Hadron Collider Accelerator Basics

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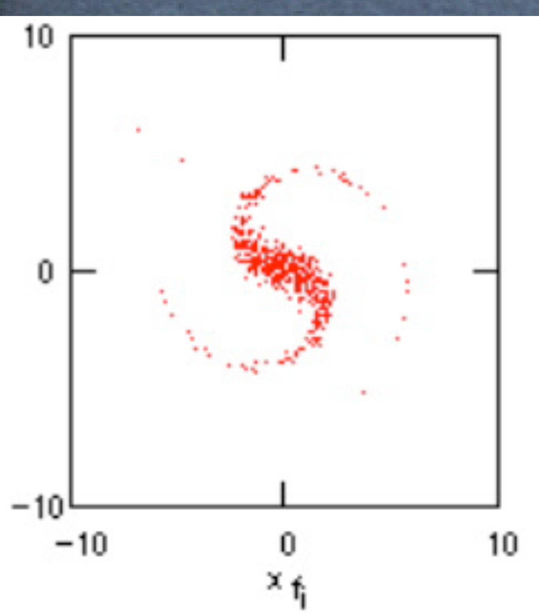
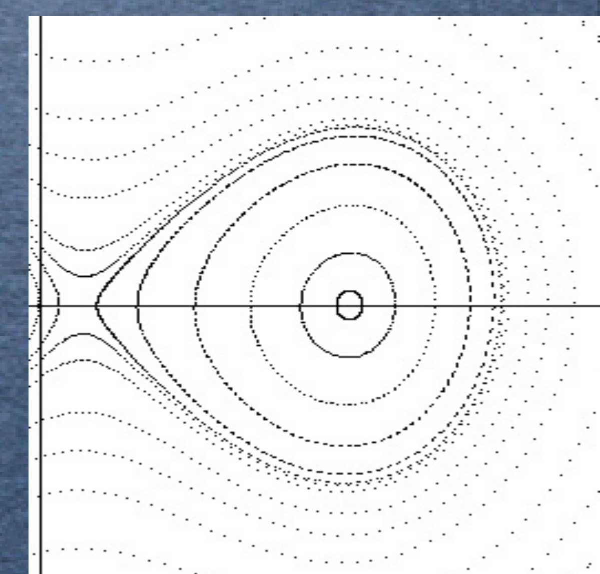
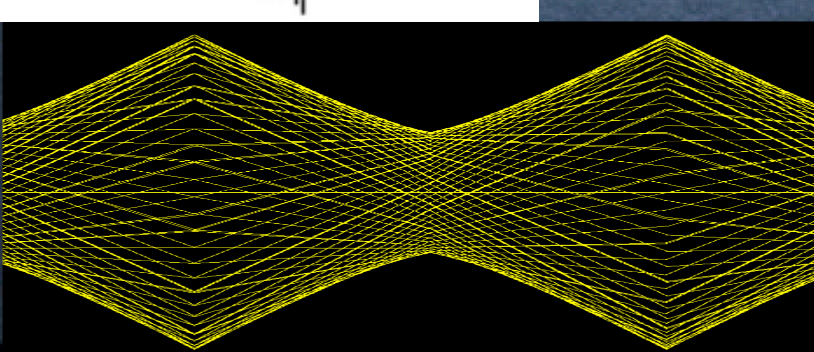
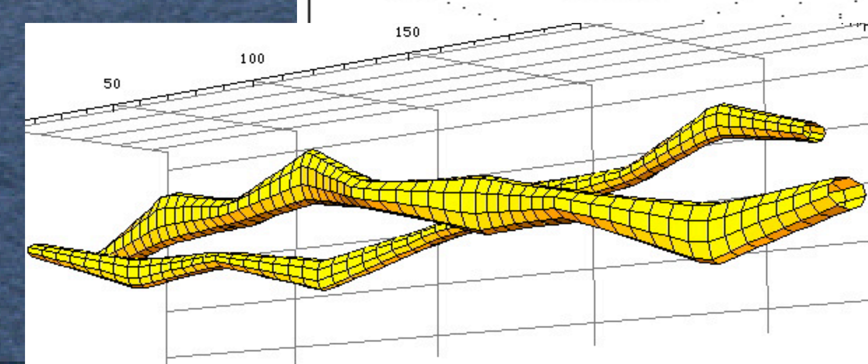




Introduction; Magnets; RF Acceleration
Transverse Motion; Accelerator Lattice
Errors and Adjustments
Challenges at High *E*/*L*
Luminosity Optimization





Introduction

- Will touch on technology, but mostly discuss the *physics* of particle accelerators, especially relevant to hadron colliding beams synchrotrons
- Will cover:
  - luminosity; how to meet the requirements?
  - basic principles; develop "the jargon"
  - a few major issues encountered at high energy, luminosity

Fixed Target Energy vs. Collider Energy

- Beam/target particles: $E_0 \equiv m_p c^2$

Fixed Target

$E,\ \vec{p} \longrightarrow \quad E_0, 0$

$E^*,\ \vec{p} \longrightarrow$

$$E^{*2} = (m^*c^2)^2 + (pc)^2 \quad = \quad [\ E_0 + E\ ]^2$$
$$= \quad E_0^2 + 2E_0E + (E_0^2 + (pc)^2)$$
$$m^*c^2 \quad = \quad \sqrt{2}\ E_0\ [1 + \gamma_{FT}]^{1/2}$$

Collider

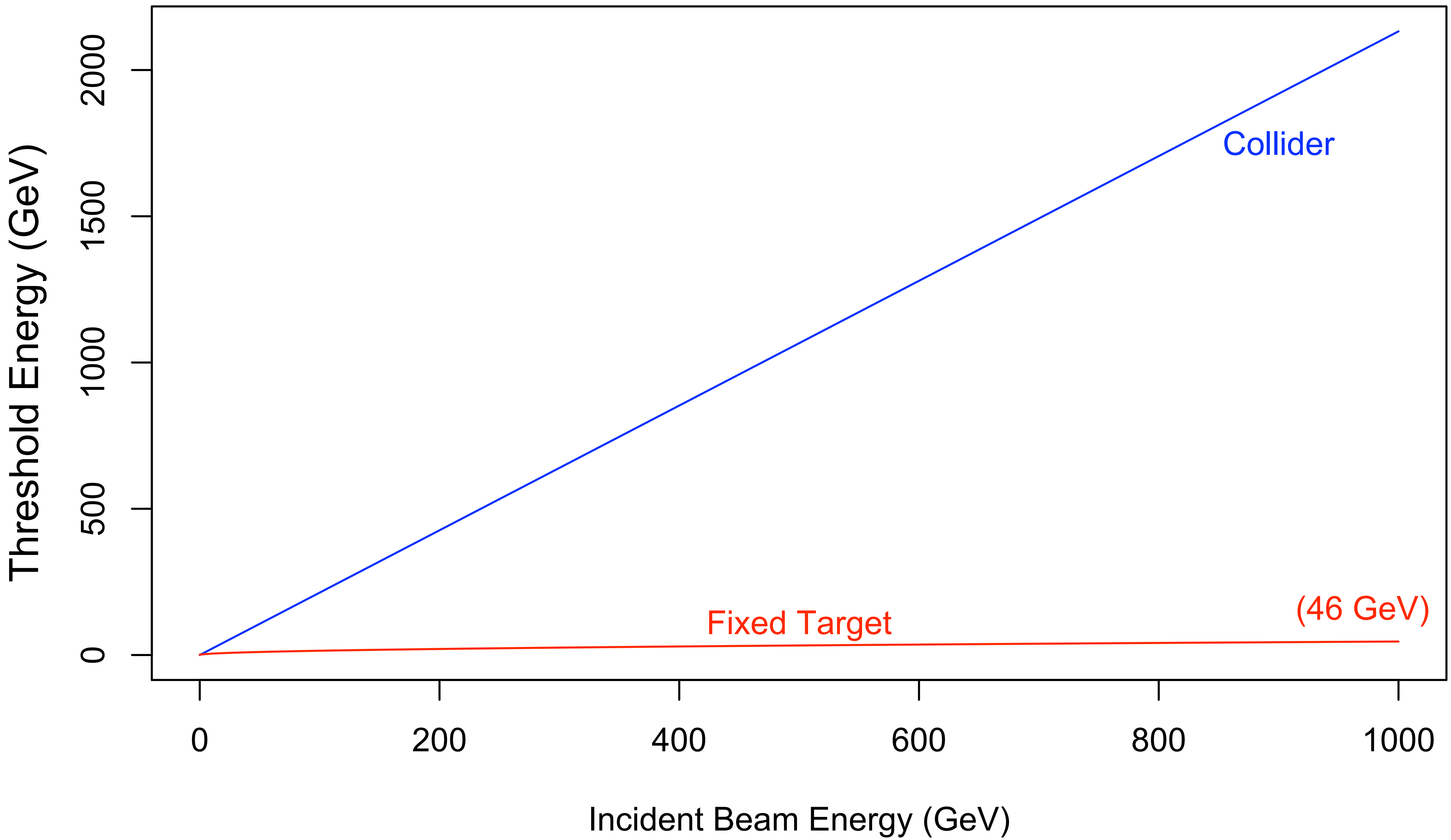
$E, \frac{\vec{p}}{2} \longrightarrow \qquad \longleftarrow E, -\frac{\vec{p}}{2}$

$E^*, 0$

$$m^*c^2 \quad = \quad 2E$$
$$= \quad 2E_0\gamma_{coll}$$

100,000 TeV FT synch. == 14 TeV LHC

Nucleon-Nucleon Collisions

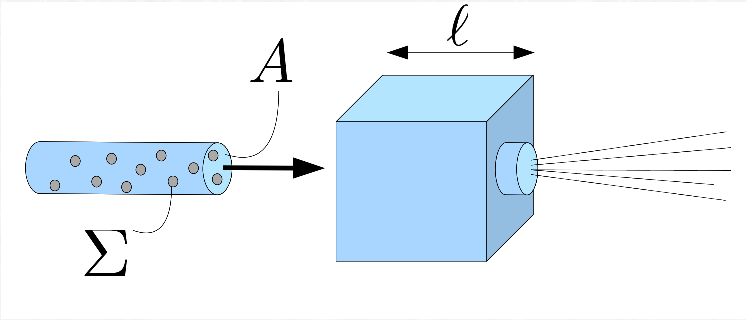


Luminosity

- Experiments want "collisions/events" -- rate?

- Fixed Target Experiment:

$$\mathcal{R} = \left(\frac{\Sigma}{A}\right) \cdot \rho \cdot A \cdot \ell \cdot N_A \cdot \dot{N}_{beam}$$
$$= \rho N_A \ell \dot{N}_{beam} \cdot \Sigma$$
$$\equiv \mathcal{L} \cdot \Sigma$$

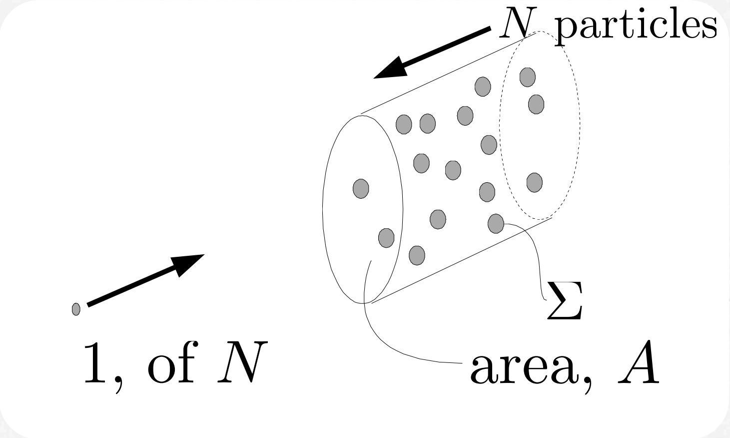


ex.: $\mathcal{L} = \rho N_A \ell \dot{N}_{beam} = 10^{24}/\mathrm{cm}^3 \cdot 100\ \mathrm{cm} \cdot 10^{13}/\mathrm{sec} = \boxed{10^{39}\mathrm{cm}^{-2}\mathrm{sec}^{-1}}$

- Bunched-Beam Collider:

$$\mathcal{R} = \left(\frac{\Sigma}{A}\right) \cdot N \cdot (f \cdot N)$$
$$= \frac{f\ N^2}{A} \cdot \Sigma$$
$$\mathcal{L} \equiv \frac{f\ N^2}{A}$$

$\boxed{(10^{34}\mathrm{cm}^{-2}\mathrm{sec}^{-1}\ \text{for LHC})}$



Integrated Luminosity

- Bunched beam is natural in collider that "accelerates" (more later)

$$\mathcal{L} = \frac{f_0 B N^2}{A}$$

$f_0 = \text{rev. frequency}$
$B = \text{no. bunches}$

- In ideal case, particles are "lost" only due to "collisions":

$$B\dot{N} = -\mathcal{L}\,\Sigma\,n$$

($n = $ no. of detectors receiving luminosity $\mathcal{L}$)

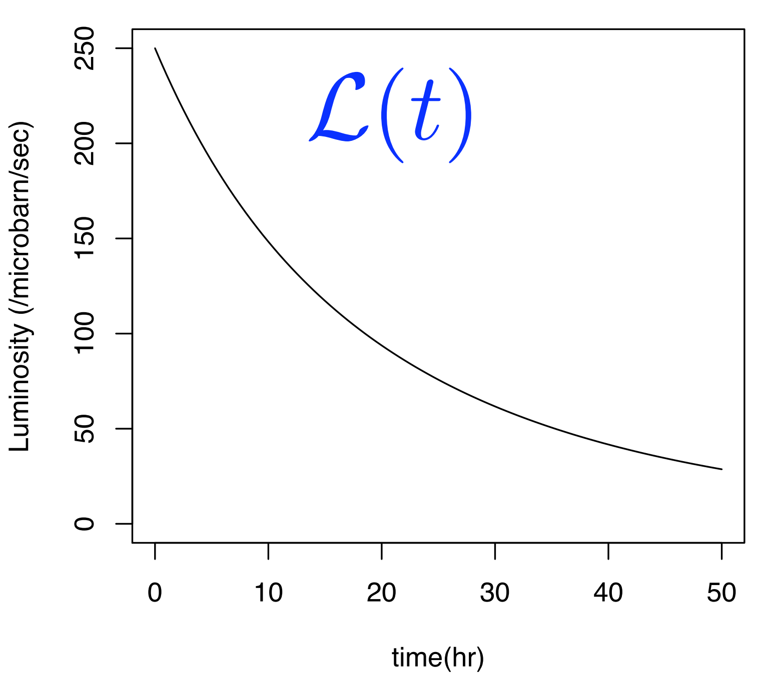
- So, in this ideal case,

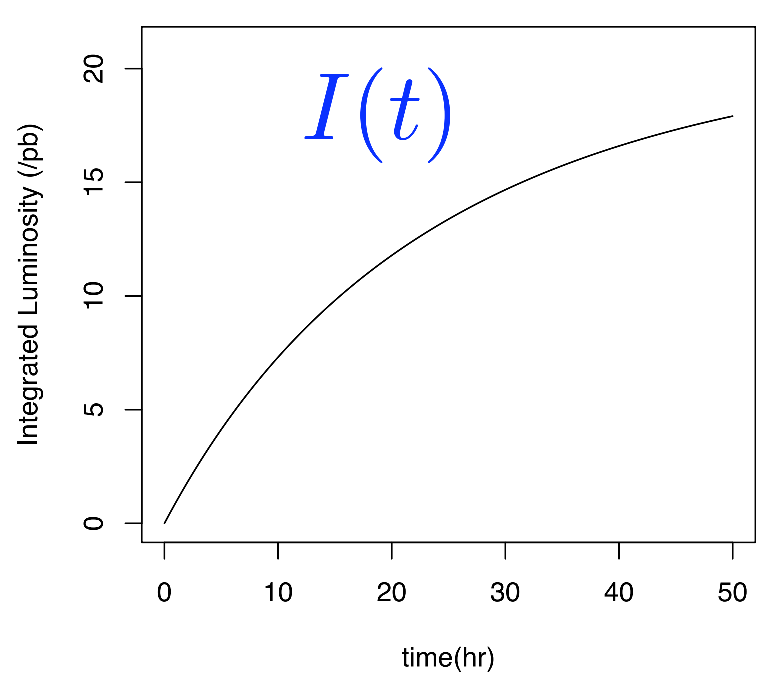
$$\mathcal{L}(t) = \frac{\mathcal{L}_0}{\left[1 + \left(\frac{n\mathcal{L}_0\Sigma}{BN_0}\right)t\right]^2}$$

Ultimate Number of Collisions

- Since $\mathcal{R} = \mathcal{L} \cdot \Sigma$ then, #events = $\int \mathcal{L}(t)dt \cdot \Sigma$
- So, our integrated luminosity is

$$I(T) \equiv \int_0^T \mathcal{L}(t)dt = \frac{\mathcal{L}_0 T}{1 + \mathcal{L}_0 T (n\Sigma / B N_0)} = I_0 \cdot \frac{\mathcal{L}_0 T / I_0}{1 + \mathcal{L}_0 T / I_0}$$





asymptotic limit:

$$\boxed{I_0 \equiv \frac{B N_0}{n\Sigma}}$$

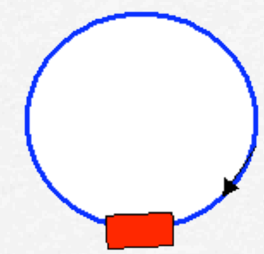
so, ...

$$\mathcal{L} = \frac{f_0 B N^2}{A}$$

(will come back to luminosity at the end)

How to Make Collisions?

- Simple Model of Synchrotron:



  - Accelerating device + magnetic field to bring particle back to accelerate again

- Field Strength -- determines size, ultimate energy of collider

  - ex:

$$\rho = \frac{p}{e\,B}\ ; \quad R = \rho/f \quad (f \approx 0.8 - 0.9)$$

"packing fraction"

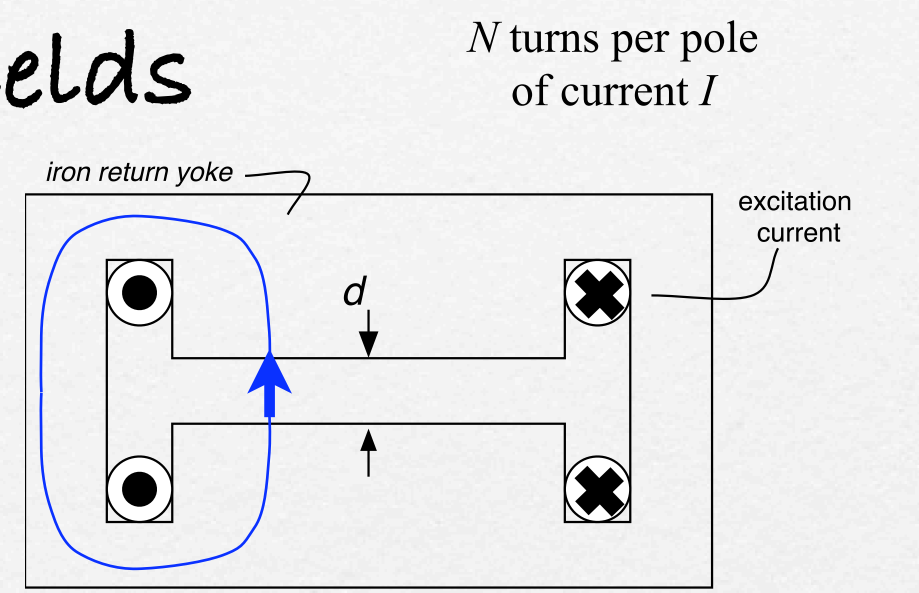
$$B = 1.8\ \mathrm{T}, \quad p = 450\ \mathrm{GeV/c} \quad f = 0.85 \rightarrow R \approx 1\ \mathrm{km}$$

Magnets

- iron-dominated magnetic fields

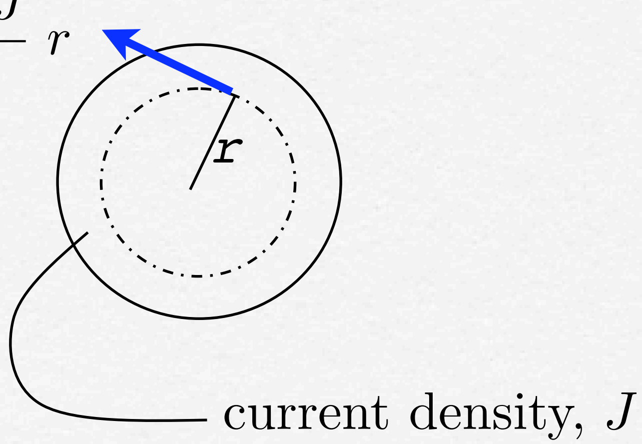
$$B = \frac{2\mu_0 N \cdot I}{d}$$

  - iron will "saturate" at about 2 Tesla



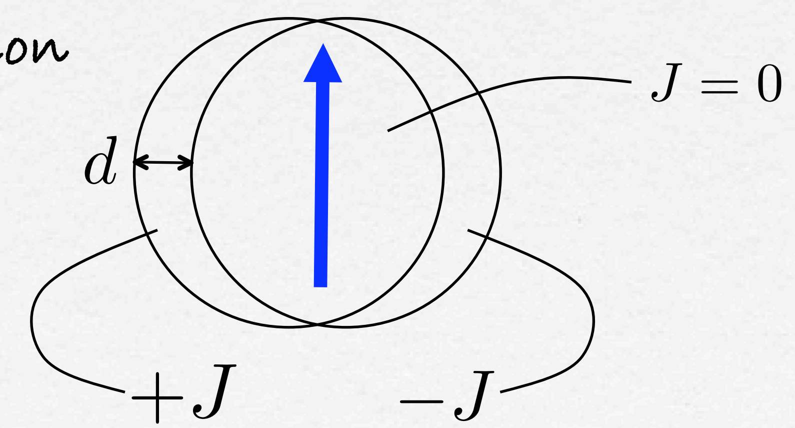
- Superconducting magnets
  - field determined by distribution of currents

