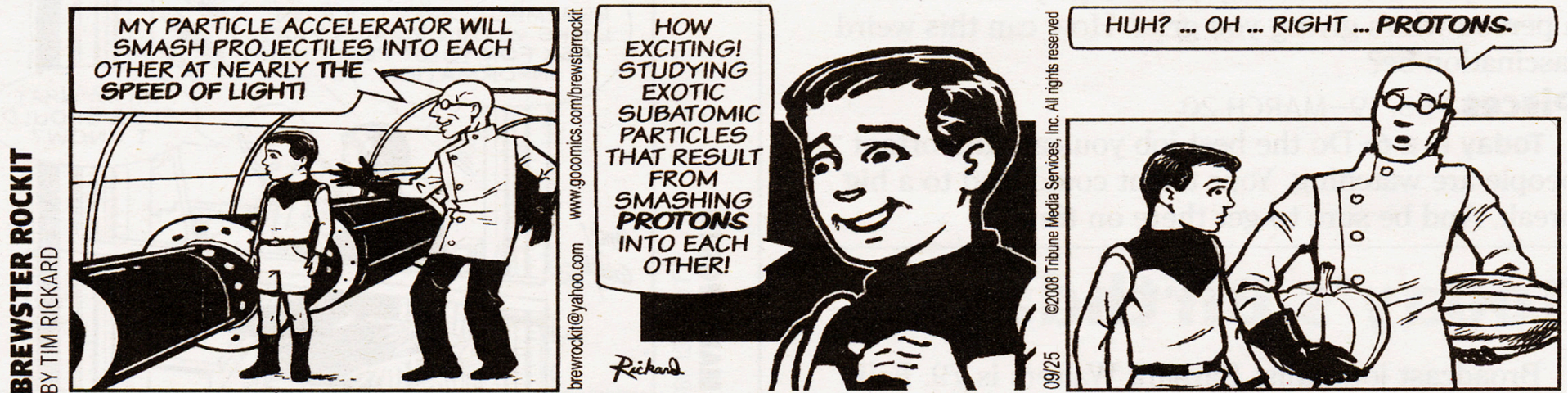


- Z. Chajęcki & MAL PRC **78** 064903 (2008)
- Z. Chajęcki & MAL arXiv:0807.3569 [nucl-th]
- Z. Chajęcki arXiv:0901.4078 [nucl-ex]
- Z. Chajęcki & MAL, to be published



Oh, right... protons

A heavy ion approach to the soft-sector in hadron-hadron collisions

Mike Lisa

Ohio State University

Outline

- Why collide watermelons rather than seeds
 - Collectivity and observable collective effects
 - spectral shapes (with PID!)
 - “elliptic flow”
 - femtoscopy (“HBT,” “B.E.C.” “GGLP”)
- } how to *really* study the soft sector
(in p+p: UE?)
- Claim (will not “prove/sell”): H.I.C. in soft sector extremely well-understood
 - first (!) apples:apples comparison: p+p versus A+A
 - spectra
 - femtoscopy
 - importance of conservation laws
 - Summary

From Peter's mail

*Your audience will instead be almost entirely unfamiliar with collective effects and... they are probably suspicious of both their theory and their modeling in heavy-ion, *as well* as their presence in pp...*

I don't think this is the right place to try to teach them hydrodynamics or convince them of its applications in heavy-ion physics...

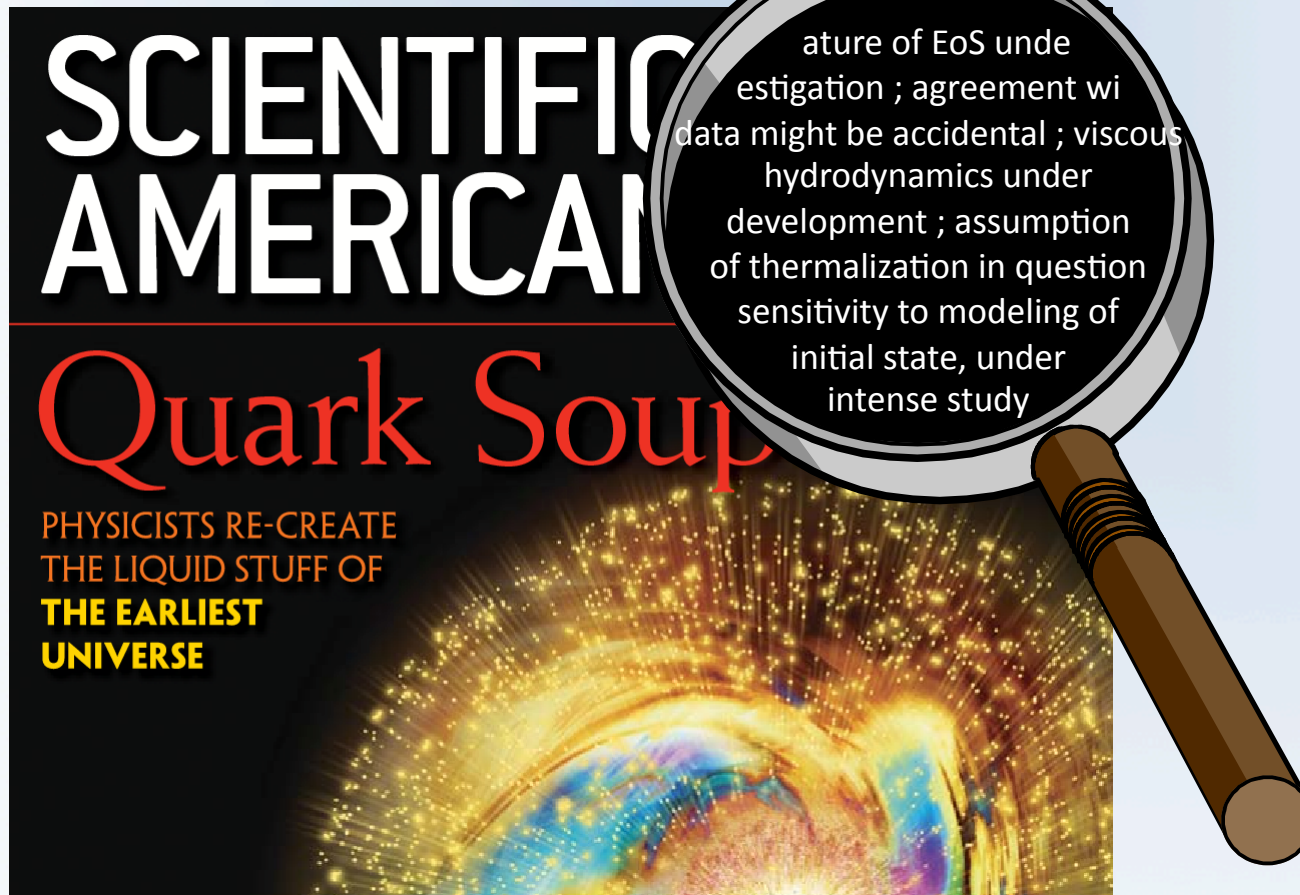
✓ I will mention hydro, but focus on generic, measurable features of bulk collectivity

I would prefer an attack angle that focuses more on the measurements themselves, the data, rather than their interpretation...

More like "this is what we see in pp, and oh by the way, it looks a lot like this effect we also see in heavy-ion, which is there interpreted as a collective effect, isn't that interesting?"

Well, a physics talk needs to go something beyond this, but we'll go a bit along this route.

Perfect press releases



- Perfect or not, creation of a **bulk, collective** system at RHIC is established - **flow**
 - This system is very color dense and largely **opaque** to partons traversing it - R_{AA}
- ? *Are these statements unique to A+A collisions?*

SCI
AM
Qu
PHYSICISTS
THE LIQUID
THE EARLIES
UNIVERSE

- Perfect or not, (
 - This system is v
- ? **Are these state**

CNN.com /technology POWERED BY Google

HOME WORLD U.S. POLITICS CRIME ENTERTAINMENT HEALTH **TECH** TRAVEL LIVING BUSINESS SPORTS

[Hot Topics](#) » [CES](#) • [iPhone](#) • [Mars Exploration](#) • [Planet In Peril](#) • [Solutions](#) • [Science](#) • [Focus on Giving](#) • more topics »

28 April 2009

New music! Join us to learn what
Watch Now: [Live on CNN.com](#) »

Big Bang machine detectors will be 'even more perfect'

updated 4 hours, 10 minutes ago

STORY HIGHLIGHTS

- The Large Hadron Collider v
- Scientists have used the ye
- CMS uses about 100,000 cc
- Experts hope to find the Hig

[Next Article in Technology](#) »

READ DETAILS MAP

By Elizabeth Landau
CNN

(CNN) -- On a recent episode of "South Park," Mr. Marsh steals a particle accelerator magnet so his son, Stan, can win the Pinewood Derby. The magnet's power results in an alien encounter, and chaos ensues.



The Compact Muon Solenoid, shown here in December, is one of six experiments inside the collider complex.

While the magnets at the real-life Large Hadron Collider may not reach extraterrestrials, scientists hope they will help lead to encounters with never-before-seen phenomena and answers to fundamental questions about the universe.

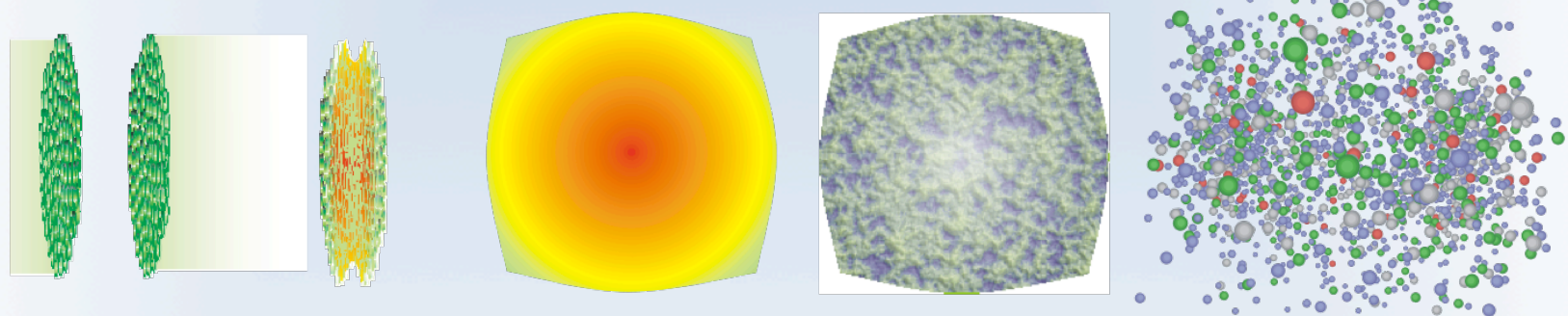
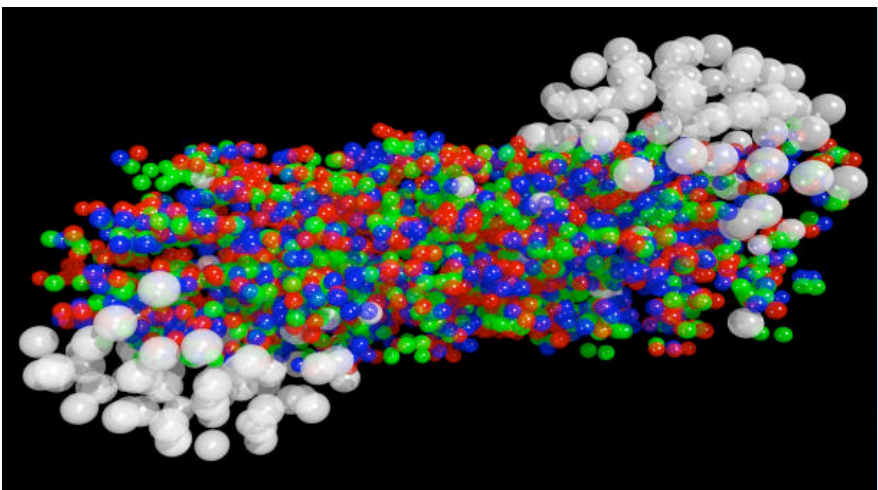
The collider, the world's most powerful particle accelerator, is being repaired after an electrical failure in September. Once it is fixed, the collider will circulate beams of particles with unprecedented energy. When these particles crash into each other, the resulting activity may help scientists figure out why the submicroscopic stuff that makes up our universe behaves the way it does.

The Large Hadron Collider will start receiving current again in July, and will circulate this year's first proton beam by the end of September, said Lyn Evans, former project leader for the collider who is currently involved with the machine's repairs. The collider is located more than 300 feet below the French-Swiss border at CERN, the European Organization for Nuclear Research.

ed - flow
ig it - R_{AA}

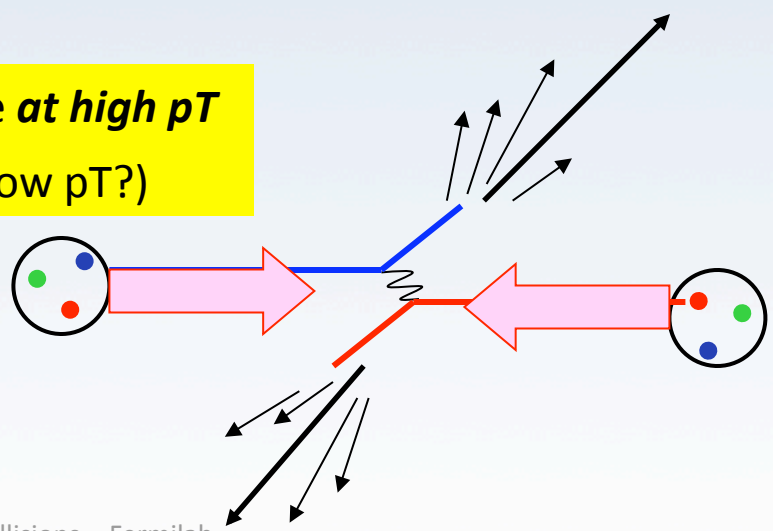
paradigms

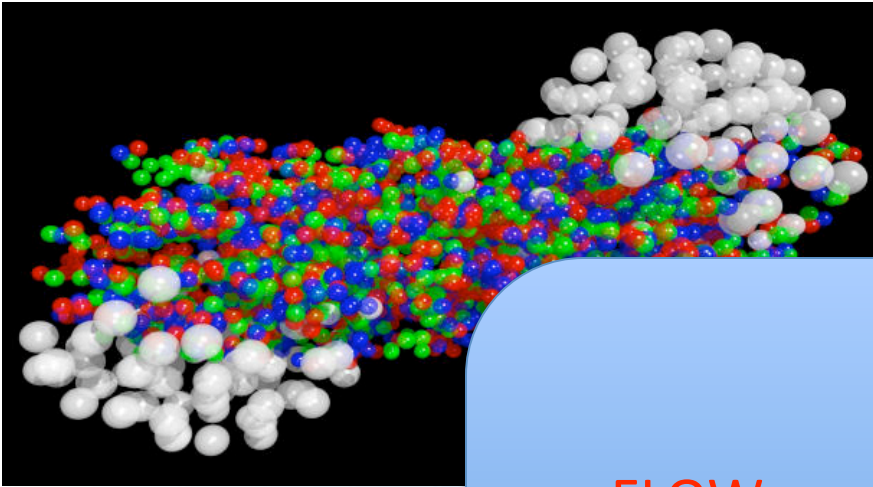
$A+A \rightarrow$ a system



“Clean” p+p– a crucial reference *at high pT*
(do we understand/care about low pT?)

$p+p$: a *process*





H. I. C. — a system

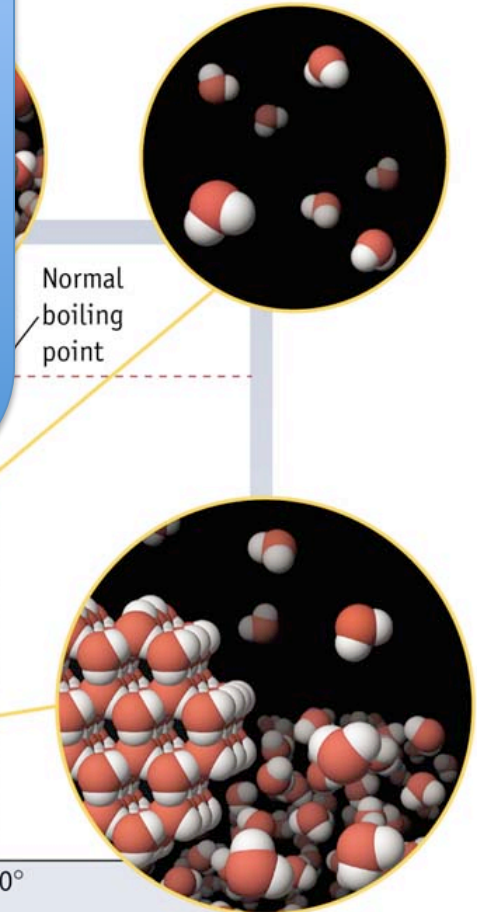
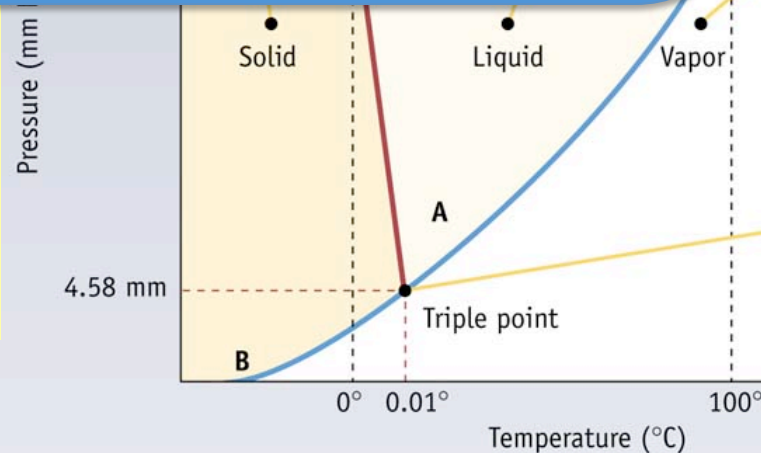
**FLOW: most direct proof of
existence of system
&
probe of its response**

bulk physics

- superfluids
- superconductors
- metal/insulator
- ...

Only for large *system*

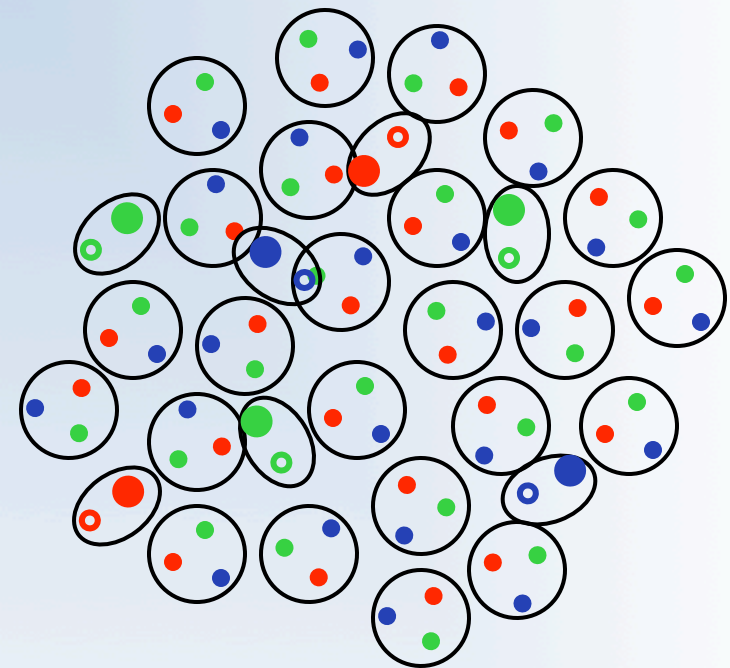
- can't melt one H_2O molecule!



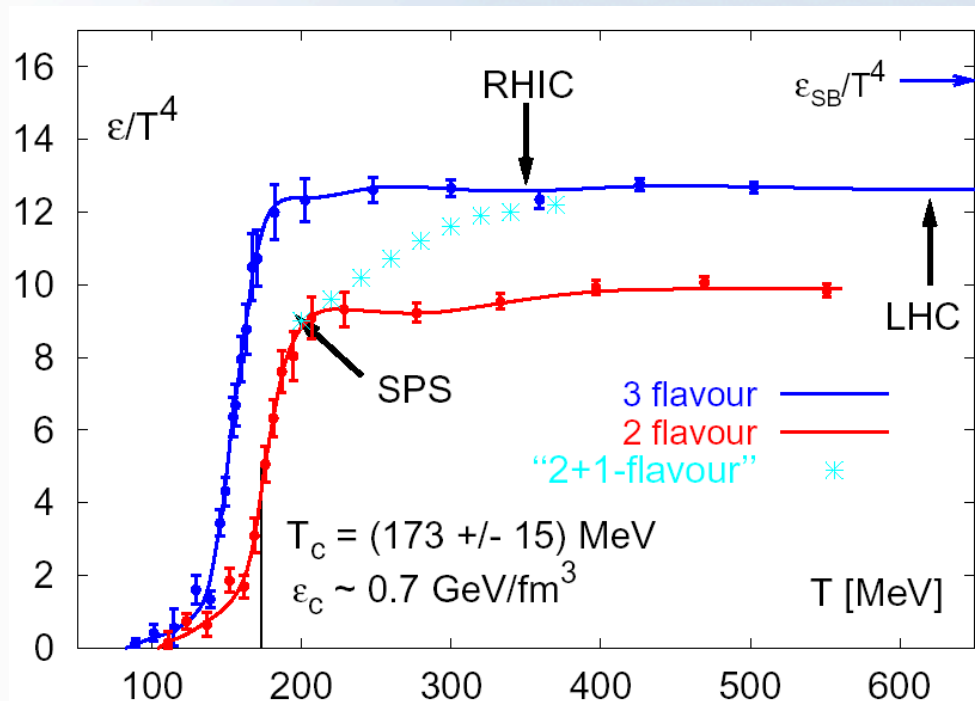
States of QCD Matter

Present understanding of Quantum Chromodynamics (QCD)

- heating
- compression
- *deconfined color matter*

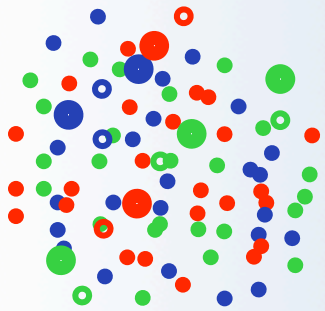


QCD Matter (deconfined)

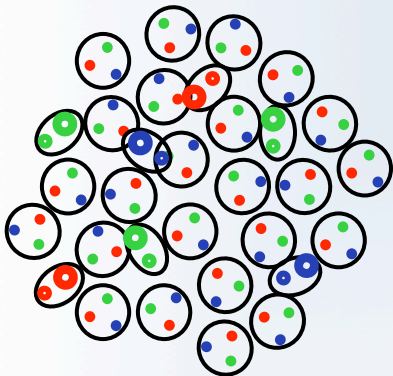


Expectations from Lattice QCD

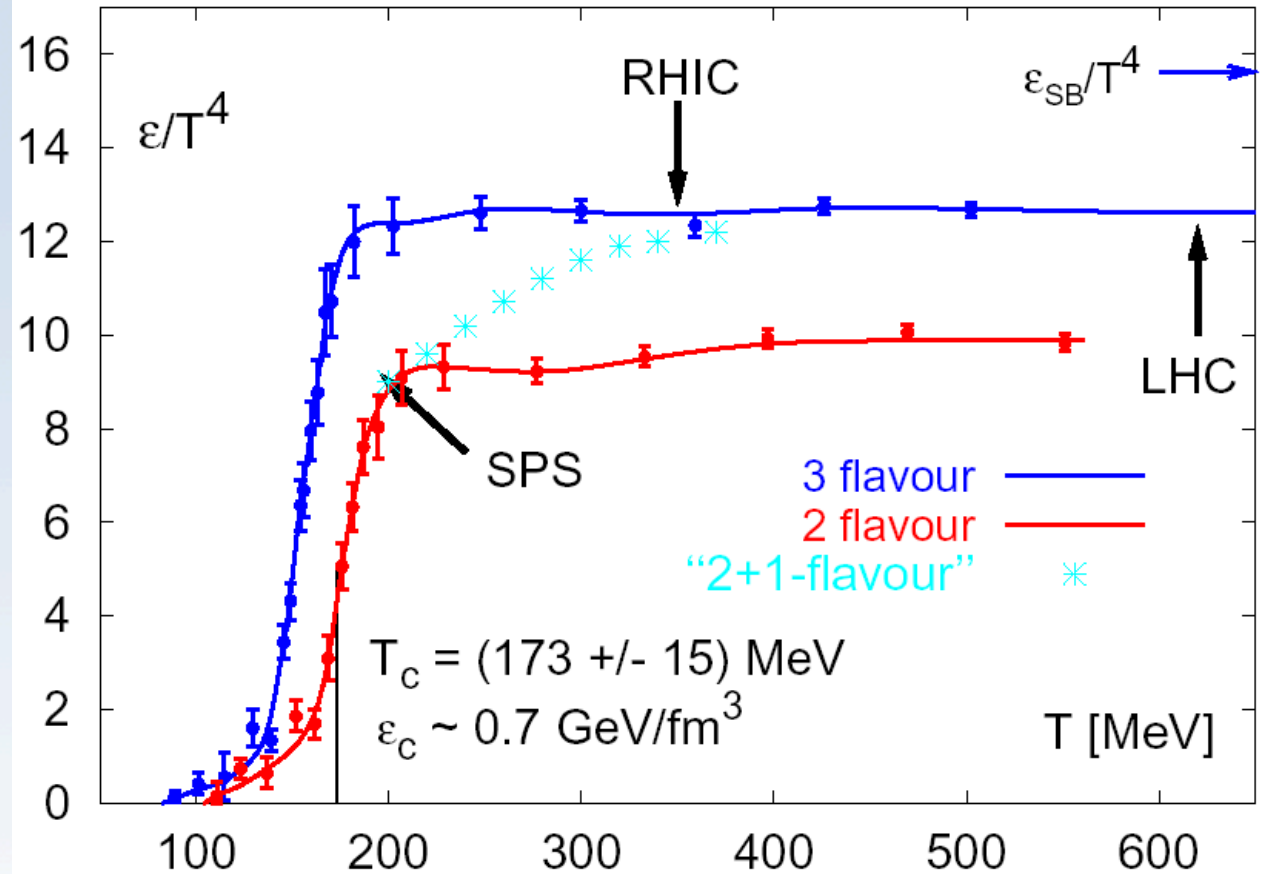
$\epsilon/T^4 \sim \#$ degrees of freedom



deconfined:
many d.o.f.



confined:
few d.o.f.



The hydrodynamics slide

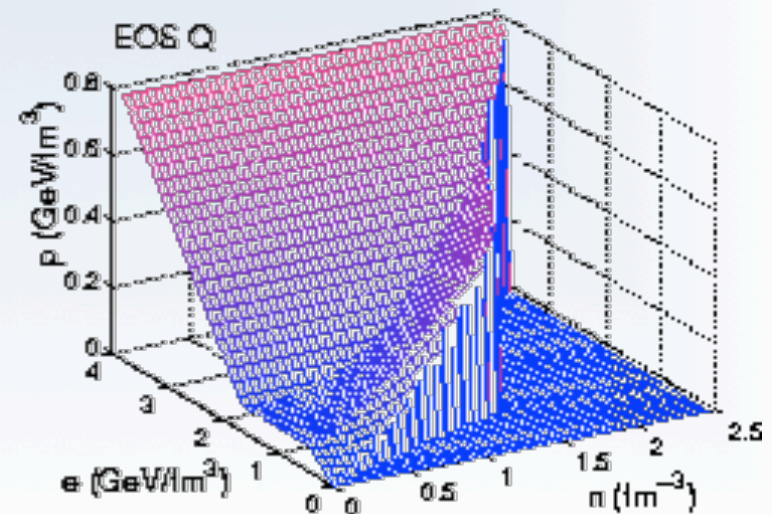
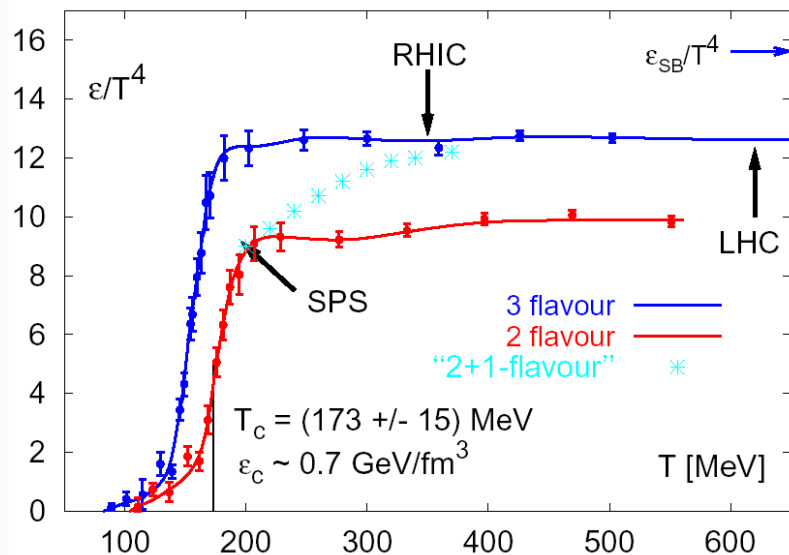
Ideal Hydrodynamics: Physics

- Equations of motion:

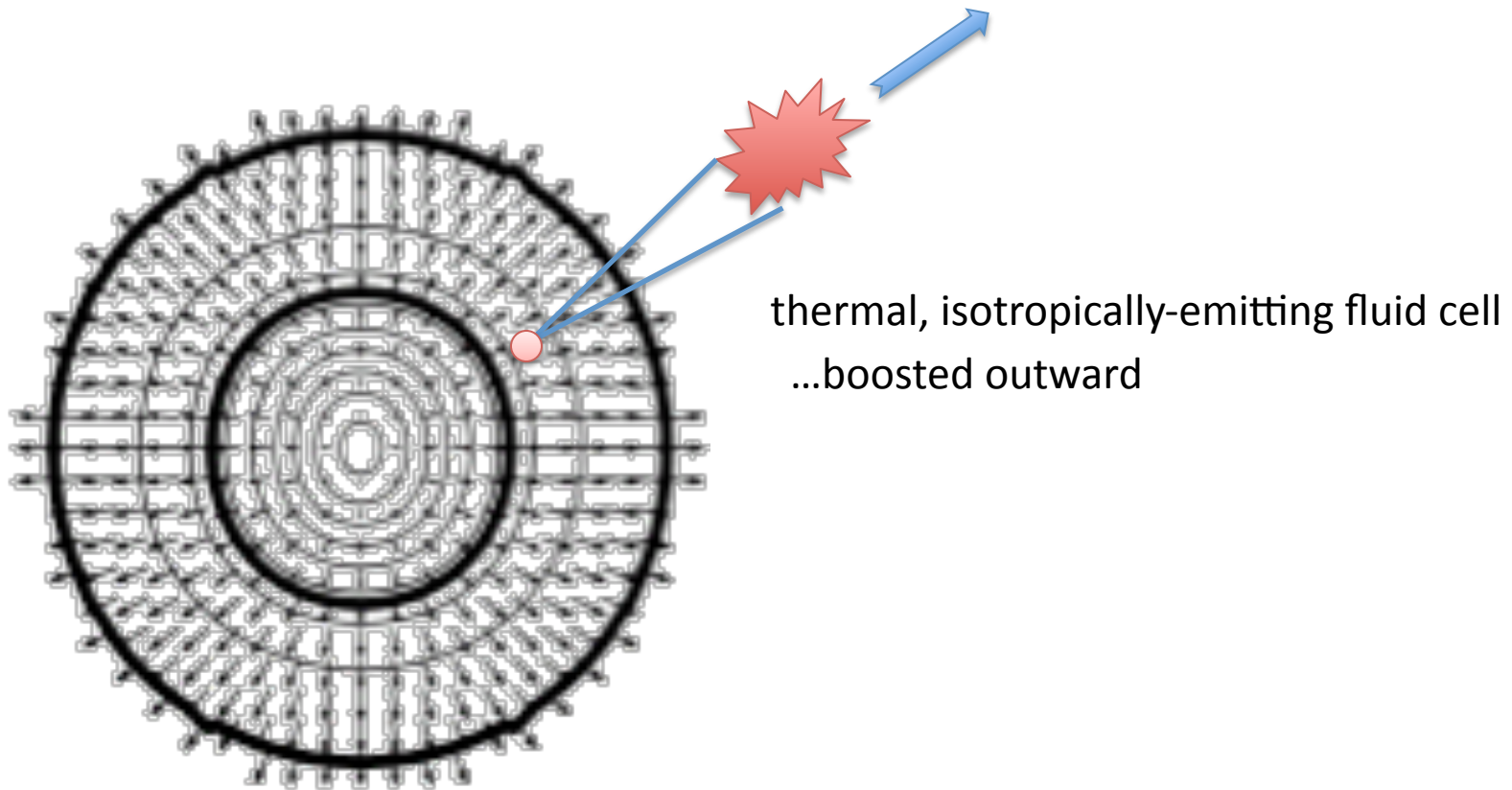
$$\partial_\mu T^{\mu\nu} = \partial_\mu (e u^\mu u^\nu + p \Delta^{\mu\nu}) = 0$$

$$\partial_\mu N^\mu = \partial_\mu (n u^\mu) = 0$$

- Equation of State (EoS): $p(e, n)$
- Five Equations of Motion (T^{00}, T^{0i}, N^0) and Five unknowns e, \mathbf{v}, n .

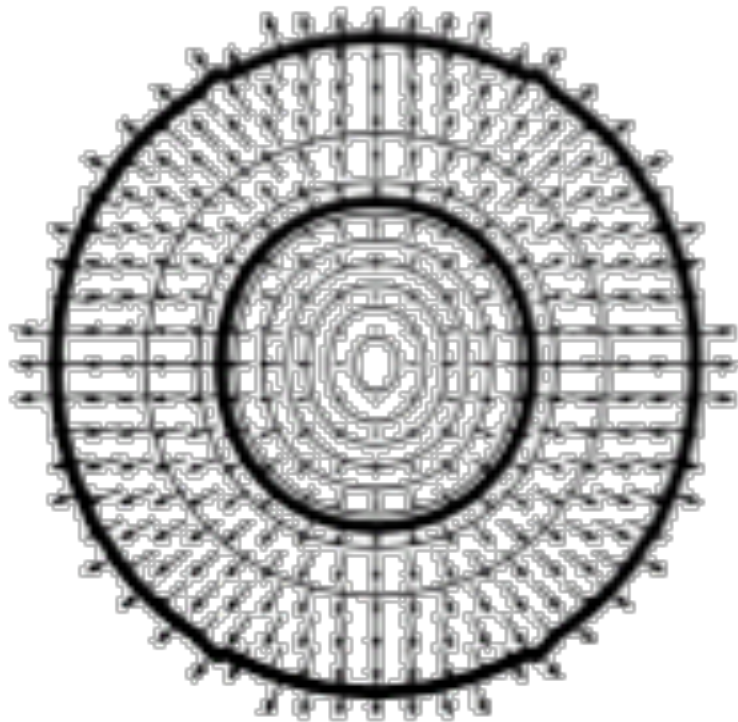


The other hydrodynamics slide

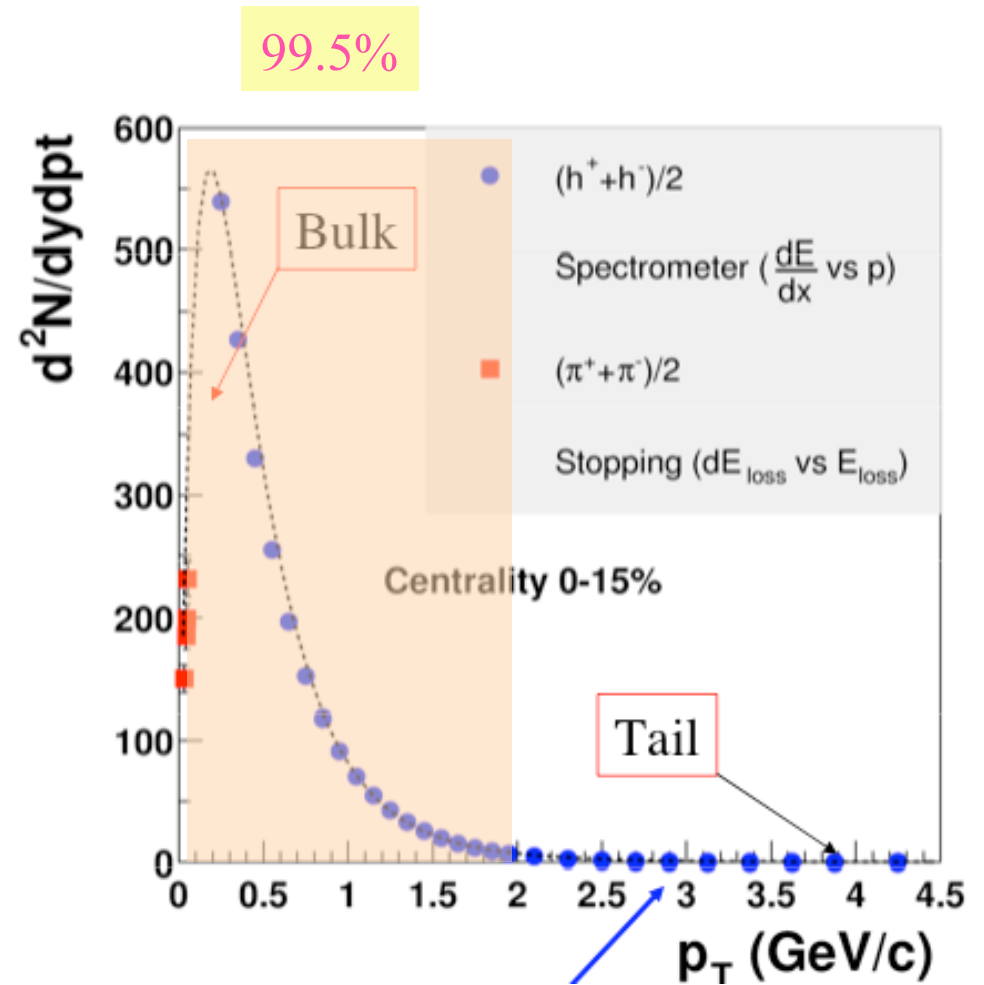


The generic outcome
“Blast-wave”

The other hydrodynamics slide



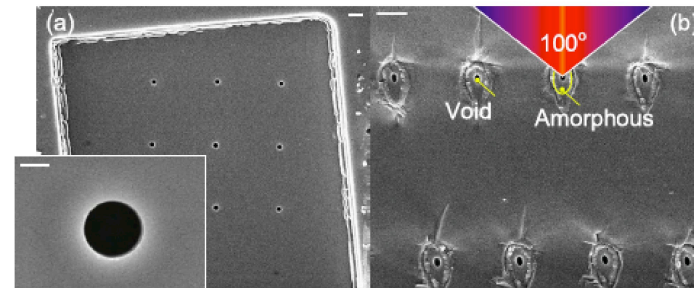
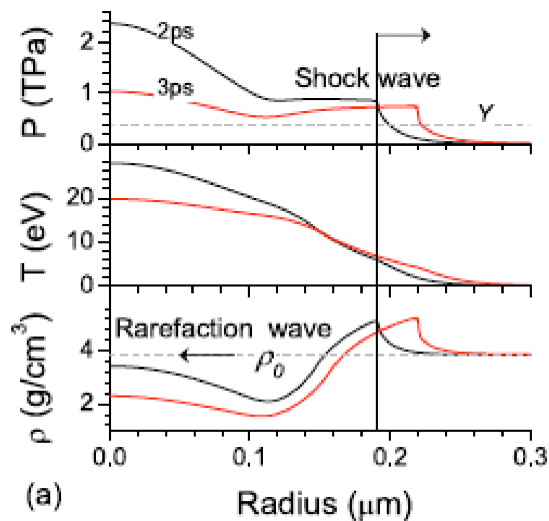
The generic outcome
"Blast-wave"



Laser-Induced Microexplosion Confined in the Bulk of a Sapphire Crystal: Evidence of Multimegabar Pressures

S. Juodkazis,¹ K. Nishimura,¹ S. Tanaka,¹ H. Misawa,¹ E. G. Gamaly,² B. Luther-Davies,²
L. Hallo,³ P. Nicolai,³ and V.T. Tikhonchuk³

Extremely high pressures (~10 TPa) and temperatures (5×10^5 K) have been produced using a single laser pulse (100 nJ, 800 nm, 200 fs) focused inside a sapphire crystal. The laser pulse creates an intensity over 10^{14} W/cm² converting material within the absorbing volume of $\sim 0.2 \mu\text{m}^3$ into plasma in a few fs. A pressure of ~10 TPa, far exceeding the strength of any material, is created generating strong shock and rarefaction waves. This results in the formation of a nanovoid surrounded by a shell of shock-affected material inside undamaged crystal. Analysis of the size of the void and the shock-affected zone versus the deposited energy shows that the experimental results can be understood on the basis of conservation laws and be modeled by plasma hydrodynamics. Matter subjected to record heating and cooling rates of 10^{18} K/s can, thus, be studied in a well-controlled laboratory environment.

