Advancement of Scaling FFAG

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Contents

- Features of scaling FFAG accelerator
- Scaling law
  - Ring
  - Linear line
- Lattice exercise
  - Insertion/Matching
  - Dispersion suppressor
- Acceleration
  - Stationary bucket : fixed frequency RF

Scaling FFAG

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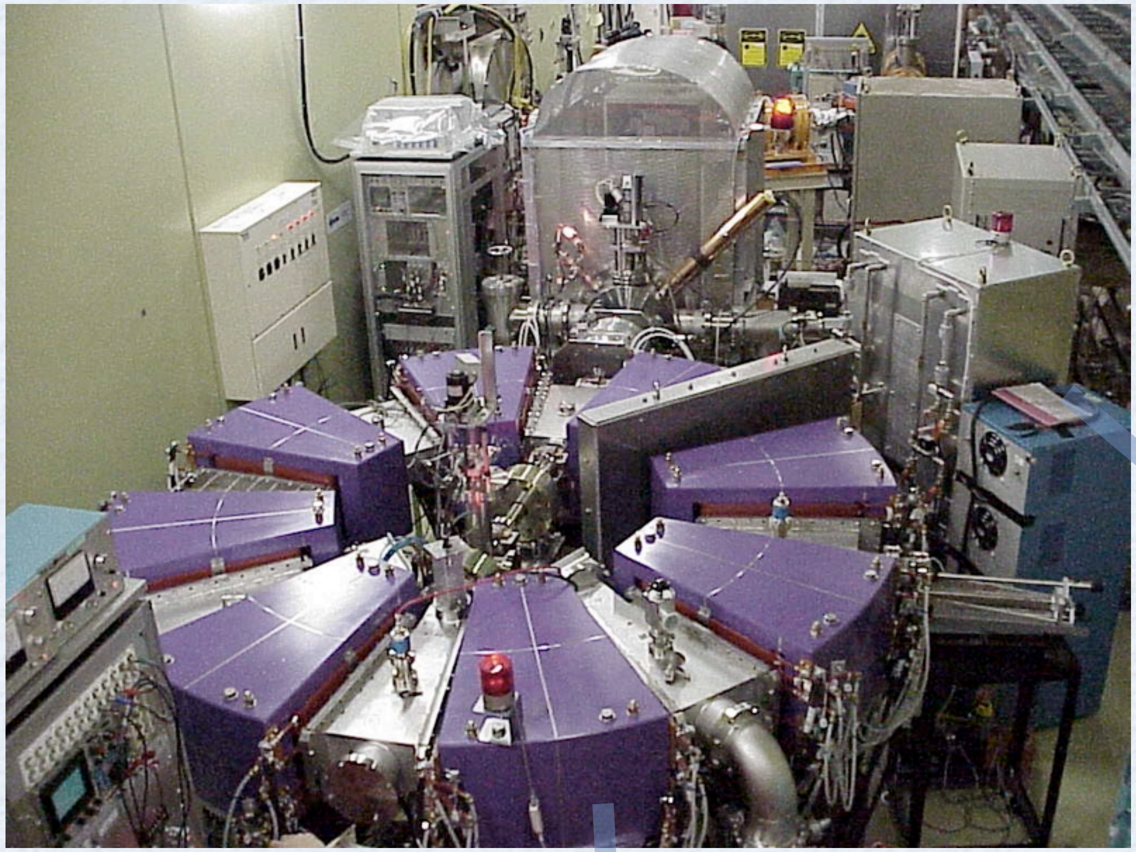
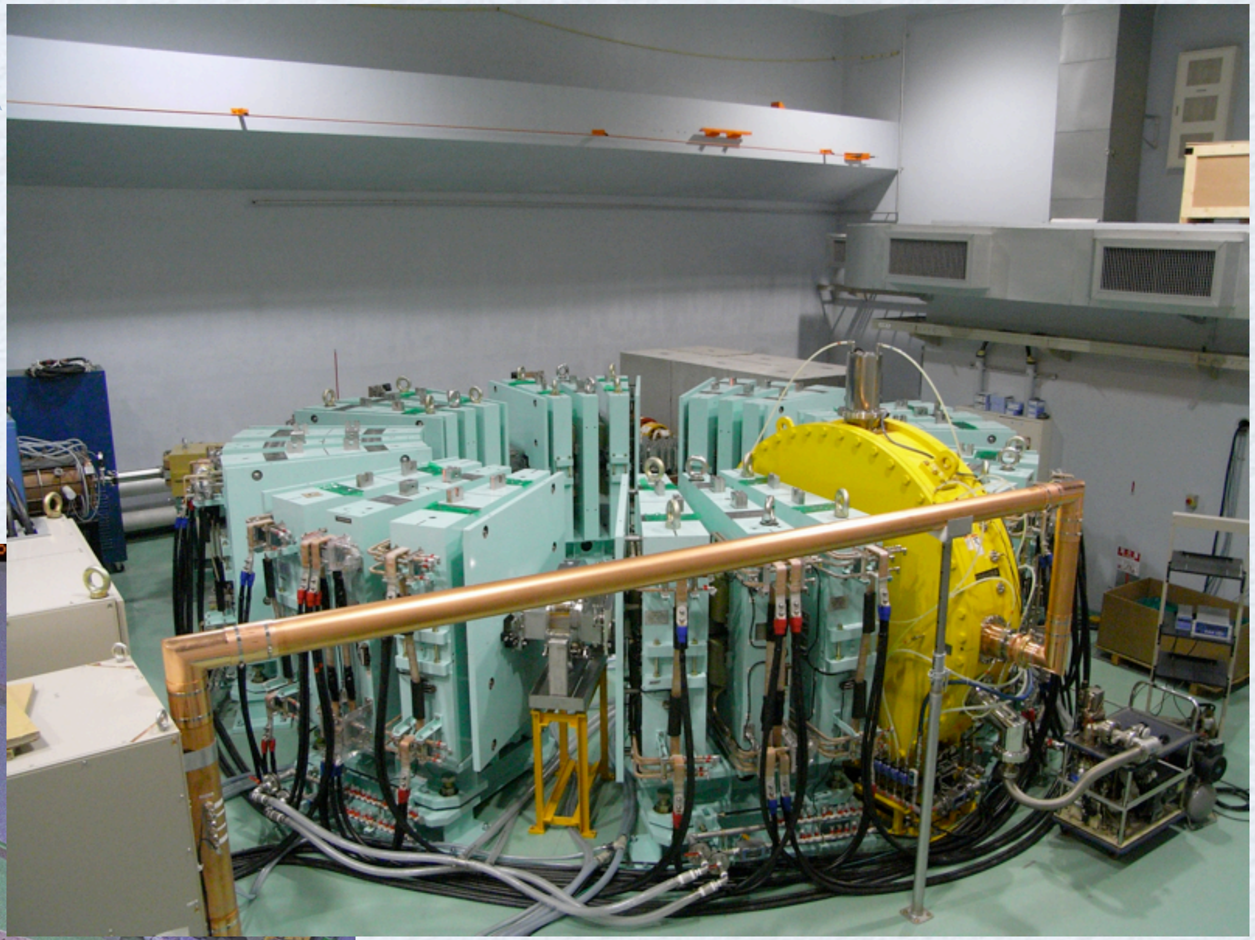
- FFAG : Fixed Field Alternating Gradient
  - Scaling: zero-chromaticity: constant tunes

Scaling FFAG

- FFAG : Fixed Field Alternating Gradient
  - Scaling: zero-chromaticity: constant tunes
- Scaling conditions for zero chromaticity
  - Orbit similarity for different beam momentum
  - Constant field index for any orbits



POP(2000)



ERIT(2008)

FFAG-KURRI(2009)

Scaling law I
circular ring

- Global coordinate: Cylindrical coordinate
- Betatron eqs.

$$\frac{d^2x}{d\theta^2}+\frac{r^2}{\rho^2}\left(1-K\rho^2\right)x=0$$

$$\frac{d^2z}{d\theta^2}+\frac{r^2}{\rho^2}\left(K\rho^2\right)z=0$$

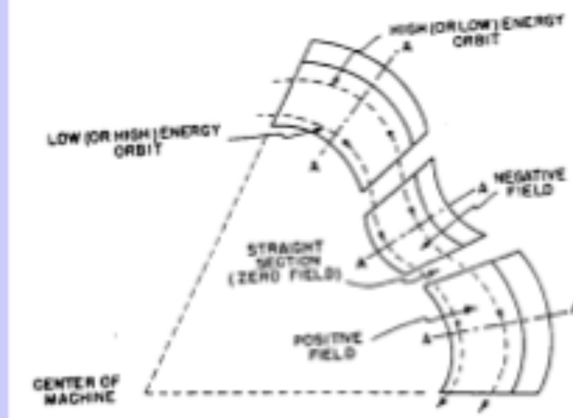


FIG. 2. Plan view of radial-sector magnets.

- Scaling condition: zero-chromaticity
  - sine qua nons: $\frac{d}{dp}\left[\frac{r^2}{\rho^2}\left(1-K\rho^2\right)\right]=0$ and $\frac{d}{dp}\left[\frac{r^2}{\rho^2}K\rho^2\right]=0$
  - sufficient conds.
  - B-field

$$\begin{cases}\dfrac{d\left(r^2/\rho^2\right)}{dp}=0\\ \dfrac{d\left(K\rho^2\right)}{dp}=0\end{cases}\longrightarrow\begin{cases}r\propto\rho\\ \dfrac{r}{B}\left[\dfrac{\partial B_z}{\partial x}\right]_{z=0}=k\end{cases}$$

$$B_z=B_0\left(\frac{r}{r_0}\right)^k f\left(\theta\right)$$

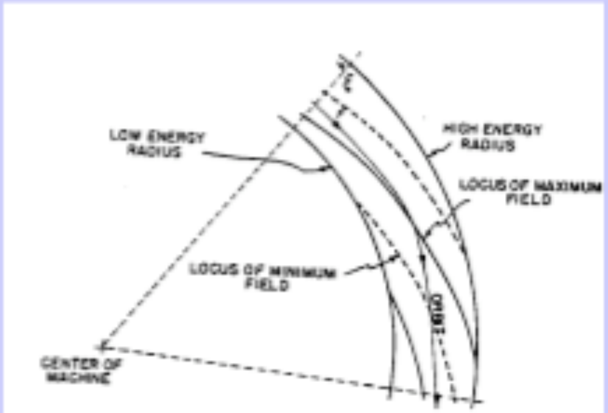


FIG. 3. Spiral-sector configuration.

