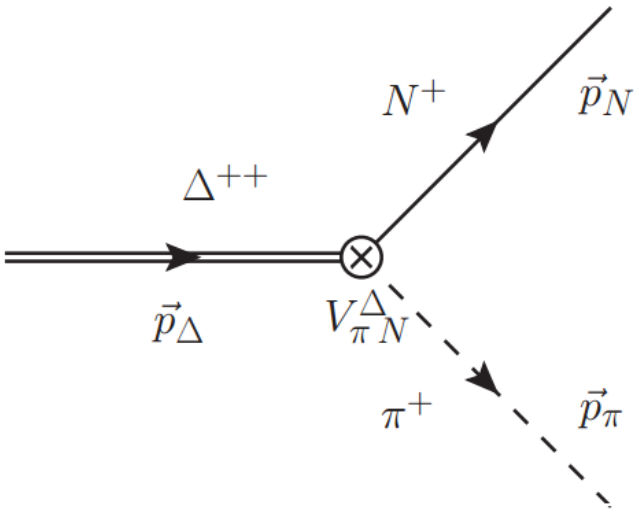
COLLABORATORS

- **Constantia Alexandrou** (University of Cyprus / The Cyprus Institute)
- **Giannis Koutsou** (The Cyprus Institute)
- **Stefan Krieg** (Forschungszentrum Jülich / University of Wuppertal)
- **Luka Leskovec** (University of Arizona)
- **Stefan Meinel** (University of Arizona / RIKEN BNL Research Center)
- **John Negele** (MIT)
- **Srijit Paul** (The Cyprus Institute / University of Wuppertal)
- **Marcus Petschlies** (University of Bonn / Bethe Center for Theoretical Physics)
- **Andrew Pochinsky** (MIT)
- **Gumaro Rendon** (University of Arizona)
- **G. S.** (Forschungszentrum Jülich / University of Wuppertal)
- **Sergey Syritsyn** (Stony Brook University / RIKEN BNL Research Center)

THE DELTA(1232)

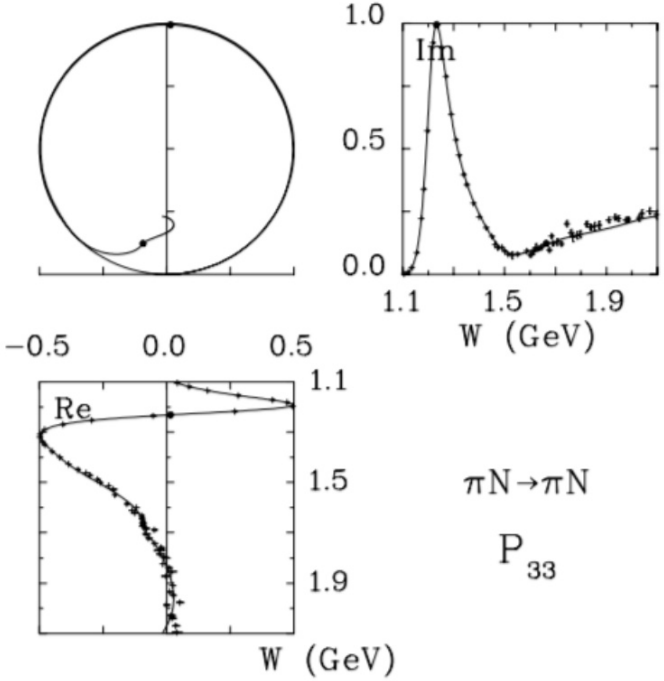
the first baryon resonance

- In nature: $\Delta^- \Delta^0 \Delta^+ \Delta^{++}$ (u,d quarks) - mass $\sim$ 1232 MeV
  - On the lattice: isospin symmetry
- The unstable $\Delta(1232)$ decay predominantly to stable $N\pi$
- Study: Pion-Nucleon scattering $J = 3/2$, $I = 3/2$, $I_3 = +3/2$
  - Orbital angular momentum: $L = 1$



EXPERIMENTAL INFO

$N\pi(\to \Delta(1232)) \to N\pi$
completely elastic...



[Shrestha,Manley (2012)]

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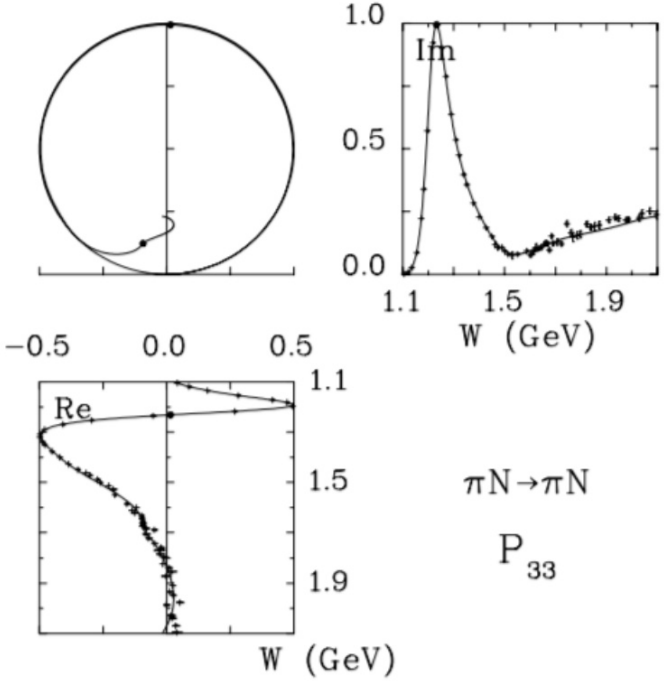

.. but there are resonances nearby.

Particle	J^P	$\Gamma_{N\pi}[MeV]$
$\Delta(1232)$	$3/2^+$	112.4(5)
$\Delta(1600)$	$3/2^+$	18(4)
$\Delta(1620)$	$1/2^-$	37(2)
$\Delta(1700)$	$3/2^-$	36(2)
...	...	

[Shrestha,Manley (2012)]

LÜSCHER METHOD

Lüscher quantization condition for baryons

$$\det[M_{Jlm,J'l'm'} - \delta_{JJ'}\delta_{ll'}\delta_{mm'}\cot\delta_{Jl}] = 0$$ [Gockeler et al. (2012)]

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Simplify!

With a proper transformation, the matrix $M_{Jlm,J'l'm'}$ can be block diagonalized in the basis of the irreps Λ of the lattice.

MOVING FRAMES

Problem

Due to quantized momenta $p = 2\pi n/L$ we have a energy levels spaced from each other. Chances of hitting the energy region of interest are low.

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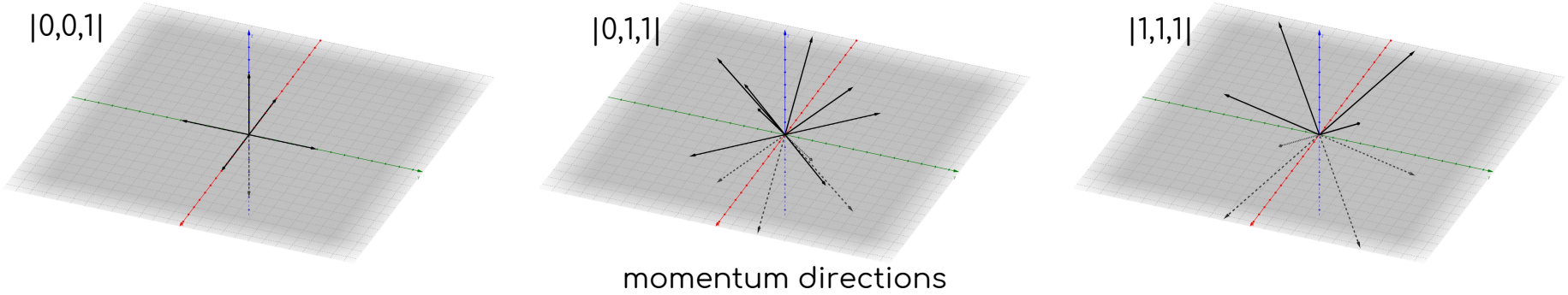
Solution: Moving frames!

The Lorentz boost contracts the box giving a different effective value of the size L. Allow access to phase shift at different energies!



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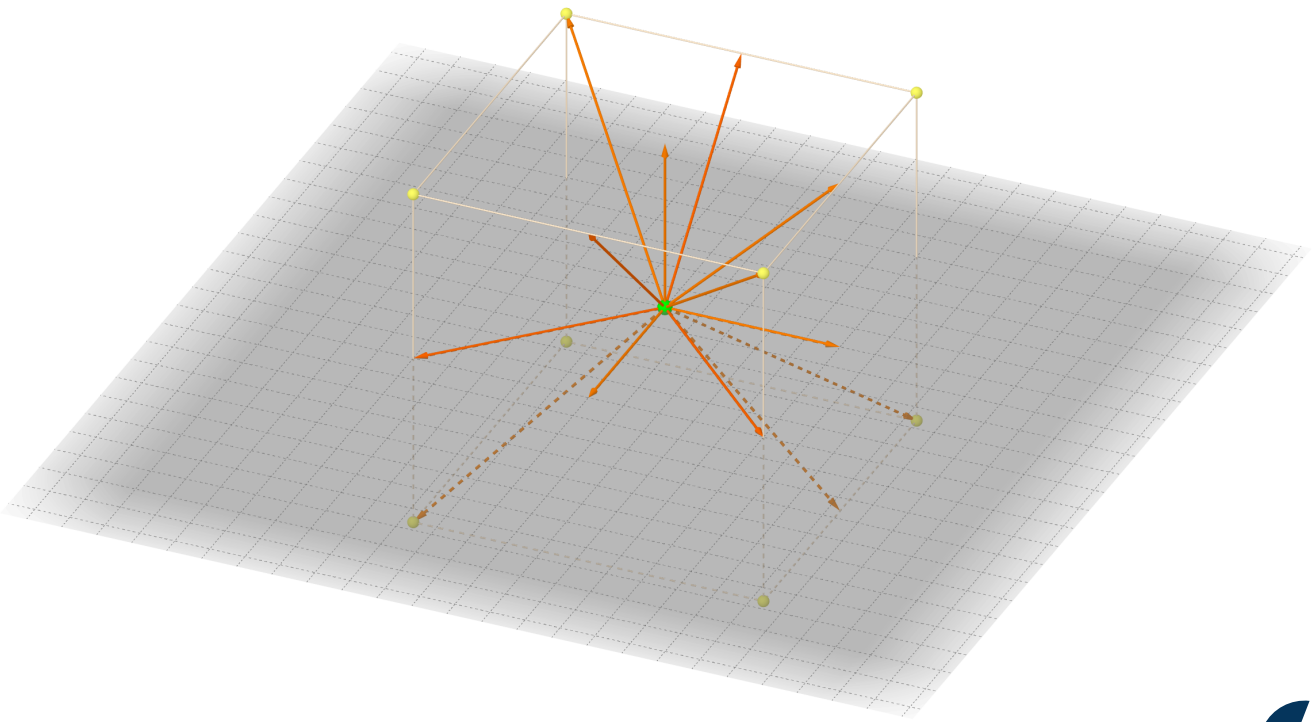

