Scaling FFAG lattices for muon acceleration

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Motivations

Use the large **transverse acceptance** of scaling FFAG lattices

while using constant RF frequency acceleration to reach **high accelerating gradient.**

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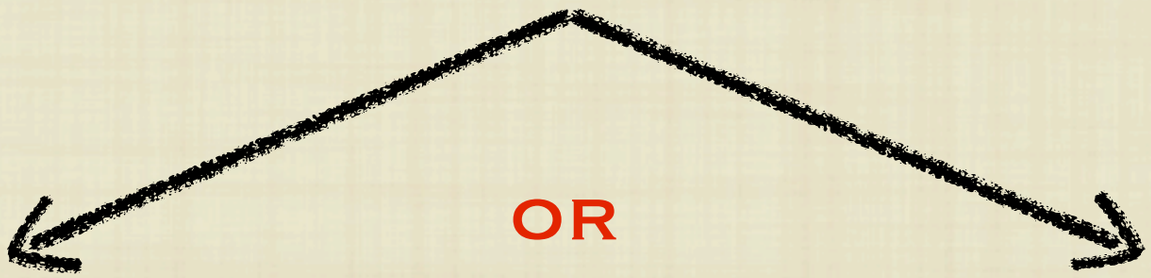
Possible with either:



or

Harmonic number jump acceleration

Stationary bucket acceleration!

Outline

I. Lattice for harmonic number jump acceleration.
   1- Reminder about harmonic number jump acceleration.
   2- Simultaneous μ- and μ+ acceleration: need for **double beam lattice.**
   3- Issue of the excursion: need **dispersion suppressors**!
   4- Lattice example and 4D tracking results.

II. Lattice for stationary bucket acceleration.
   1- Reminder about stationary bucket acceleration.
   2- Lattice details and tracking results.

Part I

Scaling FFAG lattices for Harmonic number jump acceleration

HARMONIC NUMBER JUMP ACCELERATION

To jump one harmonic every turn: $T_{i+1} - T_i = \frac{1}{f_{RF}}$

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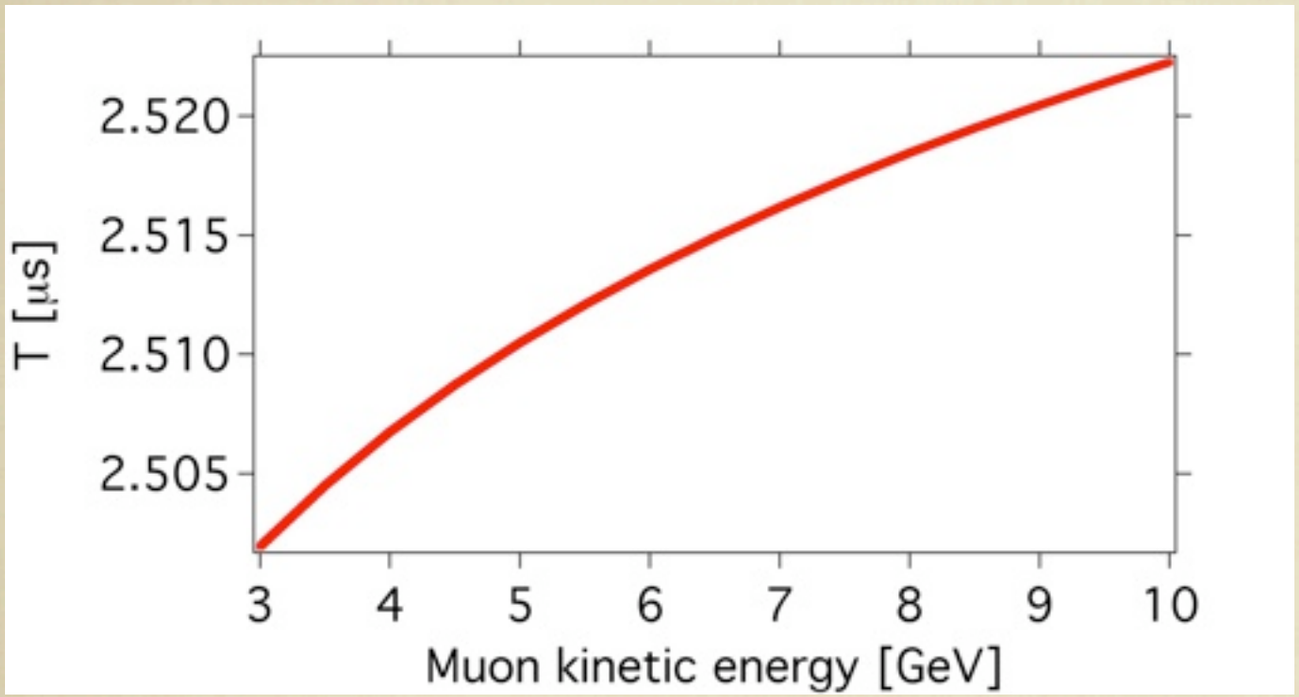


Figure 1 - Revolution time as a function of particle energy in the case of a 3 to 10 GeV scaling FFAG ring, with k = 145 and average radius = 120 m.

→ **Energy gain per turn must follow:** $$\Delta E_i = \frac{1}{f_{RF} \cdot \left[\frac{\Delta T}{\Delta E}\right]_{E_i}}$$

HNJ with cavities distributed around the ring

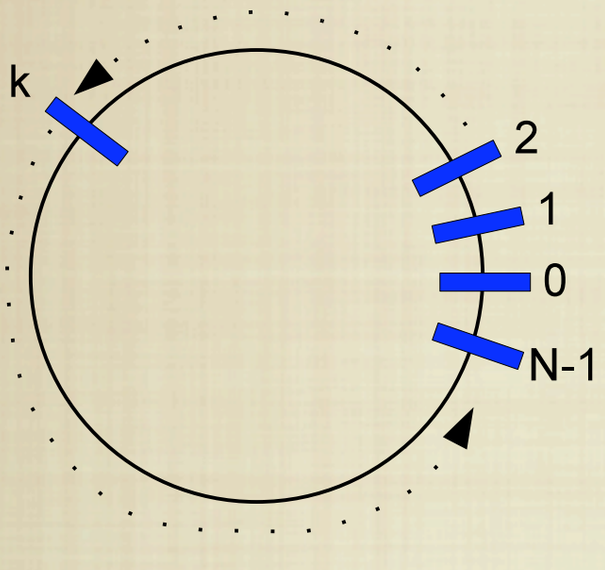


Figure 2 - N cavities homogeneously distributed around the ring.

Assuming that the initial number of harmonic h_0 is large we get(*):

$$f_k \approx f_0(1 - \frac{1}{h_0} \cdot \frac{k}{N})$$

Every cavity working at a constant frequency f_k but the frequency has to be tuned to a slightly different value!

μ+ and μ- beams cannot be accelerated simultaneously if they circulated in opposite directions...

(*)look at the proceedings of PAC'09 for all details.