Scaling FFAG ring

Scaling FFAG ring

- Pro/
  - Fixed field & Strong focusing
  - Zero chromaticity
    - constant betatron tunes → no-resonance crossing
  - Large acceptance (longitudinal & transverse)

Scaling FFAG ring

- Pro/
  - Fixed field & Strong focusing
  - Zero chromaticity
    - constant betatron tunes → no-resonance crossing
  - Large acceptance (longitudinal & transverse)
- Con/
  - Relative large dispersion:Orbit excursion is large.
    - Large aperture magnet
    - Large aperture rf cavity → Low frequency
  - Short straight section
    - Injection/Extraction difficulties → Kicker/Septum needs large apertures.
    - Available space for rf cavity is limited.

Scaling FFAG linear line

- Is it possible to make a linear FFAG straight line?
  - keeping a scaling law: zero chromaticity
  - reducing dispersion: dispersion suppressor
  - making a good match with ring: insertion
- Magnetic field configuration for FFAG linear line?
  - Obviously not:

$$B = B_0 \left(\frac{r}{r_0} \right)^k f(\theta)$$

Scaling condition II
linear (straight) transport line

- Betatron eqs.

$$\frac{d^2x}{dy^2}+\frac{1}{\rho^2}\left(1-K\rho^2\right)x=0$$

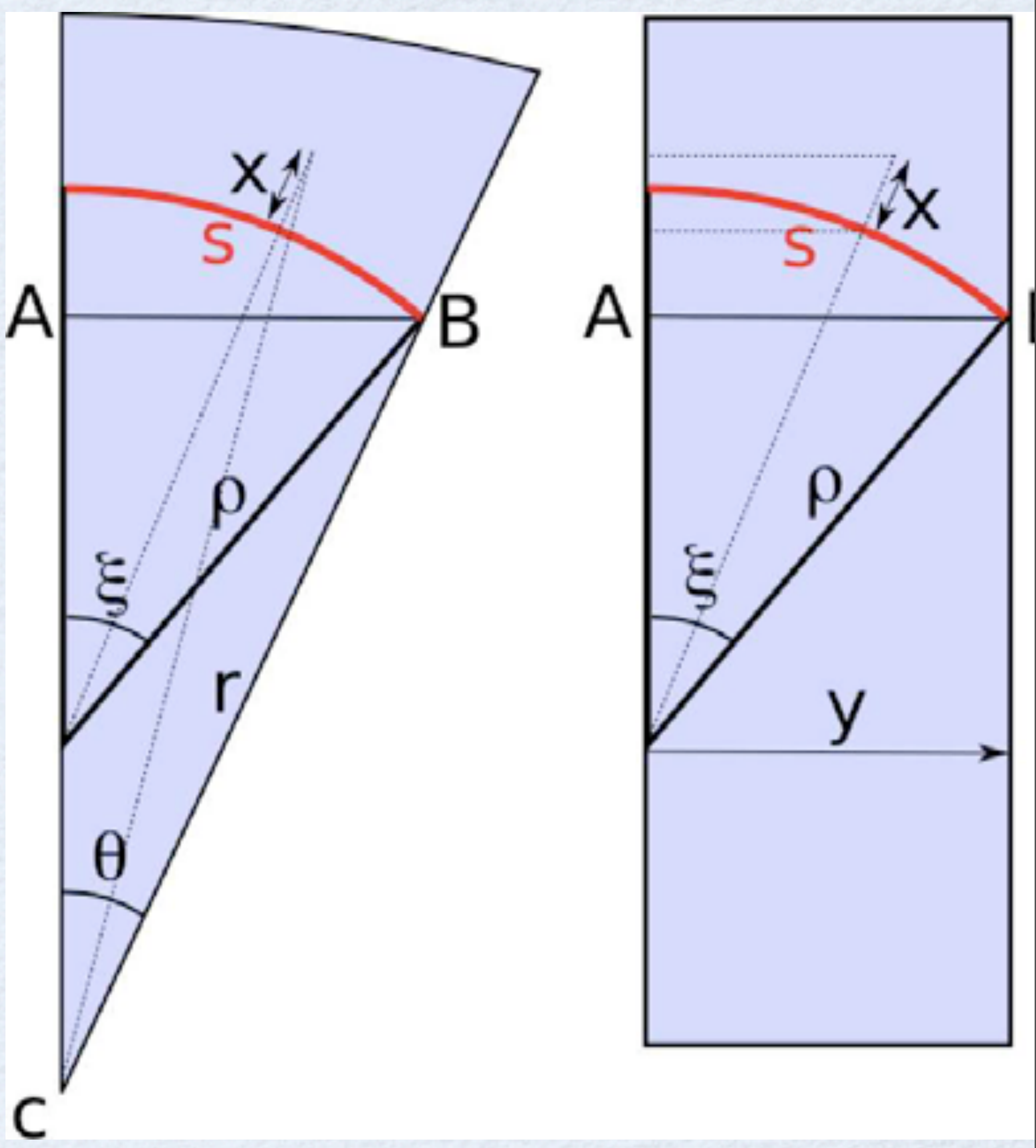
$$\frac{d^2z}{dy^2}+\frac{1}{\rho^2}\left(K\rho^2\right)z=0$$

- Scaling conditions: zero-chromaticity
  - sufficient cond.

$$\begin{cases}\dfrac{d\left(1/\rho^2\right)}{dp}=0\\[2ex] \dfrac{d\left(K\rho^2\right)}{dp}=0\end{cases}\longrightarrow\begin{cases}\rho=const.\\[1ex] \dfrac{1}{B}\left[\dfrac{\partial B_z}{\partial x}\right]_{z=0}=\dfrac{n}{\rho}\end{cases}$$

  - Magnetic field

$$B_z=B_0\exp\left[\frac{n}{\rho}x\right]$$



$$\left[\lim_{r_0\to\infty}\left(\frac{r}{r_0}\right)^k=\lim_{r_0\to\infty}\left[\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{x}{r_0}k}=\lim_{r_0\to\infty}\left[\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{n}{\rho}x}=\exp\left(\frac{n}{\rho}x\right)\right]$$

Scaling linear line

- Example (JB. Lagrange)
  - Perfect scaling(zero-chromatic) FFAG linear transport line
  - proton 80-200MeV

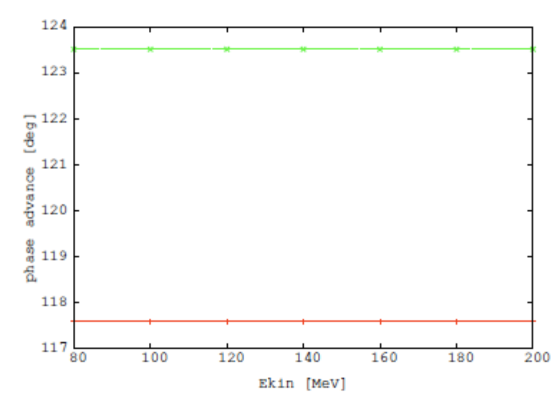
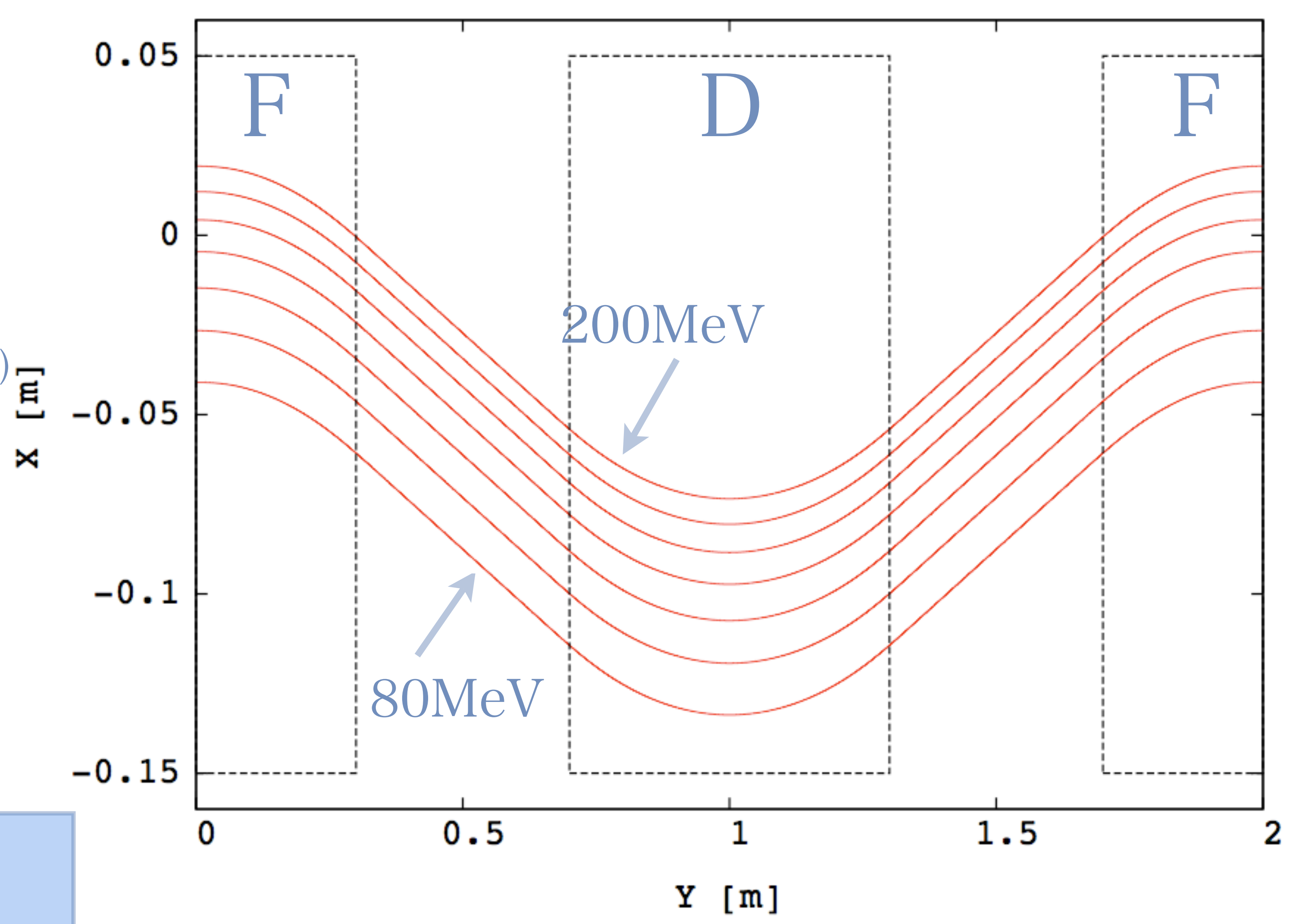


Table 1: Tracking parameters

Length of the magnets	60 cm
Drift	40 cm
Kinetic energy range	80 to 200 MeV (proton)
Field index	17
Local curvature radius	2.1 m
Step size	1 mm
Phase advances:	
horizontal μ_x	104.8 deg.
vertical μ_z	112.5 deg.



