

# Fast Integer Resonance Crossing in a Scaling FFAG



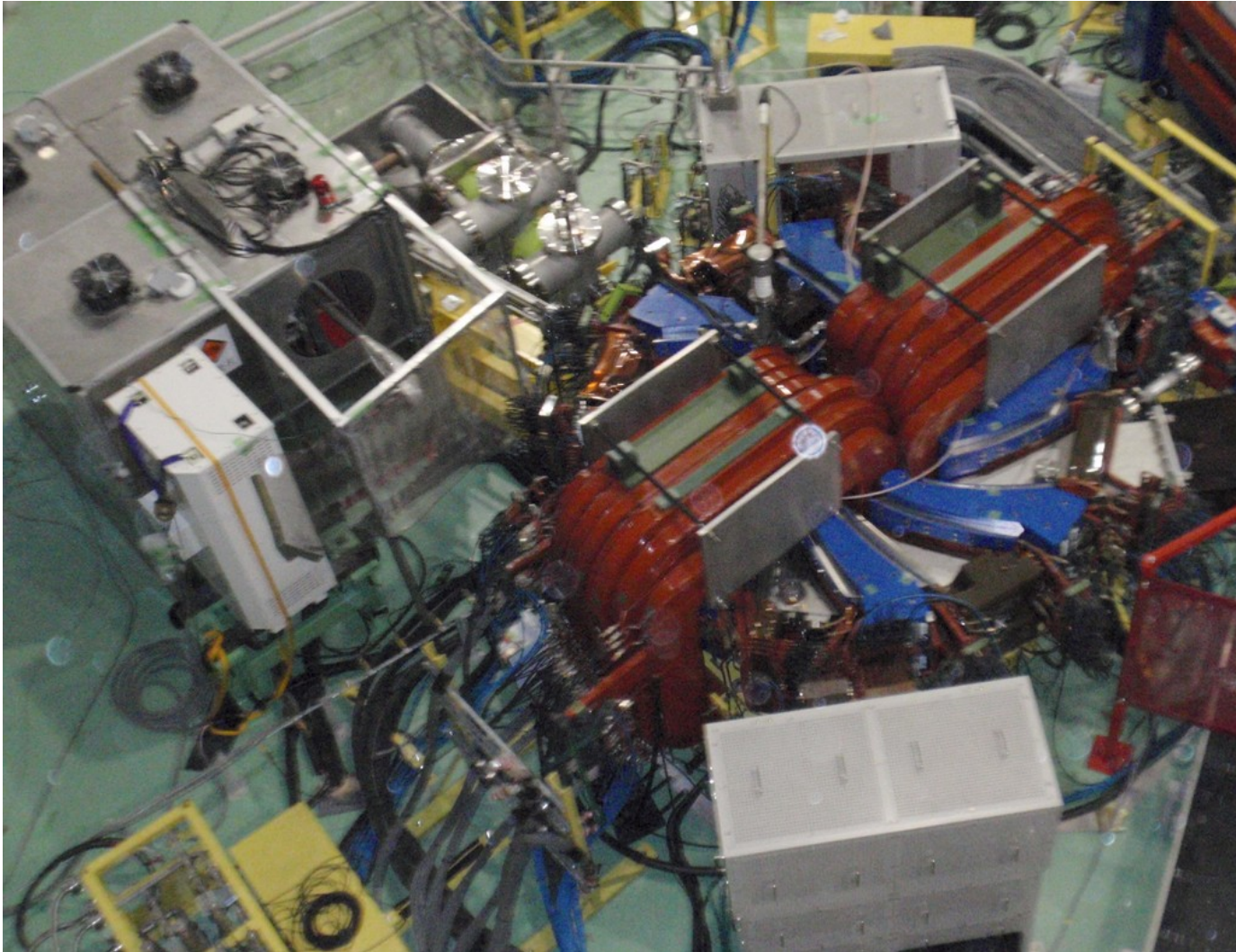
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# Purpose

To verify experimentally

Integer resonance can be crossed,  
when the crossing speed is high enough.