$$B_\theta = \frac{\mu_0 J}{2}\, r$$



"Cosine-theta" distribution

$$\boxed{B_x = 0, \quad B_y = \frac{\mu_0 J}{2}\, d}$$

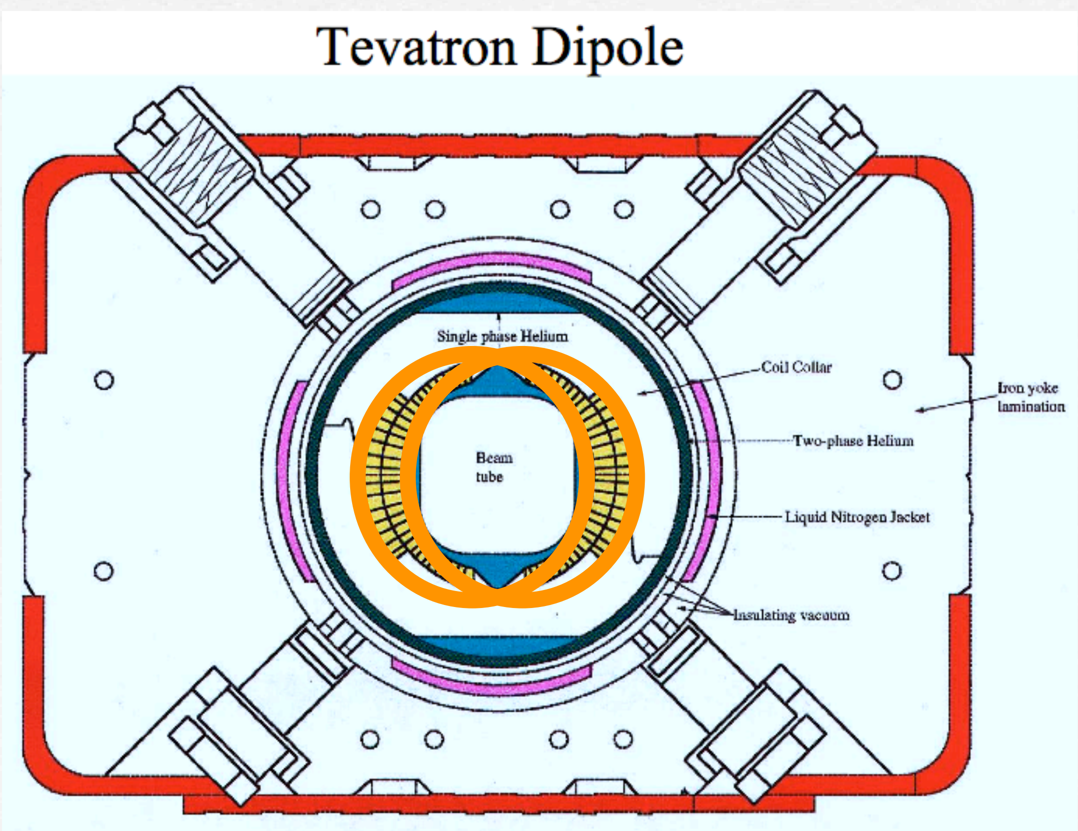


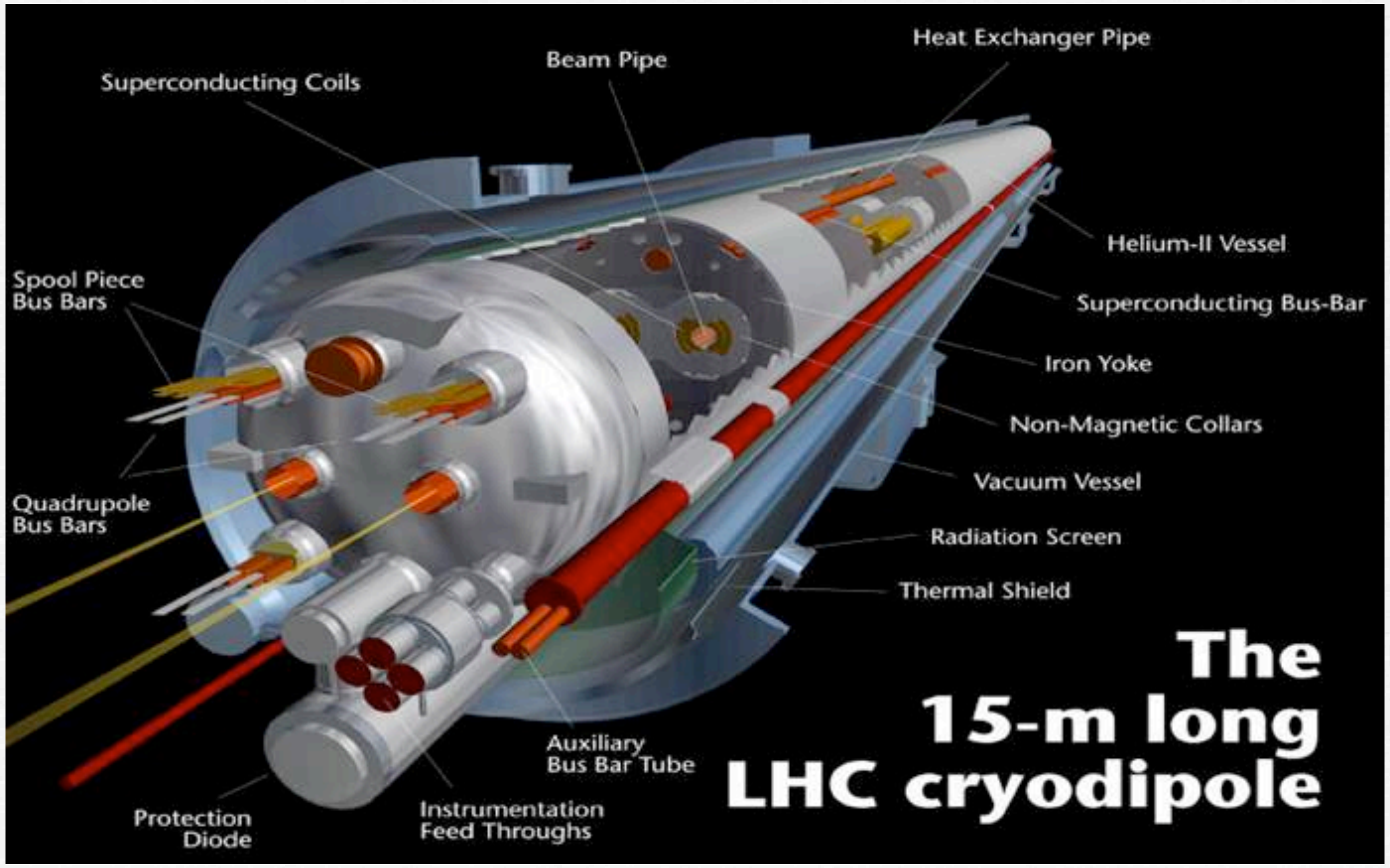
Superconducting Designs

- Tevatron
  - 1^{st} SC accelerator
  - 4.4 T; 4^0K

Numerical Example:

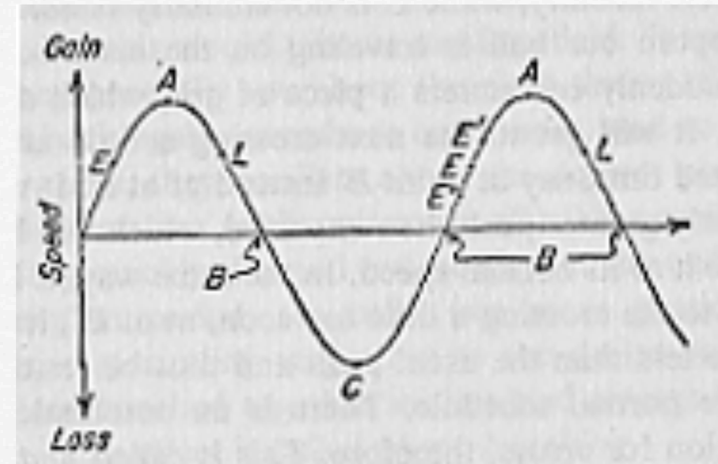
$$\begin{aligned} B &= \frac{\mu_0 J}{2} d \\ &= \frac{4\pi\ \mathrm{T\ m/A}}{10^7}\frac{1000\ \mathrm{A/mm}^2}{2}\cdot(10\ \mathrm{mm})\cdot\frac{10^3\mathrm{mm}}{\mathrm{m}} \\ &= 6\ \mathrm{T} \end{aligned}$$



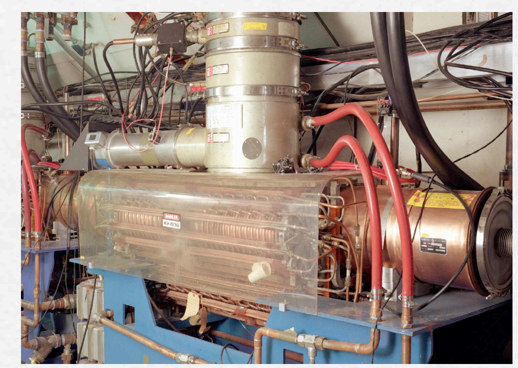


- LHC -- 8 T; 1.8^0K

Acceleration



- Principal of phase stability
  - McMillan (U. California) and Veksler (Russia)
- Imagine: particle circulating in field, B; along orbit, arrange particle to pass through a cavity with max. voltage V, oscillating at frequency $h \text{ x } f_{rev}$ (where h is an integer); suppose particle arrives near time of zero-crossing
  - net acceleration/deceleration $= eV \sin(\omega \Delta t)$



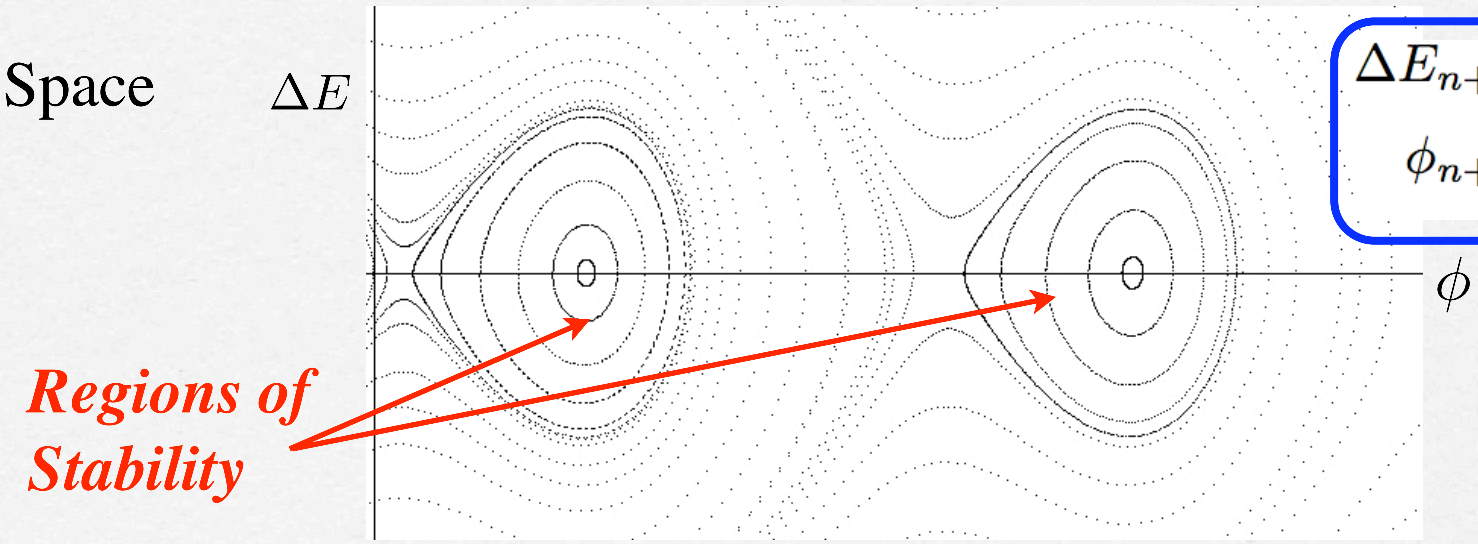
- if arrives late, more voltage is applied; arrives early, gets less
  - thus, a restoring force --> energy oscillation

"Synchrotron Oscillations"

- in general, lower momentum particles take longer, arrive late gain extra momentum
- next, slowly raise the strength of B; if raised adiabatically, oscillations continue about the "synchronous" momentum, defined by $p/e = B \cdot R$ for constant R
- This is the principle behind the synchrotron, used in all major HEP accelerators today

Longitudinal Motion

- Say ideal particle arrives at phase φ_s: $$\frac{dE_s}{dt} = f_0 eV \sin\phi_s \sim \frac{dB}{dt}$$
- Particles arriving nearby in phase, and nearby in energy will oscillate about these ideal conditions ...
  - Phase Space plot:

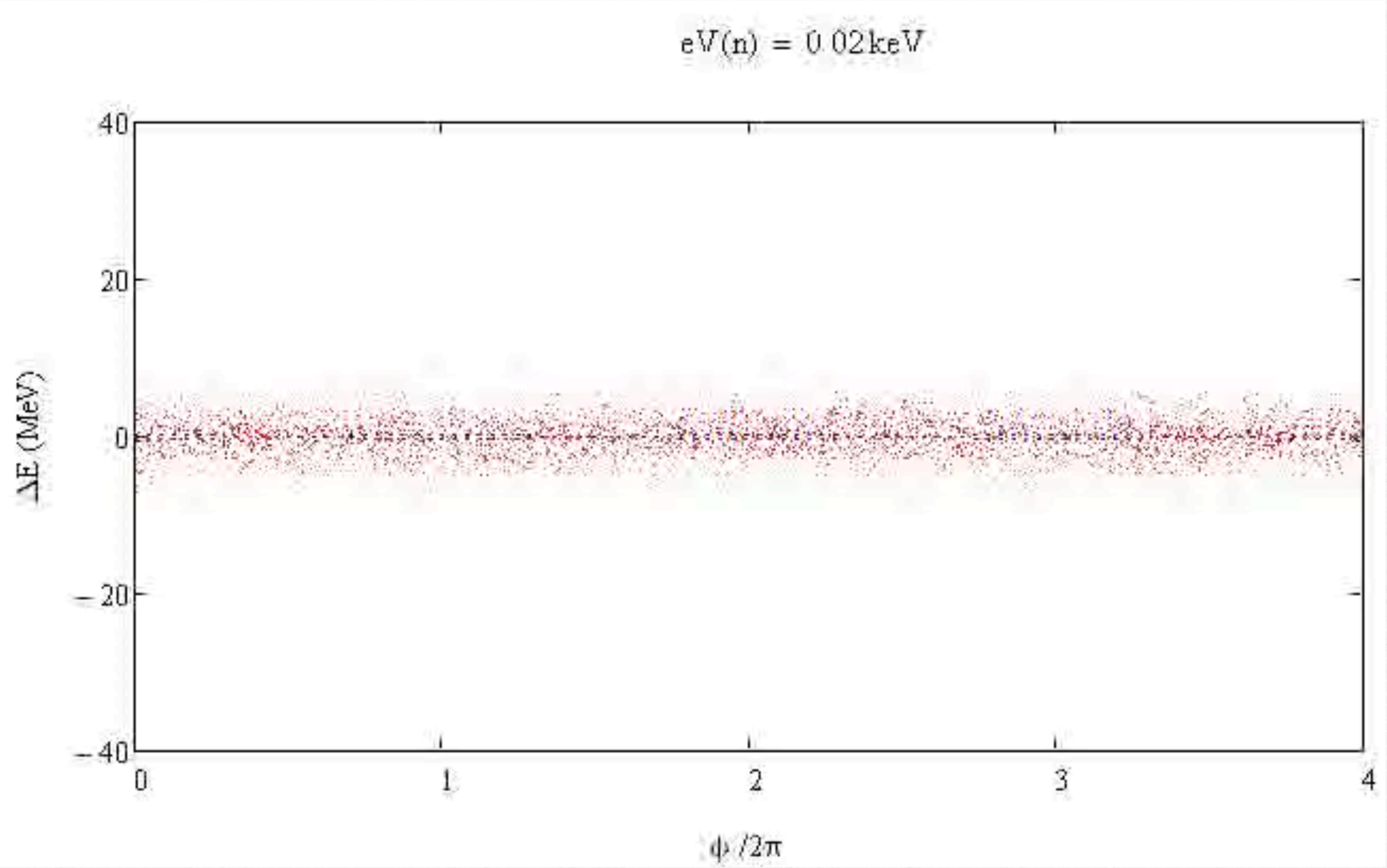


$$\begin{aligned} \Delta E_{n+1} &= \Delta E_n + eV(\sin\phi_n - \sin\phi_s) \\ \phi_{n+1} &= \phi_n + 2\pi h \cdot \frac{\eta}{\beta^2 E_s} \Delta E_{n+1} \end{aligned}$$

- Adiabatic increase of bend field generates stable phase space regions; particles oscillate, follow along
- "bunched" beam; $h = f_{rf} / f_{rev}$ = # of possible bunches

Bunched Beam

- Bunch by adiabatically raising voltage of RF cavities

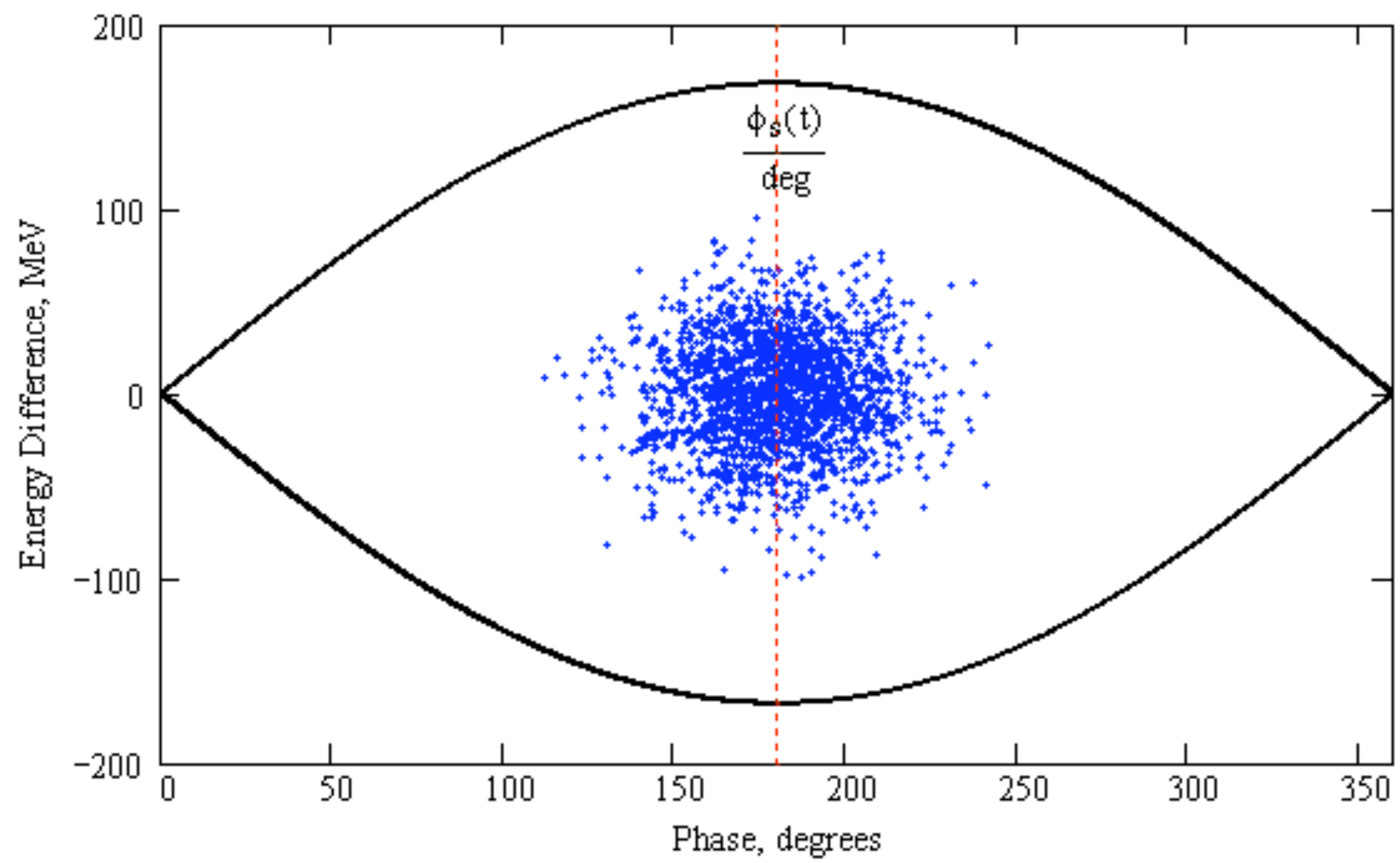


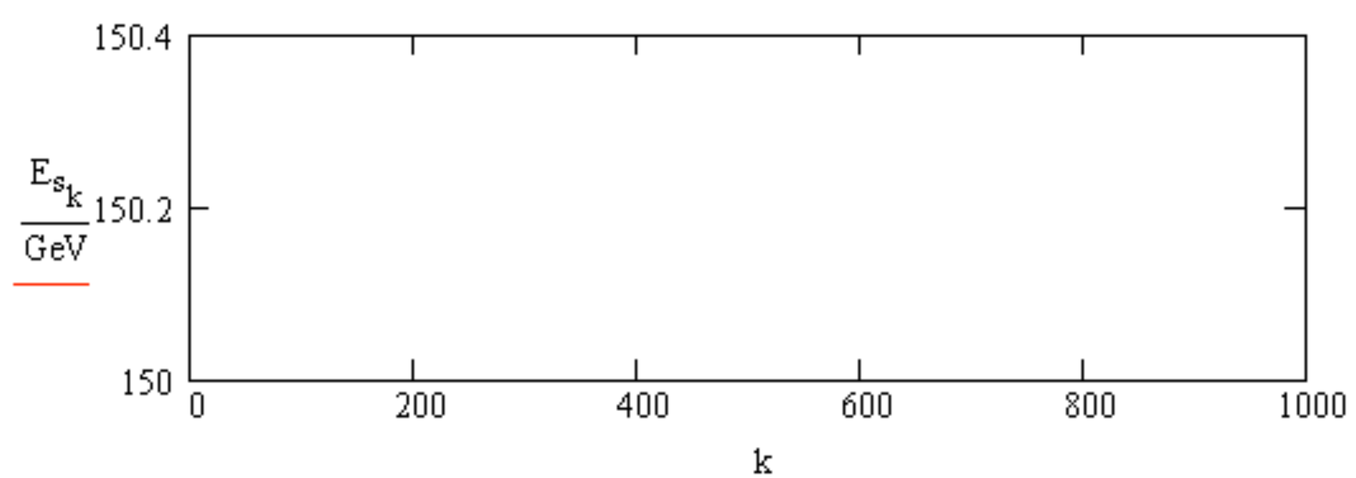
Buckets, Bunches, Batches, ...

- Stable phase space region is called a bucket.
  - Boundary is the separatrix; only an approximation
  - $\varphi_s = 0, \pi$ -- particles outside bucket remain in accelerator "DC beam"
  - For other values of φ_s -- particles outside bucket are lost
    - DC beam from injection is lost upon acceleration
- Bunches of particles occupy buckets; but not all buckets need be occupied.
- Batches (or, bunch trains) are groupings of bunches formed in specific patterns, often from upstream accelerators

Acceleration

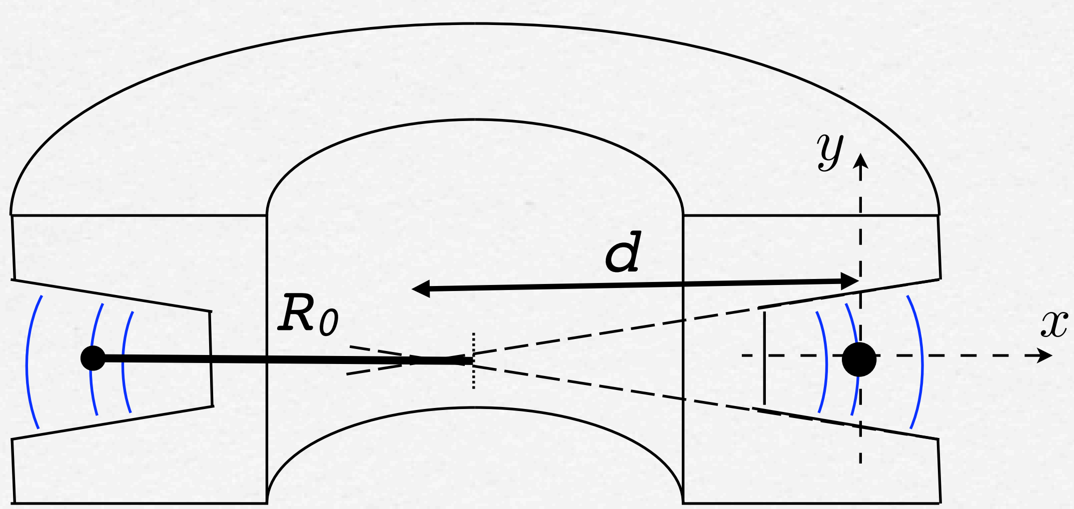
- Stable regions shirnk as begin to accelerate
- If beam phase space area is too large (or if DC beam exists), can lose particles in the process





Keeping Focused

- In addition to increasing the particle's energy, must keep the beam focused transversely along its journey
- Early accelerators employed what is now called "weak focusing"



$$B = B_0 \left(\frac{R_0}{r}\right)^n$$

n is determined by adjusting the opening angle between the poles

$d = \infty$, $n = 0$

$d = R_0$, $n = 1$

$$n \approx \frac{R_0}{d}$$

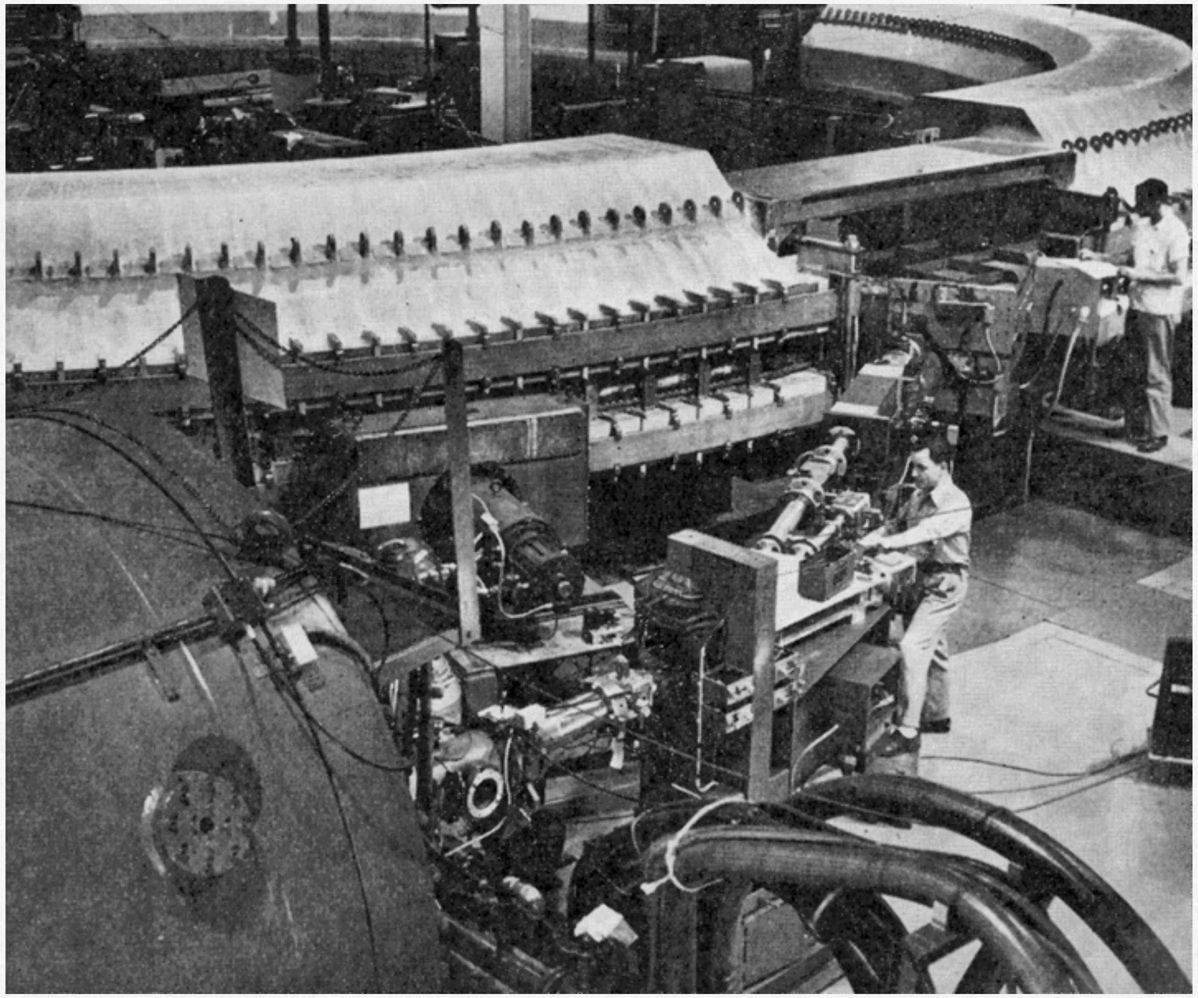
$\boxed{\text{must have } 0 \le n \le 1 \text{ for stability}}$

Room for improvement...