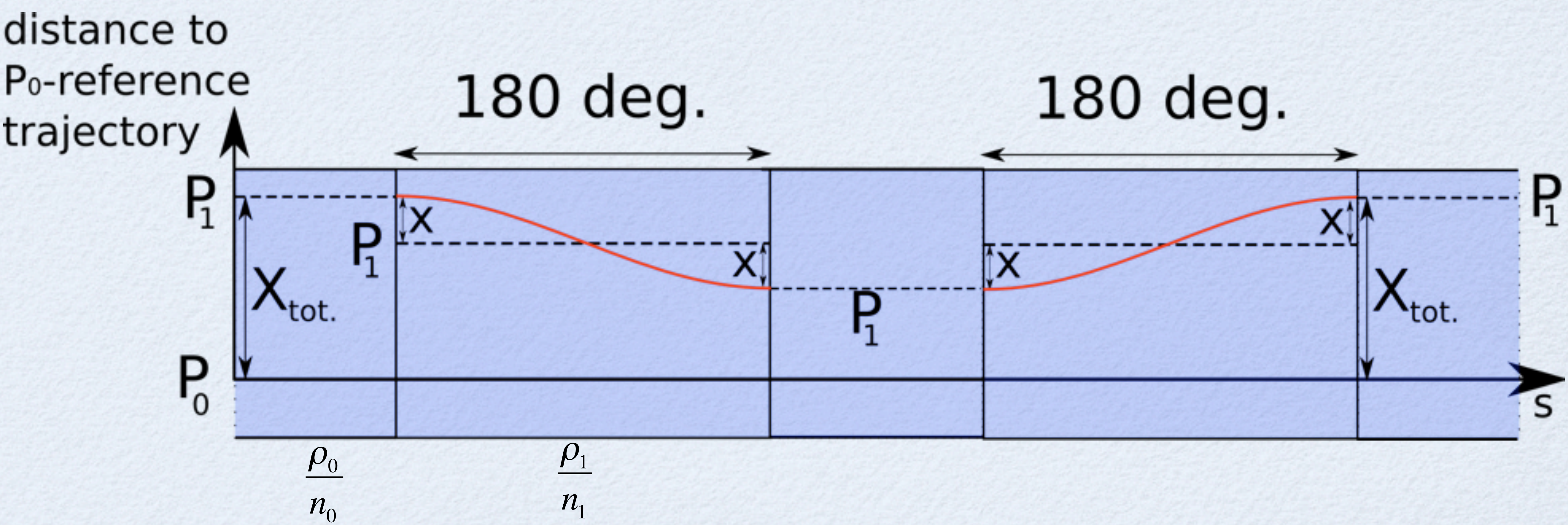
B-field

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

Dispersion suppressor

- Dispersion suppressor (Planche,Lagrange,Mori)
  - successive π-cells in the horizontal plane can suppress the dispersion.

$$X_{tot} = X_1 - X_0 = \frac{1}{n/\rho} \ln\left(\frac{P_1}{P_0}\right) \qquad x = \ln\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$