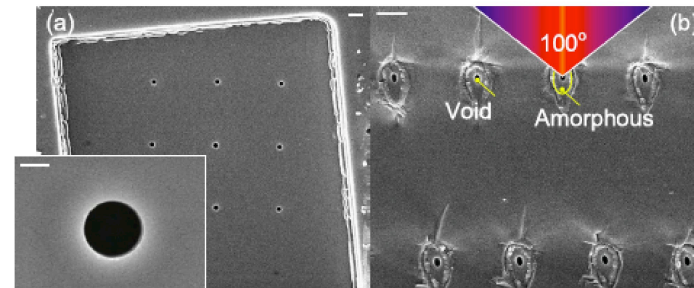
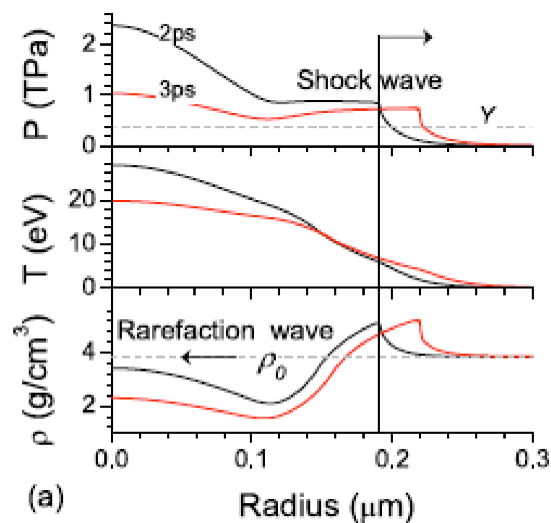


	Microexplosions	Femtoexplosions
\sqrt{s}	0.1 μJ	1 μJ
ϵ	10^{17} J/m ³	5 GeV/fm ³ = 10^{36} J/m ³
T	10^6 K	200 MeV = 10^{12} K
rate	10^{18} K/sec	10^{35} K/s

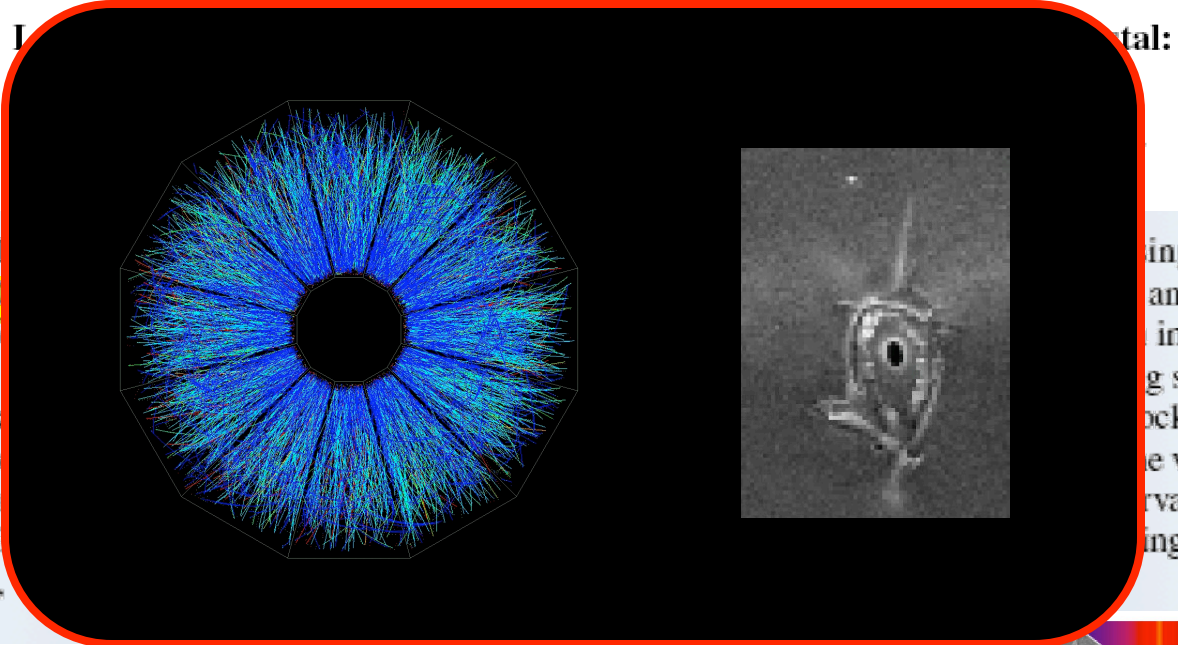
Laser-Induced Microexplosion Confined in the Bulk of a Sapphire Crystal: Evidence of Multimegabar Pressures

S. Juodkazis,¹ K. Nishimura,¹ S. Tanaka,¹ H. Misawa,¹ E. G. Gamaly,² B. Luther-Davies,²
L. Hallo,³ P. Nicolai,³ and V.T. Tikhonchuk³

Extremely high pressures (~ 10 TPa) and temperatures (5×10^5 K) have been produced using a single laser pulse (100 nJ, 800 nm, 200 fs) focused inside a sapphire crystal. The laser pulse creates an intensity over 10^{14} W/cm² converting material within the absorbing volume of ~ 0.2 μm^3 into plasma in a few fs. A pressure of ~ 10 TPa, far exceeding the strength of any material, is created generating strong shock and rarefaction waves. This results in the formation of a nanovoid surrounded by a shell of shock-affected material inside undamaged crystal. Analysis of the size of the void and the shock-affected zone versus the deposited energy shows that the experimental results can be understood on the basis of conservation laws and be modeled by plasma hydrodynamics. Matter subjected to record heating and cooling rates of 10^{18} K/s can, thus, be studied in a well-controlled laboratory environment.

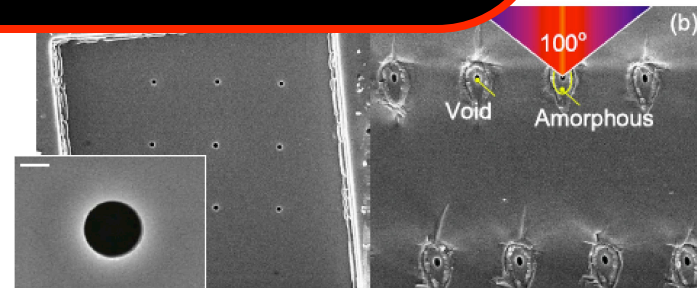
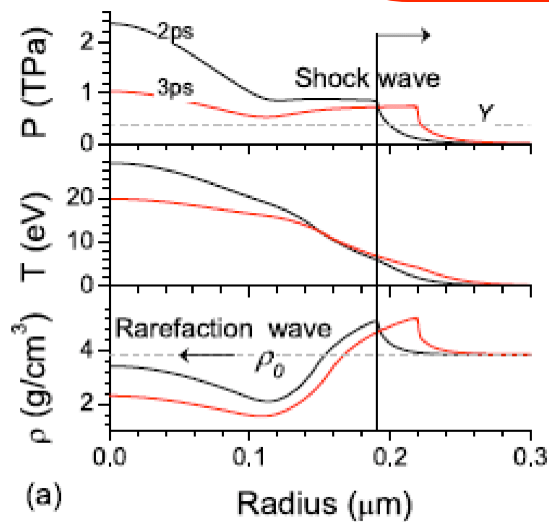


- energy quickly deposited
- enter plasma phase
- expand hydrodynamically
- cool back to original phase
- do **geometric** "postmortem" & infer momentum



Extremely
laser pulse (10
over 10^{14} W/
A pressure of
rarefaction wa
material insid
deposited ene
and be mode
 10^{18} K/s can,

tal:
ing a single
an intensity
in a few fs.
g shock and
ock-affected
e versus the
ervation laws
ing rates of

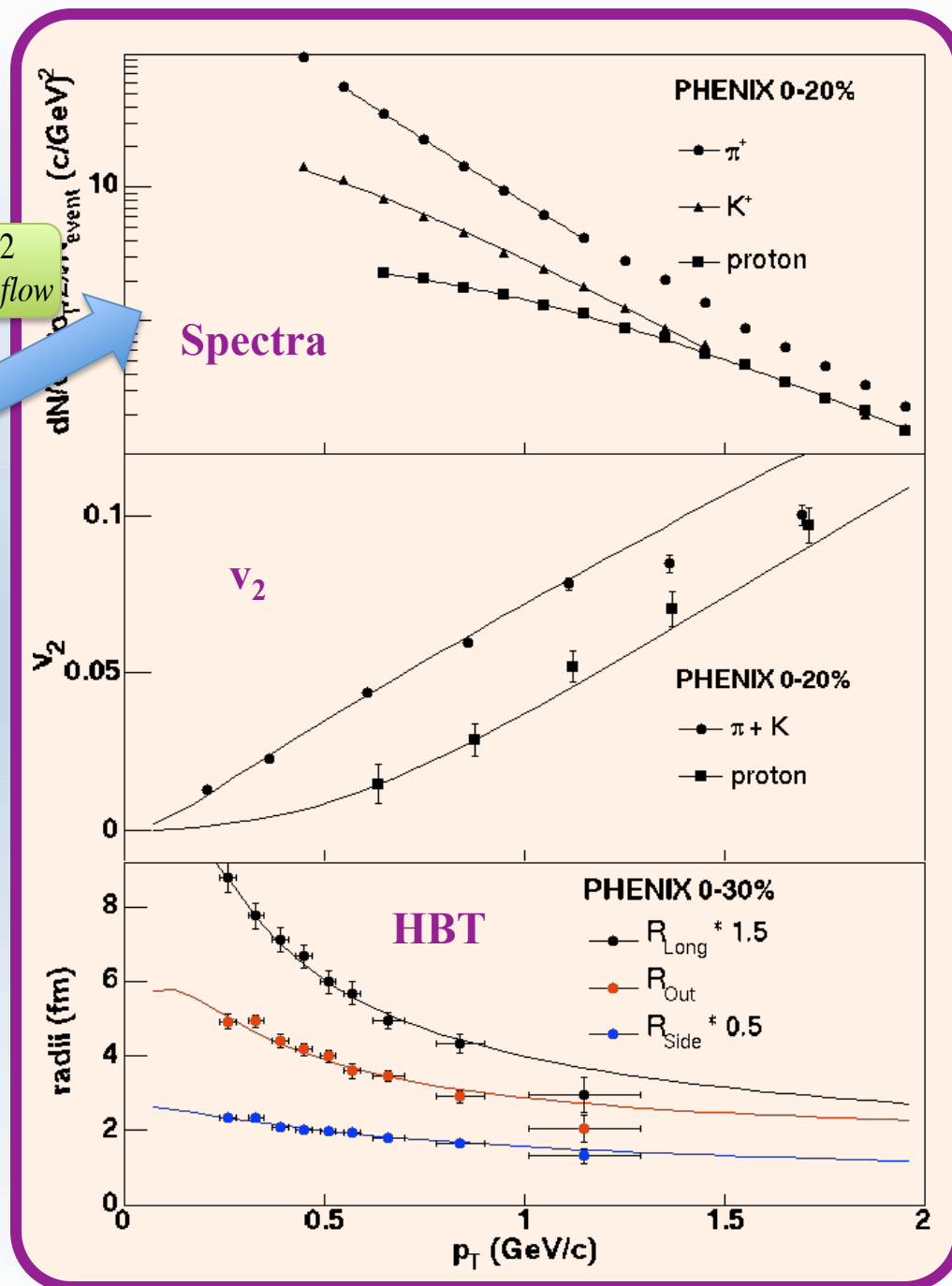
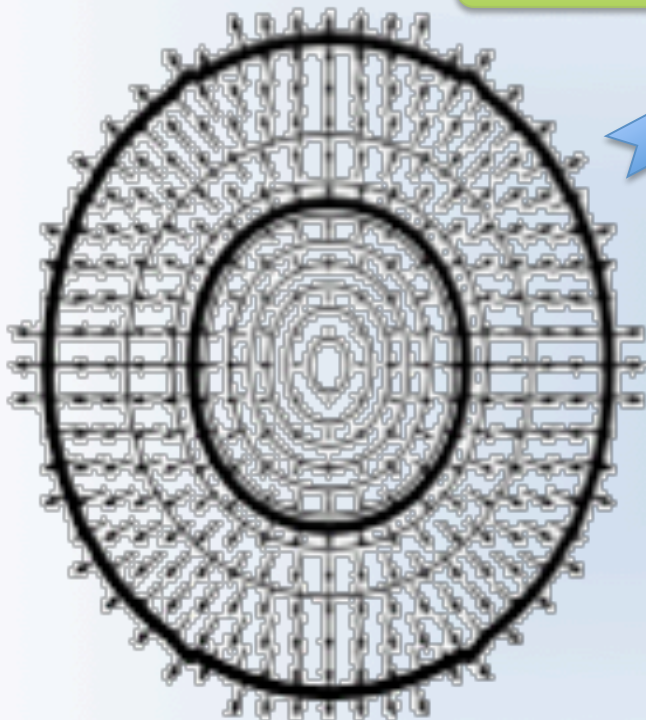


- energy quickly deposited
- enter plasma phase
- expand hydrodynamically
- cool back to original phase
- do **geometric** "postmortem" & infer momentum

Explosive flow revealed through *specific fingerprints* on soft-sector observables

calculable in hydrodynamics or toy “blast wave” models

$$m_T \approx T + m\beta_{flow}^2$$

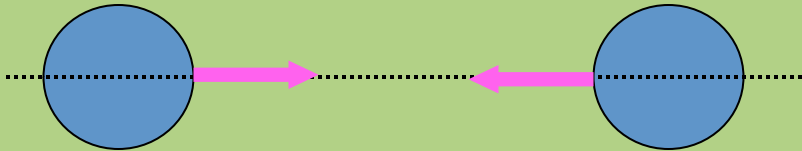


Impact parameter & Reaction plane

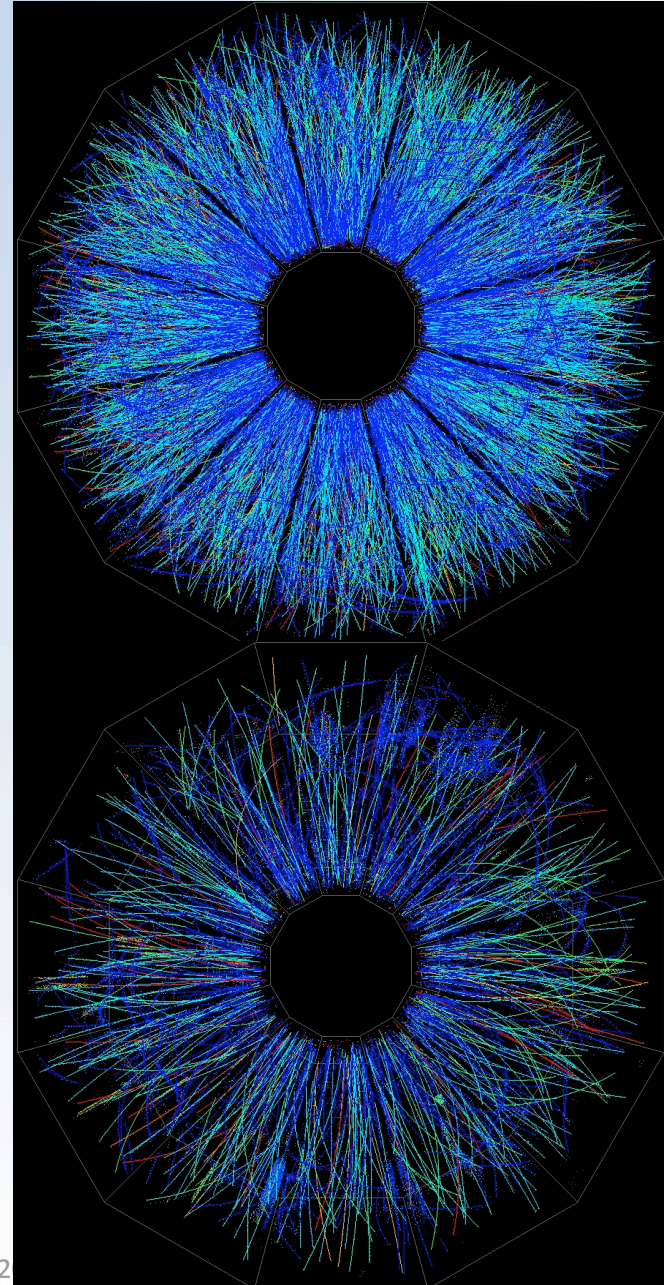
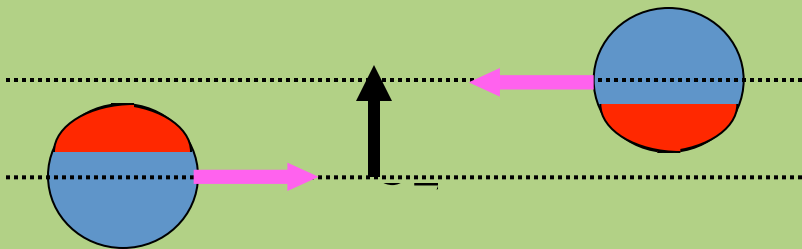
Impact parameter vector \vec{b} :

- \perp beam direction
- connects centers of colliding nuclei

$b = 0$: “central collision”
many particles produced



“peripheral collision”
fewer particles produced



Impact parameter & Reaction plane

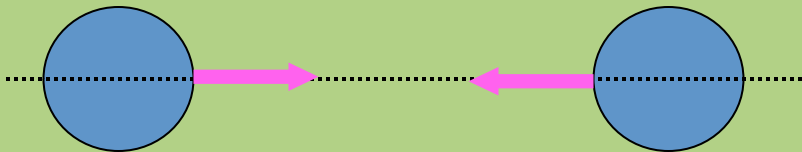
Impact parameter vector \vec{b} :

- \perp beam direction
- connects centers of colliding nuclei

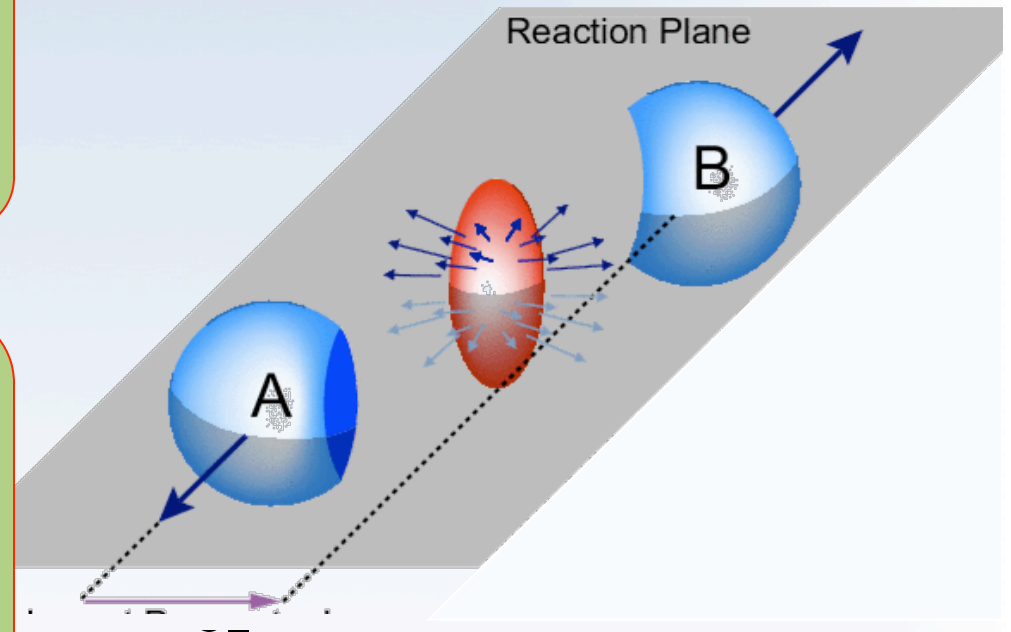
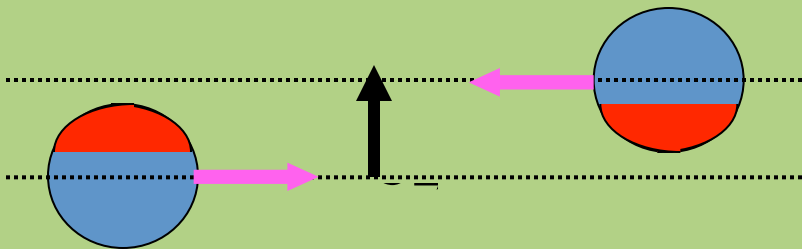
Reaction plane:

spanned by beam direction and \vec{b}

$b = 0$: "central collision"
many particles produced



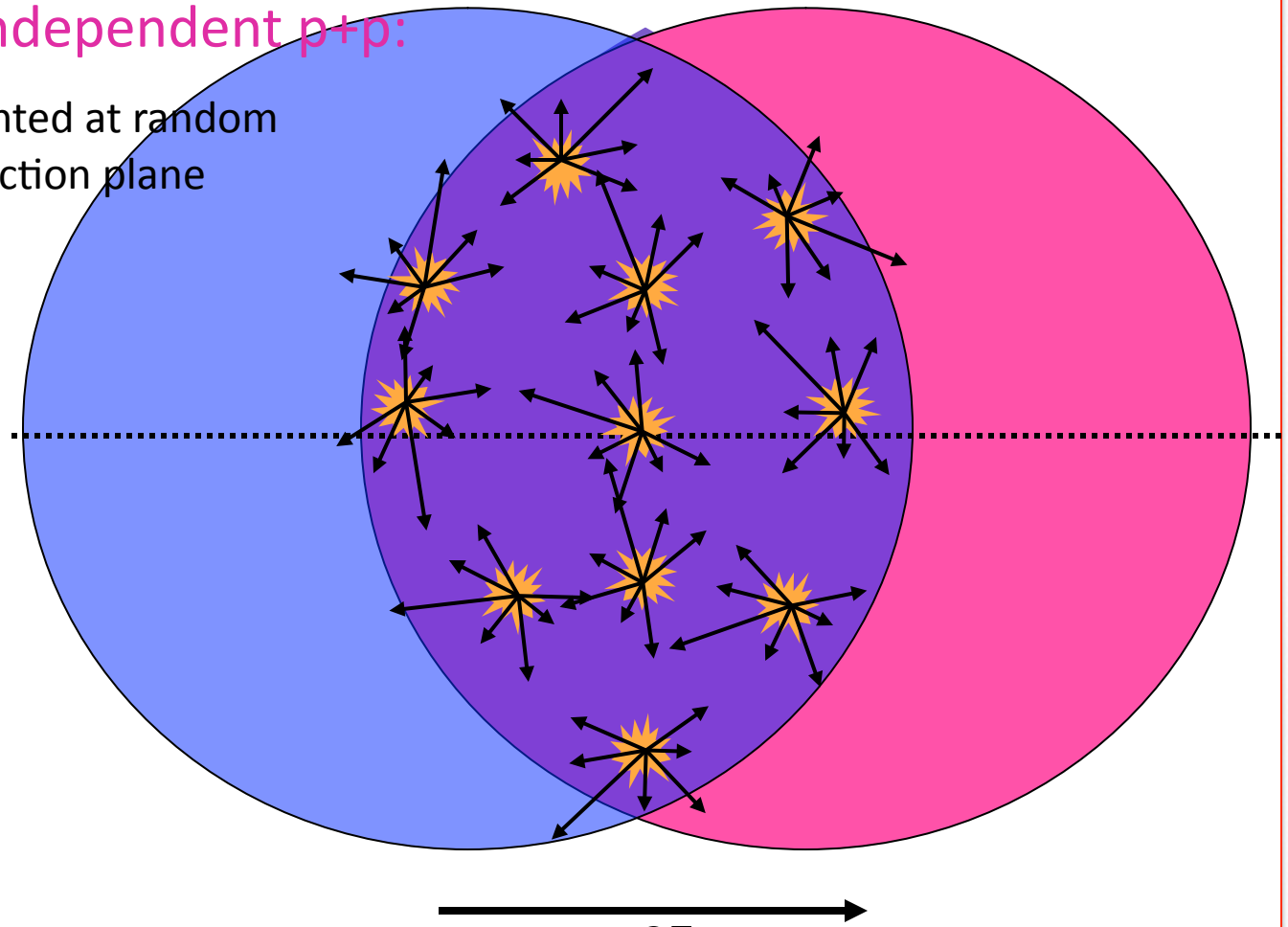
"peripheral collision"
fewer particles produced



How do semi-central collisions evolve?

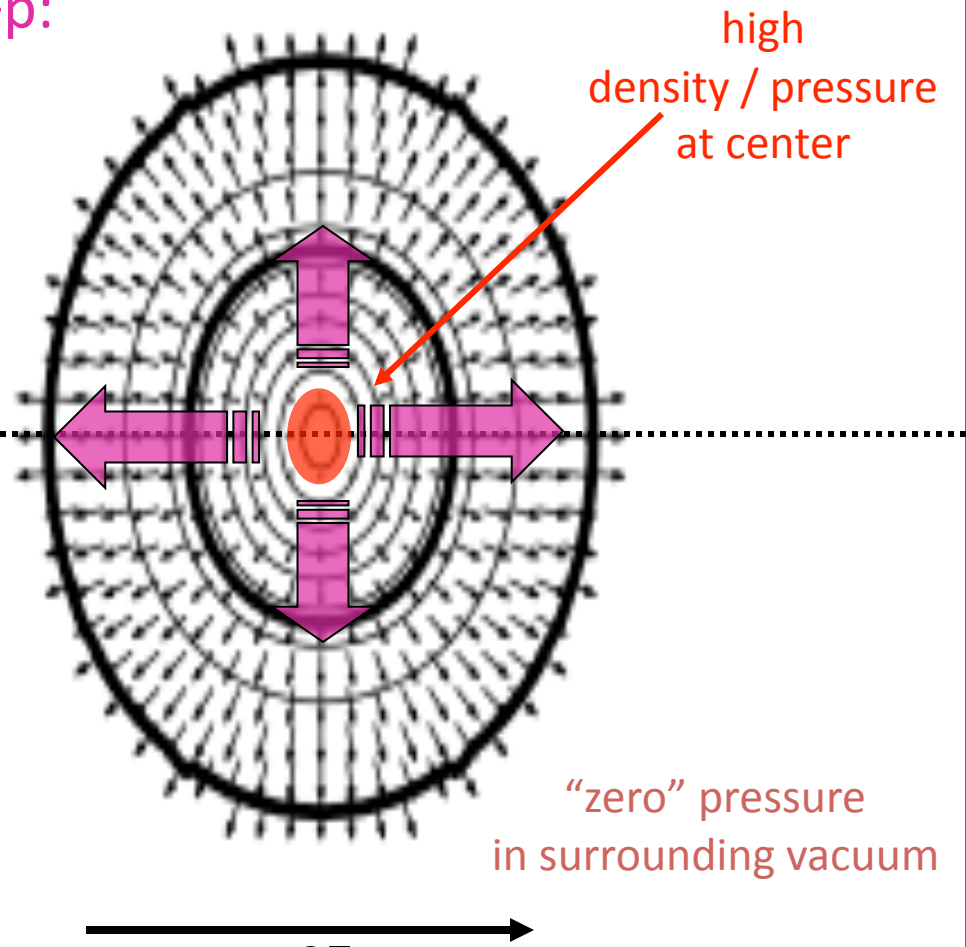
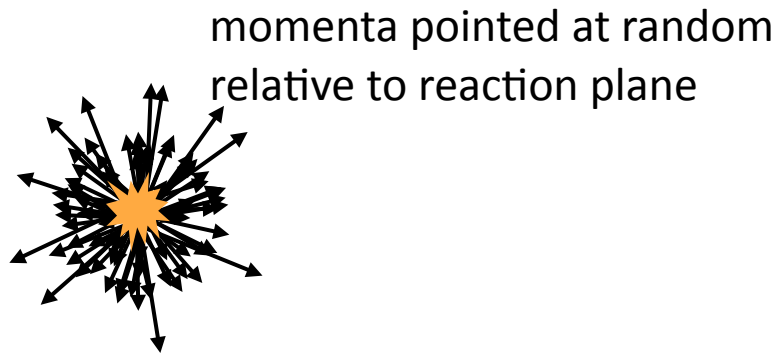
1) Superposition of independent p+p:

momenta pointed at random
relative to reaction plane



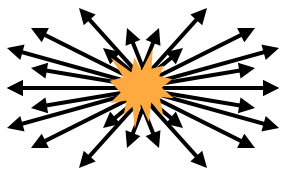
How do semi-central collisions evolve?

1) Superposition of independent p+p:



2) Evolution as a bulk system

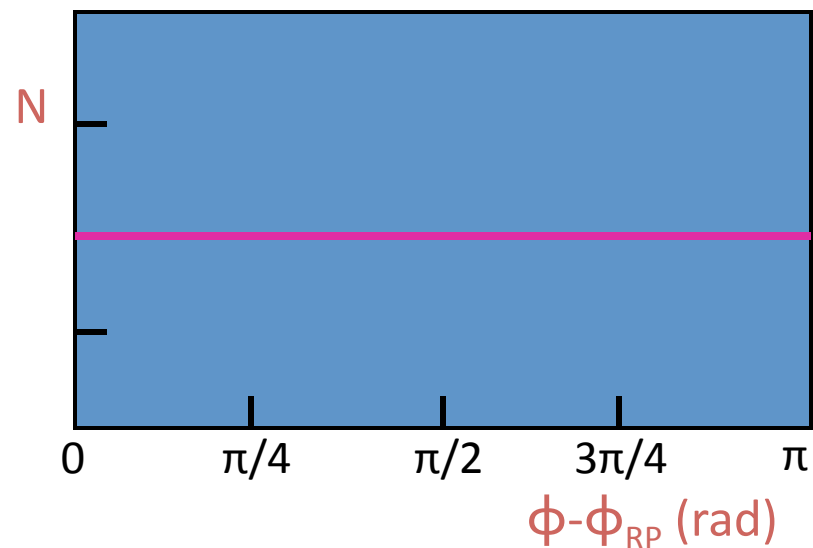
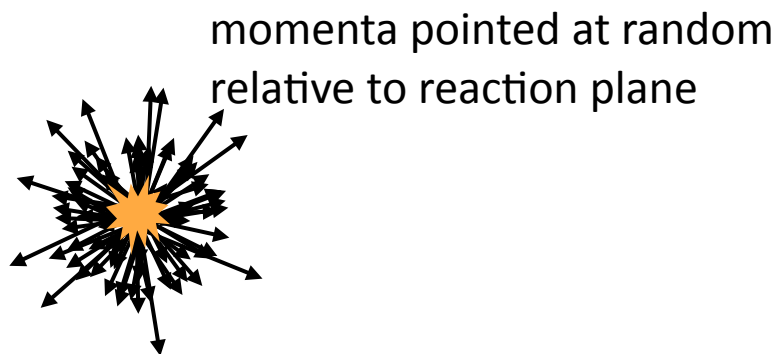
Pressure gradients (larger in-plane) push bulk "out" → "flow"



more, faster particles seen in-plane

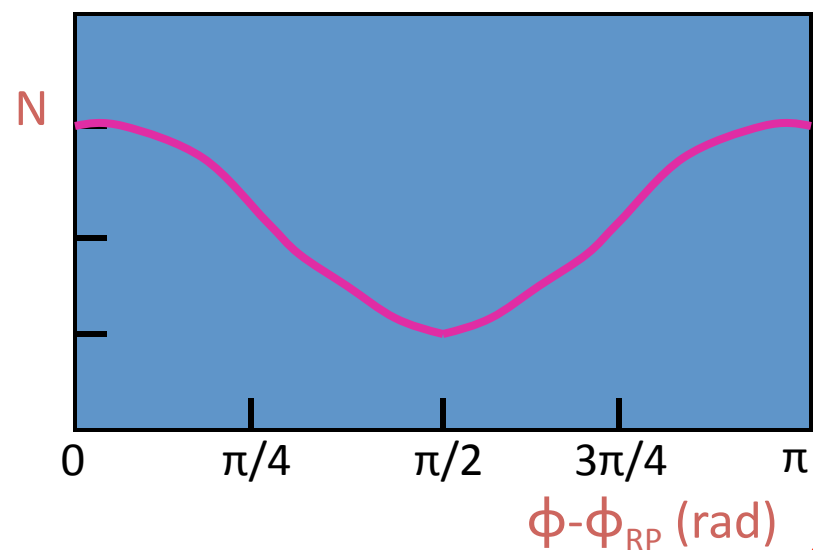
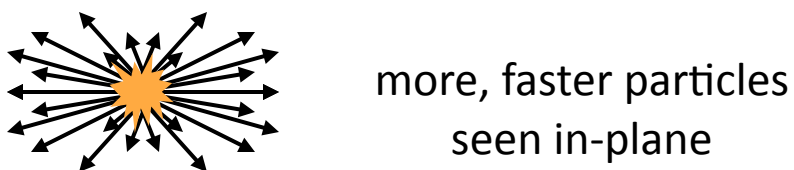
How do semi-central collisions evolve?

1) Superposition of independent p+p:



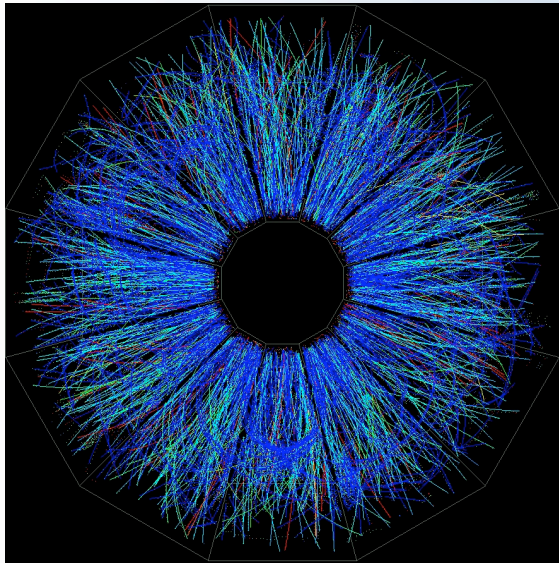
2) Evolution as a bulk system

Pressure gradients (larger in-plane) push
bulk "out" → "flow"

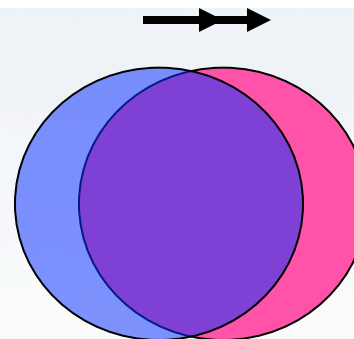
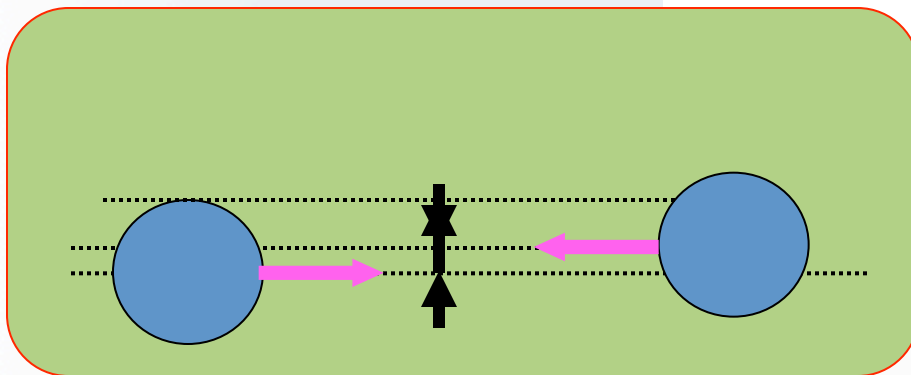
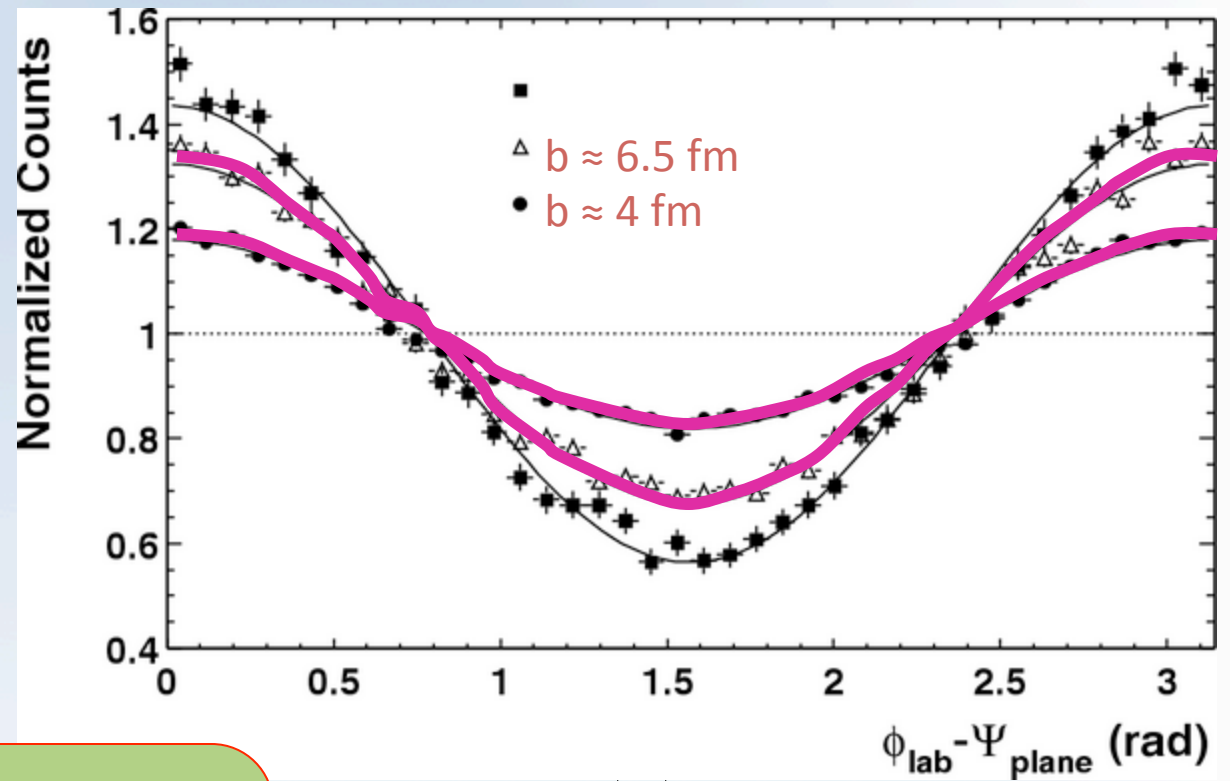


Azimuthal distributions at RHIC

STAR, PRL90 032301 (2003)