# Injector FFAG in KURRI

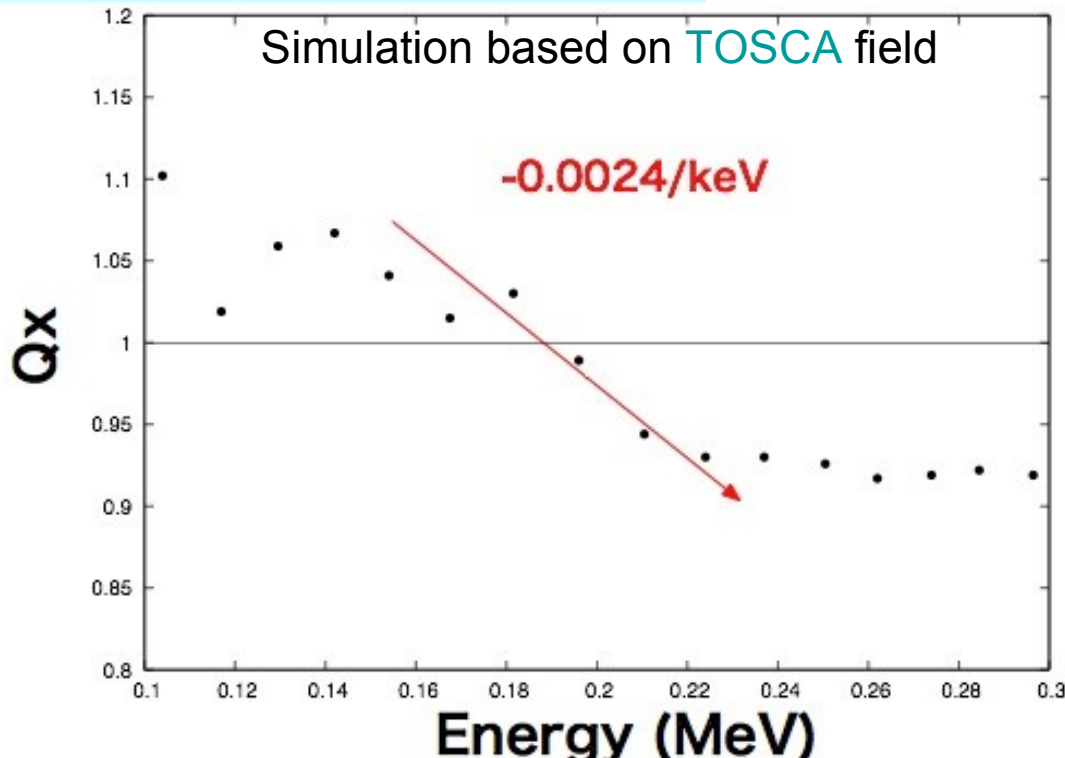




- **Variable  $k$** , by means of 32 trim-coils
  - > Hori. tune is controllable
  - > easy to demonstrate resonance crossing
- **Induction acceleration**
  - > No longitudinal focus,
  - > no energy oscillation,  
which affects the horizontal betatron oscillations