- With weak focusing, for a given transverse angular deflection,

$$x_{max} \sim \frac{R_0}{\sqrt{n}}\,\theta$$

- Thus, aperture ~ radius ~ energy



Cosmotron (1952)

(3.3 GeV)

Room for improvement...

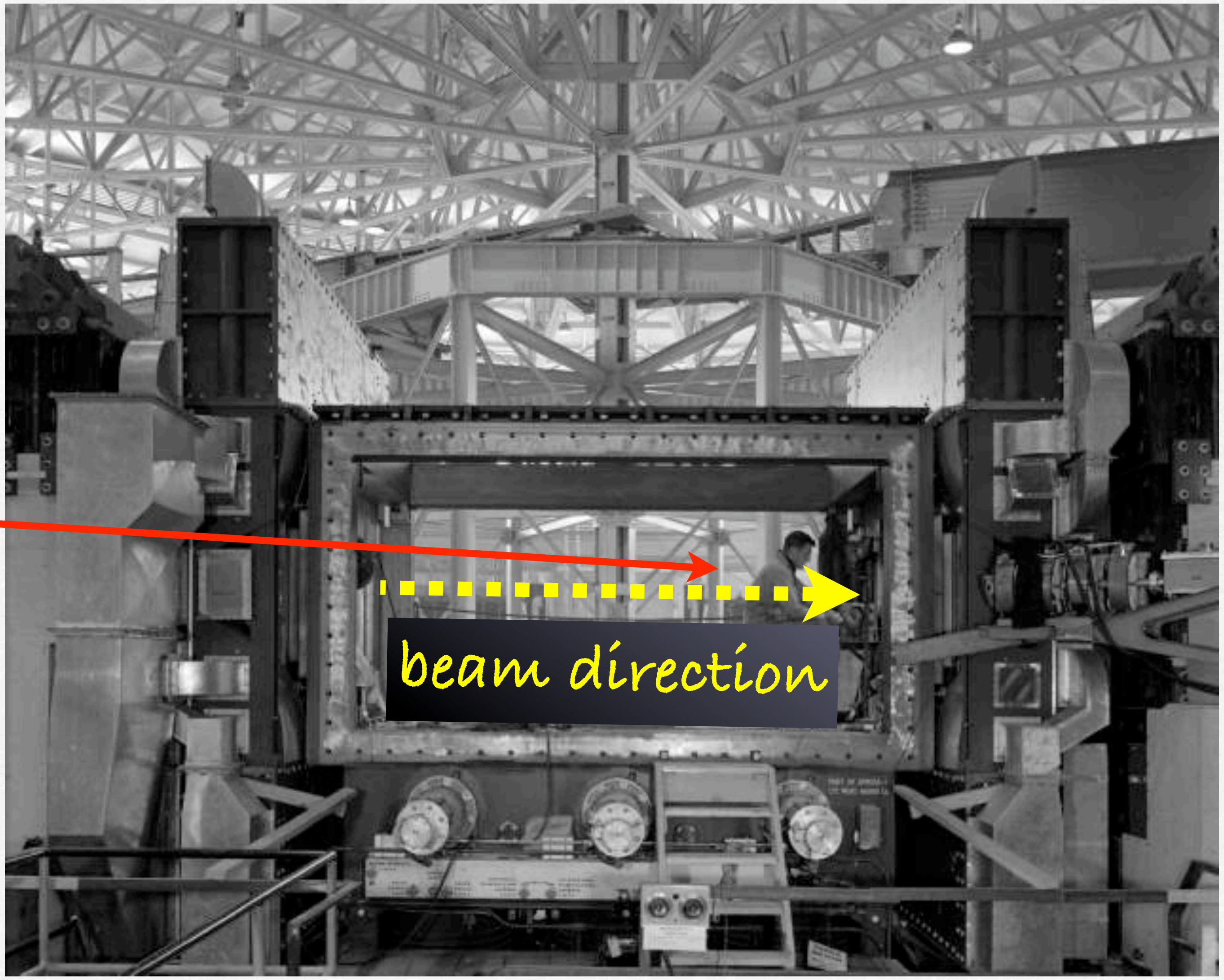
- With weak focusing, for a given transverse angular deflection,
- Thus, aperture ~ radius ~ energy

$$x_{max} \sim \frac{R_0}{\sqrt{n}}\,\theta$$

Bevatron (1954)

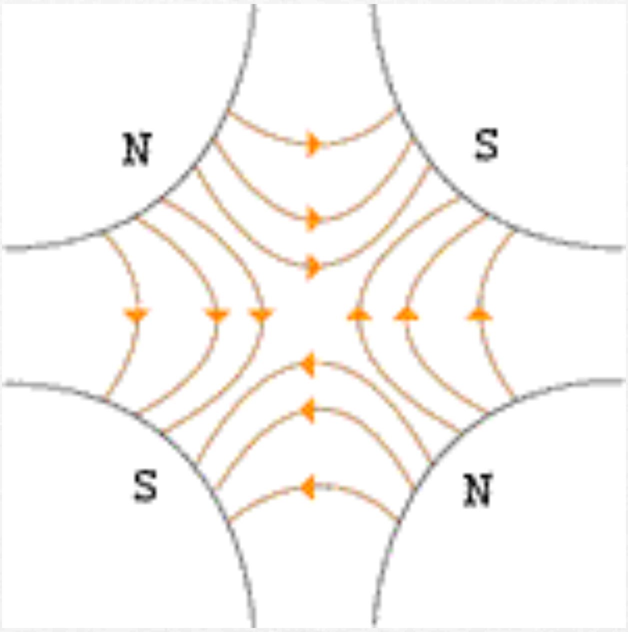
(6 GeV)

Could actually sit inside the vacuum chamber!!



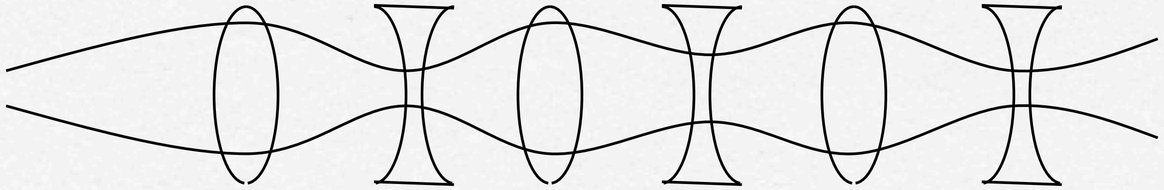
Strong Focusing

- Think of standard focusing scheme as alternating system of focusing and defocusing lenses (today, use quadrupole magnets)



- Quadrupole will focus in one transverse plane, but defocus in other; if alternate, can have net focusing in both
  - for equally spaced infinite set, net focusing requires $F > L/2$

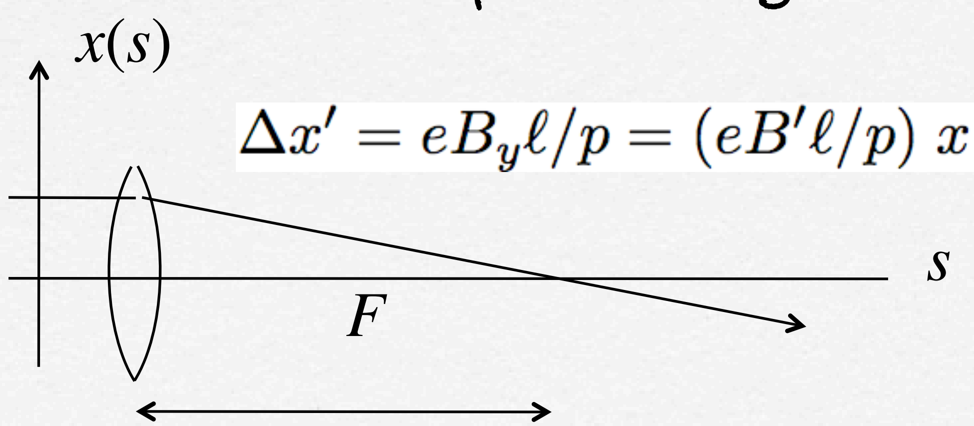
    F = focal length, L = spacing
  - FODO cells:

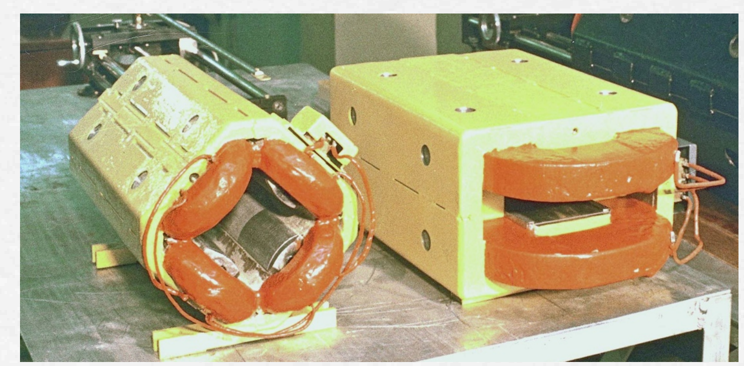


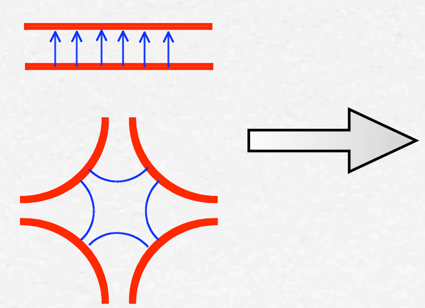
Separated Function

- Until late 60's, synchrotron magnets (wedge-shaped variety) both focused and steered the particles in a circle. ("combined function")
- With Fermilab Main Ring and CERN SpS, use "dipole" magnets to steer, and use "quadrupole" magnets to focus
- Quadrupole magnets, with alternating field gradients, "focus" particles about the central trajectory -- act like lenses
- Thin lens focal length:

$$\Delta x' = eB_y\ell/p = (eB'\ell/p)\,x \rightarrow 1/F = eB'\ell/p$$





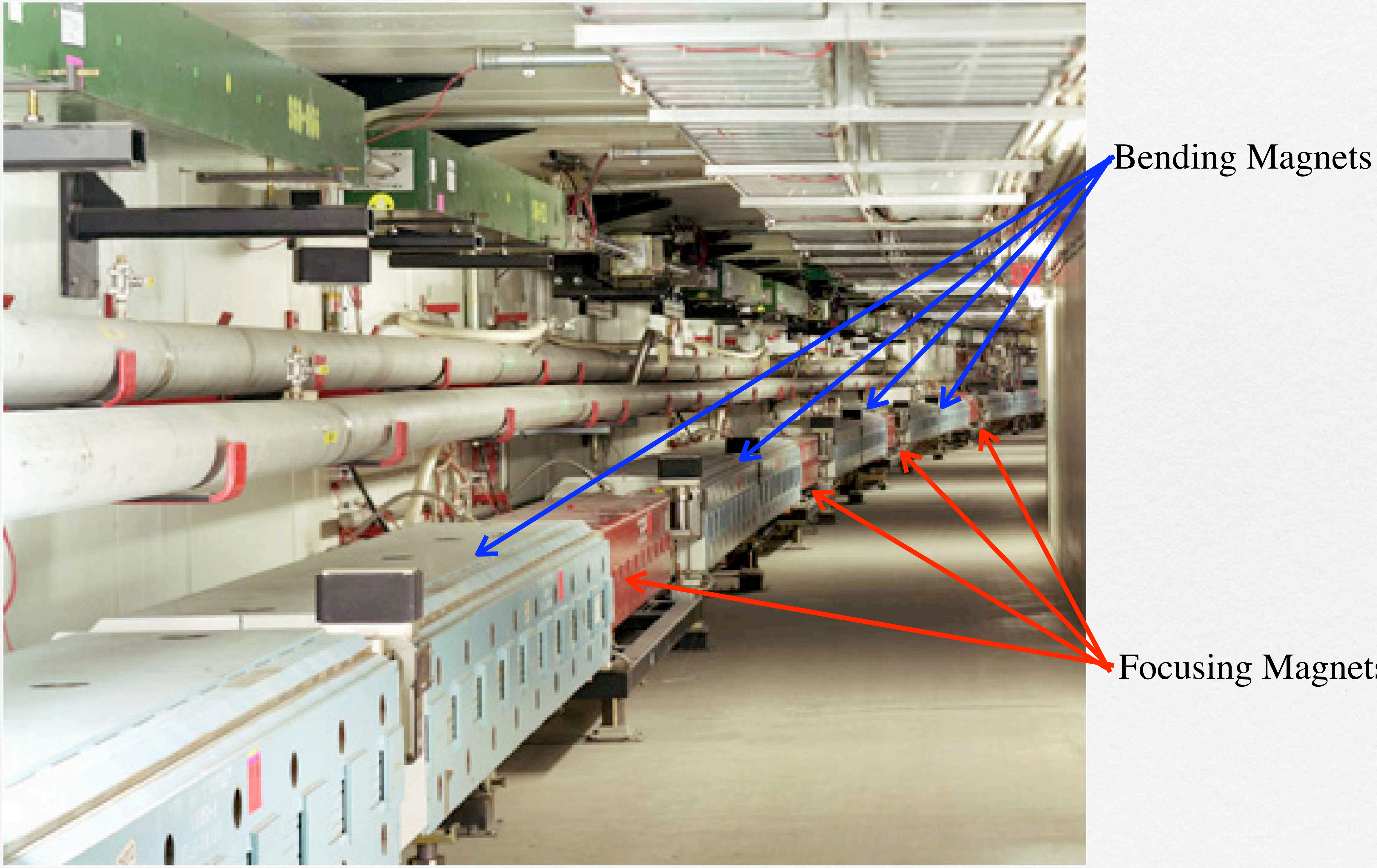


Fermilab Logo

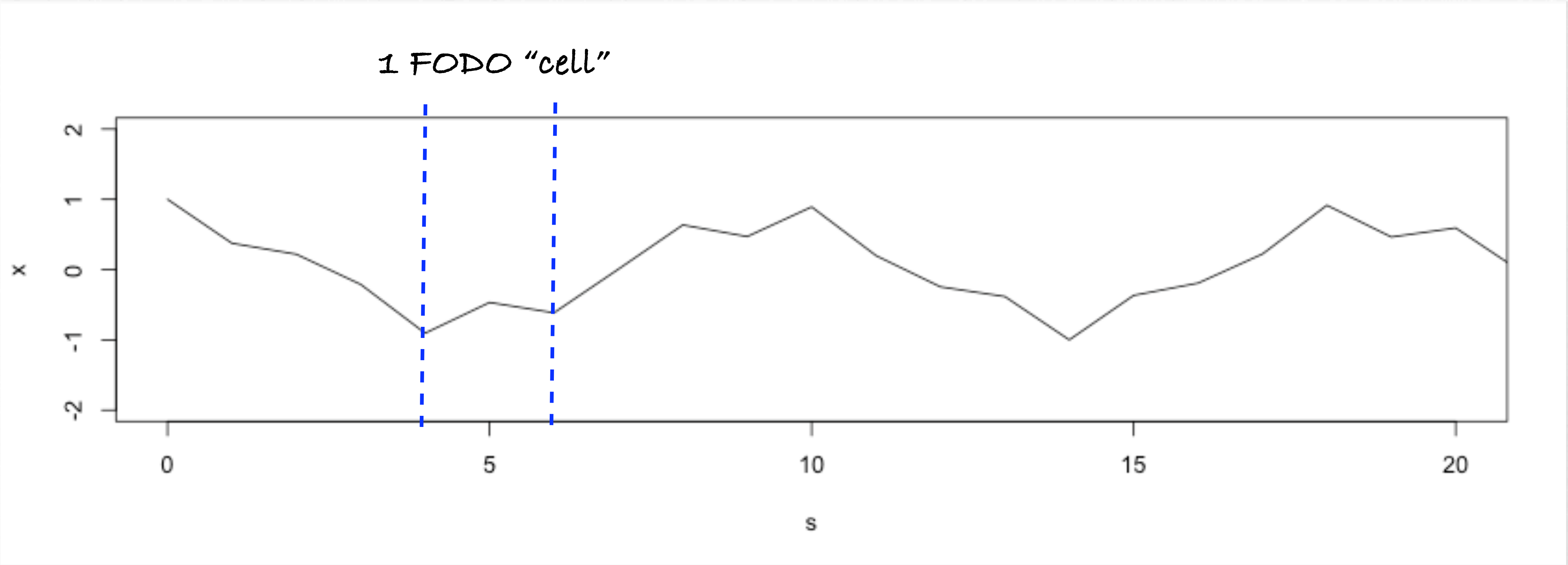
Tevatron: $B' = 77\ \mathrm{T/m}, \quad \ell = 1.7\ \mathrm{m} \rightarrow F = 25\ \mathrm{m}$

and $L = 30$ m

Example: FNAL Main Injector



Particle Trajectories



- Analytical Description: $\frac{dx'}{ds} = \frac{d^2x}{ds^2} = -\frac{eB'(s)}{p}x$ $\left[K(s) = \frac{e}{p}\frac{\partial B_y}{\partial x}(s)\right]$
  - Equation of Motion: (Hill's Equation)

$$x'' + K(s)x = 0$$

  - Nearly simple harmonic; so, assume soln.: $x(s) = A\sqrt{\beta(s)}\sin[\psi(s) + \delta]$

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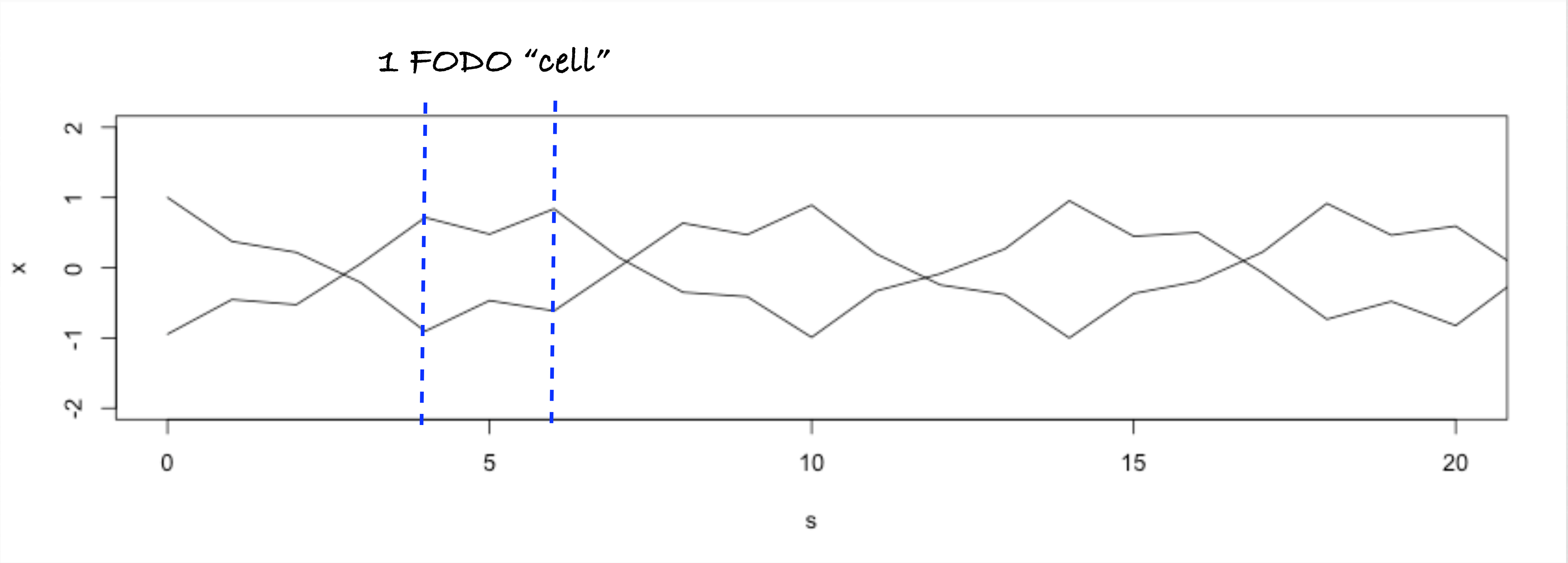

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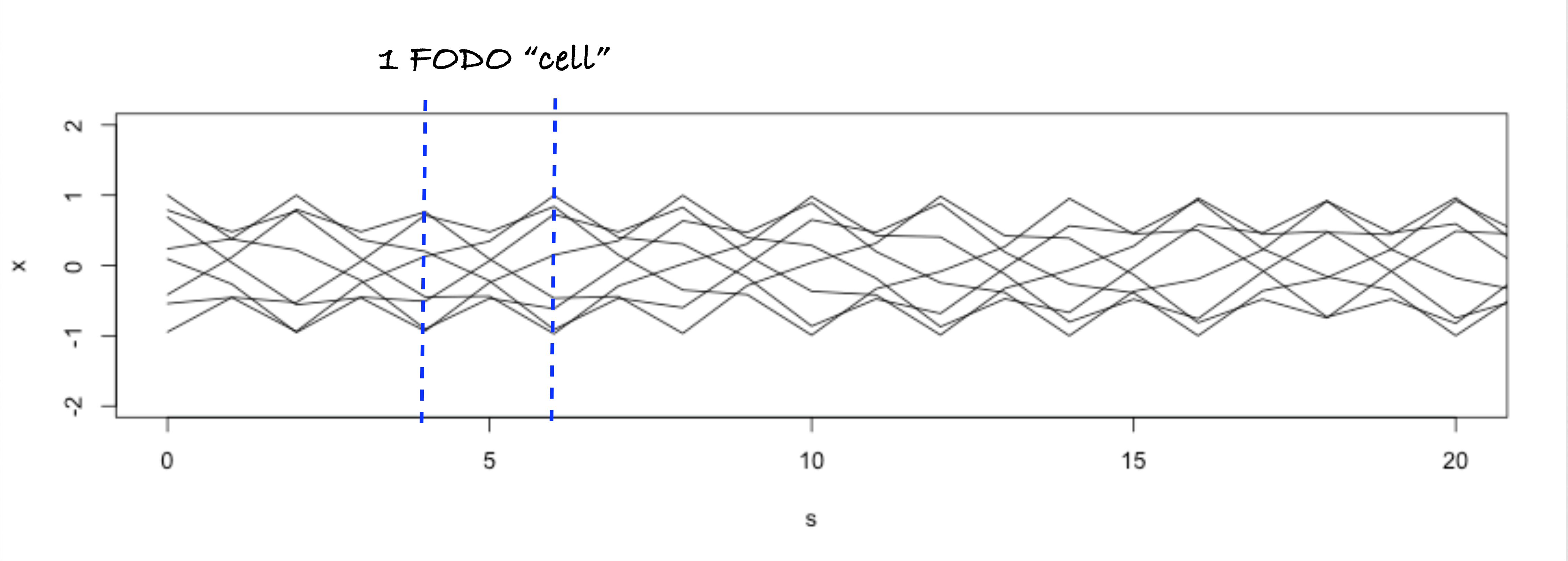

