

# INTRODUCTION TO FFAG ACCELERATORS

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With grateful acknowledgements to the colleagues who have kindly  
provided images and other material

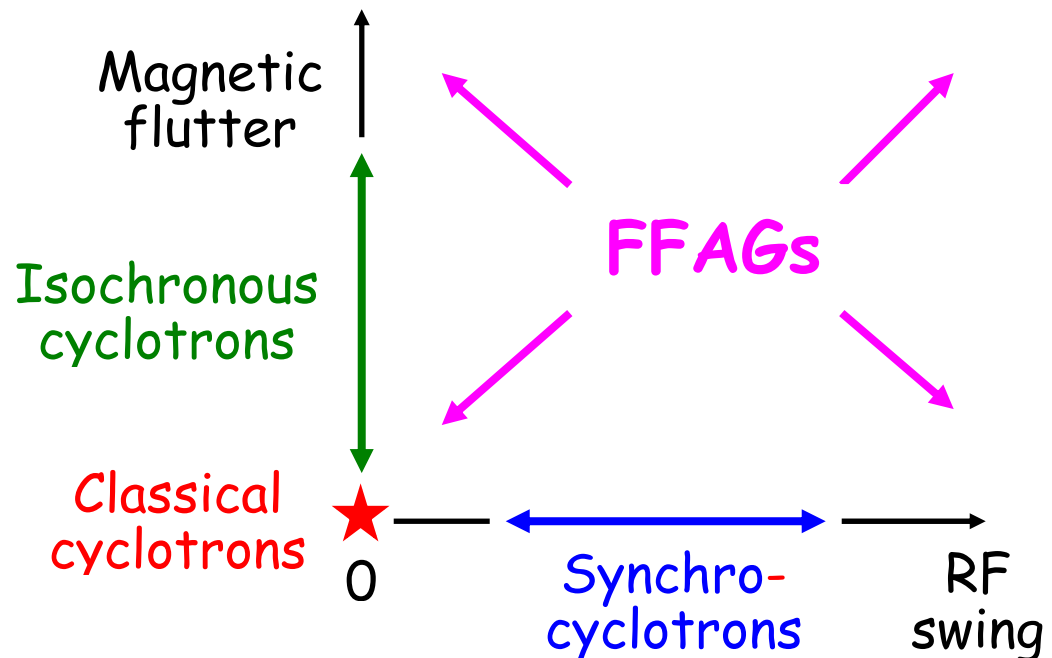
FFAG'09 Workshop, Fermilab, 21-25 September, 2009

# FFAGs - Fixed Field Alternating Gradient accelerators

**Fixed Magnetic Field** - members of the **CYCLOTRON** family<sup>1</sup>

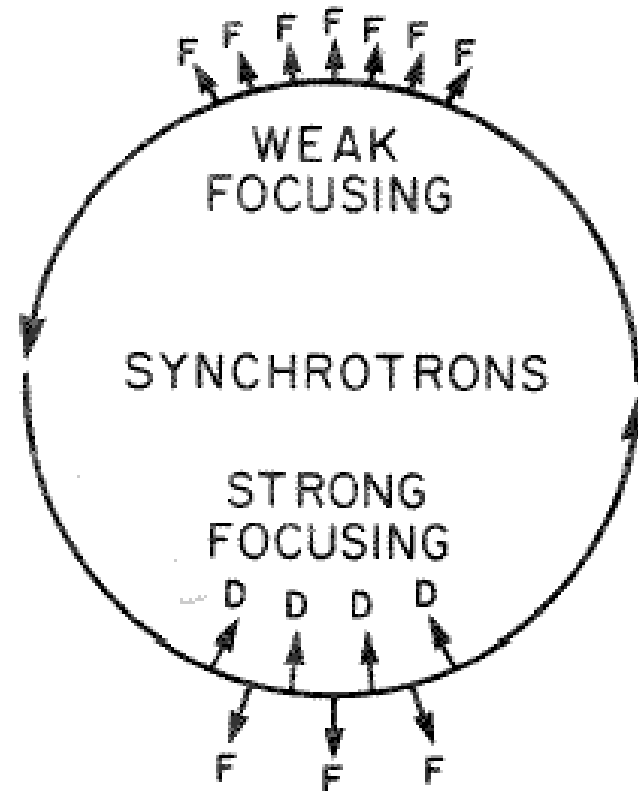
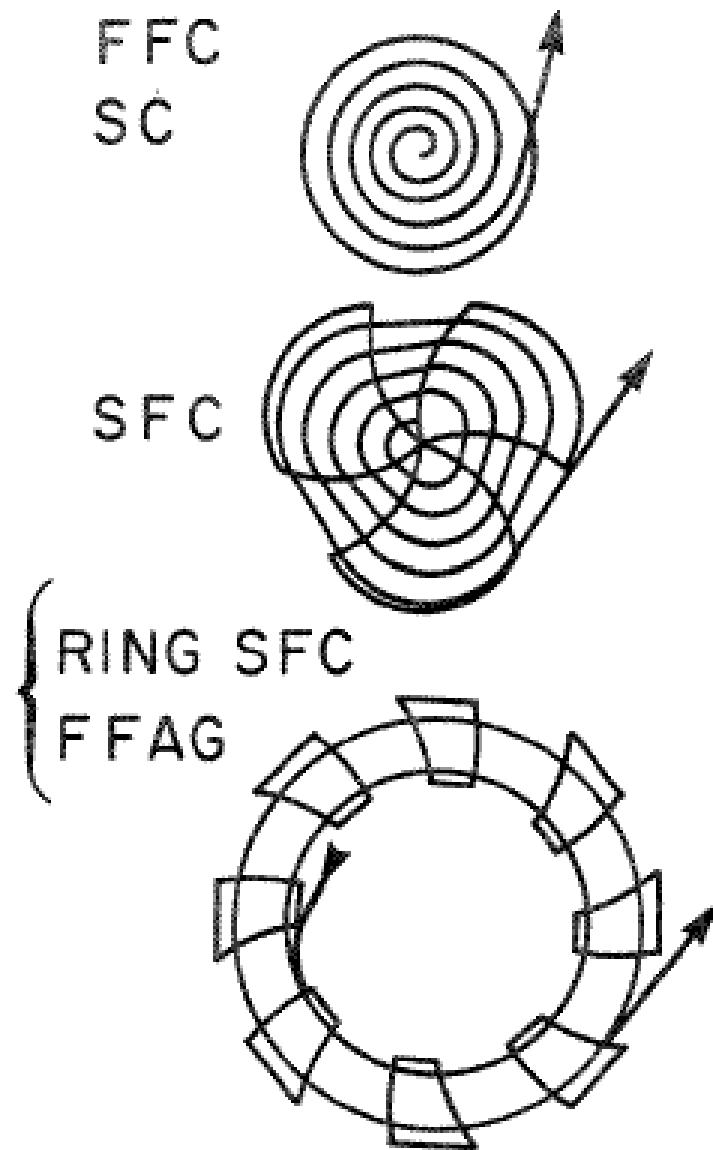
Magnetic field variation $B(\theta)$	Fixed Frequency (CW beam)	Frequency-modulated (Pulsed beam)
Uniform	Classical	Synchro-
Alternating	Isochronous	FFAG

But FFAG enthusiasts sometimes express an alternative view:  
- cyclotrons are just special cases of the FFAG!



1. E.M. McMillan, *Particle Accelerators*, in *Experimental Nuclear Physics*, **III**, 639-786 (1959)

# THE CYCLOTRON AND SYNCHROTRON FAMILIES



FFC = fixed frequency cyclotron

SC = synchrocyclotron

SFC = sector-focused cyclotron

FFAG = fixed field alternating gradient

# BASIC CHARACTERISTICS OF FFAGs

are determined by their **FIXED MAGNETIC FIELD**

- **Spiral orbits**
  - needing wider magnets, rf cavities and vacuum chambers (compared to AG synchrotrons)
- **Faster rep rates (up to kHz?)** limited only by rf capabilities
  - not by magnet power supplies
- **Large acceptances**
- **High beam current**

The last 3 factors have fuelled interest in FFAGs over 50 years!

Good reading:

- K.R. Symon, D.W. Kerst, *et al.*, *Phys. Rev.* **103**, 1837 (1956)
- C.H Prior (ed.) [\*ICFA Beam Dynamics Newsletter\* \*\*43\*\*](#), 19-133 (2007);
- FFAG Workshops - Web links at [FFAG04](#) and [FFAG 2007](#).

# BRIEF HISTORY

- FFAGs were **proposed** by **Ohkawa, Kolomensky, Symon** and **Kerst**, (1953-5)
- and **studied** intensively at **MURA** in the 1950s and 1960s
  - several **electron models** were **built** and **operated** successfully
  - but no **proton FFAG** until **Mori's** at **KEK** (1 MeV 2000, 150 MeV 2003)

Now there's an explosion of interest!

- **6 more are now operating (for p, e,  $\alpha$ ) and 3 more (e) are being built**
- **~20 designs** under study:
  - for **protons, heavy ions, electrons** and **muons**
  - many of **novel "non-scaling" design**
- with **diverse applications**:
  - **cancer therapy**
  - **industrial irradiation**
  - **driving subcritical reactors**
  - **boosting high-energy proton intensity**
  - **producing neutrinos.**

**FFAG Workshops** since 1999:- Japan (x8), CERN, USA(x3), Canada, France, UK

# SCALING DESIGNS - HORIZONTAL TUNE $\nu_r$

**Resonances** were a worry in the 1950s, because of **slow acceleration**: if, at some energy, the **betatron oscillation wavelength** matches that of a **harmonic component of the magnetic field**, the ions **may be driven into resonance**, leading to **loss of beam quality or intensity**.

The **general condition** is  $\ell \nu_x \pm m \nu_y = n$  where  $\ell, m, n$  are integers.

So "**Scaling**" designs were used, with:

- the **same orbit shape at all energies**
- the **same optics** " " " " "
- the **same tunes** " " " " "  $\Rightarrow$  no crossing of resonances!

To 1<sup>st</sup> order, the **(radial tune)<sup>2</sup>**  $\nu_r^2 \approx 1 + k$  (even with sector magnets)

where the **average field index**  $k(r) \equiv \frac{r}{B_{av}} \frac{dB_{av}}{dr}$  and  $B_{av} = \langle B(\Theta) \rangle$

So large constant  $\nu_r$  requires  $k = \text{constant} \geq 0$

$$\Rightarrow B_{av} = B_0 (r/r_0)^k \quad \text{and} \quad p = p_0 (r/r_0)^{(k+1)}$$

# SCALING FFAGs - VERTICAL TUNE $\nu_z$

In the vertical plane, with **sector magnets** and to 1<sup>st</sup> order,

$$\nu_z^2 \approx -k + F^2(1 + 2\tan^2\varepsilon)$$

where the 2<sup>nd</sup> term describes the Thomas and spiral edge focusing effects.

Note  $k > 0 \Rightarrow$  **vertical defocusing**

$\therefore$  **large constant, real  $\nu_z$  requires large, constant  $F^2(1 + 2\tan^2\varepsilon)$**

**MURA** kept (1) **magnetic flutter**  $F^2 \equiv \left\langle \left( \frac{B(\theta) - B_{av}}{B_{av}} \right)^2 \right\rangle = \text{constant}$

(most simply achieved by using **constant profile  $B(\theta)/B_{av}$** )

(2a) for spiral sectors,

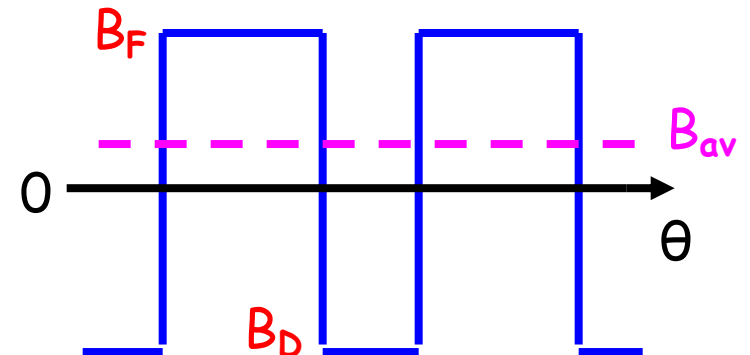
**spiral angle  $\varepsilon = \text{constant}$**  (sector axis follows  $R = R_0 e^{\theta \cot \varepsilon}$ )

(2b) for radial sectors,

**$B_D = -B_F$  to boost  $F^2$ .**

Note - **reverse fields increase average radius:**

$\Rightarrow$  **>4.5x larger** (Kerst & Symon '56 - no straights)



[Not so bad with straights: KEK 150-MeV FFAG has "circumference factor" 1.8]

In summary, scaling requires:-

- constant field index
  - constant and high flutter, with opposing F and D fields (if radial)
  - constant spiral angle (if spiral)
- meaning complex wide-aperture sector magnets

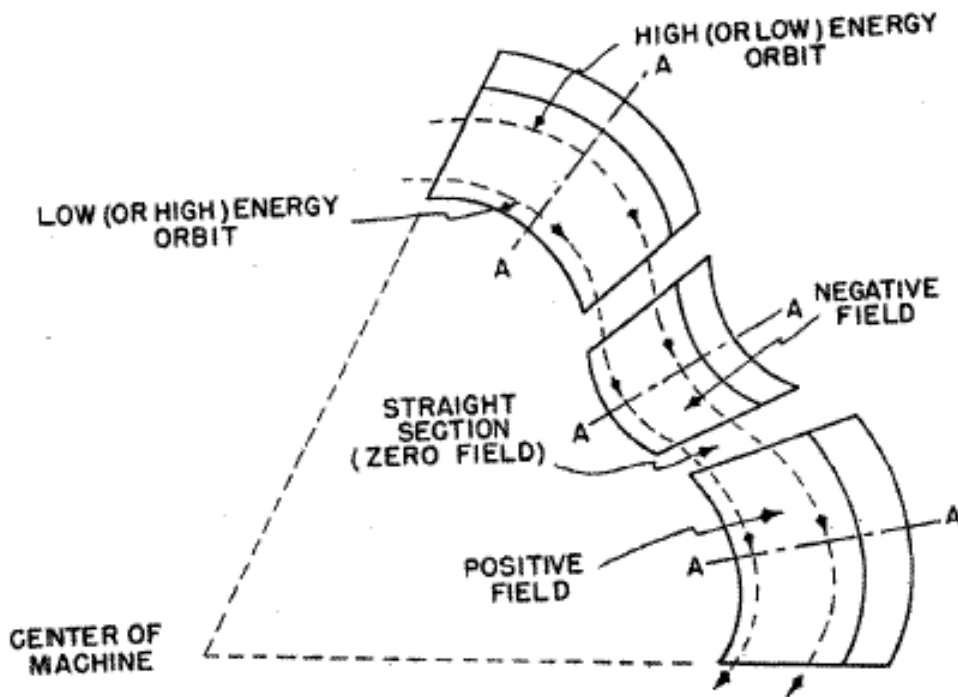


FIG. 2. Plan view of radial-sector magnets.