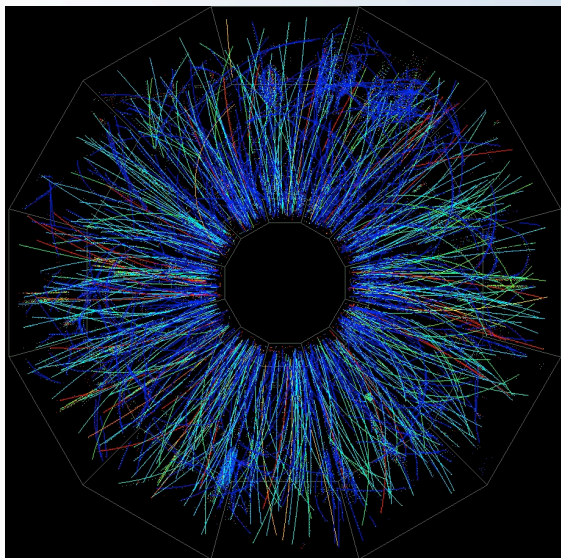


mid-central collisions

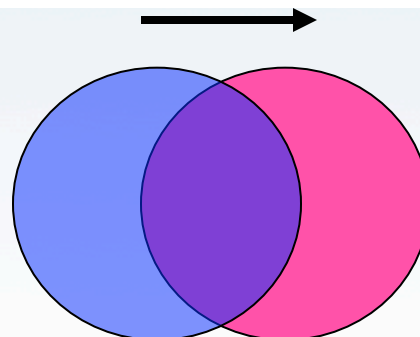
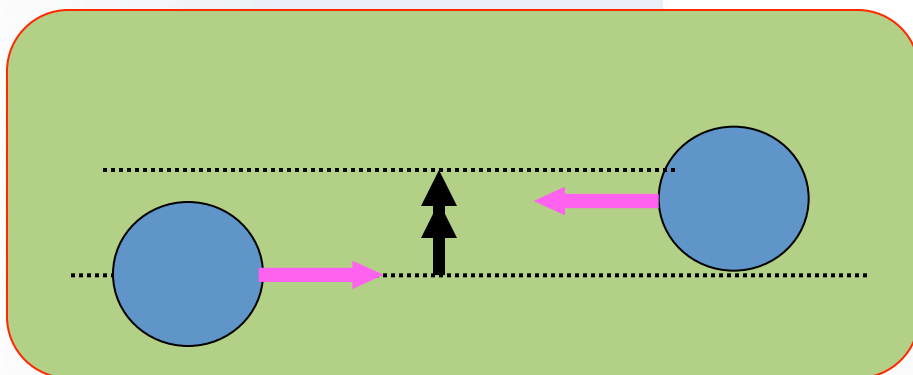
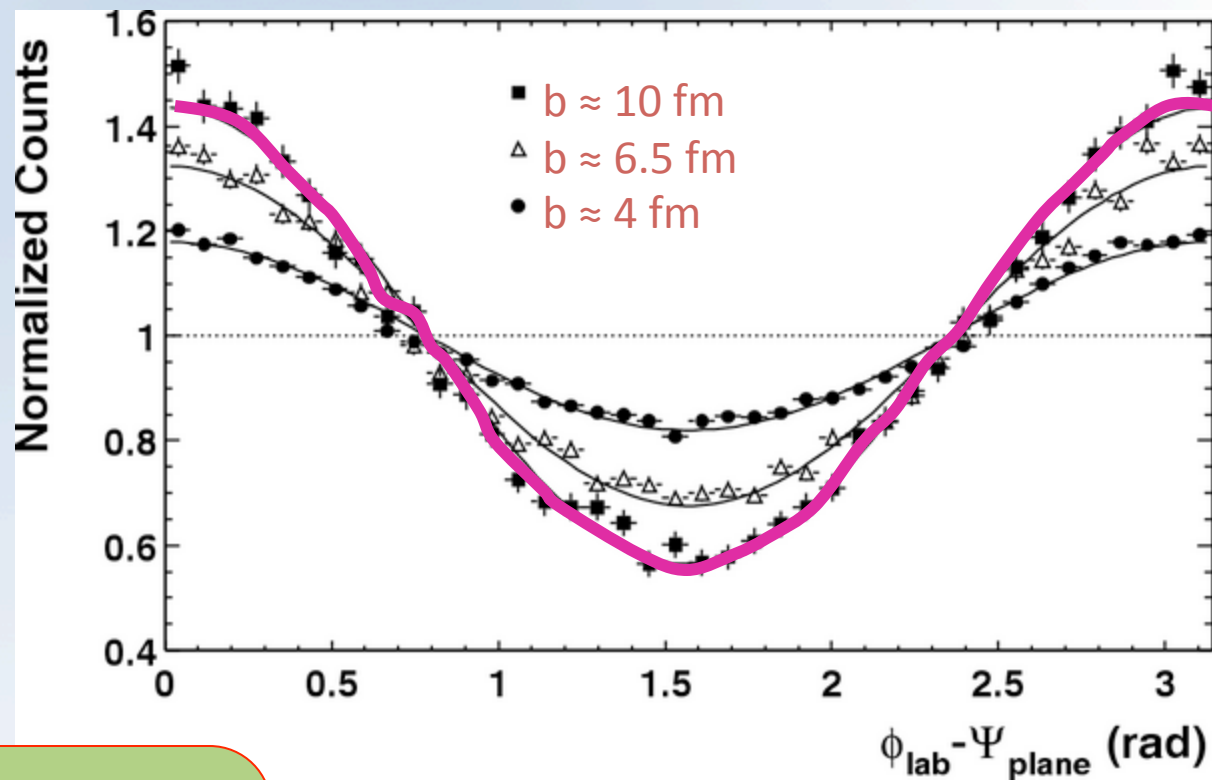


Azimuthal distributions at RHIC

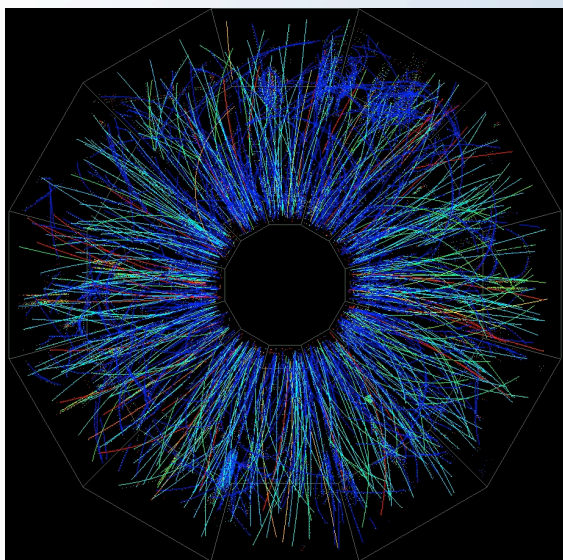
STAR, PRL90 032301 (2003)



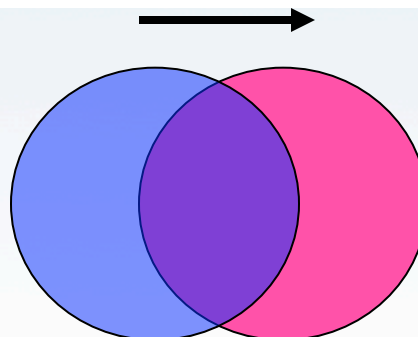
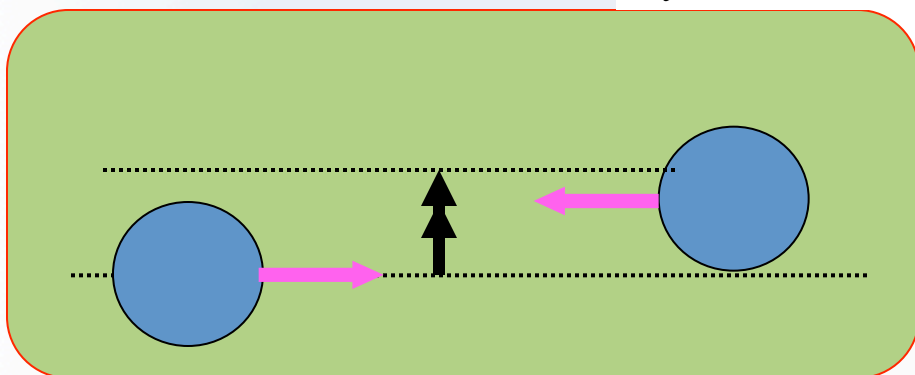
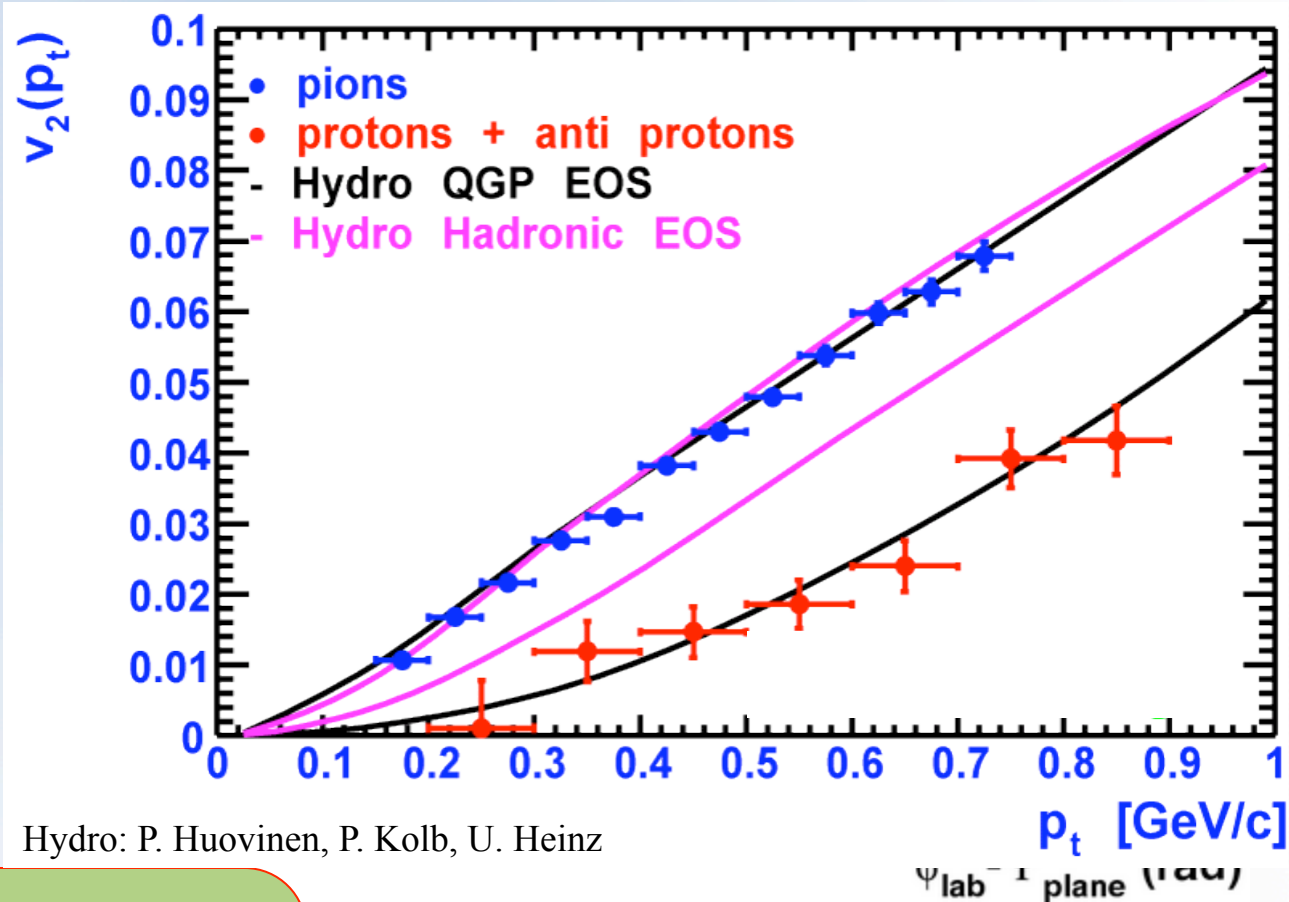
peripheral collisions



Azimuthal distributions at RHIC

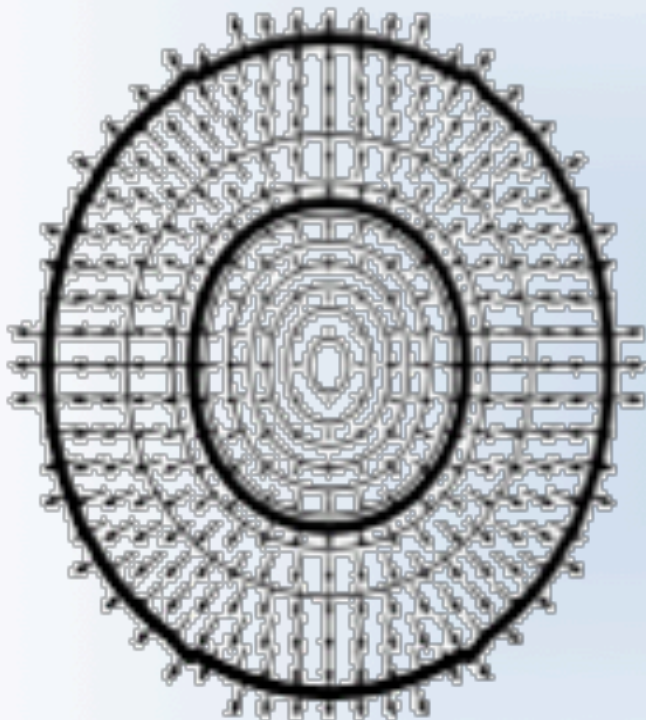


peripheral collisions

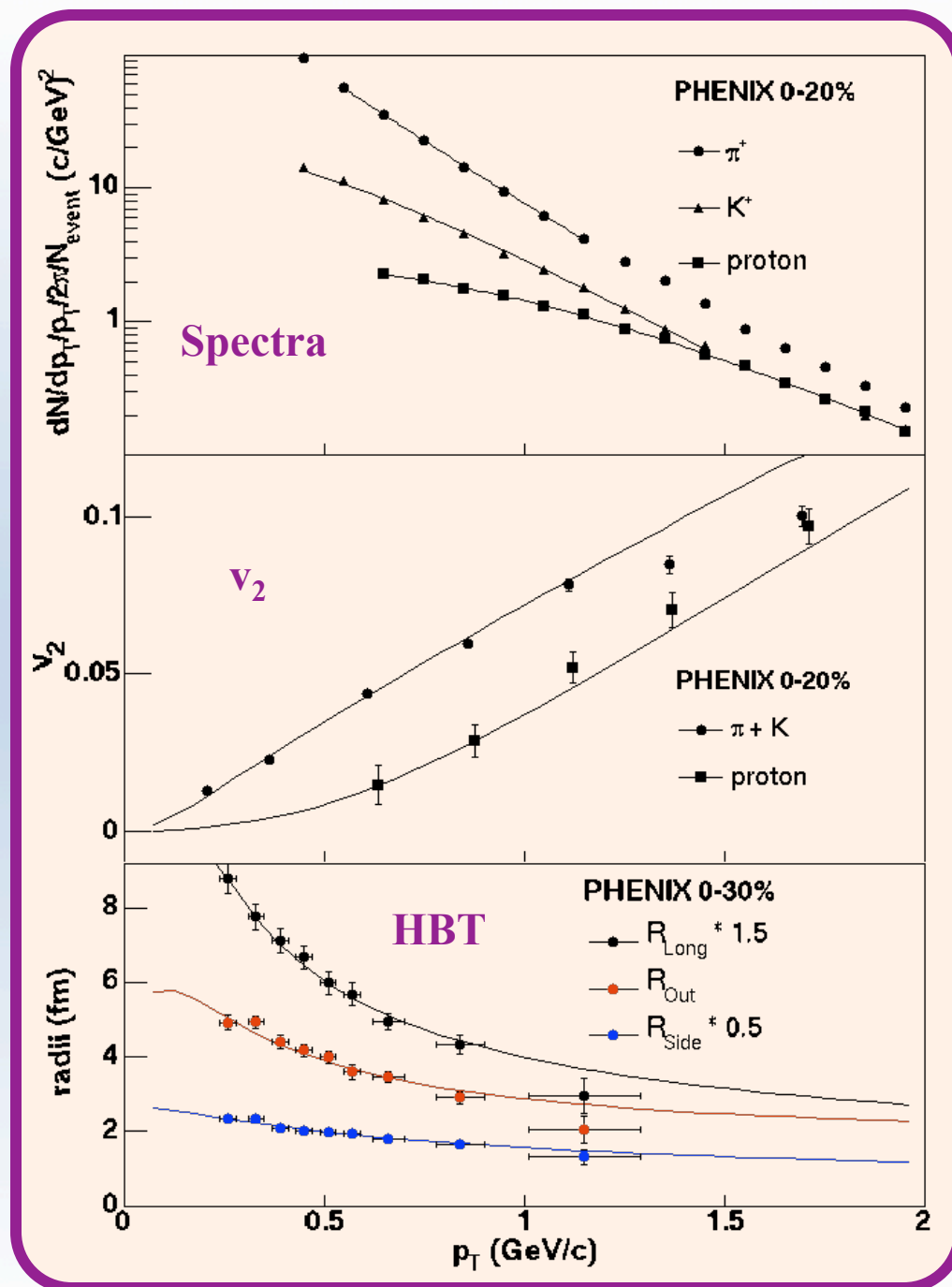


Explosive flow revealed through *specific fingerprints* on soft-sector observables

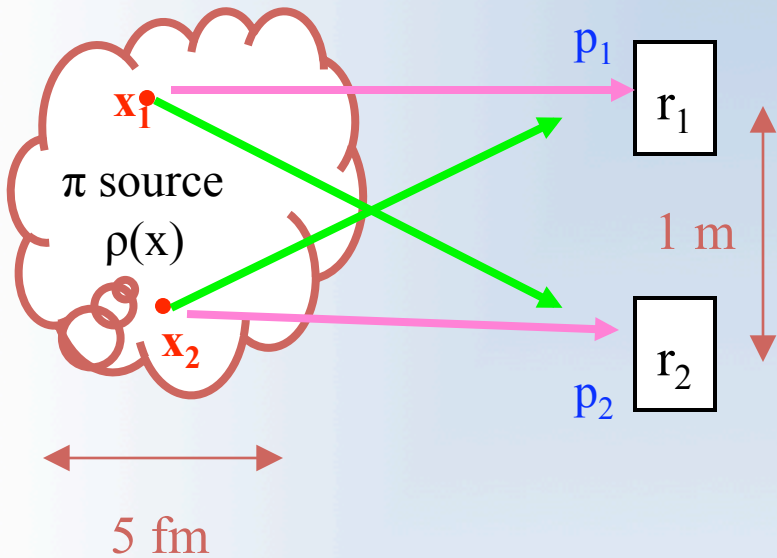
calculable in hydrodynamics or toy “blast wave” models



but the *defining* characteristic: correlated position and boost direction...



probing source geometry through interferometry



$$\Psi_T = \frac{1}{\sqrt{2}} \left\{ U(\vec{x}_1, \vec{p}_1) e^{i(\vec{r}_1 - \vec{x}_1) \cdot \vec{p}_1} U(\vec{x}_2, \vec{p}_2) e^{i(\vec{r}_2 - \vec{x}_2) \cdot \vec{p}_2} + U(\vec{x}_2, \vec{p}_1) e^{i(\vec{r}_1 - \vec{x}_2) \cdot \vec{p}_1} U(\vec{x}_1, \vec{p}_2) e^{i(\vec{r}_2 - \vec{x}_1) \cdot \vec{p}_2} \right\}$$

$$\Psi_T^* \Psi_T = U_1^* U_1 U_2^* U_2 \left(1 + e^{iq \cdot (x_1 - x_2)} \right)$$

experimentally measuring this enhanced probability:

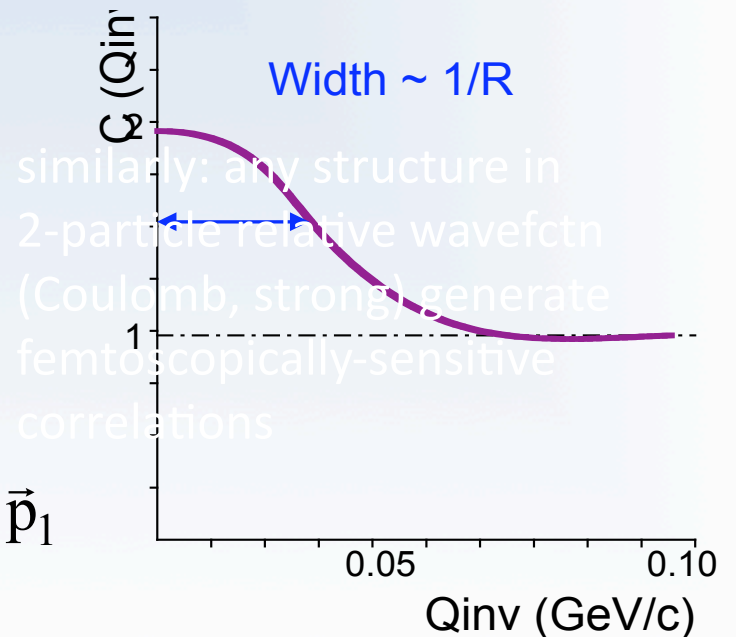
Creation probability $\Gamma(x, p) = U^* U$

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = 1 + |\tilde{\rho}(q)|^2$$

↑
Measurable!

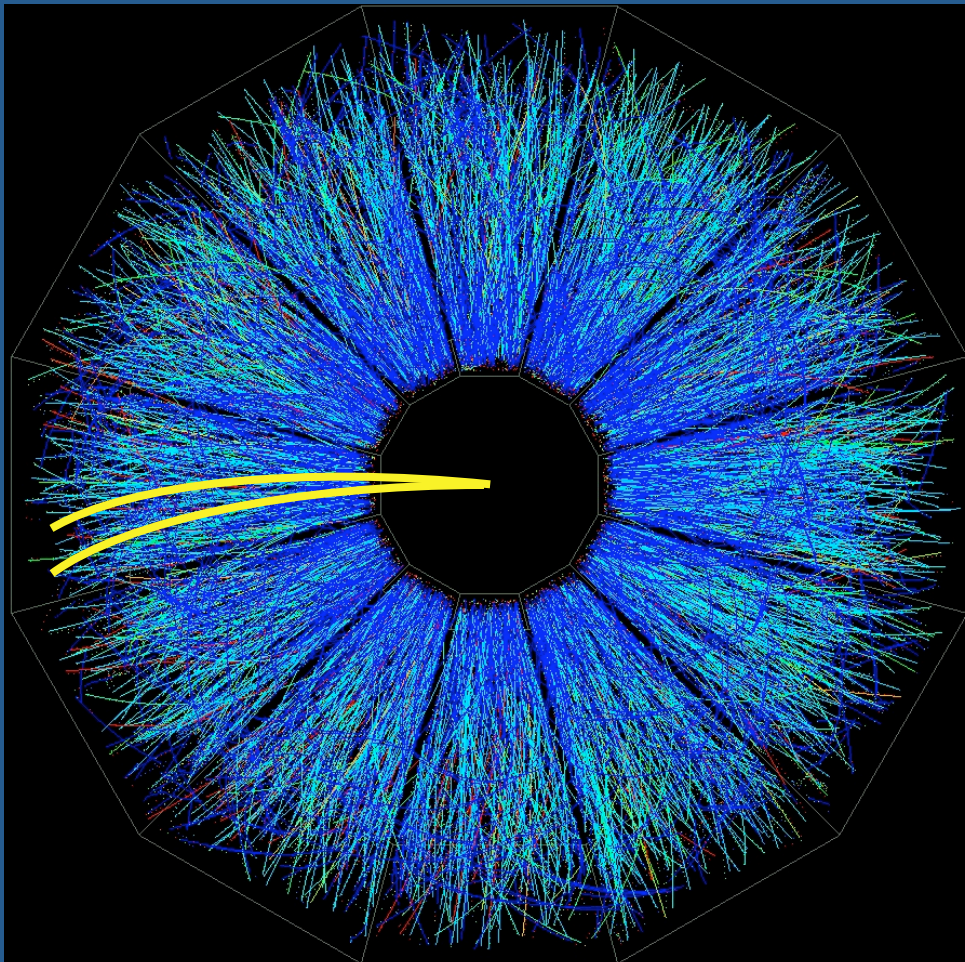
↑
F.T. of pion source

$$\vec{q} = \vec{p}_2 - \vec{p}_1$$



The Bottom line...

if a pion is emitted, it is more likely to emit another pion *with very similar momentum* if the source is small

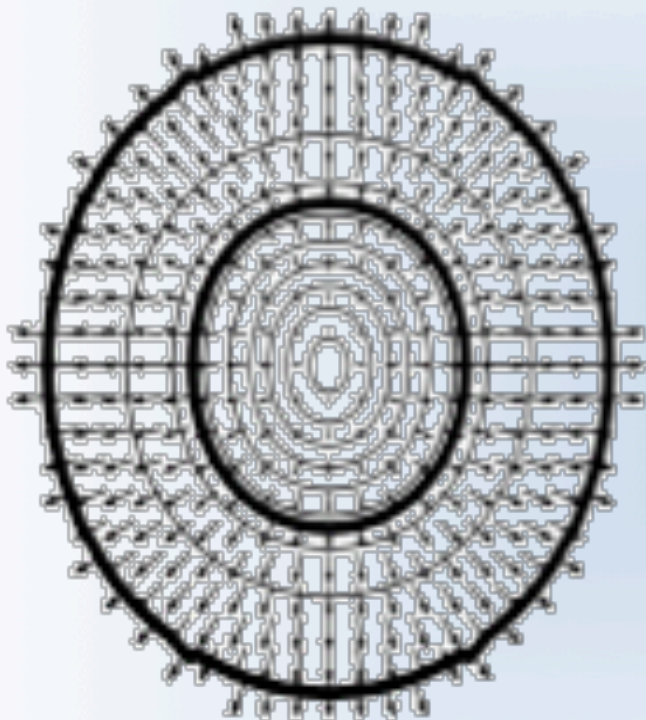


experimentally measuring this enhanced probability: quite challenging

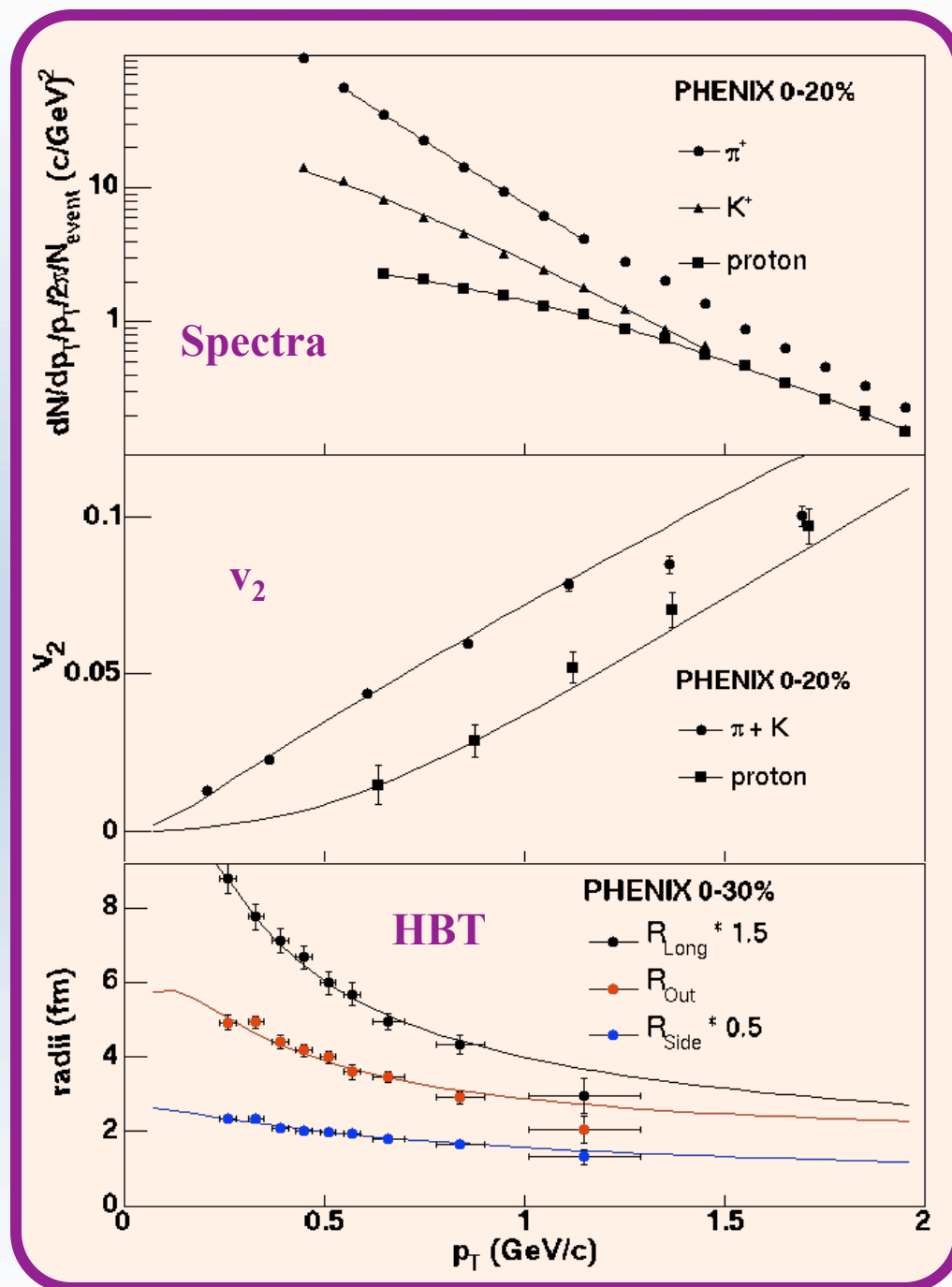
similarly: any structure in 2-particle relative wavefctn (Coulomb, strong) generate femtoscopically-sensitive correlations

Explosive flow revealed through *specific fingerprints* on soft-sector observables

calculable in hydrodynamics or toy “blast wave” models



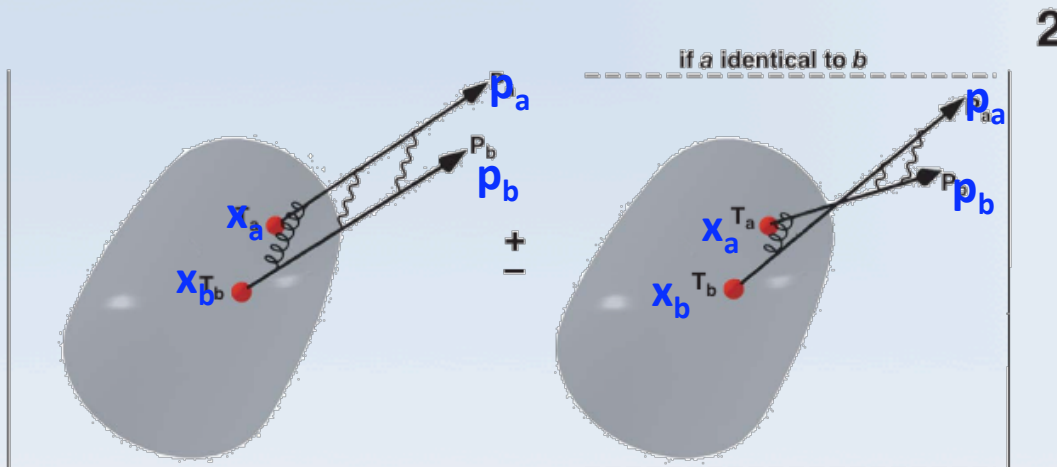
but the *defining* characteristic: correlated position and boost direction...



Femtoscopic information

$$S_{\vec{p}}^{ab}(\vec{r}') = (\vec{x}_a - \vec{x}_b) \text{ distribution}$$

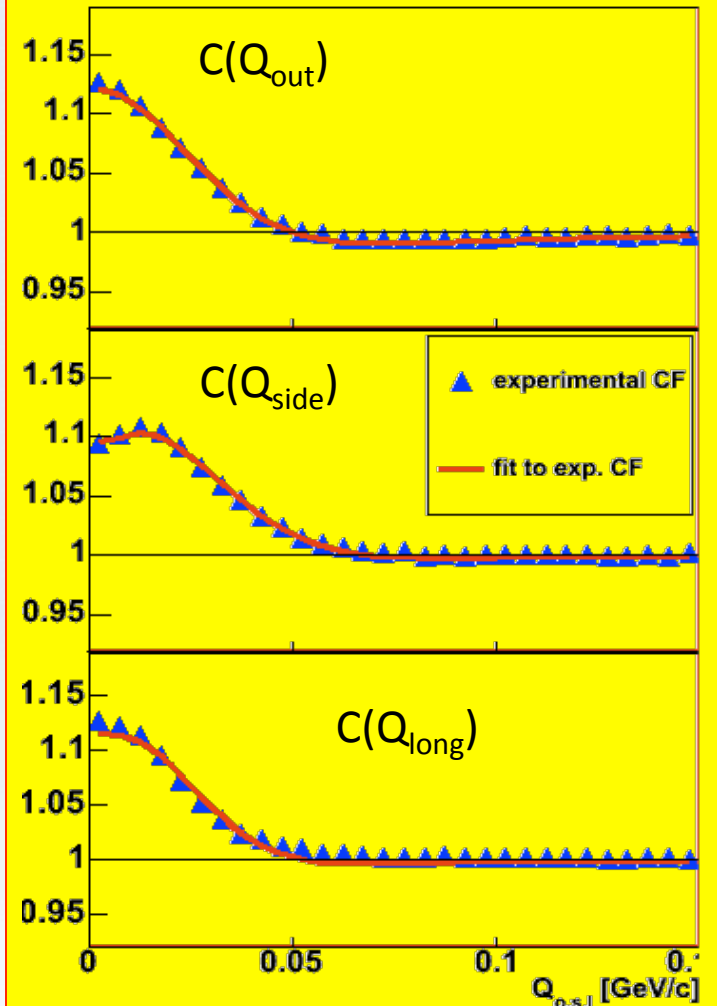
$$\phi(\vec{q}', \vec{r}') = (a,b) \text{ relative wavefctn}$$



$$C_{\vec{p}}^{ab}(\vec{q}) = \int d^3\vec{r}' \cdot S_{\vec{p}}^{ab}(\vec{r}') \cdot |\phi(\vec{q}', \vec{r}')|^2$$

- femtoscopic correlation at low $|q|$
- must vanish at high $|q|$. [indep "direction"]

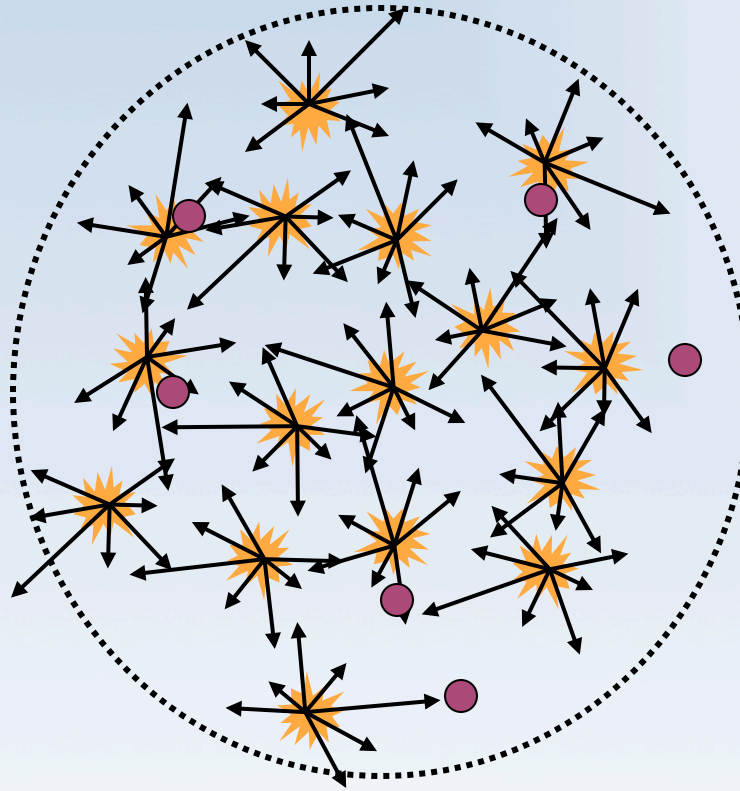
Au+Au: central collisions



3 "radii" by using
3-D vector q

Geometric substructure?

random (non-)system:
all observers measure the
“whole source”

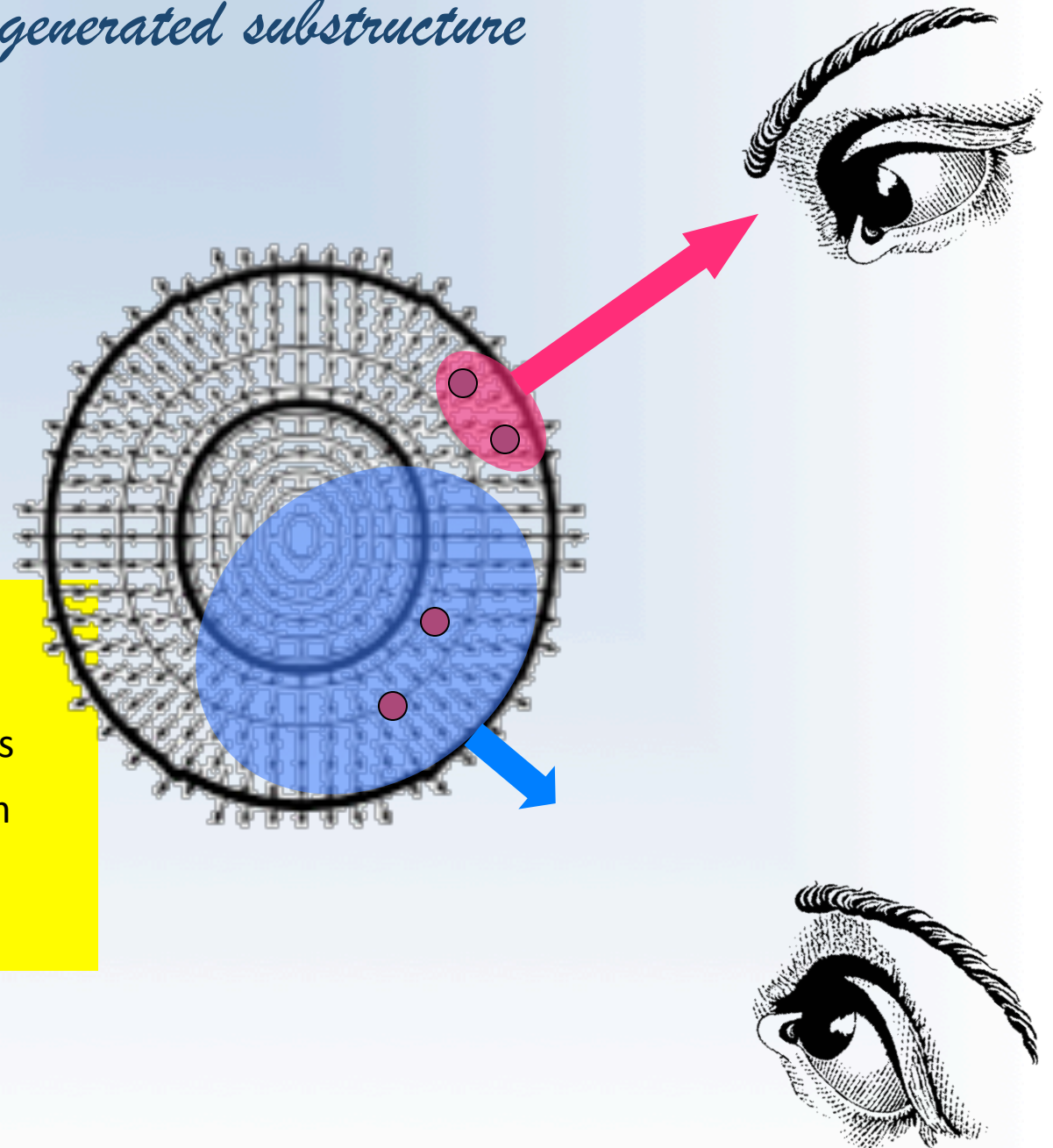


Flow-generated substructure

random (non-)system:
all observers measure the
“whole source”

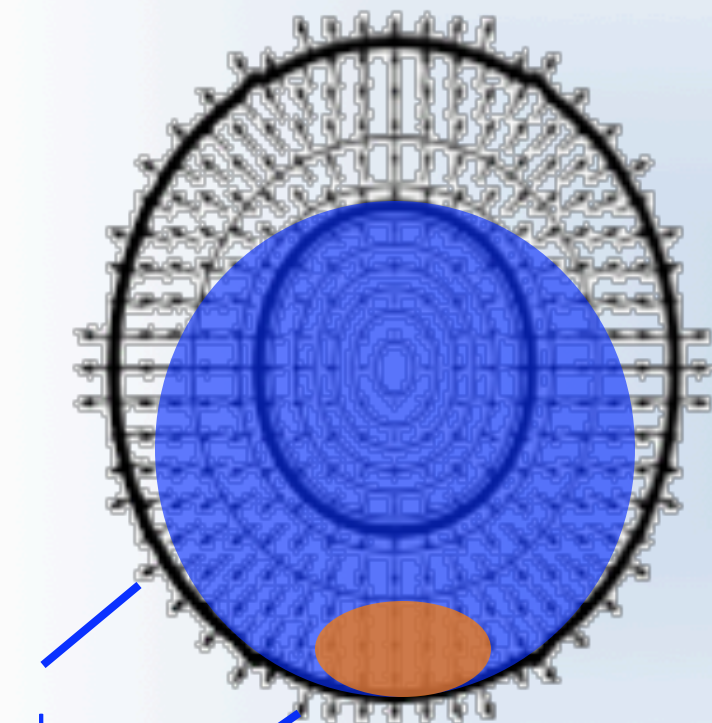
Specific predictions of
bulk **global** collective flow:

- space-momentum (x - p) correlations
- faster (high p_T) particles come from
 - **smaller** source
 - closer to “the **edge**”

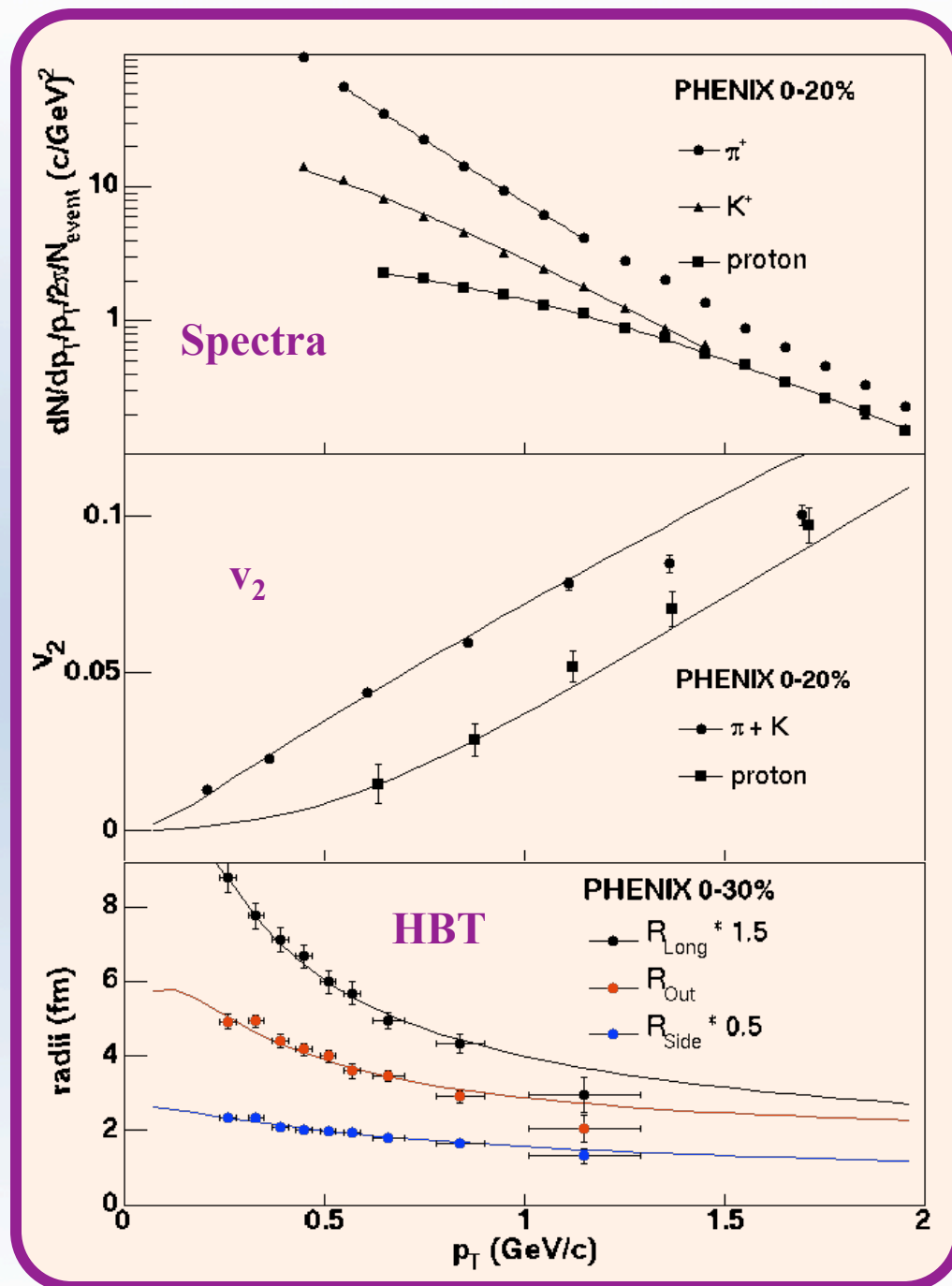


Explosive flow revealed through *specific fingerprints* on soft-sector observables

calculable in hydrodynamics or toy “blast wave” models

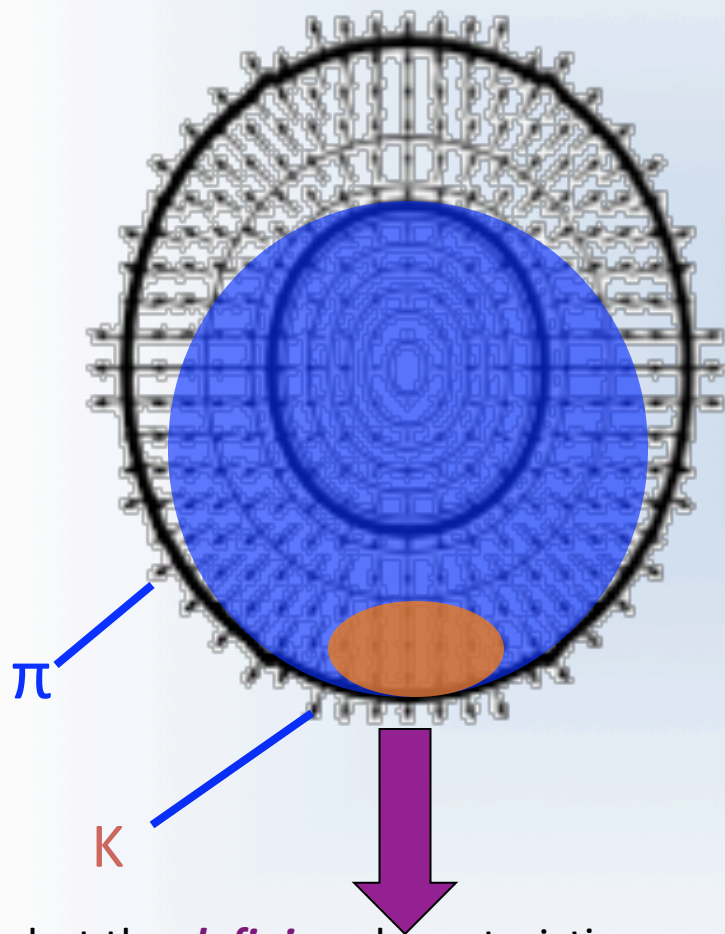


but the *defining* characteristic:
correlated position and boost direction

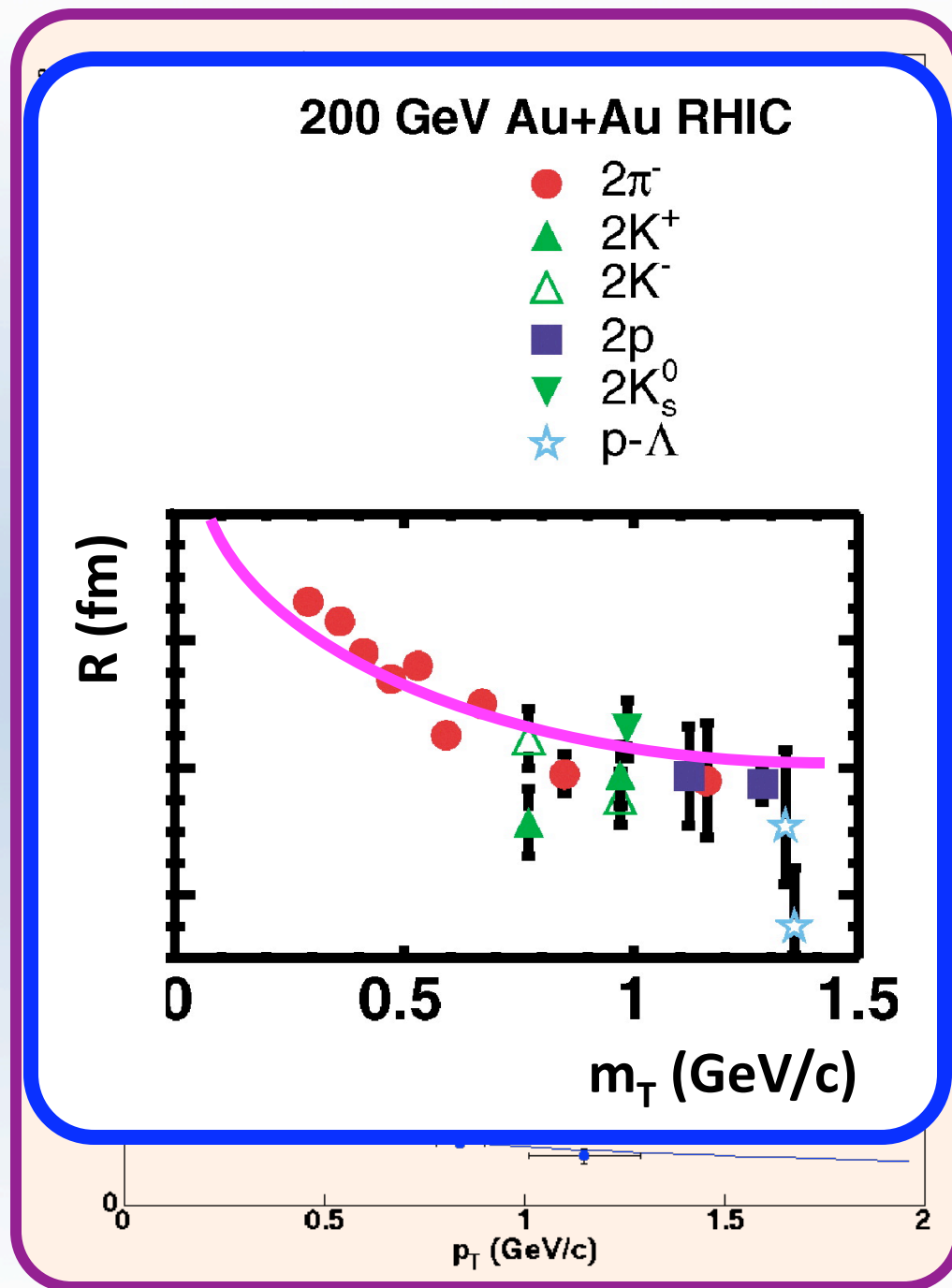


Explosive flow revealed through *specific fingerprints* on soft-sector observables

calculable in hydrodynamics or toy “blast wave” models

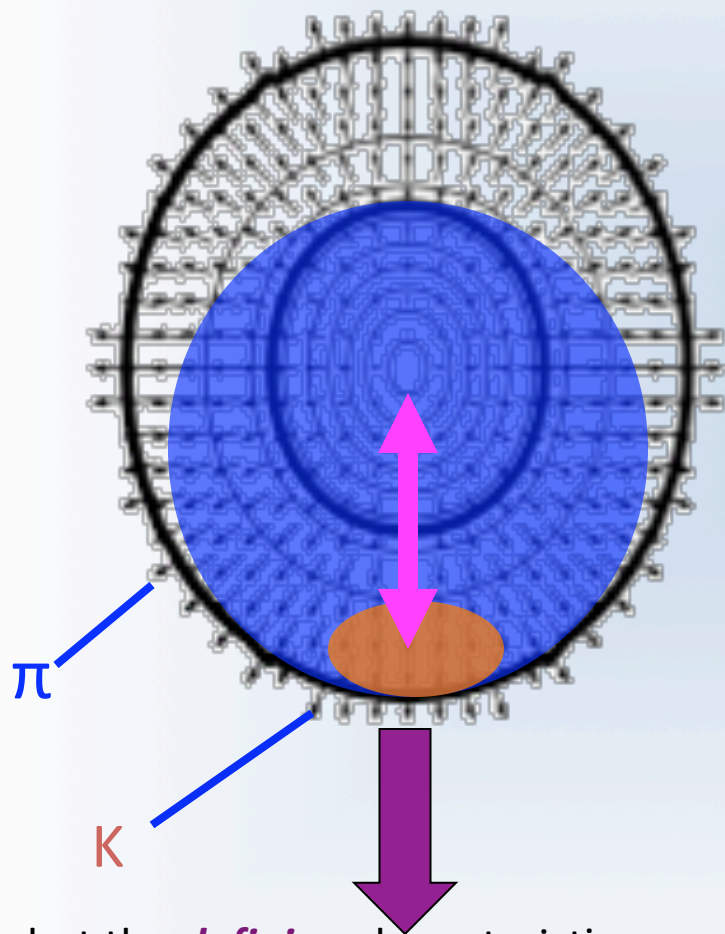


but the *defining* characteristic:
correlated position and boost direction

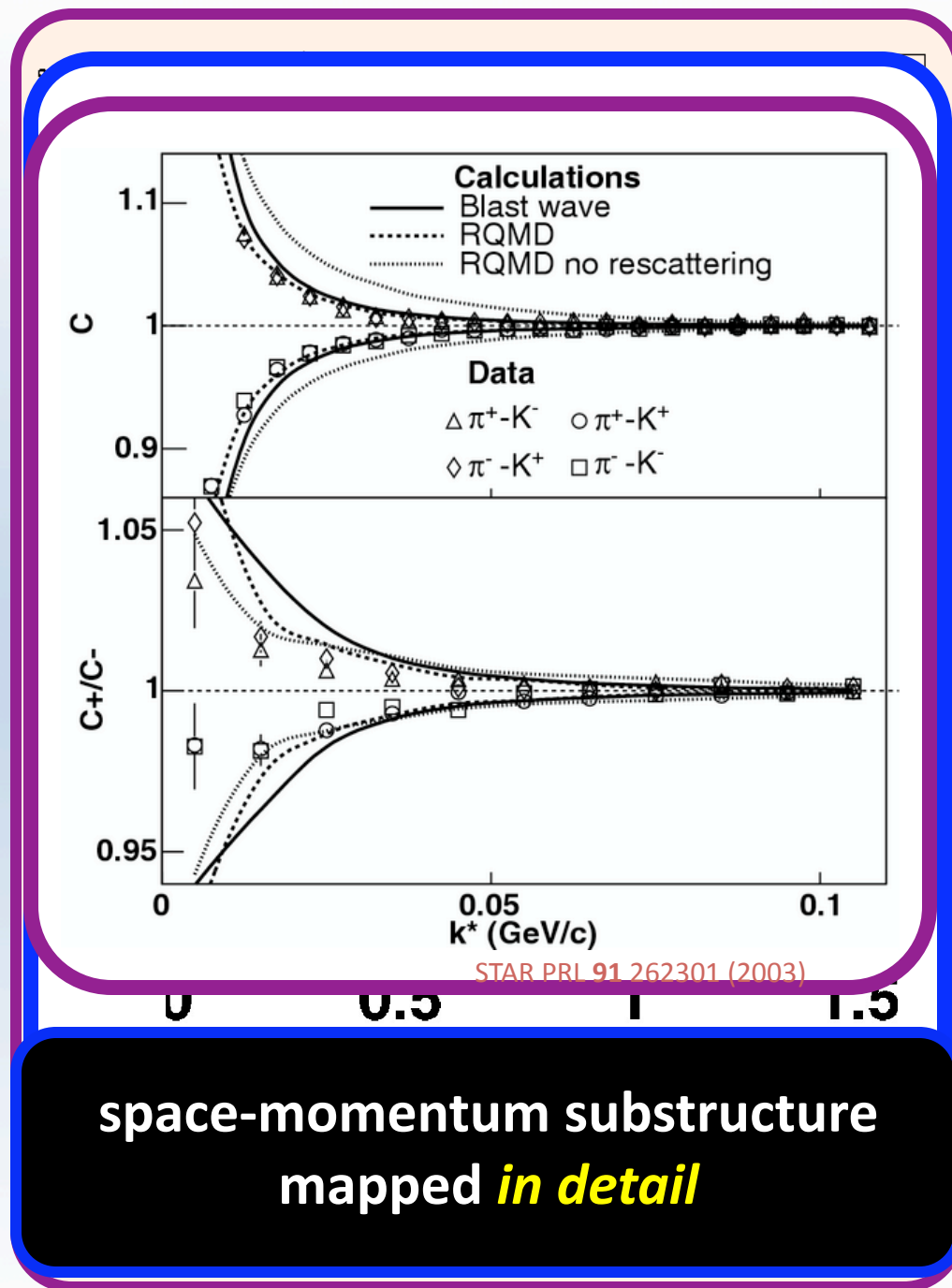


Explosive flow revealed through *specific fingerprints* on soft-sector observables

calculable in hydrodynamics or toy “blast wave” models



but the *defining* characteristic: correlated position and boost direction



space-momentum substructure mapped *in detail*

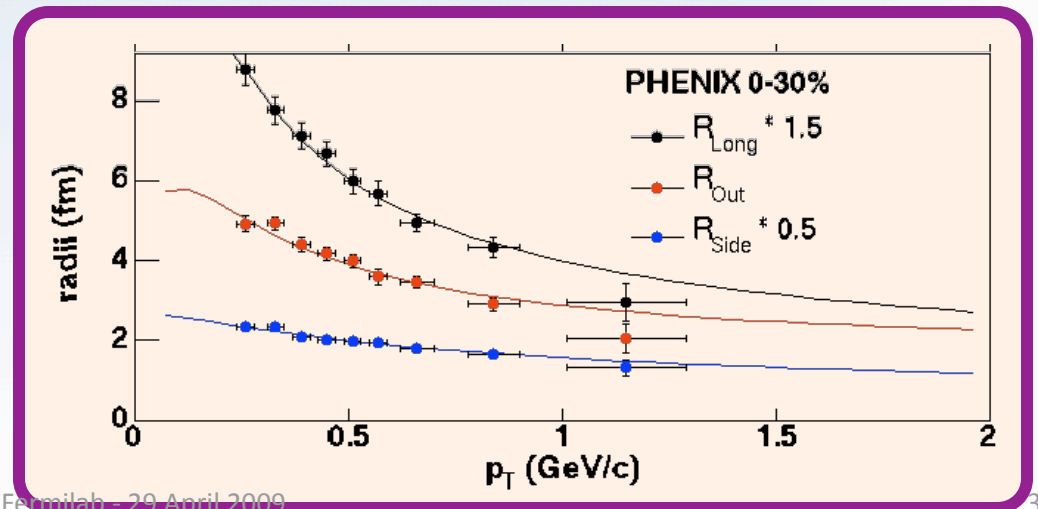
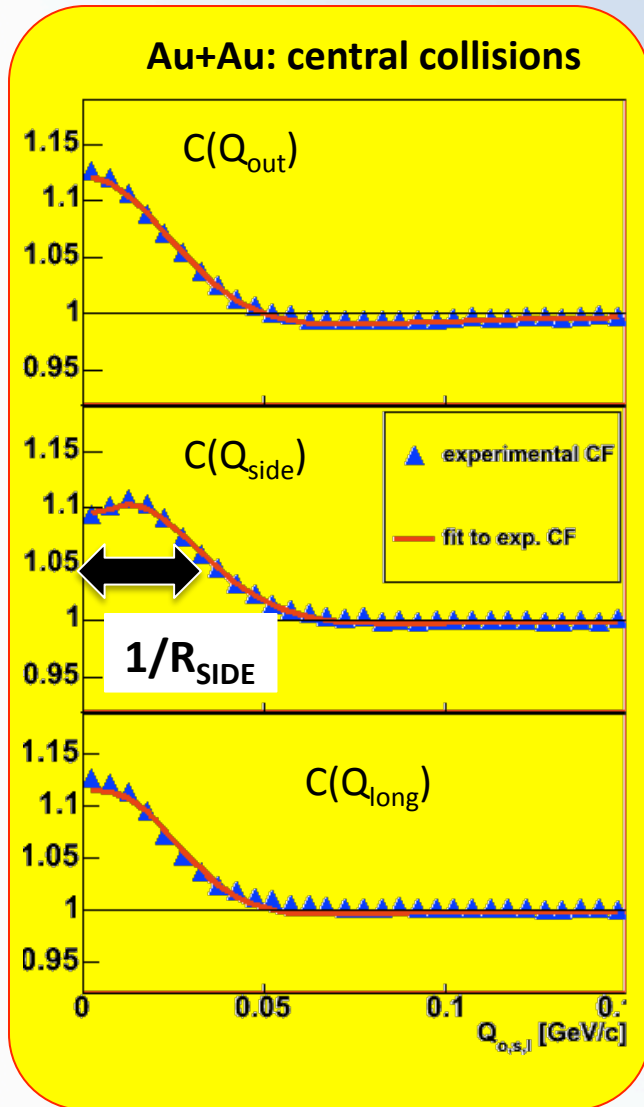
Obtaining 3D radii from 3D correlation functions

$$C(\vec{q}) = N \cdot \left[1 + \lambda \cdot \left(K_{coul}(\vec{q}) \cdot \left\{ 1 + e^{-\left(q_o^2 R_o^2 + q_s^2 R_s^2 + q_l^2 R_l^2 \right)} \right\} - 1 \right) \right]$$

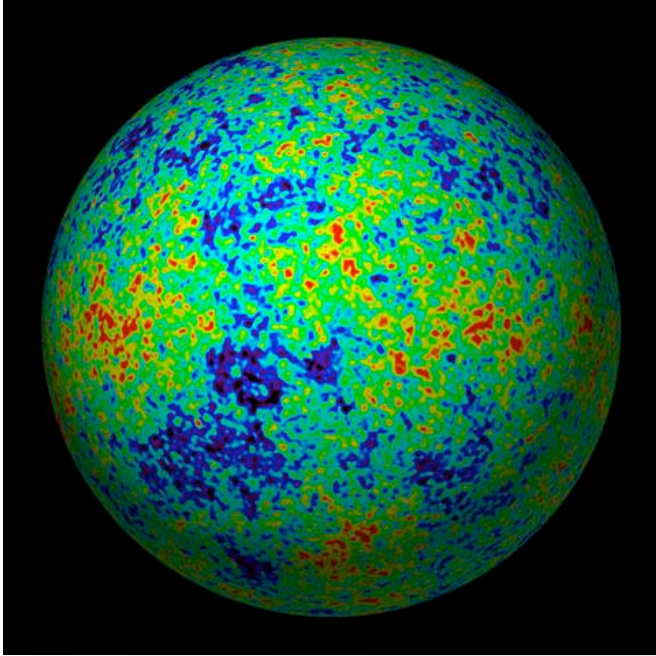
typical "Gaussian" fitting function

- Au+Au: "Gaussian" radii capture bulk scales
 - (resonance tails from imaging)
- $R(p_T)$ consistent with explosive flow

"set of zero measure" of full 3D correlation fctn



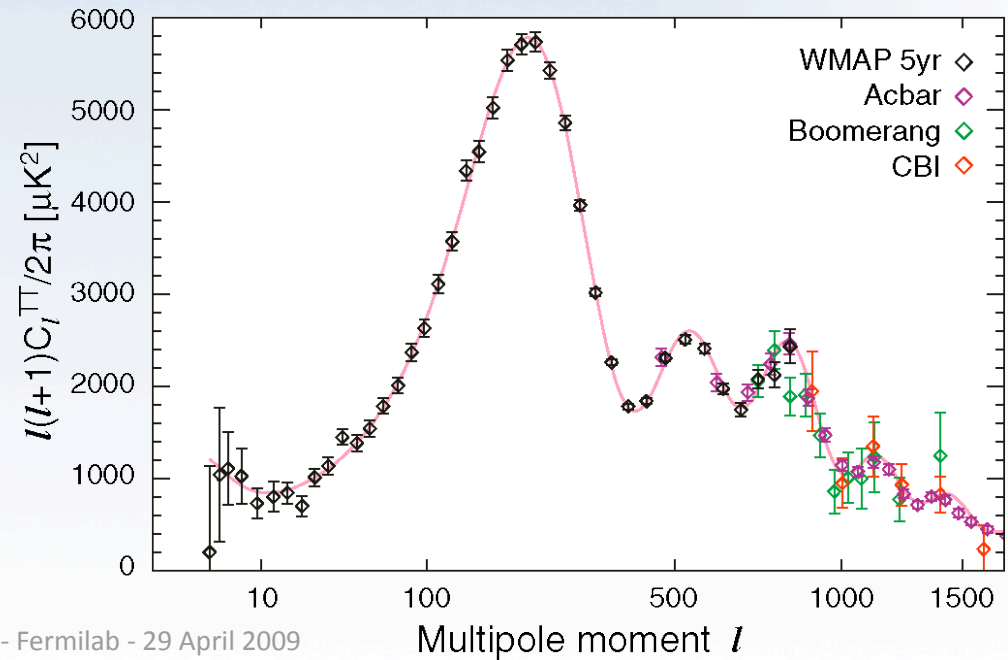
Spherical harmonic representation of 3D data



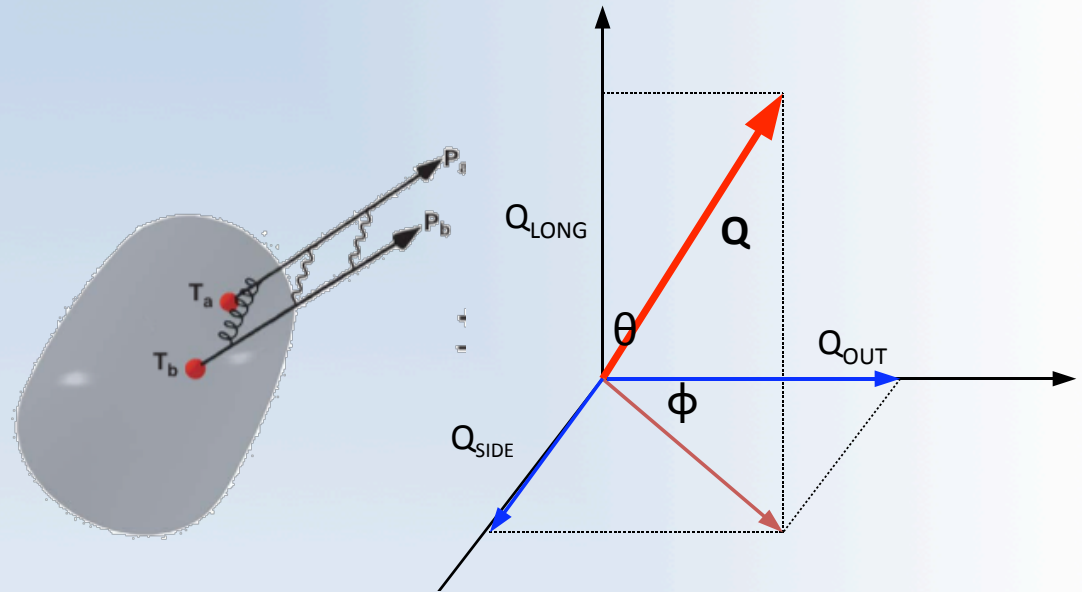
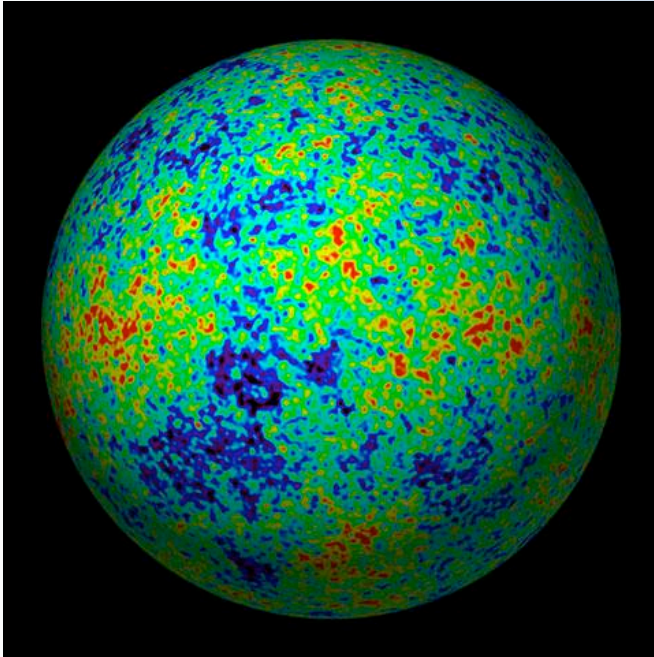
$$a_{l,m} \equiv \int d\Omega \cdot T(\theta, \phi) \cdot Y_{l,m}^*(\theta, \phi)$$

$$C_l^{TT} \equiv \left\langle |a_{l,m}|^2 \right\rangle_m$$

(average over m \square no "special" direction)



Spherical harmonic representation of 3D data



$$a_{l,m} \equiv \int d\Omega \cdot T(\theta, \phi) \cdot Y_{l,m}^*(\theta, \phi)$$

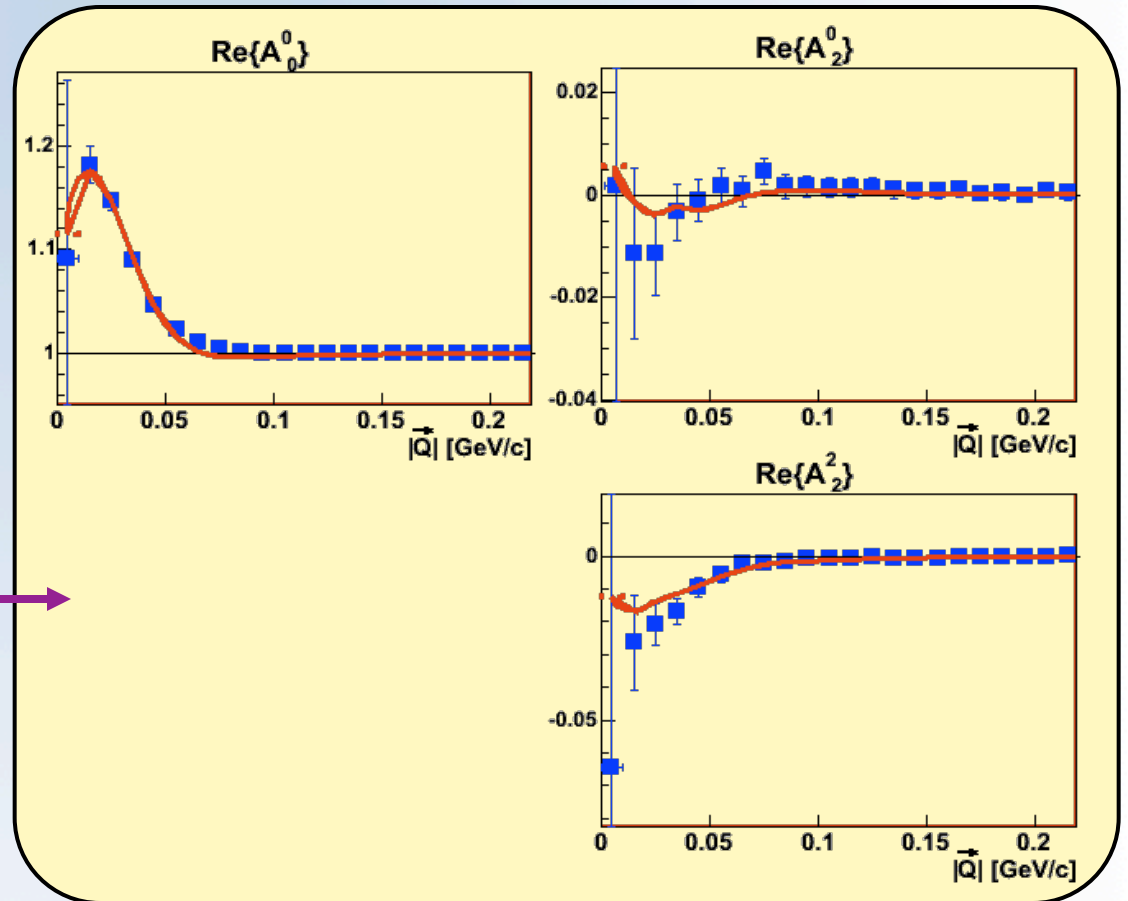
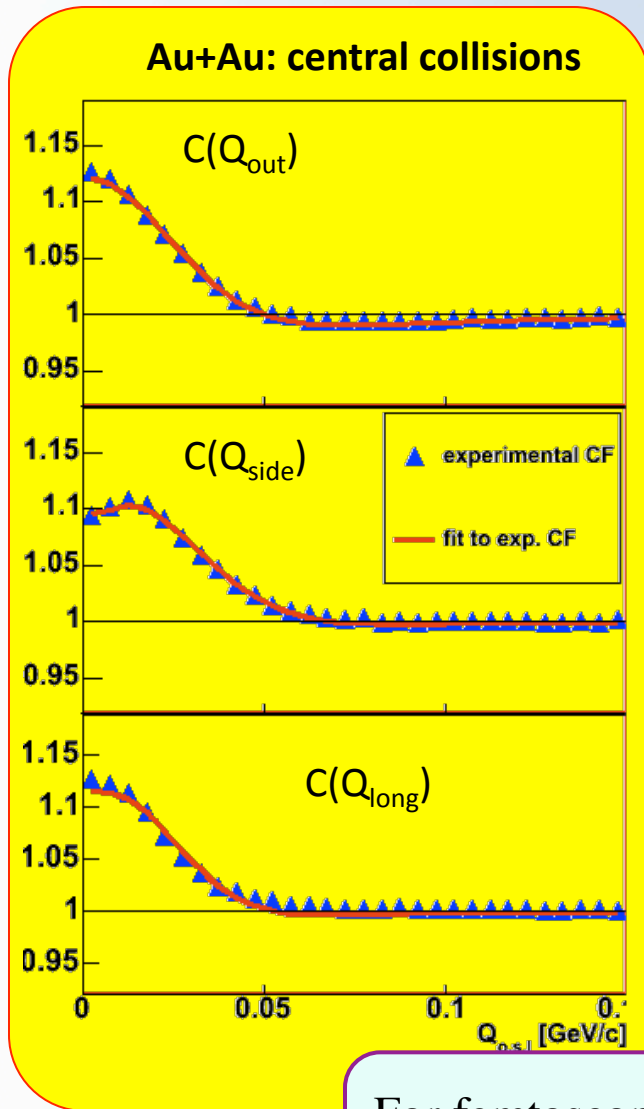
$$C_l^{TT} \equiv \left\langle |a_{l,m}|^2 \right\rangle_m$$

$$A_{l,m}(|\vec{Q}|) = \frac{\Delta_{\cos\theta} \Delta_{\phi}}{\sqrt{4\pi}} \sum_i^{bins} Y_{l,m}^*(\theta_i, \phi_i) C(|\vec{Q}|, \cos\theta_i, \phi_i)$$

Z. Chajeccki & MAL, PRC 78 064903 (2008)

(average over m \boxtimes no "special" direction)

Spherical harmonic representation of 3D data



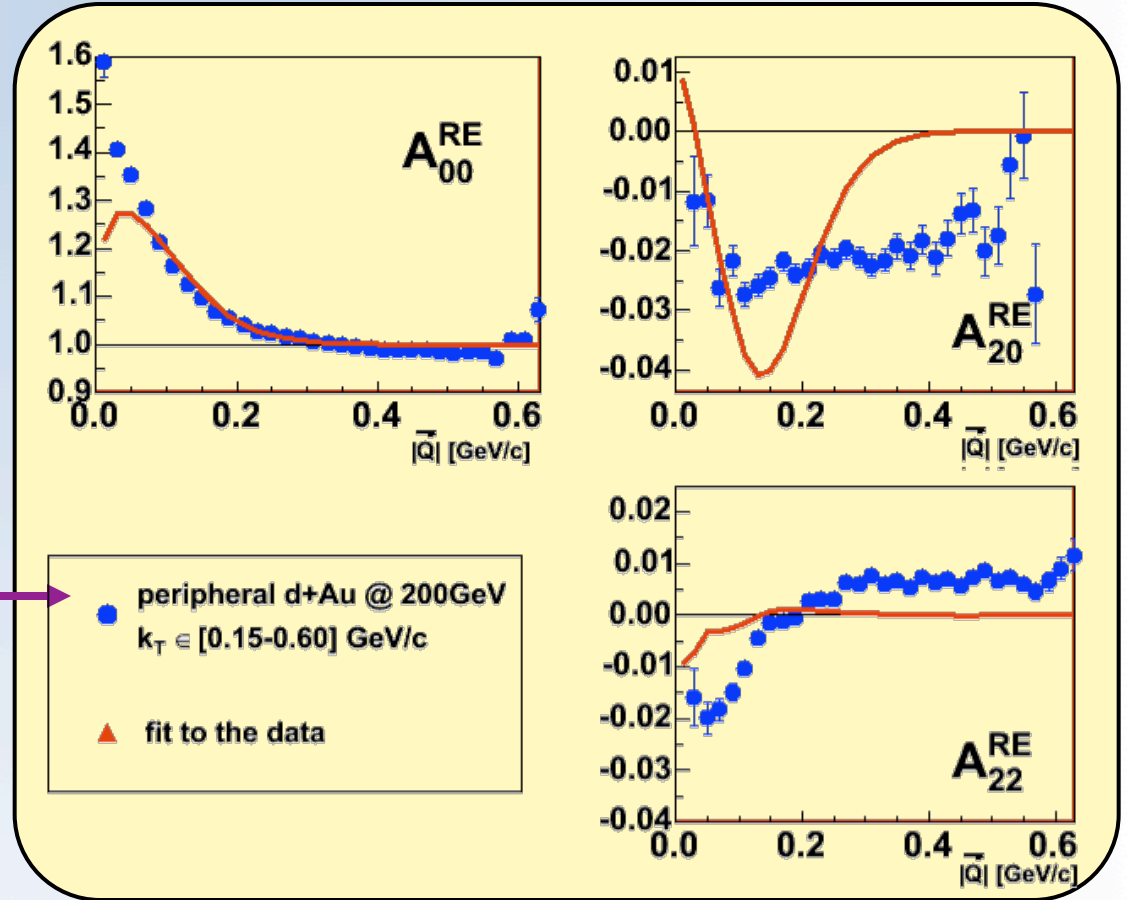
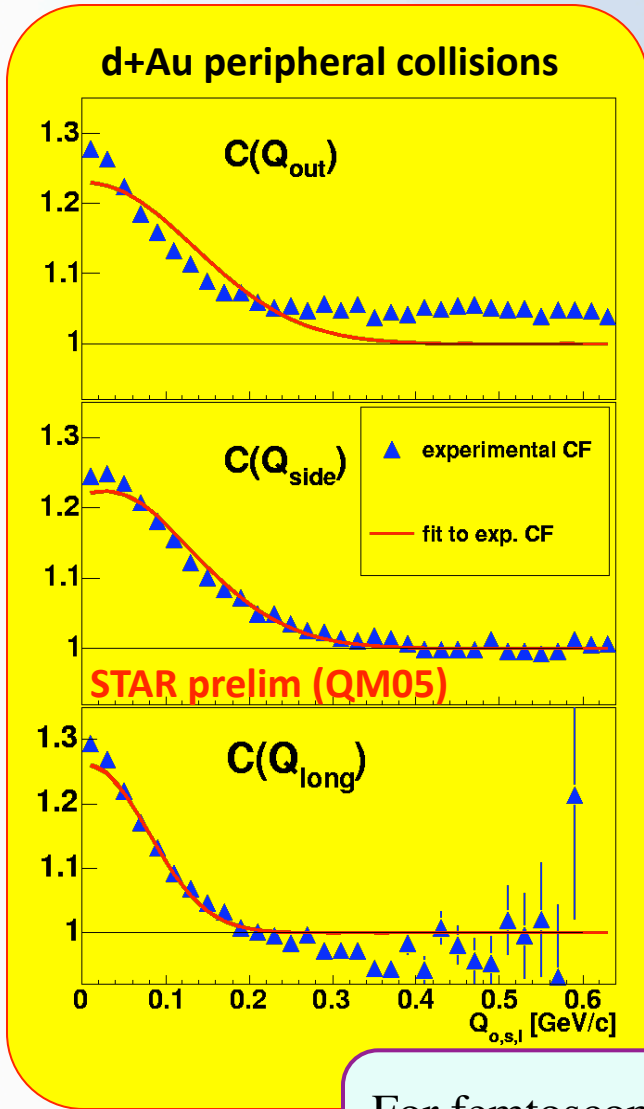
$$A_{l,m}(|\vec{Q}|) = \frac{\Delta_{\cos\theta}\Delta_{\phi}}{\sqrt{4\pi}} \sum_i^{bins} Y_{l,m}^*(\theta_i, \phi_i) C(|\vec{Q}|, \cos\theta_i, \phi_i)$$

Z. Chajęcki & MAL, PRC 78 064903 (2008)

For femtoscopic correlations:

$$C(\vec{q}; |\vec{q}| \rightarrow \infty) = C(|\vec{q}| \rightarrow \infty) \Rightarrow A_{l \neq 0}^m(|\vec{q}| \rightarrow \infty) = 0$$