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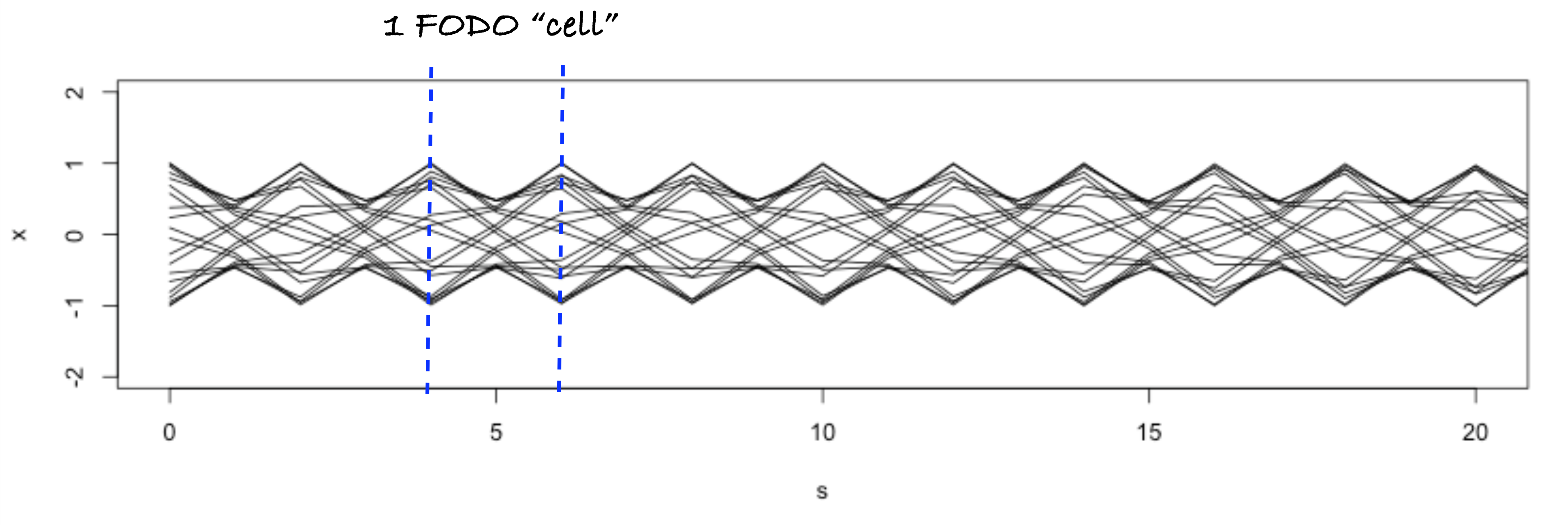

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  - Nearly simple harmonic; so, assume soln.: $x(s) = A\sqrt{\beta(s)}\sin[\psi(s)+\delta]$

Hill’s Equation and the “Beta Function”

- So, taking $x'' + K(s)x = 0$ and assuming $x(s) = A\sqrt{\beta(s)}\sin[\psi(s) + \delta]$
  - then, differentiating our solution twice, and plugging back into Hill’s Equation, we find that for arbitrary A, δ ...

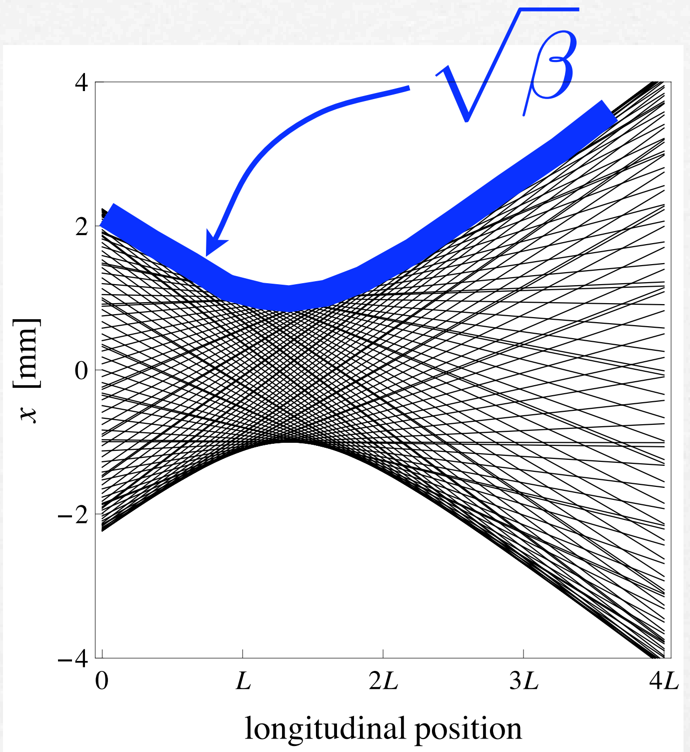
$$\begin{aligned} x'' + K(s)x &= A\sqrt{\beta}\left[\psi'' + \frac{\beta'}{\beta}\psi'\right]\cos[\psi(s)+\delta] \\ &\quad + A\sqrt{\beta}\left[-\frac{1}{4}\frac{(\beta')^2}{\beta^2} + \frac{1}{2}\frac{\beta''}{\beta} - (\psi')^2 + K\right]\sin[\psi(s)+\delta] = 0 \end{aligned}$$

- Since must have $\beta > 0$, first term --> $\psi''/\psi' = -\beta'/\beta \rightarrow$ $\boxed{\psi' = 1/\beta}$
- With this, the remaining term implies differential equation for β
  - which is, upon simplifying...

$$\boxed{\beta''' + 4K\beta' + \beta K' = 0}$$

Hill's Equation and Beta (cont'd)

- Typically, $dK/ds = 0$; so, $\beta''' + 4K\beta' = 0$
- In a "drift" region (no focusing), $\beta''' = 0$
  - Thus, beta function is a parabola in drift regions
    - If pass through a waist at $s = 0$, then, $\boxed{\beta(s) = \beta^* + \frac{s^2}{\beta^*}}$



- Through focusing region (quad, say), K = const

$$\beta'' + 4K\beta = const.$$

  - Thus, beta function is a sin/cos or sinh/cosh function, with an offset
    - "driven harmonic oscillator," with constant driving term
- So, optical properties of synchrotron (β) are now decoupled from particle properties (A, δ) and accelerator can be designed in terms of optical functions; beam size will be proportional to $\beta^{1/2}$

Tune

- Since $x(s) = A\sqrt{\beta(s)}\sin[\psi(s)+\delta]$ and $\psi' = 1/\beta$,

  then the total phase advance around the circumference is given by

$$\psi_{tot} \equiv 2\pi\nu = \oint \frac{ds}{\beta}$$

Note: β is "local wavelength/2π"

The tune, ν, is the number of transverse "betatron oscillations" per revolution. The phase advance through one FODO cell is given by

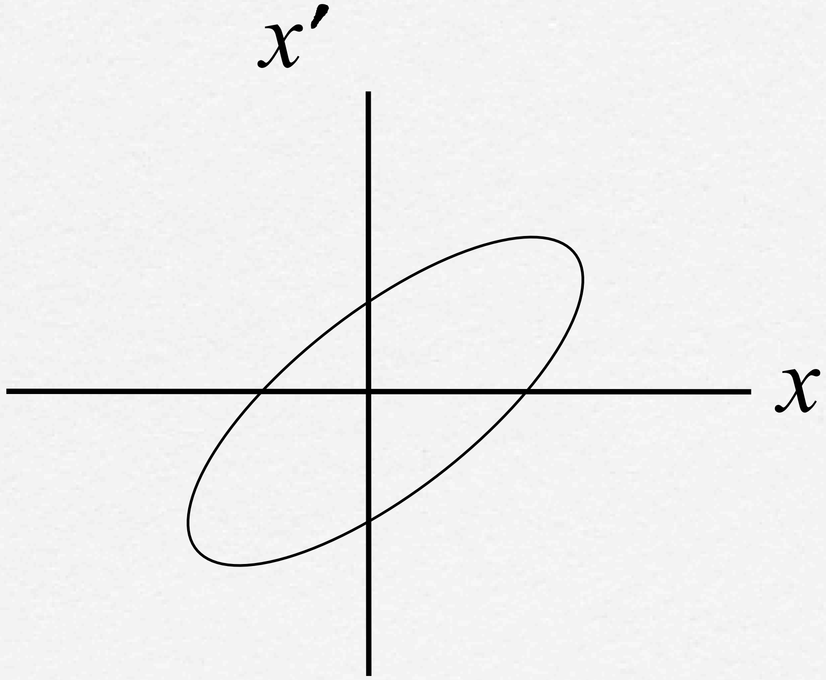
$$\psi_{cell} = 2\sin^{-1}\left(\frac{L}{2F}\right)$$

For Tevatron, $L/2F$ = 0.6, and since there are about 100 cells, the total tune is about 100 x (2 x 0.6)/2π ~ 20. The LHC tunes will be ~60.

- The function β both determines the envelope and amplitude of transverse motion, as well as the scale of the oscillation period, or wavelength

Emittance

- Just as in longitudinal case, we look at the phase space trajectories, here using transverse displacement and angle, x-x′, in transverse space.
- Viewed at one location, phase space trajectory of a particle is an ellipse:



$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$$

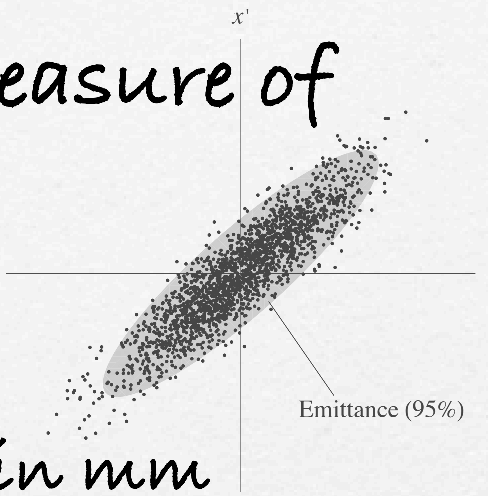
Here, $$\alpha \equiv -\frac{1}{2}\beta'$$ $$\gamma \equiv \frac{1+\alpha^2}{\beta}$$

α, β, γ are the *Courant-Snyder* parameters, or Twiss parameters

While β changes along the circumference, the area of the phase space ellipse = πA^2, and is independent of location!

So, define emittance, ϵ, of the beam as area of phase space ellipse containing some particular fraction of the particles (units = mm-mrad)

Emittance (cont'd)

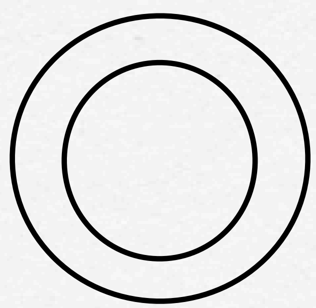


- Emittance of the particle distribution is thus a measure of beam quality.
  - At any one location... $\langle x^2 \rangle^{1/2}(s) = \sqrt{\epsilon\beta(s)/\pi}$
  - note: if β in m, ϵ in "π mm-mrad", then x will be in mm
- Variables x, x' are not canonical variables; but x, p_x are; the area in x-p_x phase space is an adiabatic invariant; so, define a *normalized emittance* as $\epsilon_N = \epsilon \cdot (\gamma v/c)$
- The normalized emittance should not change as we make adiabatic changes to the system (e.g., accelerate). Thus, beam size will shrink as $p^{-1/2}$ during acceleration.

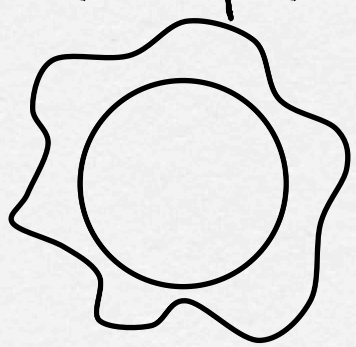
Effects due to Momentum Distribution

- Beam will have a distribution in momentum space
- Orbits of individual particles will spread out
  - B is constant; thus $\Delta R/R \sim \Delta p/p$
  - but, path is altered (focused) by the gradient fields…

Uniform field:



Synchrotron:



- These orbits are described by the Dispersion Function:

$$D(s) \equiv \Delta x_{c.o.}(s)/(\Delta p/p)$$

- Consequently, affects beam size:

$$\langle x^2 \rangle = \epsilon\beta(s)/\pi + D(s)^2 \langle (\Delta p/p)^2 \rangle$$

Chromaticity

- Focusing effects from the magnets will also depend upon momentum: $x'' + K(s,p)x = 0 \qquad K = e(\partial B_y(s)/\partial x)/p$
- To give all particles the same tune, regardless of momentum, need a "gradient" which depends upon momentum. Orbits spread out horizontally due to dispersion, can use a sextupole field:

$$\vec{B} = \frac{1}{2}B''[2xy\,\hat{x} + (x^2 - y^2)\,\hat{y}]$$

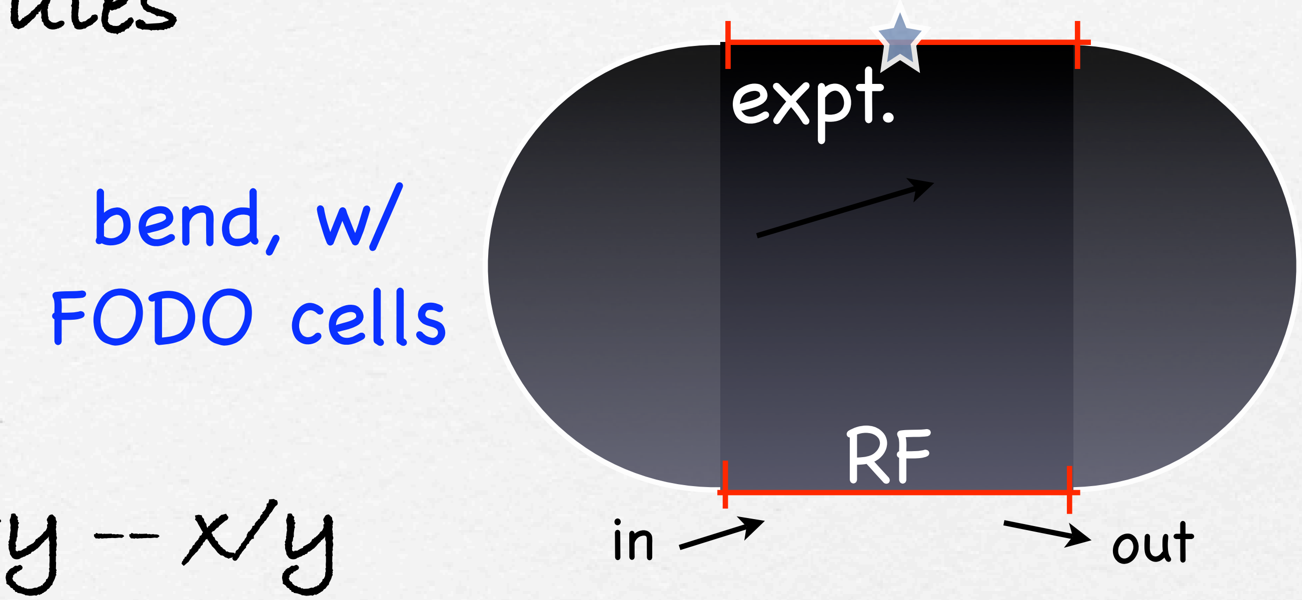
which gives $\partial B_y/\partial x = B''x = B''D(\Delta p/p)$

i.e., a field gradient which depends upon momentum

- Chromaticity is the variation of tune with momentum; use sextupole magnets to control/adjust; but, now introduces a nonlinear transverse field ... (*see part II!*)

Collider Accelerator Lattice

- can build up out of modules



- check for overall stability -- x/y
- meets all requirements of the program
  - Energy --> circumference, fields, etc.
  - spot size at interaction point: β min., $D=0$
  - etc...

FODO Cells (arcs)

$$\beta_{max,min} = 2F\sqrt{\frac{1 \pm L/2F}{1 \mp L/2F}}$$

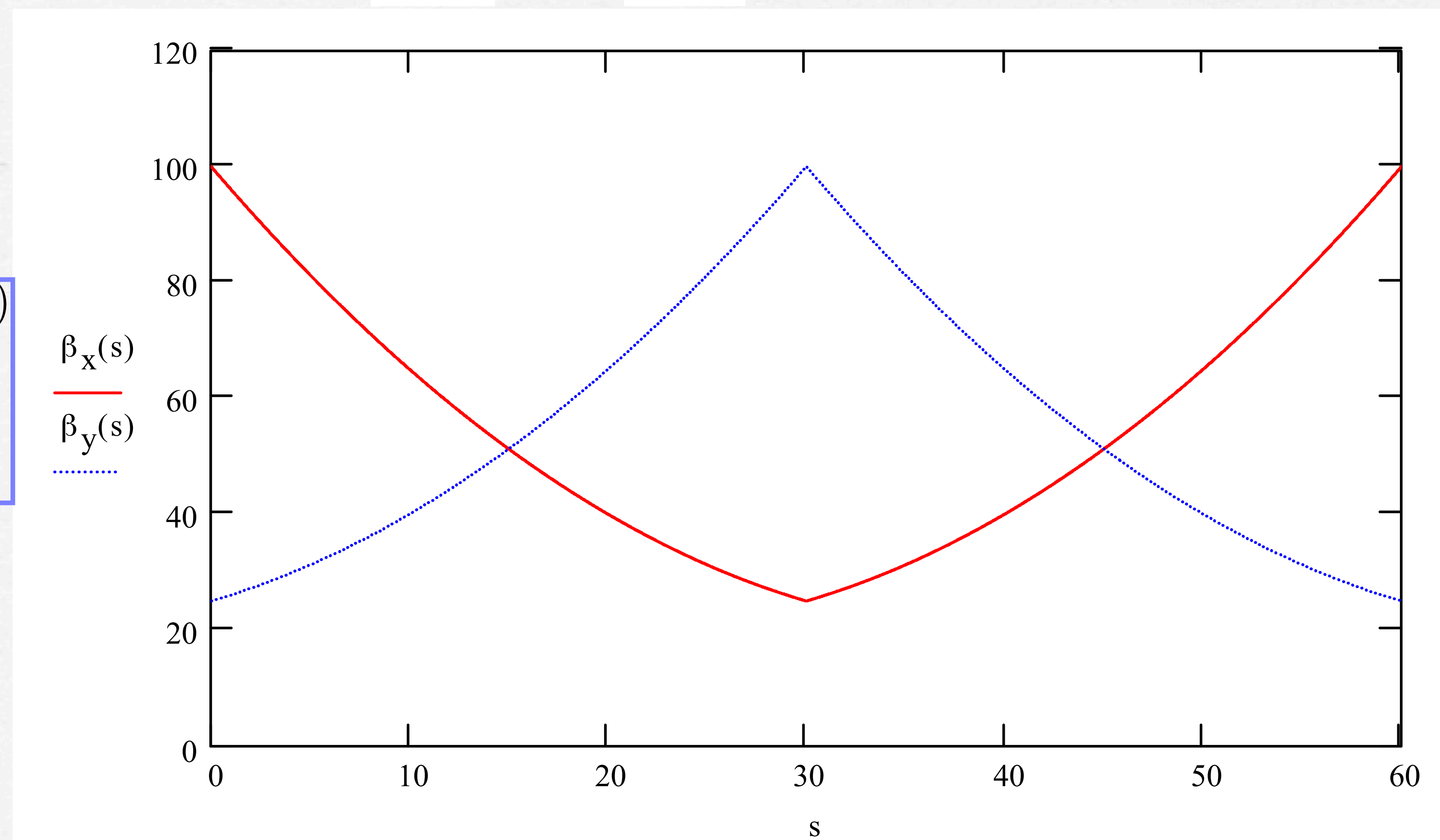
$$\Delta\beta' = \mp 2\beta/F$$ through a thin quad

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2$$ between quadrupoles

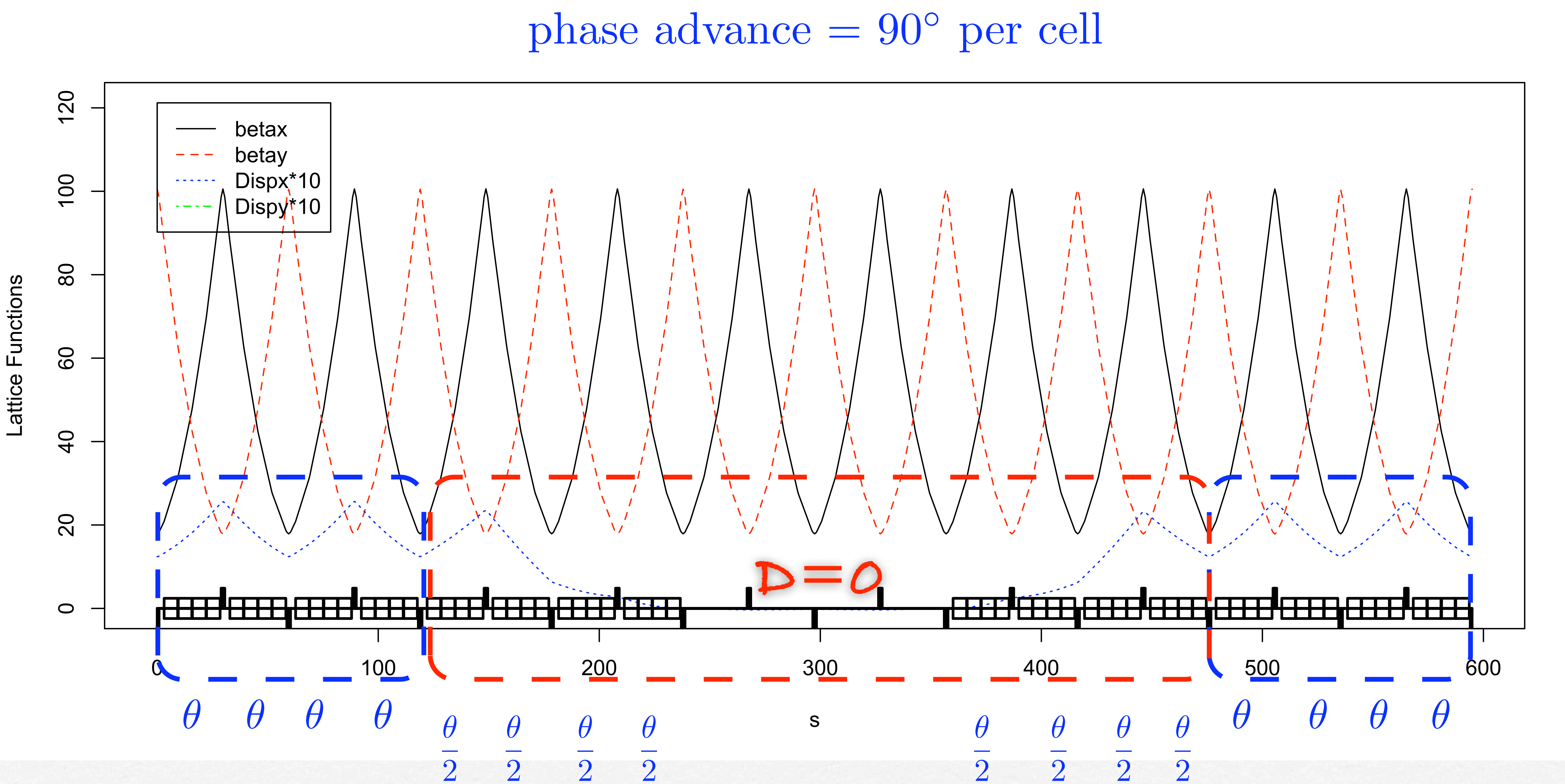
Ex: Tevatron Cell

$$\boxed{\begin{array}{l} \sin(\mu/2) = L/2F = 0.6 \longrightarrow \mu \approx 1.2(69^\circ) \\ \beta_{max} = 2(25\ \mathrm{m})\sqrt{1.6/0.4} = 100\ \mathrm{m} \\ \beta_{min} = 2(25\ \mathrm{m})\sqrt{0.4/1.6} = 25\ \mathrm{m} \\ \nu \approx 100 \times 1.2/2\pi \sim 20 \end{array}}$$