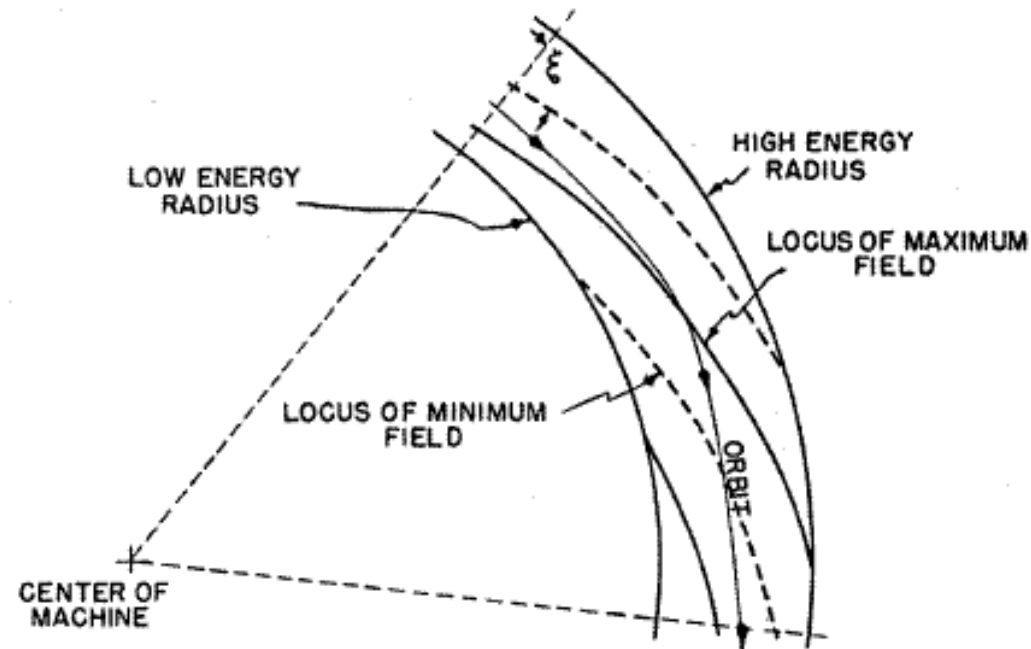


FIG. 3. Spiral-sector configuration.



# MURA Electron FFAGs

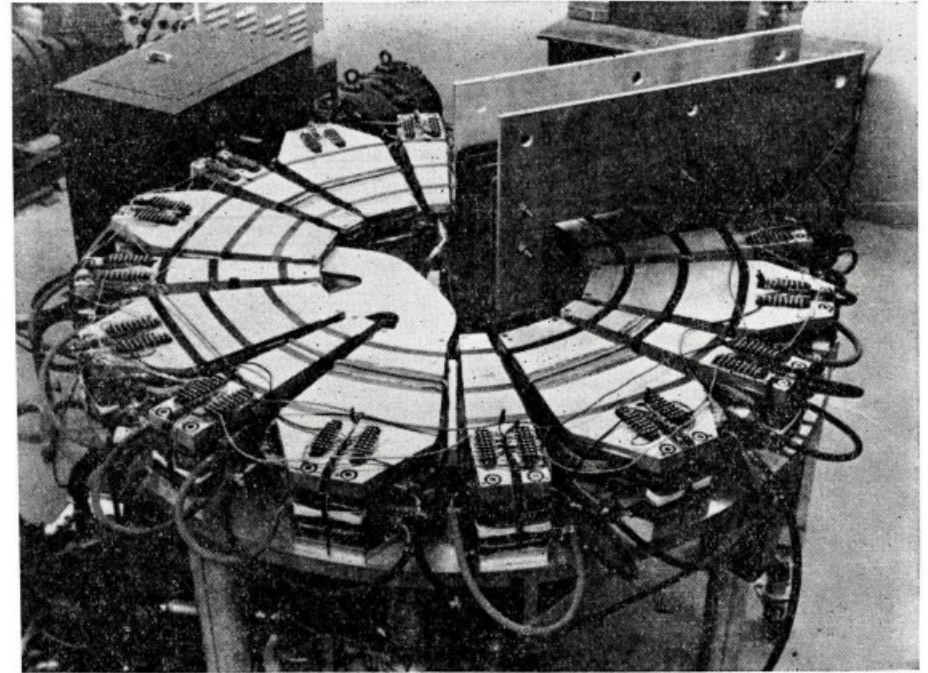
400keV radial sector →

50 MeV radial sector ↘

120 keV spiral sector ↓

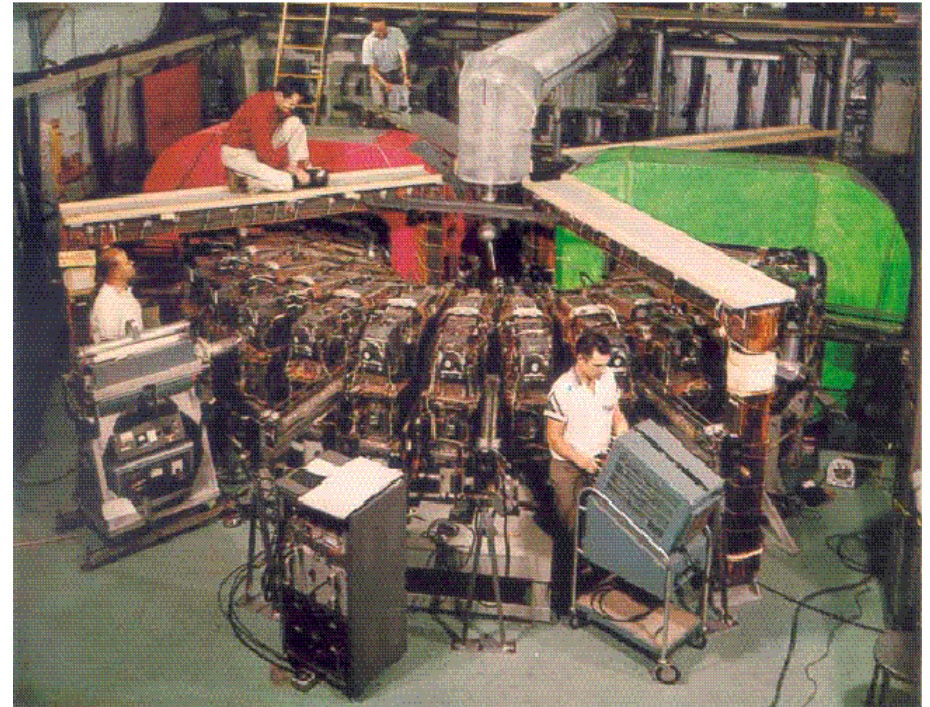


*Courtesy of MURA*



*Courtesy of MURA*

K.R. Symon, Proc PAC03, 452 (2003)





# ASPUN (ANL, 1983) 1500 MeV x 4 mA

## GENERAL

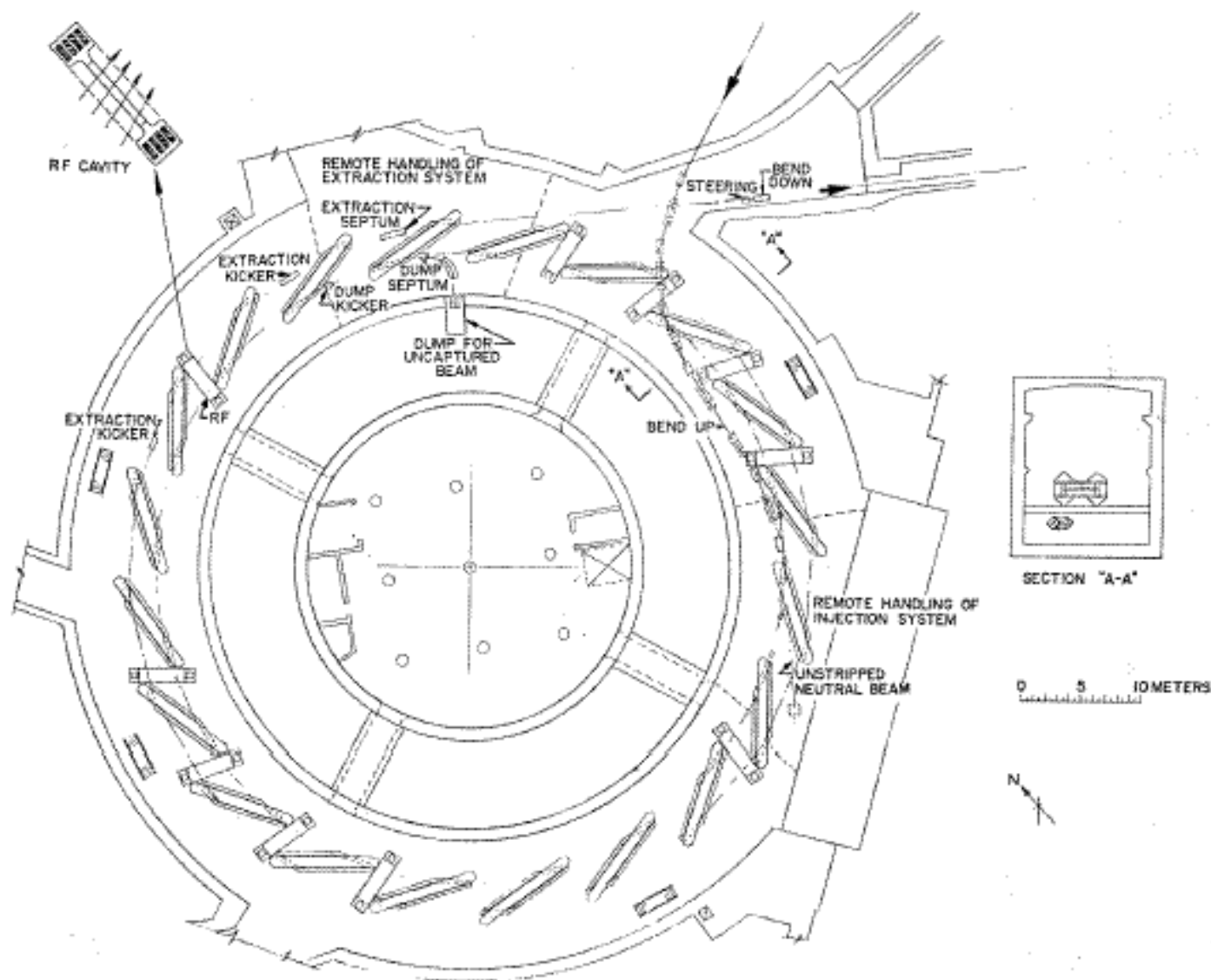
NUMBER OF MAGNETS	20
SECTOR WIDTH	3.6°
FIELD INDEX, K	14
SPIRAL ANGLE	61°
$\nu_x$	4.25
$\nu_y$	3.3
MAXIMUM REPETITION RATE	250 Hz
AVERAGE CURRENT	4 mA
SPACE CHARGE LIMIT	1014
STACK ENERGY	1250 MEV
BUNCHES/STACK	6
RF FREQUENCY	2.11-3.09 1.55-1.57

## INJECTION

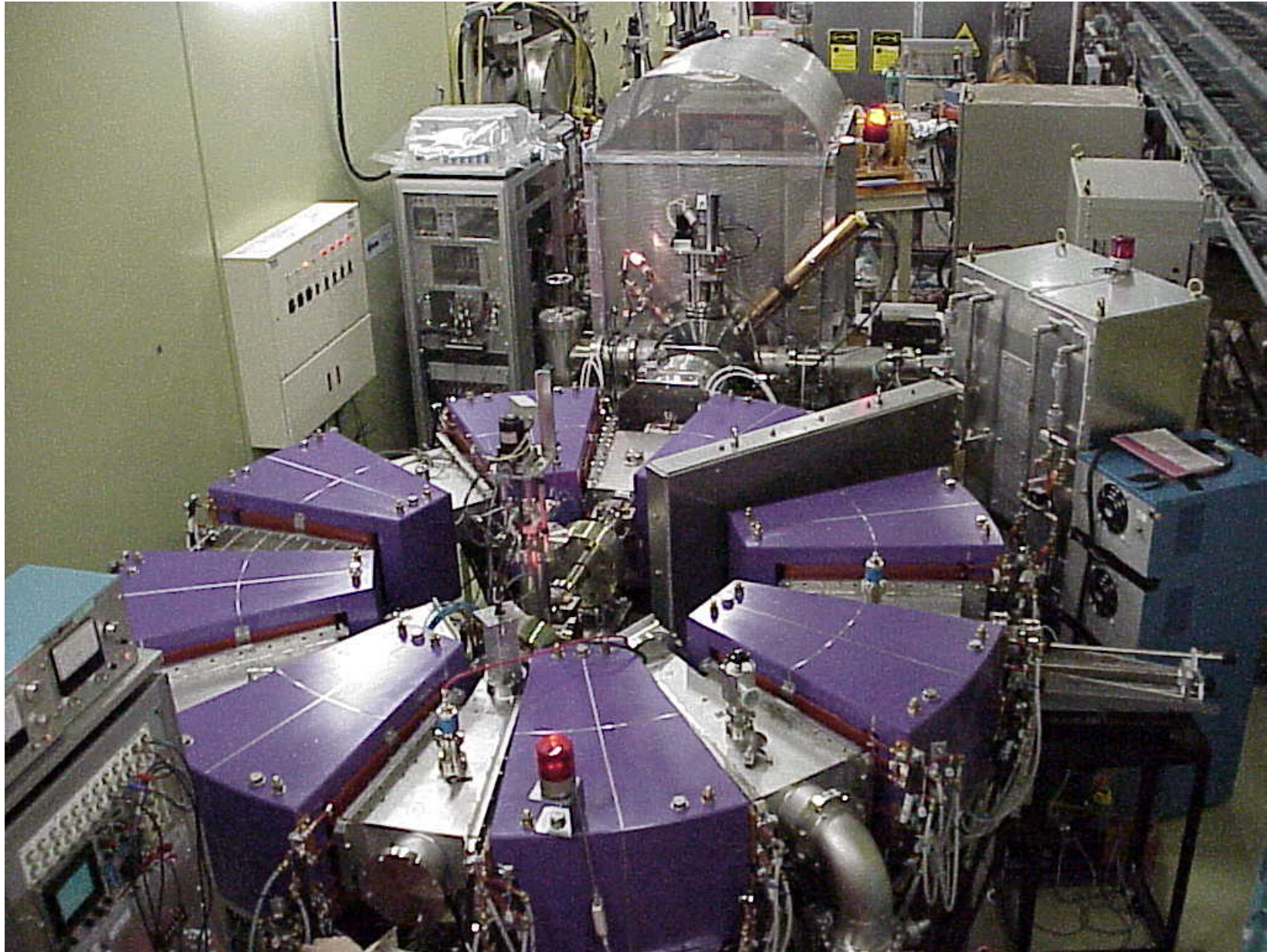
$E_{INJ}$	200 MEV
$B_p$	2.15 T-M
B	0.413 T
$\langle R \rangle_{INJ}$	25.88 M
$\epsilon_x$	650 * MMHR
$\epsilon_y$	500 * MMHR