# Tune Variation

Without exciting trim-coils,  
 $Q_x \sim 1$ , but depending on  $E$



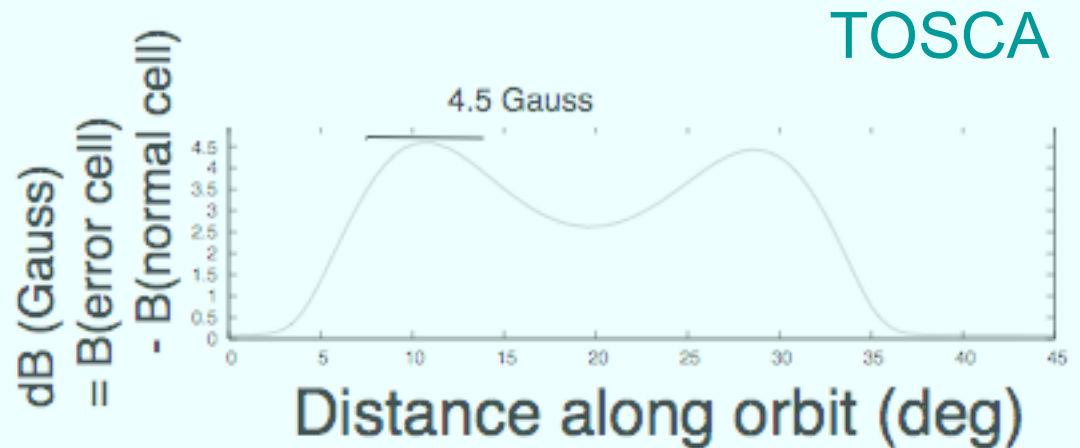
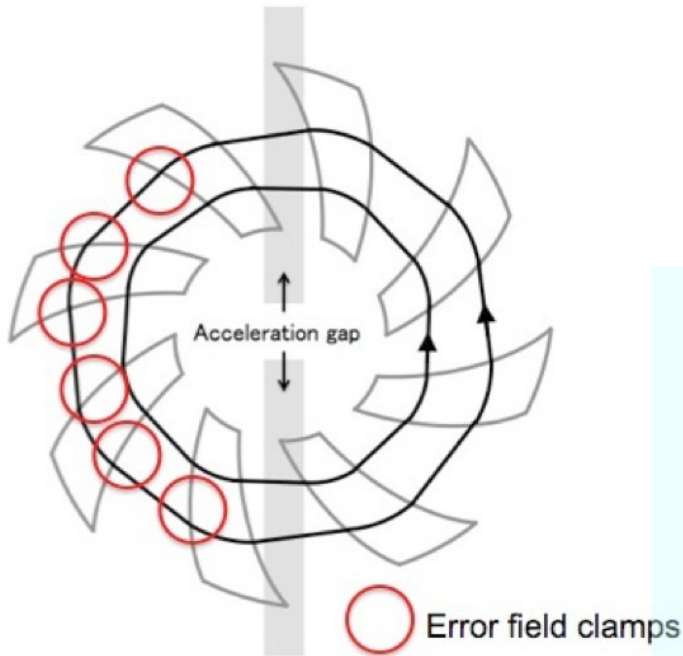
(Crossing speed)