Need for a double beam lattice

A solution to circulate a particle and its antiparticle **in the same direction** in a scaling FFAG ring is to use a FD-symmetric lattice:

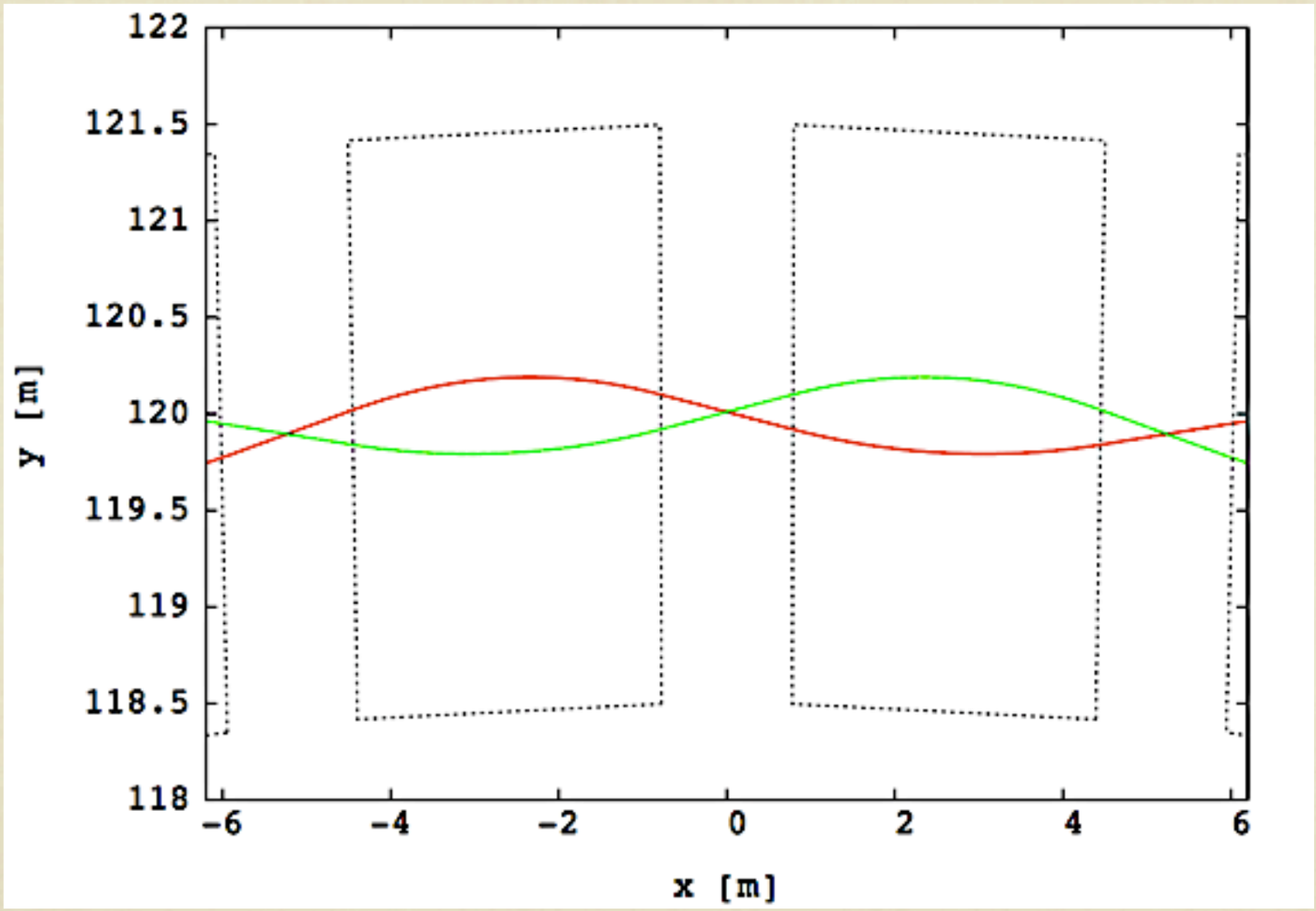


Figure 3 - Double beam FFAG lattice (k = 145). Closed orbits of μ+ and μ− circulating in the same direction. Results are obtained from Runge-Kutta stepwise tracking in hard-edge field.

NEED FOR DISPERSION SUPPRESSOR INSERTIONS!

HARMONIC JUMP CONDITION: $$T_{i+1} - T_i = \frac{1}{f_{RF}}$$

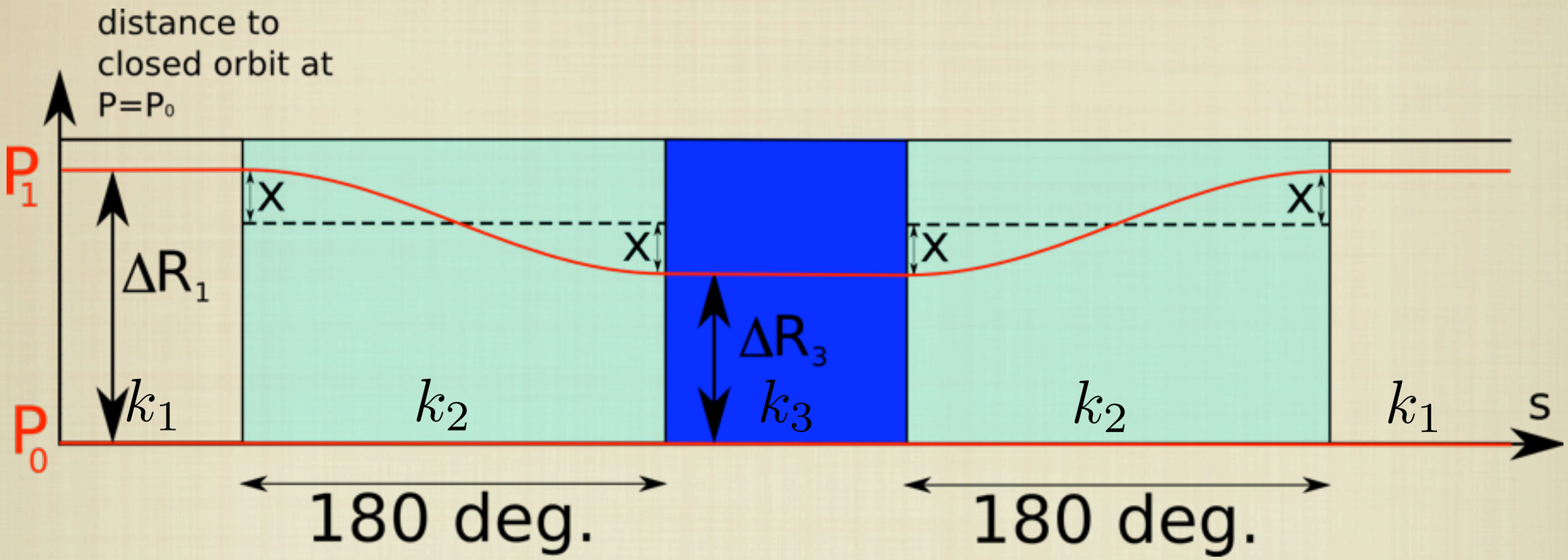
IN THE SAME TIME: $$\frac{\Delta C_i}{\beta c} = T_{i+1} - T_i$$

IN CASE OF HIGHLY RELATIVISTIC PARTICLES: $\Delta R_i \approx \frac{c}{2\pi f_{RF}} = \frac{\lambda_{RF}}{2\pi}$

$$average\ excursion = \lambda_{RF} \cdot \frac{N_{turns}}{2\pi} \longrightarrow$$

NEED FOR EXCURSION REDUCED AREAS!