## EXTRACTION

$E_{EXT}$	1500 MEV
$B_p$	7.5067 T-M
B	1.327 T
$\langle R \rangle_{EXT}$	28.139 M



# KEK Proof-of-Principle 1 MeV proton FFAG



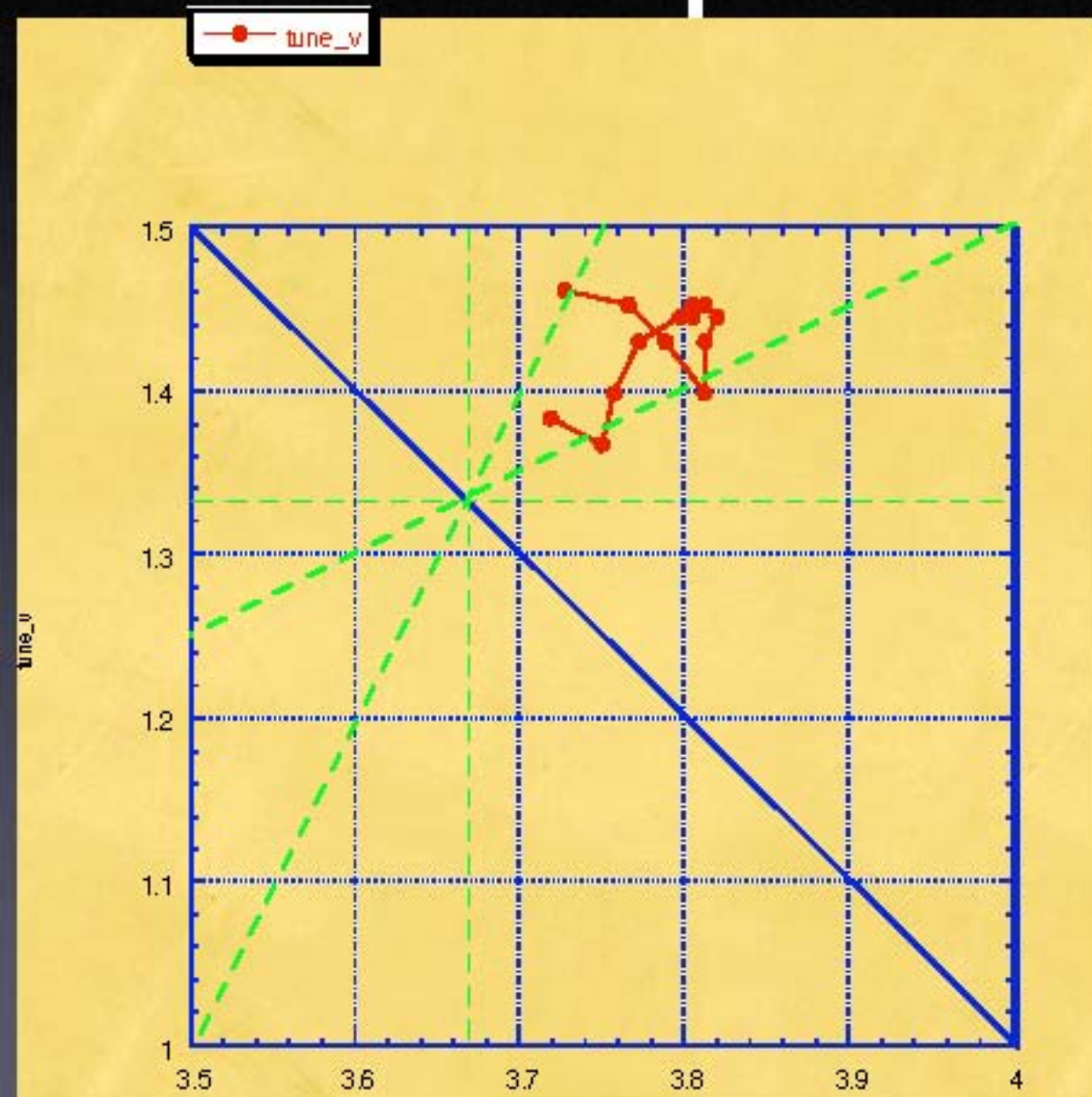


# KEK 150-MeV 12-Sector Proton FFAG





# 12-150MeV mode operation



criterion

1)  $\Delta v < 0.1$

2) avoid structure &  
linear resonances

# INNOVATIONS AT KEK

**Mori's** 1-MeV (2000) and 150-MeV **proton FFAGs** introduced two important innovations:

1. **FINEMET metallic alloy loading** in the rf cavities, allowing:
  - rf modulation at 250 Hz or more → **high beam-pulse rep rates**  
(remember the unreliable rotary capacitors on synchrocyclotrons, which operate in the same mode as FFAGs)
  - high permeability → short cavities with **high effective fields**
  - low  $Q$  ( $\cong 1$ ) → broadband operation
2. **DFD triplet sector magnets**:
  - **powered as a single unit**
  - **D acts as the return yoke**, automatically providing **reverse field**
  - modern techniques enable **accurate computation** of the **pole shape** for **constant field index  $k$**

## "Return-yoke-less" DFD Triplet for 150-MeV FFAAG





# RF system

Large Magnetic Alloy (FINEMET) Cavity

Number of core 4 pieces

Outer (Inner) size 1700x950mm(980x230mm)

Core thickness 25mm

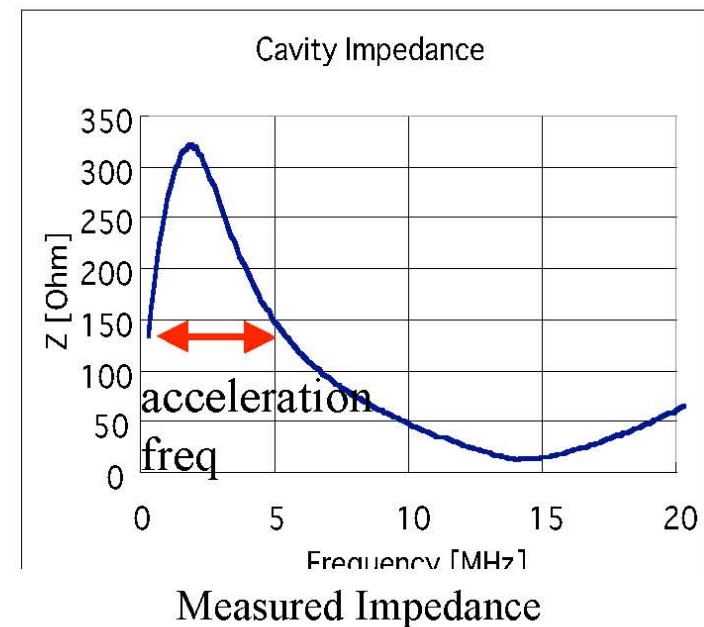
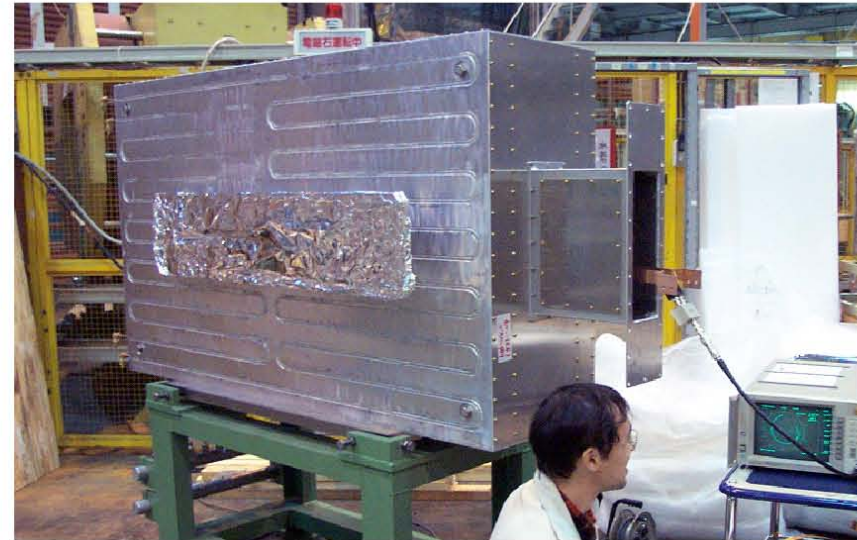
RF frequency 1.5 – 4.6 MHz