Spherical harmonic representation of 3D data



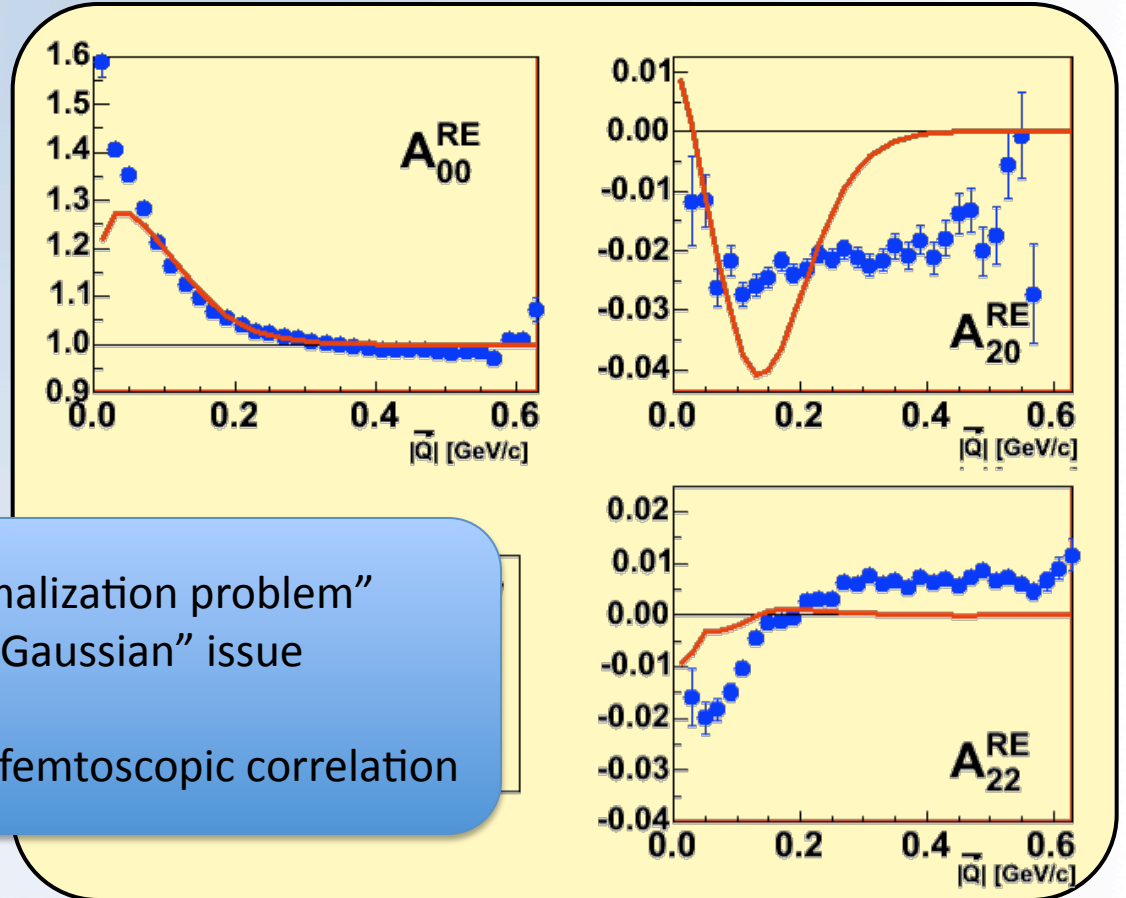
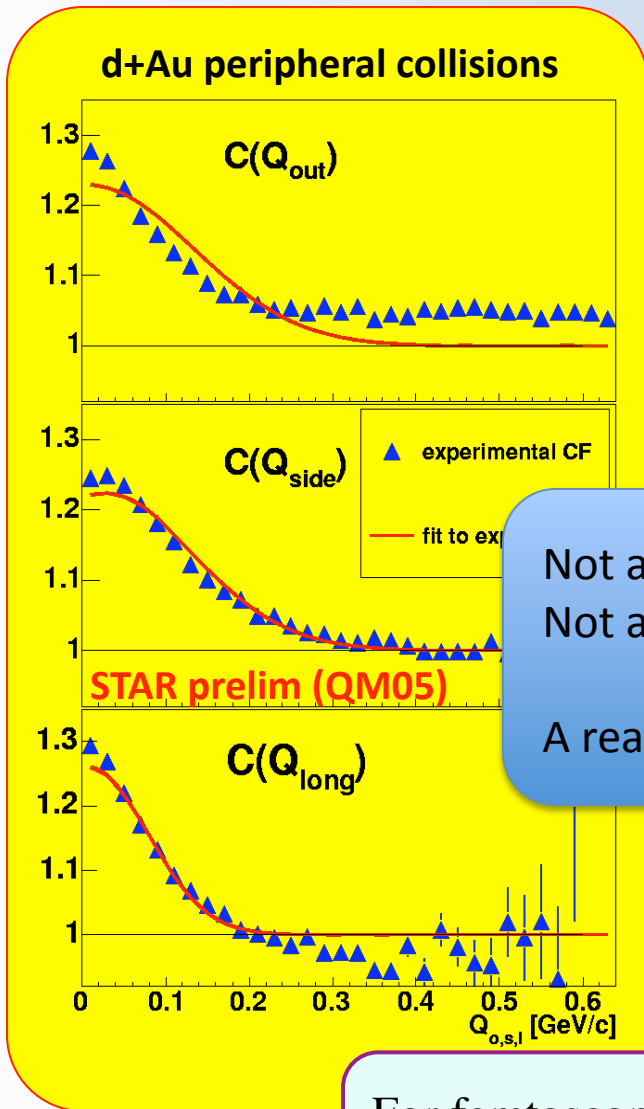
$$A_{l,m}(|\vec{Q}|) = \frac{\Delta_{\cos\theta}\Delta_{\phi}}{\sqrt{4\pi}} \sum_i^{bins} Y_{l,m}^*(\theta_i, \phi_i) C(|\vec{Q}|, \cos\theta_i, \phi_i)$$

Z. Chajeccki & MAL, PRC 78 064903 (2008)

For femtoscopic correlations:

$$C(\vec{q}; |\vec{q}| \rightarrow \infty) = C(|\vec{q}| \rightarrow \infty) \Rightarrow A_{l \neq 0}^m(|\vec{q}| \rightarrow \infty) = 0$$

Spherical harmonic representation of 3D data



Not a “normalization problem”
 Not a “non-Gaussian” issue
 A real, non-femtoscopic correlation

$$A_{l,m}(|\vec{Q}|) = \frac{\Delta_{\cos\theta}\Delta_{\phi}}{\sqrt{4\pi}} \sum_i^{bins} Y_{l,m}^*(\theta_i, \phi_i) C(|\vec{Q}|, \cos\theta_i, \phi_i)$$

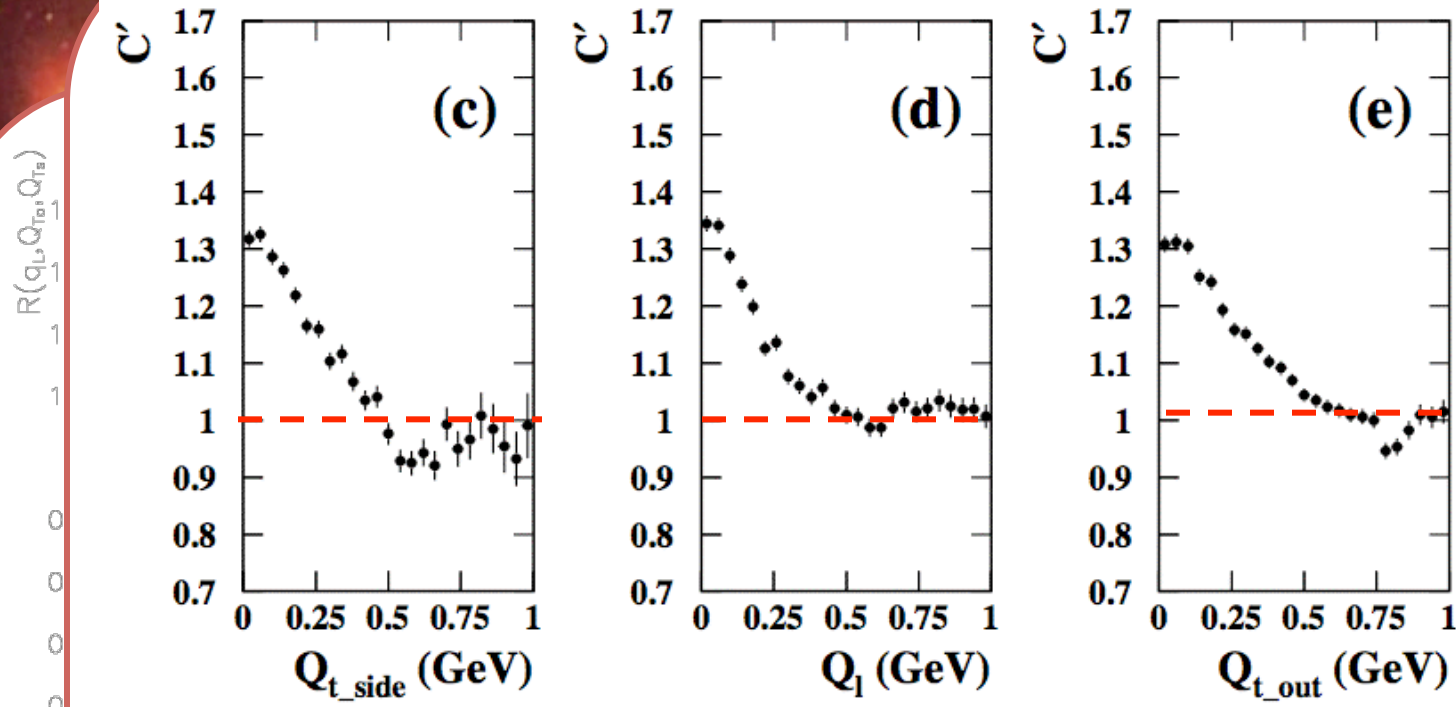
Z. Chajeccki & MAL, PRC in press, arXiv:0803.0022 [nucl-th]

For femtoscopic correlations:
 $C(\vec{q}; |\vec{q}| \rightarrow \infty) = C(|\vec{q}| \rightarrow \infty) \Rightarrow A_{l \neq 0}^m(|\vec{q}| \rightarrow \infty) = 0$



We are not alone...

Non-femto correlations in B-E analysis through the years:



$Q_x < 0.2 \text{ GeV}/c$

OPAL, CERN-PH-EP/2007-025

(submitted to Eur. Phys. J. C.)

NA22, Z. Phys. C71 (1996) 405

CLEO PRD32 (1985) 2294

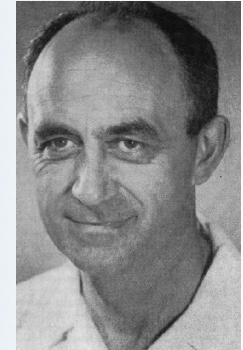
non-femto “large-2” behaviour - various approaches

- ignore it
- various ad-hoc parameterizations
- divide by $\pi^+\pi^-$ (only semi-successful, and only semi-justified)

- Can we understand it in terms of simplest-possible effect—
Energy and Momentum Conservation Induced Correlations (EMCICs)?
 - Z. Chajecki & MAL, PRC **78** 064903 (2008)

- see also
 - pT conservation effects on v_2 [Danielewicz, Ollitrault & Borghini]
 - pT conservation on 3-particle “conical emission” observables [Borghini]
 - p and E conservation effects on single particle spectra [Chajecki & MAL]

energy-momentum conservation in n-body states



spectrum of kinematic quantity α
(angle, momentum) given by

$$f(\alpha) = \frac{d}{d\alpha} (|M|^2 \cdot R_n)$$

where

M = matrix element describing interaction

(M = 1 \rightarrow all spectra given by phase space)

n-body Phase space factor R_n

$$R_n = \int^{4n} \delta^4 \left(P - \sum_{j=1}^n p_j \right) \prod_{i=1}^n \delta(p_i^2 - m_i^2) d^4 p_i$$

where

P = total 4 - momentum of n - particle system

p_i = 4 - momentum of particle i

m_i = mass of particle i

statistics: “density of states”

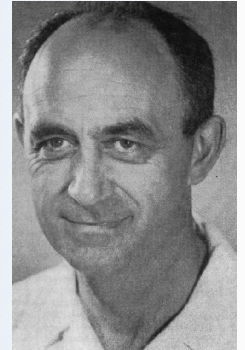
$$\delta(p_i^2 - m_i^2) d^4 p_i = \frac{|\vec{p}_i|^2}{E_i} d|\vec{p}_i| \cdot d(\cos \theta_i) \cdot d\phi_i$$

larger particle momentum \boxtimes more available states

P_{\boxtimes} conservation

$$\delta^4 \left(P - \sum_{j=1}^n p_j \right) \text{ Induces “trivial” correlations (i.e. even for } M=1)$$

Example of use of total phase space integral



- In absence of “physics” in M : (i.e. phase-space dominated)

$$\frac{\Gamma(p\bar{p} \rightarrow \pi\pi\pi)}{\Gamma(p\bar{p} \rightarrow \pi\pi\pi\pi)} = \frac{R_3(1.876; \pi, \pi, \pi)}{R_4(1.876; \pi, \pi, \pi, \pi)}$$

- single-particle spectrum (e.g. p_T):

$$W(p_i) = d^3 p_i \cdot \bar{S}_n(p_i) R_n$$

Hagedorn

- “spectrum of events”:

In limit where “ α ” = “event” = collection of momenta \vec{p}_i

$$\text{“spectrum of events”} = f(\alpha) = \frac{d}{d\alpha} R_n$$

$$\rightarrow \text{Prob}_{\text{event } \alpha} \propto \frac{d^{3n}}{\prod_{i=1}^n dp_i^3} R_n$$

Average matrix element - factorization

R. Hagedorn, *Relativistic Kinematics* 1963

Probability for an n-particle final state:

$$\begin{aligned}
 P_n &\propto \int \cdots \int \prod_{i=1}^n \delta(p_i'^2 - m^2) d^4 p_i' \times \delta^4 \left(\sum_{j=1}^n p_j' - p_1 - p_2 \right) S(p_1' \cdots p_n' | p_1, p_2) \\
 &\equiv \underbrace{\bar{S}_n \int \cdots \int \prod_{i=1}^n \delta(p_i'^2 - m^2) d^4 p_i' \times \delta^4 \left(\sum_{j=1}^n p_j' - p_1 - p_2 \right)}_{R_n} \\
 &\equiv \bar{S}_n R_n
 \end{aligned}$$

Single-particle spectrum

$$\begin{aligned}
 W(p_1') d^3 p_1' &\propto d^3 p_1' \int \cdots \int \delta(p_1'^2 - m^2) dp_{01} \prod_{i=2}^n \delta(p_i'^2 - m^2) d^4 p_i' \times \\
 &\quad \delta^4 \left(\sum_{j=1}^n p_j' - p_1 - p_2 \right) S(p_1' \cdots p_n' | p_1, p_2) \\
 &\equiv d^3 p_1' \cdot \bar{S}_n(p_1') R_F
 \end{aligned}$$

Correlations arising (only) from conservation laws (PS constraints)

$$\tilde{f}(p_i) = 2E_i \frac{dN}{d^3 p_i}$$

single-particle “parent” distribution
w/o P.S. restriction

what we
measure

$$\tilde{f}_c(p_1, \dots, p_k) \equiv \left(\prod_{i=1}^k \tilde{f}(p_i) \right) \cdot \frac{\int \left(\prod_{i=k+1}^N \frac{d^3 p_i}{2E_i} \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^N p_i - P \right)}{\int \left(\prod_{i=1}^N \frac{d^3 p_i}{2E_i} \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^N p_i - P \right)}$$

no other
correlations

$$= \left(\prod_{i=1}^k \tilde{f}(p_i) \right) \cdot \frac{\int \left(\prod_{i=k+1}^N d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^N p_i - P \right)}{\int \left(\prod_{i=1}^N d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^N p_i - P \right)}$$


k-particle distribution (k<N) with P.S. restriction

Simplification for “large” $N-k$ (1)

Numerator is the probability distribution of a sum of many ($N-k$) uncorrelated vectors
(i.e. the probability that they will add up to $P - \sum_{i=1}^k p_i$)

If ($N-k$) big \rightarrow Multivariate Central Limit Theorem

$$\sum_{i=k+1}^N p_i - \left(P - \sum_{i=1}^k p_i \right)$$



$$= \left(\prod_{i=1}^k \tilde{f}(p_i) \right) \cdot \frac{\int \left(\prod_{i=k+1}^N d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^N p_i - P \right)}{\int \left(\prod_{i=1}^N d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^N p_i - P \right)}$$

Denominator is “just” a constant normalization (indep p_i)

Simplification for "large" $N-k$ (2)

Numerator is the probability distribution of a sum of many ($N-k$) uncorrelated vectors
(i.e. the probability that they will add up to $P - \sum_{i=1}^k p_i$)

If ($N-k$) big \rightarrow Multivariate Central Limit Theorem

$$\left(\prod_{i=1}^k \tilde{f}(p_i) \right) \cdot \frac{\int \left(\prod_{i=k+1}^N d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^N p_i - P \right)}{\int \left(\prod_{i=1}^N d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^N p_i - P \right)}$$

$\xrightarrow{\text{large } N-k}$

$$\left(\prod_{i=1}^k \tilde{f}(p_i) \right) \cdot \left(\frac{N}{N-k} \right)^2 \exp \left[- \left(\sum_{i=1}^k (p_i^\mu - \langle p^\mu \rangle) \right) \frac{b_{\mu\nu}}{2(N-k)} \left(\sum_{i=1}^k (p_i^\nu - \langle p^\nu \rangle) \right) \right]$$

averages $\langle X \rangle \equiv \int d^4 p \cdot X \cdot \delta(p^2 - m^2) \tilde{f}(p)$

covariance matrix $(b^{-1})_{\mu\nu} = \langle p_\mu p_\nu \rangle - \langle p_\mu \rangle \langle p_\nu \rangle$

Fortunately, diagonal covariance matrix!

$$(b^{-1})_{\mu\nu} = \langle p_\mu p_\nu \rangle - \langle p_\mu \rangle \langle p_\nu \rangle$$

✓ Work in global C.O.M. frame: $\langle p^\mu \rangle = \delta_{\mu,0} \langle E \rangle$

✓ No elliptic flow, etc: $(b^{-1})_{1,2} = \langle p_x p_y \rangle = 0$

✓ No directed flow, etc: $(b^{-1})_{1,3} = (b^{-1})_{2,3} = 0$

✓ On - shell means momentum and energy obviously correlated...

but covariance (second moment) vanishes. For $i \neq 0$:

$$(b^{-1})_{0,i} = \langle E p_i \rangle - \langle E \rangle \langle p_i \rangle = \langle E p_i \rangle = \frac{\int dE \int d^3 p \cdot E \tilde{f}(p) \delta(p^2 - m) \cdot p_i}{\int dE \int d^3 p \cdot \tilde{f}(p) \delta(p^2 - m)} = 0$$

since $E \tilde{f}(p) \delta(p^2 - m)$ is even and p_i is odd

Using central limit theorem ("large* N-k")

k-particle distribution in N-particle system

$$\tilde{f}_c(p_1, \dots, p_k) = \left(\prod_{i=1}^k \tilde{f}(p_i) \right) \left(\frac{N}{N-k} \right)^2 \exp \left(- \sum_{\mu=0}^3 \frac{\left(\sum_{i=1}^k (p_{i,\mu} - \langle p_\mu \rangle) \right)^2}{2(N-k)\sigma_\mu^2} \right)$$

where

$$\sigma_\mu^2 = \langle p_\mu^2 \rangle - \langle p_\mu \rangle^2$$

$$\langle p_\mu \rangle = 0 \quad \text{for } \mu = 1, 2, 3$$

N.B.
relevant later

$$\langle p_\mu^2 \rangle \equiv \int d^3p \cdot p_\mu^2 \cdot \underbrace{\tilde{f}(p)}_{\text{unmeasured parent distrib}} \neq \int d^3p \cdot p_\mu^2 \cdot \underbrace{\tilde{f}_c(p)}_{\text{measured}}$$

- Danielewicz *et al*, PRC**38** 120 (1988)
- Borghini, Dinh, & Ollitraut PRC**62** 034902 (2000)
- Borghini Eur. Phys. J. C**30**:381-385, (2003)
- Chajeccki & MAL, PRC **78** 064903 (2008)

Effects on single-particle distribution

$$\begin{aligned}\tilde{f}_c(\mathbf{p}_i) &= \tilde{f}(\mathbf{p}_i) \left(\frac{N}{N-1} \right)^2 \exp \left(- \sum_{\mu=0}^3 \frac{(\mathbf{p}_{i,\mu} - \langle \mathbf{p}_\mu \rangle)^2}{2(N-1)\sigma_\mu^2} \right) \\ &= \tilde{f}(\mathbf{p}_i) \left(\frac{N}{N-1} \right)^2 \exp \left(- \frac{1}{2(N-1)} \left(\frac{\mathbf{p}_{x,i}^2}{\langle \mathbf{p}_x^2 \rangle} + \frac{\mathbf{p}_{y,i}^2}{\langle \mathbf{p}_y^2 \rangle} + \frac{\mathbf{p}_{z,i}^2}{\langle \mathbf{p}_z^2 \rangle} + \frac{(\mathbf{E}_i - \langle \mathbf{E} \rangle)^2}{\langle \mathbf{E}^2 \rangle - \langle \mathbf{E} \rangle^2} \right) \right)\end{aligned}$$

We will return to this....

k-particle correlation function

$$\begin{aligned}
 C(\mathbf{p}_1, \dots, \mathbf{p}_k) &\equiv \frac{\tilde{f}_c(\mathbf{p}_1, \dots, \mathbf{p}_k)}{\tilde{f}_c(\mathbf{p}_1) \dots \tilde{f}_c(\mathbf{p}_k)} \\
 &= \frac{\left(\frac{N}{N-k}\right)^2 \exp\left[-\frac{1}{2(N-k)} \sum_{i=1}^k \left(\frac{\left(\sum_{i=1}^k p_{x,i}\right)^2}{\langle p_x^2 \rangle} + \frac{\left(\sum_{i=1}^k p_{y,i}\right)^2}{\langle p_y^2 \rangle} + \frac{\left(\sum_{i=1}^k p_{z,i}\right)^2}{\langle p_z^2 \rangle} + \frac{\left(\sum_{i=1}^k (E_i - \langle E \rangle)\right)^2}{\langle E^2 \rangle - \langle E \rangle^2} \right)}{\left(\frac{N}{N-1}\right)^{2k} \exp\left[-\frac{1}{2(N-1)} \sum_{i=1}^k \left(\frac{p_{x,i}^2}{\langle p_x^2 \rangle} + \frac{p_{y,i}^2}{\langle p_y^2 \rangle} + \frac{p_{z,i}^2}{\langle p_z^2 \rangle} + \frac{(E_i - \langle E \rangle)^2}{\langle E^2 \rangle - \langle E \rangle^2} \right)}\right]}
 \end{aligned}$$

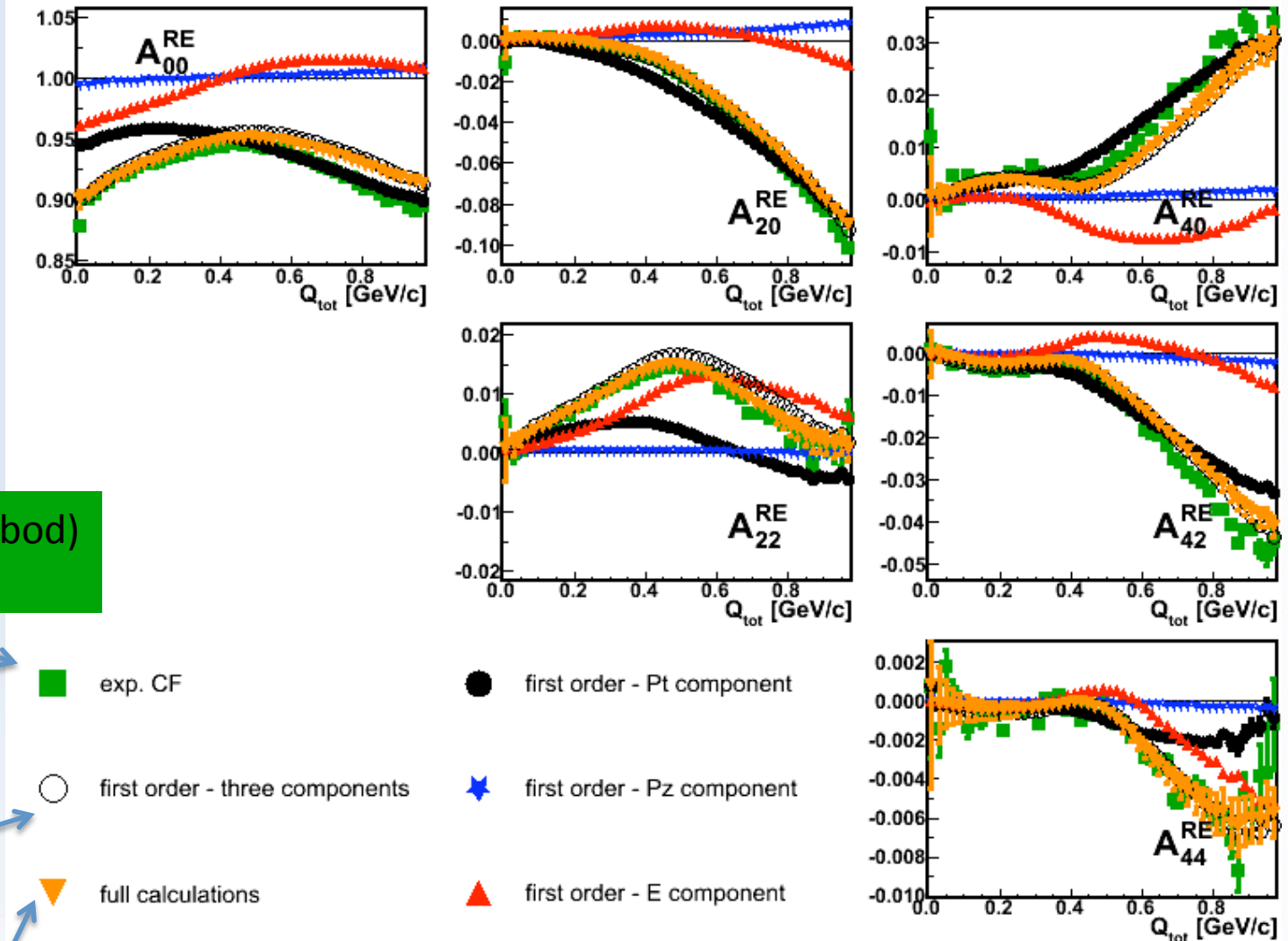
Dependence on “parent” distrib *f* vanishes,
except for energy/momentum means and RMS

2-particle correlation function (1st term in 1/*N* expansion)

$$C(\mathbf{p}_1, \mathbf{p}_2) \cong 1 - \frac{1}{N} \left(2 \frac{\vec{p}_{T,1} \cdot \vec{p}_{T,2}}{\langle p_T^2 \rangle} + \frac{p_{z,1} \cdot p_{z,2}}{\langle p_z^2 \rangle} + \frac{(E_1 - \langle E \rangle) \cdot (E_2 - \langle E \rangle)}{\langle E^2 \rangle - \langle E \rangle^2} \right)$$

How do EMCFCs look? – nontrivial!

Genbod N=18 $\langle K \rangle = 0.9$ GeV; PRF - $|\mathbb{W}| < 0.5$



event generator (genbod) with only EMCFCs

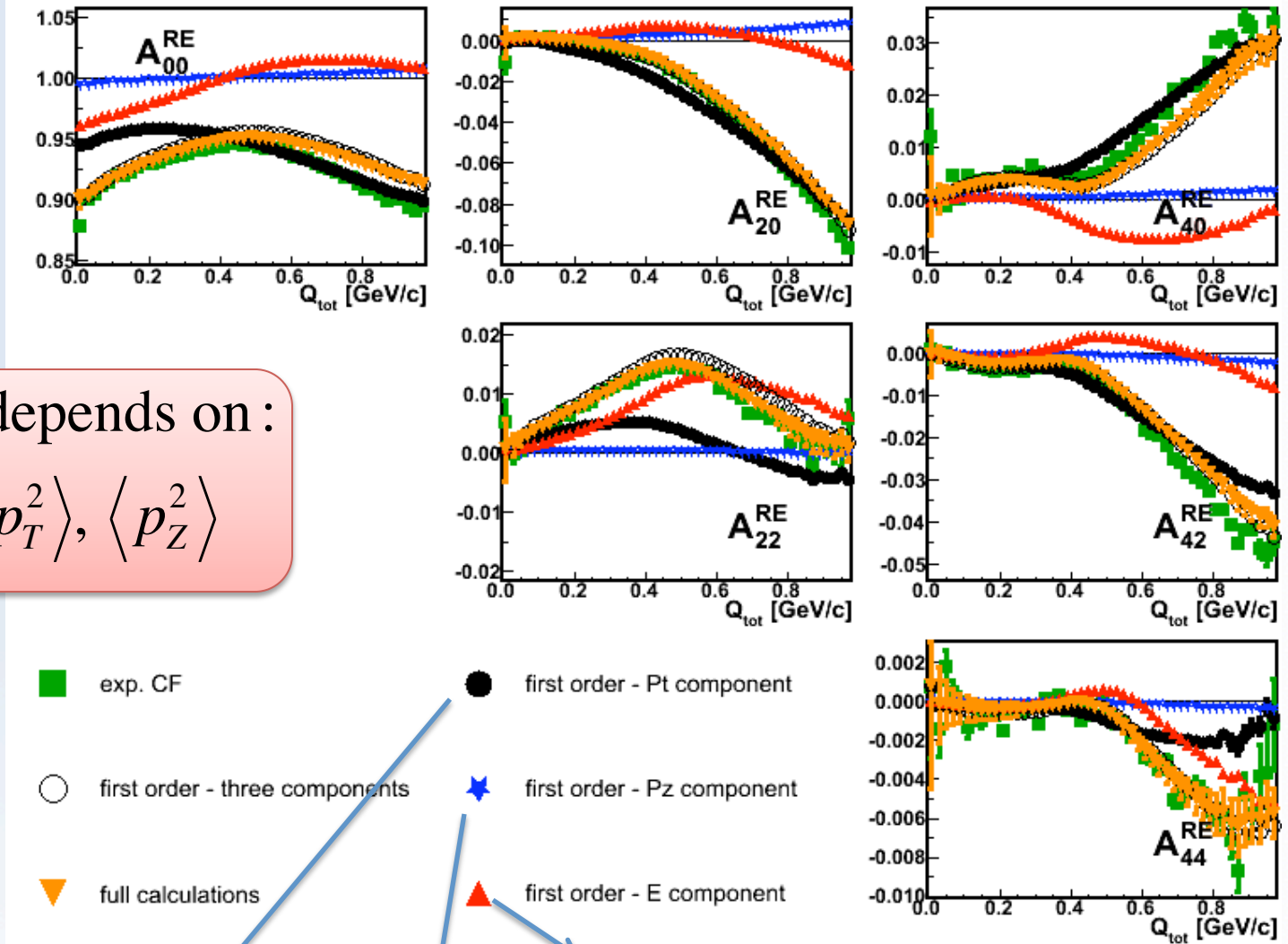
$O(1/N)$ term in CLT approximation

CLT approximation

- exp. CF
- first order - Pt component
- first order - three components
- ★ first order - Pz component
- ▼ full calculations
- ▲ first order - E component

• structure not confined to large Q
 • kinematic cuts have strong effect

How do EMCFCs look? – nontrivial!



Detailed shape depends on:
 $N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_Z^2 \rangle$

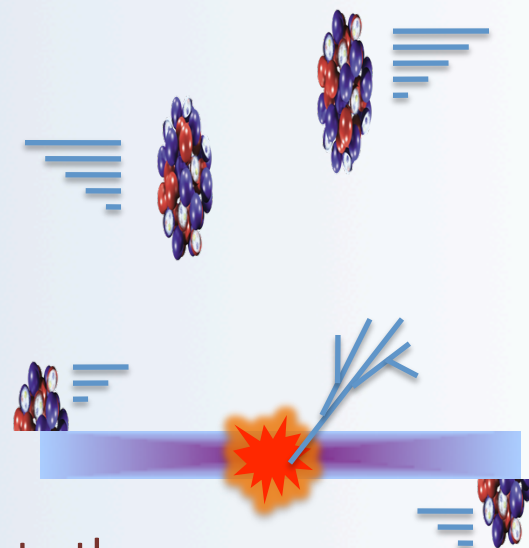
- exp. CF
- first order - three components
- ▼ full calculations
- first order - Pt component
- ★ first order - Pz component
- ▲ first order - E component

$$C(p_1, p_2) \cong 1 - \frac{1}{N} \left(2 \frac{\vec{p}_{T,1} \cdot \vec{p}_{T,2}}{\langle p_T^2 \rangle} + \frac{p_{z,1} \cdot p_{z,2}}{\langle p_z^2 \rangle} + \frac{(E_1 - \langle E \rangle) \cdot (E_2 - \langle E \rangle)}{\langle E^2 \rangle - \langle E \rangle^2} \right)$$

“the system” ... a nontrivial concept

$$N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_Z^2 \rangle$$

Characteristic scales of relevant system in which limited energy-momentum is shared



- Not known a priori
- should *track* measured quantities, but not be identical to them

1. N includes all primary particles (including unmeasured γ 's etc)
2. secondary decay (resonances, fragmentation) smears connection b/t $\langle E^2 \rangle$ and measured one

3. $\langle E^2 \rangle$ etc: averages of the *parent* distribution

$$\langle p_\mu^2 \rangle \equiv \int d^3p \cdot p_\mu^2 \cdot \underbrace{\tilde{f}(p)}_{\text{unmeasured parent distrib}} \neq \int d^3p \cdot p_\mu^2 \cdot \underbrace{\tilde{f}_c(p)}_{\text{measured}}$$

4. “relevant system” almost certainly not the “whole” (4π) system

- e.g. beam fragmentation probably not relevant to system emitting at midrapidity
 - characteristic physical processes (strings etc): $\Delta y \sim 1 \div 2$
- jets: “of the system” ??
 - or just stealing energy *from* “the system?”

- if “relevant system” \neq “whole system” then total energy-momentum *will fluctuate* e-by-e

The underlying event appears indep of jet – CDF

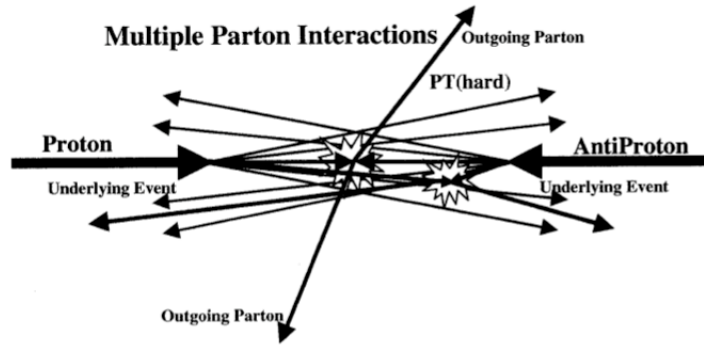
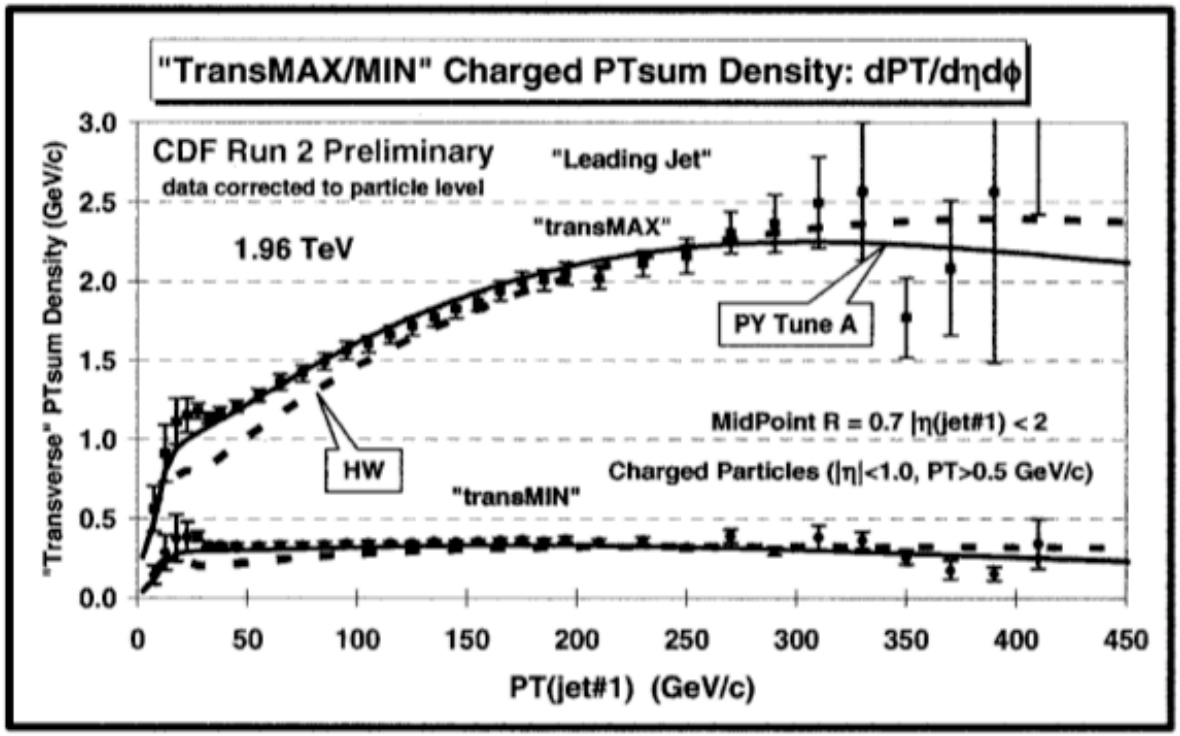
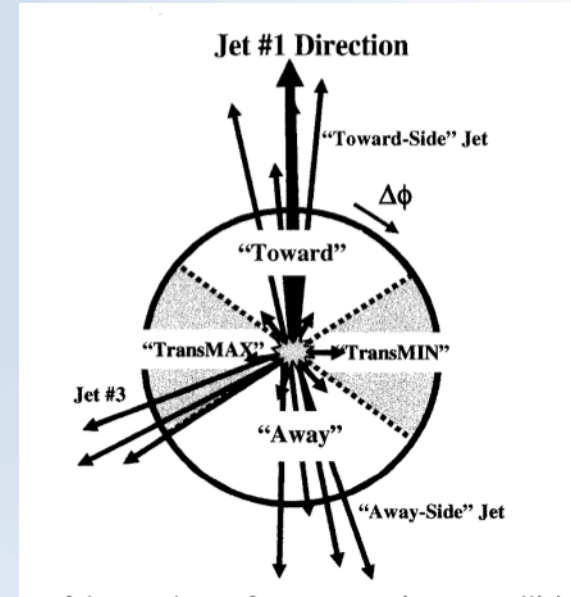


Fig. 1-5. Illustration of the way PYTHIA models the “underlying event” in proton-antiproton collision by including multiple parton interactions. In addition to the hard 2-to-2 parton-parton scattering with transverse momentum, $P_T(\text{hard})$, there is a second “semi-hard” 2-to-2 parton-parton scattering that contributes particles to the “underlying event”.



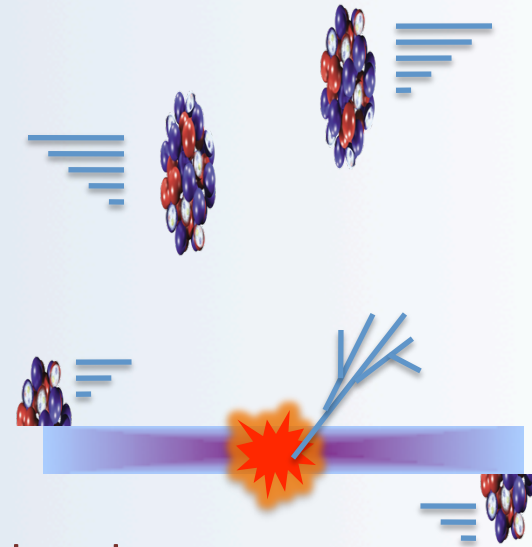
L.A. Cruz, PhD Thesis, U. Florida
 “Underlying event at Tevatron” (2005)

“the system” ... a nontrivial concept

$$N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_Z^2 \rangle$$

Characteristic scales of relevant system in which limited energy-momentum is shared

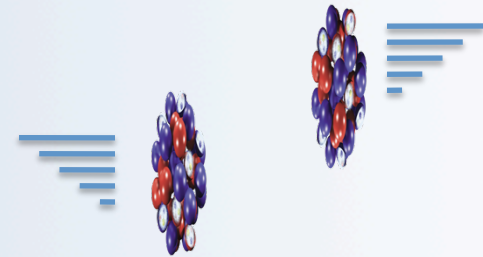
- Not known a priori
- should *track* measured quantities, but not be identical to them
- We will treat them as parameters: what to expect?



