

$$\begin{aligned}
A_s x_s^{(k+1)} &= b_s - A_l x_l^{(k)} - A_r x_r^{(k)} \\
&= b_s - \left(Ax^{(k)} - A_s x_s^{(k)} \right) \\
&= r^{(k)} + A_s x_s^{(k)} \equiv \hat{b}_s^{(k)}
\end{aligned}$$