Insertion Matching

btw. ring & straight line

- B(closed orbit) matching condition

$$\left(1+\frac{x}{r_m}\right)^{k+1} = \exp\left(\frac{n}{\rho}x\right)$$

ring linear line

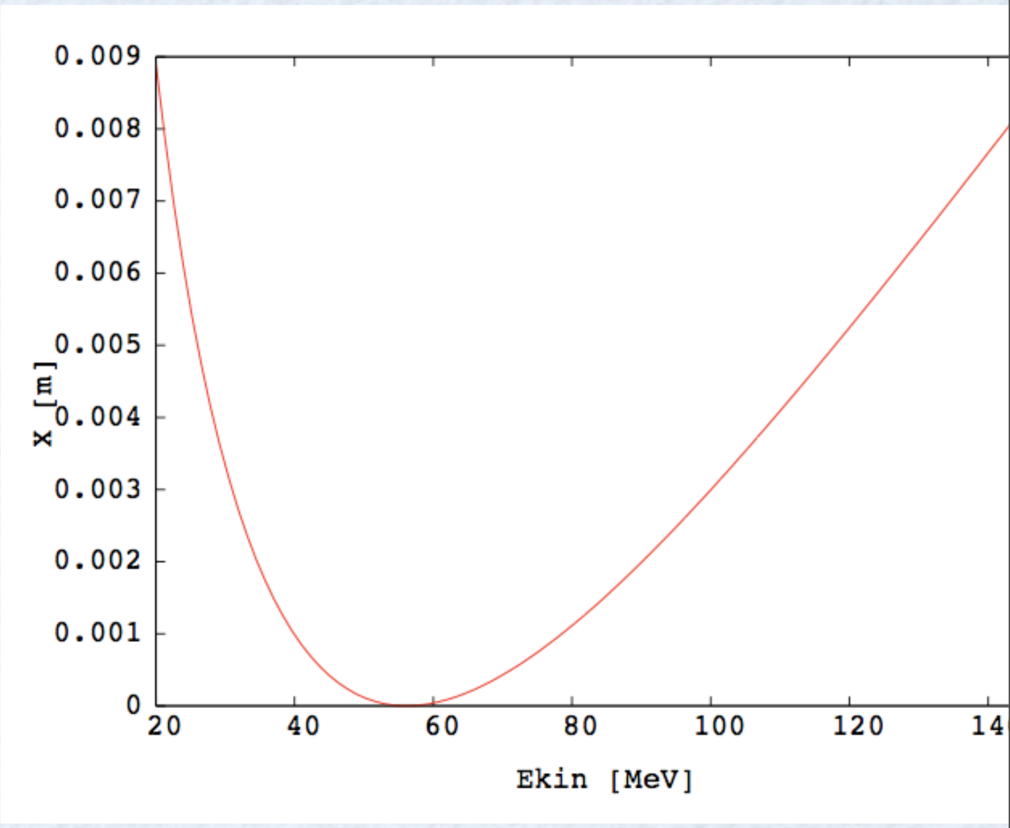
$$\frac{k+1}{r_m} = \frac{n}{\rho}$$

← 1st order

CO mismatch

higher order error: $\sim \frac{1}{k}x$

→ smaller for larger ring

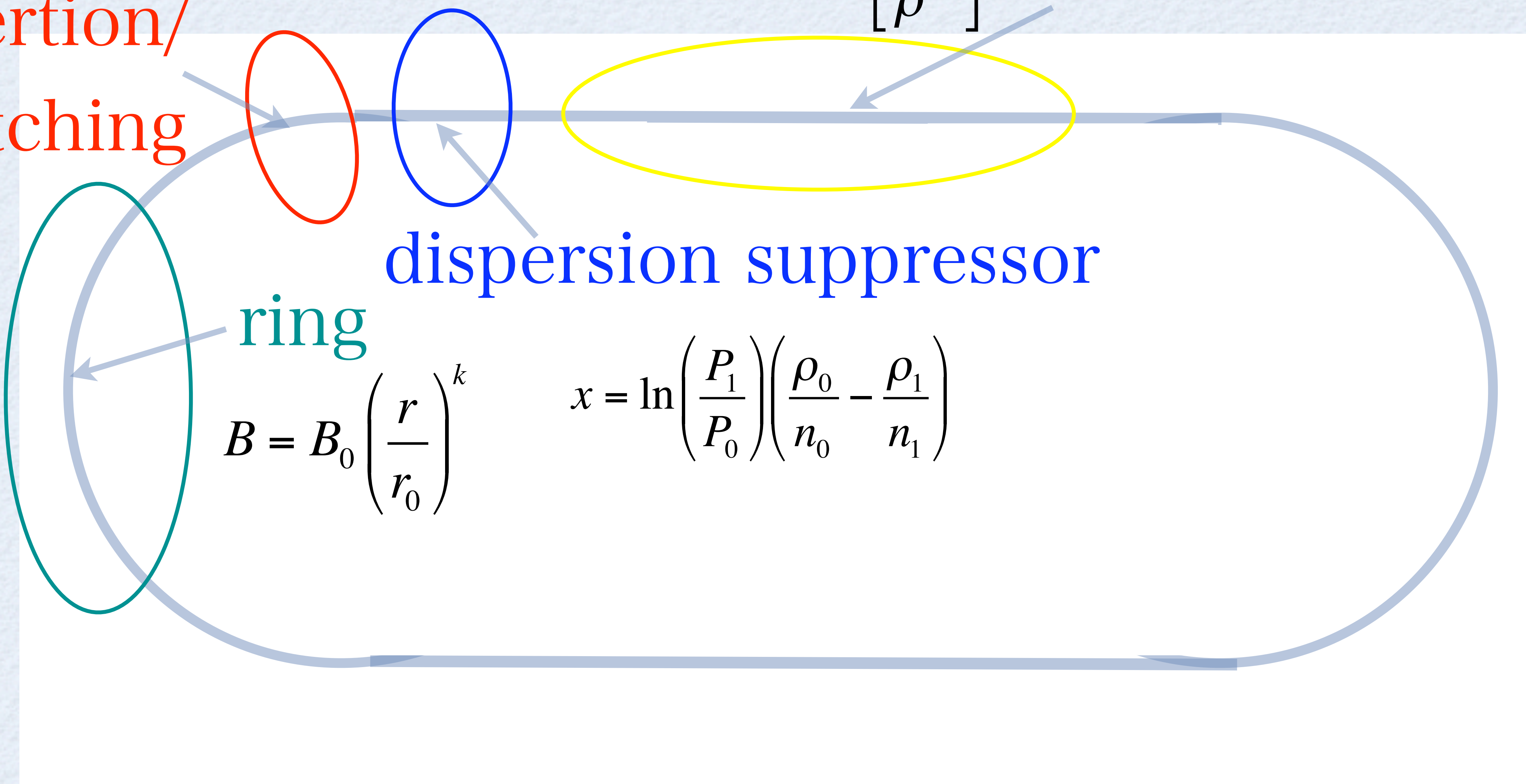


Example: 150MeV p-FFAG ring(KURRI) with insertion

Advanced scaling FFAG

$$\frac{k+1}{r_m} = \frac{n}{\rho}$$

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$



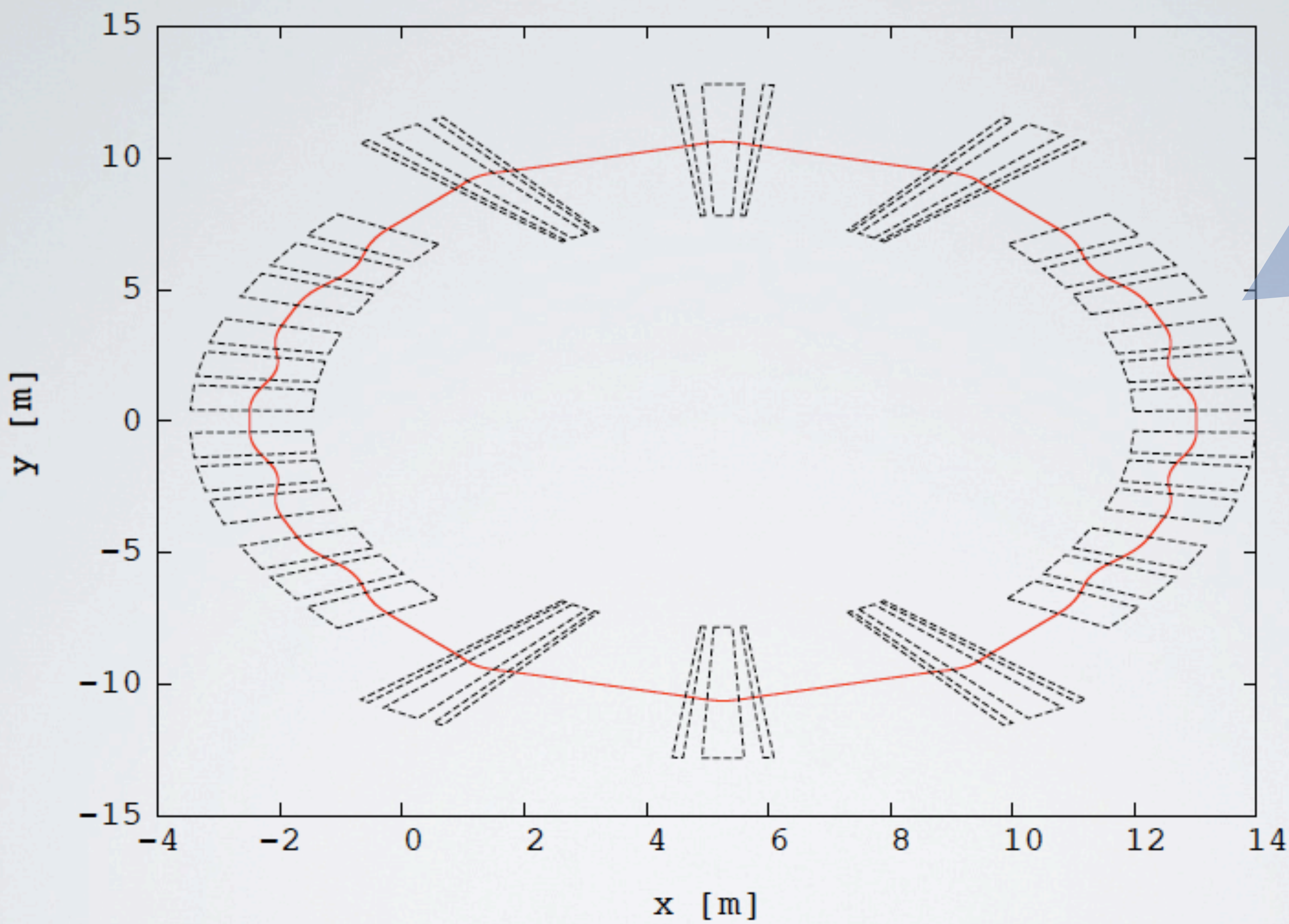
$$B = B_0 \left(\frac{r}{r_0}\right)^k$$

$$x = \ln\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$

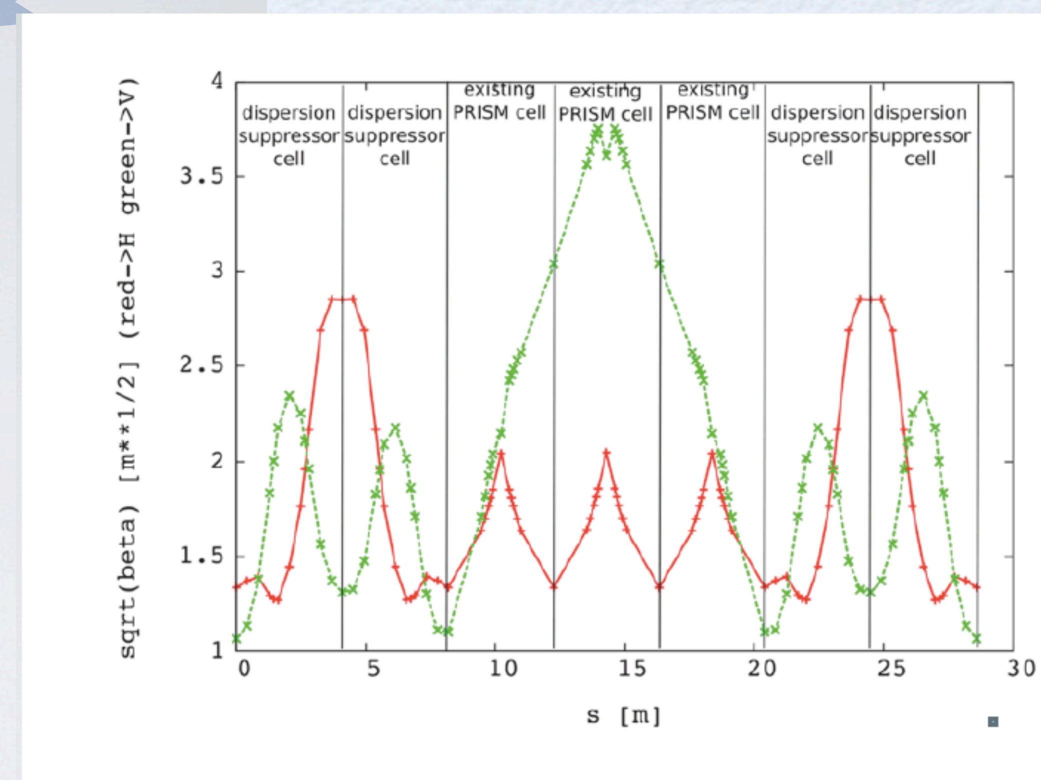
Muon phase rotation ring:PRISM

- J.B. Lagrange (talk at this workshop)

π section for dispersion suppressor



PRISM ring (without straight parts) with dispersion suppressor and existing PRISM magnets.



Muon accelerator for neutrino factory

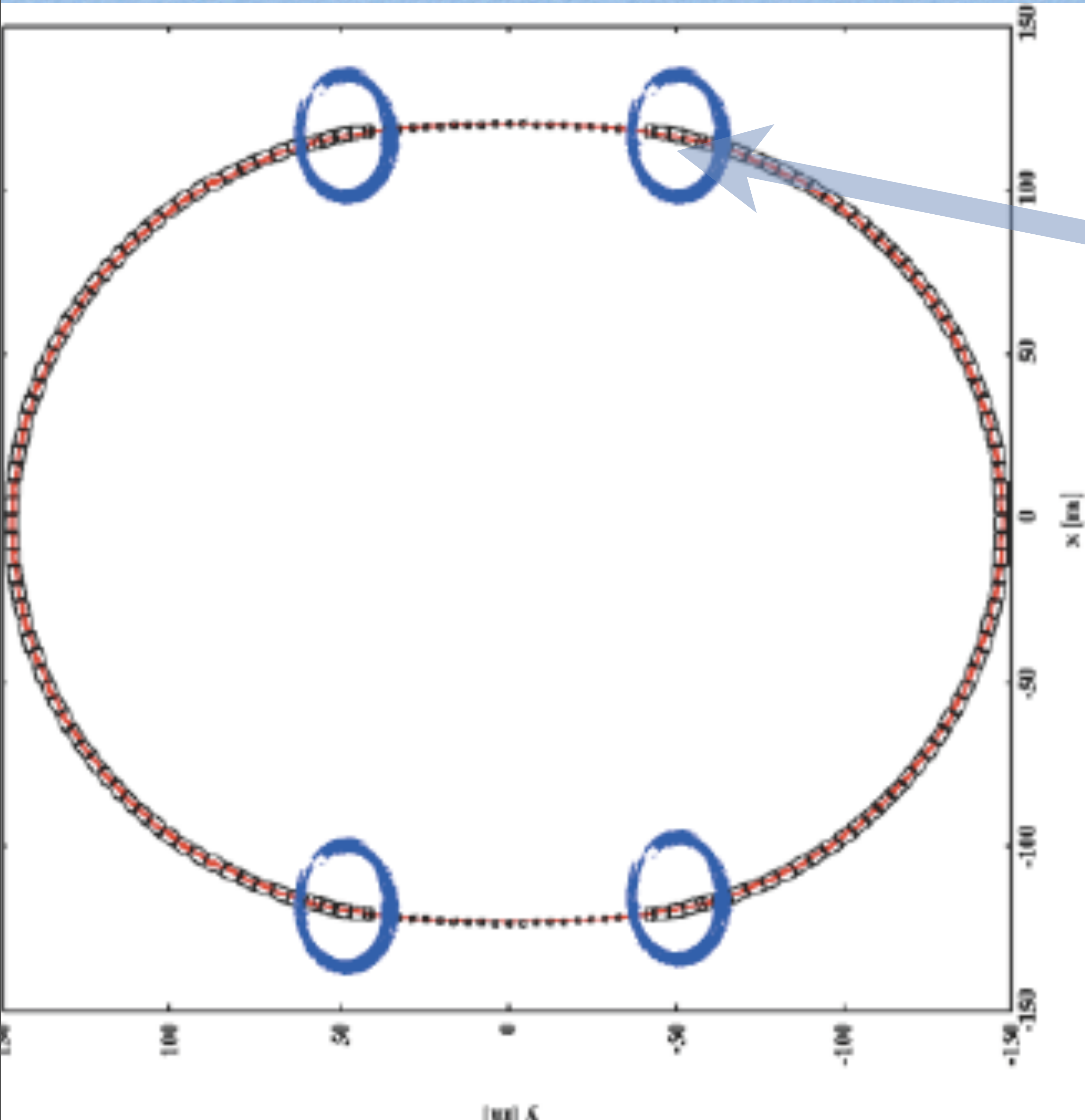
Table 1: 3 to 10 GeV Muon Ring Parameters