Dispersion suppressor with FFAG magnets



$$with\ \frac{2}{k_2+1} = \frac{1}{k_1+1} + \frac{1}{k_3+1}$$

3 to 10 GeV muon double beam FFAG + **excursion reduced areas**

B_{max}	3 T
Horizontal tune	23.52
Vertical tune	7.12

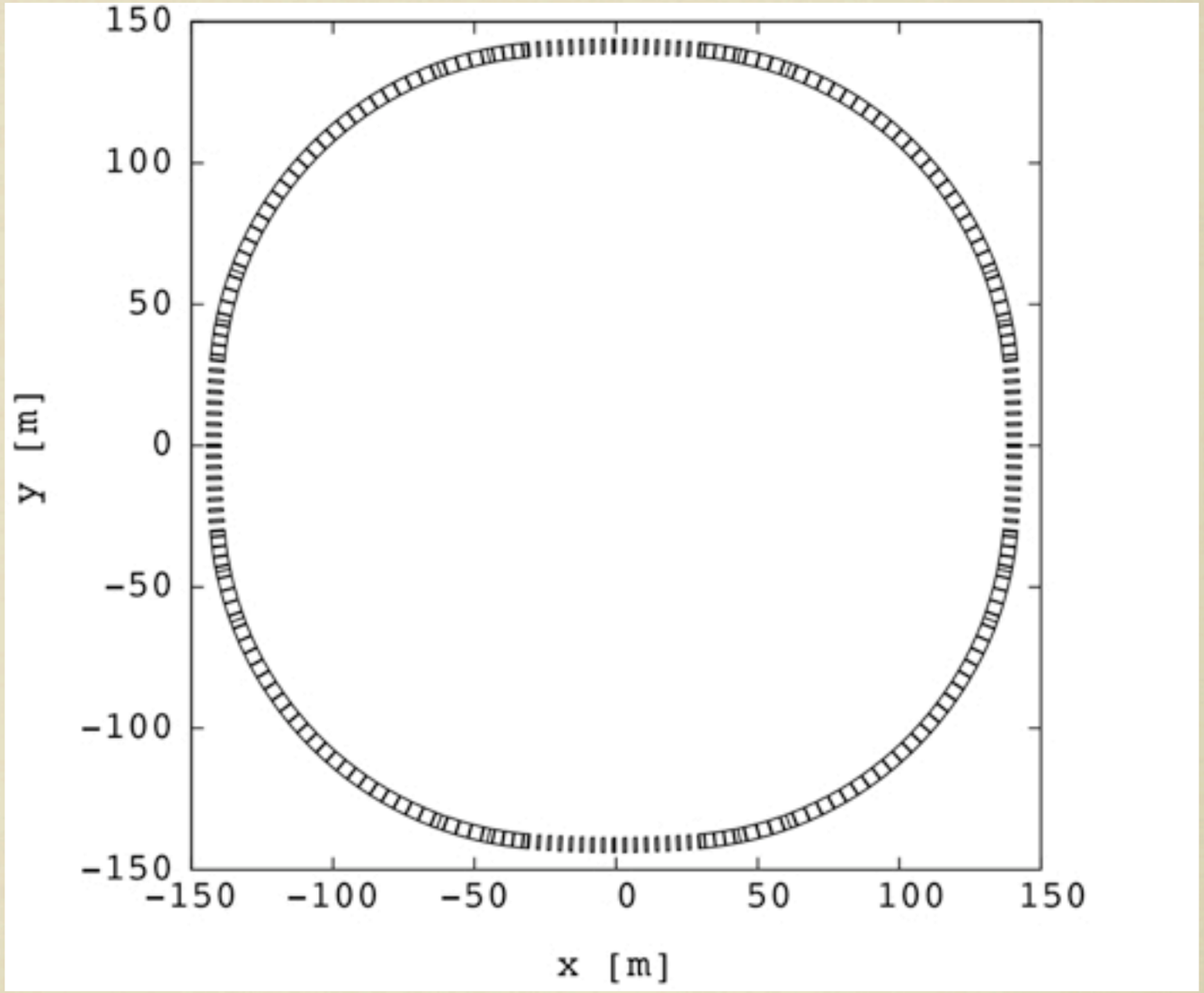


Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.

3 to 10 GeV muon double beam FFAG + **excursion reduced areas**

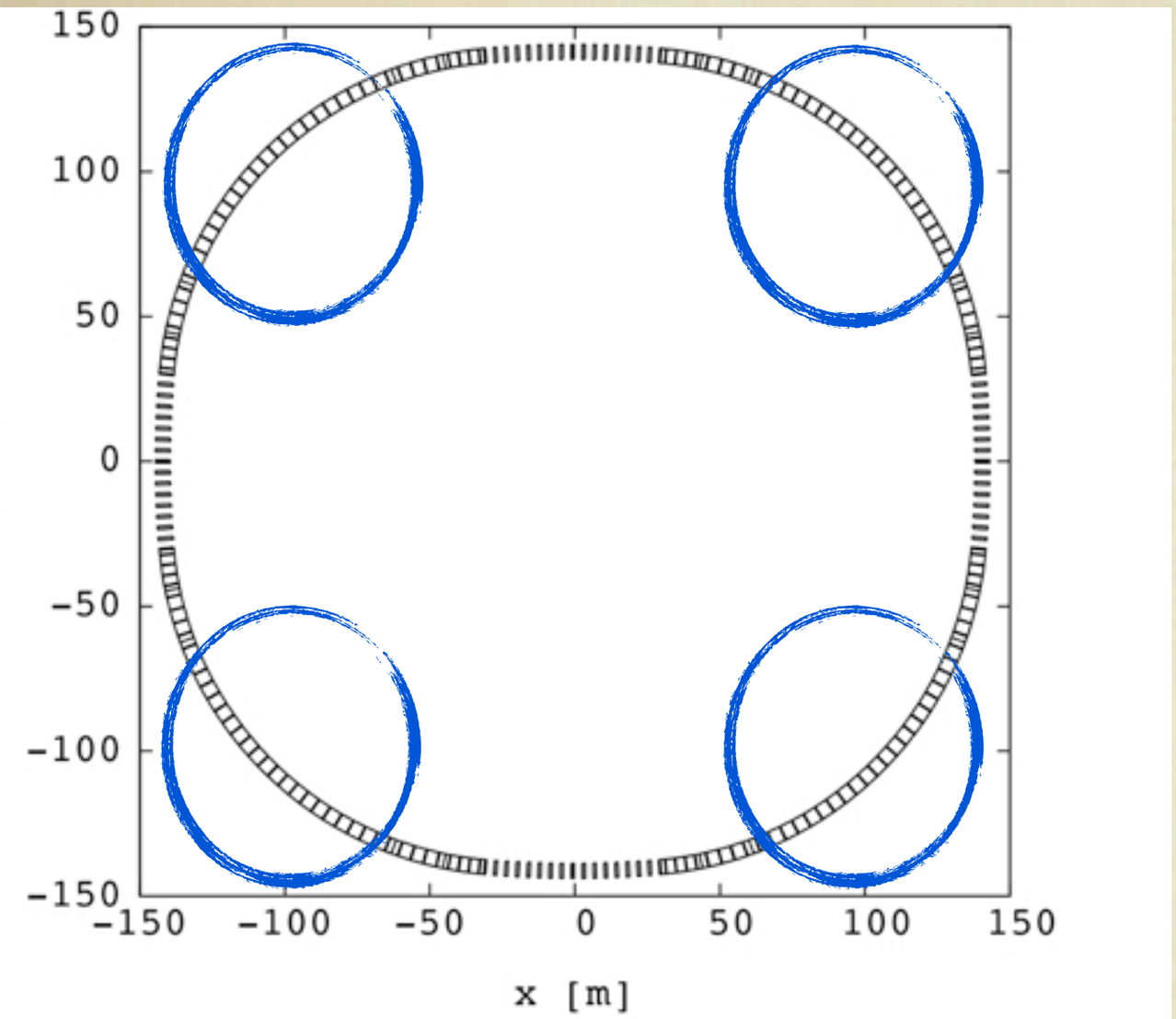


Table 2 - Ring main cells parameters

Mean radius	120 m
Number of cells	4×11
Cell opening angle	4.5 deg.
Field index k	145
B_{max}	3 T
Horiz. phase adv. per cell	82.1 deg.
Vert. phase adv. per cell	31.8 deg.

Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.