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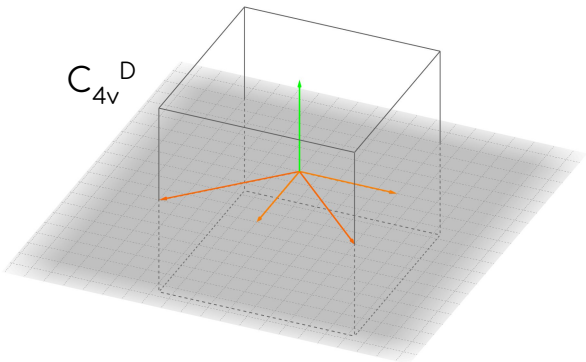
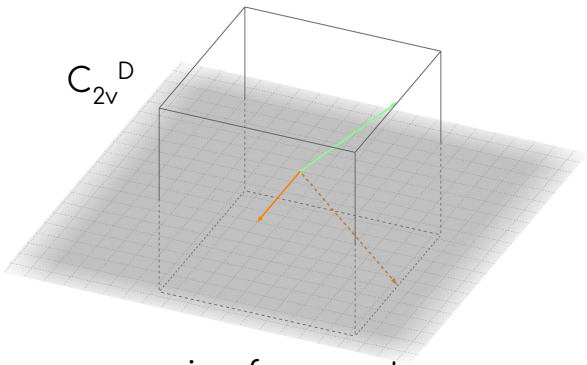
  - For half-integer J we need the double cover O_h^D(96 elements) which include the negative identity (2π rotation)
- Each of the infinite irreps J^P in the continuum get mapped to one of the finite irreps Λ of the group O_h^D on the lattice.

GROUND PLAN

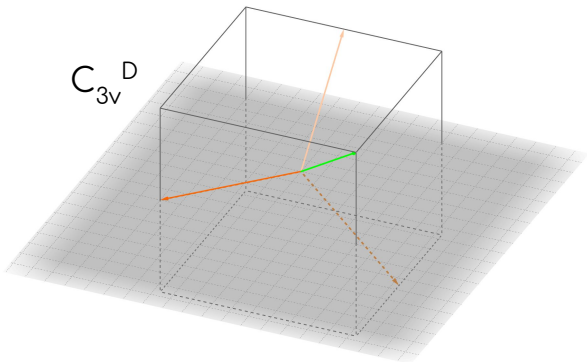
Frames, Groups & Irreps Λ (with ang. mom. content)

$P_{ref}[N_{dir}]$	Group	N_{elem}	$\Lambda(J) : \pi(\ 0^{-}\ )$	$\Lambda(J) : N(\ \frac{1}{2}^{+}\ )$	$\Lambda(J) : \Delta(\ \frac{3}{2}^{+}\ )$
$(0,0,0)$ [1]	O_h^D	96	$A_{1u}(\ 0\ ,4,\ldots)$	$G_{1g}(\ \frac{1}{2}\ ,\frac{7}{2},\ldots) \oplus G_{1u}(\ \frac{1}{2}\ ,\frac{7}{2},\ldots)$	$H_g(\ \frac{3}{2}\ ,\frac{5}{2},\ldots) \oplus H_u(\ \frac{3}{2}\ ,\frac{5}{2},\ldots)$
$(0,0,1)$ [6]	C_{4v}^D	16	$A_2(\ 0\ ,1,\ldots)$	$G_1(\ \frac{1}{2}\ ,\frac{3}{2},\ldots)$	$G_1(\frac{1}{2},\ \frac{3}{2}\ ,\ldots) \oplus G_2(\ \frac{3}{2}\ ,\frac{5}{2},\ldots)$
$(0,1,1)$ [12]	C_{2v}^D	8	$A_2(\ 0\ ,1,\ldots)$	$G(\ \frac{1}{2}\ ,\frac{3}{2},\ldots)$	$G(\frac{1}{2},\ \frac{3}{2}\ ,\ldots)$
$(1,1,1)$ [8]	C_{3v}^D	12	$A_2(\ 0\ ,1,\ldots)$	$G(\ \frac{1}{2}\ ,\frac{3}{2},\ldots)$	$G(\frac{1}{2},\ \frac{3}{2}\ ,\ldots) \oplus F_1(\ \frac{3}{2}\ ,\frac{5}{2},\ldots) \oplus F_2(\ \frac{3}{2}\ ,\frac{5}{2},\ldots)$





axis of symmetry



SINGLE HADRON OPERATORS

Delta interpolators:

$$\Delta^{(1)}_{i\mu} = \epsilon_{abc} u^a_\mu (u^{bT} C \gamma_i u^c) \tag{1}$$

$$\Delta^{(2)}_{i\mu} = \epsilon_{abc} u^a_\mu (u^{bT} C \gamma_i \gamma_0 u^c) \tag{2}$$

Nucleon interpolators:

$$\mathcal{N}^{(1)}_\mu = \epsilon_{abc} u^a_\mu (u^{bT} C \gamma_5 d^c) \tag{3}$$

$$\mathcal{N}^{(2)}_\mu = \epsilon_{abc} u^a_\mu (u^{bT} C \gamma_0 \gamma_5 d^c) \tag{4}$$

Pion interpolator:

$$\pi = \bar{d} \gamma_5 u \tag{5}$$

PROJECTION METHOD

how it works...

$$O^{G^D,\Lambda,r,m}(p) = \frac{d_\Lambda}{g_{G^D}} \sum_{\tilde{R}\in G^D} \Gamma^{\Lambda}_{r,r}(\tilde{R}) U_{\tilde{R}} \phi(p) U_{\tilde{R}}^{-1}$$ [C. Morningstar et al. (2013)]

- to get an operators $O^{G^D,\Lambda,r,m}(p)$ for a specific:

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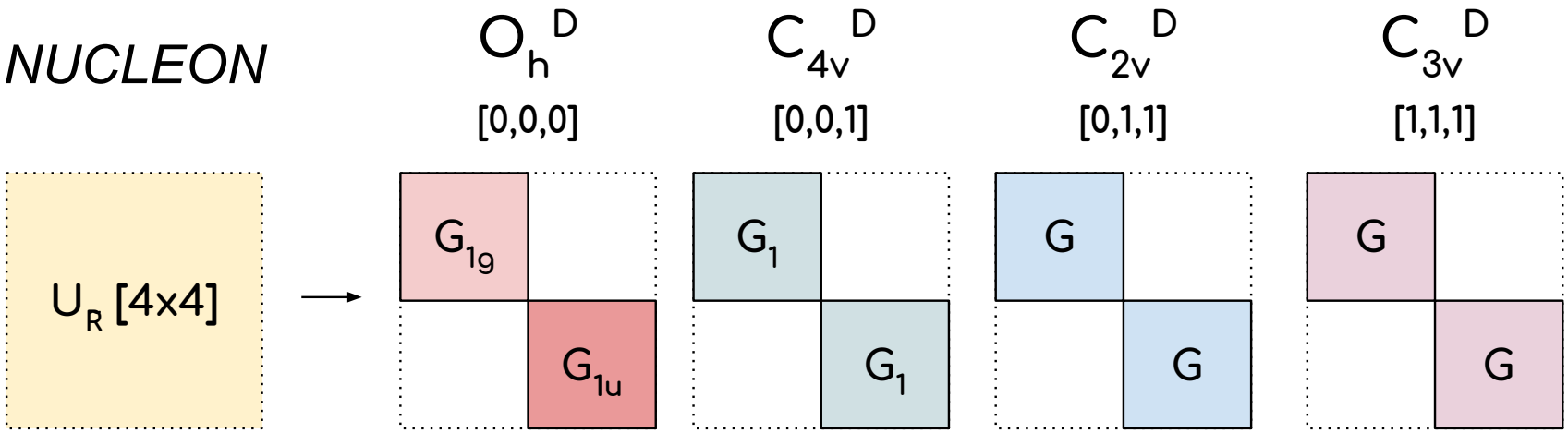
  3. single/multi hadron operator $\phi(p)$
  4. proper transformation matrices $U_{\tilde{R}}$

OCCURENCES OF IRREPS

It is possible to find the occurence ($\sim$multiplicity) m of the irrep Γ^Λ in the transformation matrices $U_{\tilde{R}}$ using the character χ: [Moore,Fleming (2006)]

$$m = \frac{1}{g_{G^D}} \sum_{\tilde{R} \in G^D} \chi^{\Gamma^\Lambda}(\tilde{R}) \chi^U(\tilde{R})$$