L = 30 F = 25

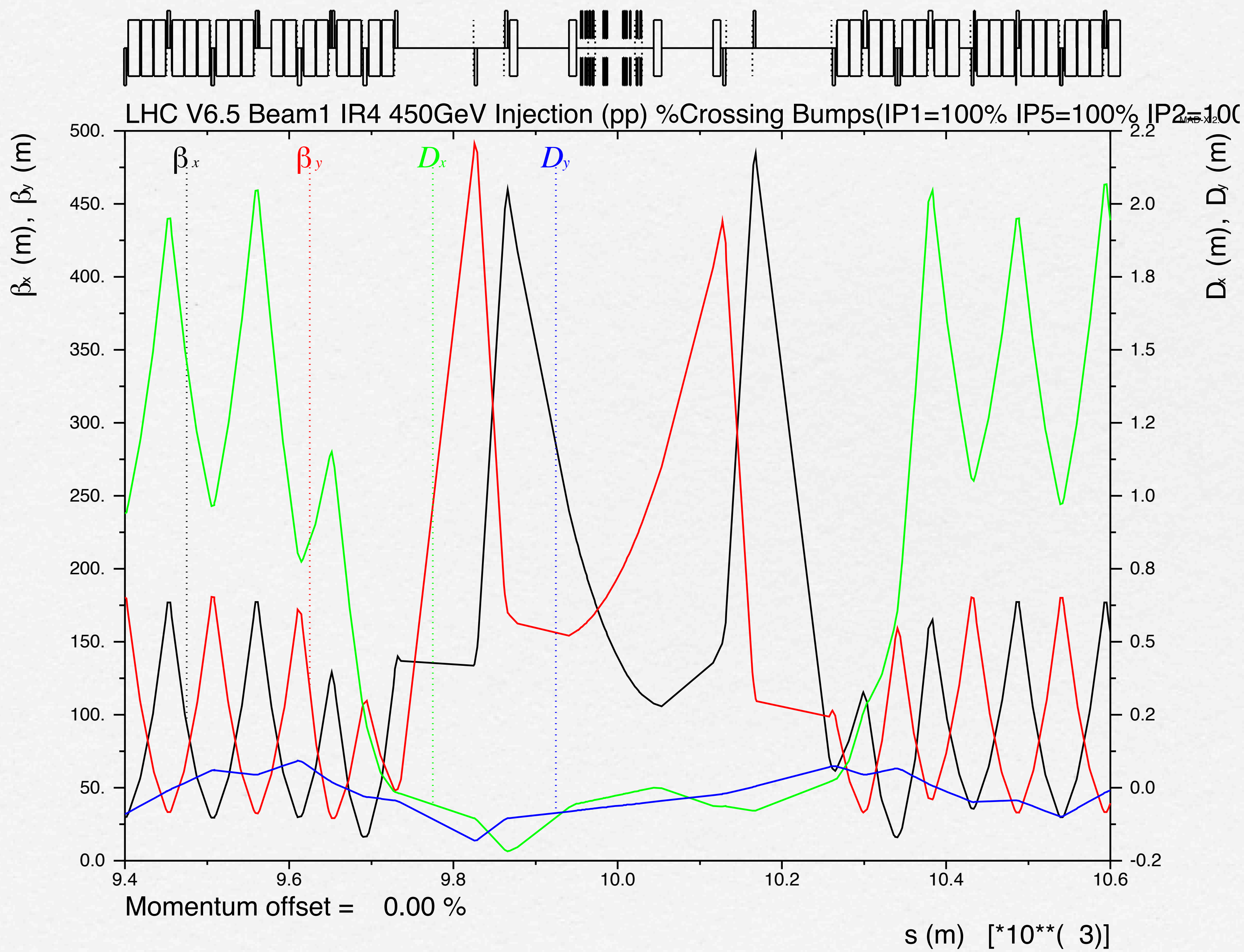


Dispersion Suppression



Long Straight Section

- a "matched insertion" that propagates the amplitude functions from their FODO values, through the new region, and reproduces them on the other side
- Here, we see an LHC section used for beam scraping



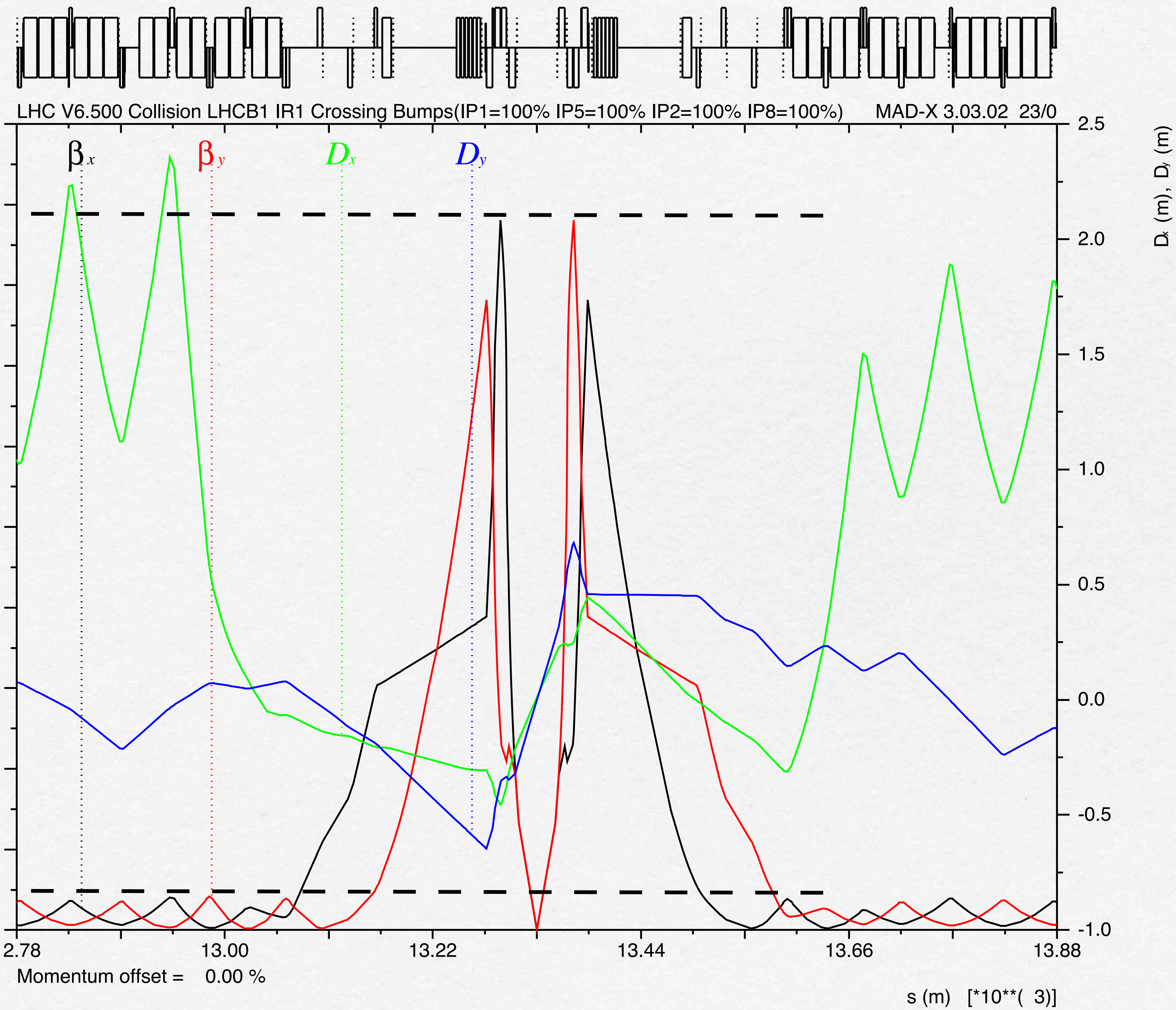
Interaction Region

LHC
high
lumi
IR

Note scales!

Triplets

FODO

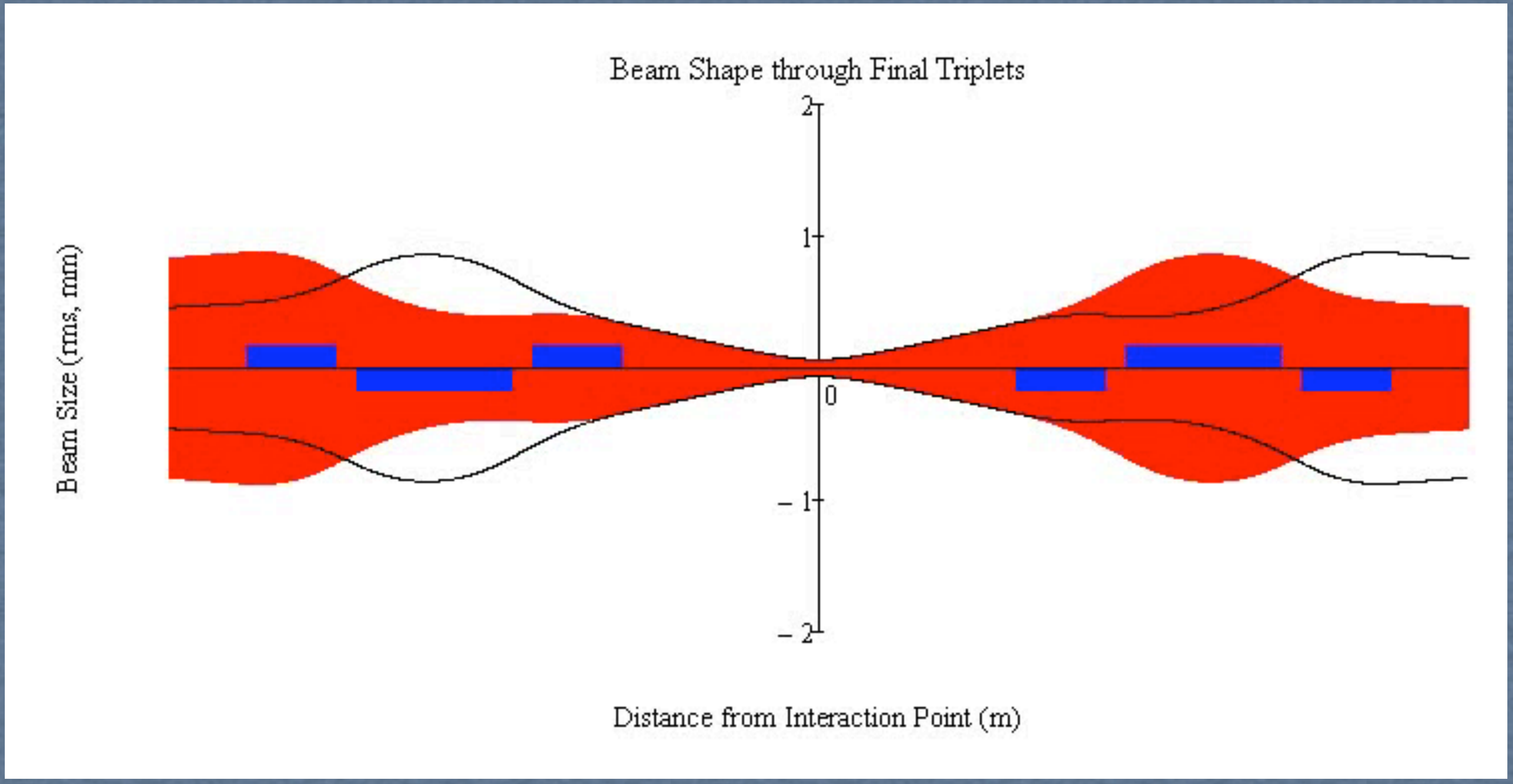


Low-Beta "Squeeze"

- A triplet of quadrupoles located on either side of detector region provide the final focusing of beam
- Triplet and other quadrupoles, located outside the region, used to adjust beam size at the focus

(Tevatron Example)

– quads outside of this region do most of work; like "changing the eyepiece" of a telescope to adjust magnification



Put it all Together

- make up a synchrotron out of FODO cells for bending, a few matched straight sections for special purposes…

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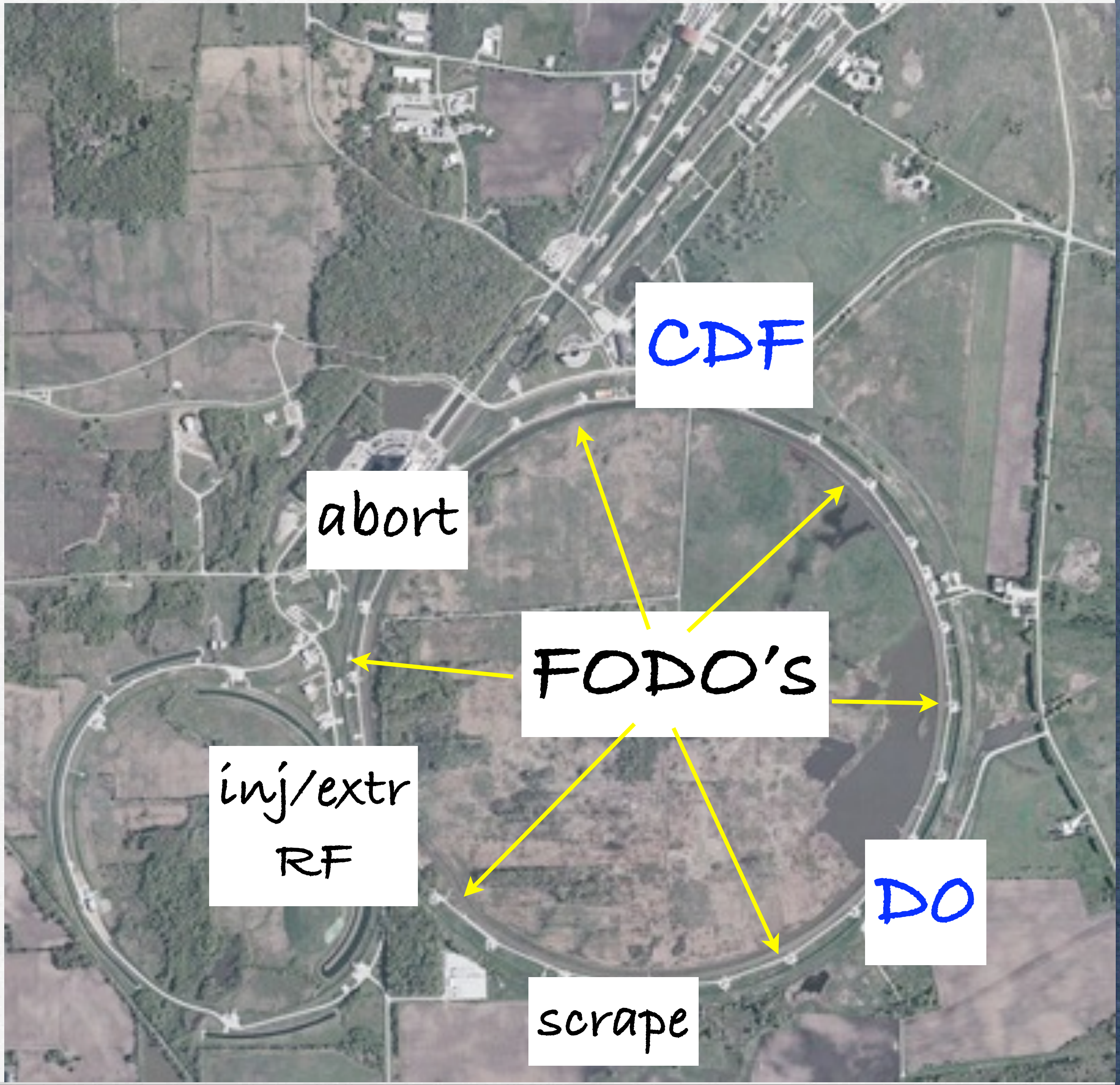
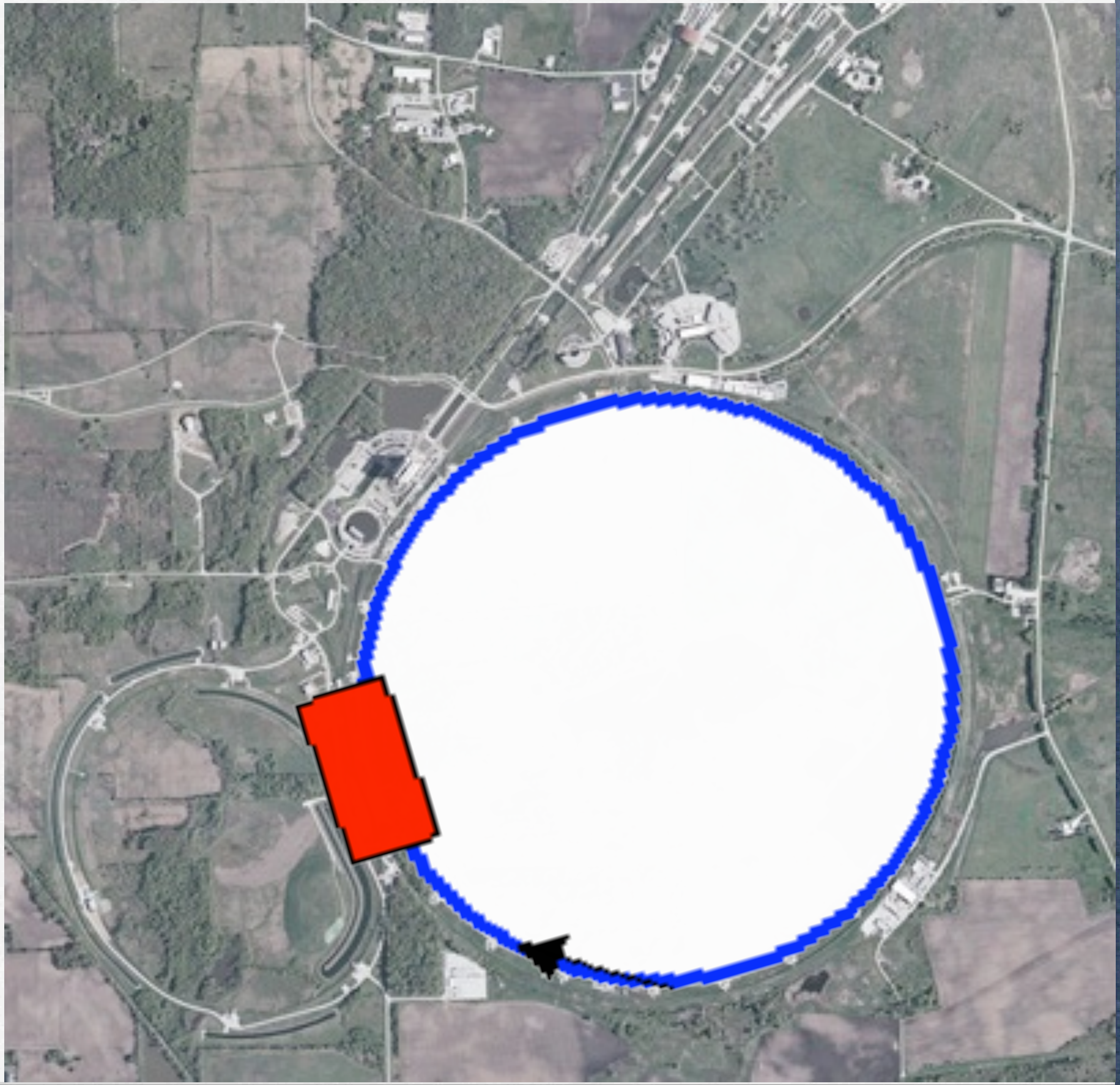


Put it all Together

- make up a synchrotron out of FODO cells for bending, a few matched straight sections for special purposes...



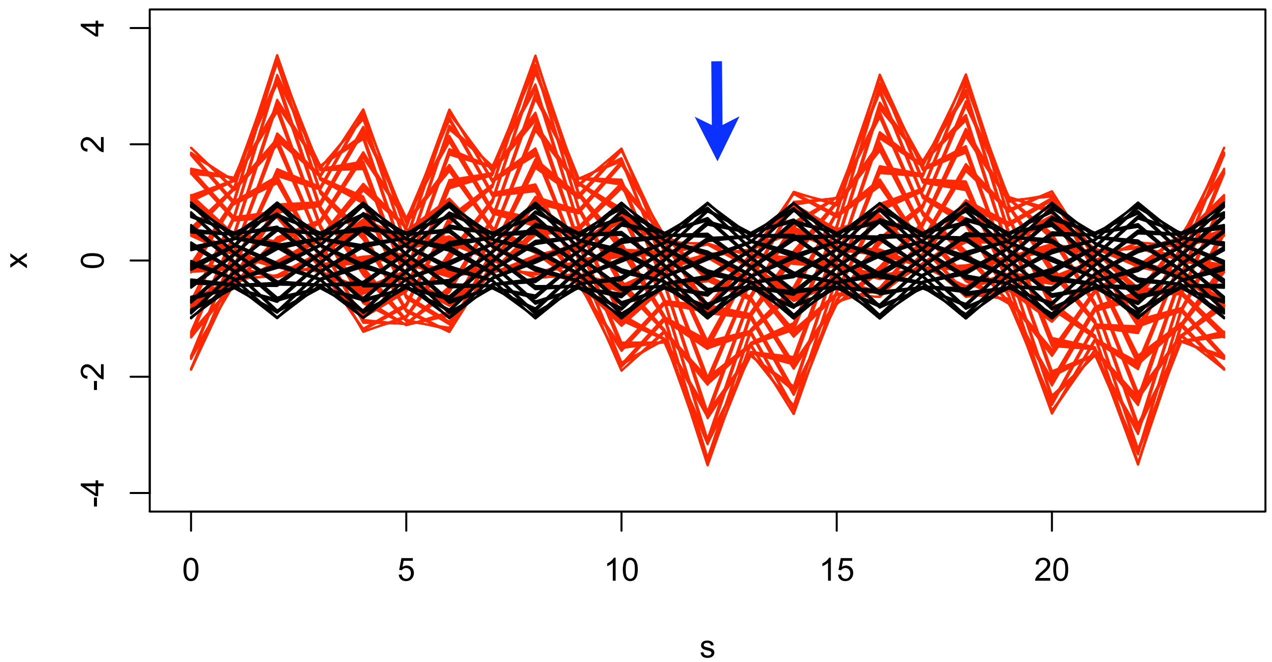
Part II...

Now, add more realism...

Corrections and Adjustments

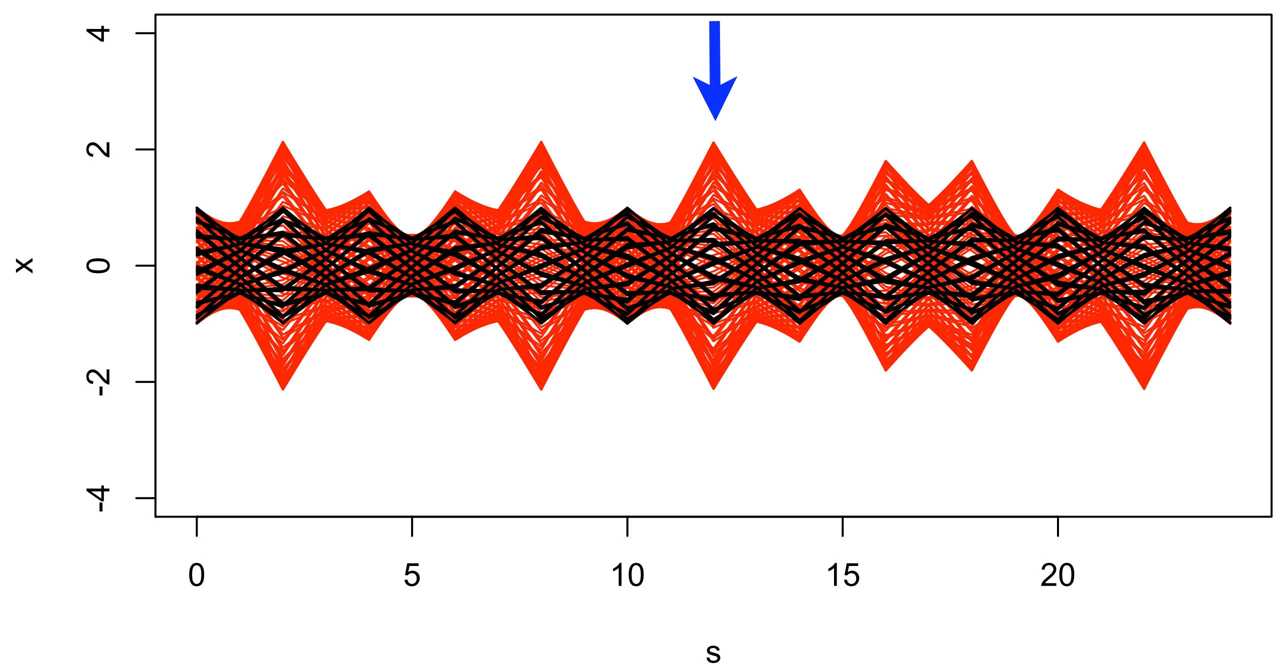
- Correction/adjustment systems required for fine control of accelerator:
  - correct for misalignment, construction errors, drift, etc.
  - adjust operational conditions, tune up
- typically, place correctors and instrumentation near quads -- "corrector package"
  - control steering, tunes, chromaticity, etc.