3 to 10 GeV muon double beam FFAG + **excursion reduced areas**

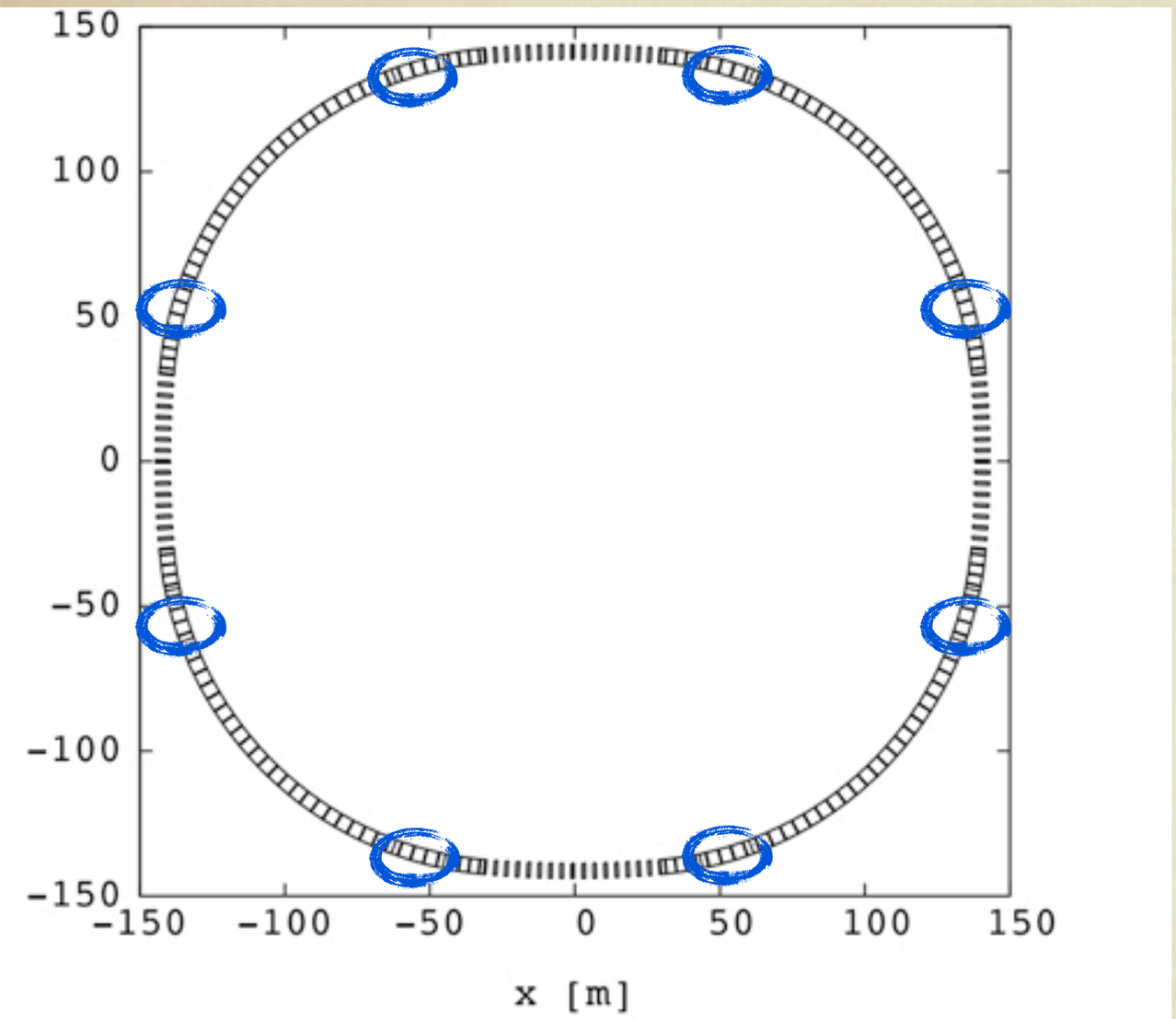


Table 2 - 1st dispersion suppressor

Mean radius	120 m
Number of cells	4×4
Cell opening angle	4.3 deg.
Field index k	183.6
B_{max}	3 T
Horiz. phase adv. per cell	90 deg.
Vert. phase adv. per cell	27.6 deg.

Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.

3 to 10 GeV muon double beam FFAG + **excursion reduced areas**

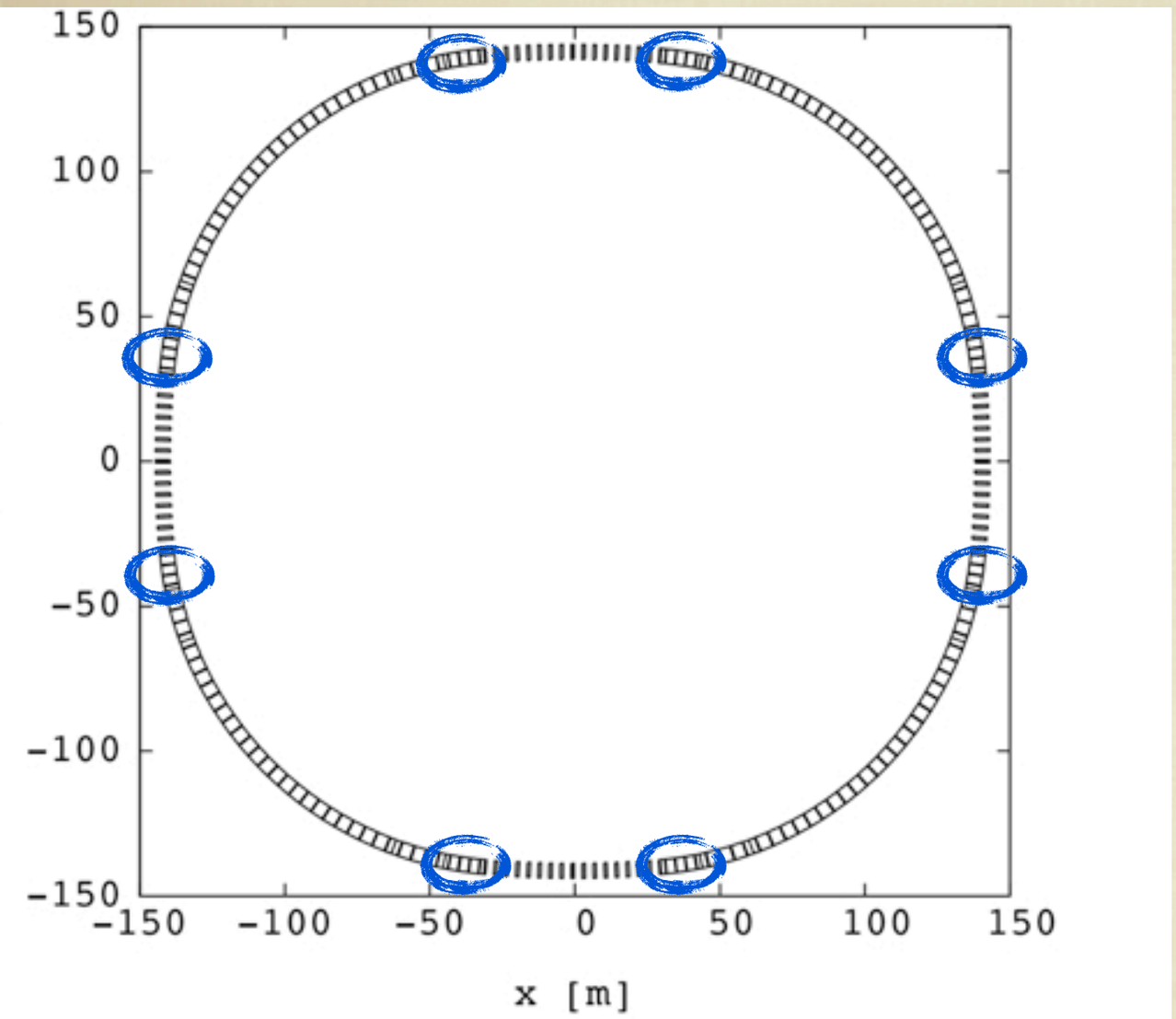


Table 2 - 1st dispersion suppressor

Mean radius	120 m
Number of cells	4×4
Cell opening angle	3.34 deg.
Field index k	307.7
B_{max}	3 T
Horiz. phase adv. per cell	90 deg.
Vert. phase adv. per cell	20.4 deg.

Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.