RF voltage 9kV

RF output 55kW

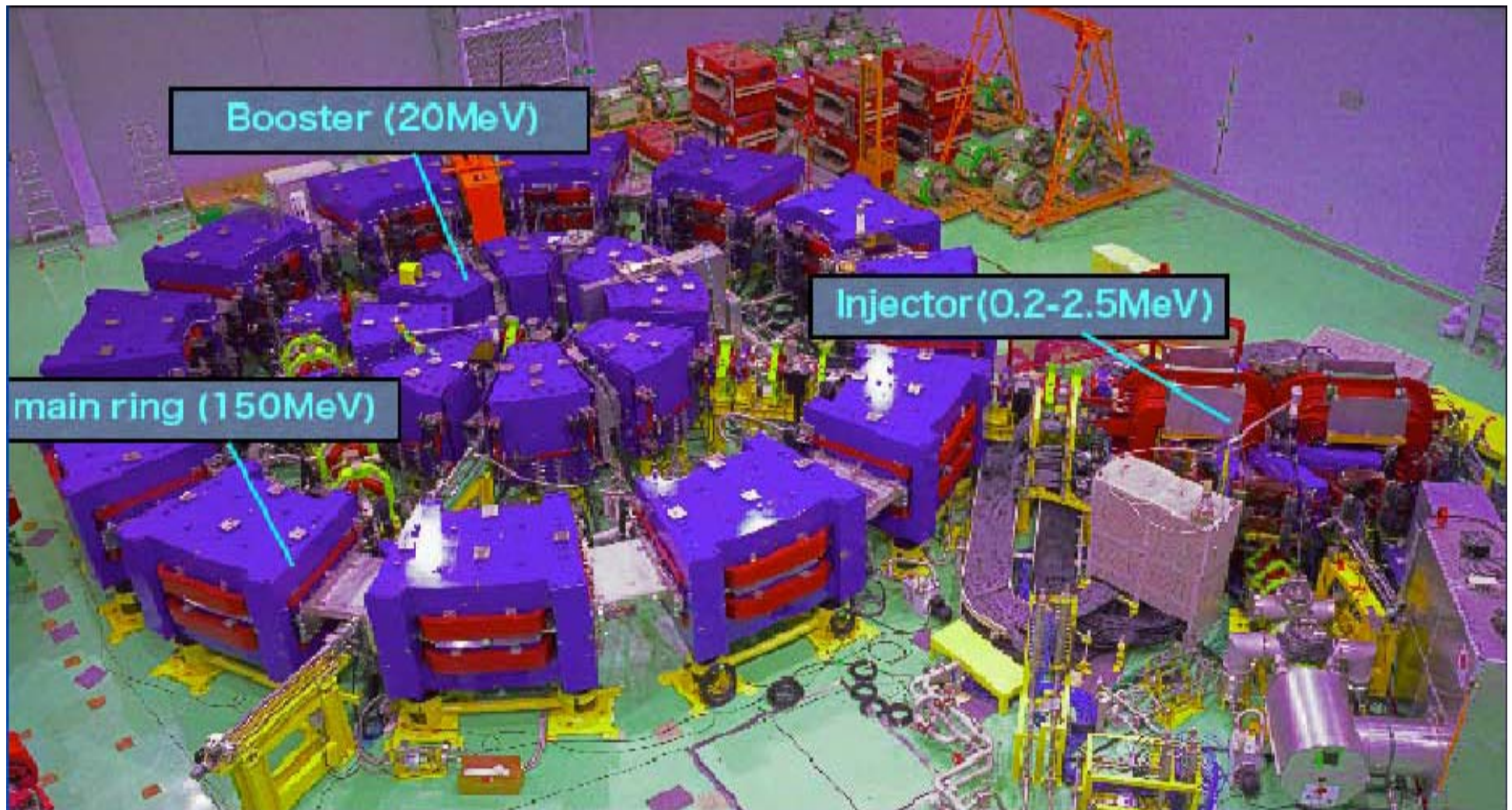
Power density 1W/cm<sup>3</sup>

Cooling water 70 L/min





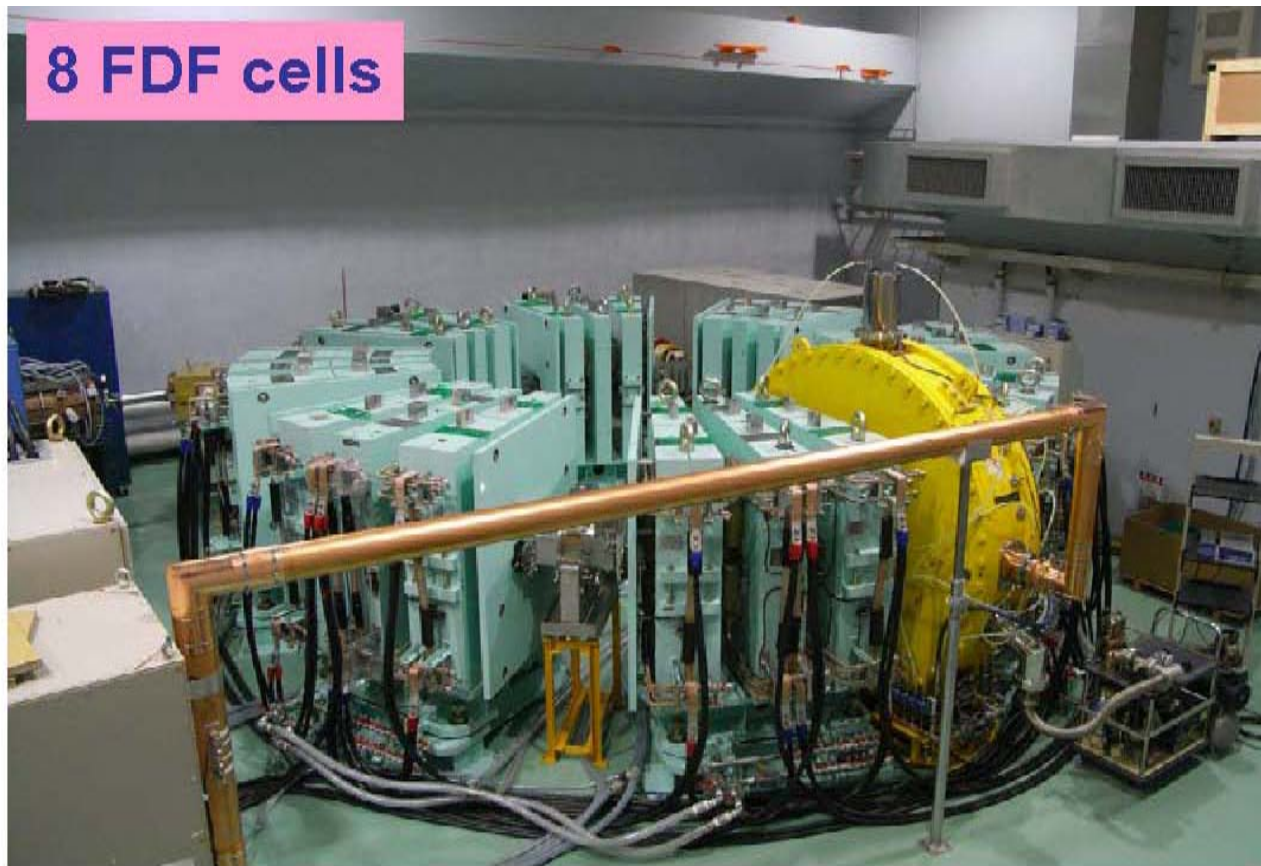
# FFAG Complex at Kyoto University Research Reactor Inst.



- to test Accelerator-Driven Sub-critical Reactor (ADSR) operation

# KURRI ERIT STORAGE RING FOR BNCT

(ERIT = Energy/Emittance Recovery Internal Target)



70-mA of circulating 11-MeV protons produce an intense neutron beam ( $>10^9/\text{cm}^2/\text{s}$  at the patient) via the  $\text{Be}(p,n)$  reaction.

$V_{\text{rf}} = 250 \text{ kV}$  plus large FFAG acceptances ( $>3000 \text{ mm-mrad}$ ,  $\pm 5\% \delta p/p$ ) allow ionization cooling to maintain stable beam over 1000 turns.



# $\alpha$ -PARTICLE TEST RING FOR PRISM AT RCNP OSAKA



Using 6 of the PRISM storage ring's 10 sectors  
to demonstrate bunch rotation in phase space

# SCALING FFAGs

- IN OPERATION OR UNDER CONSTRUCTION -

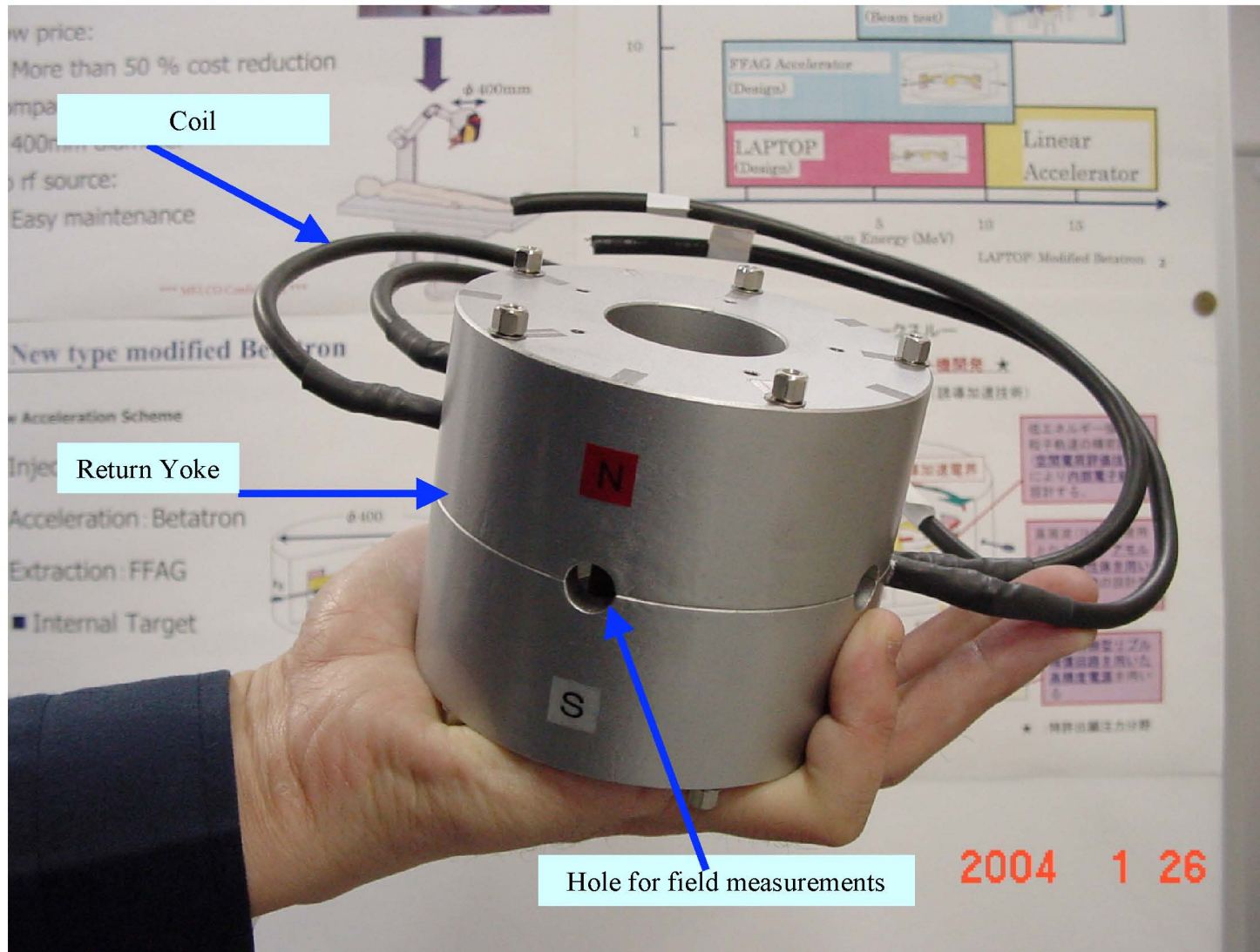
	Energy (MeV/u)	Ion	Cells	Spiral angle	Radius (m)	1 <sup>st</sup> beam
KEK - POP	1	p	8	0°	0.8-1.1	2000
KEK	150	p	12	0°	4.5-5.2	2003
KURRI - ADSR	150	p	12	0°	4.5-5.1	2006
(Accelerator-Driven	20	p	8	0°	1.3-1.9	2006
Subcritical Reactor)	2.5	p	8	40°	0.6-1.0	2008
KURRI-ERIT (BNCT)	11	p	8	0°	2.35	2008
PRISM study	0.8	$\alpha$	6	0°	3.3	2008
PRISM*	20	$\mu$	10	0°	6.5	
NHV	0.5	e	6	30°	0.19-0.44	2008
RadiaBeam Radiatron	5	e	12	0°	0.3-0.7	(2009)

\* storage ring for  $\mu$  bunch rotation in phase space

# SCALING FFAGs - DESIGN STUDIES

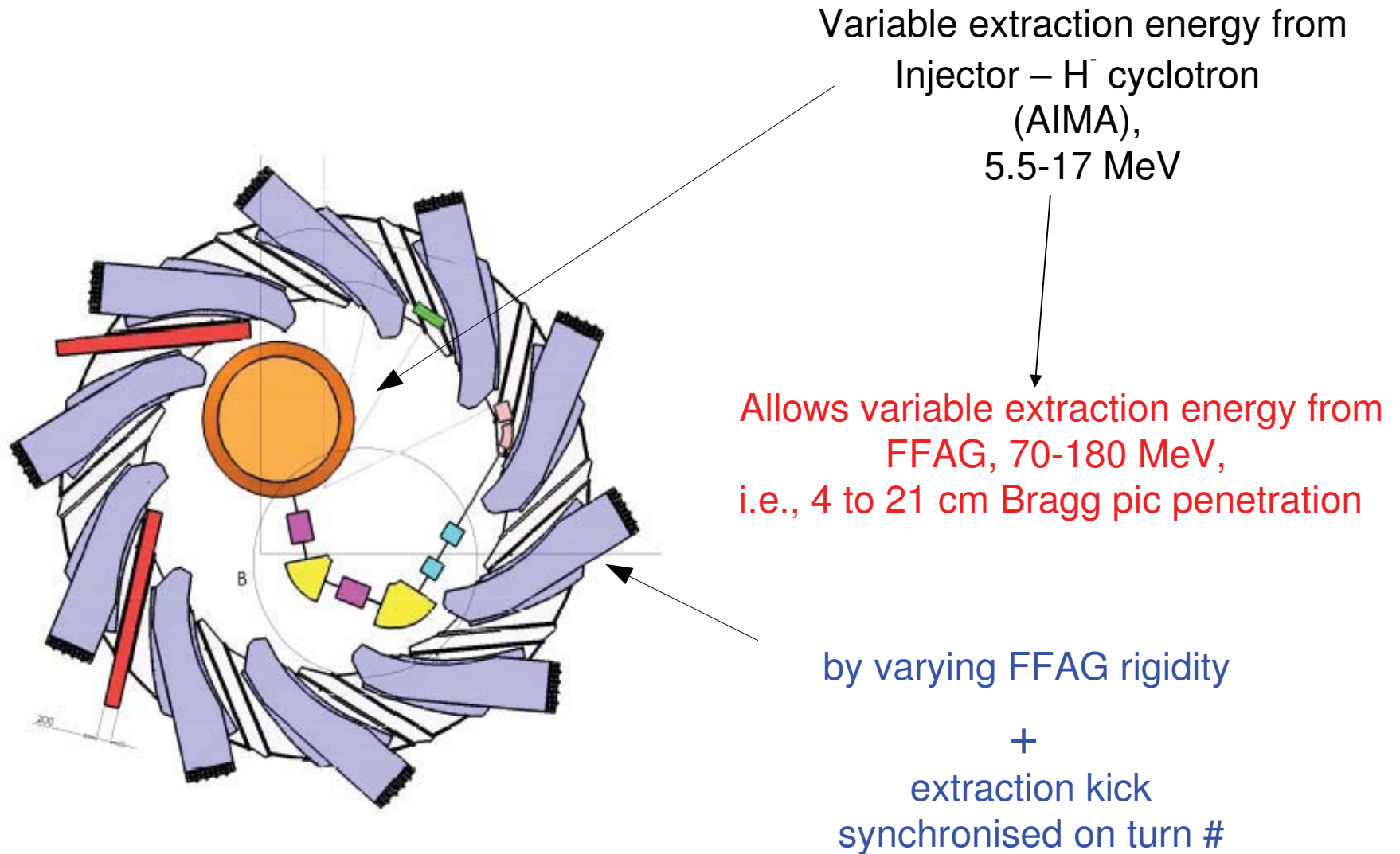
	Energy (MeV/u)	Ion	Cells	Spiral angle	Radius (m)	Rep rate (Hz)	Comments
MEICo - Laptop	1	e	5	35°	.023 - .028	1,000	Hybrid - <u>Magnet built</u>
eFFAG	10	e	8	47°	0.26 - 1.0	5,000	20-100 mA
LPSC RACCAM	180	p	10	54°	3.2 - 3.9	>20	<u>Magnet sector 2008</u>
Ibaraki Med.Acc.	230	p	8	50°	2.2 - 4.1	20	0.1 $\mu$ A
MEICo - p Therapy	230	p	3	0°-60°	0 - 0.7	2,000	<u>SC</u> , Quasi-isochronous
MEICo - Ion Therapy (Mitsubishi Electric)	{ 400 7	$C^{6+}$ $C^{4+}$	16 8	64° 0°	7.0 - 7.5 1.35 - 1.8	0.5 0.5	Hybrid (FFAG/synch <sup>n</sup> ) " " " "
NIRS Chiba	{ 400	$C^{6+}$	12	0°	10.1 - 10.8	200	Compact
- Hadron	{ 100	"	12	0°	5.9 - 6.7	"	radial
Therapy	{ 7	$C^{4+}$	10	0°	2.1 - 2.9	"	sectors
Mu Cooling Ring	160	$\mu$	12	0°	0.95 $\pm$ 0.08		Gas-filled
J-PARC	{ 20,000	$\mu$	120	0°	200		<u><math>\Delta r = 0.5</math> m</u> , ~10 turns.
Neutrino	{ 10,000	"	64	0°	90		
Factory	{ 3,000	"	32	0°	30		$Q \approx 1$ rf cavities allow
Accelerators	{ 1,000	"	16	0°	10		<u>broadband operation</u>