$$= 0.0024/\text{kev} * (\text{Accel. Voltage}) < 0.0084/\text{turn}$$



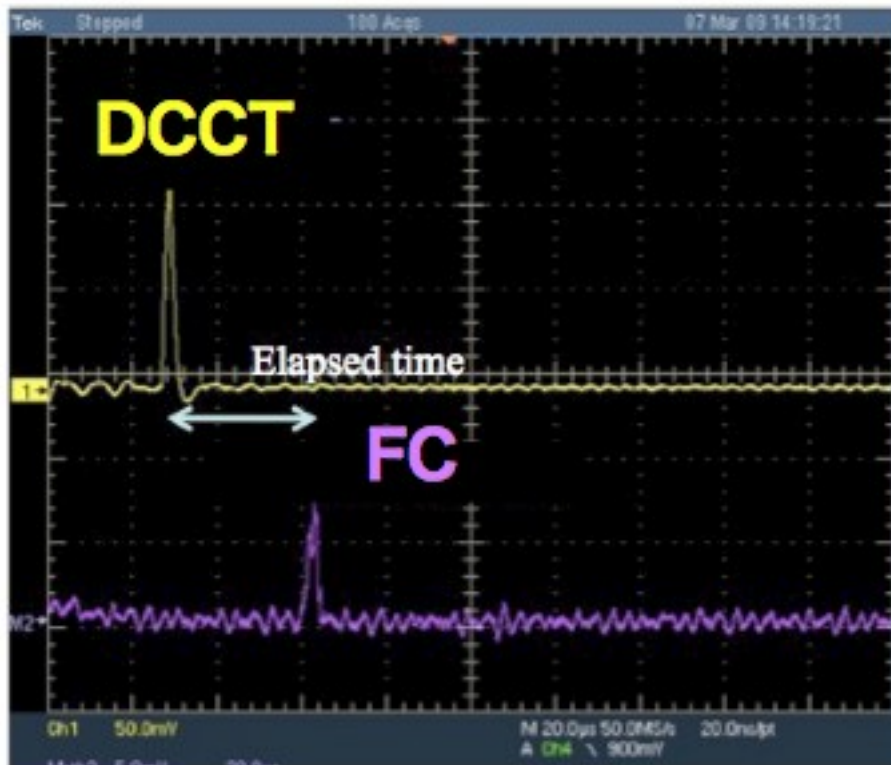
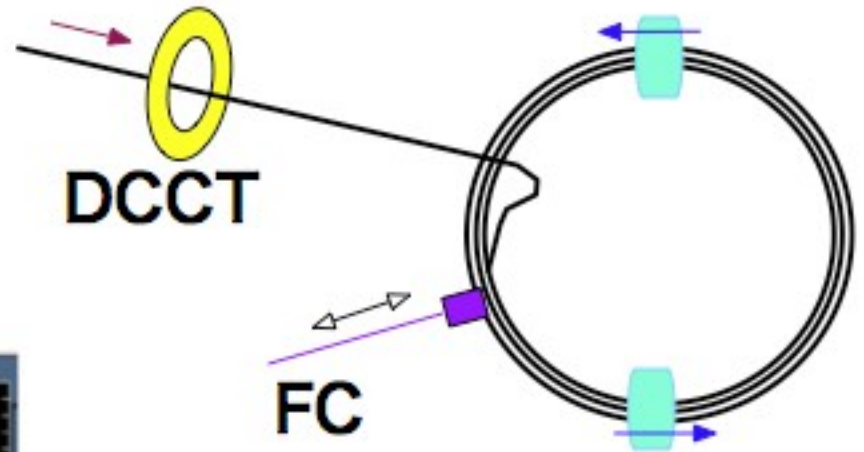
# First Harmonic Force