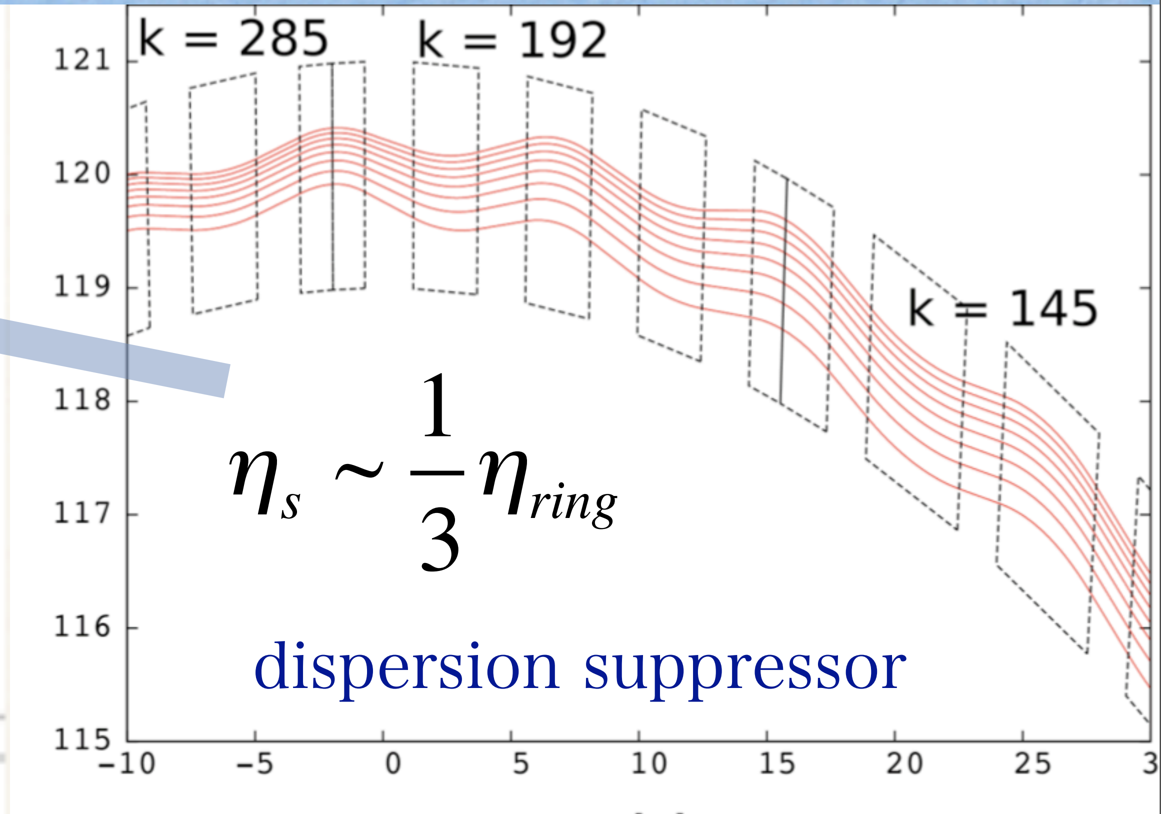
Lattice type	scaling FFAG - double beam
Mean radius	120 m
Number of cells	72
Field index k	145
Packing factor	0.7
B_{max}	2.6 T
Horiz. phase adv. per cell	93.2 deg.
Verti. phase adv. per cell	30.2 deg.
Mean RF frequency	$\sim$ 400 MHz
RF peak voltage	1.6 GV/turn
Number of RF cavities	72