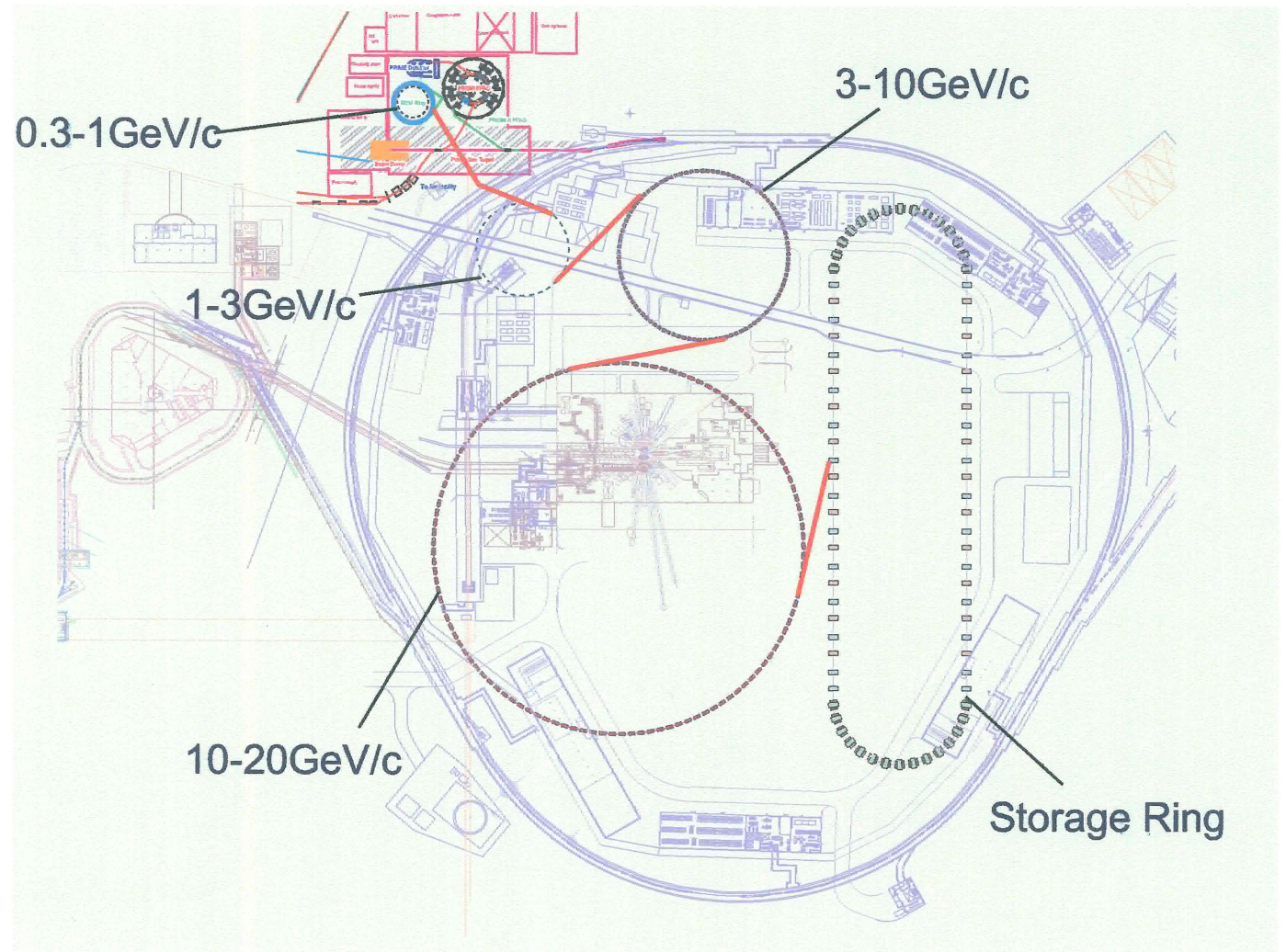


The present study is partially supported by the REIMEI Research Resources of Japan Atomic Energy Research Institute.

# Principle of Energy Variability for RACCAM System



# Neutrino Factory : FFAG based





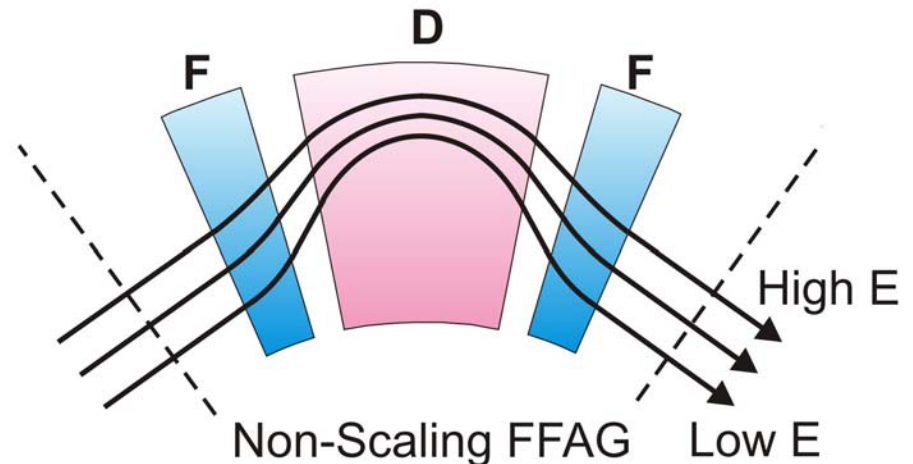
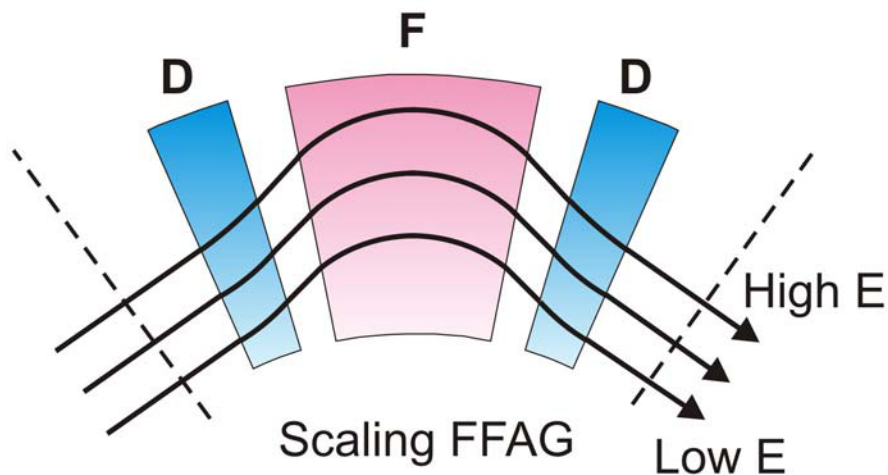
# LINEAR NON-SCALING (LNS) FFAGs

FFAGs look attractive for accelerating muons in  $\mu$  Colliders or  $\nu$  Factories

- Large acceptance (in  $r$  &  $p$ ) eliminates cooling & phase rotation stages
- Rapid acceleration (<20 turns) makes resonance crossing ignorable (Mills '97)
- Less expensive than recirculating linacs.

NON-SCALING approach first tried by Carol Johnstone (arc 1997, ring 1999)

- strong positive-bending Ds + negative Fs - i.e. negative field gradients!
- "LINEAR" constant-gradient magnets.



This leads to:

- Greater momentum compaction (& hence narrower radial apertures);
- No multipole field components to drive betatron resonances  $>1^{\text{st}}$  order;
- Simpler construction ( $B \propto r$  rather than  $r^k$ ).

# SCALING v. LINEAR NON-SCALING FFAGs

Note that for LNS-FFAGs, orbit circumference  $C$  varies quadratically with energy rather than rising monotonically:

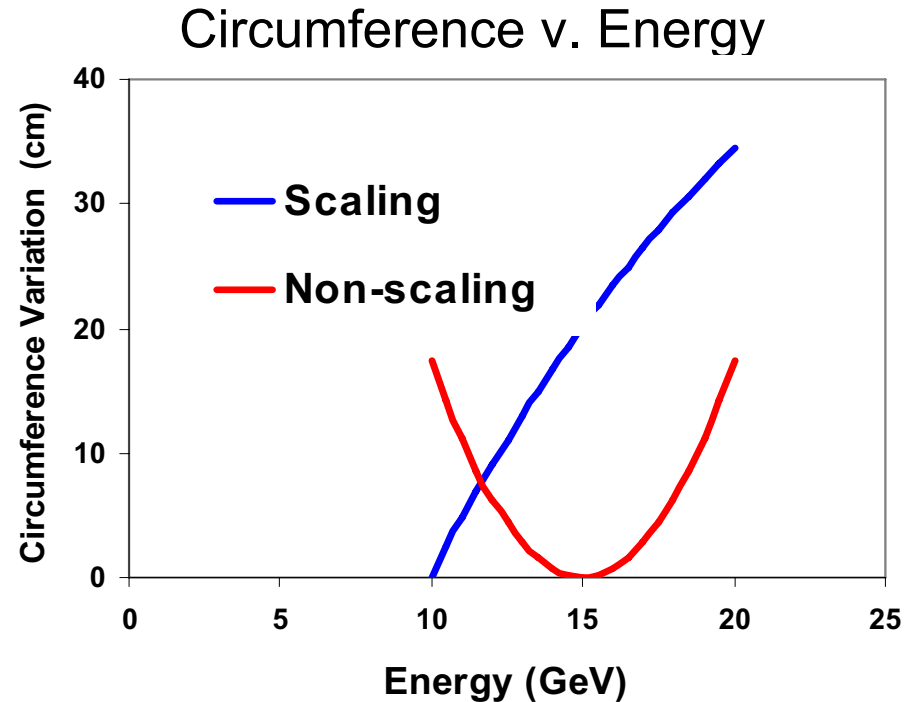
$$C(p) = C(p_m) + \frac{12\pi^2}{e^2 q^2 N L_{FD}} (p - p_m)^2$$

So less variation in  $C$  and orbit period, enabling fixed rf frequency operation when  $v \approx c$ .

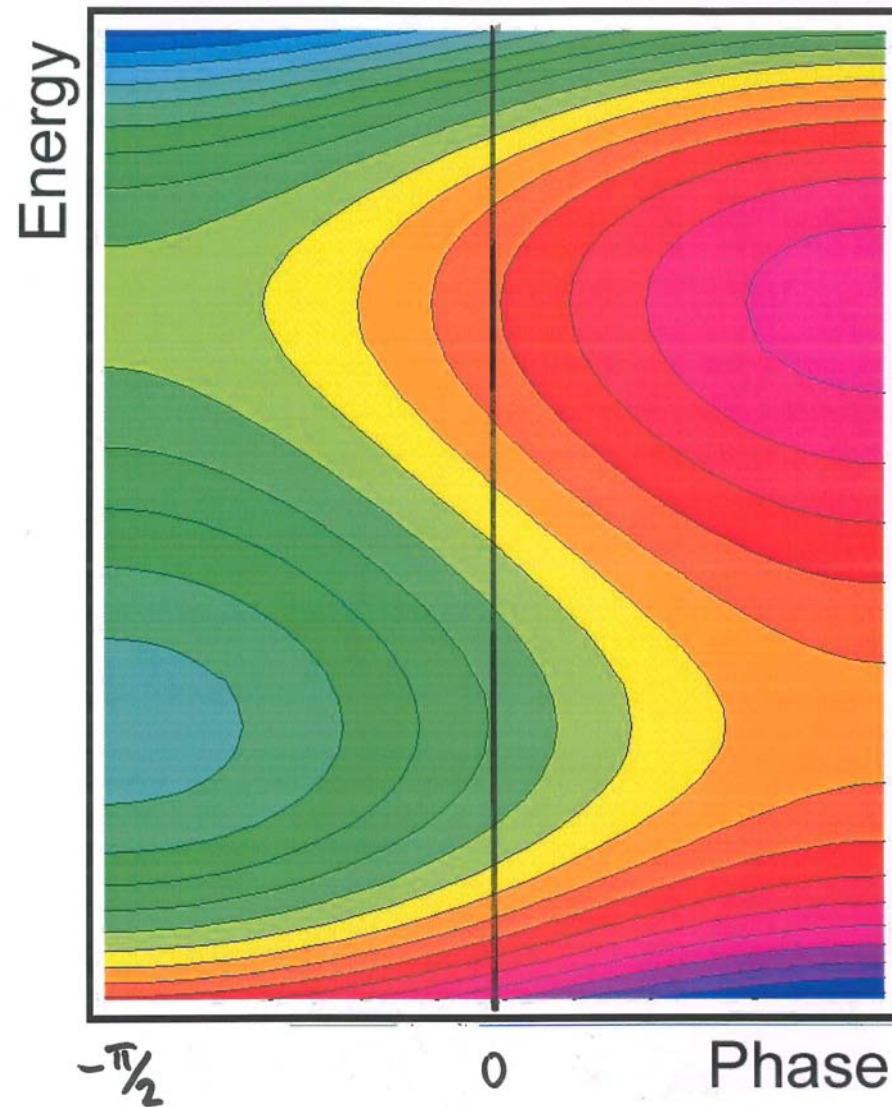
- The muons oscillate in phase across the rf voltage peak (3 crossings)
  - just as in a real, imperfectly isochronous, cyclotron!

The International Design Study for a Neutrino Factory chose LNS-FFAGs of 12.6-25 GeV and 25-50 GeV for the final stages of muon acceleration - with designs developed by a consortium led by Johnstone (FNAL), Berg (BNL), and Koscielniak (TRIUMF).

Non-linear NS-FFAGs are also being explored.

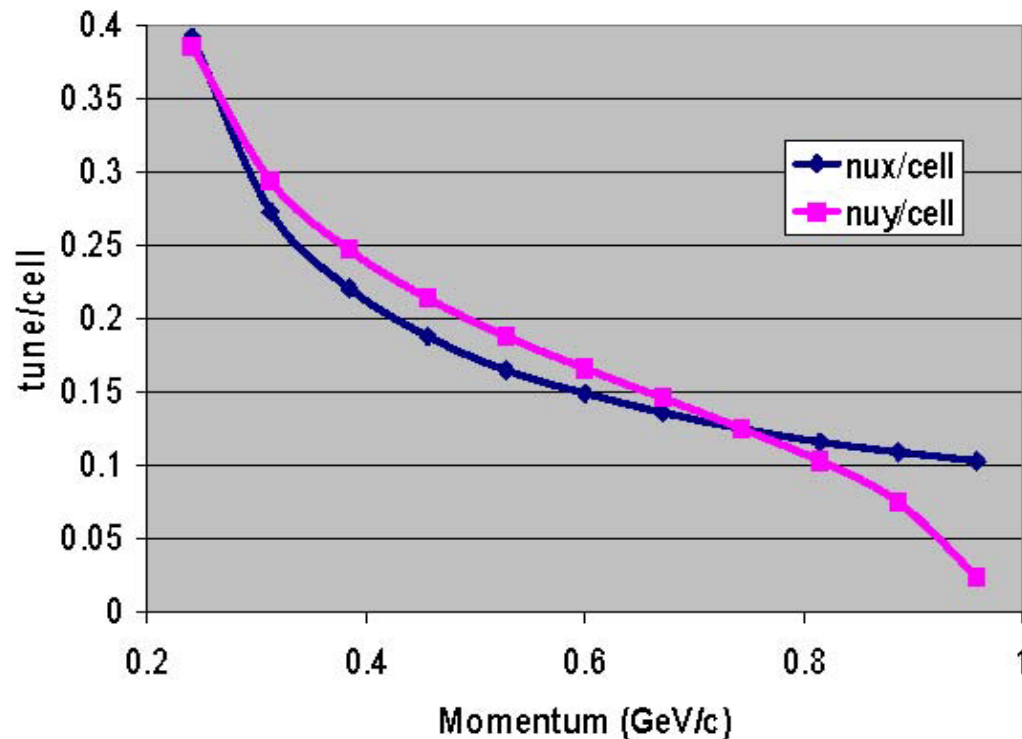


# SERPENTINE ACCELERATION IN LNS-FFAGs



- Not within the buckets - but between them
- Follow the golden trail!