was applied by 'Error field clamps'  
which has wider gap than the others.



- \* Effects of accelerations at two gaps work in counter-phase when  $Q_x \sim 1$ .

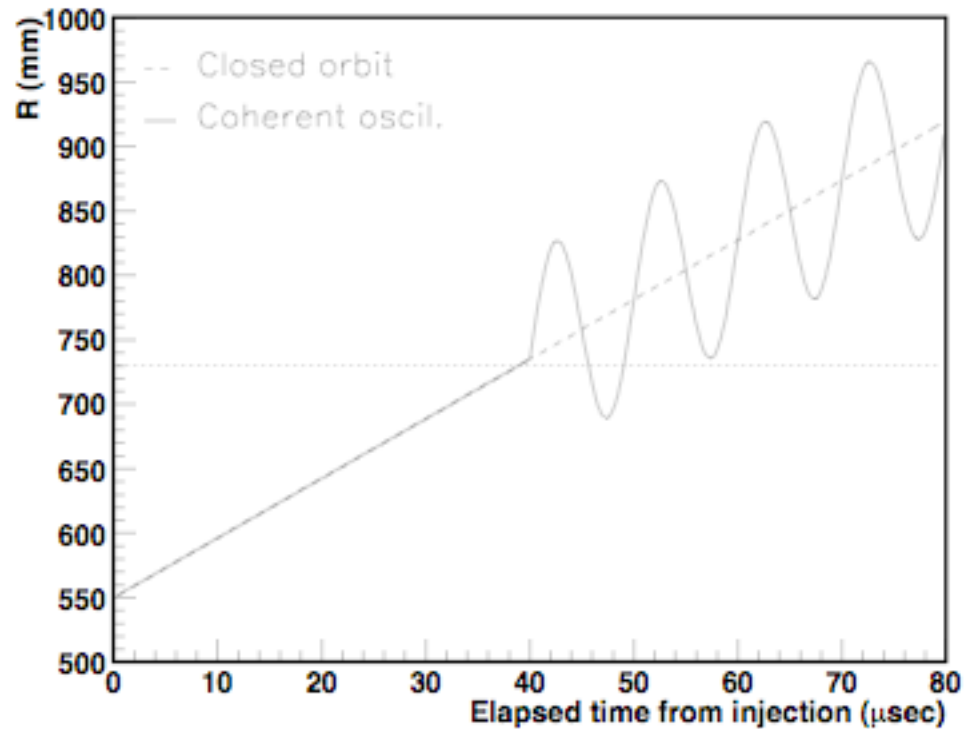
# Observing Coherent Oscillations



Elapsed time was measured  
at different radius

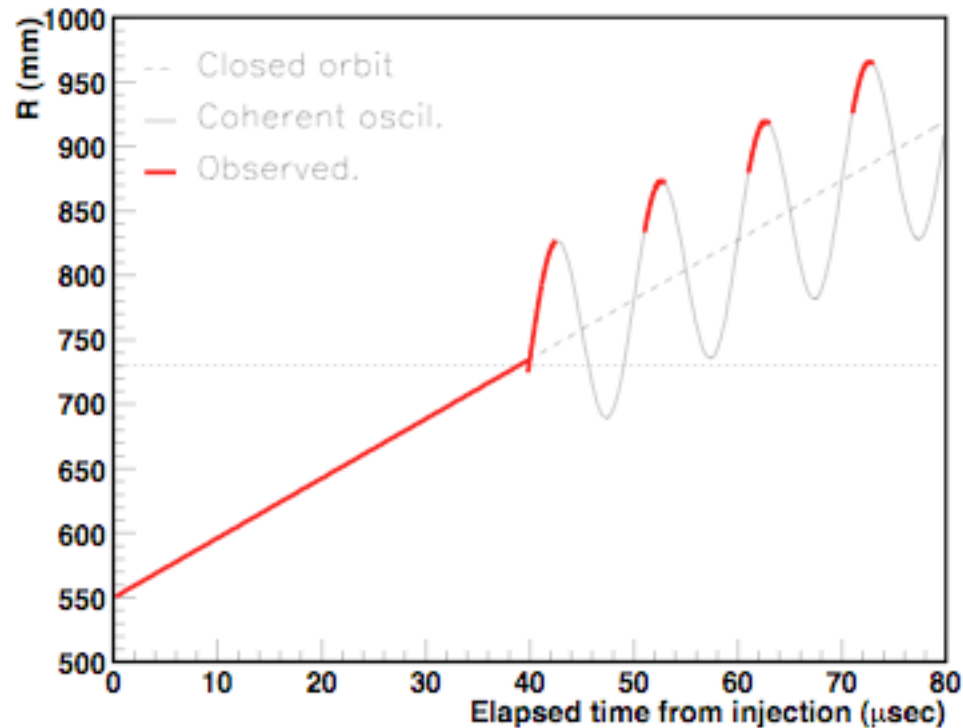
then .....