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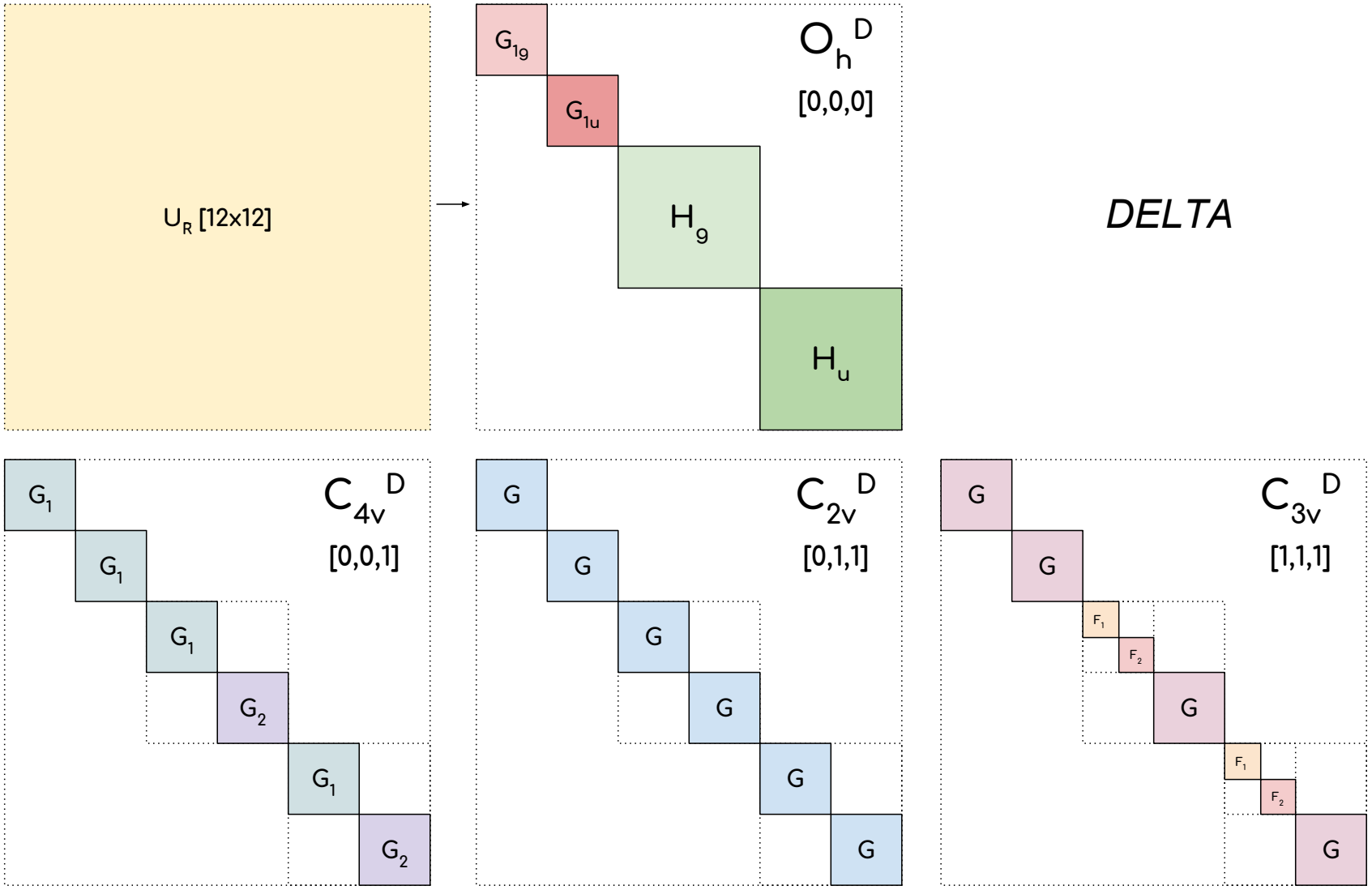


EXAMPLE OF PROJECTION: NUCLEON

C4v - G1 - first instance (row 1 , row 2)

$$\begin{pmatrix} \{\mathrm{Nc}(0,0,1)(3)\} & \{\mathrm{Nc}(0,0,1)(4)\} \\ \{\mathrm{Nc}(0,0,-1)(3)\} & \{\mathrm{Nc}(0,0,-1)(4)\} \\ \{\mathrm{Nc}(1,0,0)(4)-\mathrm{Nc}(1,0,0)(2)\} & \{\mathrm{Nc}(1,0,0)(2)+\mathrm{Nc}(1,0,0)(4)\} \\ \{\mathrm{Nc}(-1,0,0)(4)-\mathrm{Nc}(-1,0,0)(2)\} & \{\mathrm{Nc}(-1,0,0)(2)+\mathrm{Nc}(-1,0,0)(4)\} \\ \{\mathrm{Nc}(0,-1,0)(4)-\mathrm{Nc}(0,-1,0)(2)\} & \{\mathrm{Nc}(0,-1,0)(2)+\mathrm{Nc}(0,-1,0)(4)\} \\ \{\mathrm{Nc}(0,1,0)(4)-\mathrm{Nc}(0,1,0)(2)\} & \{\mathrm{Nc}(0,1,0)(2)+\mathrm{Nc}(0,1,0)(4)\} \end{pmatrix}$$

C4v - G1 - second instance (row 1 , row 2)

$$\begin{pmatrix} \{\mathrm{Nc}(0,0,1)(1)\} & \{\mathrm{Nc}(0,0,1)(2)\} \\ \{\mathrm{Nc}(0,0,-1)(1)\} & \{\mathrm{Nc}(0,0,-1)(2)\} \\ \{\mathrm{Nc}(1,0,0)(1)+\mathrm{Nc}(1,0,0)(3)\} & \{\mathrm{Nc}(1,0,0)(3)-\mathrm{Nc}(1,0,0)(1)\} \\ \{\mathrm{Nc}(-1,0,0)(1)+\mathrm{Nc}(-1,0,0)(3)\} & \{\mathrm{Nc}(-1,0,0)(3)-\mathrm{Nc}(-1,0,0)(1)\} \\ \{\mathrm{Nc}(0,-1,0)(1)+\mathrm{Nc}(0,-1,0)(3)\} & \{\mathrm{Nc}(0,-1,0)(3)-\mathrm{Nc}(0,-1,0)(1)\} \\ \{\mathrm{Nc}(0,1,0)(1)+\mathrm{Nc}(0,1,0)(3)\} & \{\mathrm{Nc}(0,1,0)(3)-\mathrm{Nc}(0,1,0)(1)\} \end{pmatrix}$$

EXAMPLE OF PROJECTION: DELTA

C4v - G1 - row 1 - instance 1

$$\Delta(0,0,1)(3,1)$$
$$\frac{\Delta(1,0,0)(1,1)}{\sqrt{2}} - \frac{\Delta(1,0,0)(1,3)}{\sqrt{2}}$$
$$\frac{1}{2}\Delta(0,-1,0)(1,1) - \frac{1}{2}\Delta(0,-1,0)(1,3) - \frac{1}{2}\Delta(0,-1,0)(3,2) + \frac{1}{2}\Delta(0,-1,0)(3,4)$$

C4v - G1 - row 1 - instance 2

$$\frac{\Delta(0,0,1)(1,2)}{\sqrt{2}} + \frac{i\Delta(0,0,1)(2,2)}{\sqrt{2}}$$
$$\frac{1}{2}\Delta(1,0,0)(2,1) - \frac{1}{2}\Delta(1,0,0)(2,3) - \frac{1}{2}i\Delta(1,0,0)(3,2) + \frac{1}{2}i\Delta(1,0,0)(3,4)$$
$$\frac{\Delta(0,-1,0)(2,1)}{\sqrt{2}} - \frac{\Delta(0,-1,0)(2,3)}{\sqrt{2}}$$

C4v - G1 - row 1 - instance 3

$$\Delta(0,0,1)(3,3)$$
$$\frac{1}{2}i\Delta(1,0,0)(2,2) + \frac{1}{2}i\Delta(1,0,0)(2,4) + \frac{1}{2}\Delta(1,0,0)(3,1) + \frac{1}{2}\Delta(1,0,0)(3,3)$$
$$\frac{1}{2}\Delta(0,-1,0)(1,2) + \frac{1}{2}\Delta(0,-1,0)(1,4) + \frac{1}{2}\Delta(0,-1,0)(3,1) + \frac{1}{2}\Delta(0,-1,0)(3,3)$$

C4v - G1 - row 1 - instance 4

$$\frac{\Delta(0,0,1)(1,4)}{\sqrt{2}} + \frac{i\Delta(0,0,1)(2,4)}{\sqrt{2}}$$
$$\frac{\Delta(1,0,0)(1,2)}{\sqrt{2}} + \frac{\Delta(1,0,0)(1,4)}{\sqrt{2}}$$
$$\frac{\Delta(0,-1,0)(2,2)}{\sqrt{2}} + \frac{\Delta(0,-1,0)(2,4)}{\sqrt{2}}$$

COMPLETE SET OF OPERATOR: Δ, $N\pi$

Group	P_{ref}[dir. used]	Irrep[rows]	Op. type	n		Op. per irrep
O_h^D	$(0,0,0)[1]$	$H_g[4]$	$\Delta(\gamma_i)$	1		
			$\Delta(\gamma_i\gamma_0)$	1		
			$N\pi(\gamma_5)$	8		
			$N\pi(\gamma_0\gamma_5)$	8		
				18	$\times 4 rows$	72
		$H_u[4]$		18	$\times 4 rows$	72
C_{4v}^D	$(0,0,1)[3]$	$G_1[2]$	Δ	4×2		
			$N\pi$	20×2		
				48	$\times 2 rows \times 3 dir$	288
		$G_2[2]$	Δ	2×2		
			$N\pi$	16×2		
				36	$\times 2 rows \times 3 dir$	216
C_{2v}^D	$(0,1,1)[6]$	$G[2]$	Δ	6×2		
			$N\pi$	24×2		
				60	$\times 2 rows \times 6 dir$	720
C_{3v}^D	$(1,1,1)[4]$	$G[2]$	Δ	4×2		
			$N\pi$	12×2		
				32	$\times 2 rows \times 4 dir$	256
		$F_1[1]$	Δ	2×2		
			$N\pi$	4×2		
				12	$\times 4 dir$	48
		$F_2[1]$		12	$\times 4 dir$	48
Total						1720

PLANNING

Ensemble from BMW collaboration

N_s	N_t	β	$am_{u,d}$	am_s	c_{sw}	$a(fm)$	$L(fm)$	$m_\pi(MeV)$	$m_\pi L$
24	48	3.31	-0.0953	-0.040	1.0	0.116	2.8	254	3.6

[ensemble description in S. Durr et al.(2011)