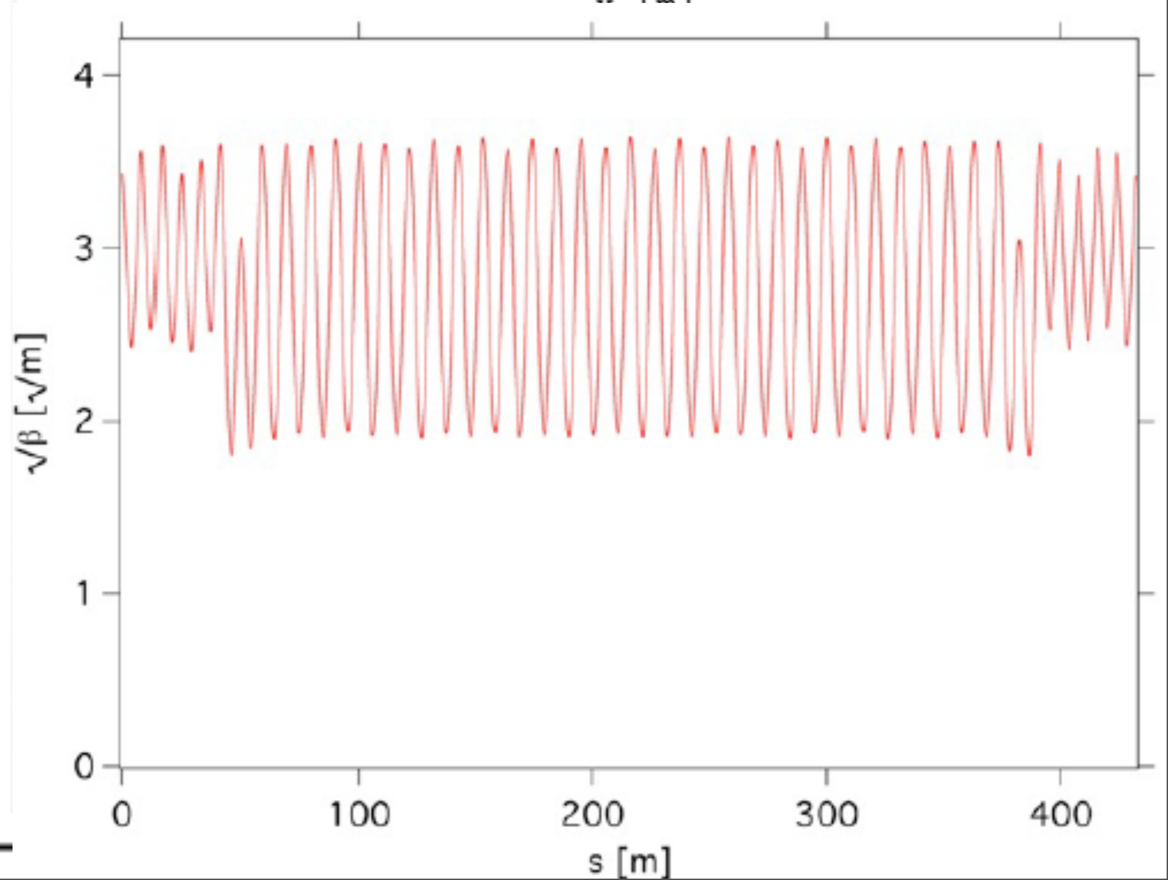
Harmonic Number Jump

→ require higher harmonics

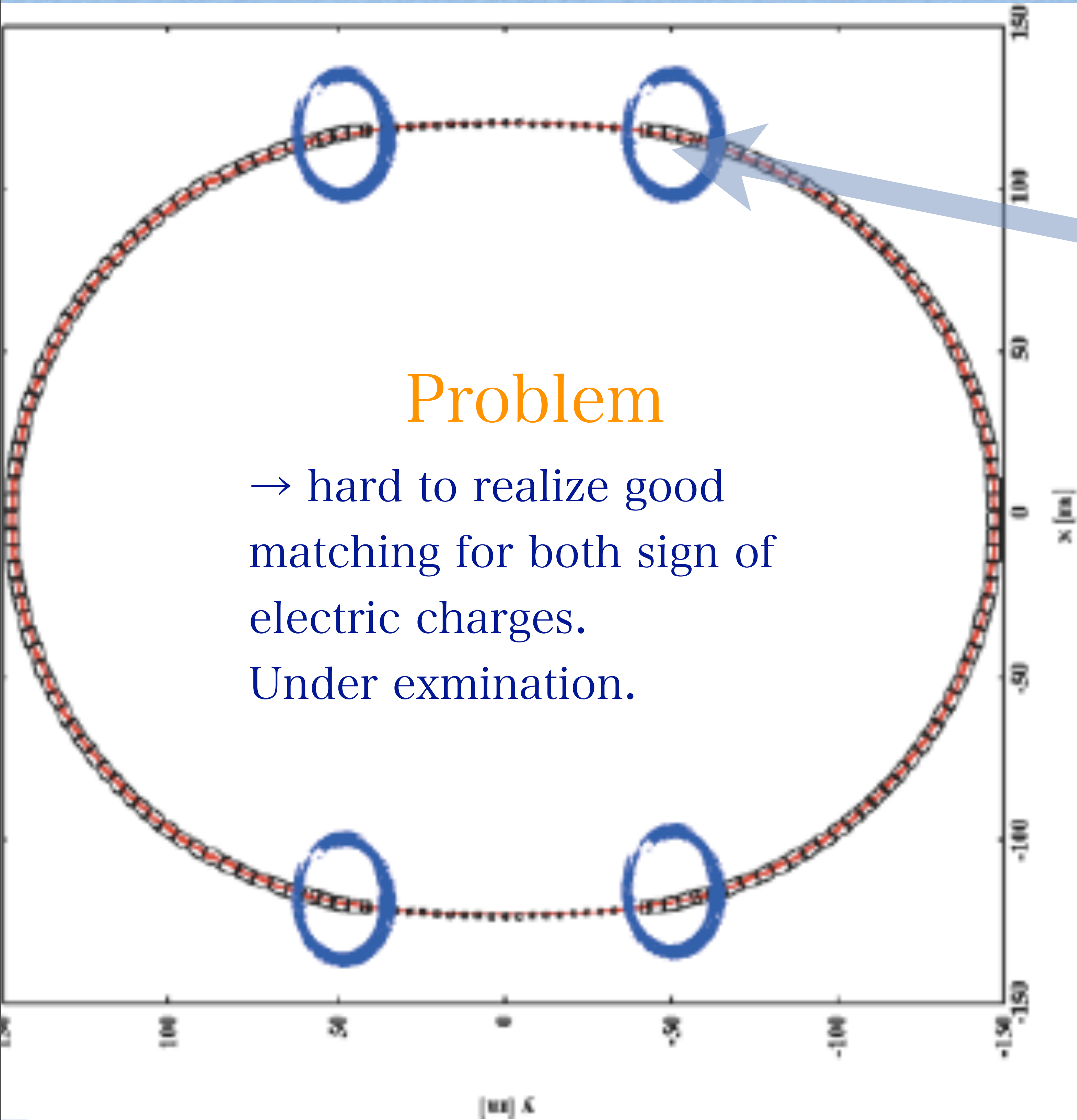
Muon accelerator for neutrino factory





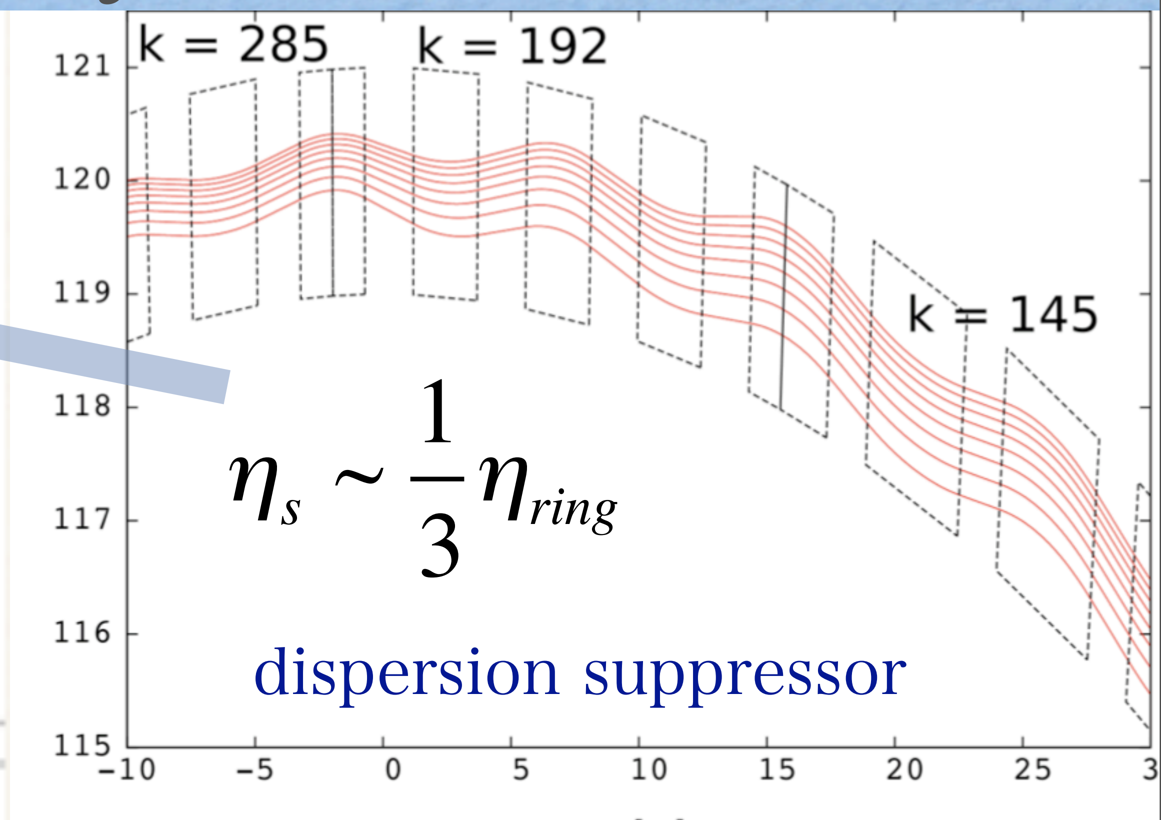


Muon accelerator for neutrino factory



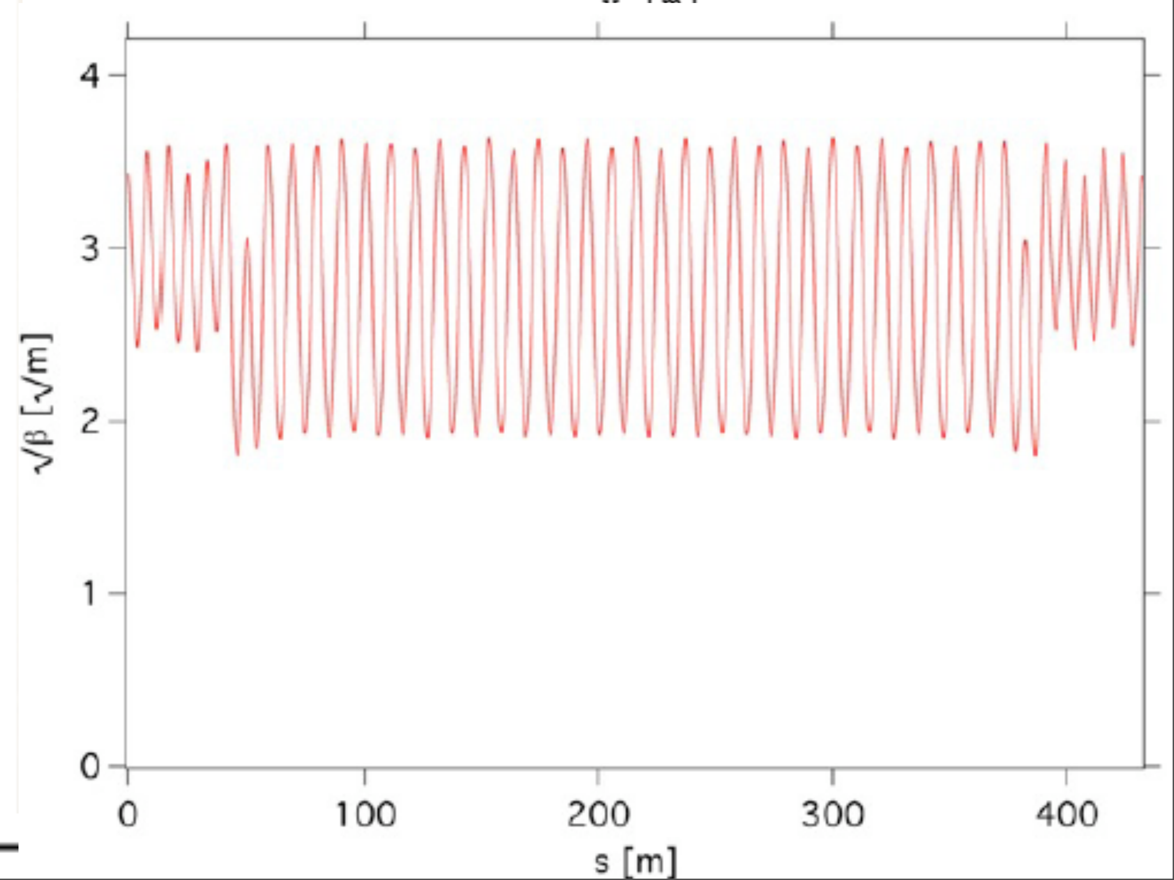
Problem

→ hard to realize good matching for both sign of electric charges.
Under exmination.