# What is expected





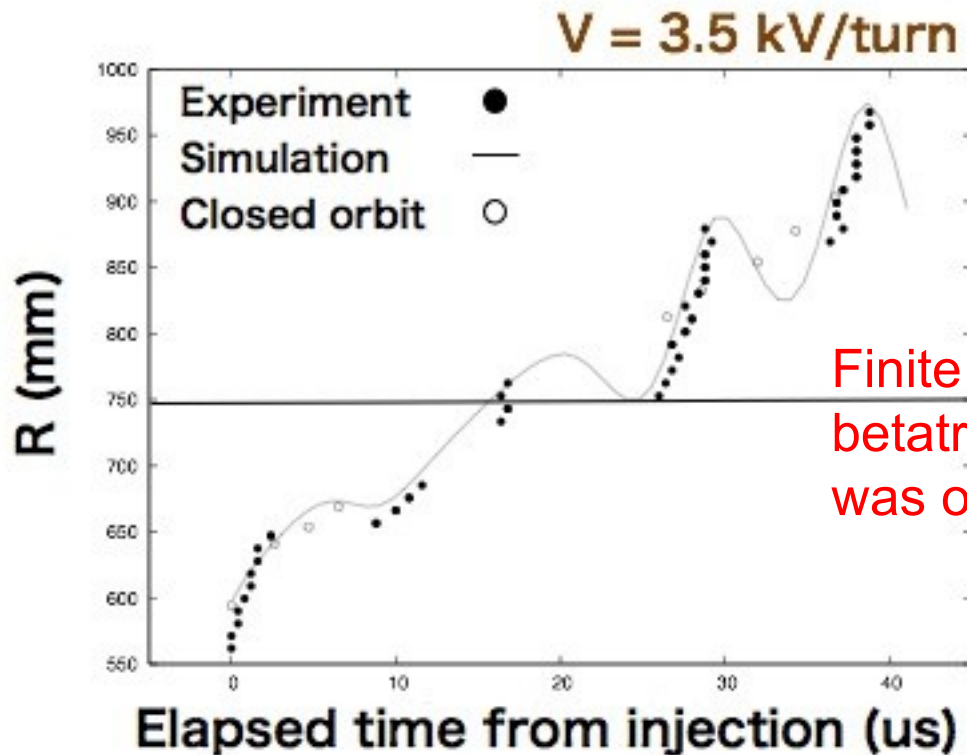
# What is expected



Coherent oscillations will be observed

# Experimental Results

A part of beam survived after resonance crossing !



Finite amplitude of  
betatron oscillations  
was observed.

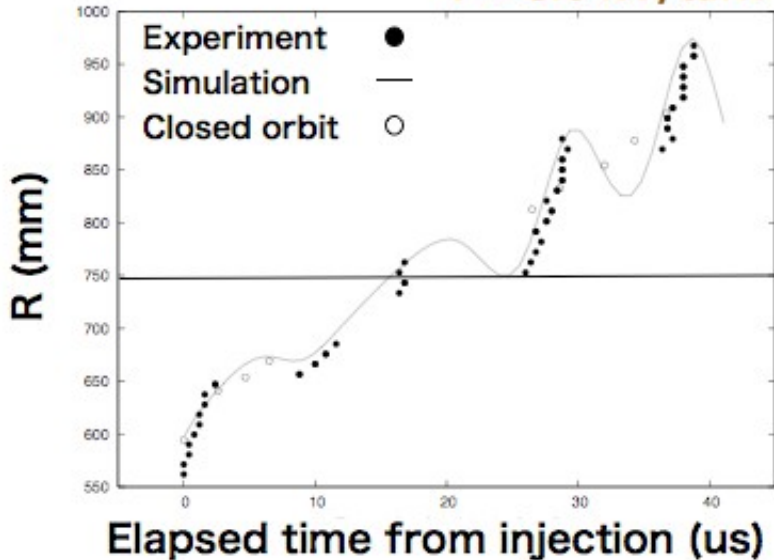


The curve shows  
simulation results with  
fitting Initial condition  
(amplitude and phase).

# Dependence on Crossing speed

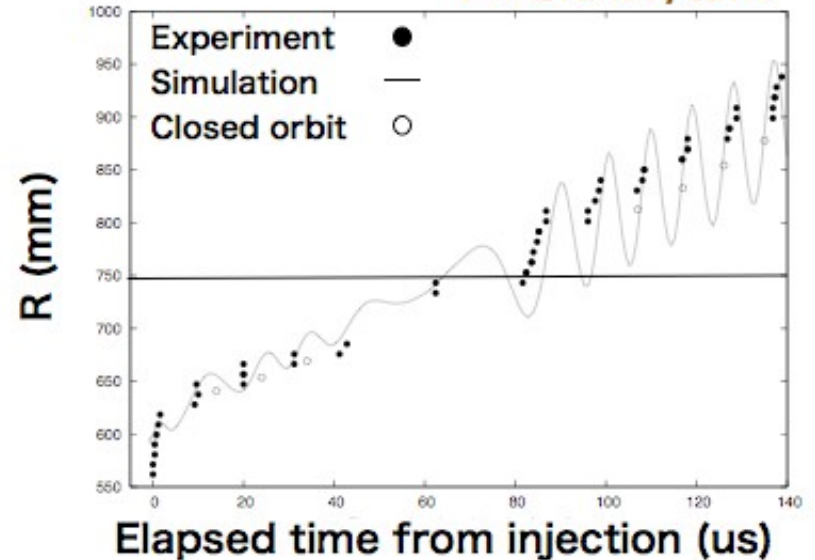
FAST

$V = 3.5$  kV/turn



SLOW

$V = 0.9$  kV/turn



No difference in the final amplitude ?