3 to 10 GeV muon double beam FFAG + **excursion reduced areas**

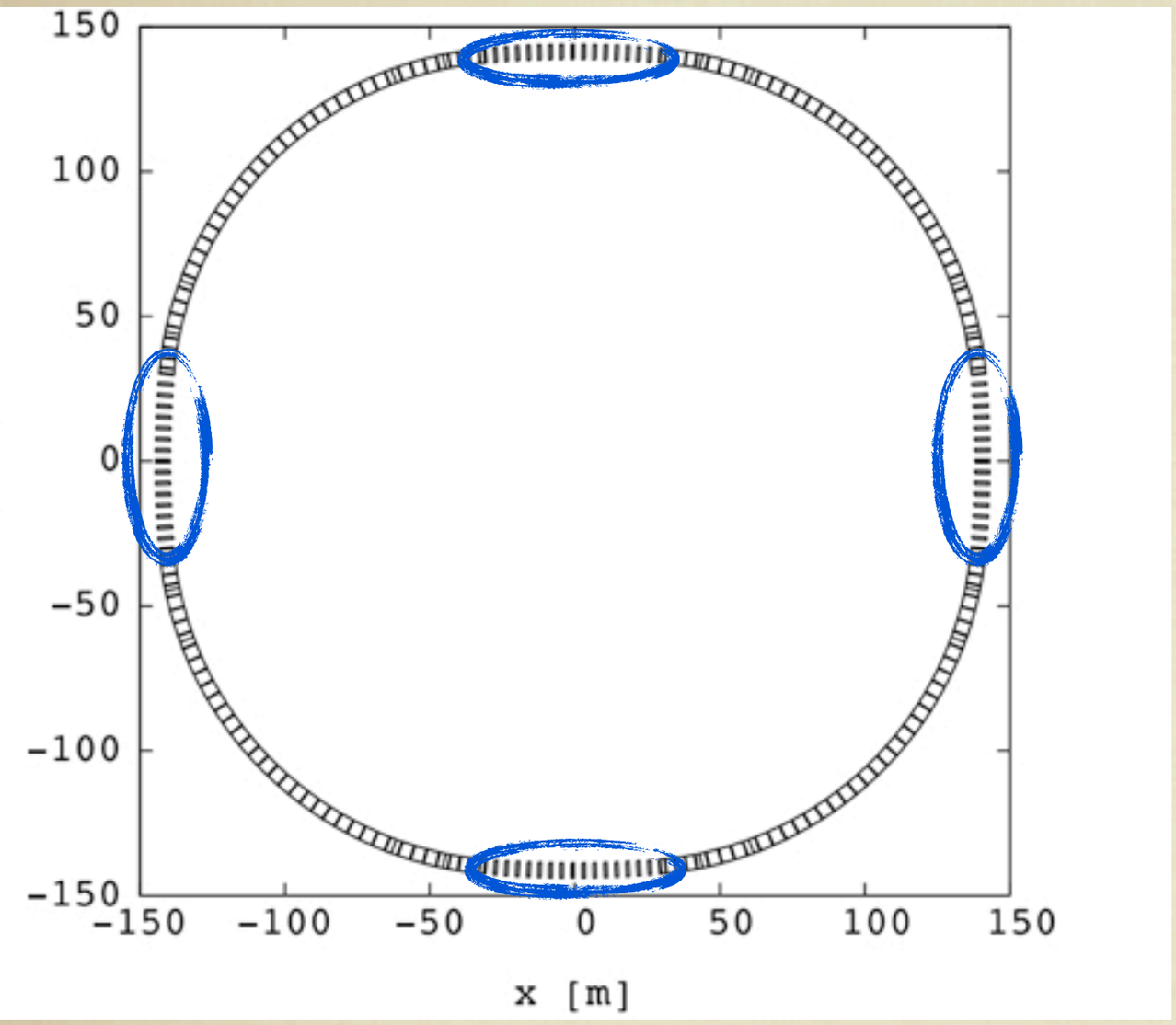


Figure 4 - Schematic view of a 3 to 10 GeV double beam muon FFAG ring with 4 excursion reduced insertions.

Table 2 - 1st dispersion suppressor

Mean radius	350 m
Number of cells	4×8
Cell opening angle	1.2425 deg.
Field index k	1168.6
B_{max}	3 T
Horiz. phase adv. per cell	64.6 deg.
Vert. phase adv. per cell	12.6 deg.

3 to 10 GeV muon double beam FFAG + **excursion reduced areas**

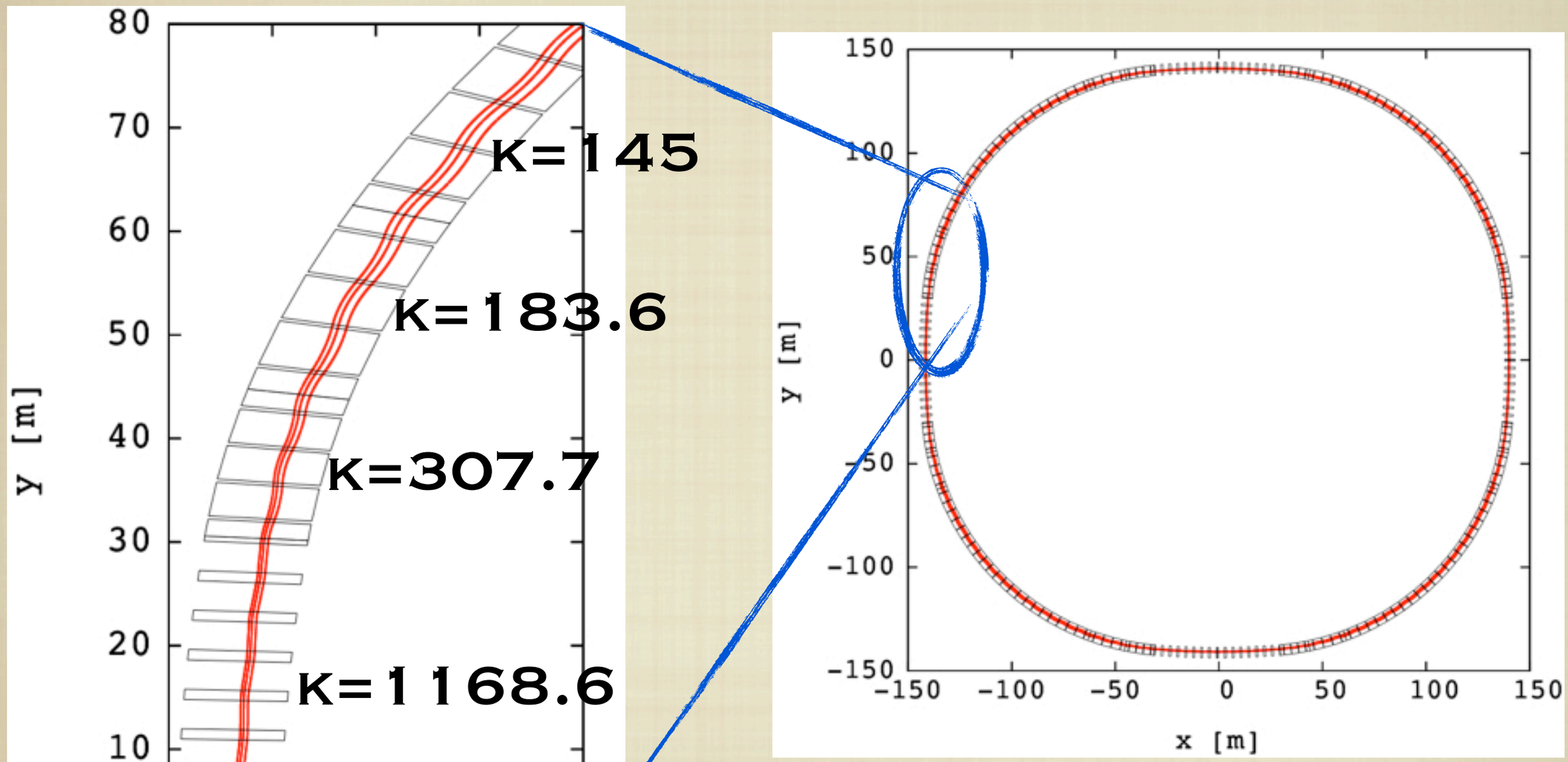


Figure 5 - μ- closed orbits at 3, 6 and 10 GeV.