Maxwell - Boltzmann parent $\frac{d^3 N}{d^3 p} \sim e^{-E/T}$

	non - rel	ultra - rel	if $T = .15 \div .35$
$\langle p_T^2 \rangle$	$2mT$	$8T^2$	$0.045 \div 0.98 \text{ (GeV/c)}^2$
$\langle E^2 \rangle$	$\frac{15}{4} T^2 + m^2$	$12T^2$	$0.10 \div 1.5 \text{ GeV}^2$
$\langle E \rangle$	$\frac{3}{2} T + m$	$3T$	$0.36 - 1 \text{ GeV}$

“the system” ... a nontrivial concept



$$N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_Z^2 \rangle$$

Characteristic scales of relevant system in which limited energy-momentum is shared

- Not known a priori
- should *track* measured quantities, k
- What to expect?

Blastwave, $T = 100 \text{ MeV}$ $\rho_0 = 0.9$

$$\langle p_T^2 \rangle_\pi = 0.240 \text{ GeV}^2 \quad (\langle p_T \rangle_\pi = 0.405 \text{ GeV})$$

$$\langle m_T \rangle_\pi = 0.435 \text{ GeV}$$

$$\langle m_T^2 \rangle_\pi = 0.259 \text{ GeV}^2$$

Maxwell - Boltzmann parent $\frac{d^3 N}{d^3 p} \sim e^{-E/T}$

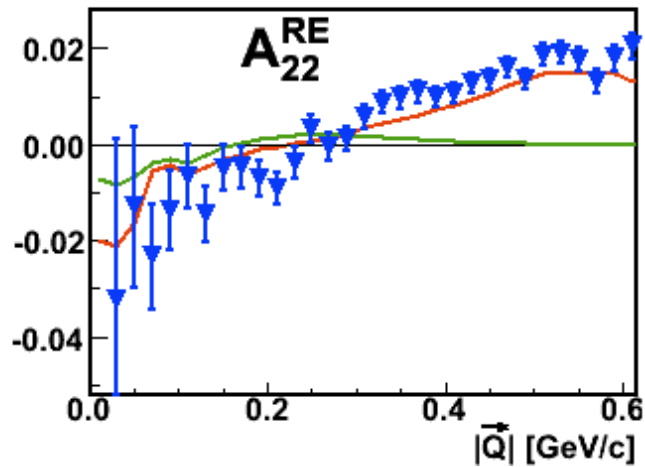
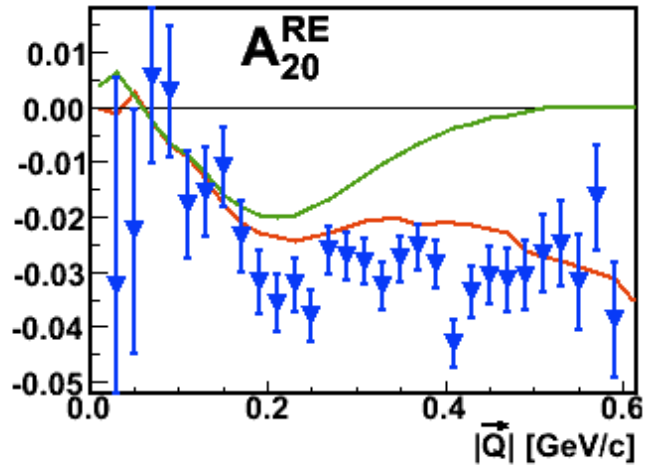
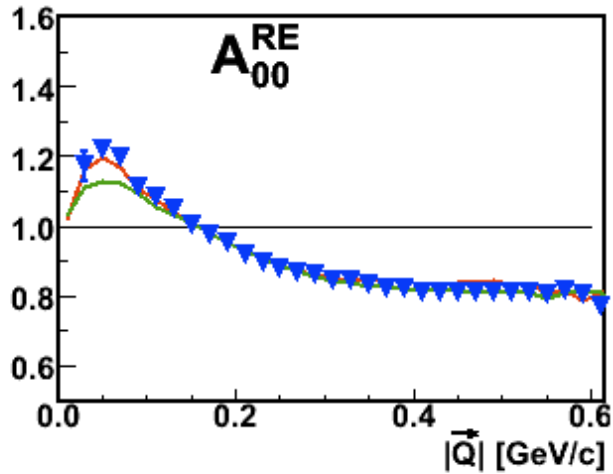
	non - rel	ultra - rel	if $T = .15 - .2$
$\langle p_T^2 \rangle$	$2mT$	$8T^2$	$0.045 \div 0.9$
$\langle E^2 \rangle$	$\frac{15}{4} T^2 + m^2$	$12T^2$	$0.10 \div 1.5$
$\langle E \rangle$	$\frac{3}{2} T + m$	$3T$	$0.36 - 1 \text{ GeV}$

η_{max}	$\langle N \rangle$	$\langle p_T^2 \rangle_c$	$\langle p_z^2 \rangle_c$	$\langle E^2 \rangle_c$	$\langle E \rangle_c$
1.0	16	0.20	0.11	0.40	0.44
2.0	29	0.21	0.76	1.05	0.68
3.0	39	0.21	3.5	3.8	1.2
4.0	47	0.21	24	25	2.2
5.0	51	0.22	88	89	3.7

TABLE I: For a given selection on pseudorapidity $|\eta| < \eta_{max}$, the number and kinematic variables for primary particles from a PYTHIA simulation of $p + p$ collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ are given. Units are GeV/c or $(\text{GeV}/c)^2$, as appropriate.

p+p minbias

STAR poster QM09



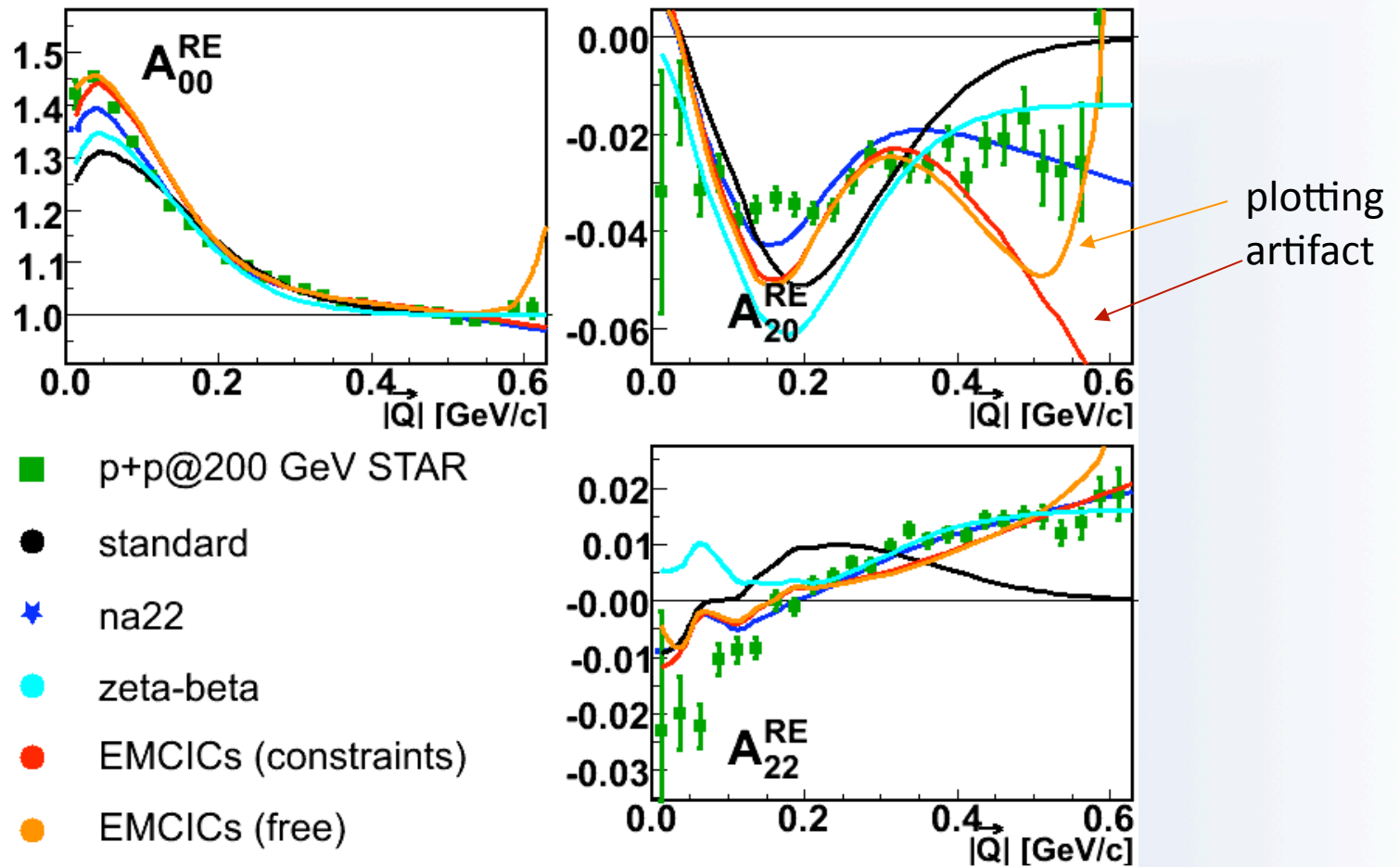
▼ CF
— standard fit
— EMCIC fit

$k_T = [0.35, 0.45] \text{ GeV/c}$

$\lambda = 0.38 \pm 0.01$
 $R_o = 0.81 \pm 0.01 \text{ fm}$
 $R_s = 0.84 \pm 0.02 \text{ fm}$
 $R_l = 1.29 \pm 0.02 \text{ fm}$

$\lambda = 0.69 \pm 0.01$
 $R_o = 0.96 \pm 0.04 \text{ fm}$
 $R_s = 0.98 \pm 0.03 \text{ fm}$
 $R_l = 1.26 \pm 0.02 \text{ fm}$
 $N = 13.6$
 $\langle E \rangle = 0.68 \text{ GeV}$
 $\langle E^2 \rangle = 0.54 \text{ GeV}^2$
 $\langle p_T^2 \rangle = 0.17 \text{ GeV}^2$
 $\langle p_z^2 \rangle = 0.33 \text{ GeV}^2$

Various fits to the pion correlation function (p+p)



fit method	R_{out} [fm]	R_{side} [fm]	R_{long} [fm]
standard	0.65 +/- 0.01	0.85 +/- 0.01	1.42 +/- 0.02
"NA22"	1.18 +/- 0.02	1.05 +/- 0.02	1.75 +/- 0.03
"zeta-beta"	1.01 +/- 0.03	0.79 +/- 0.03	1.52 +/- 0.05
EMCICs (constr.)	1.05 +/- 0.02	1.06 +/- 0.02	1.66 +/- 0.03
EMCICs(free)	1.06 +/- 0.02	1.08 +/- 0.02	1.69 +/- 0.03

femtoscscopy in $p+p$ @ STAR

1. Heisenberg uncertainty?

2. String fragmentation? (Lund)

3. Resonance effects?

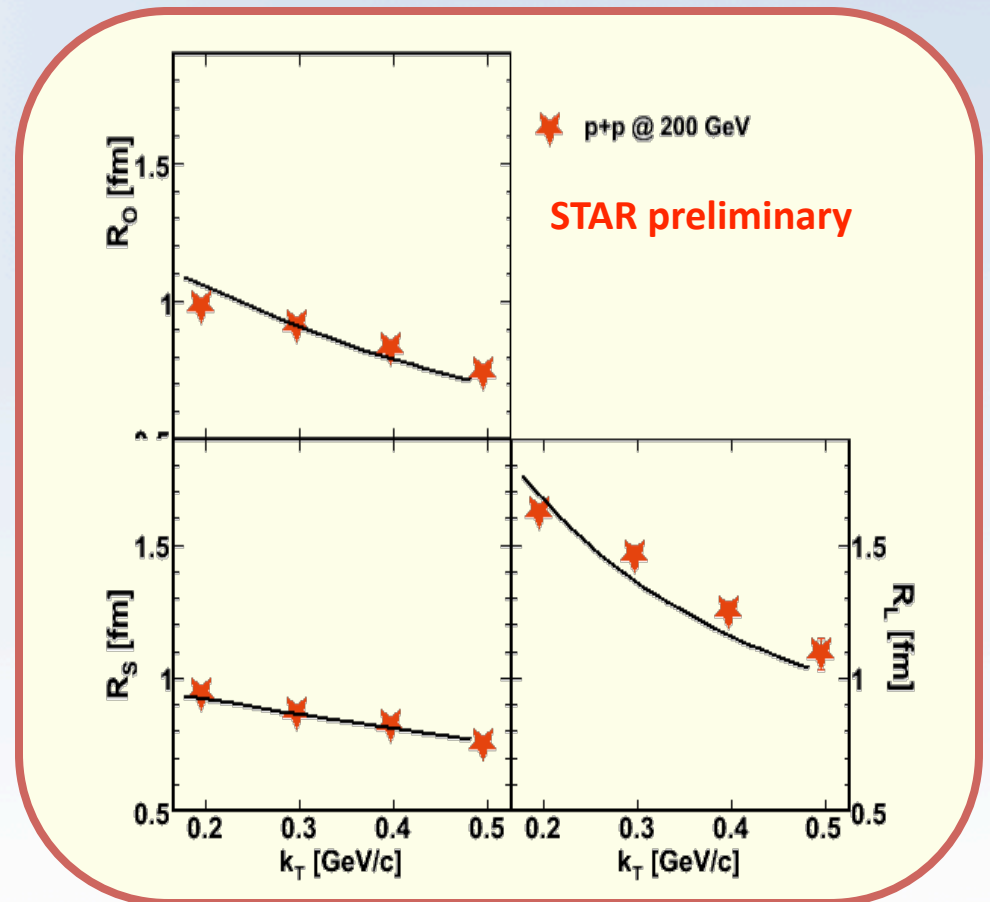
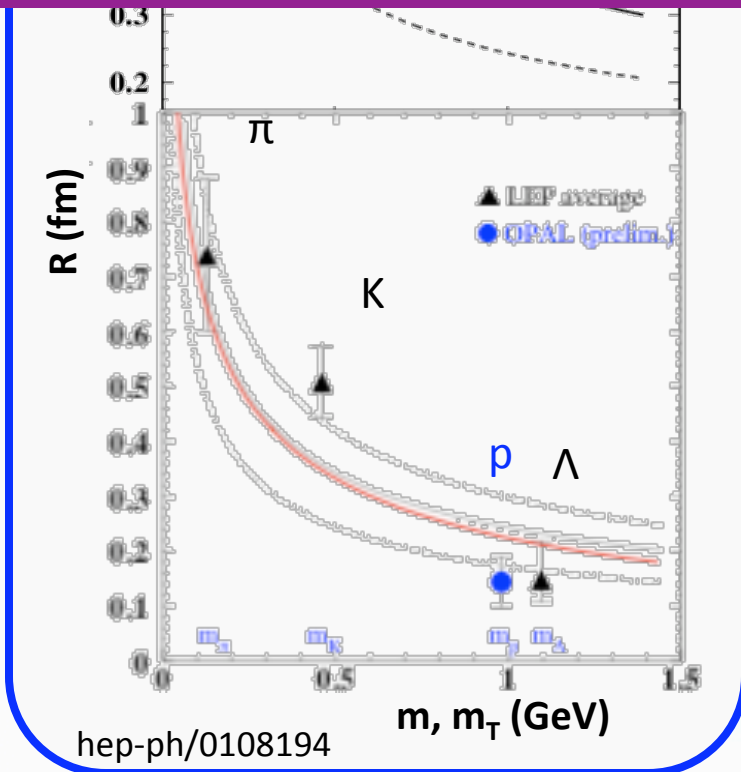
4. Flow???

- Increasingly suggested in HEP experiments

$p+p$ and $A+A$ measured in *same* experiment, *same* acceptance, *same* techniques

- **unique** opportunity to compare physics
- what causes p_T -dependence in $p+p$?
- same cause as in $A+A$?

c.f. Z. Chajecki arXiv:0901.4078 [nucl-ex]



1. Heisenberg uncertainty?

2. String fragmentation? (Lund)

3. Resonance effects?

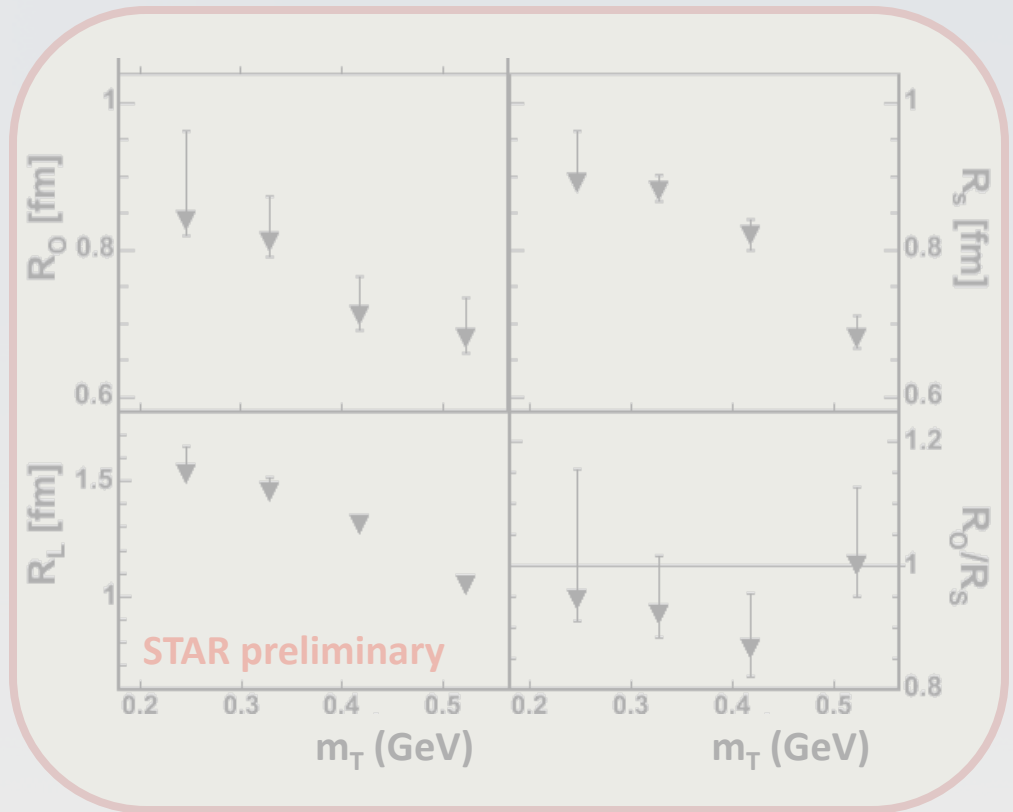
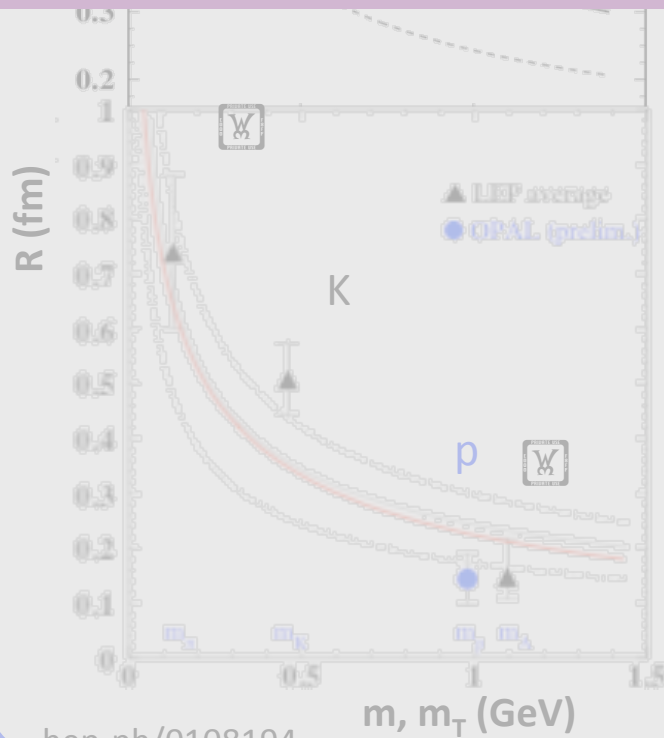
4. Flow???

- Increasingly suggested in recent experiments

NA22 Collaboration Z. Phys. C 71, 405–414 (1996)
(hadron-hadron collisions)

[based on shape of $C(q)$...]

Our data do not confirm the expectation from the string type model... A good description of our data is, however, achieved in the framework of the hydrodynamical expanding source model.



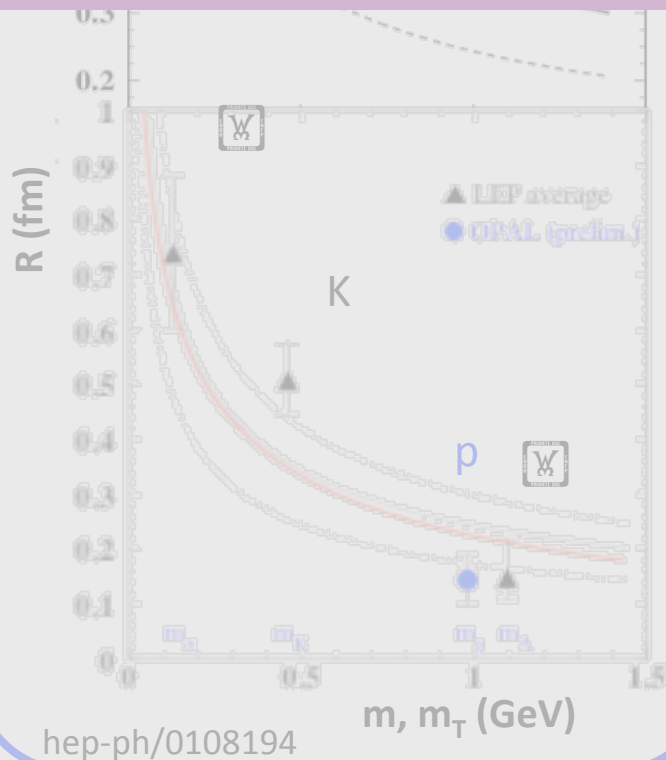
1. Heisenberg uncertainty?

2. String fragmentation? (Lund)

3. Resonance effects?

4. Flow???

- Increasingly suggested in recent experiments



NA22 Collaboration Z. Phys. C 71, 405–414 (1996)
(hadron-hadron collisions)

[based on shape of $C(q)$...]

Our data do not confirm the expectation from the string type model... A good description of our data is, however, achieved in the framework of the hydrodynamical expanding source model.

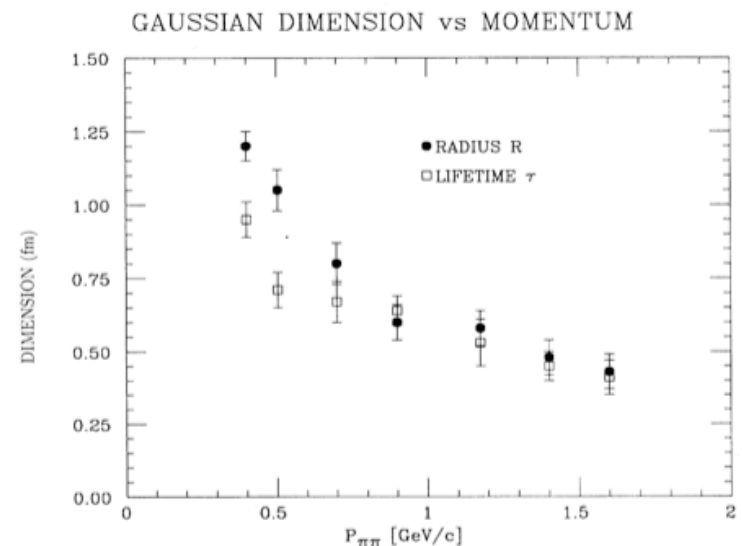


FIG. 9. Interaction “radius” and lifetime as a function of the total momentum $P_{\pi\pi}$ of the pion pair. R_G is primarily a source dimension along the beam direction. τ might possibly be interpreted as a source dimension transverse to the beam. Data are from Table III.

E735 Collaboration, PRD48 1931 (1993)

also PLB 2002

consistent with an expanding shell model.

1. Heisenberg uncertainty?

2. String fragmentation? (Lund)

3. Resonance effects?

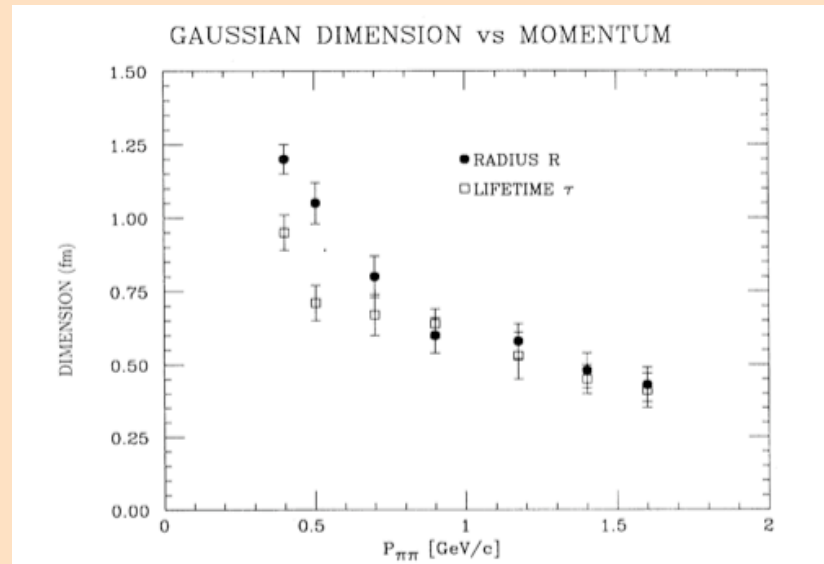
4. Flow???

- Increasingly suggested in recent experiments

NA22 Collaboration Z. Phys. C 71, 405–414 (1996)
(hadron-hadron collisions)

[based on shape of $C(q)$...]

Our data do not confirm the expectation from the string type model... A good description of our data is, however, achieved in the framework of the hydrodynamical expanding source model.



W. Kittel Acta Phys.Polon. B32 (2001) 3927 [Review article]
... and suggests the existence of an important “collective flow”, even in the system of particles produced in $e+e^-$ annihilation!

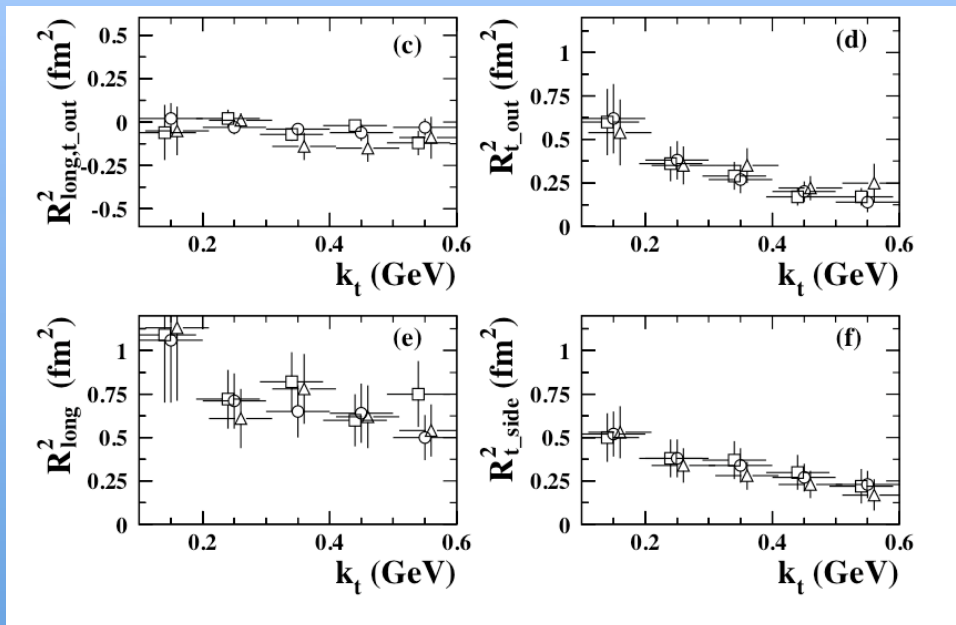
A $1/\sqrt{m} T$ scaling first observed in heavy-ion collisions is now also observed in Z fragmentation and may suggest a “transverse flow” even there!

time as a function of the
 G is primarily a source
might possibly be inter-
to the beam. Data are

8 1931 (1993)

ing shell model.

OPAL Collaboration, Eur.Phys.J.C52:787-803,2007; arXiv:0708.1122 [hep-ex]



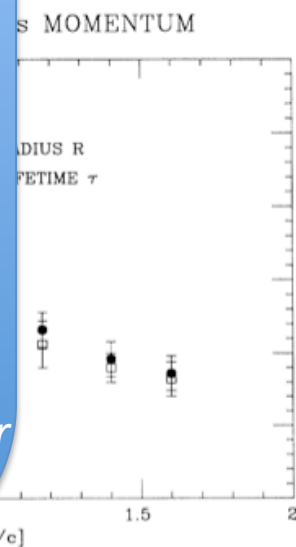
$R_{t_{side}}$, $R_{t_{out}}$ and, less markedly, R_{long} decrease with increasing k_t . The presence of correlations between the particle production points and their momenta is an indication that the pion source is not static, but rather expands during the particle emission process.

W. Kittel Acta Phys.Polon. B32 (2001) 3927 [Review article]

... and suggests the existence of an important “collective flow”, even in the system of particles produced in $e+e^-$ annihilation!

A $1/\sqrt{m} T$ scaling first observed in heavy-ion collisions is now also observed in Z fragmentation and may suggest a “transverse flow” even there!

Expectation from description of in the framework of pion source model.



time as a function of the G is primarily a source might possibly be inter- to the beam. Data are

B 1931 (1993)

ing shell model.

OPAL Collaboration, Eur.Phys.J.C52:787-803,2007; arXiv:0708.1122 [hep-ex]



RHIC: “comparison machine”

Vary size. All else fixed. [acceptance, technique...]

- spectra
- femtoscopy

compare with a system we “know” is flowing

R2ts
The p
their
expans

W. Kittel Acta Phys.Polon. B32 (2001) 3927 [Review article]

... and suggests the existence of an important “collective flow”, even in the system of particles produced in $e+e^-$ annihilation!

A $1/\sqrt{m}$ T scaling first observed in heavy-ion collisions is now also observed in Z fragmentation and may suggest a “transverse flow” even there!

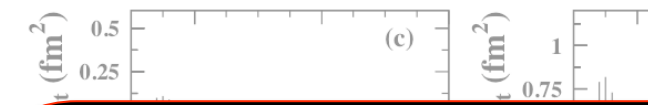
estimation from
f
rk
el.

(GeV/c)

ime as a function of the
g is primarily a source
ight possibly be inter-
to the beam. Data are

8 1931 (1993)

ing shell model.



RHIC: "comparison m

Vary size. All else fixe

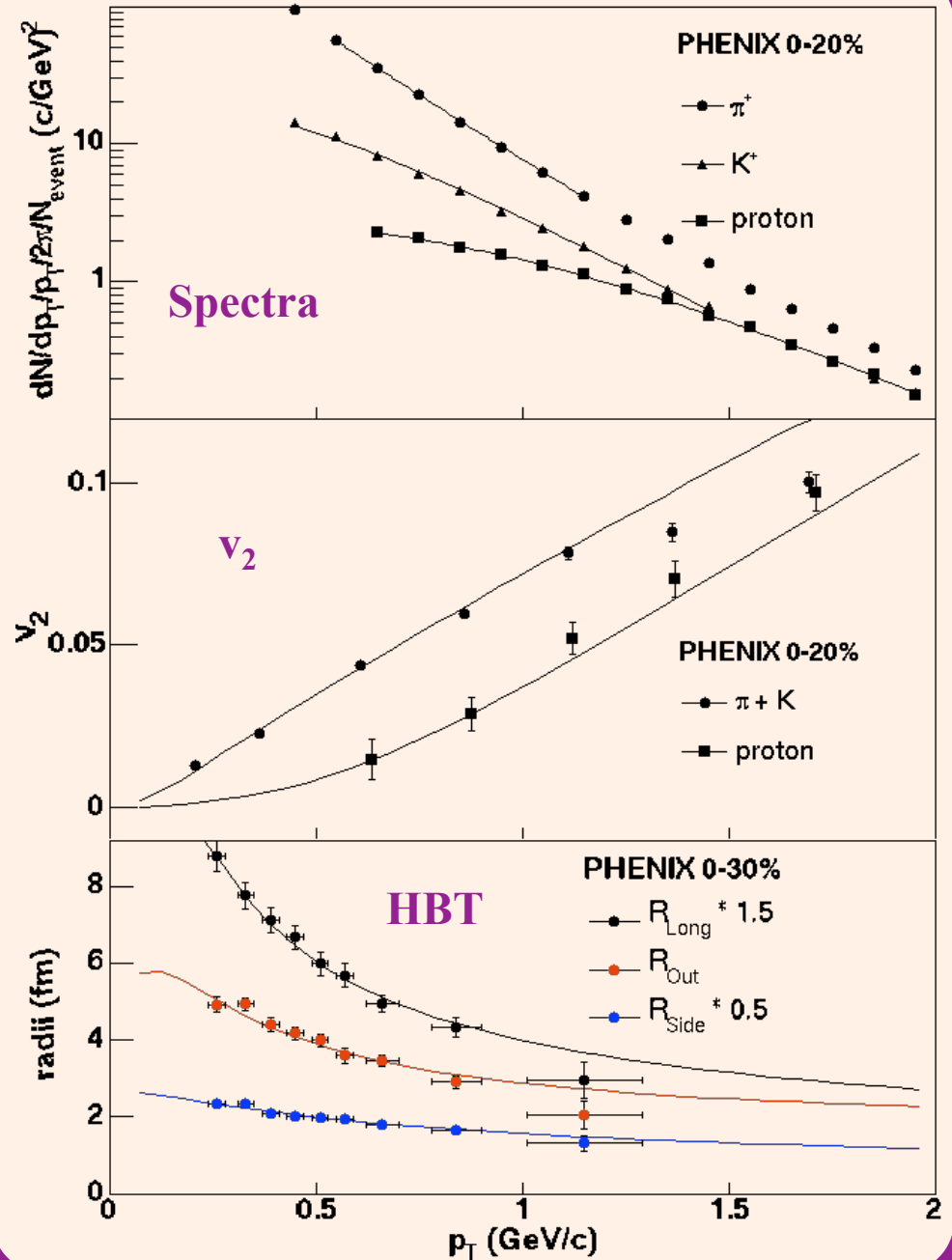
- spectra
- femtoscopy

compare with a syste

R2ts
The p
their
expans

W. Kittel Acta Phys.Polon. B32 (2001) 3
... and suggests the existence of an impor
system of particles produced in e+e- ann

A 1/vm T scaling first observed in heavy-
Z fragmentation and may suggest a "transverse flow" even there.



Apples: apples comparison...

Z. Chajecski, QM05

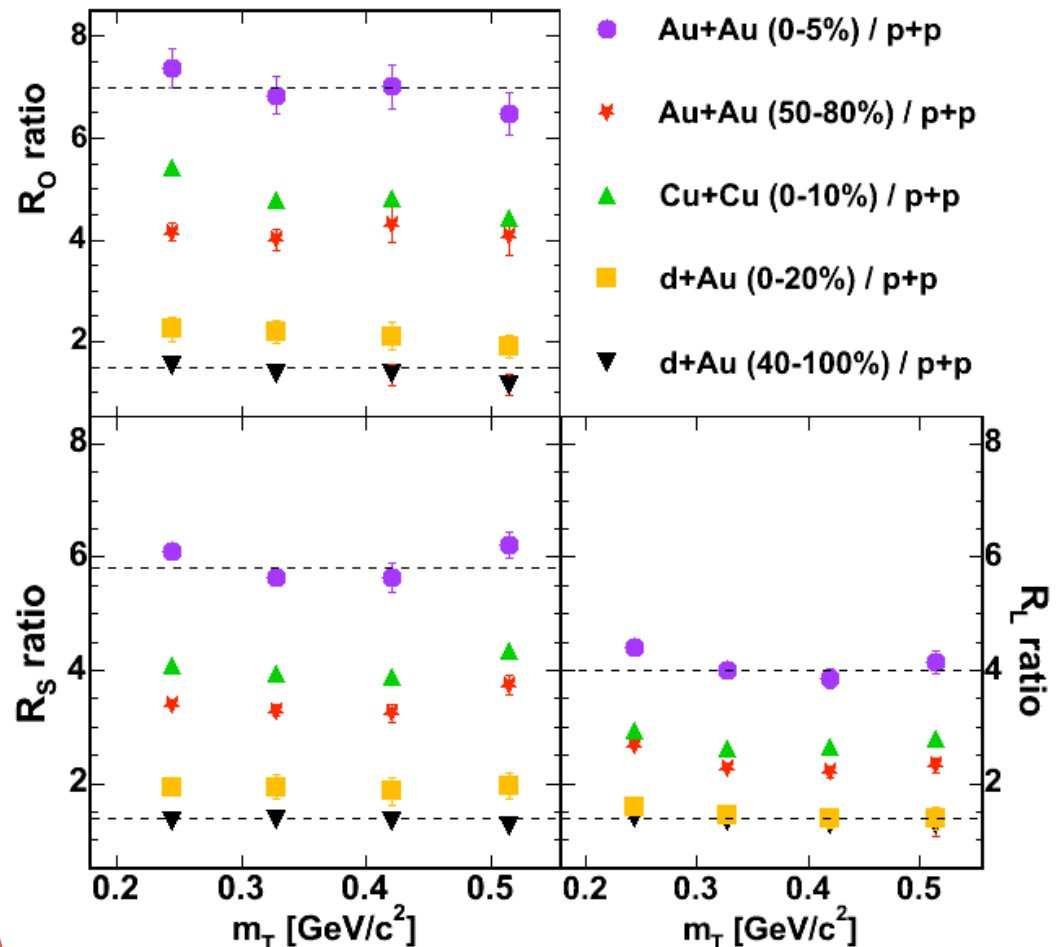
$R(p_T)$ taken as strong space-time evidence of flow in Au+Au

- clear, quantitative consistency predictions of BlastWave

“Identical” signal seen in p+p

- cannot be of “identical” origin? (other than we “know it cannot”...)