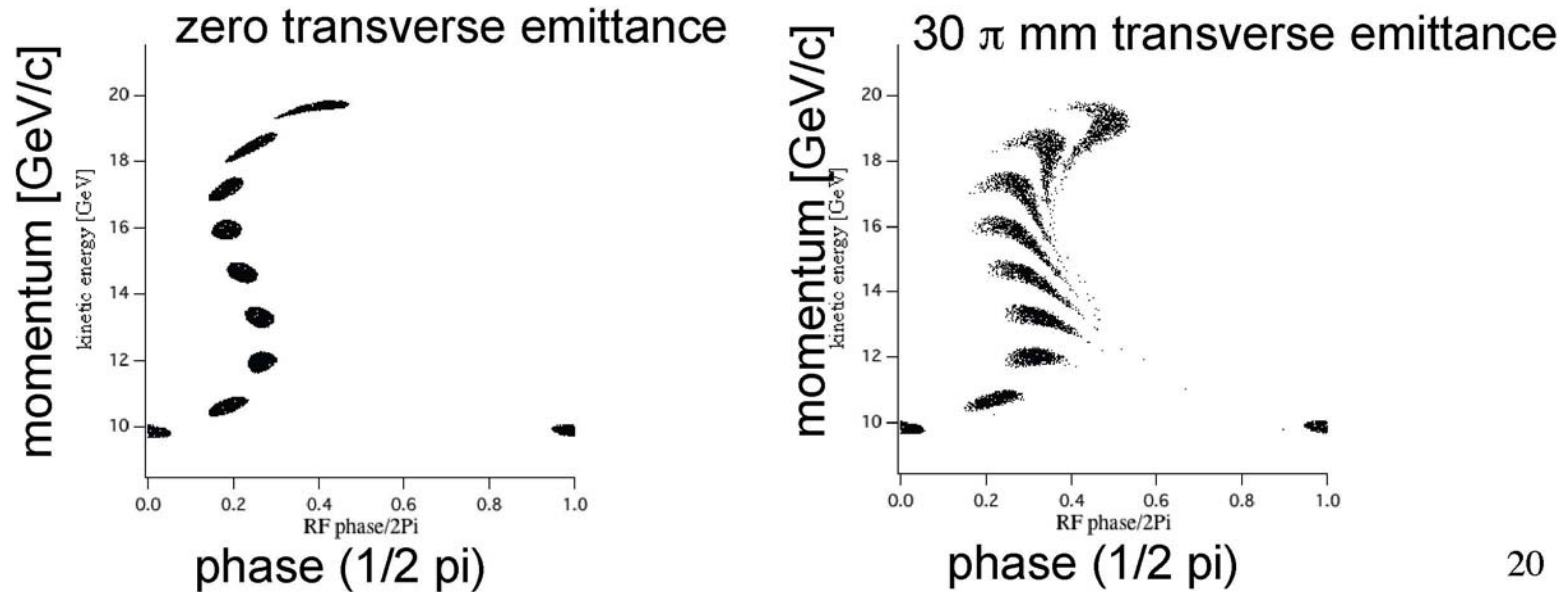
# TUNES IN LNS-FFAGs



If the orbits cross the magnet ends perpendicularly:

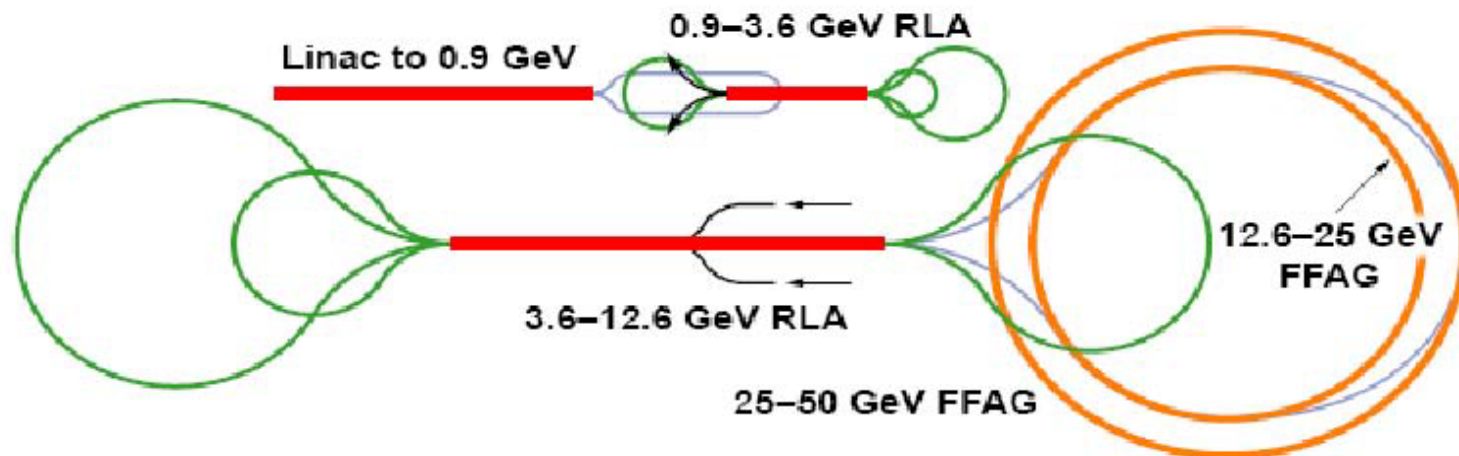
- the tunes fall sharply with energy, crossing betatron resonances
- possibly leading to loss of beam quality/quantity
- danger lessened by rapid energy gain, but very expensive
- for muons ( $\tau = 2 \mu\text{s}$ ): expensive but essential anyhow
- for ions: just expensive

# MATCHING LNS-FFAGs



20

Unfortunately, for large-emittance beams, the **radial longitudinal coupling** in LNS-FFAGs makes **transfer matching difficult**. Mitigation techniques exist, but the  **$\nu$  Factory ISS** concluded that **>2 LNS-FFAGs would not be practical** - and opted for the more costly recirculating linacs below 12.6 GeV.





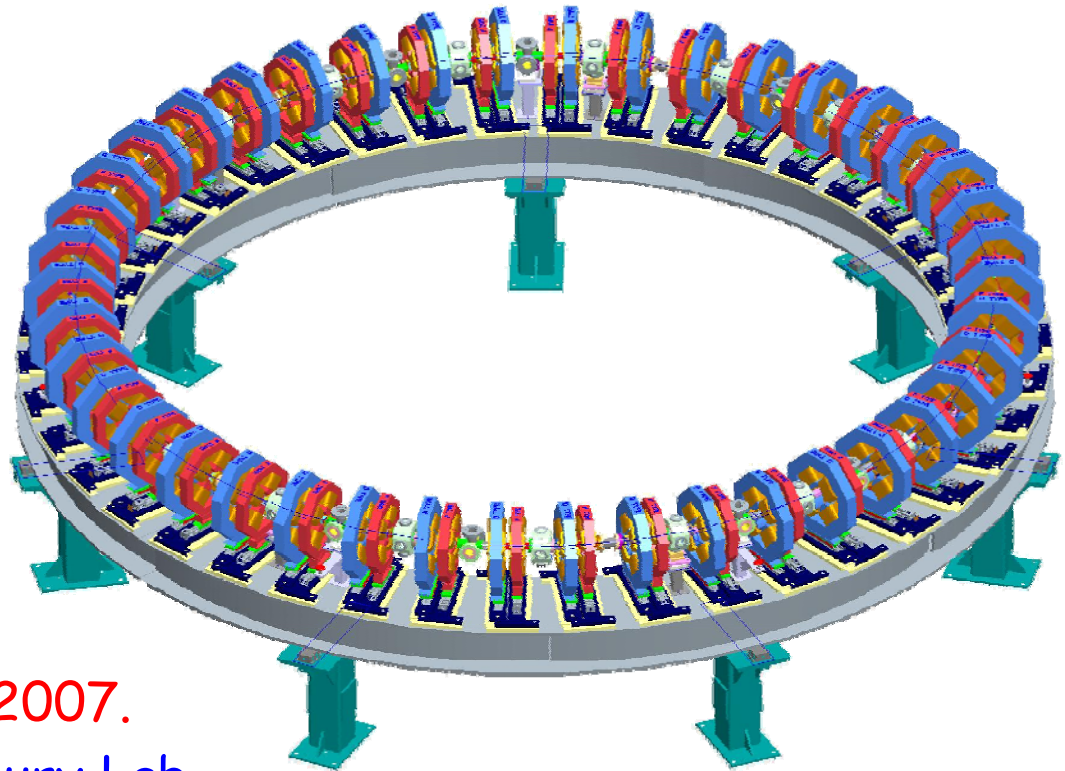
# ELECTRON MODEL LNS-FFAG "EMMA"

A **Proof of Principle** machine for **linear non-scaling FFAGs** to demonstrate their **two novel features**:

- **safe passage through many low-order structural resonances**
- **acceleration outside buckets.**

EMMA has relativistic parameters similar to those of a **10-20 GeV muon FFAG**, with a **doublet lattice** based on **offset quadrupoles**:

Energy	10-20 MeV
Circumference	16.57 m
Cells	42
N.T. Acceptance	3 mm
F quad length	5.88 cm
D quad length	7.57 cm
RF frequency	1.3 GHz
Cavities	19 x 120 kV
Injector	ALICE (7-35 MeV)



**UK funding (\$16M) started April 2007.**  
**Construction under way at Daresbury Lab.**

# NON-SCALING LATTICES FOR HADRONS

To **accelerate hadrons**, where  $v \ll c$ , the **wider range of speeds** and **orbit times  $\tau$**  requires either:

- **frequency modulation**, or **broadband operation**,
  - both requiring **pulsed beam operation**, or
- **harmonic number jumping (HNJ)** - as in **microtrons**
  - where the energy gain is adjusted to give  $\Delta\tau = \text{-integer} \times \tau_{\text{rf}}$
  - allowing **cw fixed-frequency operation** and **higher beam intensity**
  - but requiring **precise variation of rf cavity voltage with radius**.

With the **small radial orbit spread**, **variable-energy extraction** can be realized by **timing the kicker pulse**, even with fixed kicker and septum.

**Three groups** are actively designing NS-FFAGs for **cancer treatment**:

1. Keil (CERN), Trbojevic (BNL) and Sessler (LBNL)
2. Johnstone (FNAL) and Koscielniak (TRIUMF)
3. Yokoi, Peach et al. (Adams Inst.) and Machida (RAL).

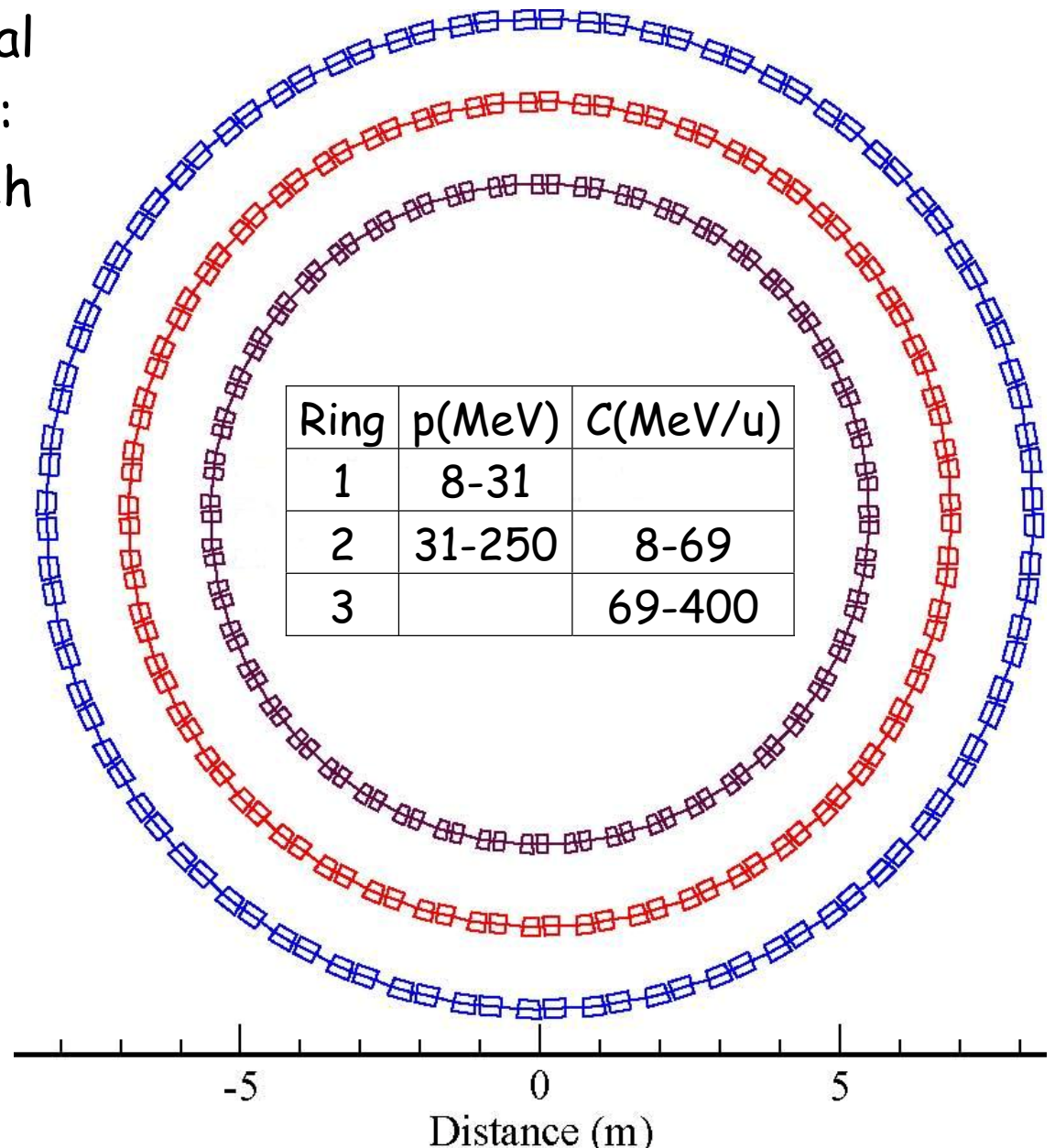
# Keil-Sessler-Trbojevic LNS-FFAG Therapy Complex

The first LNS-FFAG proposal for ion beam cancer therapy:  
- three concentric rings, each of **48 doublet cells**.