3 to 10 GeV muon double beam FFAG + **excursion reduced areas**

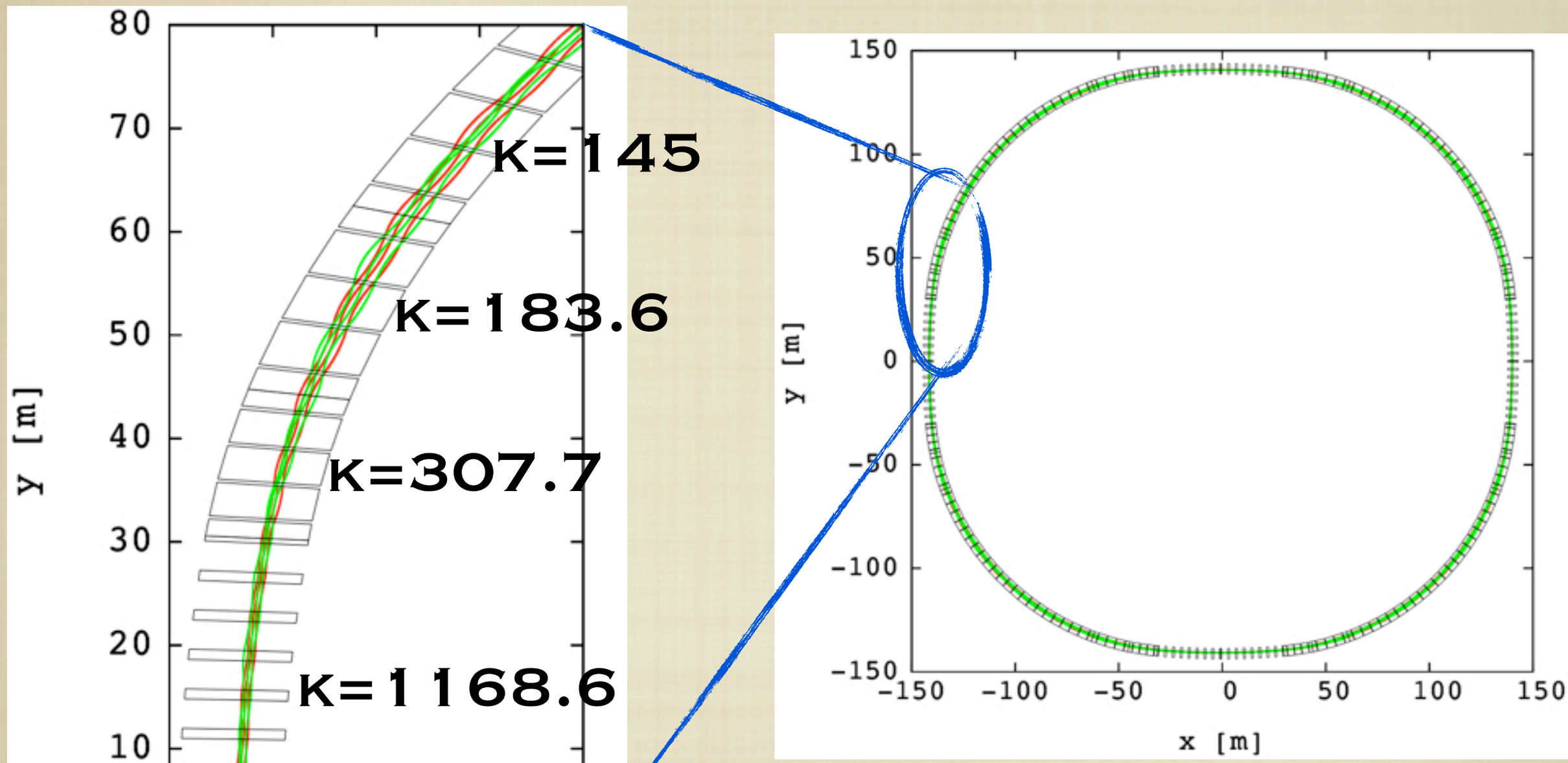
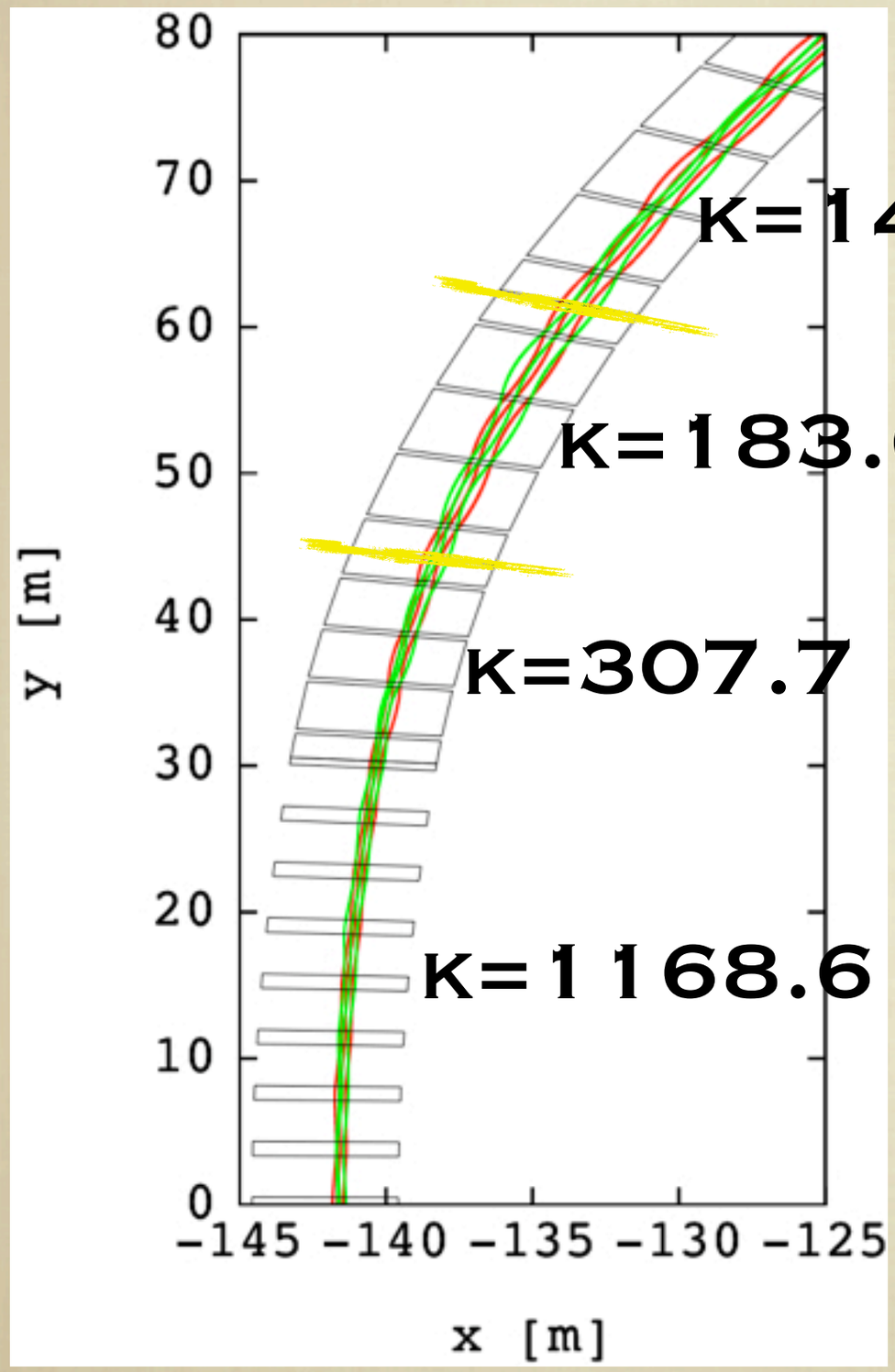
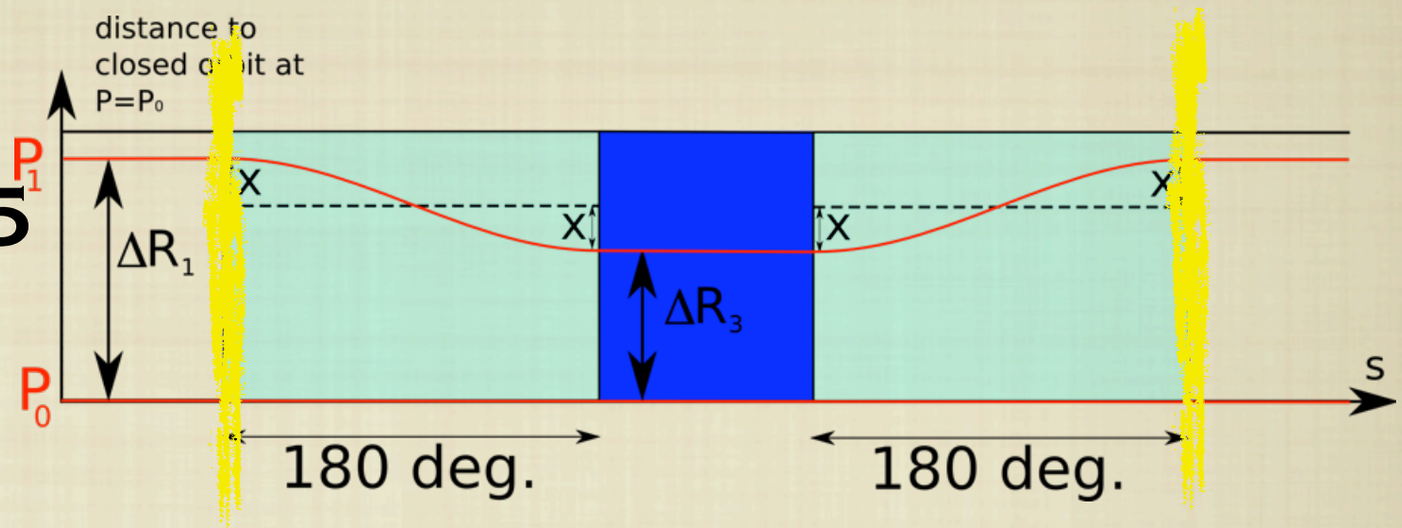