Linear Distortions



- ☐ Orbit distortion due to single dipole field error

- ☐ Envelope Error (Beta-beat) and tune shift due to gradient error

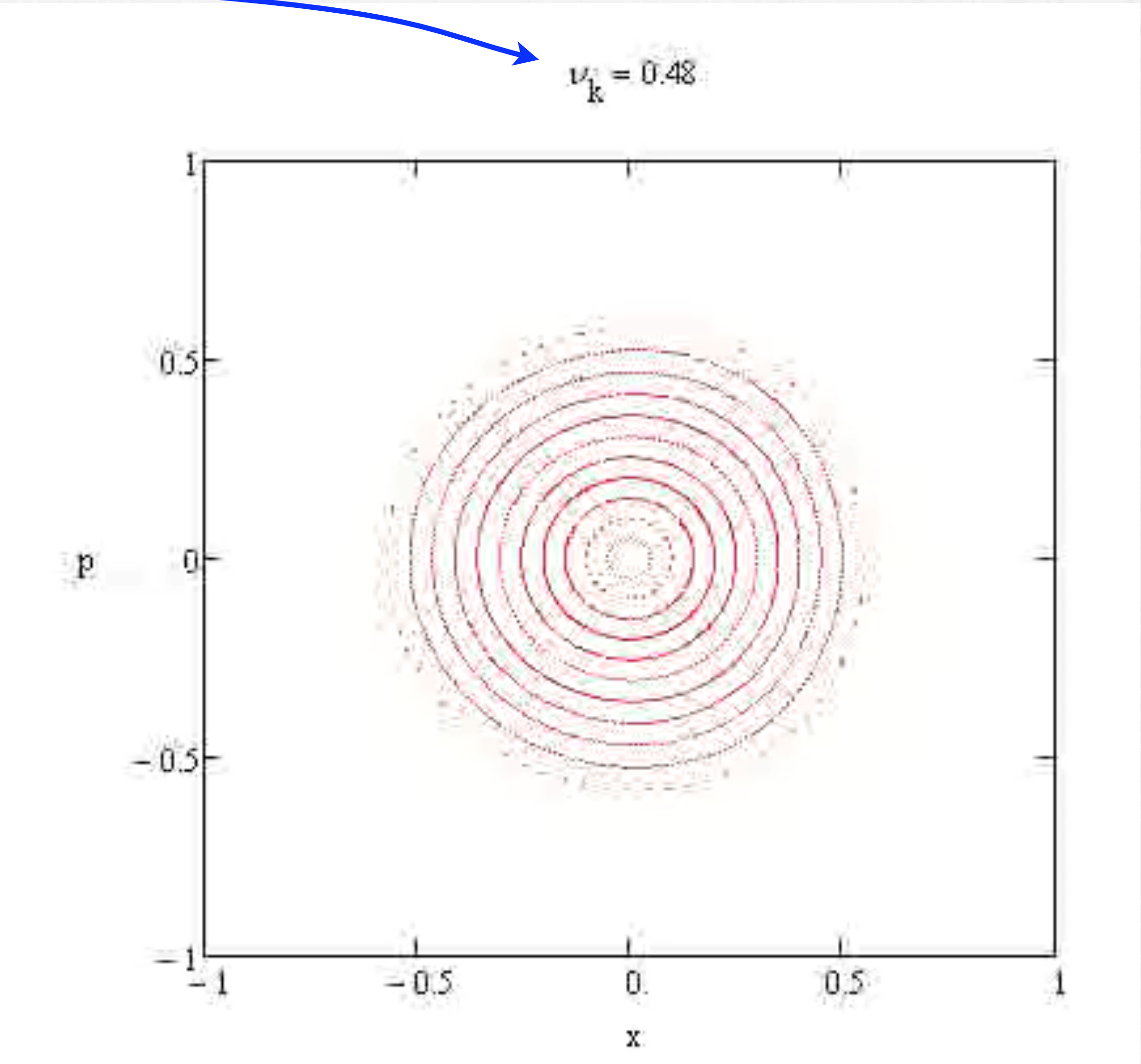


Resonances and Tune Space

- Error fields are encountered repeatedly each revolution -- can be resonant with tune
  - repeated encounter with a steering (dipole) error produces an orbit distortion: $$\boxed{\Delta x \sim \frac{1}{\sin \pi\nu}}$$
    - thus, avoid integer tunes
  - repeated encounter with a focusing (quad) error produces distortion of amplitude fcn:
    - thus, avoid half-integer tunes $$\boxed{\Delta\beta/\beta \sim \frac{1}{\sin 2\pi\nu}}$$

Nonlinear Resonances

- Phase space w/ sextupole field present ($\sim x^2$)
  - tune dependent:
  - "dynamic aperture"
- Thus, avoid tune values:
  - k, k/2, k/3, ...



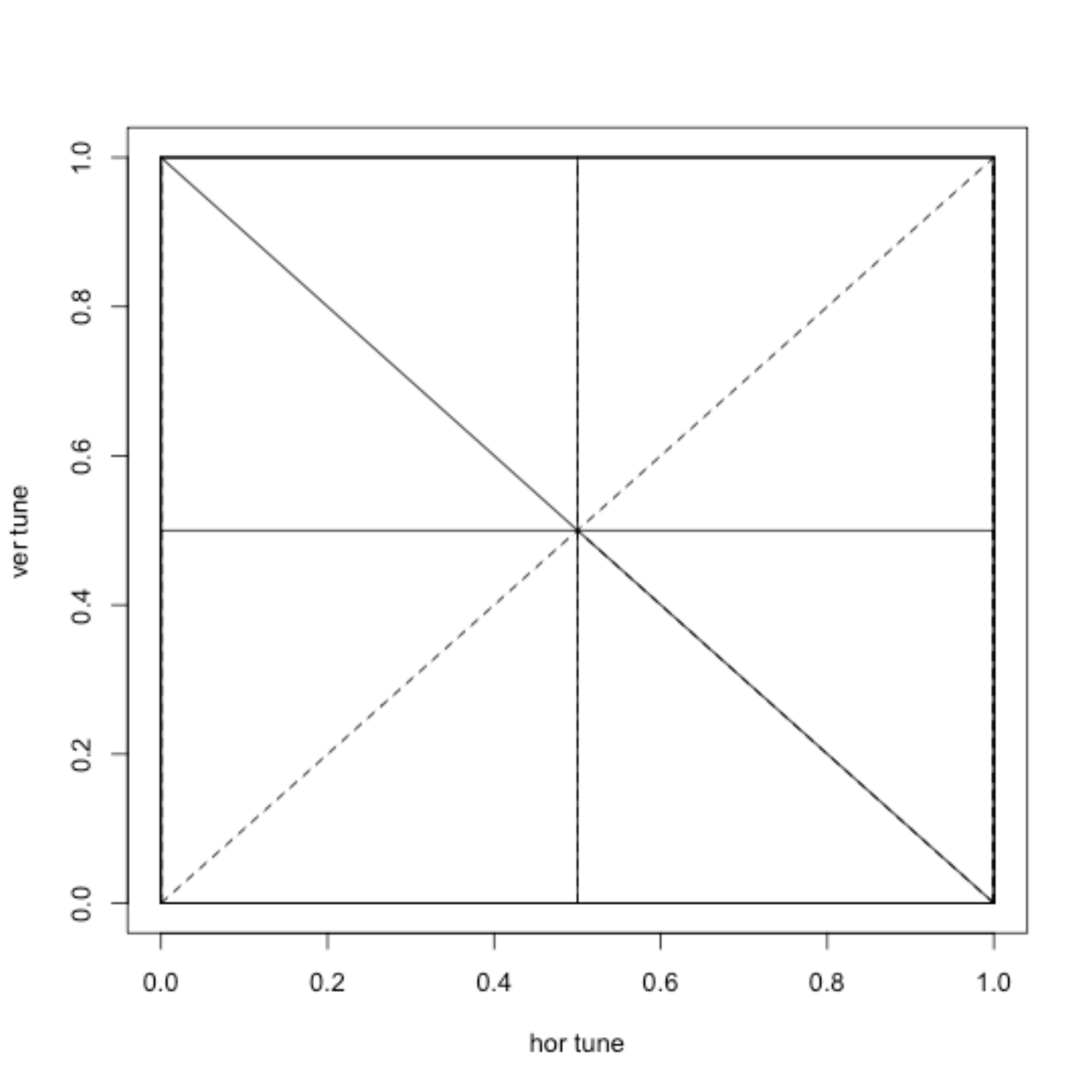
Tune Diagram

- Always "error fields" in the real accelerator
- Coupled motion also generates resonances (sum/difference resonances)
  - in general, should avoid: $m\,\nu_x \pm n\,\nu_y = k$

avoid ALL rational tunes???

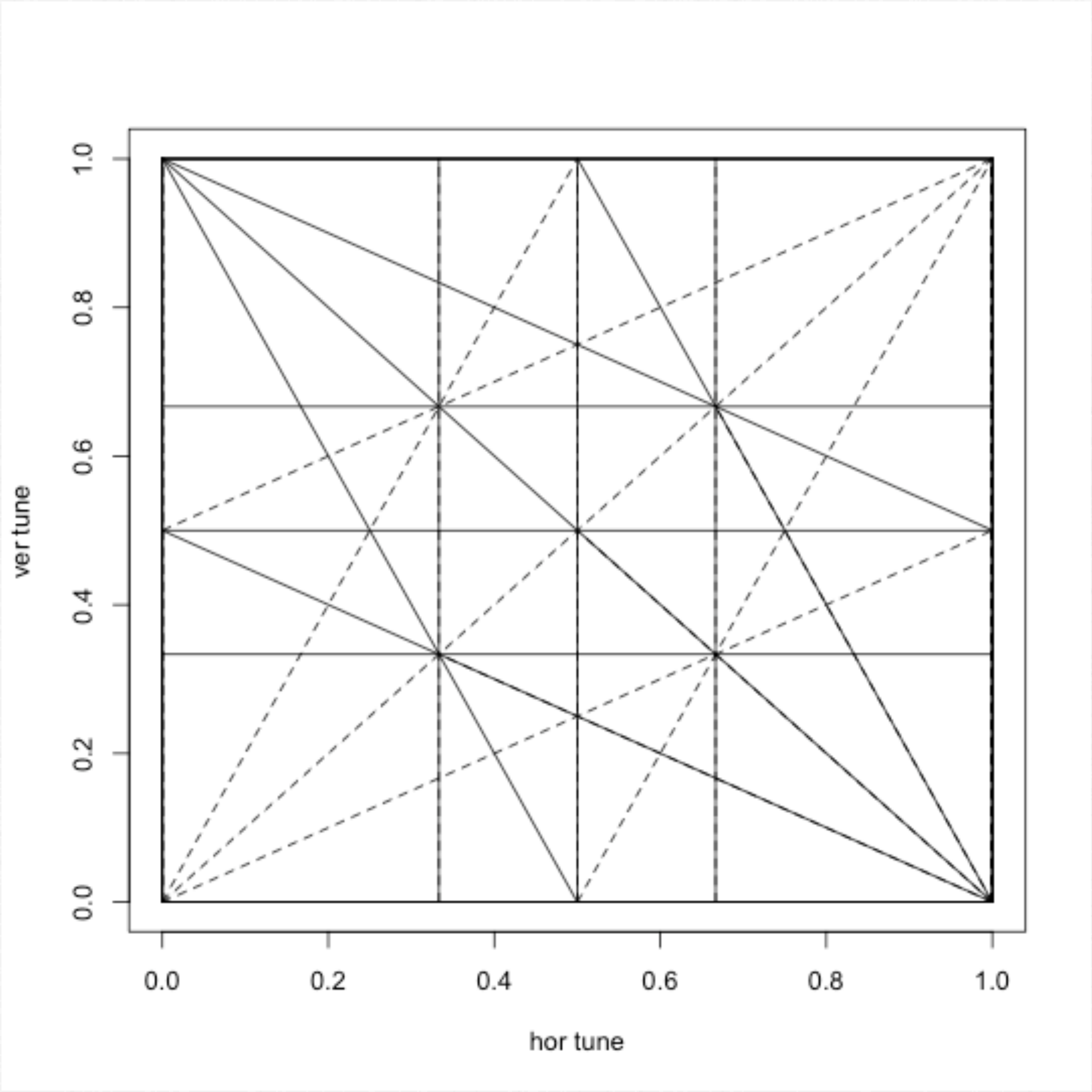
Tune Diagram

Through order
k =2

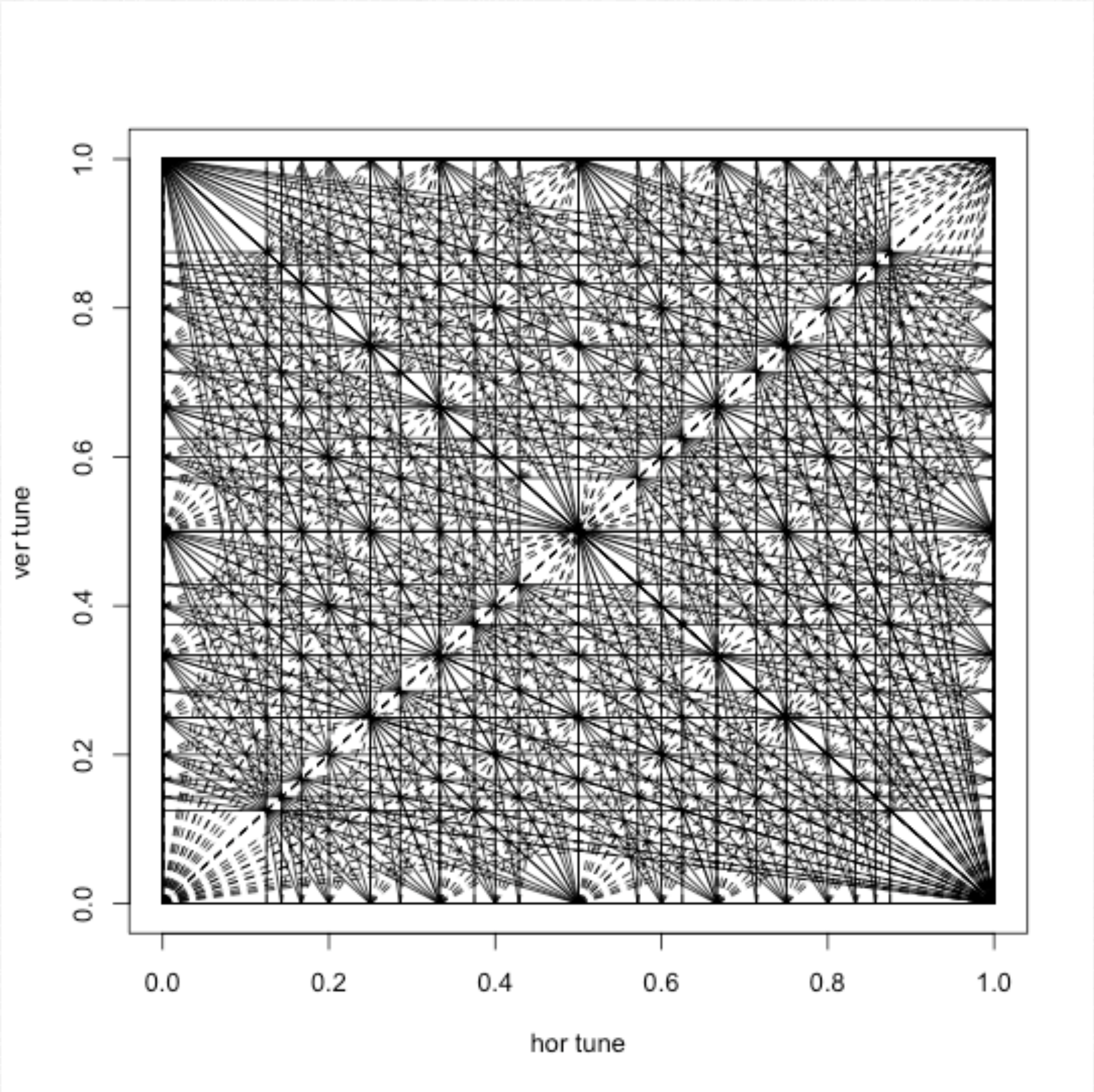


Tune Diagram

Through order
$k = 3$

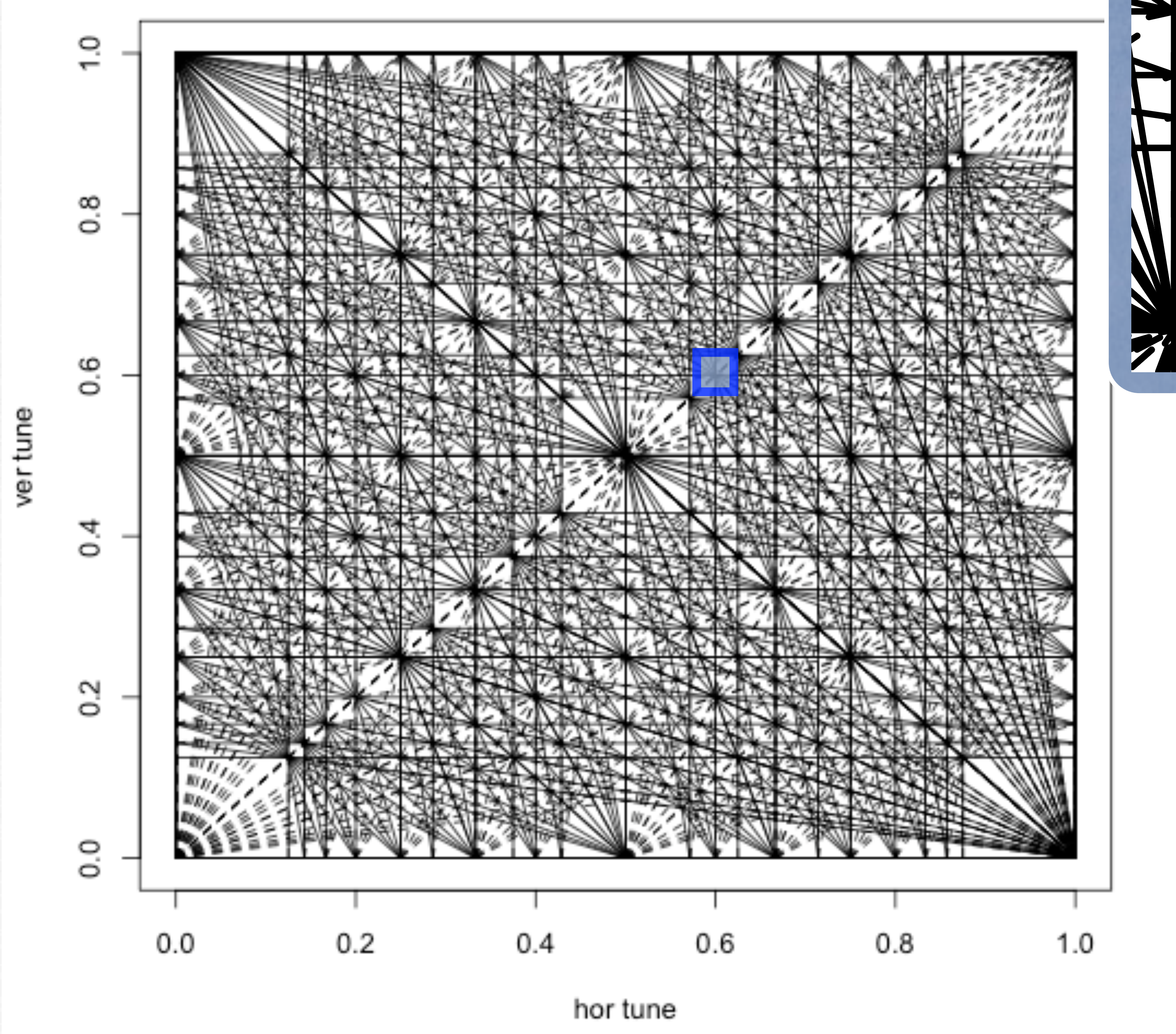


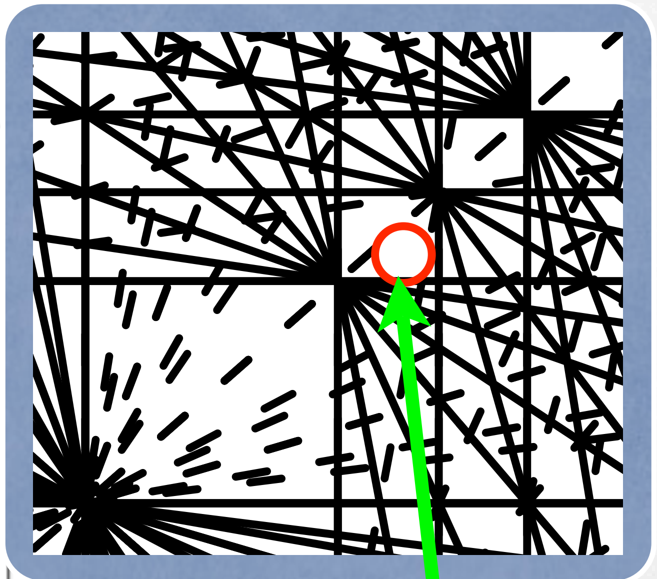
Tune Diagram



Through order
k =8

Tune Diagram



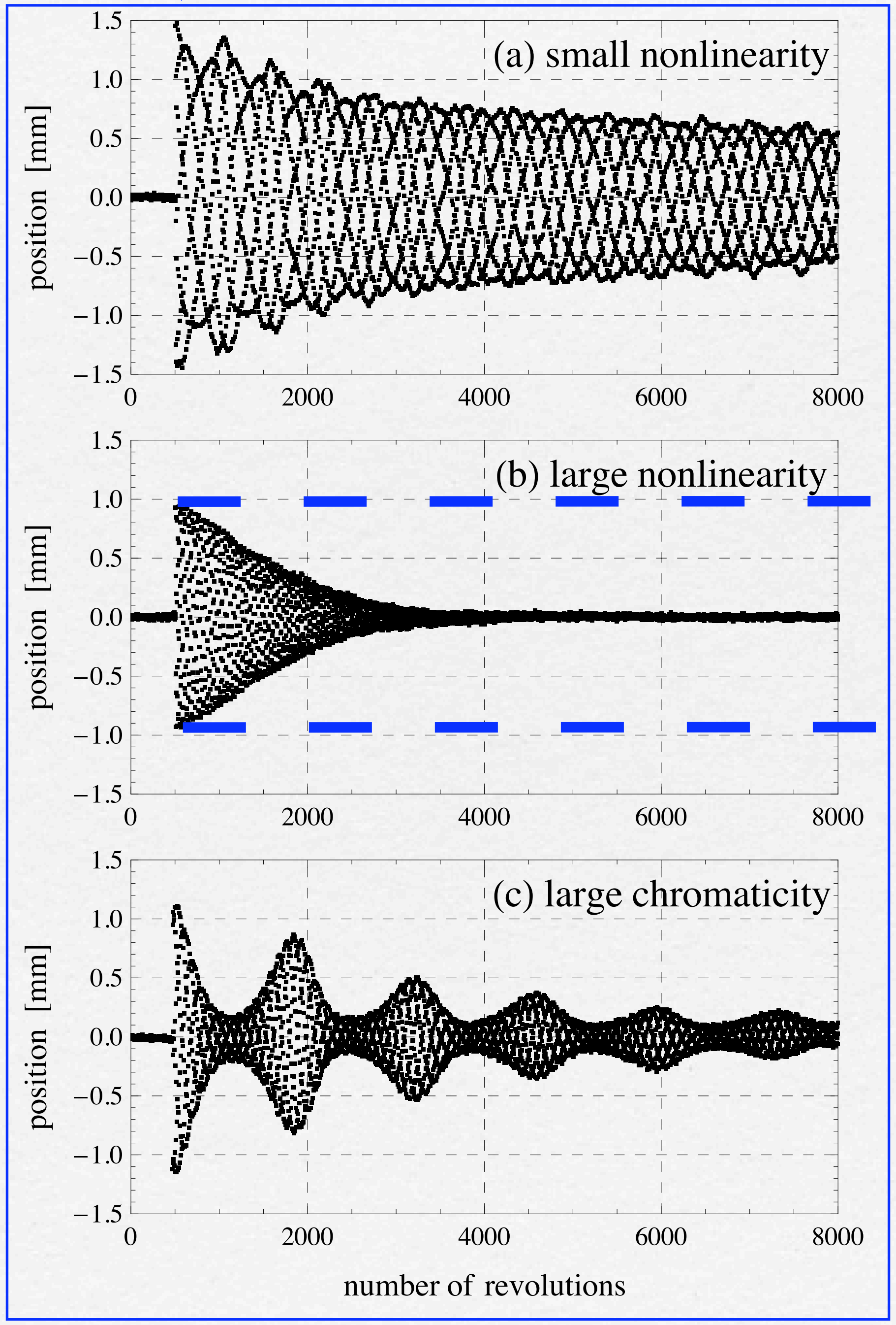


width ~ 0.025

Tune Spread

- momentum -- chromaticity
  - "natural"; field errors in magnets ~ x^2 where Disp.
- nonlinear tune spread
  - field terms ~ x^2, x^3, etc.
- --> "decoherence" of beam position signal

"kick" the beam



Beam-Beam Force

- As particle beams "collide" (very few particles actually "interact" each passage), the fields on one beam affect the particles in the other beam. This "beam-beam" force can be significant.
  - on-coming beam can act as a "lens" on the particles, thus changing focusing characteristics of the synchrotron, tunes, etc.