The tunes fall with energy, crossing several  $n$  &  $n/2$  imperfection resonances - but no intrinsic resonances below 3<sup>rd</sup> order - so good beam quality is maintained.

RF is frequency-modulated (in the range 9-25 MHz).

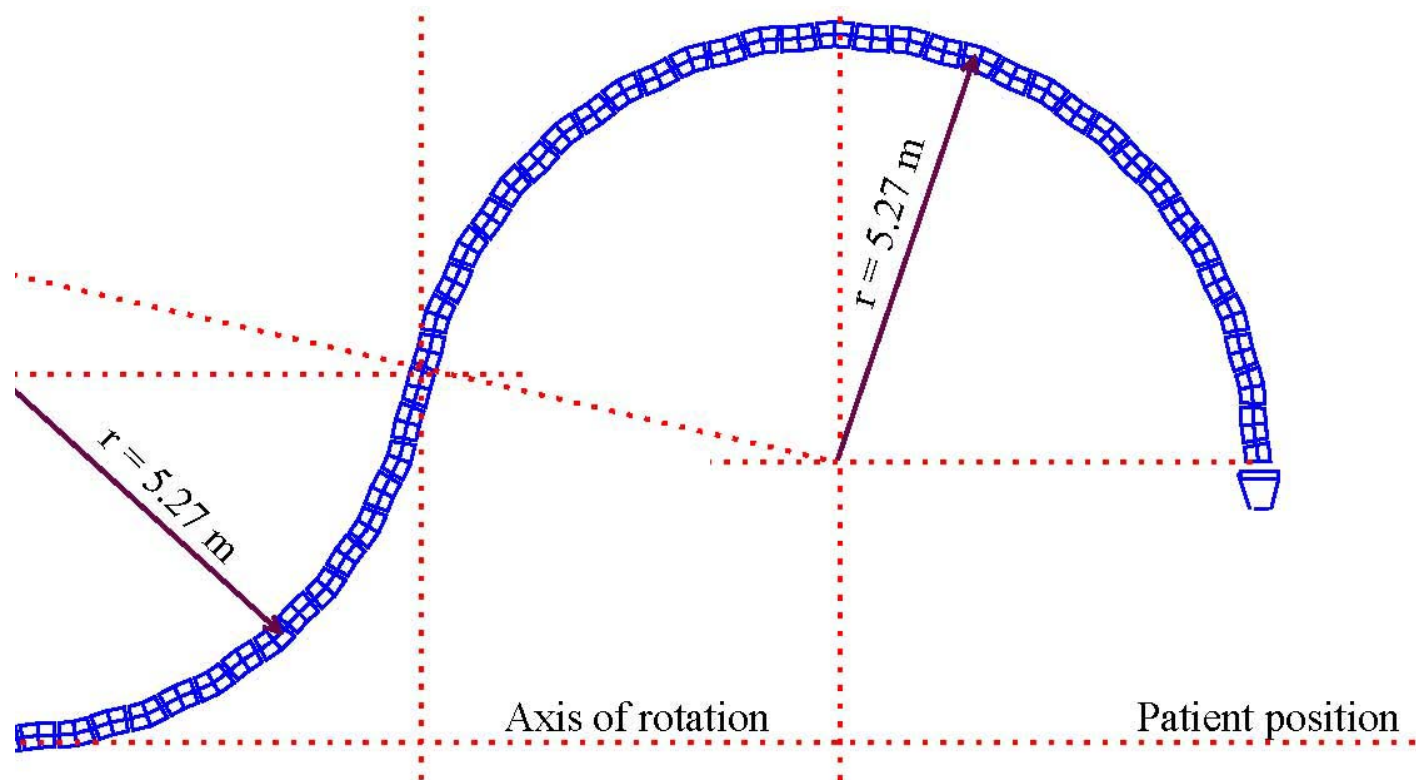
Note the **small magnets** (cf. NIRS 3-ring S-FFAG).





## Keil-Sessler-Trbojevic Lightweight FFAG Gantry

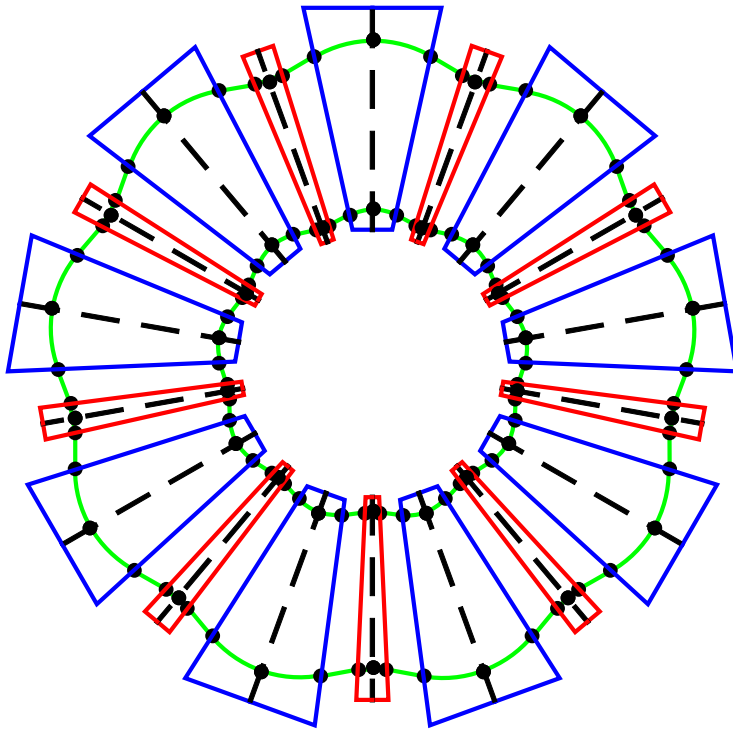
This group has also proposed a lightweight LNS-FFAG gantry, composed of superconducting magnets (either high-temperature or cryogenic) in a close-packed triplet lattice.



The acceptance is large enough to transmit  $C^{6+}$  ions of 150-400 MeV/u at one excitation, and protons of 90-250 MeV at another.

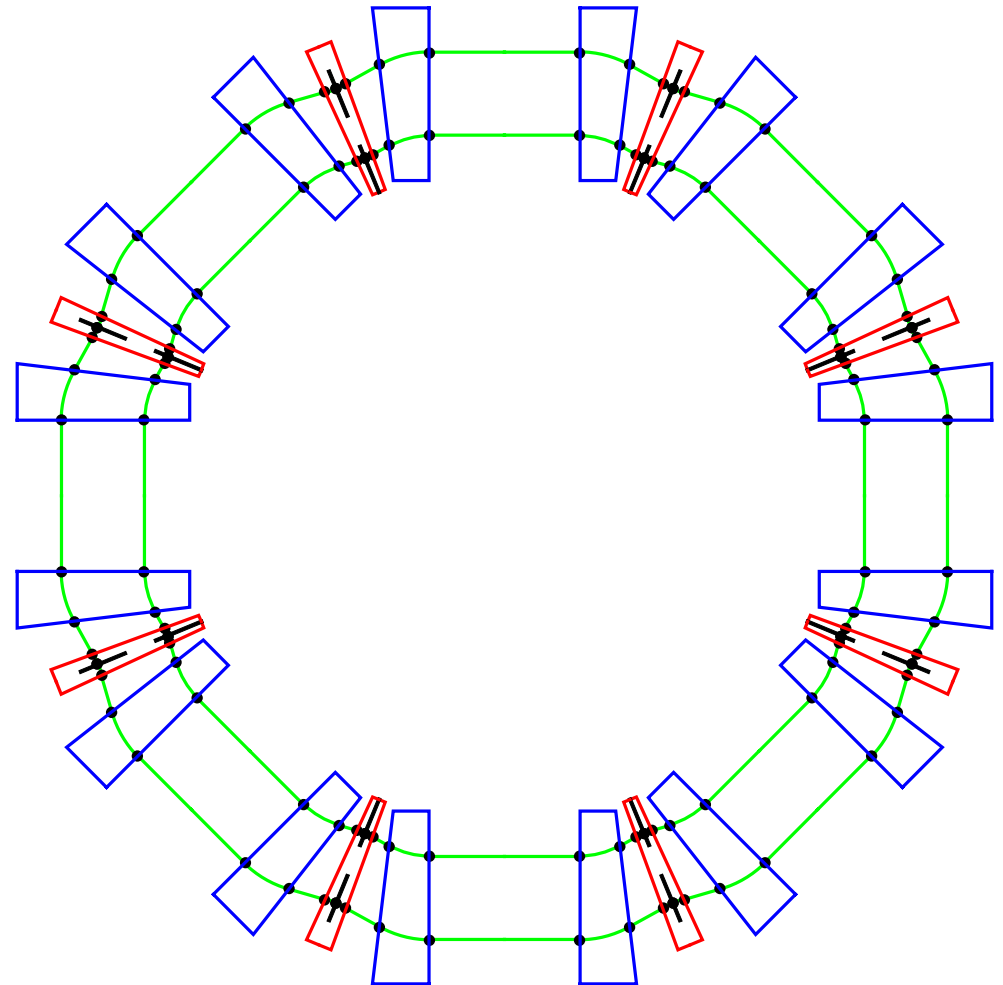
# Johnstone-Koscielniak Tune Stabilized NLNS-FFAGs (1)

Two designs are being considered for 30-250 MeV protons  
- roughly to scale



**9-cell FODO**

Orbit radii 1.98-2.49 m



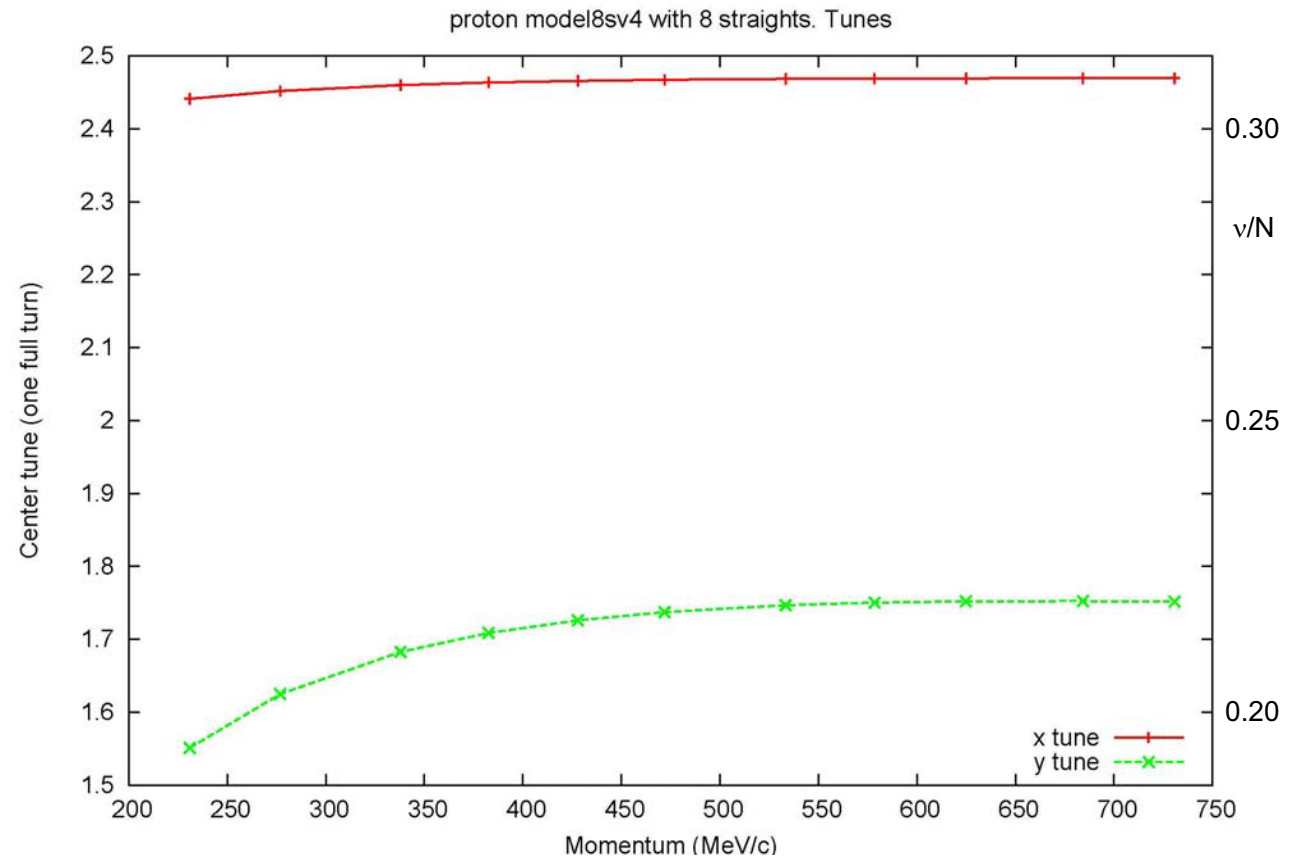
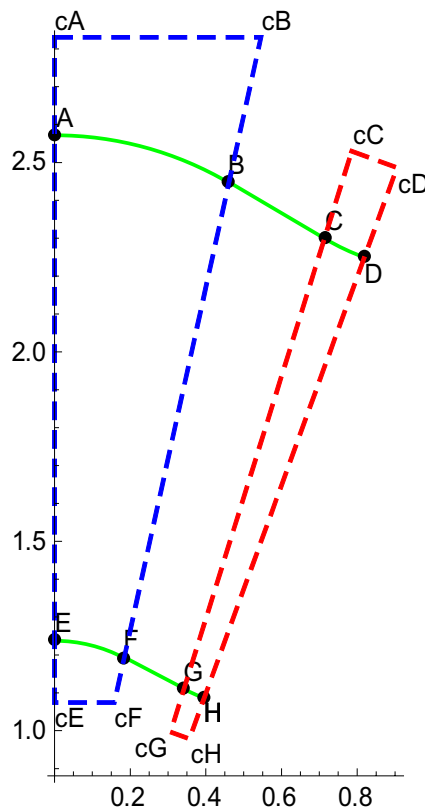
**8-cell FDF**