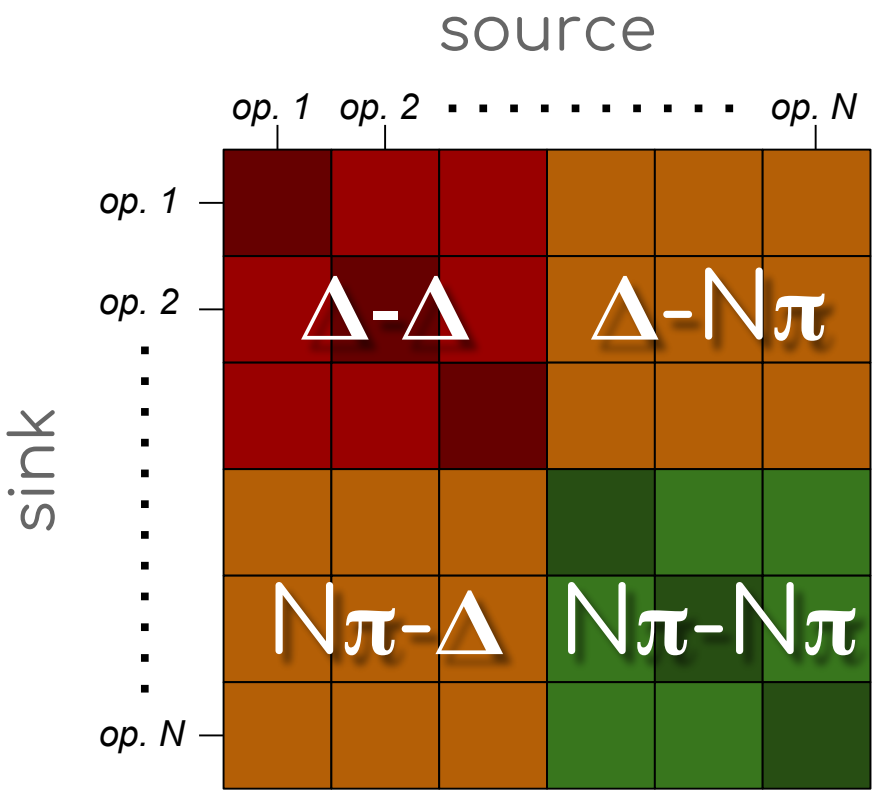
- Lattice action: Wilson-Clover with $N_f = 2 + 1$ dynamical fermions
- Two-point correlators built from a combination of smeared forward, sequential and stochastic propagators
- This talk: 192 configurations,16 source location per conf.

Beginning

- Project the operators on all relevant irreps (Wolfram Mathematica)
- Compute contraction for all diagrams and momentum direction
- Apply the projected operators on correlators

CORRELATION MATRICES

After projectioning the correlators...



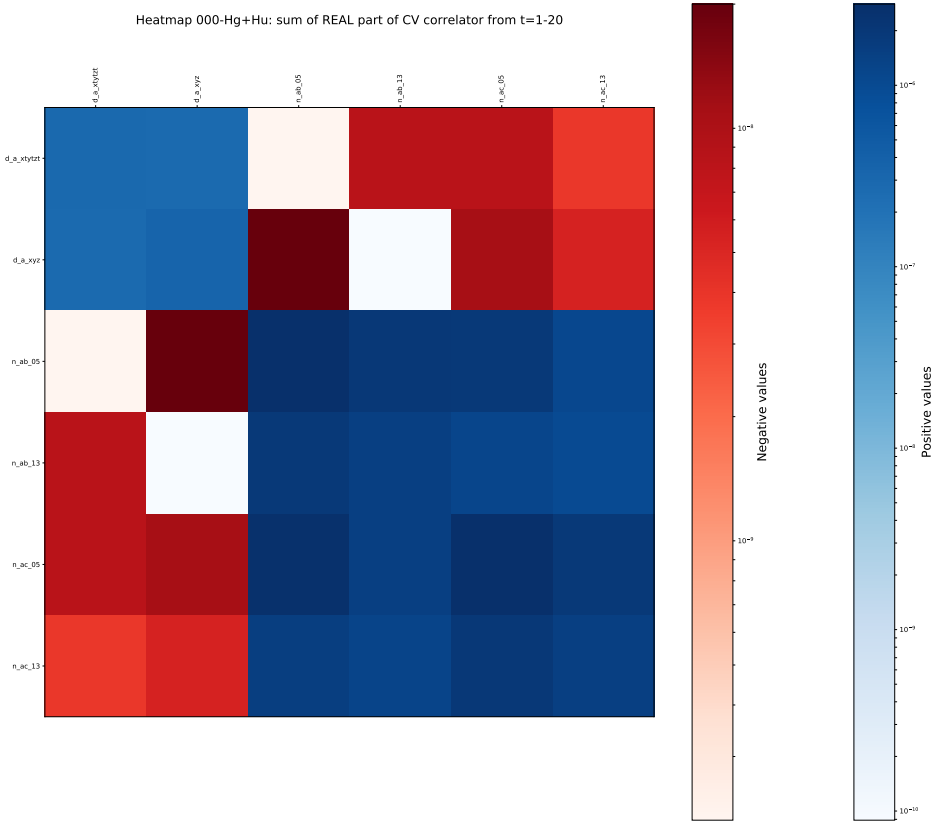
✖ N irreps

(H,G1,G2,G,F1,F2,G)

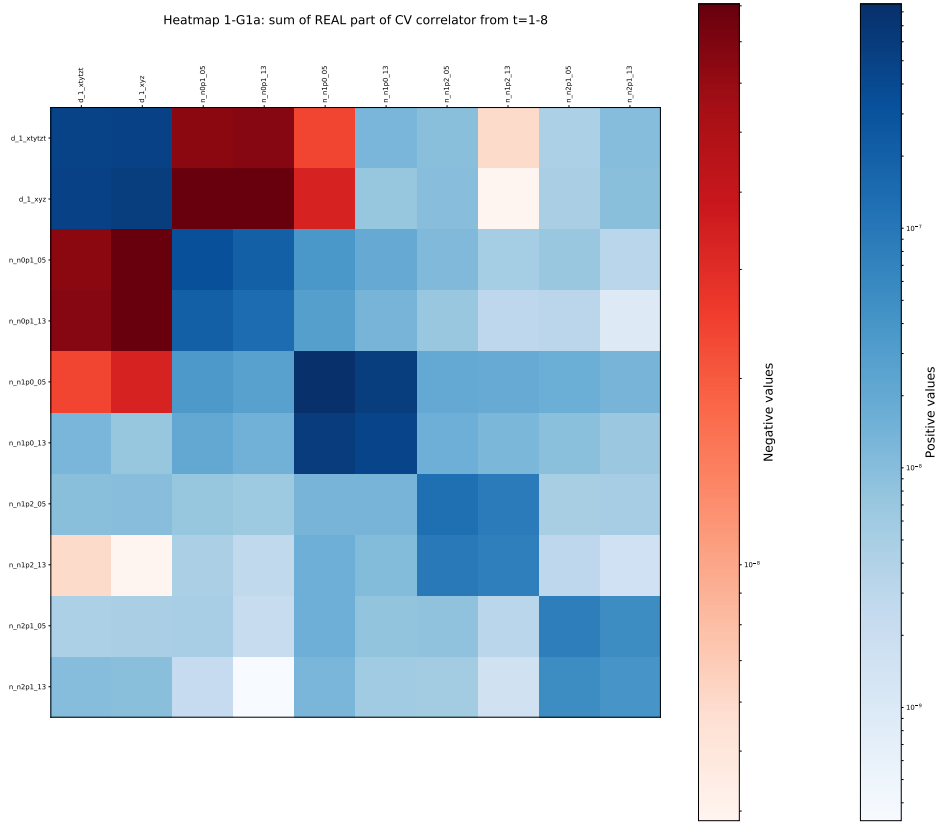
- Use GEVP to determine the spectra
- Search the best basis of operators