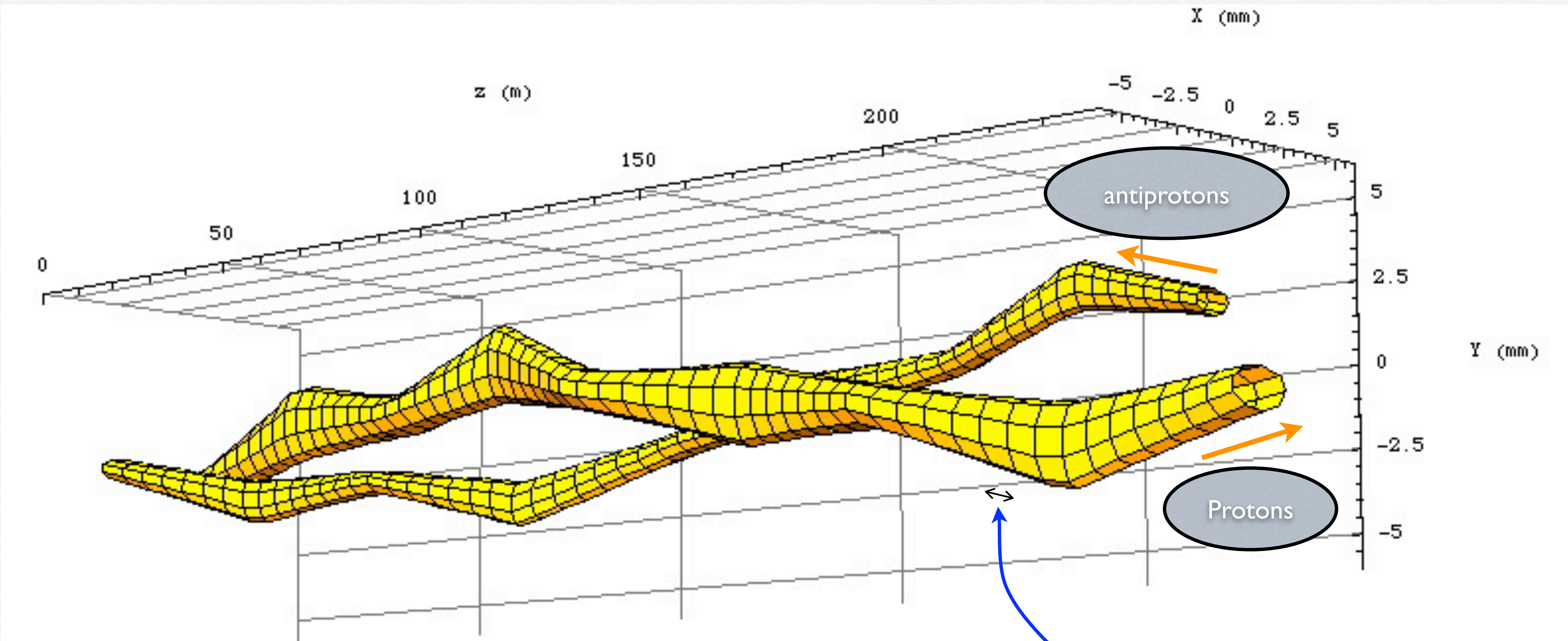
$$\boxed{\text{Force} \propto \frac{1-e^{-x^2/2\sigma^2}}{x} \approx \frac{x}{2\sigma^2} \quad \text{, for small } x}$$

- Head-On: core sees ~linear force; rest of beam, nonlinear force --> tune spread, nonlinear resonances, etc.
- Long-Range: force ~ 1/r --> for large enough separation, mostly coherent across the bunch, but still some nonlinearity
- Bunch structure (train) means some bunches will experience different effects, increasing the tune spread, etc., of the total beam

Beam-Beam Mitigation

- Beams are "separated" (if not in separate rings of magnets) by electrostatic fields so that the bunches interact only at the detectors
  - "Pretzel" or "helical" orbits separate the beams around the ring
  - However, the "long-range" interactions can still affect performance
  - new "electron lenses" and current-carrying wires are being investigated which can mitigate the effects of beam-beam interactions, both head-on and long-range

Tevatron: 2 Beams in 1 Pipe



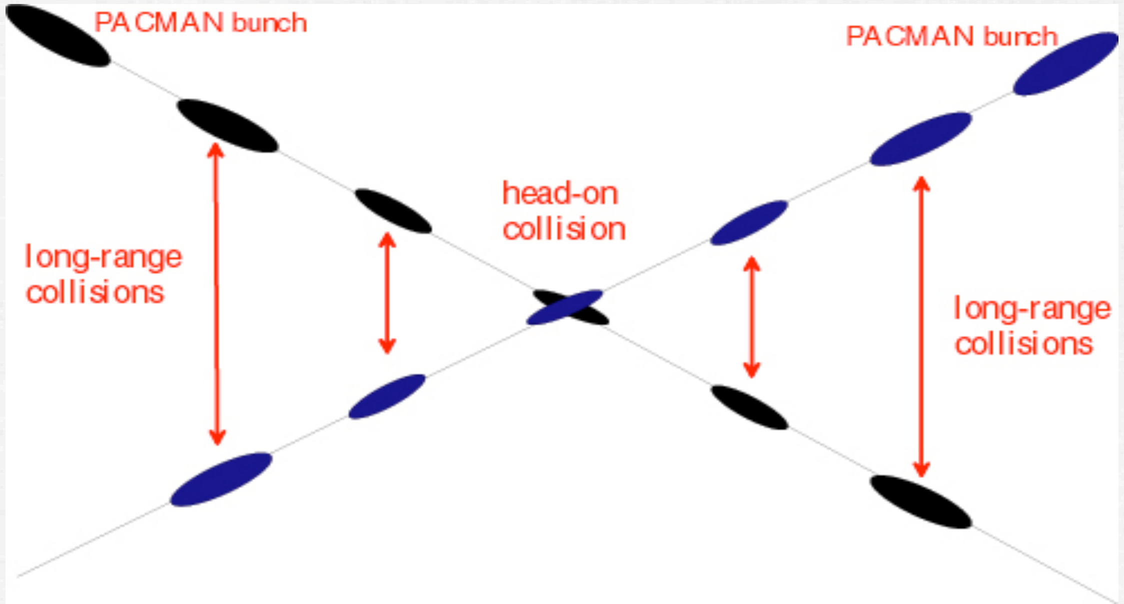
bunch length

Helical orbits through 4 standard arc cells of the Tevatron

LHC: 2 Beams in 2 Pipes

- Across each interaction region, for about 120 m, the two beams are contained in the same beam pipe
- This would give ~ 30 bunch interactions through the region
- Want a single Head-on collision ***at*** the IP, but will still have long-range interactions on either side
- Beam size grows away from IP, and so does separation; can tolerate beams separated by ~10 sigma

$$d/\sigma = \theta \cdot (\beta^*/\sigma^*) \approx 10$$
$$\longrightarrow \qquad \theta = 10 \cdot (0.017)/(550) \approx 300\ \mu\text{rad}$$



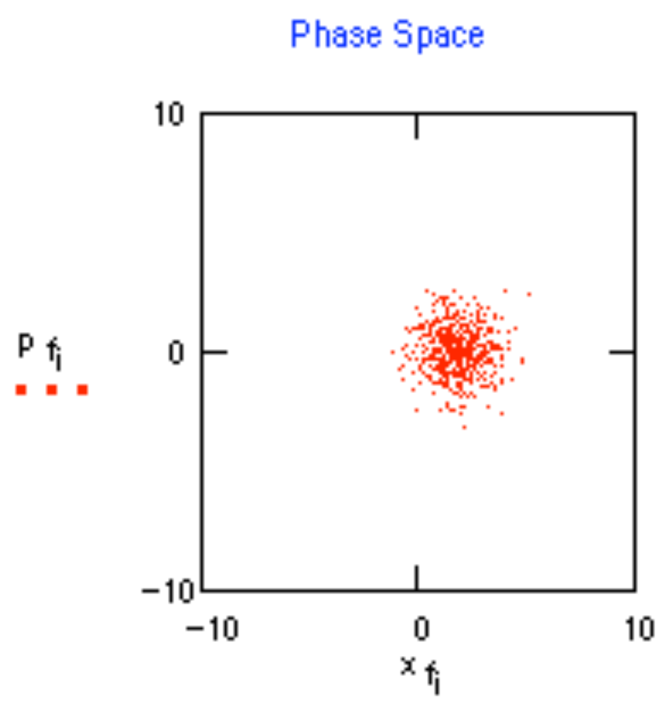
Emittance Control

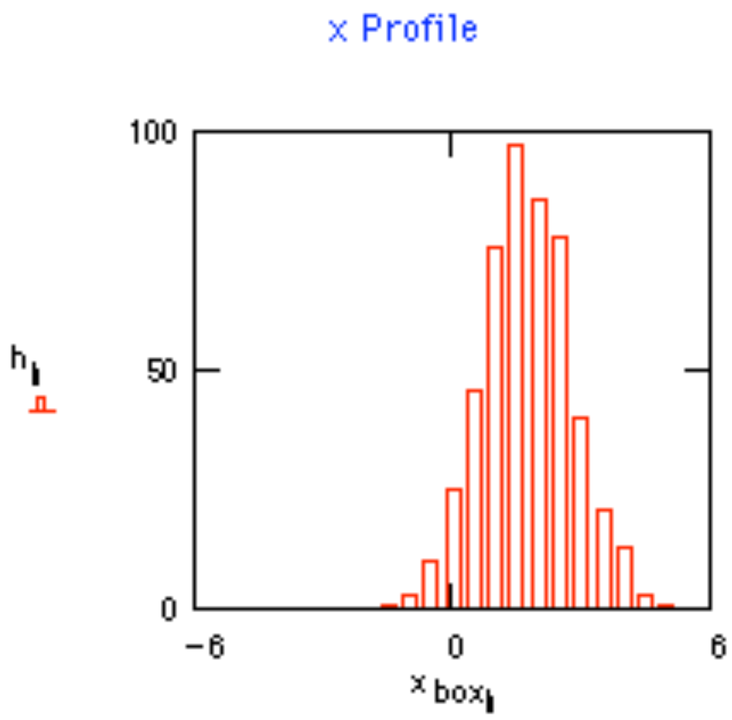
- Electrons radiate extensively at high energies; combined with energy replenishment from RF system, small equilibrium emittances result
  - in Hadron Colliders; ϵ at collision energy determined by proton source, and its control through the injectors
- larger emittance -- smaller luminosity
- larger emittance growth rates during collisions result in particle loss
  - less particles for luminosity!

Injection Errors

- Emittance growth from trajectory errors at injection -- more sensitive at higher energy injection (beam size is smaller)
- Similarly, energy/phase mismatch at injection (injection into "center" of buckets)
- damper systems
  - fast corrections of turn-by-turn trajectory
  - correct offsets before "decoherence" sets in

Decoherence and Emittance Growth





$\mathrm{mean}(x_f) = 1.985$ $\qquad$ $\mathrm{stdev}(x_f) = 1.039$

Emittance Increase: $\mathrm{stdev}(x_f)^2 = 1.08$

Predicted "typical" values: (Steering Mismatch) $1 + \frac{1}{2}\cdot\Delta x^2 = 3$

FRAME = 0 (Amplitude function Mismatch) $\dfrac{r_\beta^2 + 1}{2\cdot r_\beta} = 1$

Diffusion

- Random sources (power supply noise; beam-gas scattering in vacuum tube; ground motion) will alter the oscillation amplitudes of individual particles
  - will grow like $\sqrt{N}$, amplitudes will eventually reach aperture
- Thus, beam lifetime will develop, affecting beam intensity, emittance, and thus luminosity

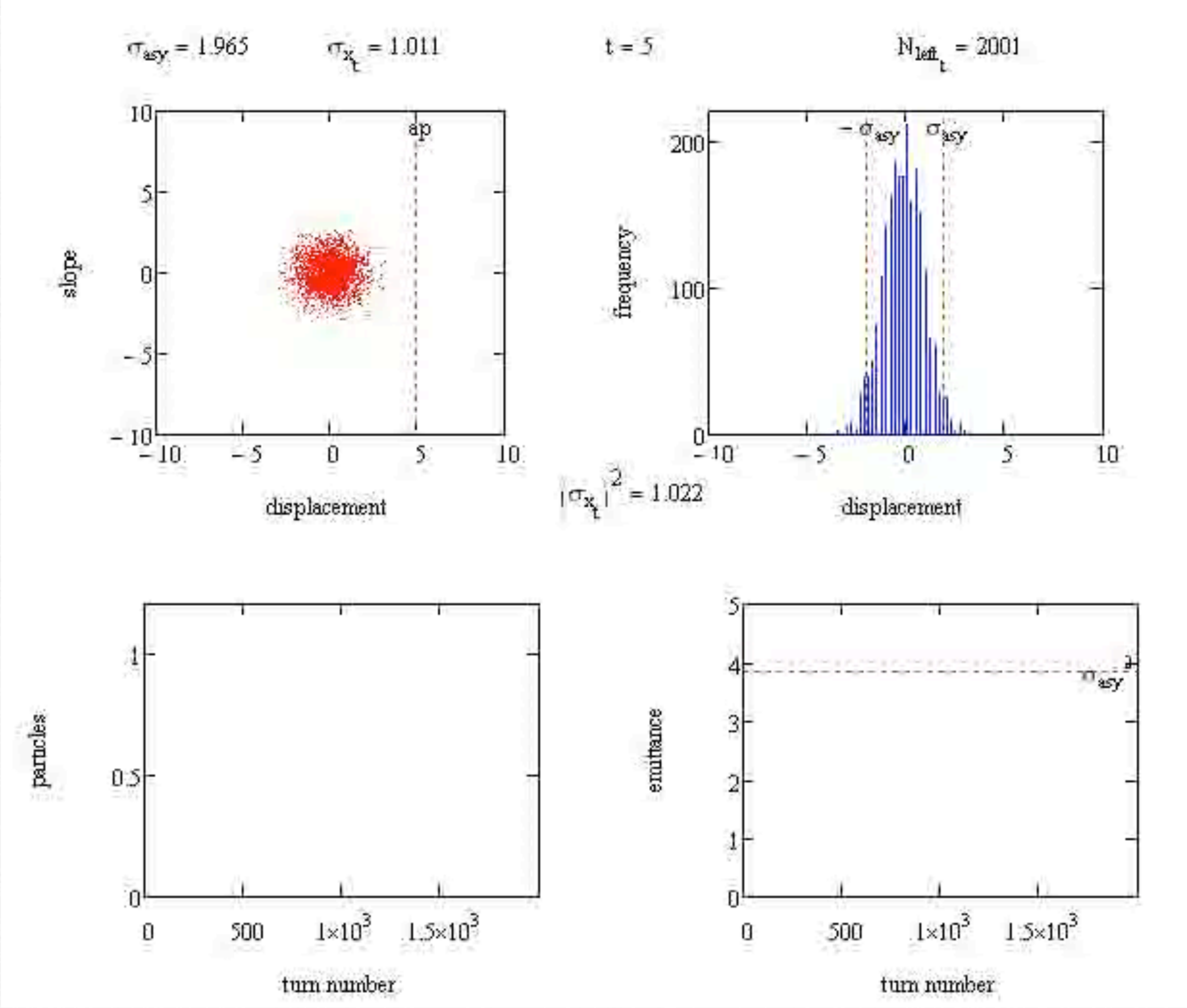
Diffusion Example

Phase Space

Beam Profile

Beam Intensity

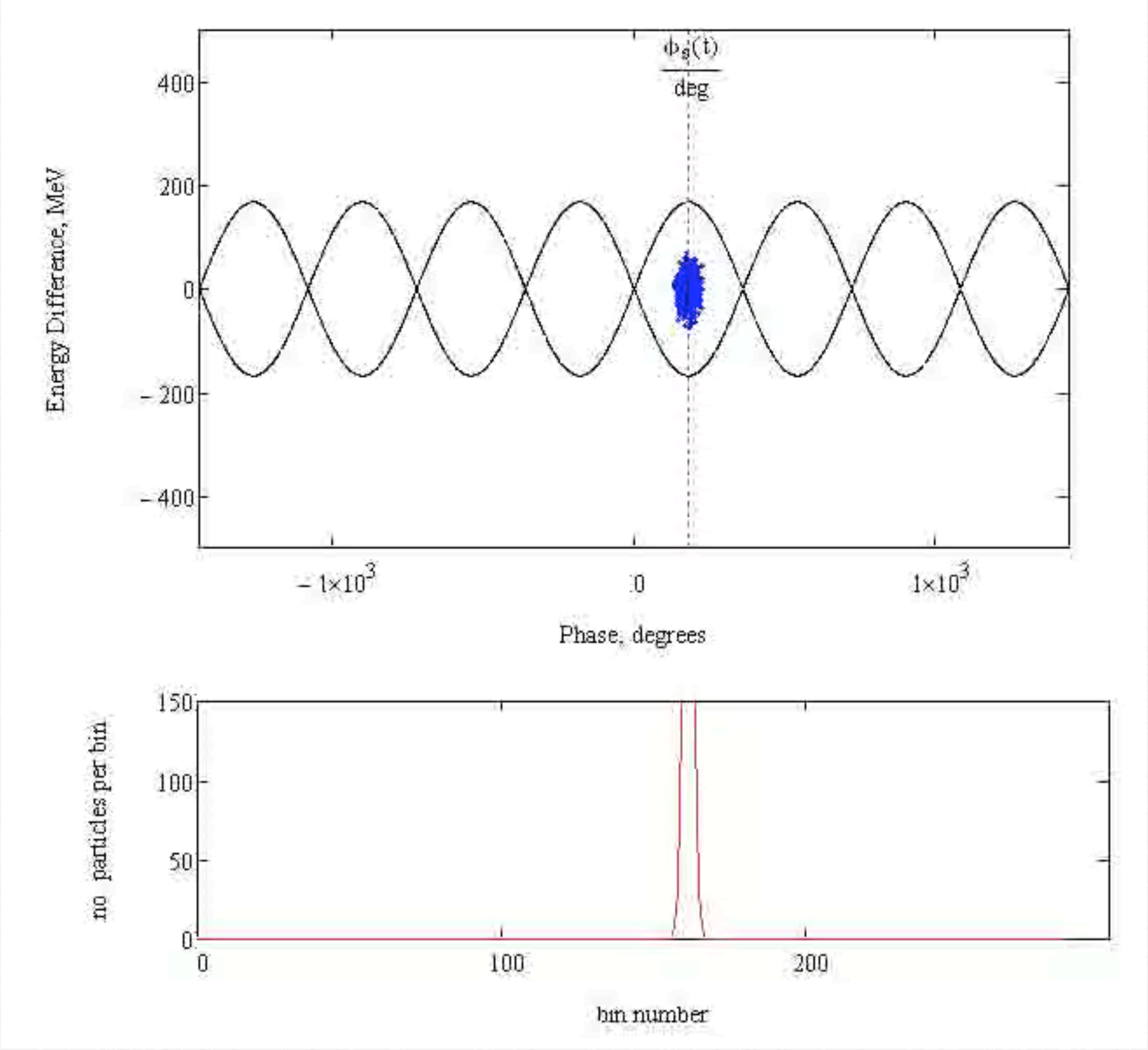
Beam Emittance



DC Beam

- Noise from RF system (phase noise, voltage noise) will increase the beam longitudinal emittance
- Particles will "leak" out of their original bucket, and circulate around the circumference out of phase with the RF
  - "DC Beam"
- Hence, collisions can occur between nominal bunch crossings; of concern for the experiments

DC Beam Generation



Energy Deposition

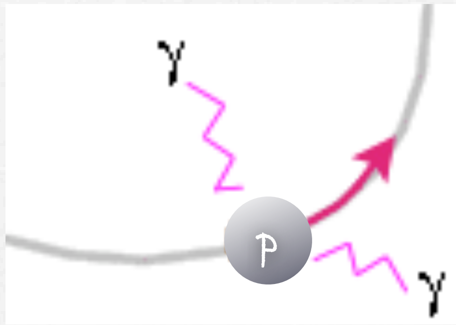
- 1-10 TeV is high energy, but actually less than one micro-Joule; multiply by 10^{13}-10^{14} particles, total energy quite high
- Sources of energy deposition
  - Synchrotron Radiation
  - Particle diffusion (above)
  - Beam abort
  - Collisions!