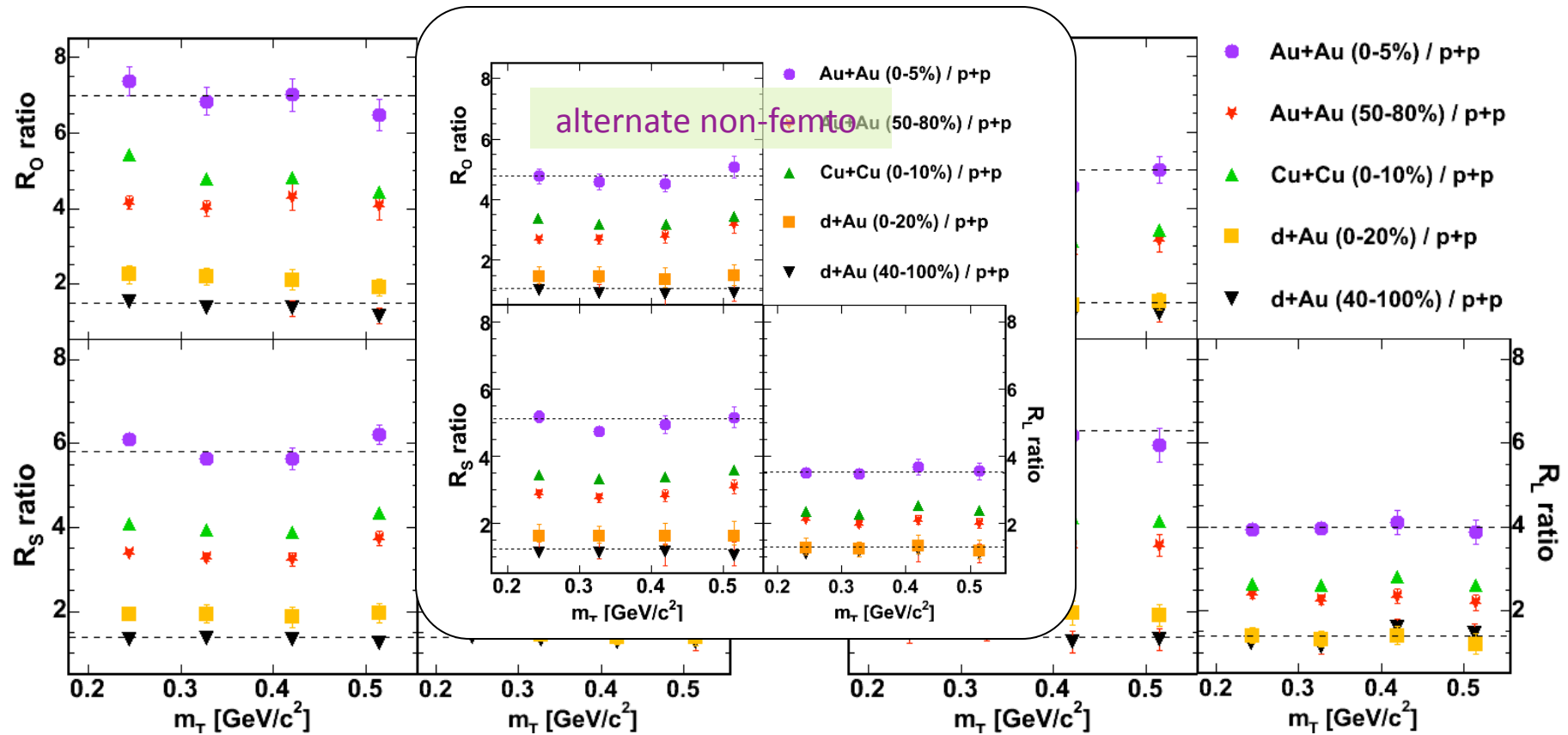
Ratio of (AuAu, CuCu, dAu) HBT radii by pp



pp, dAu, CuCu - STAR preliminary

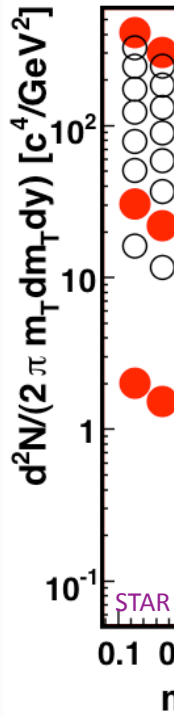
Significant non-femto correlations, but little effect on "message"

STAR preliminary rather, "suggestion": explosive flow in p+p?
radii by pp



Fit w/o baseline parameterization

NEW fit w/ baseline parameterization



NO.....?

Au+Au 0-5%

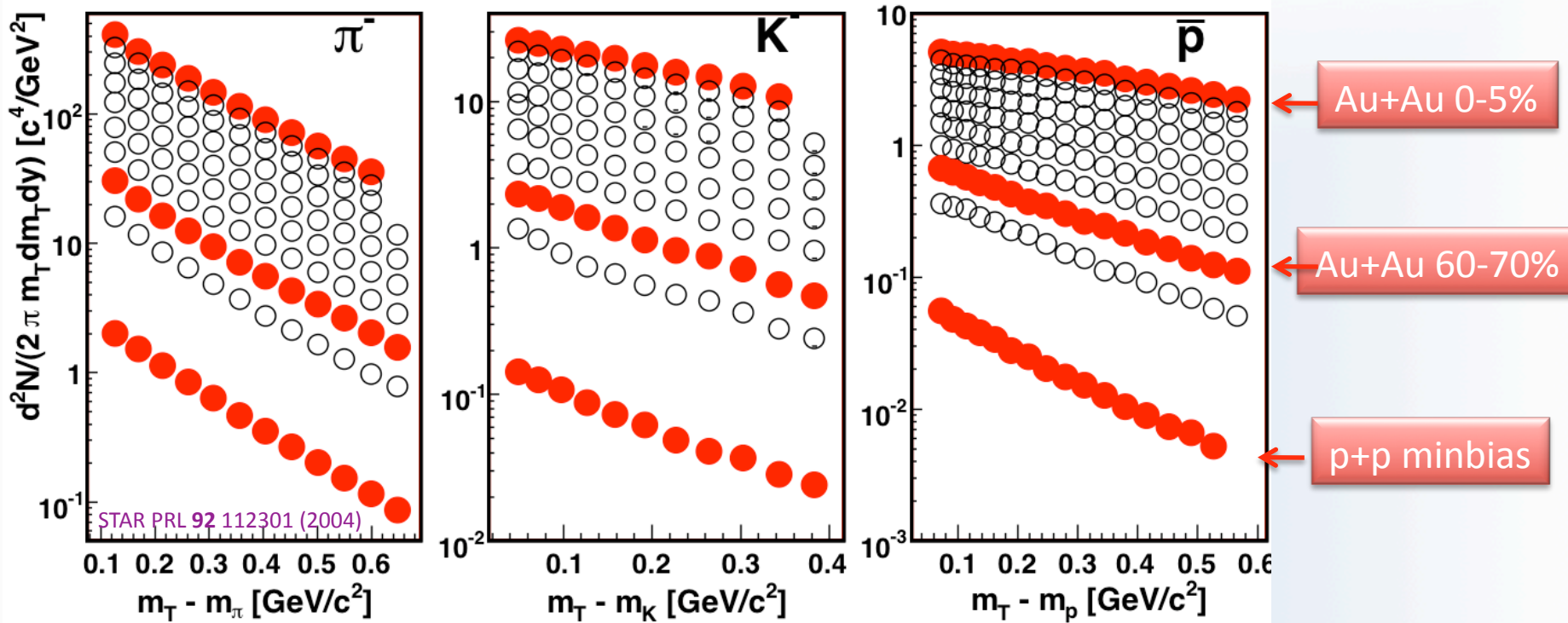
Au+Au 60-70%

p+p minbias

Blast-wave

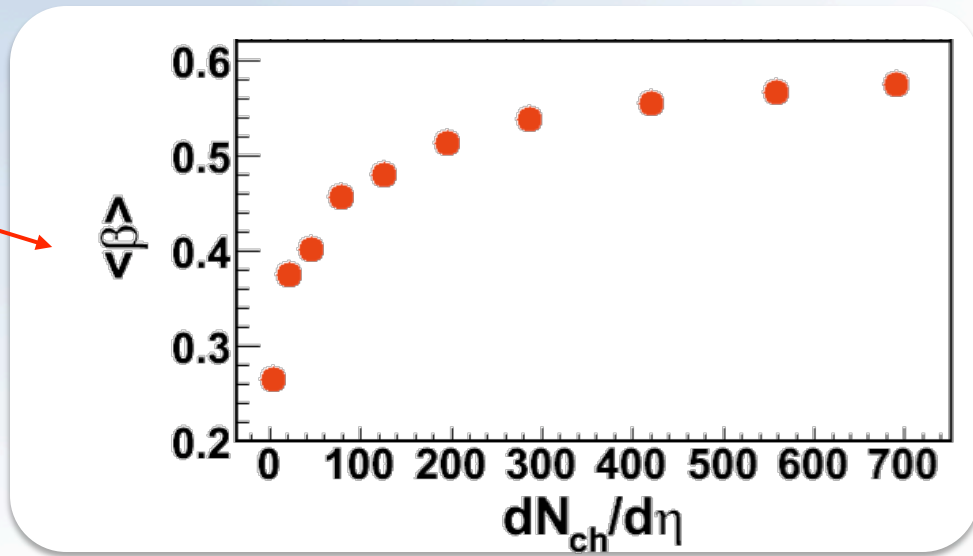
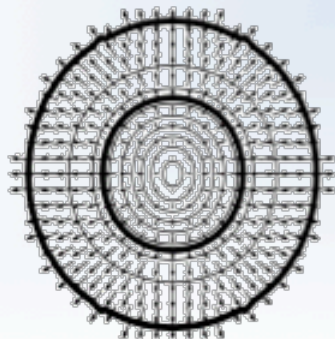
- much less





Blast-wave fit to spectra:

- much less explosive flow in p+p collisions



Don't forget - EMCFCs even for $k=1$

measured

$$\tilde{f}_c(p_i) = \tilde{f}(p_i) \left(\frac{N}{N-1} \right)^2 \exp \left(- \frac{1}{2(N-1)} \left(\frac{2p_{T,i}^2}{\langle p_T^2 \rangle} + \frac{p_{z,i}^2}{\langle p_z^2 \rangle} + \frac{(E_i - \langle E \rangle)^2}{\langle E^2 \rangle - \langle E \rangle^2} \right) \right)$$

“matrix element”

“distortion” of single-particle spectra

$$N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_z^2 \rangle$$

Characteristic scales of relevant system in which limited energy-momentum is shared

EMC effects even for $k=1$

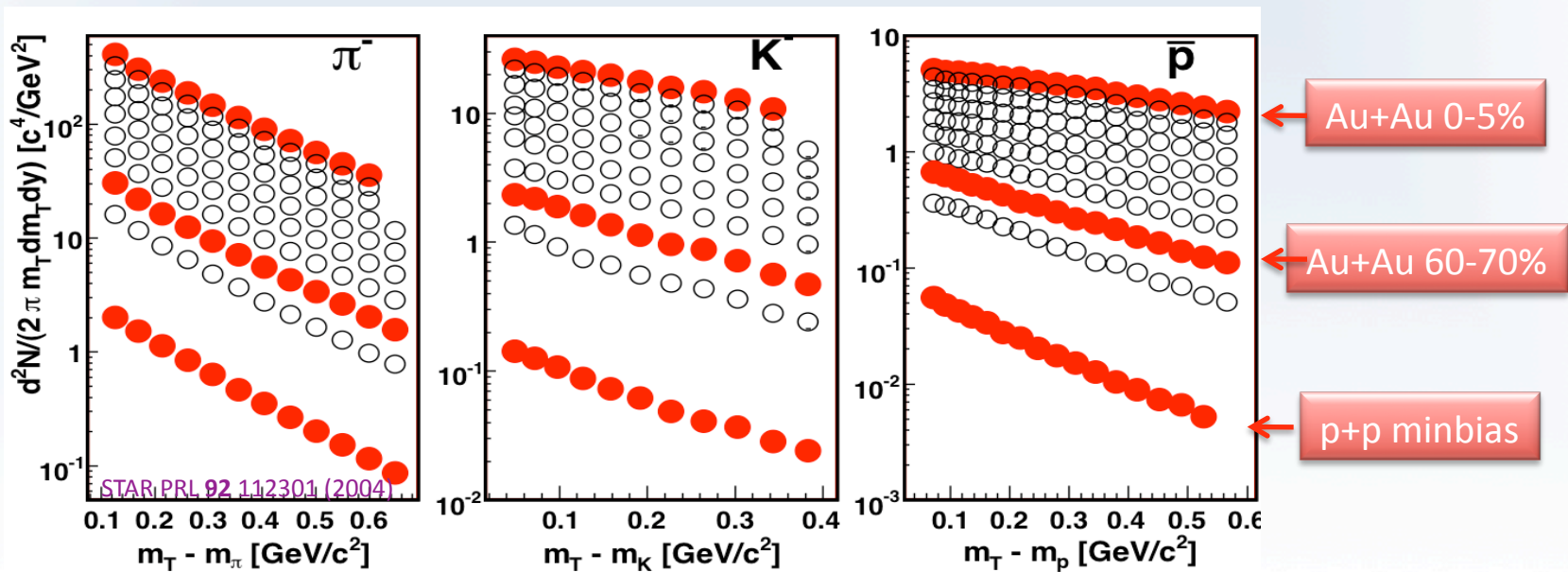
measured

$$\tilde{f}_c(p_i) = \tilde{f}(p_i) \underbrace{\left(\frac{N}{N-1} \right)^2 \exp \left[-\frac{1}{2(N-1)} \left(\frac{2p_{Ti}^2}{\langle p_T^2 \rangle} + \frac{p_{zi}^2}{\langle p_z^2 \rangle} + \frac{(E_i - \langle E \rangle)^2}{\langle E^2 \rangle - \langle E \rangle^2} \right) \right]}_{\text{"distortion" of single-particle spectra}}$$

"matrix element"

What if the only difference between p+p and A+A collisions was N ?

same $\tilde{f}(p)$, $\langle p_T^2 \rangle$, $\langle E \rangle$, $\langle E^2 \rangle$



EMCFs even for $k=1$

measured

$$\tilde{f}_c(p_i) = \tilde{f}(p_i) \underbrace{\left(\frac{N}{N-1} \right)^2 \exp \left(-\frac{1}{2(N-1)} \left(\frac{2p_{T,i}^2}{\langle p_T^2 \rangle} + \frac{p_{z,i}^2}{\langle p_z^2 \rangle} + \frac{(E_i - \langle E \rangle)^2}{\langle E^2 \rangle - \langle E \rangle^2} \right) \right)}_{\text{"distortion" of single-particle spectra}}$$

"matrix element"

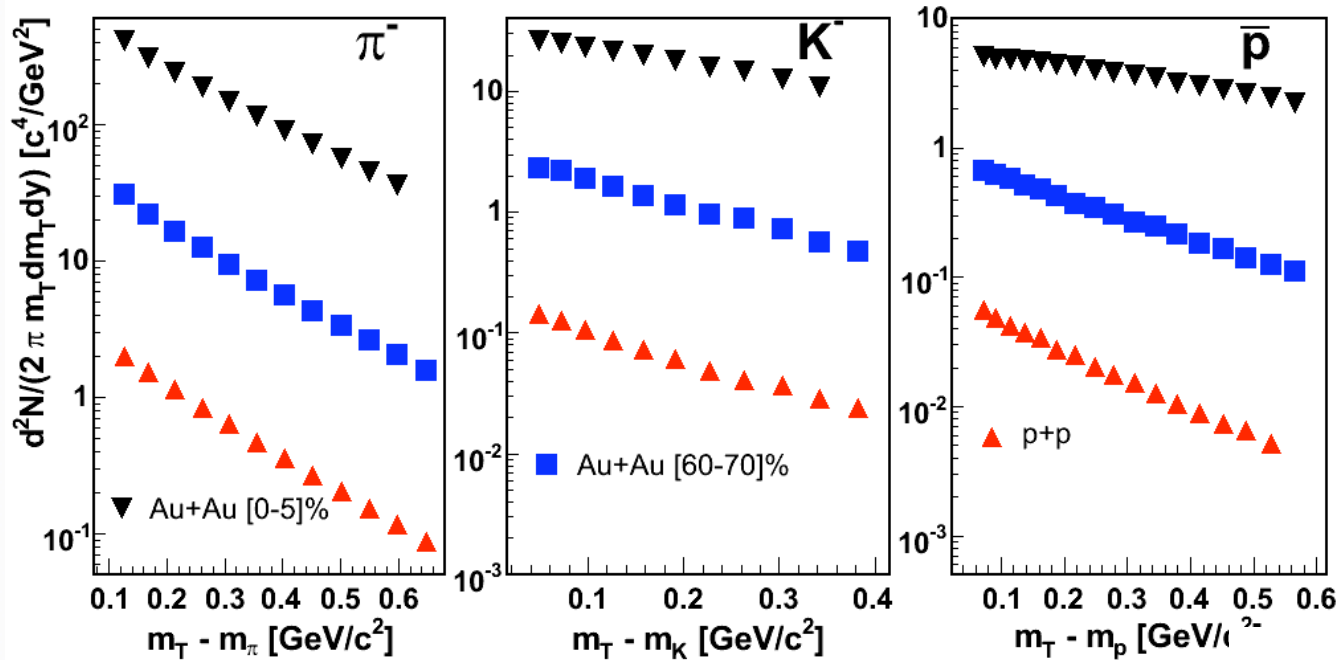
What if the only difference between p+p and A+A collisions was N ?

same $\tilde{f}(p)$, $\langle p_T^2 \rangle$, $\langle E \rangle$, $\langle E^2 \rangle$

Then we would measure:

$$\frac{\tilde{f}_c^{pp}(p_{T,i})}{\tilde{f}_c^{AA}(p_{T,i})} = \left(\frac{(N_{AA} - 1)N_{pp}}{(N_{pp} - 1)N_{AA}} \right)^2 \exp \left(\left(\frac{1}{2(N_{AA} - 1)} - \frac{1}{2(N_{pp} - 1)} \right) \left(\frac{2p_{T,i}^2}{\langle p_T^2 \rangle} + \frac{(E_i - \langle E \rangle)^2}{\langle E^2 \rangle - \langle E \rangle^2} \right) \right)$$

Multiplicity evolution of spectra - $p+p$ to $A+A$ (soft sector)

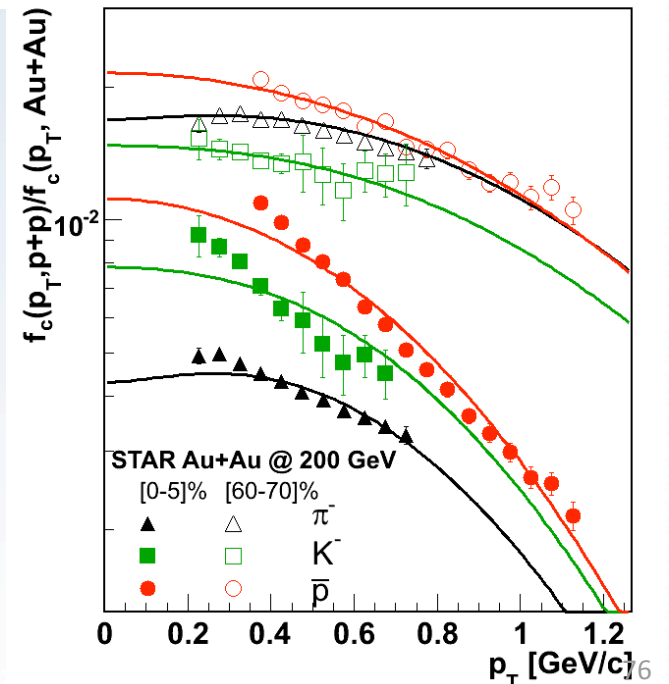


$$\frac{\tilde{f}_c^{pp}(p_{T,i})}{\tilde{f}_c^{AA}(p_{T,i})} \propto \exp\left(\left(\frac{1}{2(N_{AA}-1)} - \frac{1}{2(N_{pp}-1)}\right)\left(\frac{2p_{T,i}^2}{\langle p_T^2 \rangle} + \frac{(E_i - \langle E \rangle)^2}{\langle E^2 \rangle - \langle E \rangle^2}\right)\right)$$

N evolution of spectra dominated by PS “distortion”

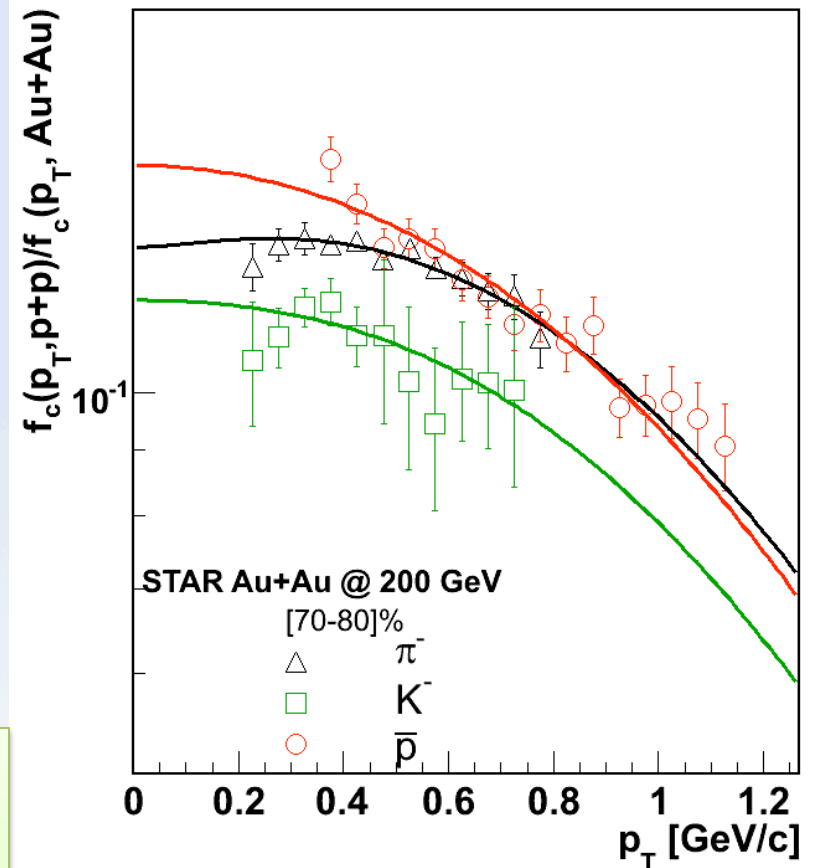
$p+p$ system samples *same* parent distribution, but under stronger PS constraints

$K \sim$ unity. driven by conservation of discrete quantum #s (strangeness, etc)



MPT: What changes with multiplicity...? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c) ²]	$\langle E^2 \rangle$ [GeV ²]	$\langle E \rangle$ [GeV]
<i>p + p</i> min-bias	10.3	0.12	0.43	0.61
<i>Au + Au</i> 70-80%	15.2	"	"	"
<i>Au + Au</i> 60-70%	18.3	"	"	"
<i>Au + Au</i> 50-60%	27.3	"	"	"
<i>Au + Au</i> 40-50%	38.7	"	"	"
<i>Au + Au</i> 30-40%	67.6	"	"	"
<i>Au + Au</i> 20-30%	219	"	"	"
<i>Au + Au</i> 10-20%	> 300	"	"	"
<i>Au + Au</i> 5-10%	> 300	"	"	"
<i>Au + Au</i> 0-5%	> 300	"	"	"



postulate of *same* parent consistent with *all* spectra

- magnitude
- p_T dependence (shape)
- mass dependence

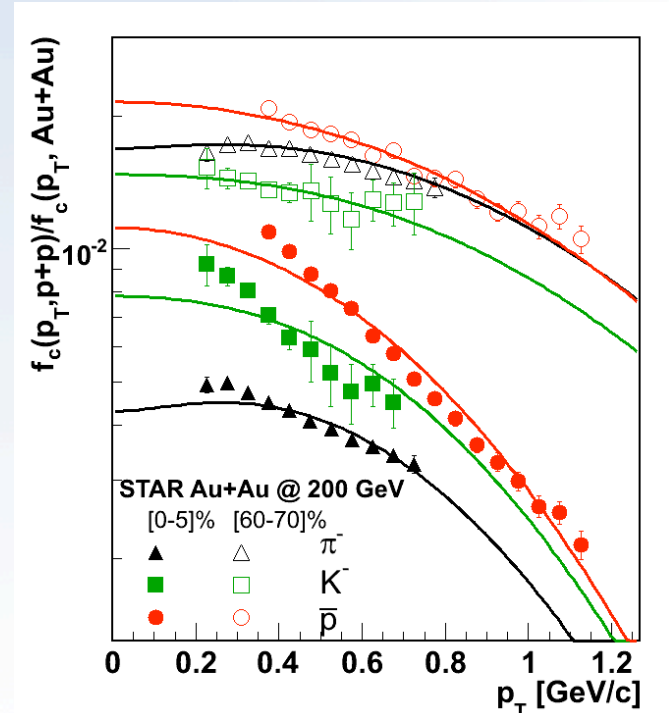
Kinematic scales of "the system"

$$\frac{\tilde{f}_c^{E_1}(p_{T,i})}{\tilde{f}_c^{E_2}(p_{T,i})} = \left(\frac{(N_2 - 1)N_1}{(N_1 - 1)N_2} \right)^2 \exp \left(\left(\frac{1}{2(N_2 - 1)} - \frac{1}{2(N_1 - 1)} \right) \left(\frac{2p_{T,i}^2}{\langle p_T^2 \rangle} + \frac{(E_i - \langle E \rangle)^2}{\langle E^2 \rangle - \langle E \rangle^2} \right) \right)$$

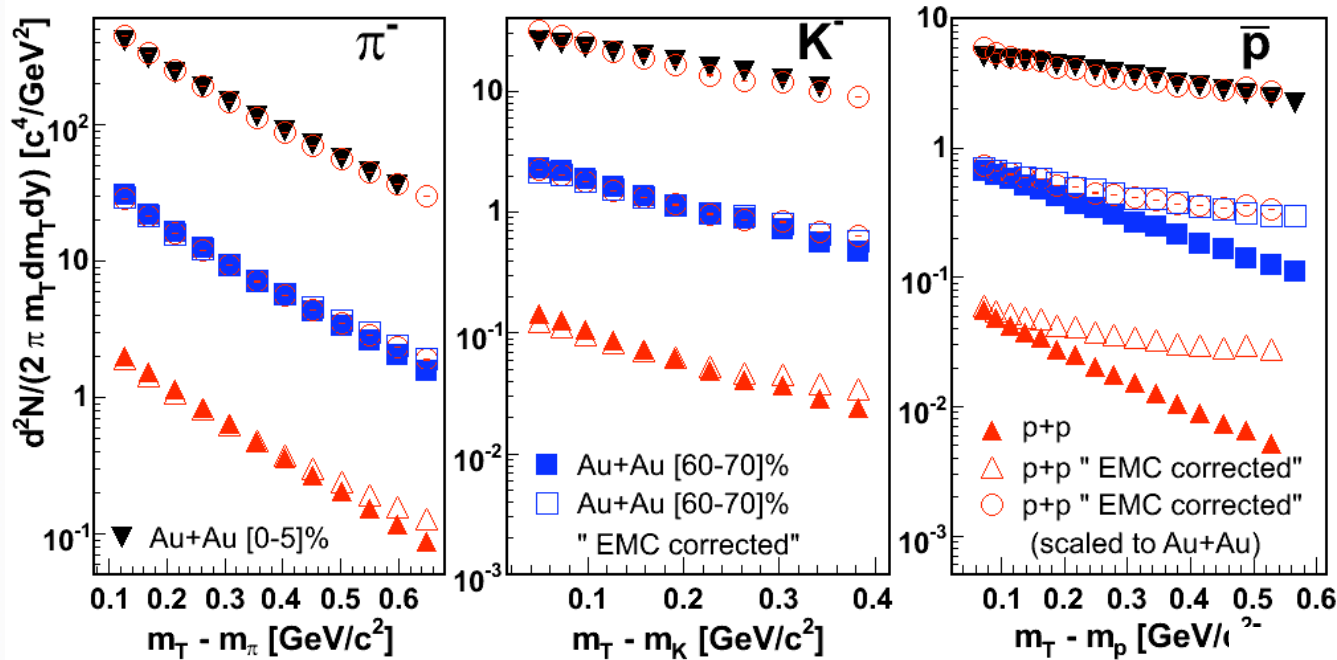
	non - rel	ultra - rel	if $T = .15 \div .35$	What we find
$\langle p_T^2 \rangle$	$2mT$	$8T^2$	$0.045 \div 0.98 \text{ (GeV/c)}^2$	0.12 (GeV/c)^2
$\langle E^2 \rangle$	$\frac{15}{4}T^2 + m^2$	$12T^2$	$0.10 \div 1.5 \text{ GeV}^2$	0.43 GeV^2
$\langle E \rangle$	$\frac{3}{2}T + m$	$3T$	$0.36 - 1 \text{ GeV}$	0.61 GeV

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c) ²]	$\langle E^2 \rangle$ [GeV ²]	$\langle E \rangle$ [GeV]
$p + p$ minbias	10.3	0.12	0.43	0.61
$Au + Au$ 70-80%	15.2	"	"	"
$Au + Au$ 60-70%	18.3	"	"	"
$Au + Au$ 50-60%	27.3	"	"	"
$Au + Au$ 40-50%	38.7	"	"	"
$Au + Au$ 30-40%	67.6	"	"	"
$Au + Au$ 20-30%	219	"	"	"
$Au + Au$ 10-20%	> 300	"	"	"
$Au + Au$ 5-10%	> 300	"	"	"
$Au + Au$ 0-5%	> 300	"	"	"

TABLE II: Multiplicity and parent-distribution kinematic parameters which give a reasonable description of the spectrum ratios for identified particles in the soft sector. See text for details. Note that the multiplicity changes with event class; the parent distribution is assumed identical.



Multiplicity evolution of spectra - $p+p$ to $A+A$ (soft sector)

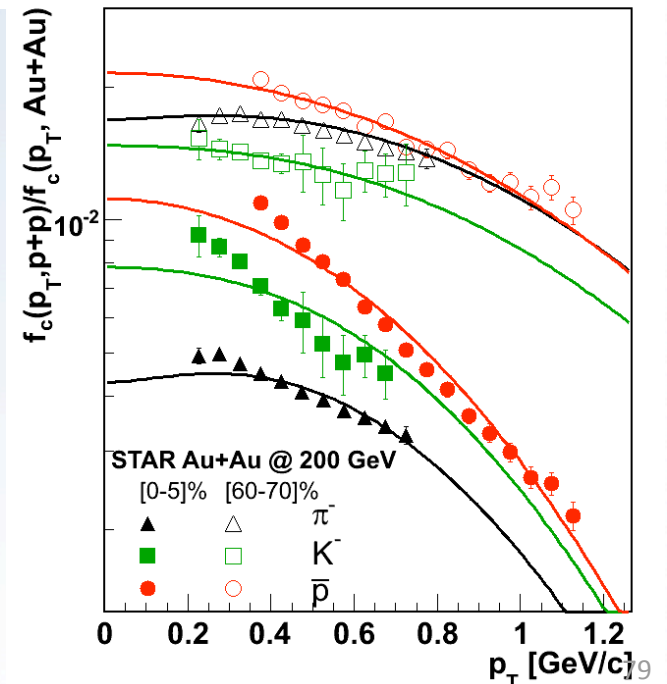


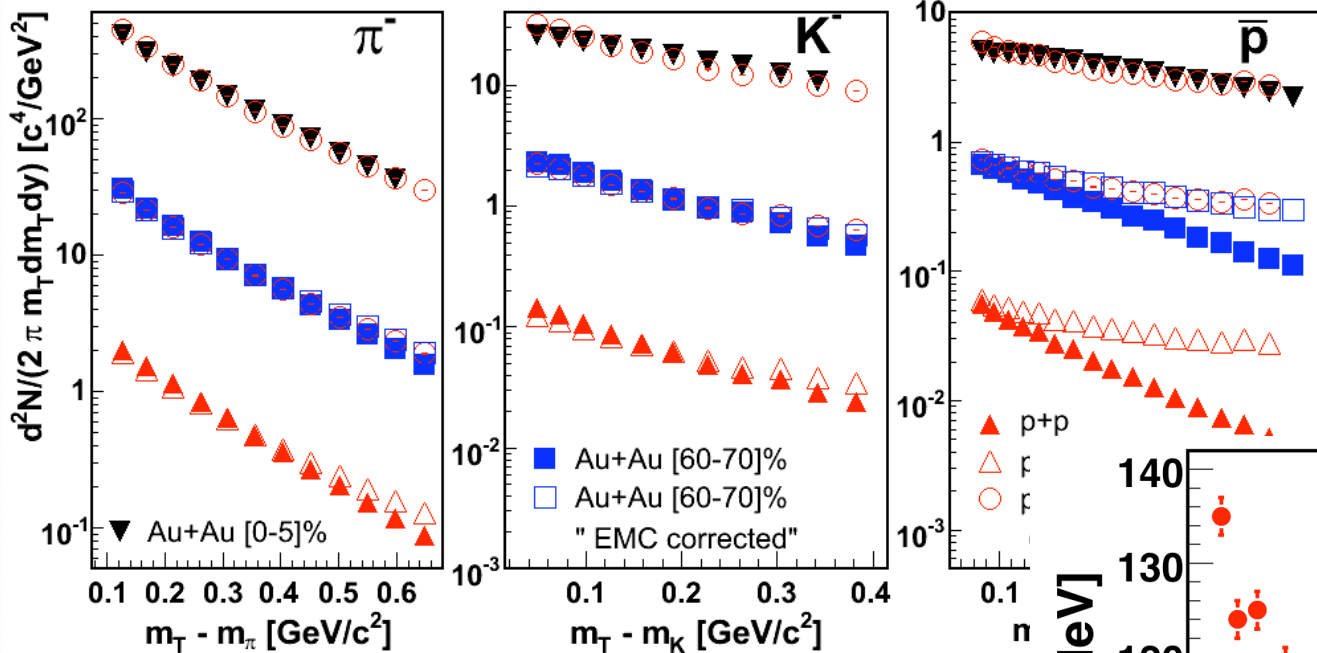
$$\frac{\tilde{f}_c^{pp}(p_{T,i})}{\tilde{f}_c^{AA}(p_{T,i})} \propto \exp\left(\left(\frac{1}{2(N_{AA}-1)} - \frac{1}{2(N_{pp}-1)}\right)\left(\frac{2p_{T,i}^2}{\langle p_T^2 \rangle} + \frac{(E_i - \langle E \rangle)^2}{\langle E^2 \rangle - \langle E \rangle^2}\right)\right)$$

N evolution of spectra dominated by PS "distortion"

$p+p$ system samples *same* parent distribution, but under stronger PS constraints

$K \sim$ unity. driven by conservation of discrete quantum #s (strangeness, etc)

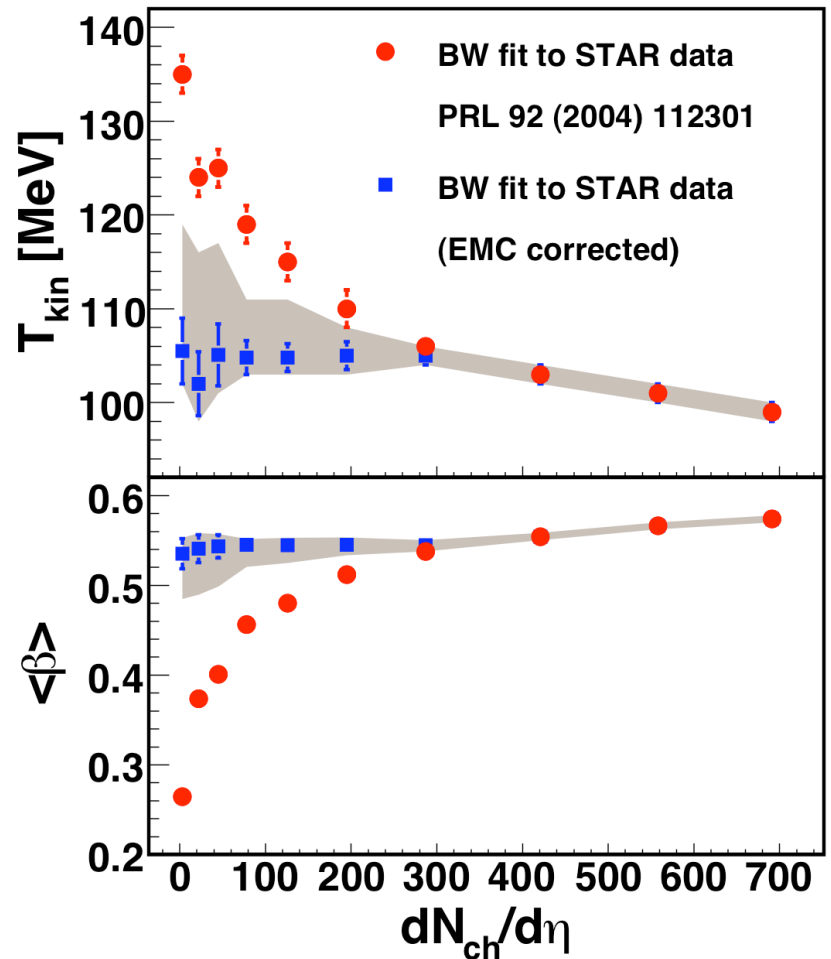




By popular demand

Almost universal “flow” & “temperature” parameters in a BlastWave fit

Apparent changes in β , T with $dN/d\eta$ caused by EMCICs*



* EMCIC = Energy & Momentum Conservation Induced *Constraint*

Blast-wave in $p+p$: simultaneous description of spectra, \sqrt{s}

$T = 105.5$ MeV

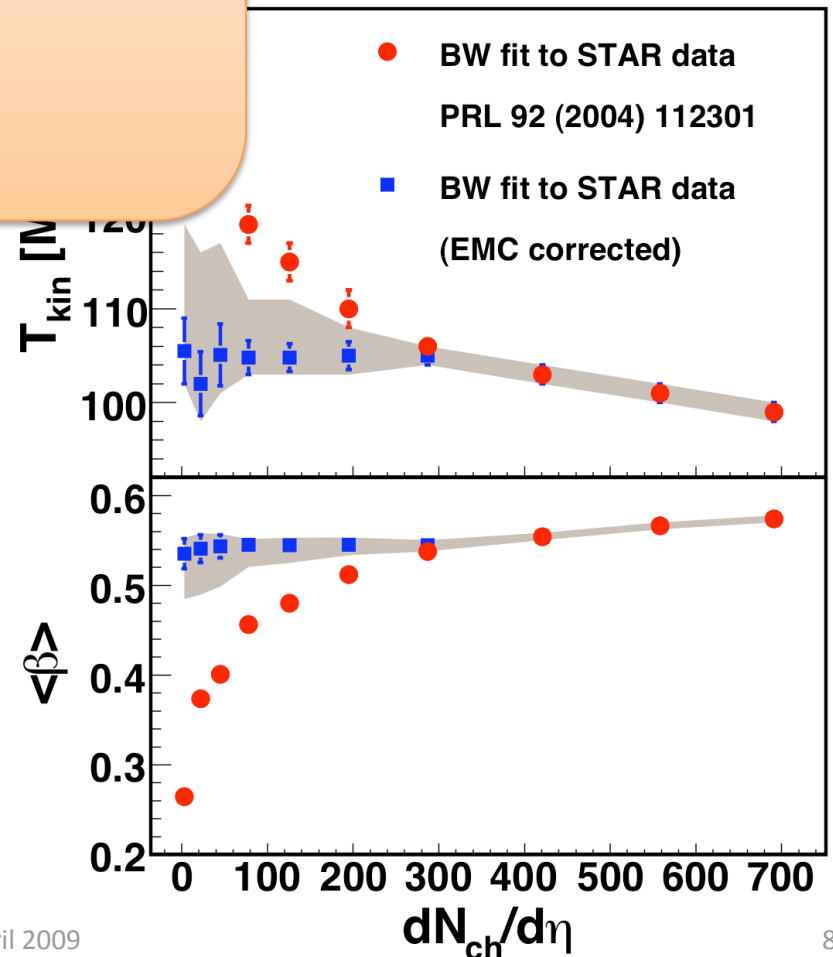
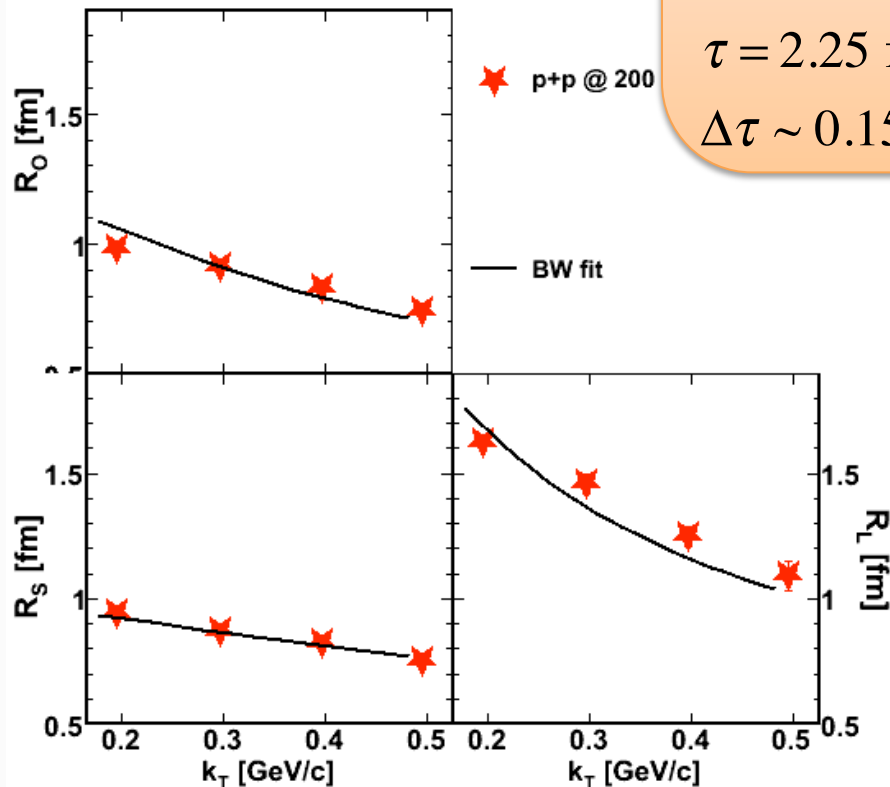
$\rho_0 = 0.934$ ($\langle\beta\rangle = 0.535$)