HEATMAP

Rest frame / Moving frame



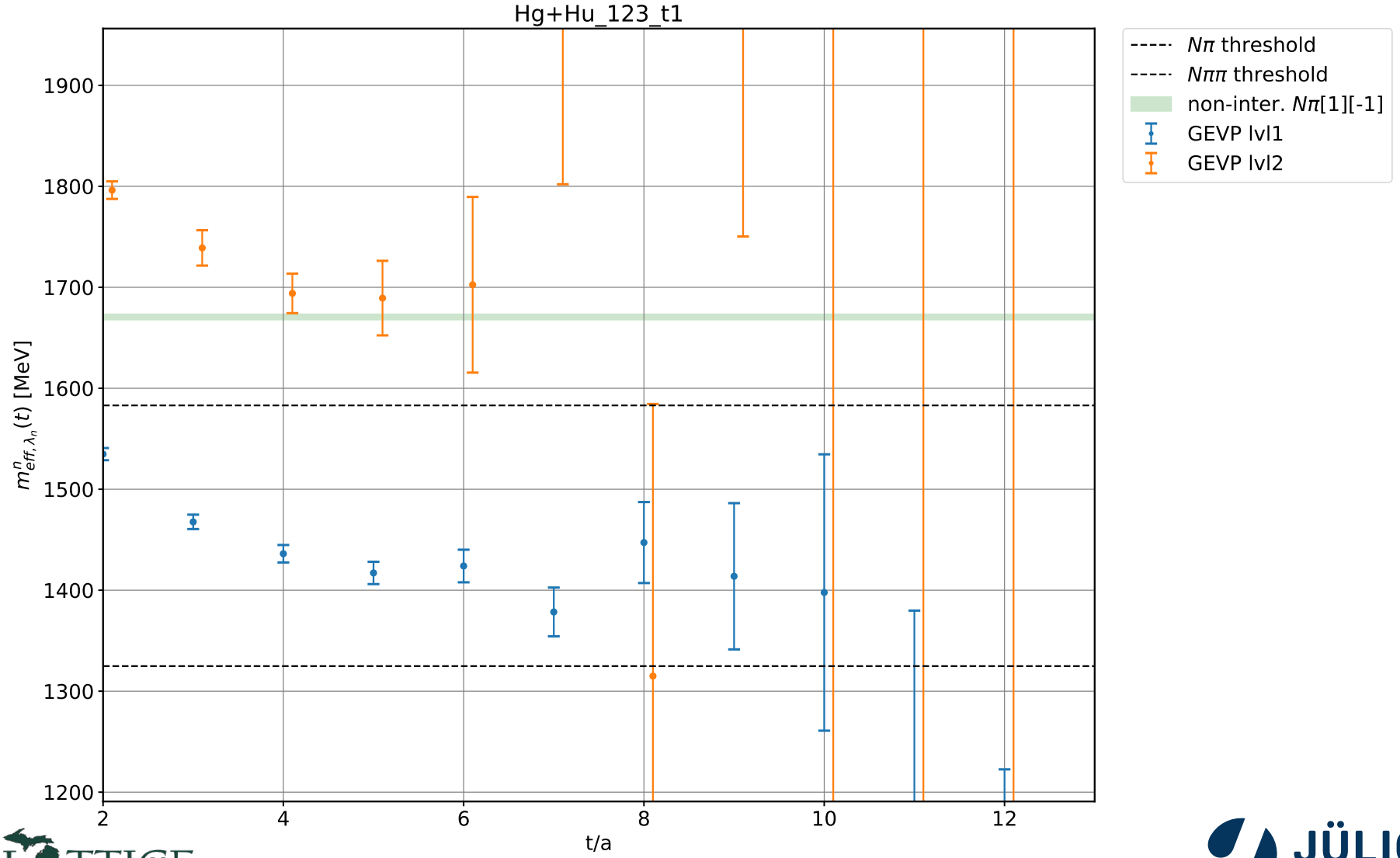
$O_h^D - irrepHg$



$C_{4v}^D - irrepG1$

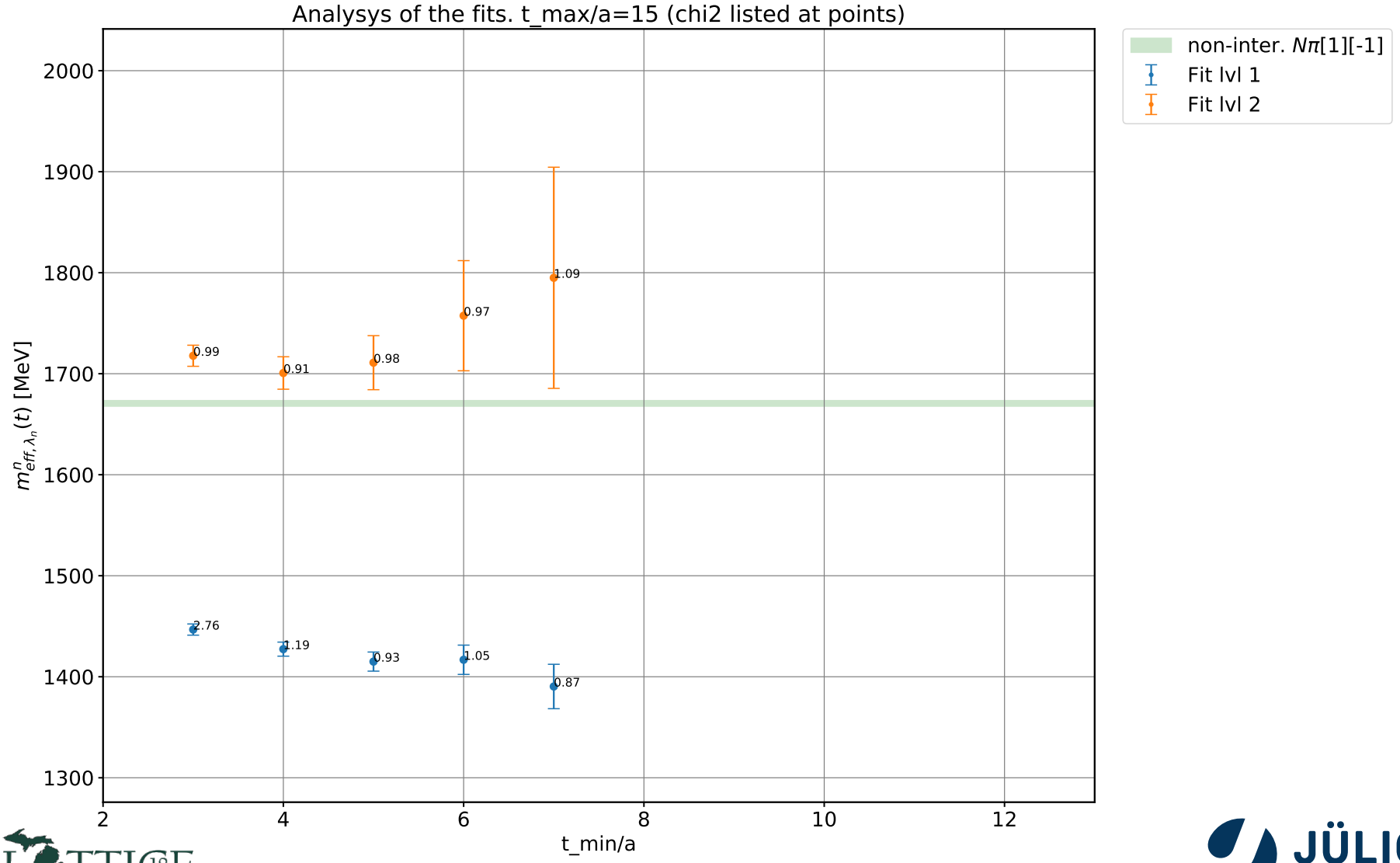
SPECTRA - REST FRAME

$O_h^D - irrepHg(+Hu)$



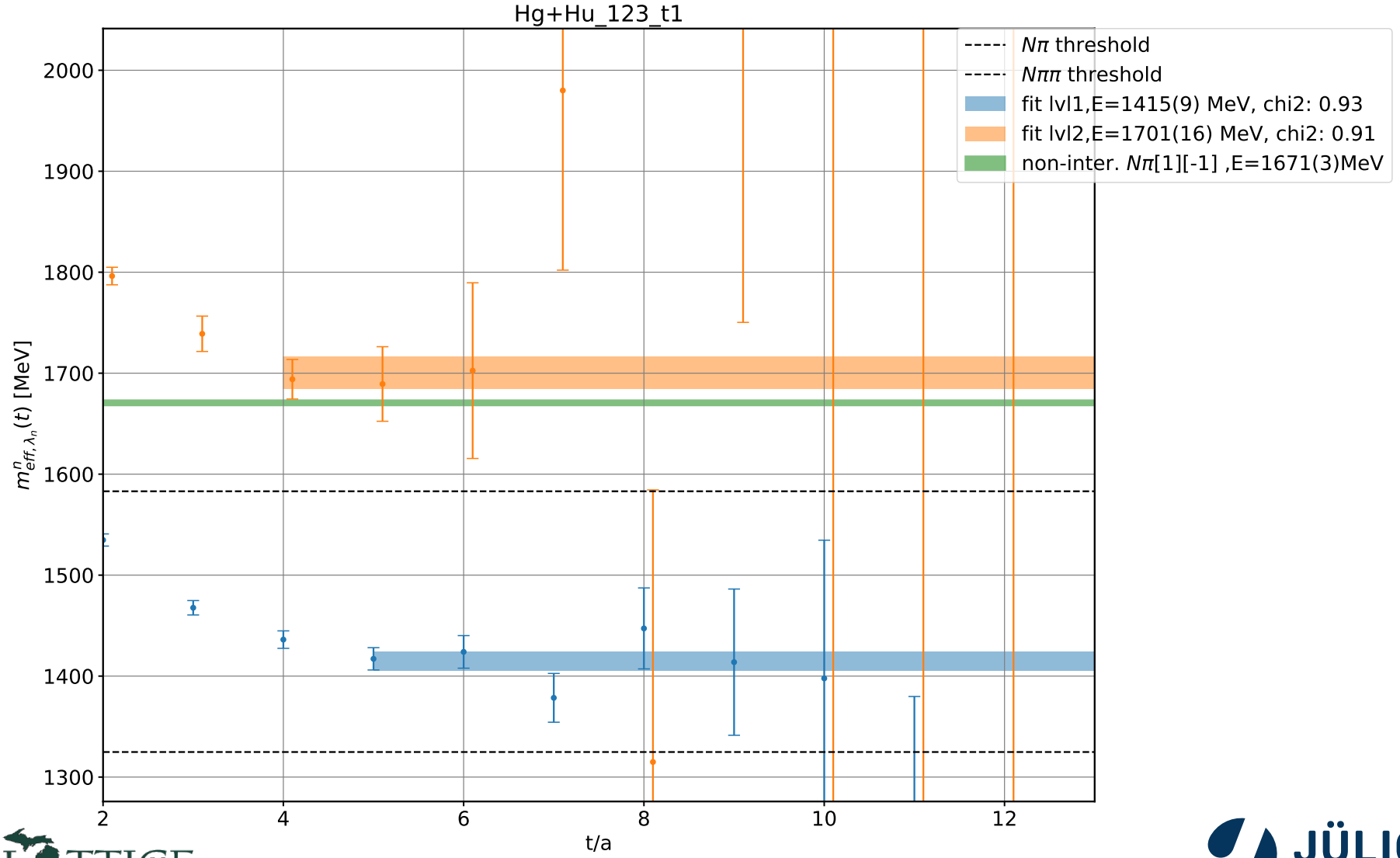
ANALYSIS OF THE FITS - REST FRAME

$O_h^D - irrepHg(+Hu)$



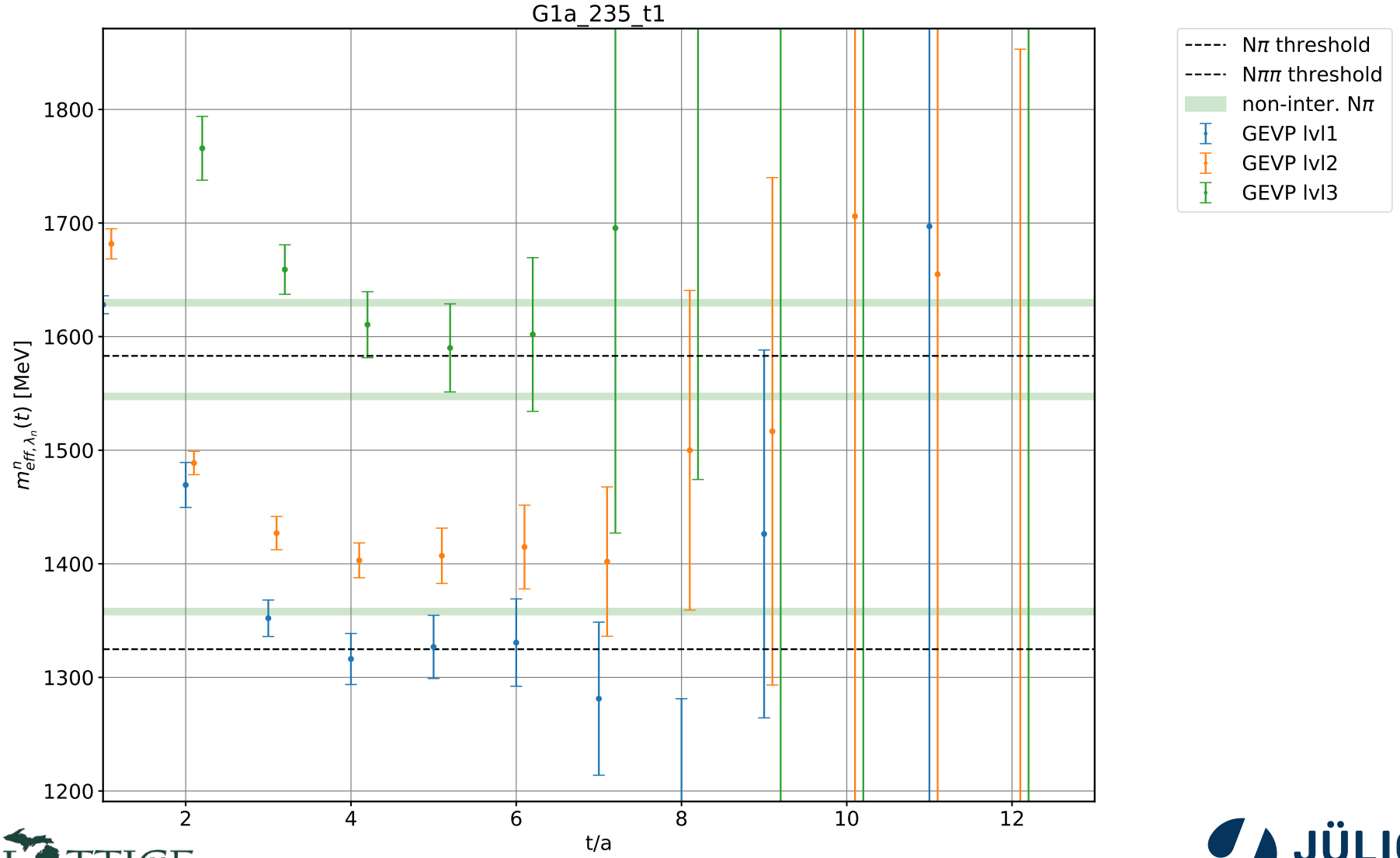
SPECTRA - REST FRAME

$O_h^D - irrep : Hg(+Hu)$



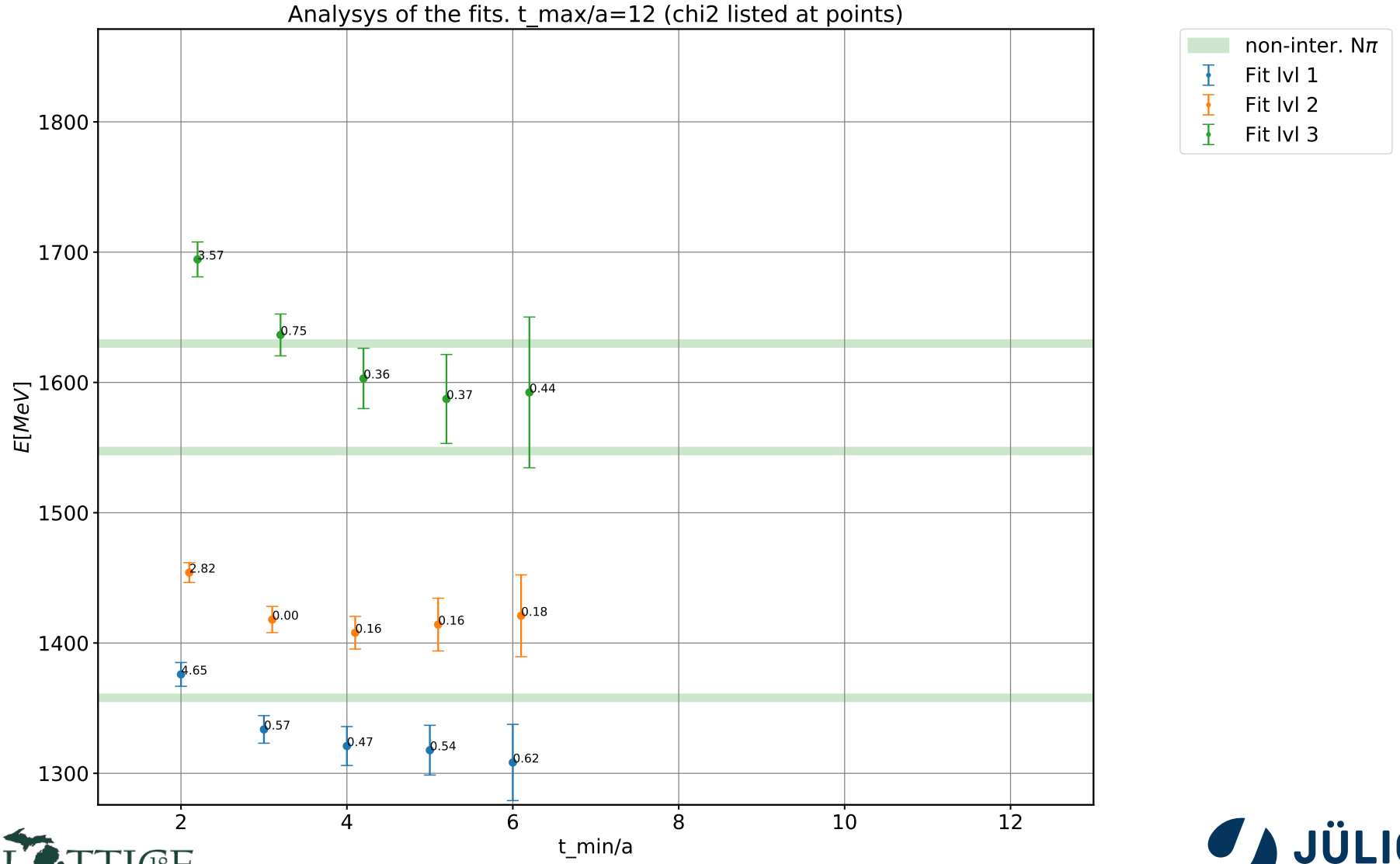
SPECTRA - MOVING FRAME