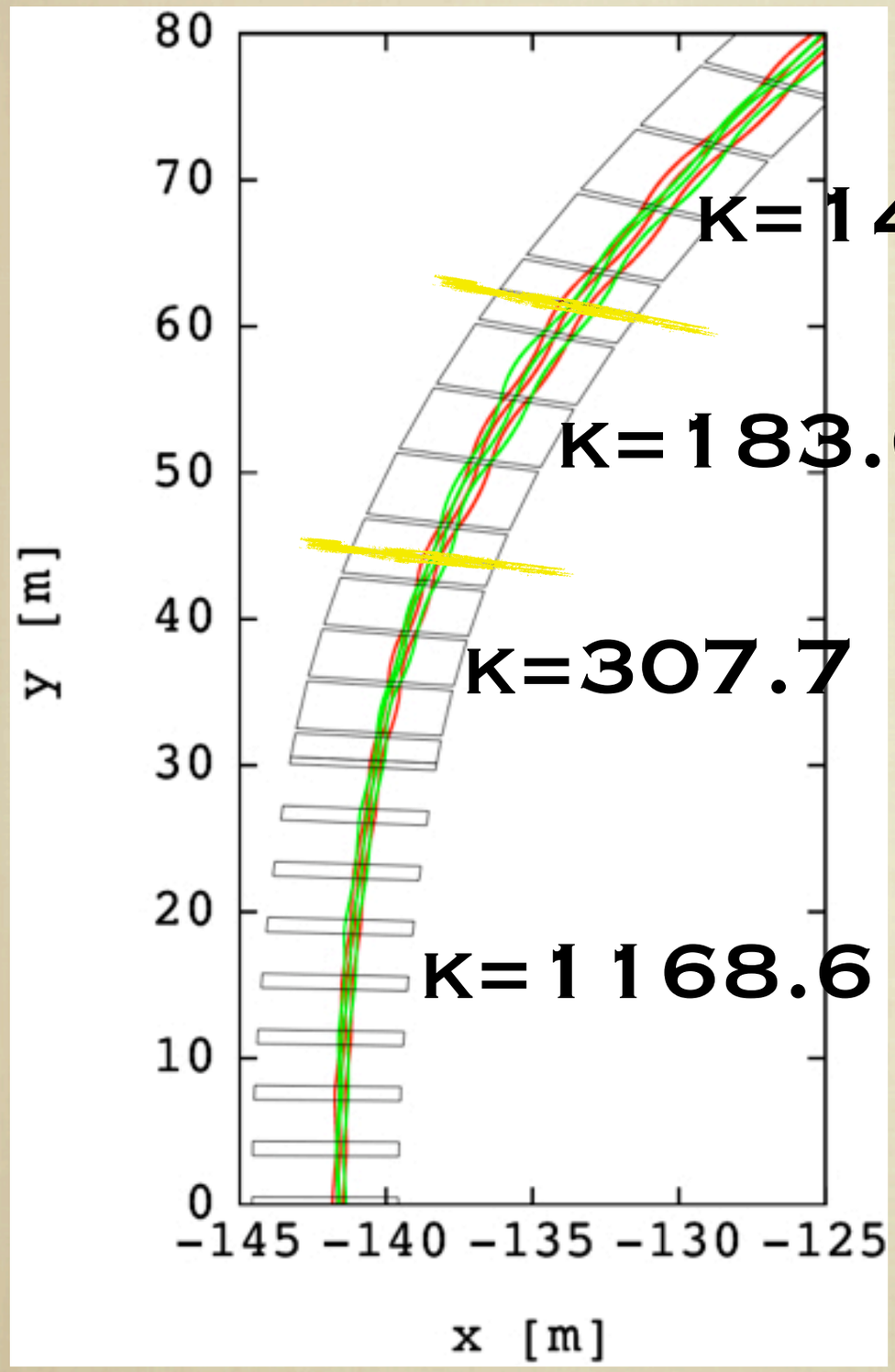
Figure 6 - μ- (red) and μ+ (green) closed orbits at 3, 6 and 10 GeV.