$R = 2.19$ fm

$\tau = 2.25$ fm/c

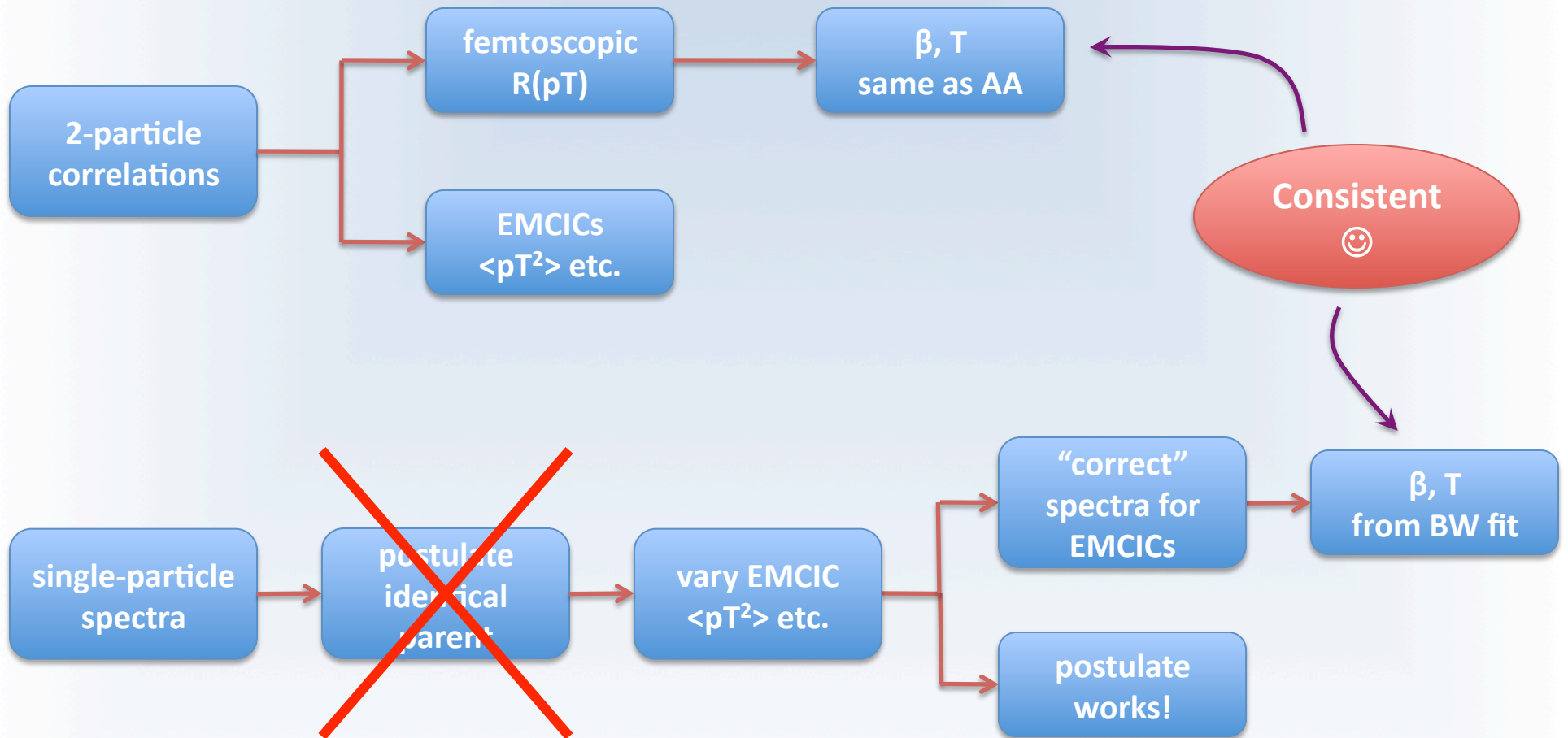
$\Delta\tau \sim 0.15$

determined entirely
by spectra



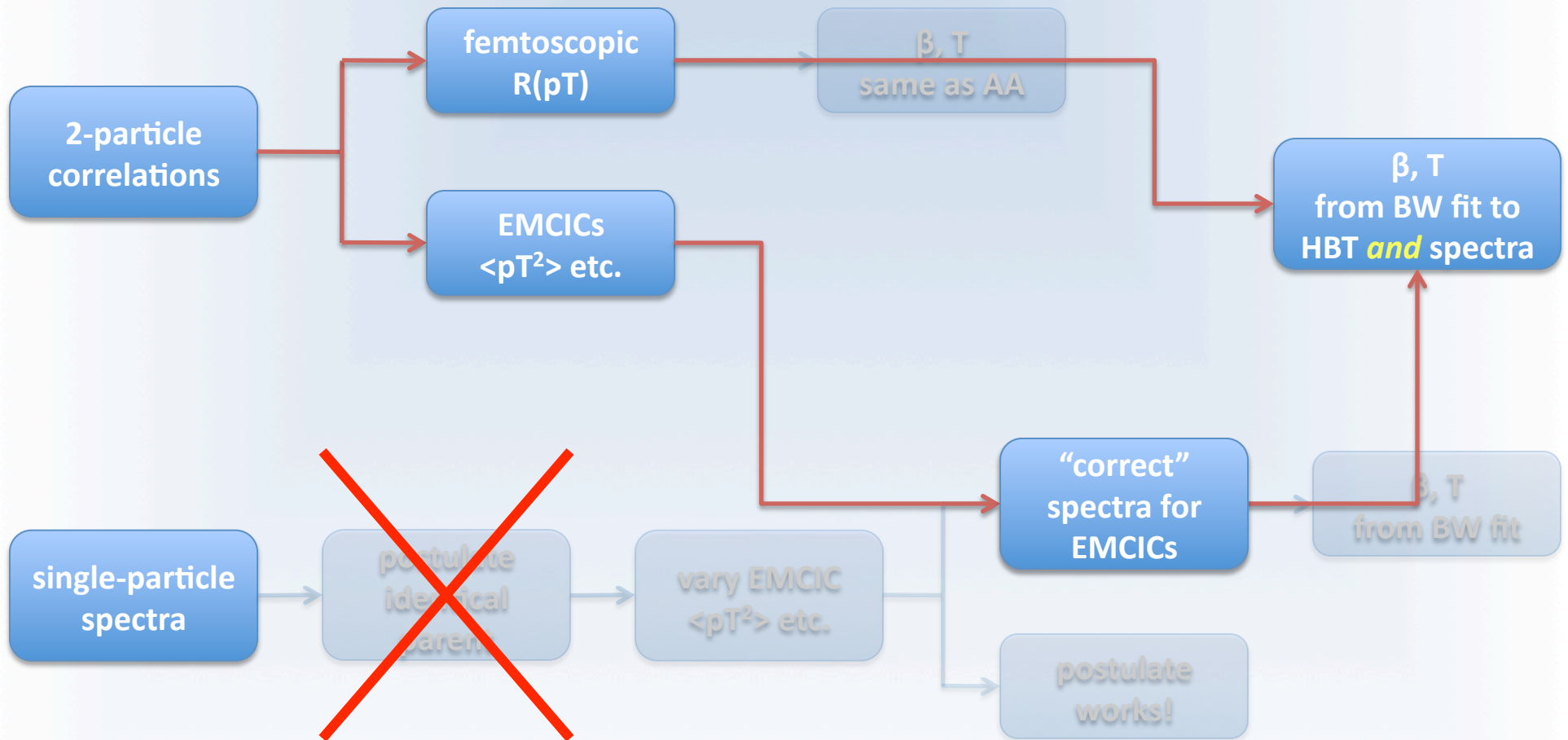
Can't follow the game without a program...

PHASE I

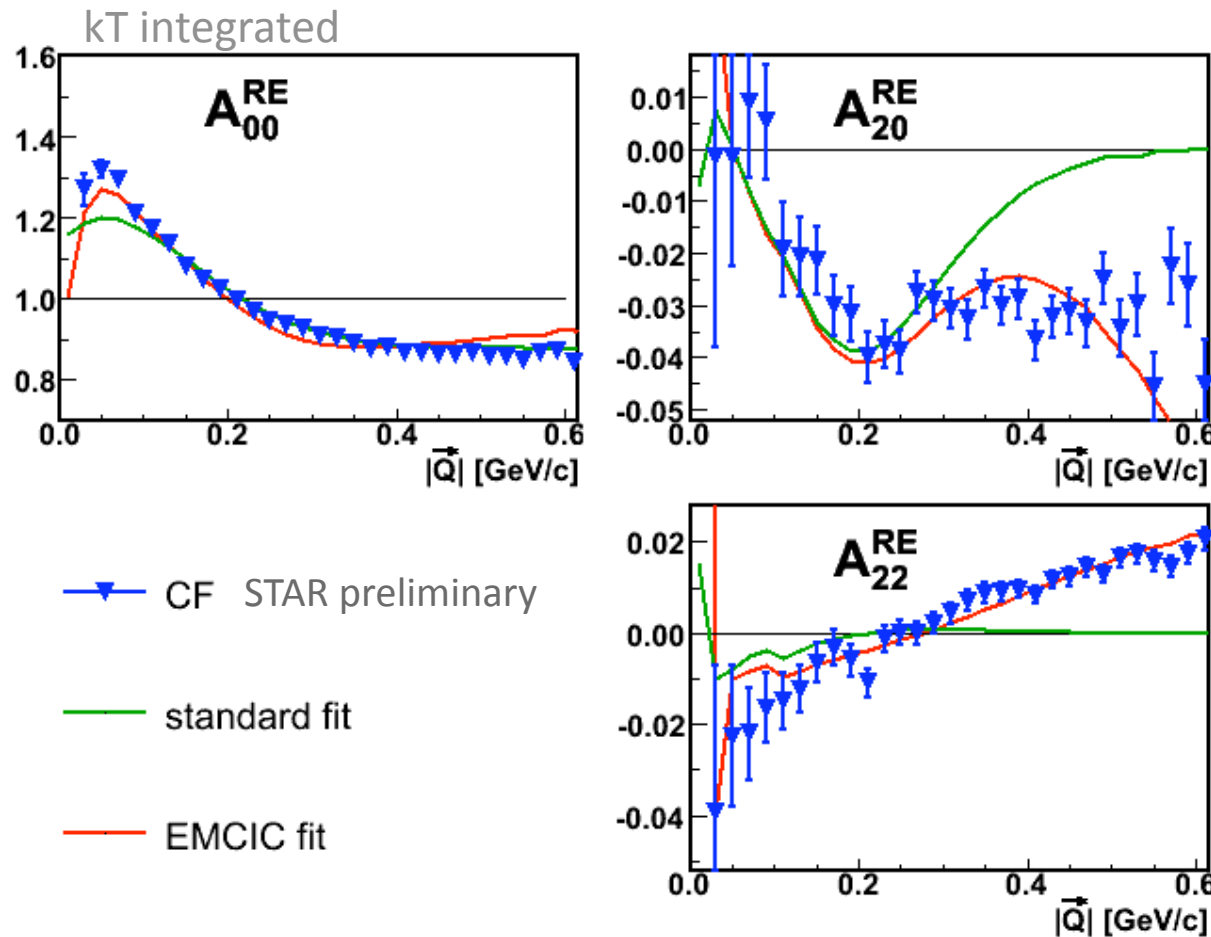


Can't follow the game without a program...

PHASE II



Femto and "system" parameters from 2-particle correlations

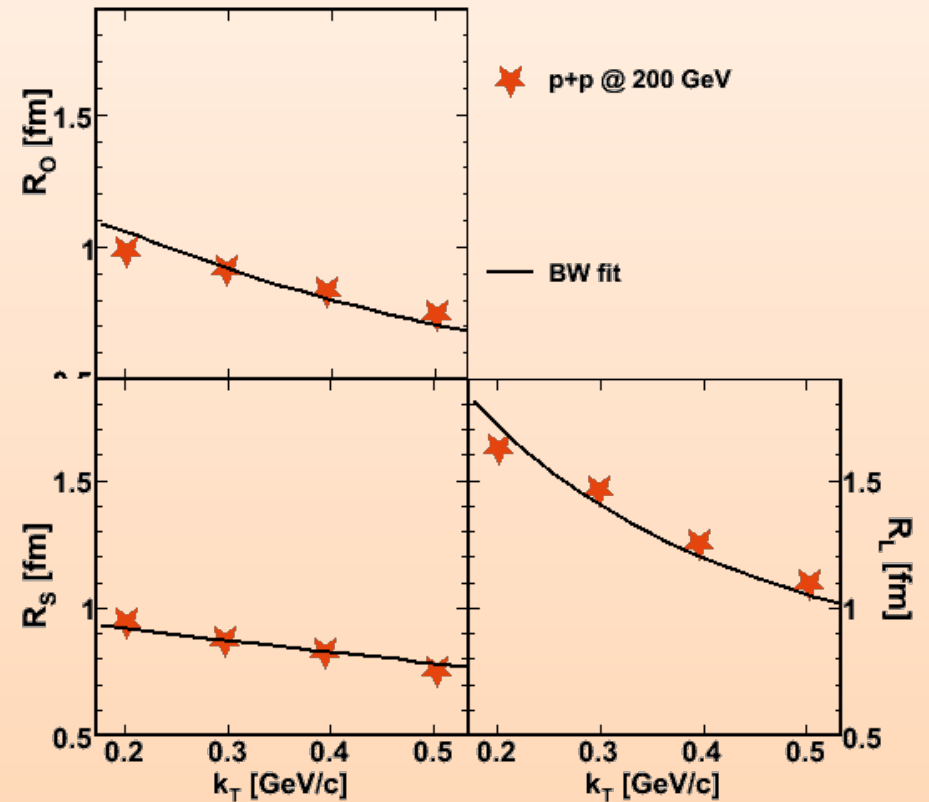
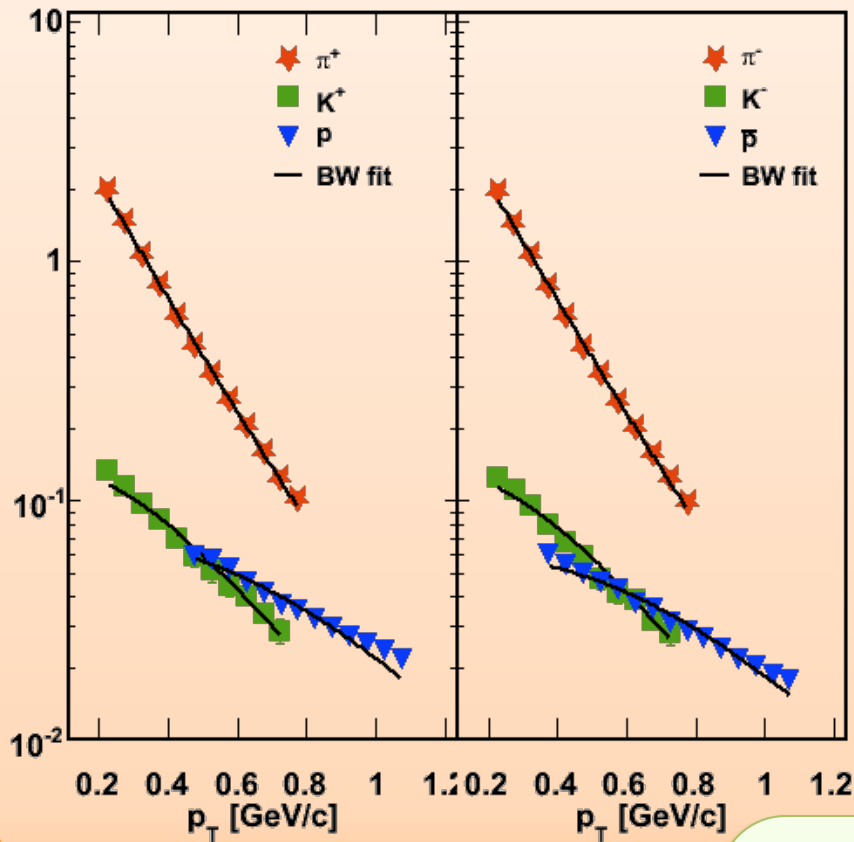


$\lambda = 0.38 \pm 0.01$
 $R_o = 0.81 \pm 0.01$ fm
 $R_s = 0.84 \pm 0.02$ fm
 $R_l = 1.29 \pm 0.02$ fm

$\lambda = 0.69 \pm 0.01$
 $R_o = 0.96 \pm 0.04$ fm
 $R_s = 0.98 \pm 0.03$ fm
 $R_l = 1.26 \pm 0.02$ fm

$N = 13.6$
 $\langle E \rangle = 0.68$ GeV
 $\langle E^2 \rangle = 0.54$ GeV²
 $\langle p_T^2 \rangle = 0.17$ GeV²
 $\langle p_z^2 \rangle = 0.33$ GeV²

Combined fit: consistent flow-based description



$$T = 106 \pm 3 \text{ MeV}$$

$$\langle \beta \rangle = 0.48 \pm 0.03$$

$$R = 2.09 \pm 0.04 \text{ fm}$$

$$\tau_0 = 2.25 \pm 0.05 \text{ fm/c}$$

$$\Delta\tau = 0.1 \pm 0.2 \text{ fm/c}$$

Combined fit: consistent flow-based description

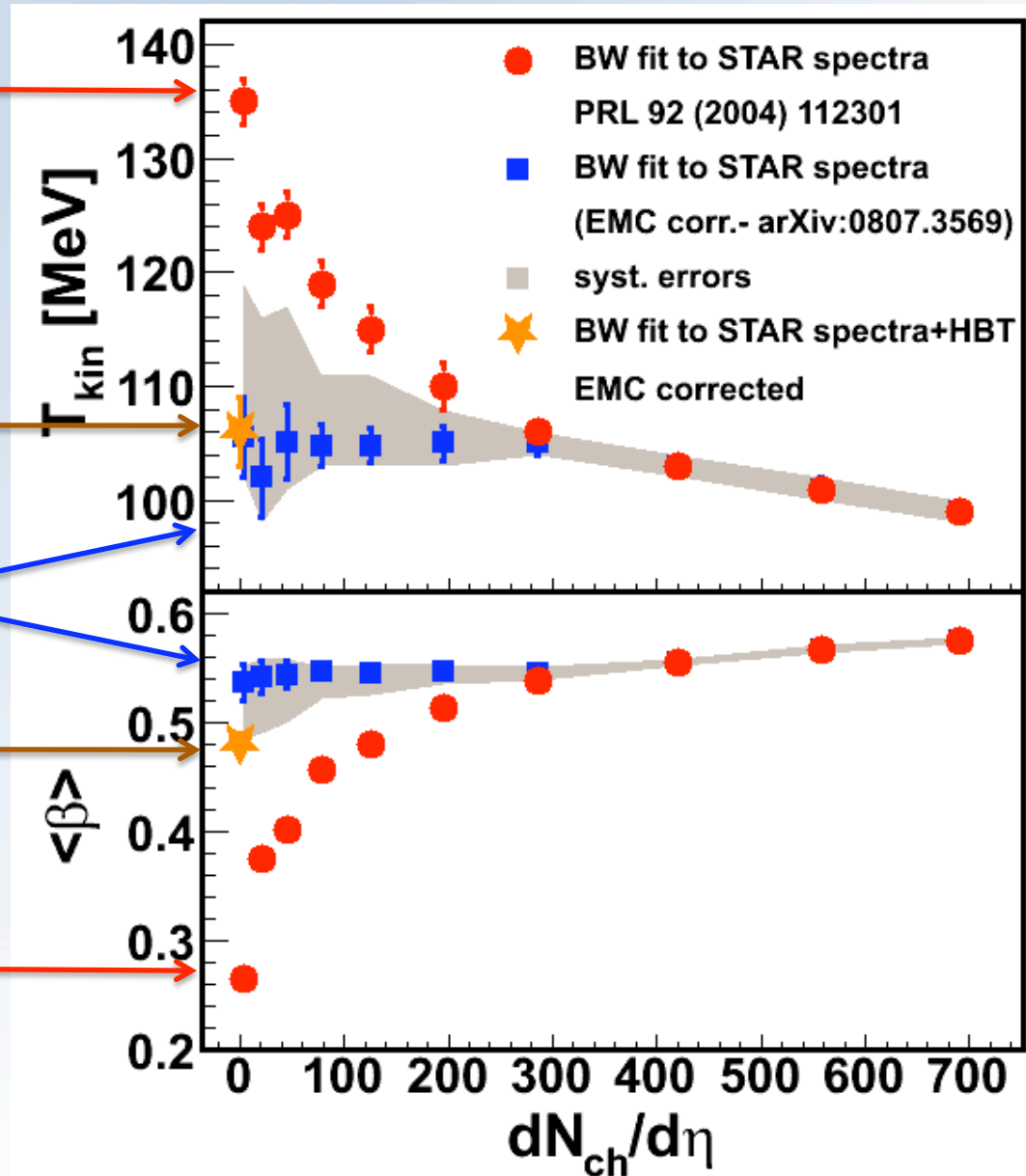
“raw” (ignoring EMCICs)

EMCICs **fixed by correlations**
Joint spectra/HBT BW fit

EMCICs **free adjusted**
to spectra & fit to spectra

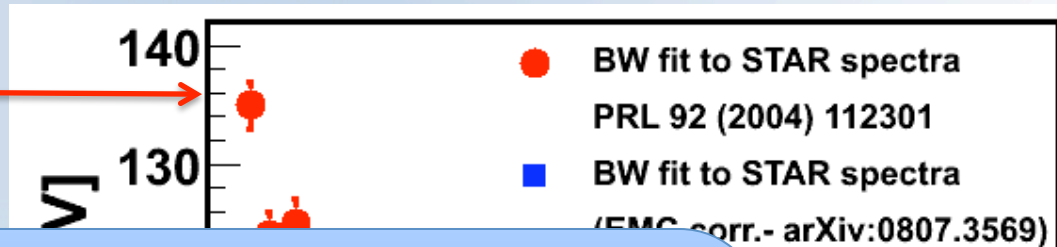
EMCICs **fixed by correlations**
Joint spectra/HBT BW fit

“raw” (ignoring EMCICs)



Combined fit: consistent flow-based description

“raw” (ignoring EMCICs)



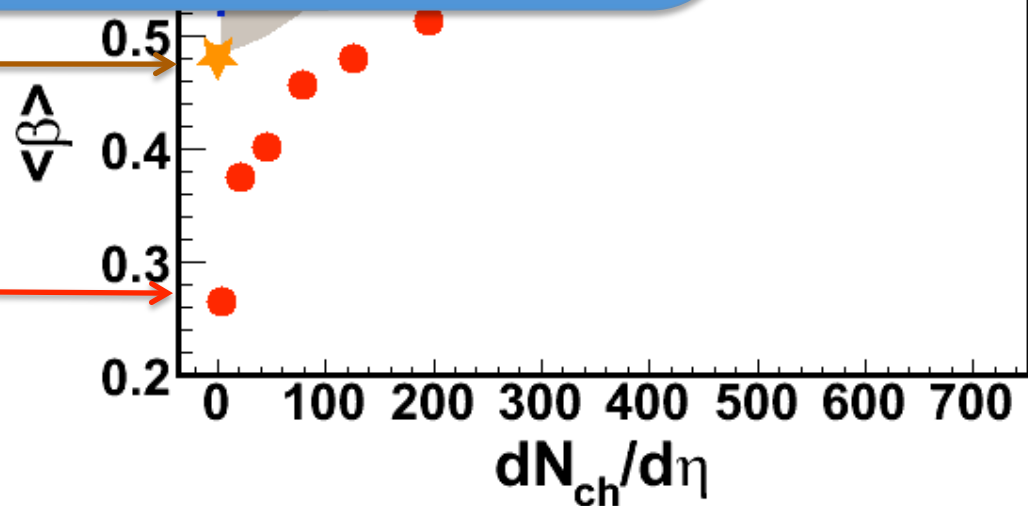
p+p collisions show same flow signals as A+A collisions

EMCICs **fixed by correlations**
 Joint spectra/HBT BW fit

EMCICs **free adjusted**
 to spectra & fit to spectra

EMCICs **fixed by correlations**
 Joint spectra/HBT BW fit

“raw” (ignoring EMCICs)



Implication: $A+A$ is just a collection of flowing $p+p$?

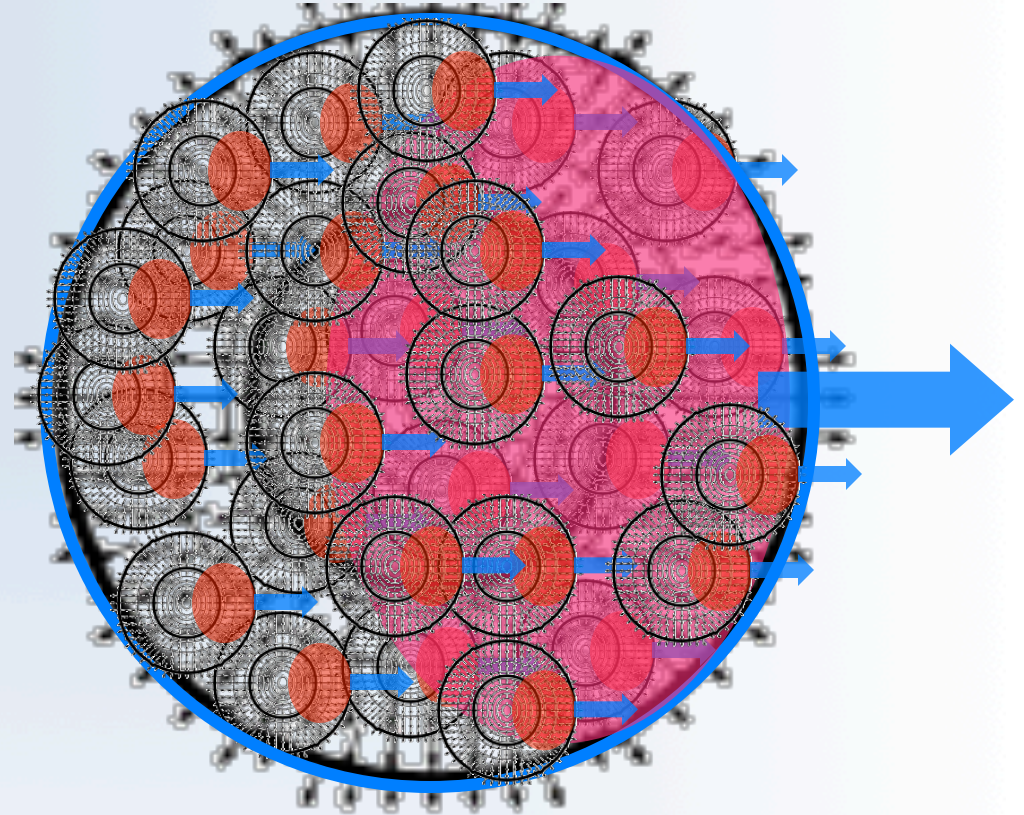
- **No! Quite the opposite.**

– **femtoscopically**

- $A+A$ looks like a big BlastWave
- *not* superposition of small BlastWaves
- $A+A$ has thermalized globally

– **spectra**

- superposition of spectra from $p+p$ has same shape as a spectrum from $p+p$!
- relaxation of P.S. constraints indicates $A+A$ has thermalized globally

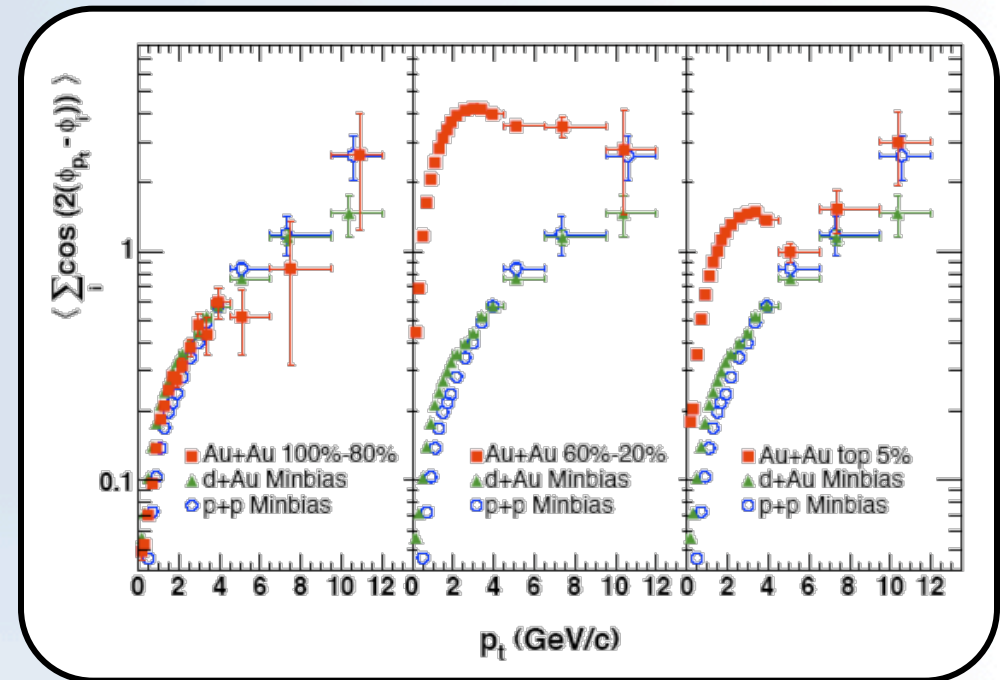


- **rather, $p+p$ looks like a “little $A+A$ ”**

1st Joint Workshop on Energy Scaling of Hadron Collisions

Implication: A+A is just a collection of flowing p+p?

- **No! Quite the opposite.**
 - **femtoscopically**
 - A+A looks like a big BlastWave
 - *not* superposition of small BlastWaves
 - A+A has thermalized globally
 - **spectra**
 - superposition of spectra from p+p has same shape as a spectrum from p+p!
 - relaxation of P.S. constraints indicates A+A has thermalized globally
- rather, p+p looks like a “little A+A”

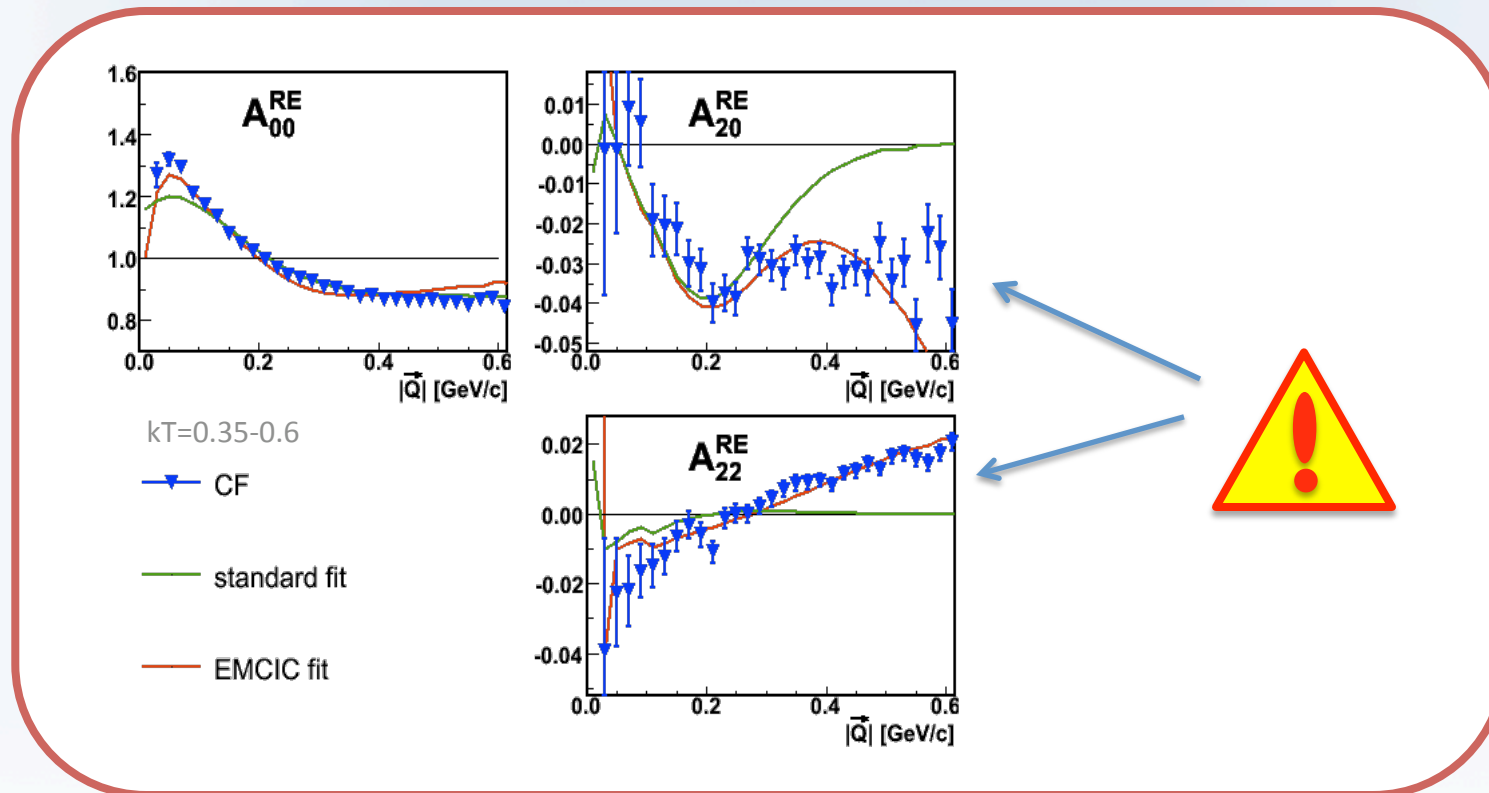


anisotropic flow

- A+A shows increased signal over superposition of p+p
- is the p+p signal “flow” ??

Summary

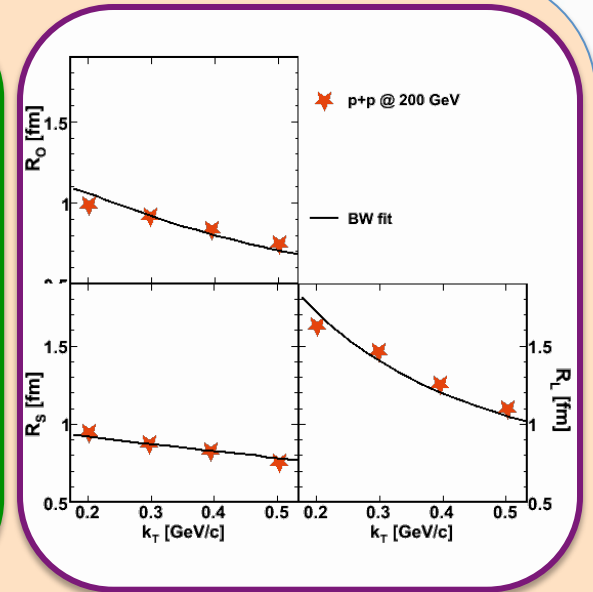
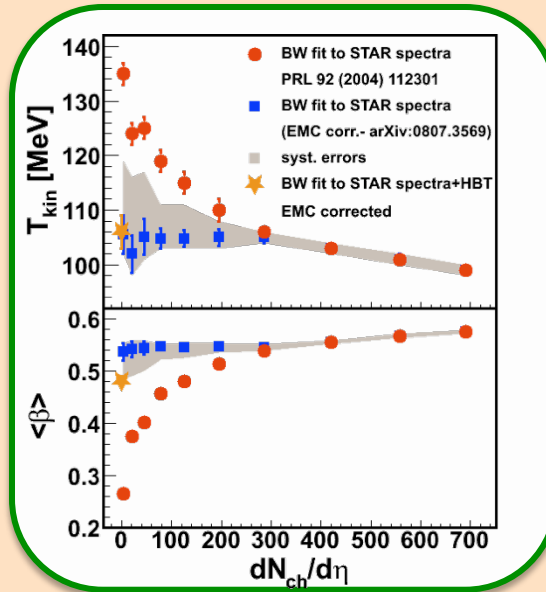
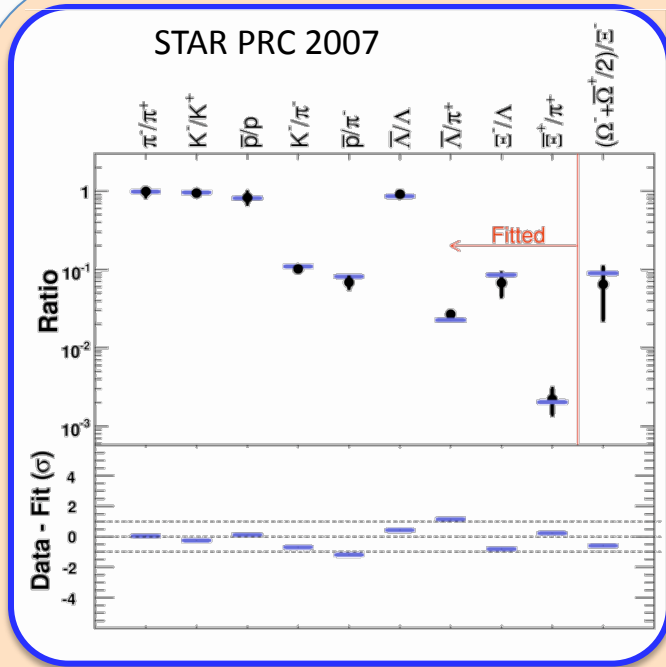
- E&M conservation induces phase-space constraints w/ explicit N dependence
 - should not be ignored in (crucial!) N-dependent comparisons
 - significant effect on 2- (and 3-) particle correlations [c.f. Ollitrault, Borghini, Voloshin...]
 - ...and single-particle spectra (often neglected because no “red flags”)



Summary

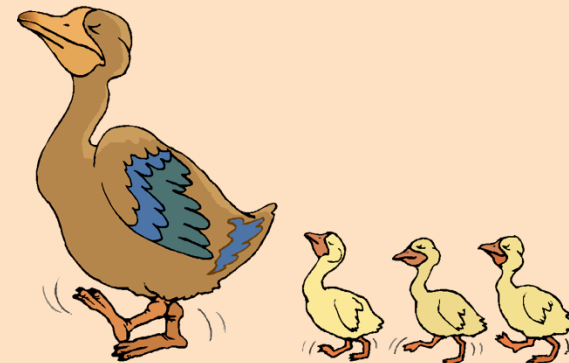
- E&M conservation induces phase space constraints w/ explicit N dependence
 - should not be ignored in (crucial!) N-dependent comparisons
 - significant effect on 2- (and 3-) particle correlations [c.f. Ollitrault, Borghini, Voloshin...]
 - ...*and* single-particle spectra (often neglected because no “red flags”)
- Femtoscopy & Spectra
 - in H.I.C., well understood, detailed fingerprint of flow
 - RHIC – first opportunity for direct comparison with p+p
 - accounting for EMCICs, identical flow signals in p+p
- is pp/AA physics very similar, or are measurements insensitive to diff physics?
- **Has AA become the reference system for pp in non-perturbative sector???**
- Thermalization, hadronization, very early color dynamics...

Summary



“It is a capital mistake to theorize before one has data.”
 – Sir Arthur Conan Doyle [*Debasish Das @ WWND09*]

“It is even worse, when one *has* data, to insist upon two orthogonal theoretical interpretations for the same systematics.”
 – Prof. Mike Lisa



Summary

- E&M conservation induces phase space constraints w/ explicit N dependence

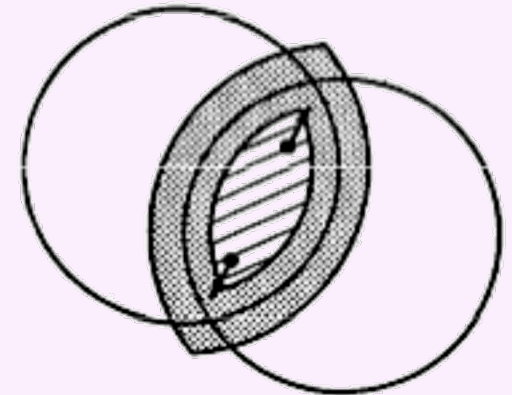


Fermi National Accelerator Laboratory

FERMILAB-Pub-82/59-THY
August, 1982

Energy Loss of Energetic Partons in Quark-Gluon Plasma:
Possible Extinction of High p_T Jets in Hadron-Hadron Collisions.

J. D. BJORKEN
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510



- Well, anyway, account for the “trivial”
 - EMCICs do *not* explain N-dependence of spectra – para
 - EMCIC effects in p+p big, may modify physics conclusions