$C_{4v}^{D} - irrep : G1$



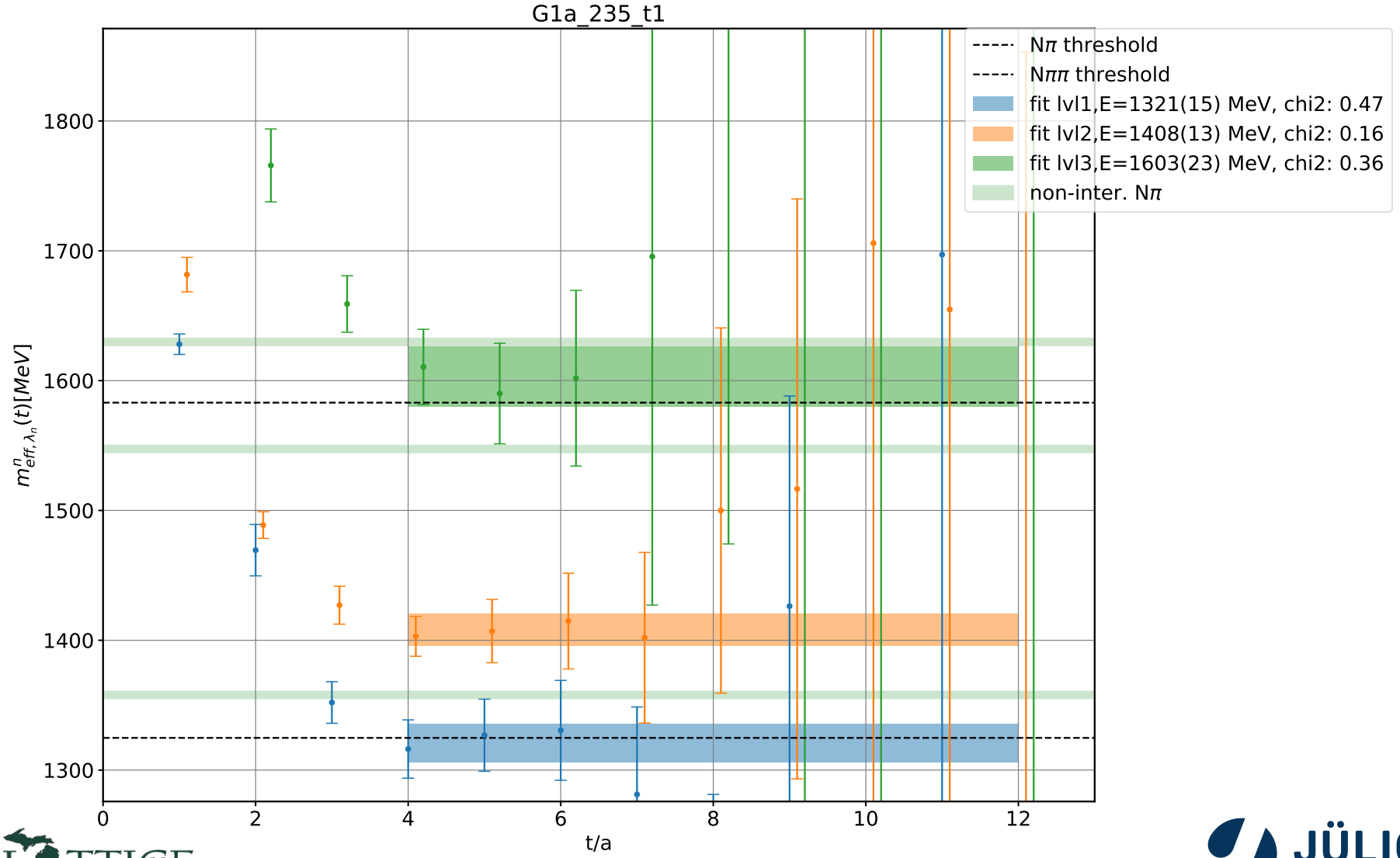
ANALYSIS OF THE FITS - MOVING FRAME

$C_{4v}^{D} - irrepG1$



SPECTRA - MOVING FRAME

C_{4v}^{D} – *irrepG1*



CONCLUSION

- Complete projection of operators for N, Δ and $N\pi$ in relevant irreps
- Room for improvement of spectra with a thorough search for best basis
- Additional configurations coming($3\times$)
- In the future aim to add a bigger box, more m_π and smaller lattice spacing