It's possible because .....

# Final Amplitude depends on Initial Conditions

$$\frac{d^2x}{d\phi^2} + \nu(\phi)^2 x = f \sin(n\phi)$$

General Solution

$$x(\phi) = \underbrace{A(x_0, x'_0; \phi)}_{\text{blue arrow}} + \underbrace{S(\phi)}_{\text{red arrow}}$$

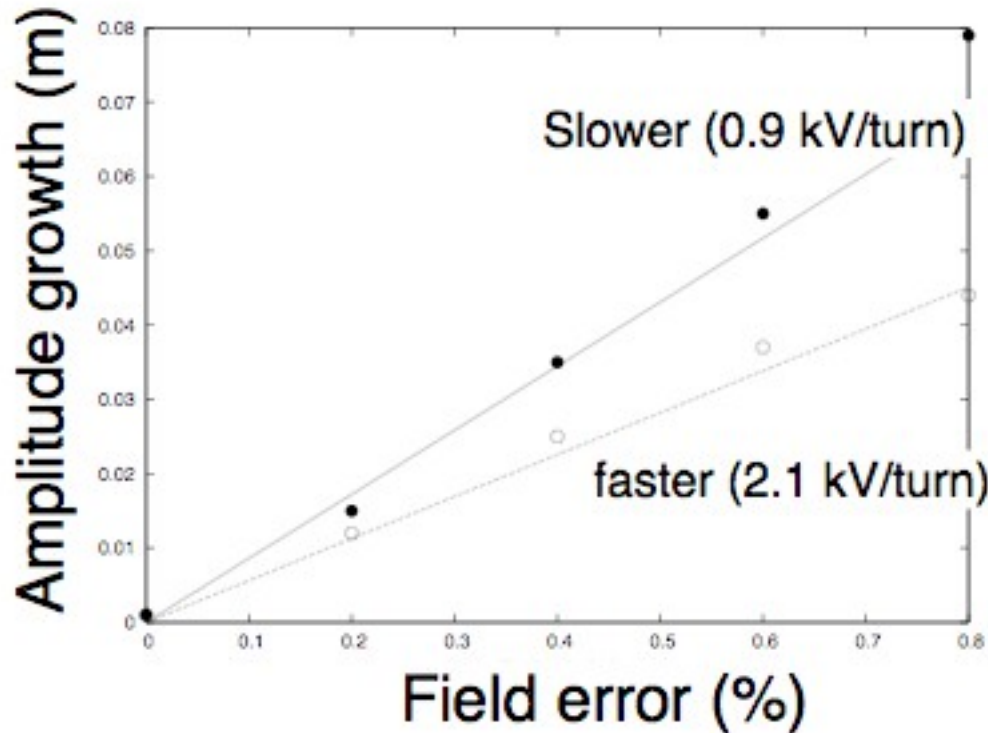
Solutions of homogeneous eq.;  
Oscillating in **freq  $\nu$** ,  
Little resonant blow up,  
Initial amplitude, phase ..  
If this part is not negligible

A solution of inhomogeneous eq.;  
Oscillating in **freq  $\nu$** ,  
Big resonant blow up

Phase difference ?

- . In phase --> Maximum amplitude growth  $|S|+|A|$
- . Counter phase --> Minimum growth  $|S|-|A|$

# Maximum Amplitude Growth (Simulation)



Worst cases of simulations  
with different initial phase

- simul. (single kick approx. of driving force)
- model  $(\text{field err}) / \sqrt{\text{crossing speed}}$

# Summary

- Fast crossing of  $Q_x=1$  resonance has been examined in Injector FFAG of KURRI.
- The beam survived after the crossing, because of the fast tune variation (and large horizontal acceptance).
- The measured oscillation was reproduced by Runge-Kutta simulations.
- Simulated amplitude growth was proportional to  $1/\sqrt{dQ/dt}$