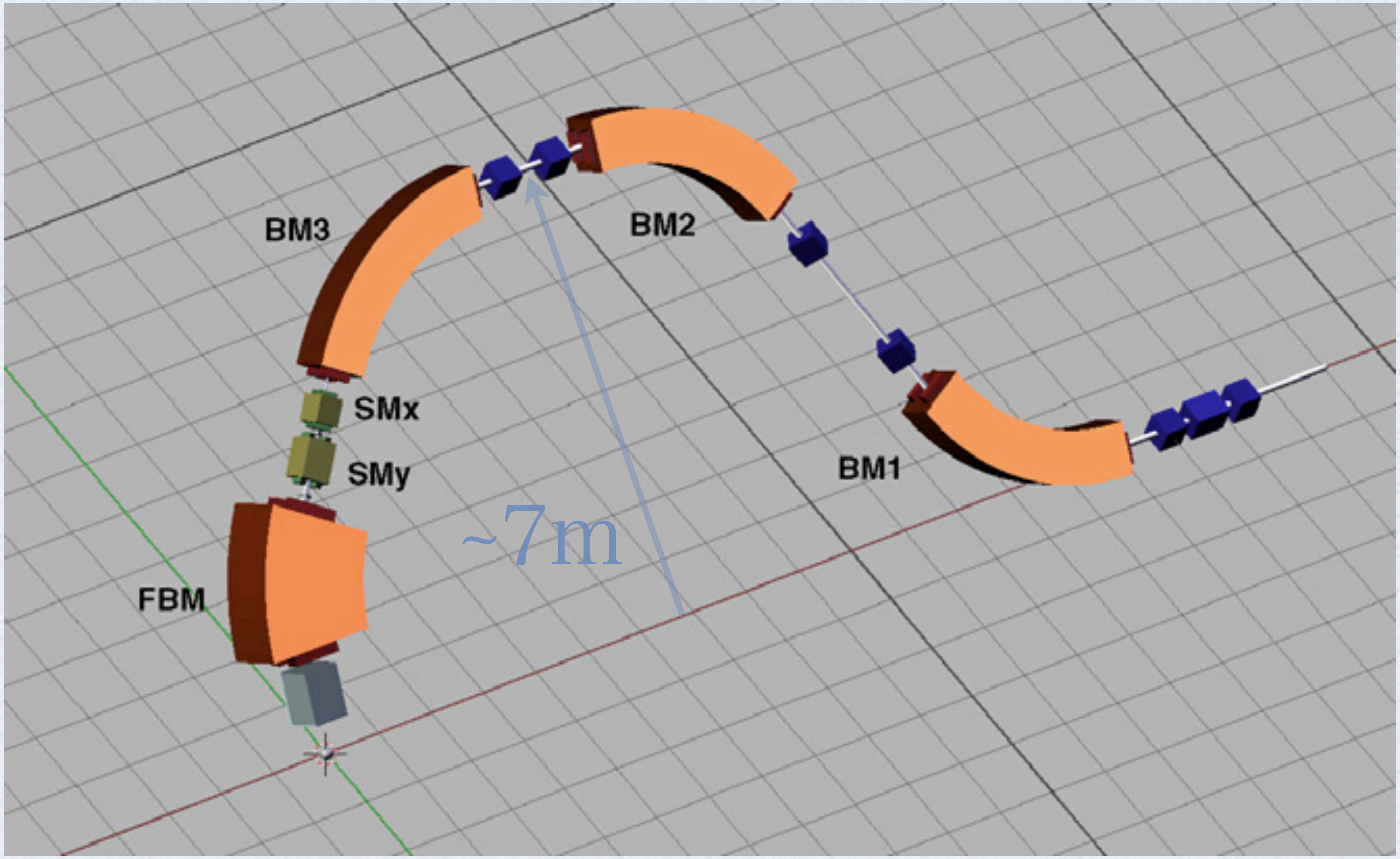
$$\eta_s \sim \frac{1}{3}\eta_{ring}$$

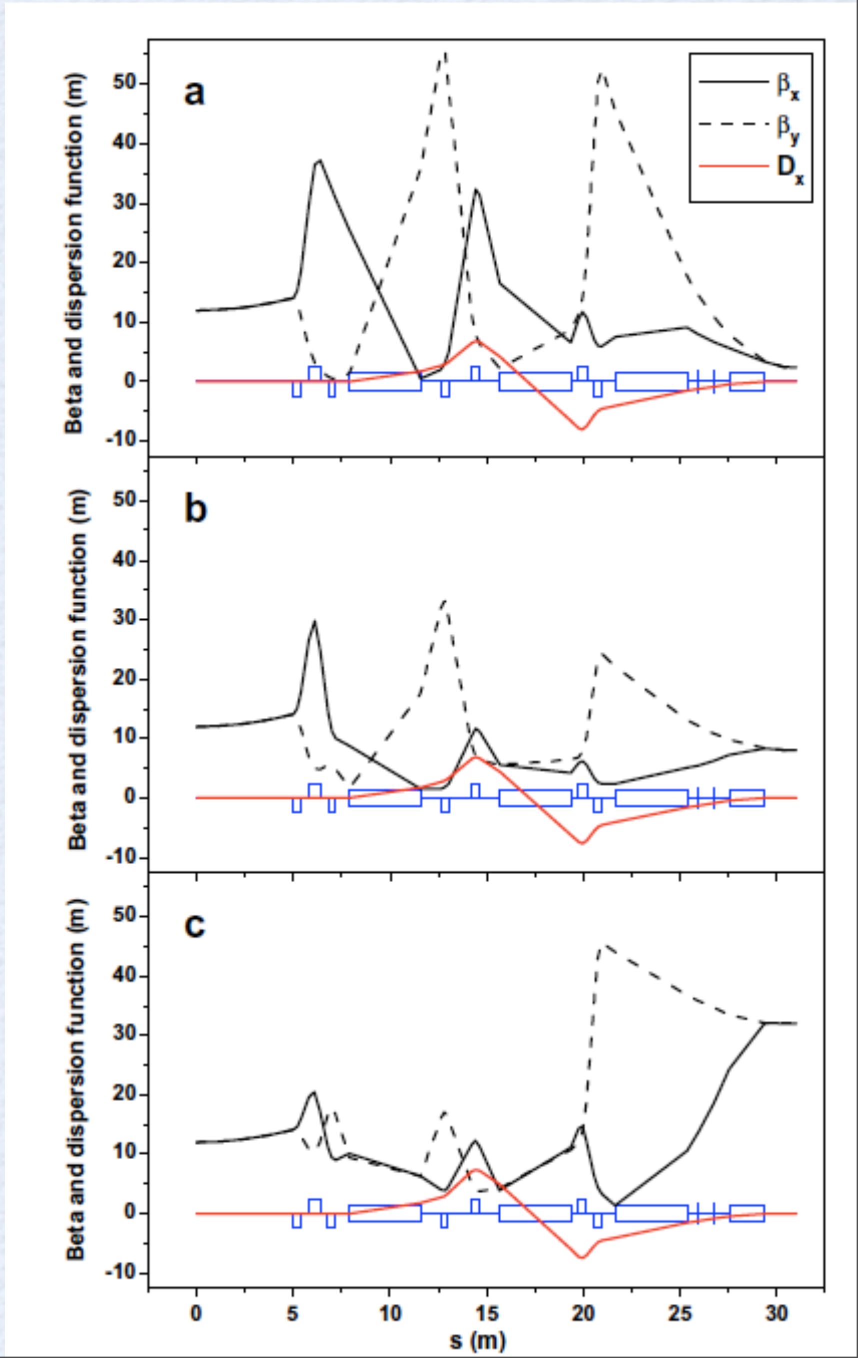
dispersion suppressor



Application: Gantry for C-beam therapy

- Gantry design C-beam therapy : T.Furukawa et al.,:Nucl. Instr.Meth. PRB 266(2008)2186-2189



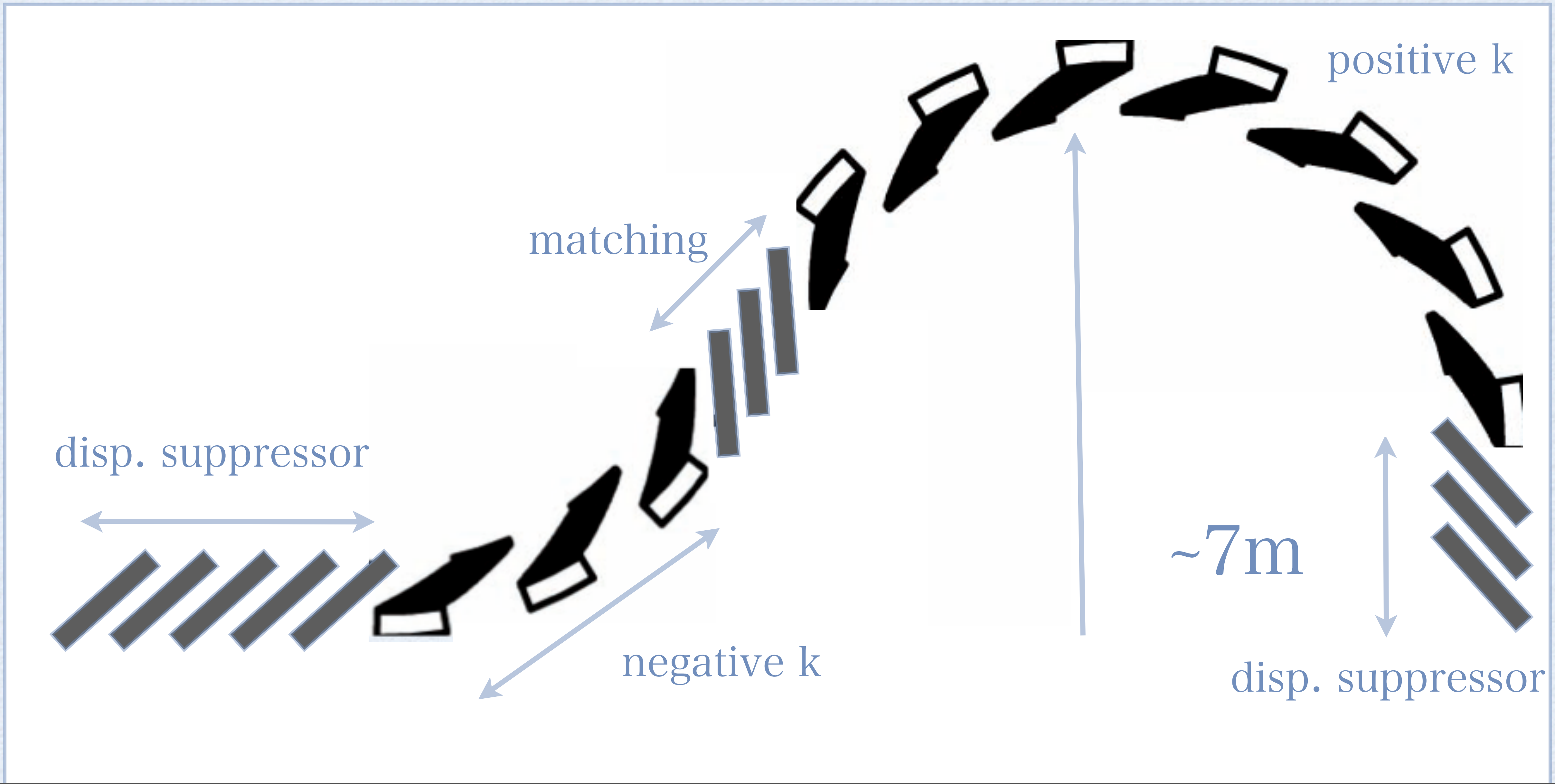


Gantry for spot scanning

- 3-D conformal spot scanning
  - Large momentum acceptance required in gantry lattice
    - 250MeV/c <-> 400MeV/c (p/p_0~1.3)
    - In ordinary gantry lattice, momentum acceptance is small (<few %)
- FFAG lattice has very large acceptance!
- Non-scaling FFAG lattice proposed by Trbojevic et al.)
- Scaling FFAG lattice with dispersion suppressor

Gantry with scaling FFAG lattice

- Energy(max.) 400MeV/c (C)
- k(arc) ~12
- B(max.) 1.9 T
- Orbit excursion 14cm (250-400MeV/c)



RF acceleration

- Longitudinal beam dynamics in scaling FFAG
  - Orbit lengths are scaled as beam momentum

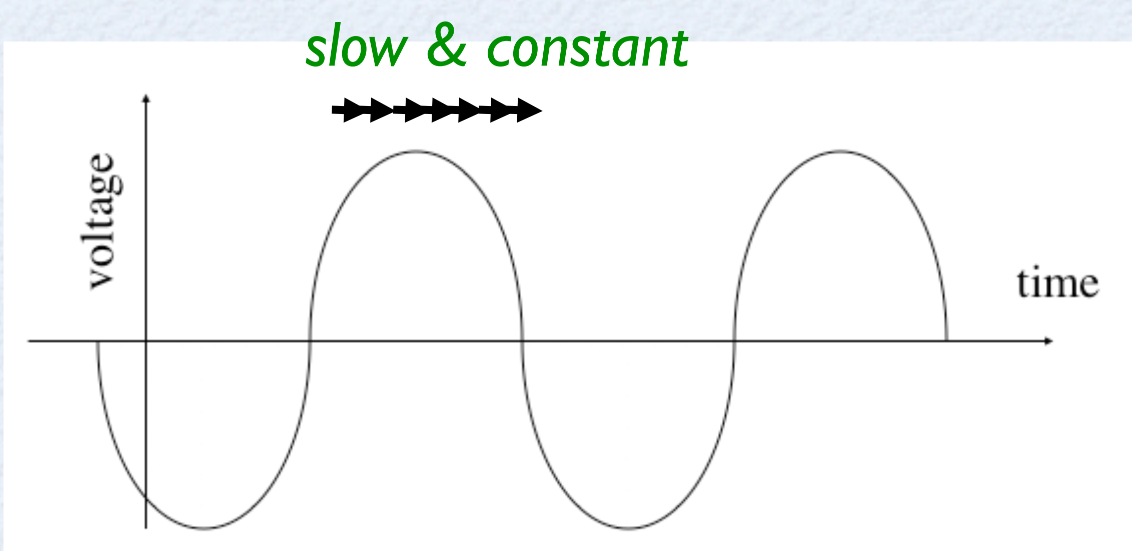
$$r \propto p^{\frac{1}{k+1}}; \alpha_p = \frac{1}{k+1}$$

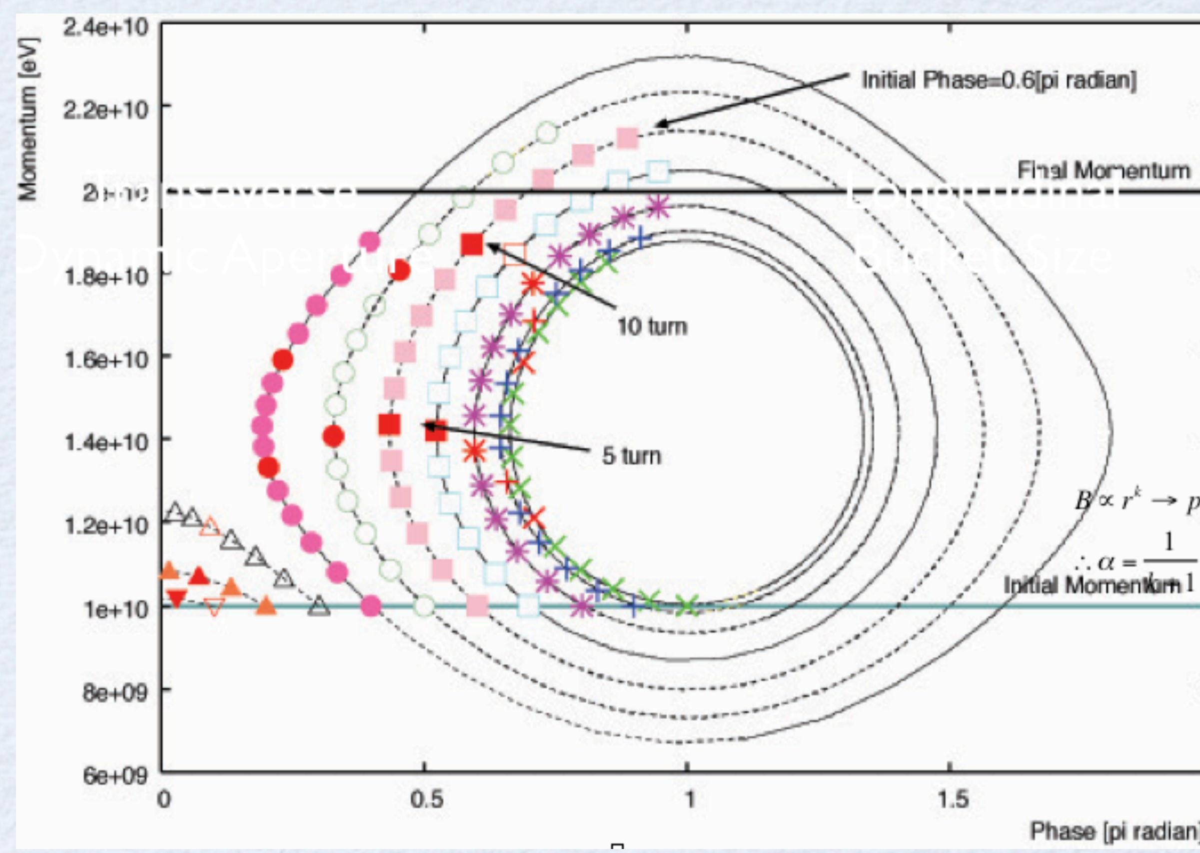
- Beam acceleration in scaling FFAG accelerator
  - Variable rf frequency
  - Fixed rf frequency
    - Stationary bucket
    - Harmonic number jump