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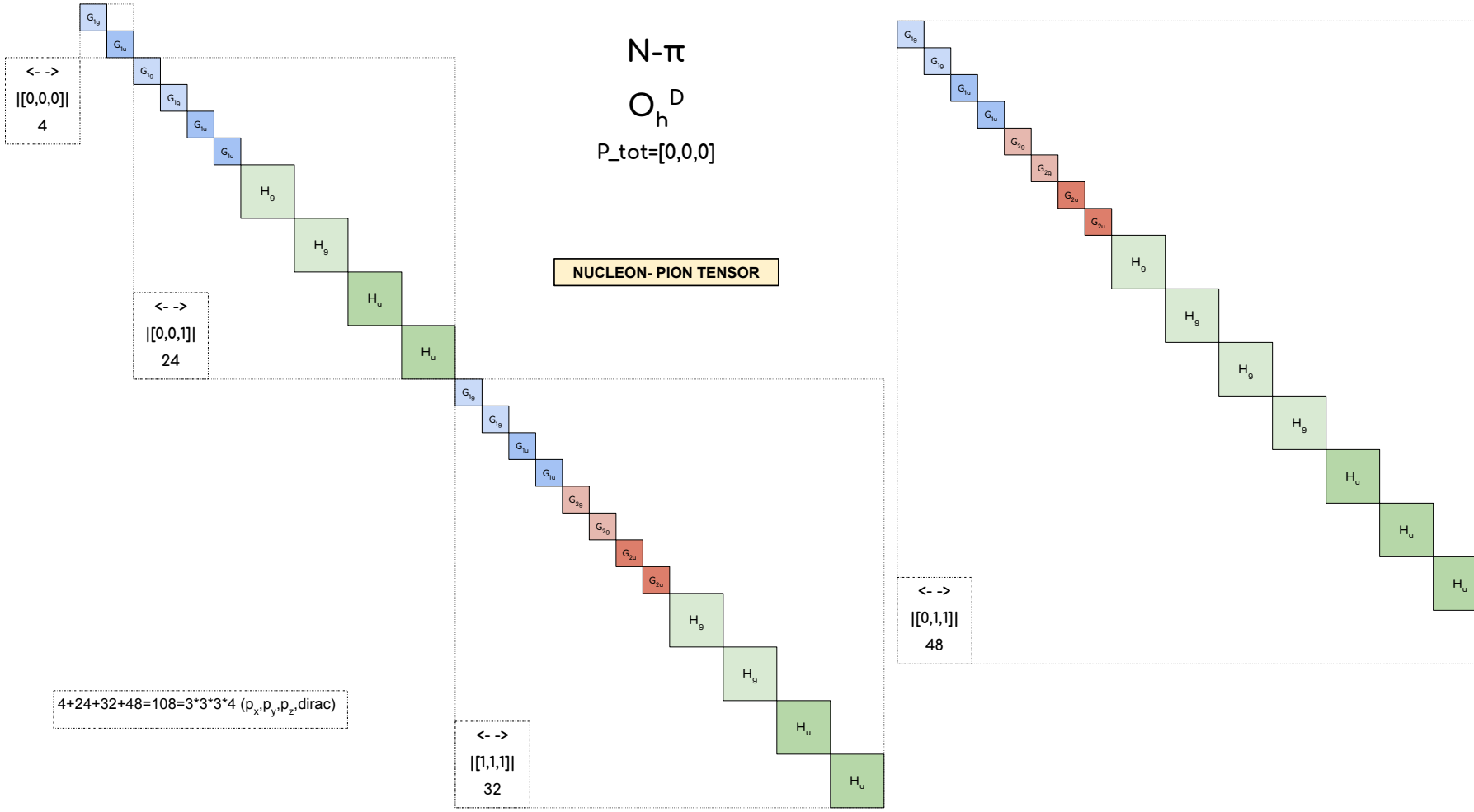
Thank you for the attention!

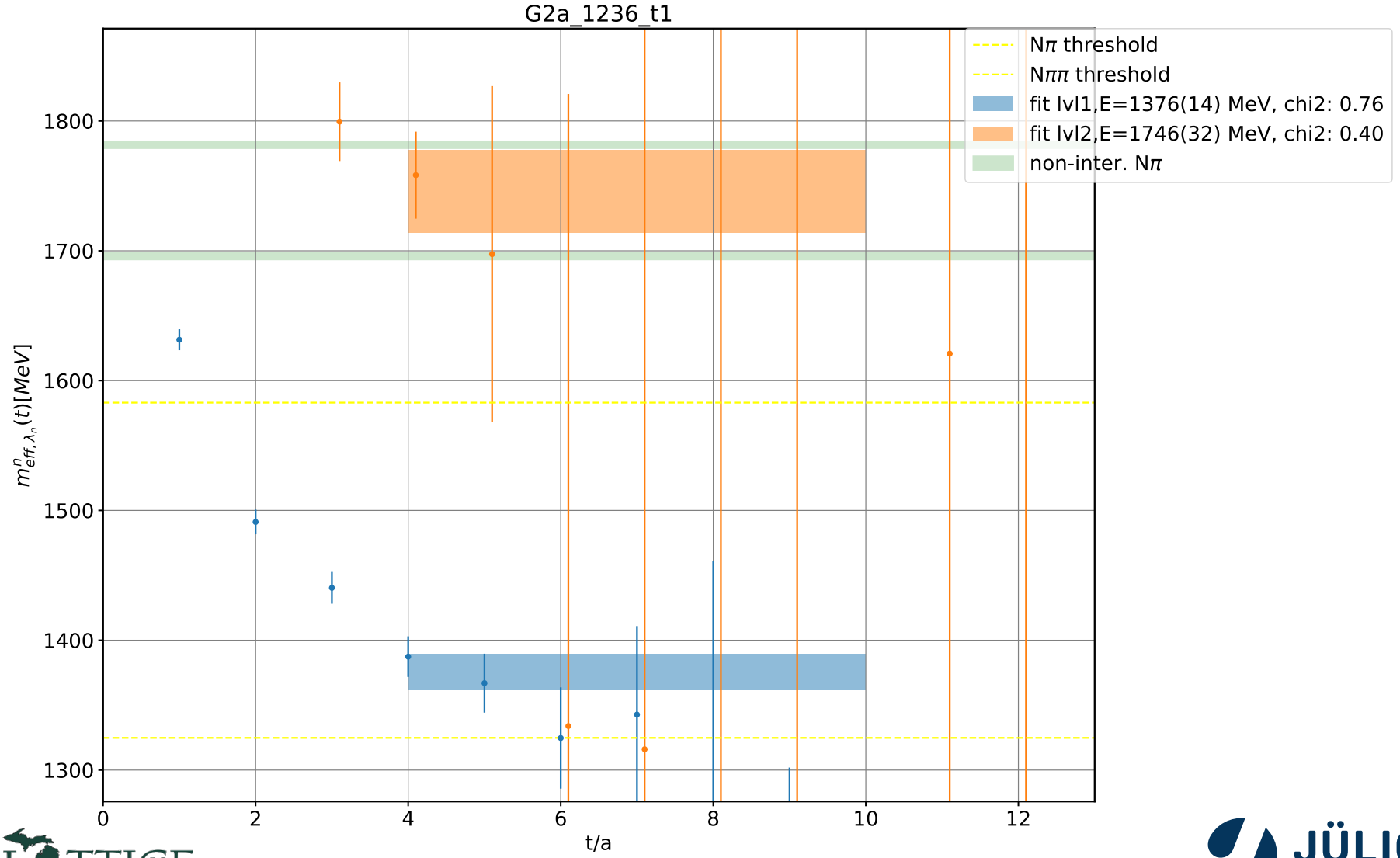
(second part in the next talk by Srijit)