Beam Stored Energy

- Tevatron
  - $10^{13} \cdot 10^{12} eV \cdot 1.6 \cdot 10^{-19} J/eV \sim$ 2 MJ
- LHC
  - $3 \cdot 10^{14} \cdot 7 \cdot 10^{12} eV \cdot 1.6 \cdot 10^{-19} J/eV \sim$ 300 MJ each beam!
- Power at IP's -- rate of lost particles x energy: $\mathcal{L} \cdot \Sigma \cdot E$
  - Tevatron (at 4K) -- ~4 W at each detector region
  - LHC (at 1.8K) -- ~1300 W at each detector region

Synchrotron Radiation

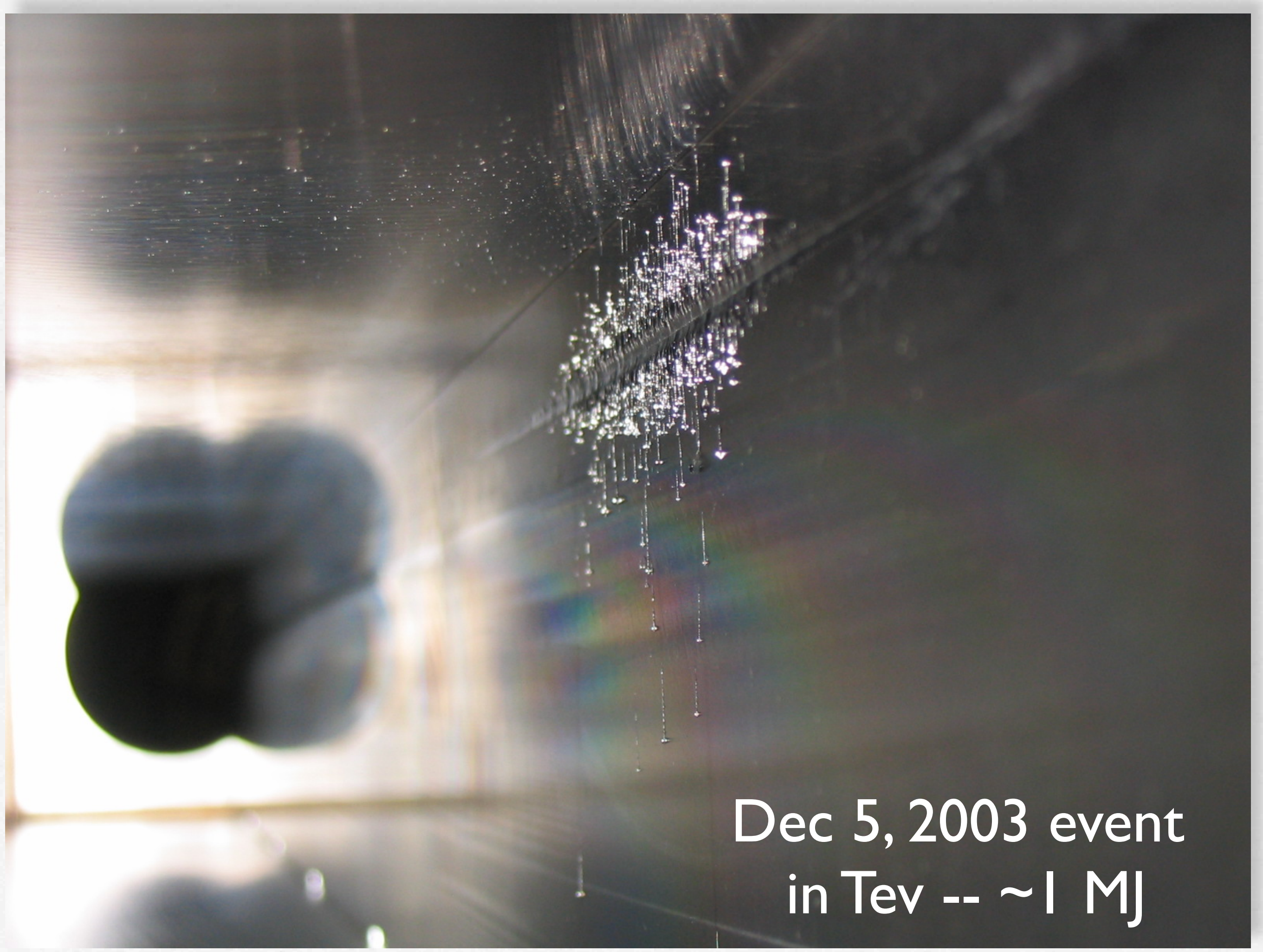
- loss per turn: $$\Delta E_{s.r.} = \frac{4\pi r_0}{3(mc^2)^3} E^4 R \langle \frac{1}{\rho} \rangle$$
  - For Tevatron:
    - ~ 9 eV/turn/particle; ~ 1 W/ring
  - for LHC:
    - ~6700 eV/turn/particle; 3.6 kW/ring



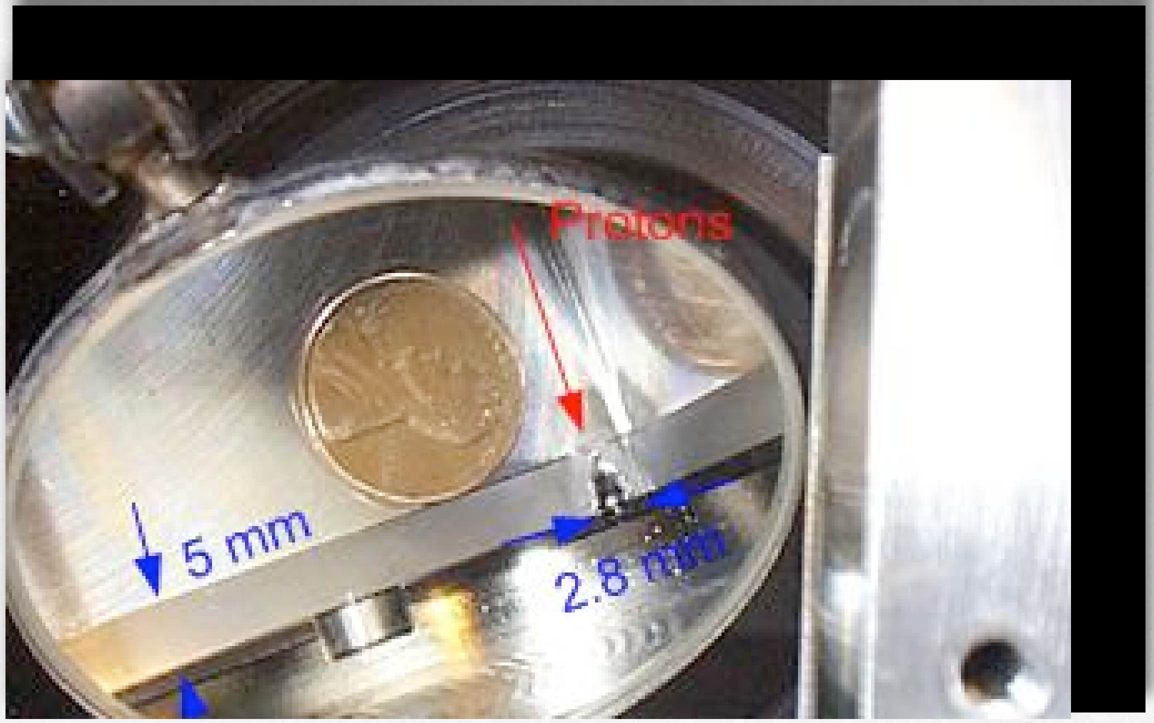
- vacuum instability -- "electron cloud"
  - requires liner for LHC beam tube

Collimation Systems

- Tevatron -- several collimators/scrapers
- LHC -- ~ 100 collimators


Dec 5, 2003 event in Tev -- ~1 MJ

Careful control of collimators, beam trajectory, envelope required



Back to Luminosity...

- Can now express in terms of beam physics parameters; ex.: for short, round beams...

$$\mathcal{L} = \frac{f_0 B N^2}{4\pi\sigma^{*2}} = \frac{f_0 B N^2 \gamma}{4\epsilon\beta^*}$$

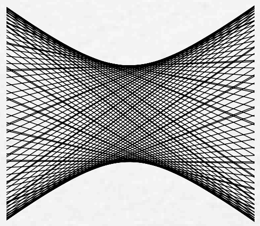
- If different bunch intensities, different transverse beam emittances for the two beams,

$$\mathcal{L} = \frac{f_0 B N_1 N_2}{2\pi(\sigma_1^{*2} + \sigma_2^{*2})} = \frac{f_0 B N_1 N_2 \gamma}{2\beta^*(\epsilon_1 + \epsilon_2)}$$

and assorted other variations...

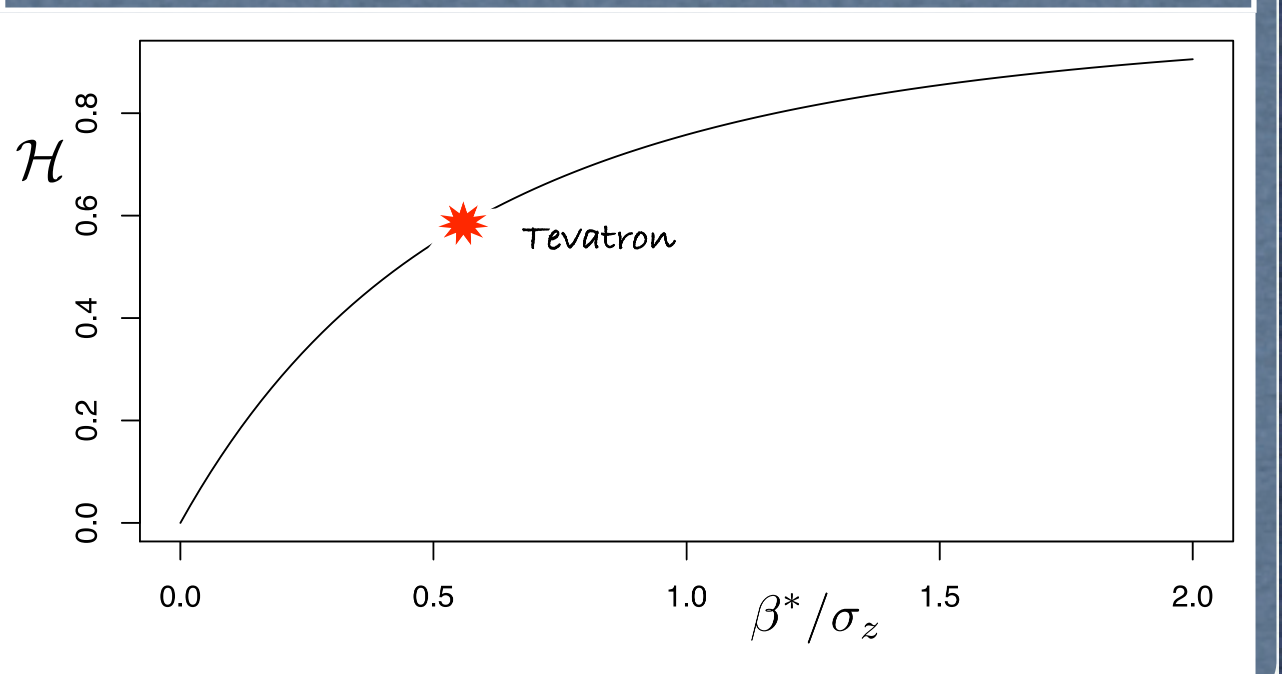
Hour Glass

- If bunches are too long, the rapid increase of the amplitude function away from the interaction "point" reduces luminosity



  - Tevatron:
    - $\sigma_s \approx 2\beta^*$
  - LHC:
    - $\sigma_s << \beta^*$

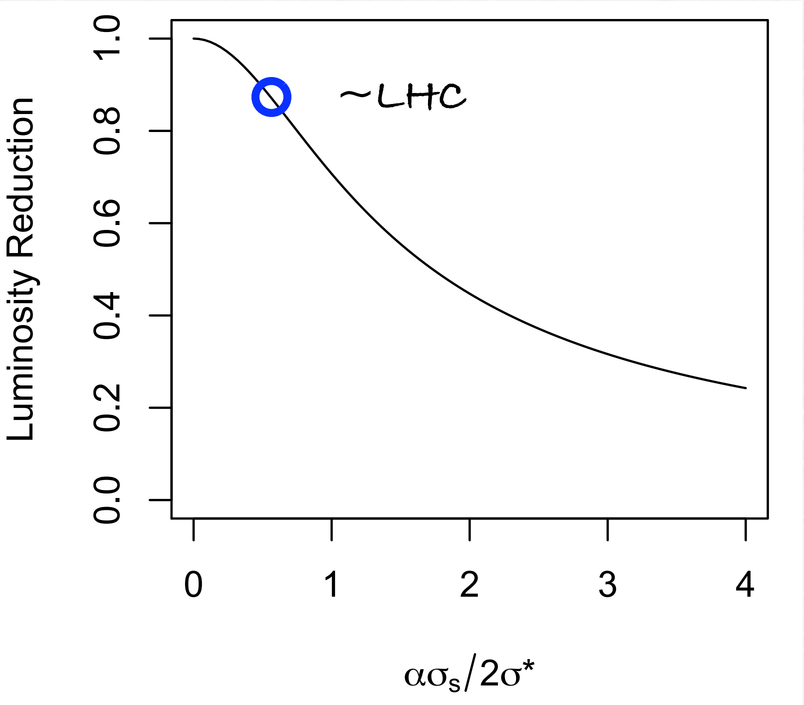
$$\mathcal{H} = \sqrt{\pi}\left(\frac{\beta^*}{\sigma_z}\right) e^{(\beta^*/\sigma_z)^2}\left[1 - \mathrm{erf}(\beta^*/\sigma_z)\right]$$



Crossing Angle

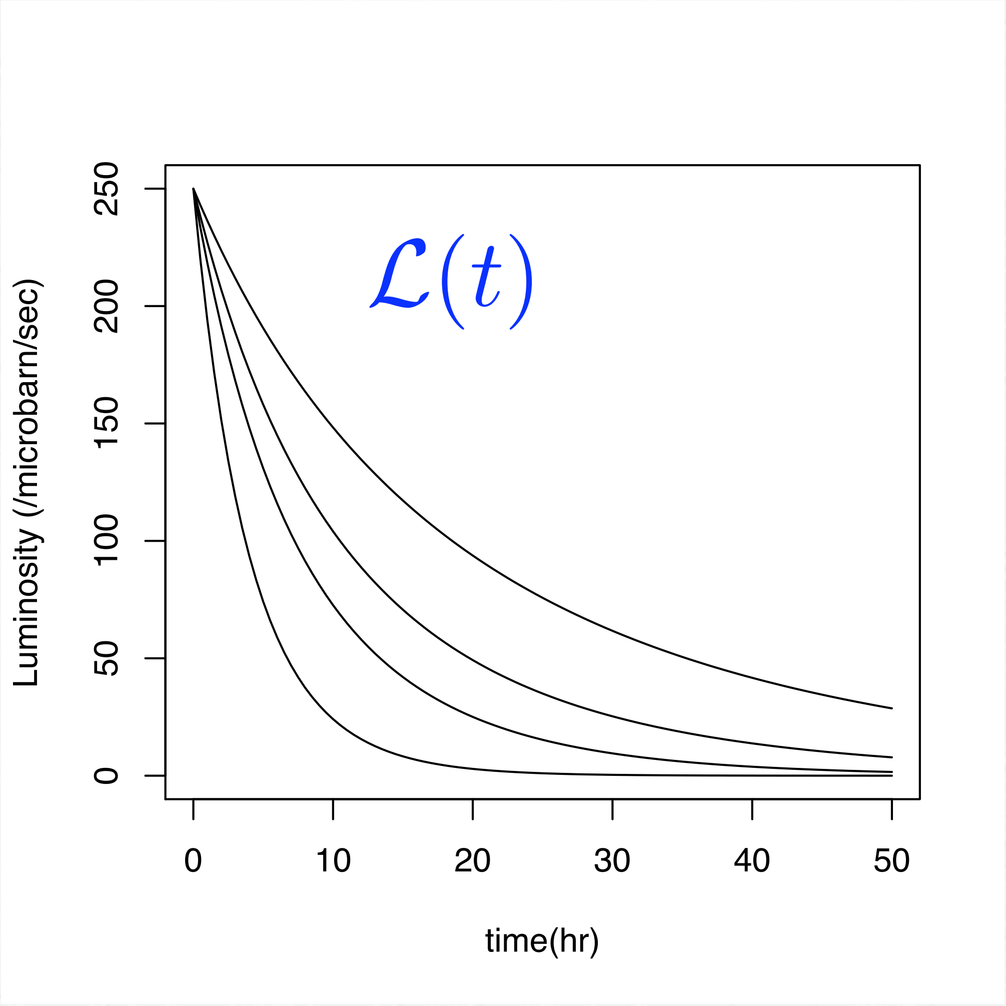
- in Tevatron, bunches spaced far enough apart that next passage by another bunch is outside detector region, after put on separate trajectories.
- in LHC, many more bunches, shorter spacing; if not a crossing angle, would have MANY head-on collisions throughout detector region.
  - reduces luminosity somewhat:

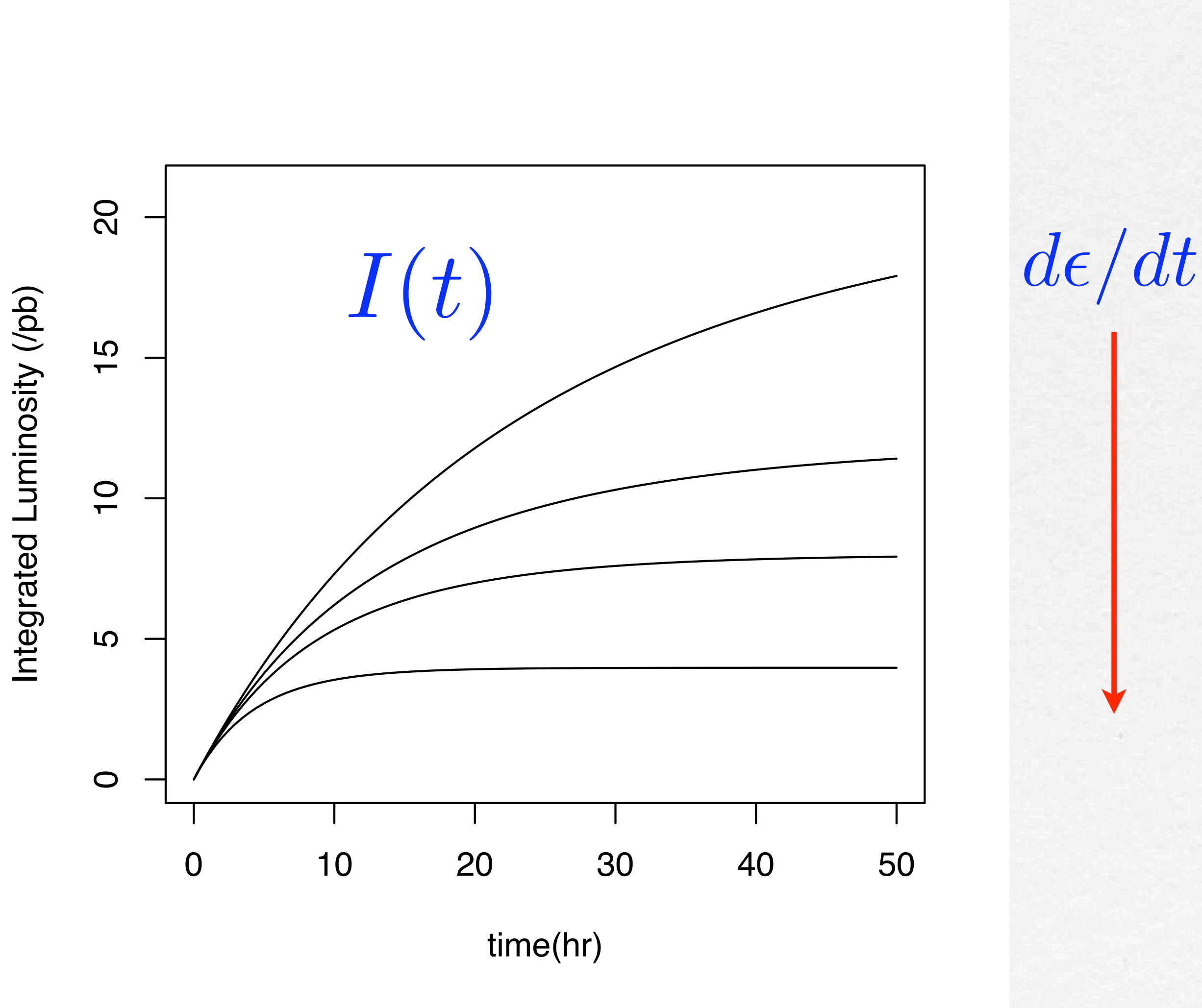
$$\mathcal{L} = \mathcal{L}_0 \cdot \frac{1}{\sqrt{1 + (\alpha\sigma_s/2\sigma^*)^2}}$$



Back to Integrated Luminosity...

- need to include effect of emittance growth, etc.
  - particles will be lost by means other than collisions
- suppose diffusion effects cause $d\epsilon/dt$ (they do!):



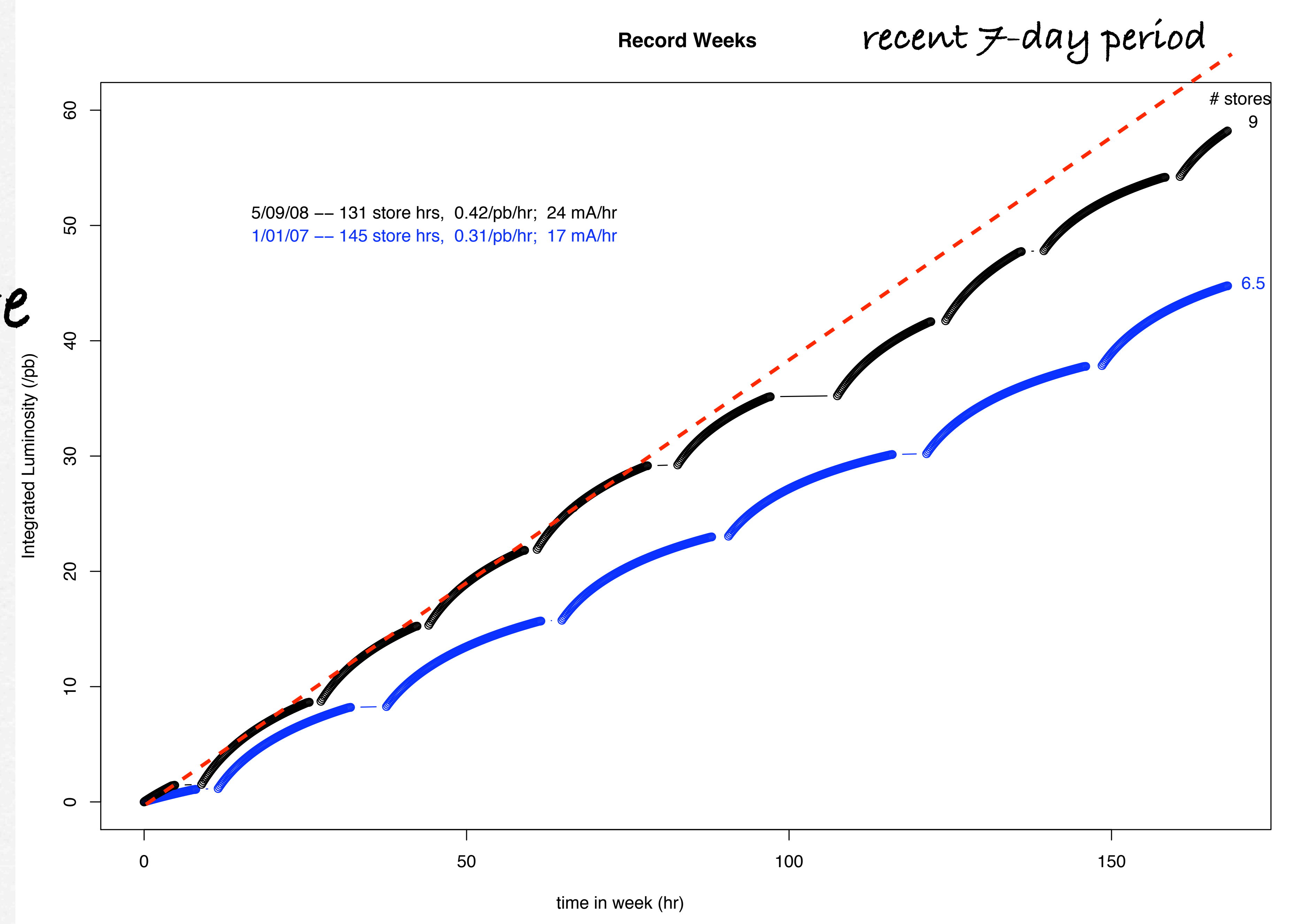


Optimization of Integrated Luminosity

- The ultimate goal for the accelerator -- provide largest total number of collisions possible
- So, optimize initial luminosity, according to turn-around time, emittance growth rates, etc. to produce most integrated luminosity per week (say)
  - example: recent Tevatron running

Tevatron Operation

Here, need to balance the above with the production rate of antiprotons to find optimum running conditions



What's been left out?

- Hope have gotten a glimpse of the process…
- What, there's more??
  - Coupling of degrees-of-freedom transverse x/y, trans. to longitudinal
  - Space charge interactions (mostly low-energies)
  - Wake fields, impedance, coherent instabilities
  - Beam cooling techniques
  - RF manipulations
  - Resonant extraction
  - Crystal collimation
  - Magnet, cavity design
  - Beam Instrumentation and diagnostics
  - …

Further Reading

- D. A. Edwards and M. J. Syphers, An Introduction to the Physics of High Energy Accelerators, John Wiley & Sons (1993)
- S. Y. Lee, Accelerator Physics, World Scientific (1999)
- E. J. N. Wilson, An Introduction to Particle Accelerators, Oxford University Press (2001)

and many others...

- Conference Proceedings --
  - Particle Accelerator Conference (2007, 2005, ...)
  - European Particle Accelerator Conference (2006, 2004, ...)
  - Asian Particle Accelerator Conference (2007, 2004, ...)

Further Schooling...

- US Particle Accelerator School:
  - http://uspas.fnal.gov
  - Twice yearly, January / June
- CERN Accelerator School:
  - http://cas.web.cern.ch
    - Spring (specialized topics)
    - autumn (intro/intermediate)

email: *syphers@fnal.gov*