Fixed rf frequency

- Stationary bucket (EPAC06-TUXFI01)
  - Constant & small enough phase-slip → large energy gain
    - relativistic particle
    - constant Momentum compaction
  - Adequate for scaling FFAG

$$\eta = \frac{1}{\gamma^2} - \alpha \cong -\alpha = -\frac{1}{k+1}$$



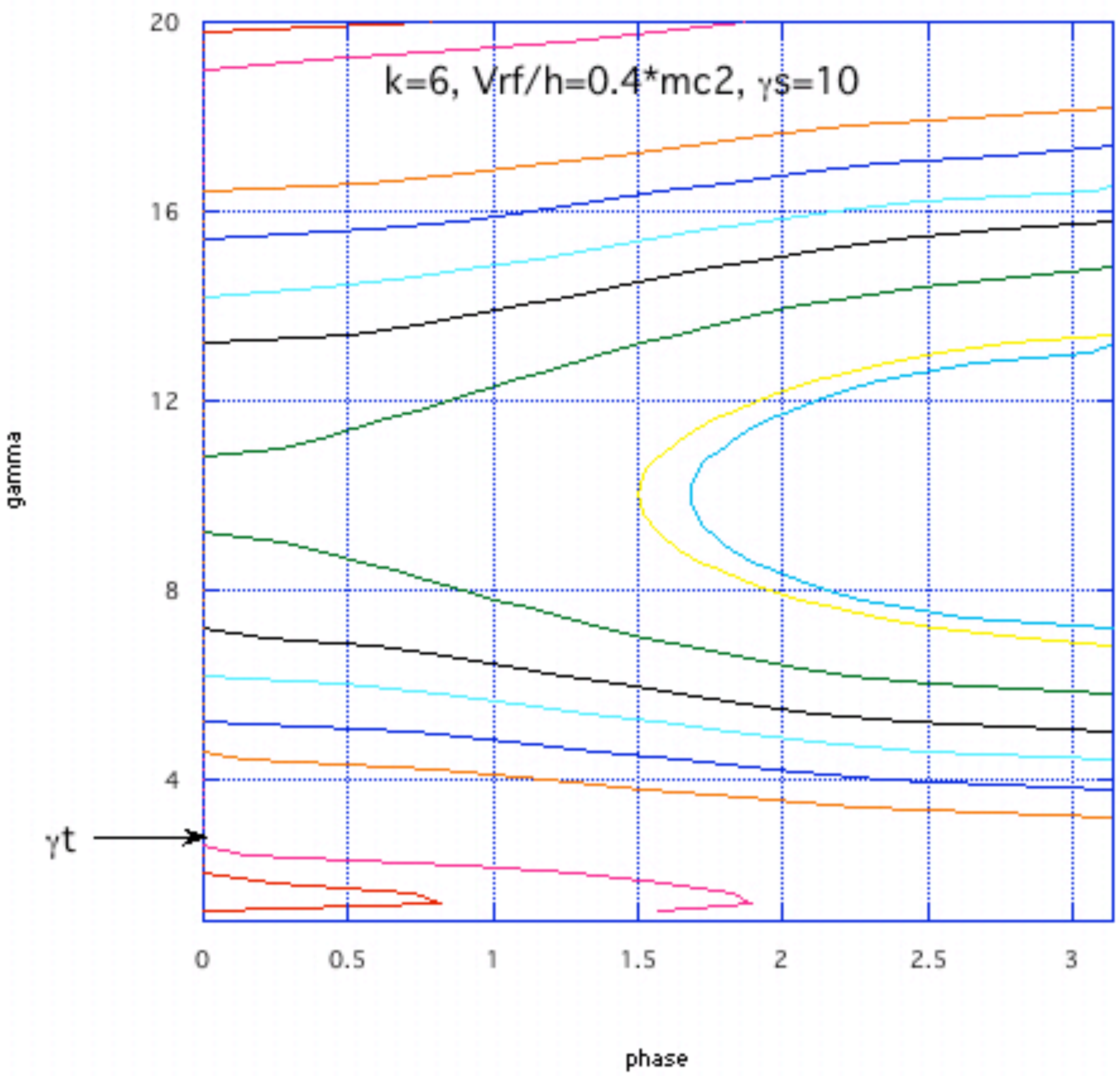


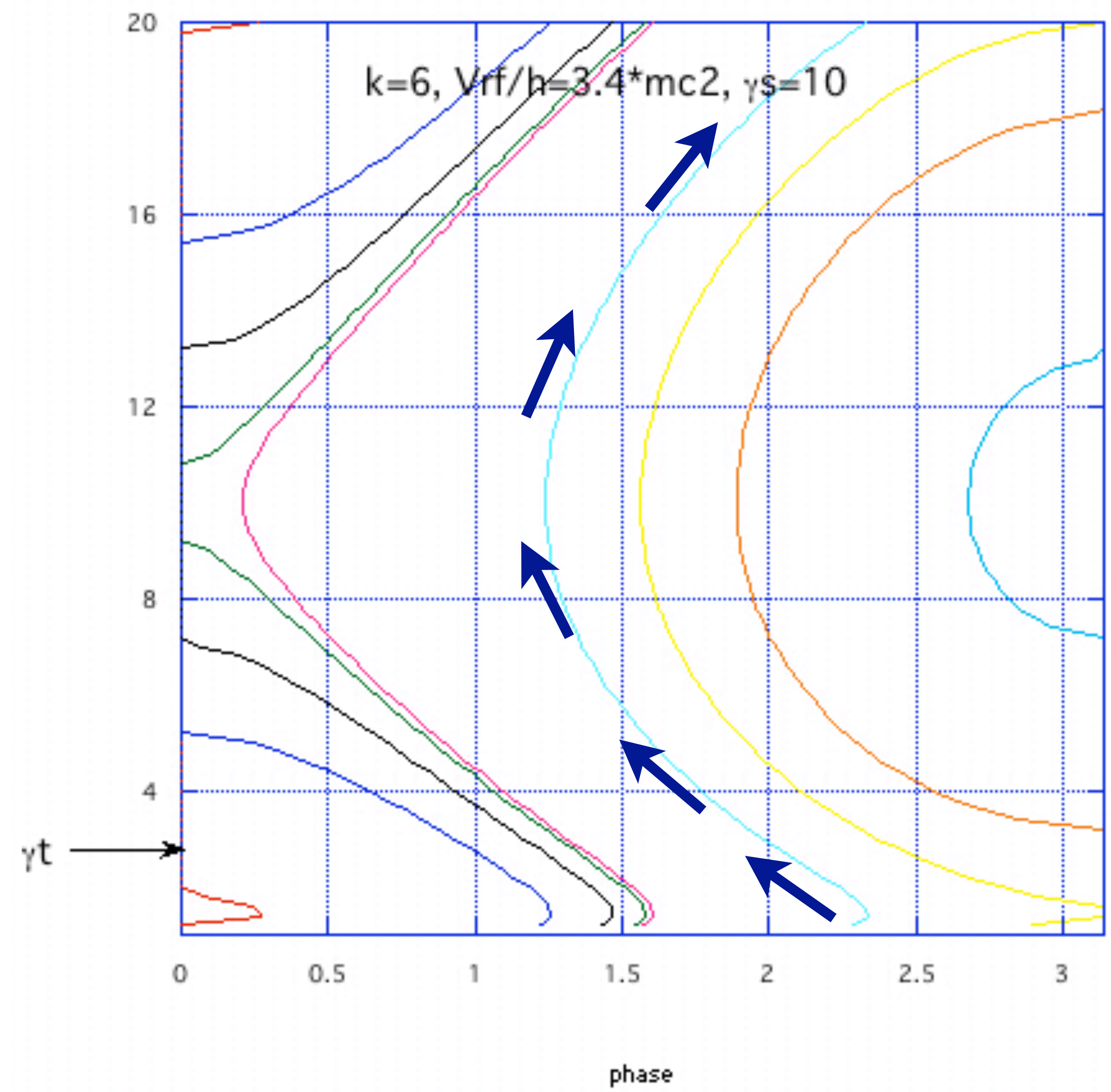
Stationary bucket

- Longitudinal beam dynamics in scaling FFAG
  - Hamiltonian is given in analytical form.
    - E.Yamakawa (in this workshop)
- Acceleration from low energy (non-relativistic) to high energy (relativistic) with a fixed-frequency rf becomes possible.

Stationary bucket

$$H = 2\pi m_0 c^2 \left[\frac{\left(\gamma_s^2 - 1\right)^{\lambda}}{2\gamma_s} \frac{\left(\gamma^2 - 1\right)^{-\lambda+1}}{(1-\lambda)} + \gamma \right] + e\frac{V_{rf}}{h} f_0 \cos\phi, \lambda = \frac{k}{2(k+1)}$$





Summary

- Advanced scaling FFAG scheme has been developed.
- Scaling linear system requires,
  - Scaling law
  - Insertion/Matching
  - Dispersion suppressor
- Race-track FFAG ring is in reality.
  - Muon acceleration → Neutrino Factory(T. Planche)
  - Muon phase rotation → PRISM(J.B. Lagrange)
- Acceleration with fixed-frequency RF
  - Acceleration with Stationary bucket (E. Yamakawa)