BACK-UP SLIDE



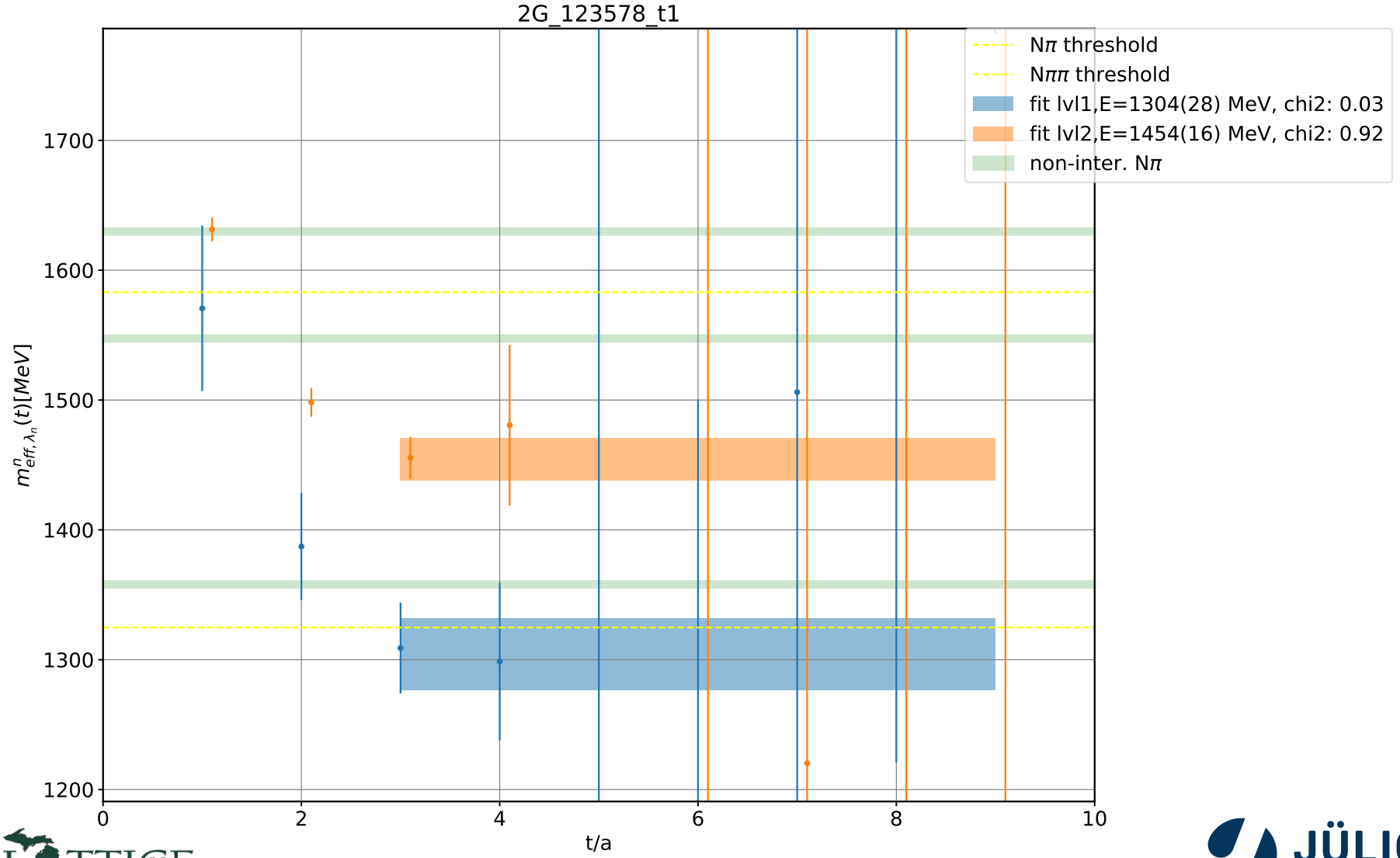
BACK-UP SLIDE

C_{4v}^{D} – *irrepG2*



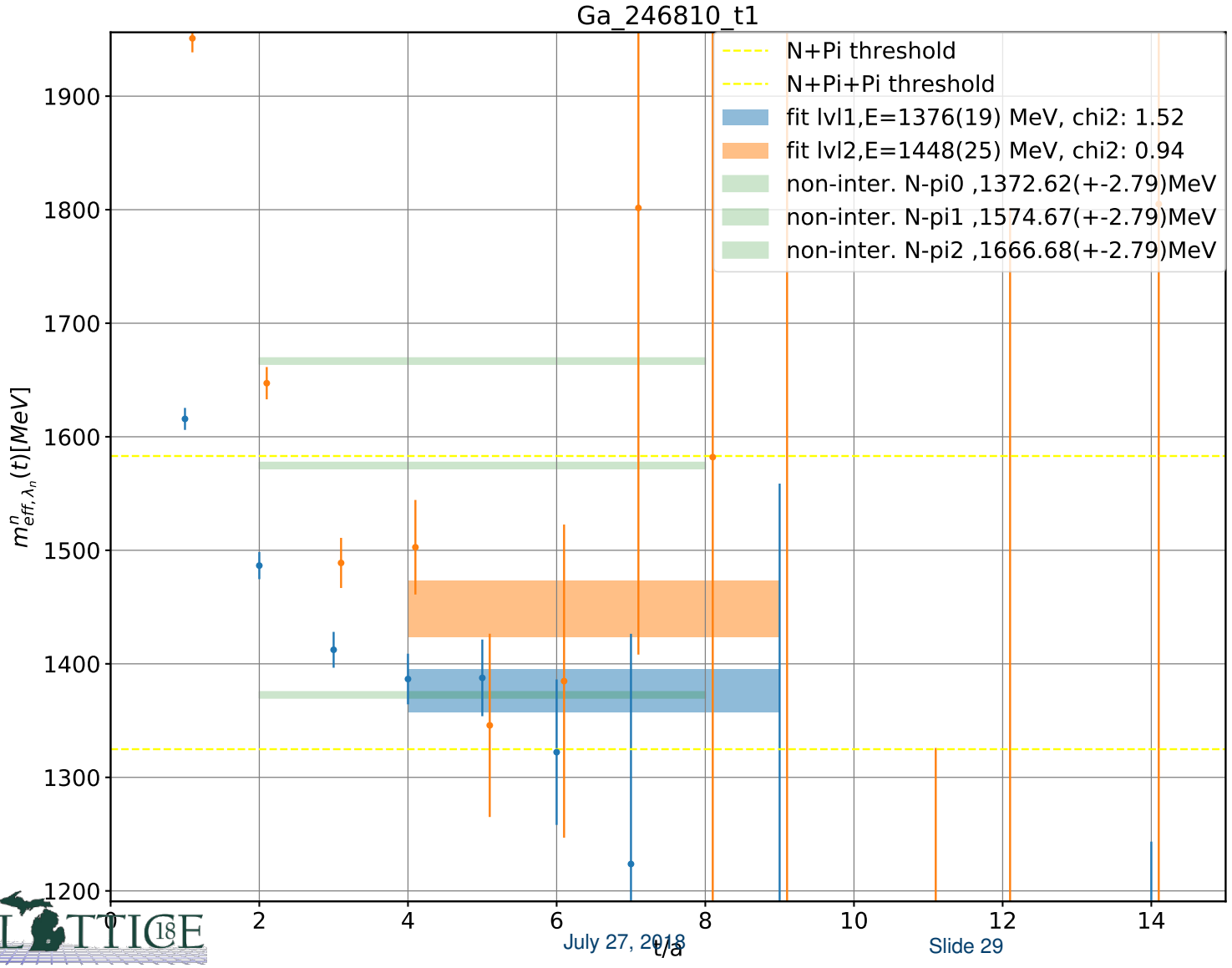
BACK-UP SLIDE

C_{2v}^{D} – *irrepG*



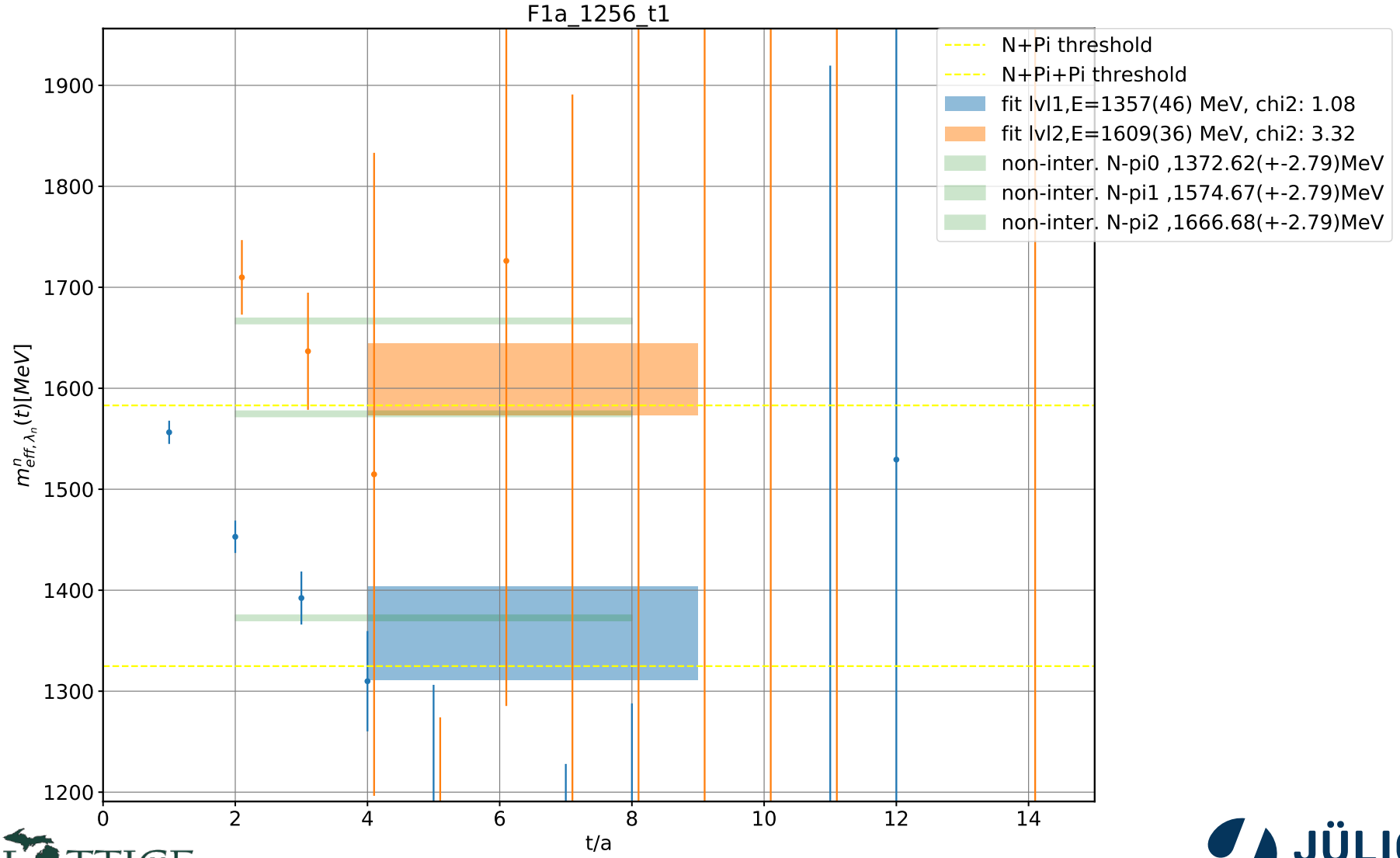
BACK-UP SLIDE

C_{3v}^{D} – *irrepG* **preliminary**



BACK-UP SLIDE

C_{3v}^{D} – *irrepF*1 **preliminary**



BACK-UP SLIDE

C_{3v}^{D} – *irrepF2* **preliminary**