Orbit radii 2.75-3.39 m

## Tune Stabilized NLNS-FFAGs (2)

Tune drop-off with energy is avoided by:

- employing the “**edge focusing**” that occurs for **non-perpendicular magnet entry/exit**
- allowing a **non-linear  $B(r)$  field variation**



**Nearly flat tunes** are obtained, with **large dynamic apertures**.

# PAMELA (Adams Inst. - Yokoi, Machida, Peach, *et al.*)

31 - 250 MeV protons

12-cell FDF

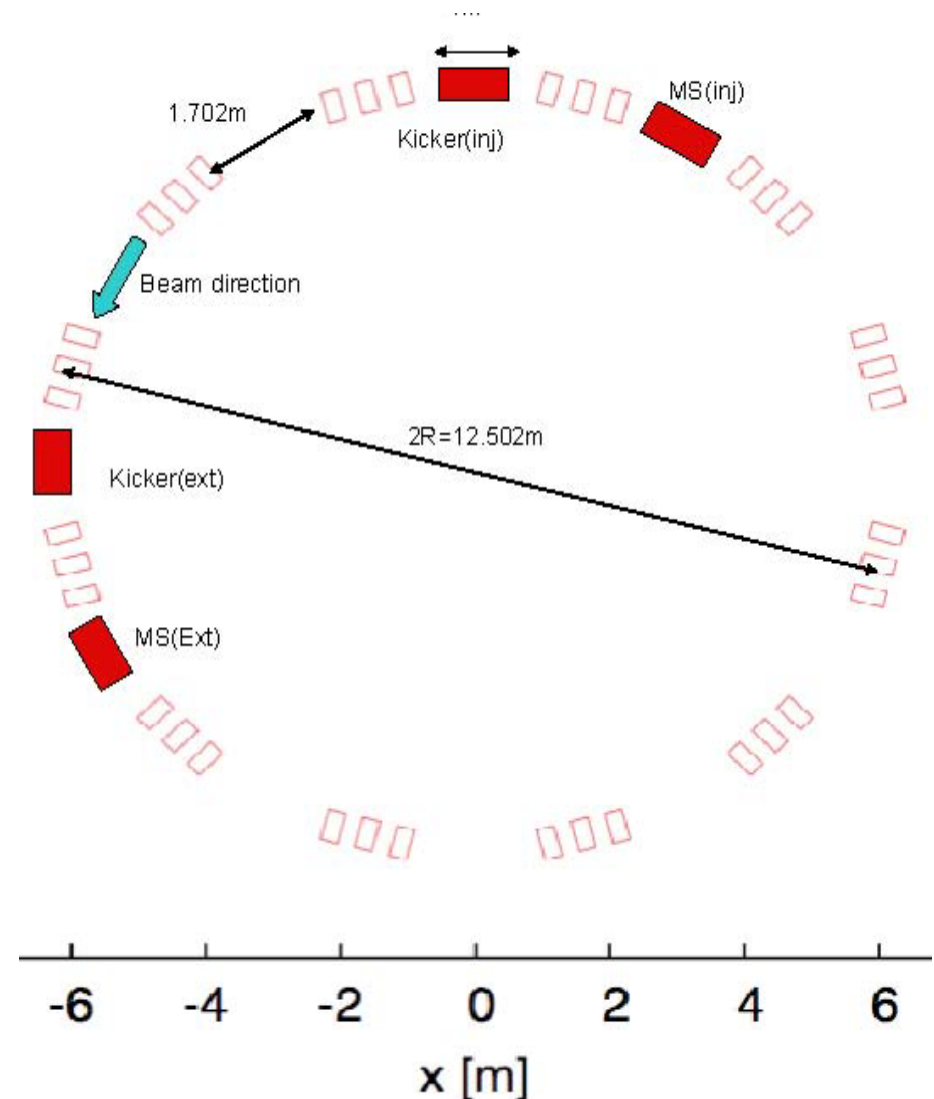
Radius  $\approx 6.25$  m

4-T magnets

## Machida semi-scaling lattice

- High field index  $k$  (i.e.  $B \sim r^k$ ) for small orbit excursions
- approximate  $r^k$  locally by  $\sum b_n x^n$  with  $n = 0, 1, 2, 3$  only
- flat tunes, good dynamic aperture

400-MeV/u  $C^+$  version is being prepared



# CURRENT FFAG CANCER THERAPY STUDIES

<u>SCALING</u>		Energy (MeV/u)	Ion	Cells	Spiral angle	Radius (m)	Pulse rep. rate (Hz)
KURRI: ERIT		11	p	8	0°	2.35	200
LPSC: RACCAM		17-180	p	10	54°	3.2-3.9	130
<u>NON-SCALING</u>							
Keil, Sessler & Trbojevic		8-31	p	48	0°	5.49-5.52	≤1000
		31-250	p	48	0°	6.86-6.95	≤1000
		8-69	C <sup>6+</sup>	48	0°	6.86-6.95	≤1000
		69-400	C <sup>6+</sup>	48	0°	8.23-8.32	≤1000
Trbojevic		28-250	p	24	0°	4.18-4.42	cw (HNJ)
Johnstone <i>et al.</i>	FODO	30-250	p	9	0°	1.98-2.49	
	FDF			8	0°	2.75-3.39	
PAMELA		30-250	p	12	0°	≈6.25	≤1000 or
(Machida lattice)		7-450	C <sup>+</sup>				cw (HNJ)

# LINEAR NON-SCALING LATTICES FOR HADRONS (3)

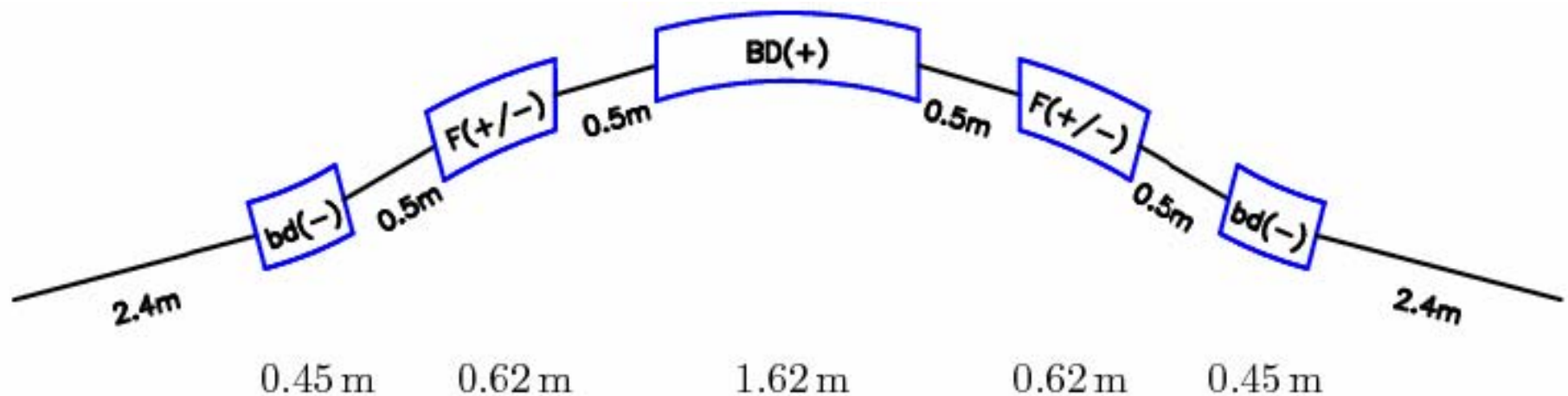
**Sandro Ruggiero** (BNL) has proposed a number of **LNS-FFAGs** using FDF triplet cells and HNJs as **proton or heavy-ion drivers**:

Project	Energy (GeV)	Cells	Circumf. (m)	No. of rings	Rep. rate (Hz)	Current ( $\mu\text{A}$ - avg.)	Power (MW - avg.)
AGS Booster replacement	0.4 - 1.5	136	807	1	2.5 - 5	33	0.05
Proton Driver I for $\nu$ Factory	0.4 - 12	136	807	3	50	330	4
Proton Driver II for $\nu$ Factory	0.4 - 12	136	807	3	cw	8,500	100
MINHA electron model	$2-8 \times 10^{-4}$	48	18	Octant under construction			
Proton Driver for ADSR	0.05 - 1	80	204	2	1,000 - cw	10,000	10
$\text{U}^{238}$ Driver for Radioactive Ions	0.015 - 0.4	80	204	2	1,000 - cw	4.2	0.4

Note that the same cell structure may be used for more than one application!

# NON-LINEAR NON-SCALING LATTICES

**G.H. Rees** has designed several FFAGs using novel **5-magnet “pumpet” cells**, in which variations in field gradient and sign enable each magnet's function to vary with radius - providing great flexibility - even allowing **well-matched insertions**!



- an **isochronous “IFFAG”** for **muons** (**8-20 GeV**,  $N = 123$ ,  $C = 1255$  m, 16 turns, - as illustrated - or **with insertions**,  $N = 4 \times (20 \text{ arc} + 10 \text{ str.})$ ,  $C = 905$  m)
- an **IFFAG muon booster** (**3.2-8 GeV**, 8 turns)
- an **IFFAG electron model** (**11-20 MeV**,  $N = 45$ ,  $C = 29.3$  m)
- a **v Factory proton driver** (**3-10 GeV**,  $N = 66$ ,  $C = 801$  m, 50 Hz, 4 MW)
- a **vF driver electron model** (**3.0-5.45 MeV**,  $N = 27$ ,  $C = 23.8$  m)

# SUMMARY

- Last 10 years have seen **rebirth of interest** in FFAGs world-wide
- 8 built, 3 under way, ~20 designs proposed
- Interest stems from applications needing the **FFAG's unique characteristics**:
  - high rep rate
  - high acceptance
- A whole **new class of "non-scaling" FFAGs** has been discovered
  - several varieties are being studied
  - perhaps scope for more?



# SERPENTINE ACCELERATION IN CYCLOTRONS

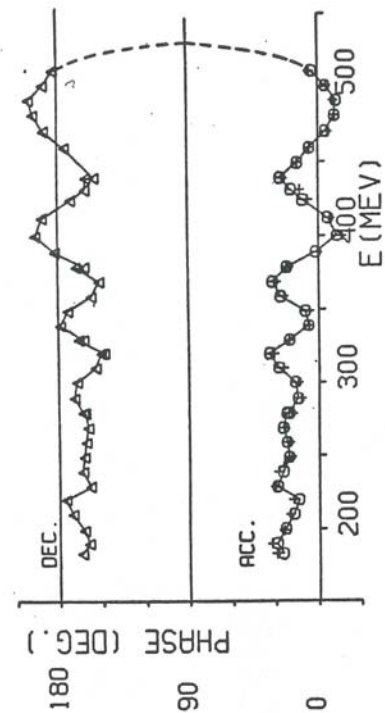
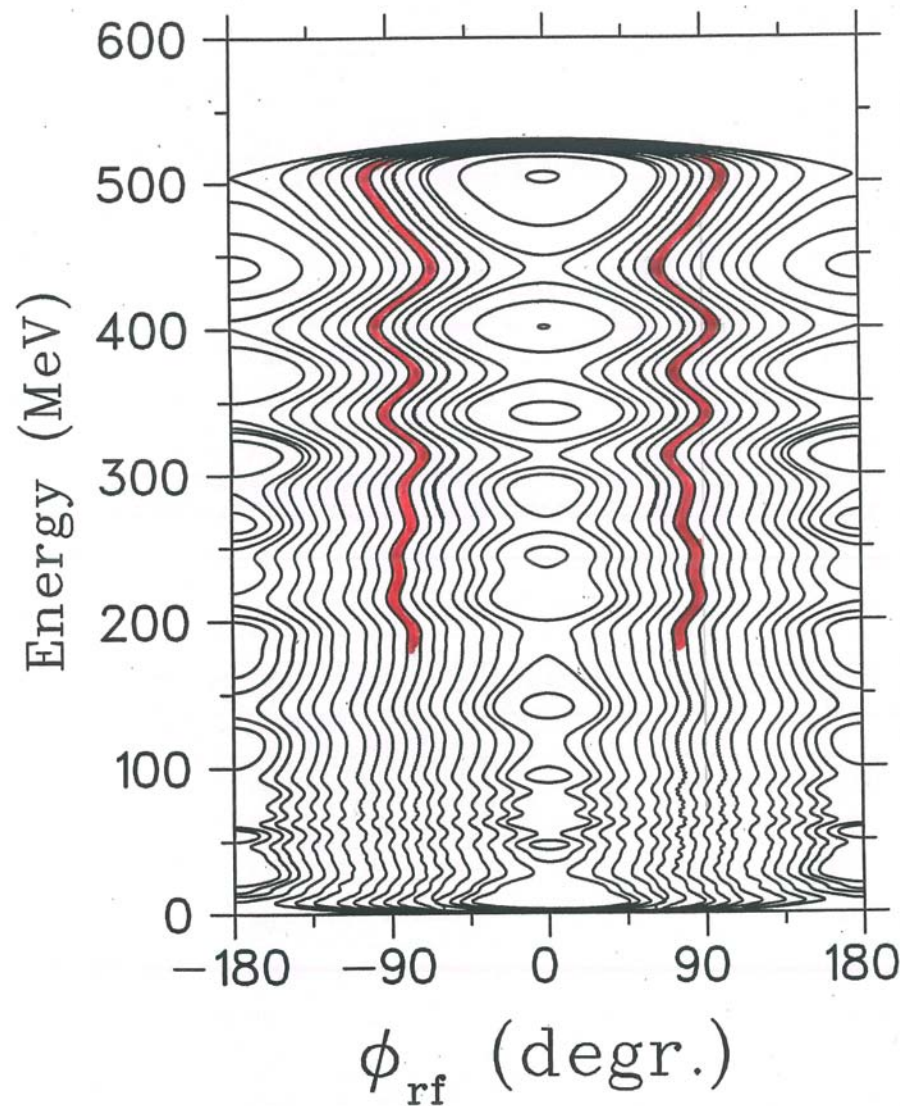


Fig. 5. Measured phases of accelerating and decelerating beams in the TRIUMF cyclotron.

Measured phase history  
in the TRIUMF cyclotron

- Real cyclotrons are only imperfectly isochronous
- Acceleration occurs along a serpentine path