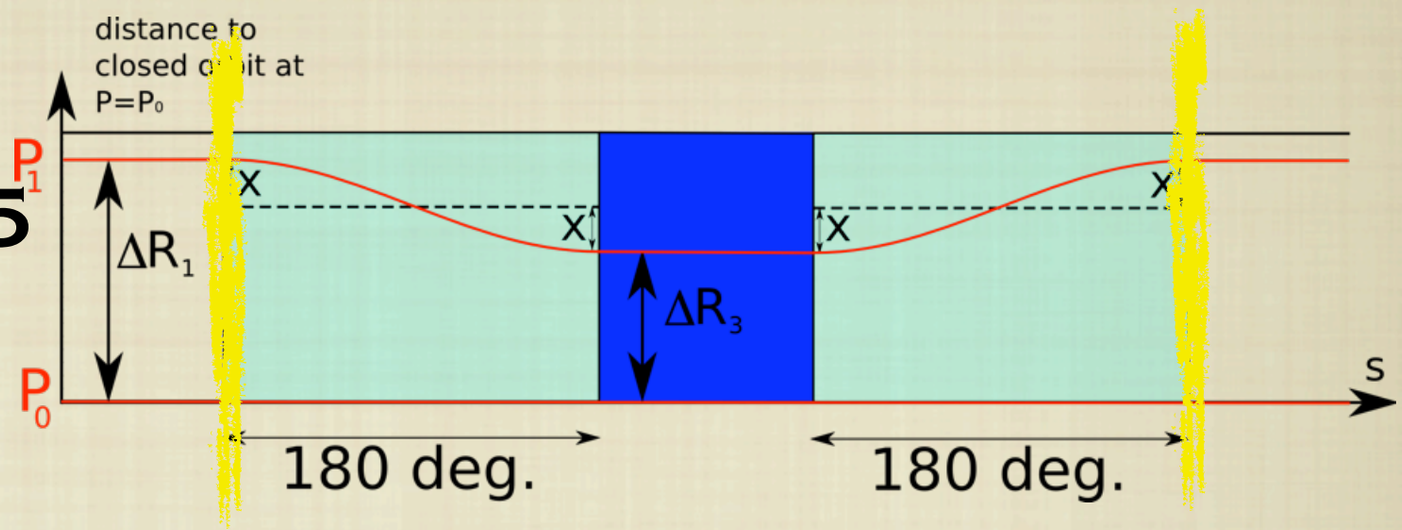
Asymmetry between μ- and μ+ behavior

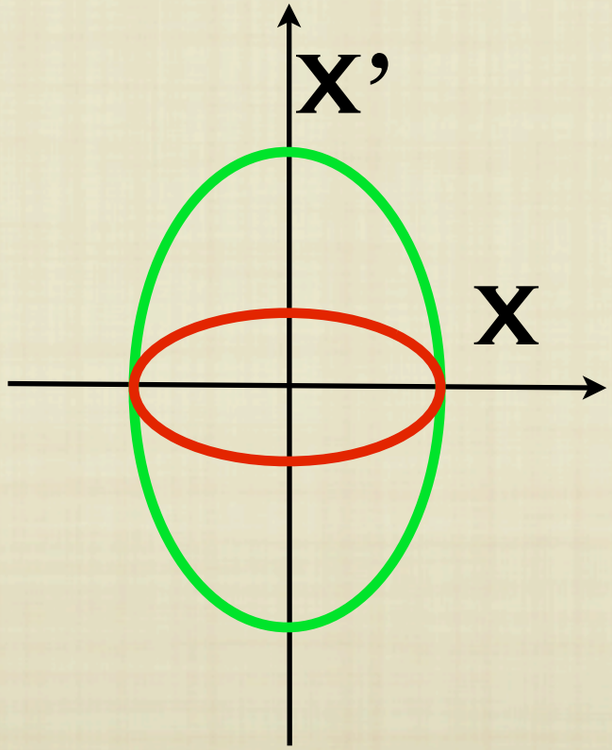




Asymmetry between μ- and μ+ behavior







In dispersion suppressor cells: **same amplitude** but different **beta function** at the matching point!

μ-: Matching done at the center of F magnets

Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

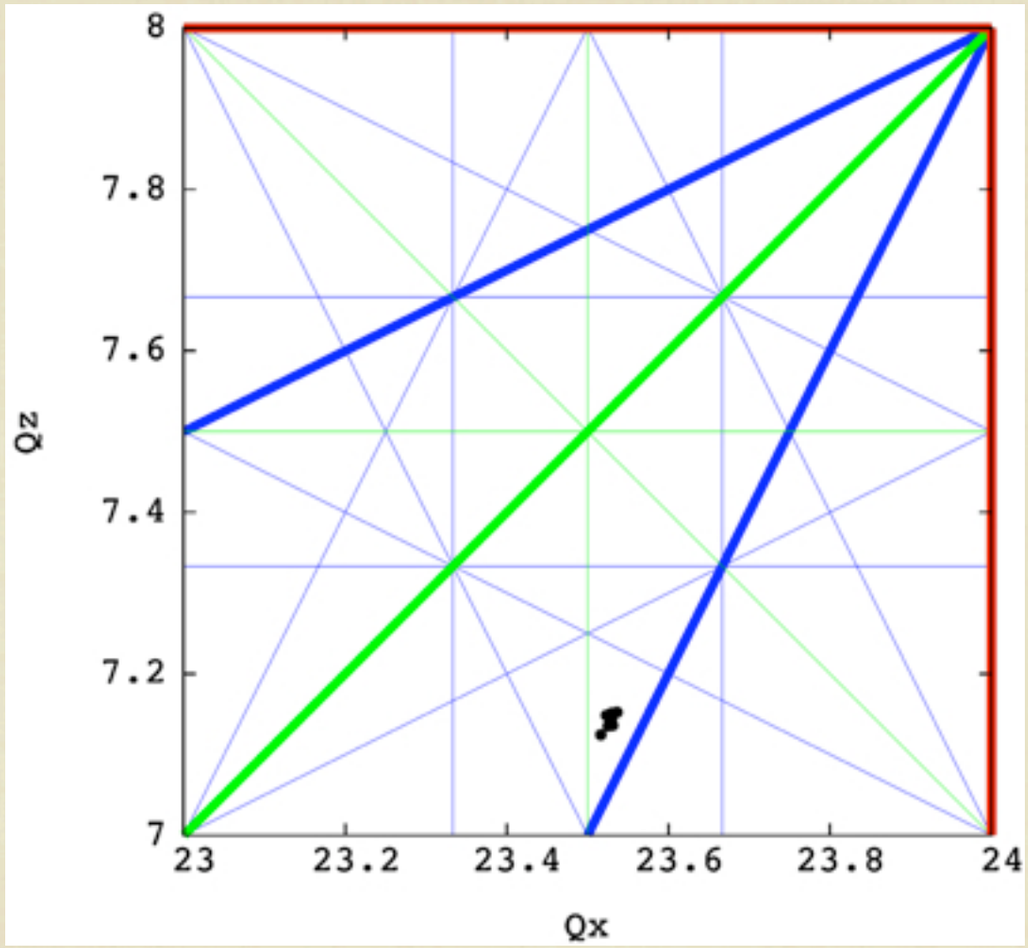


Figure 7 - μ- Tune variation between 3 and 10 GeV in the lattice with insertions (from stepwise tracking in a soft edge field model).

μ-: matching done at the center of F magnets

Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

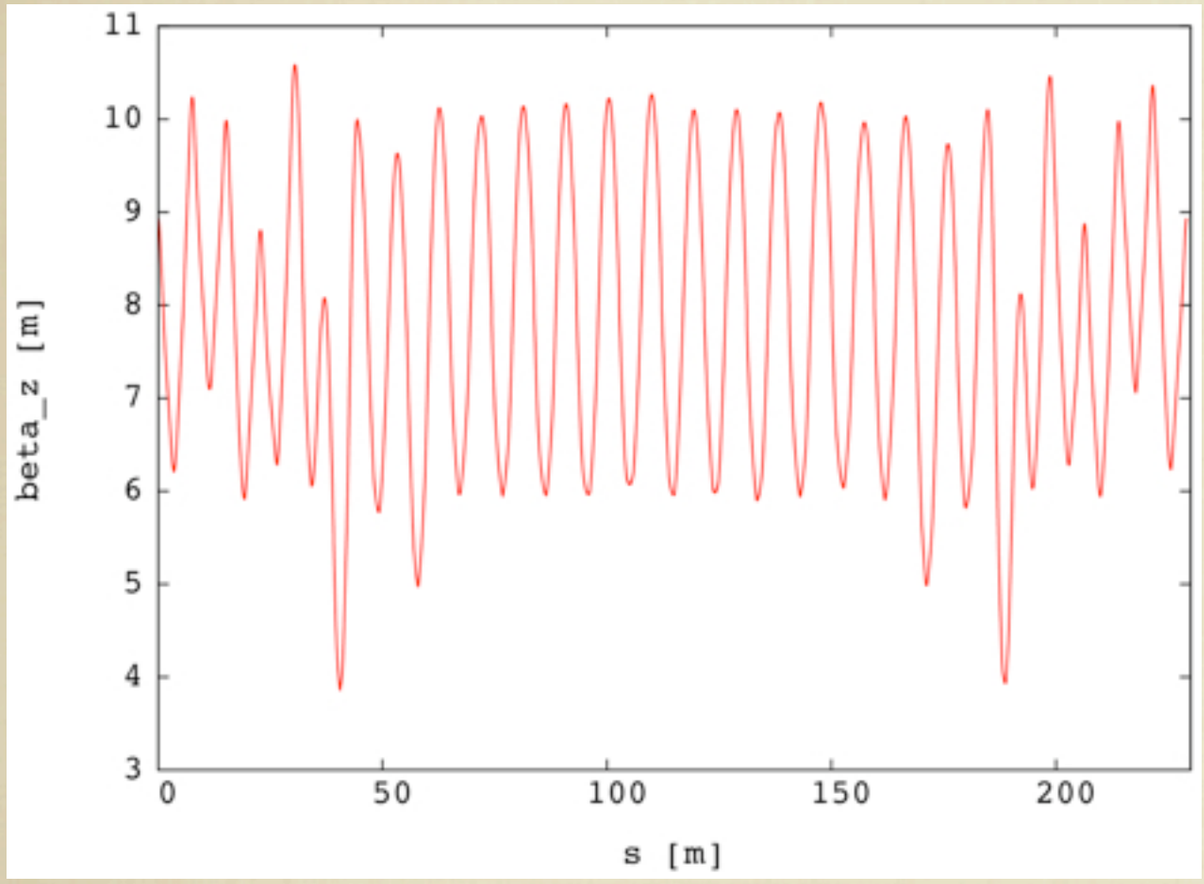


Figure 8 - μ- : Horizontal beta function at 6 GeV (quarter of a turn).

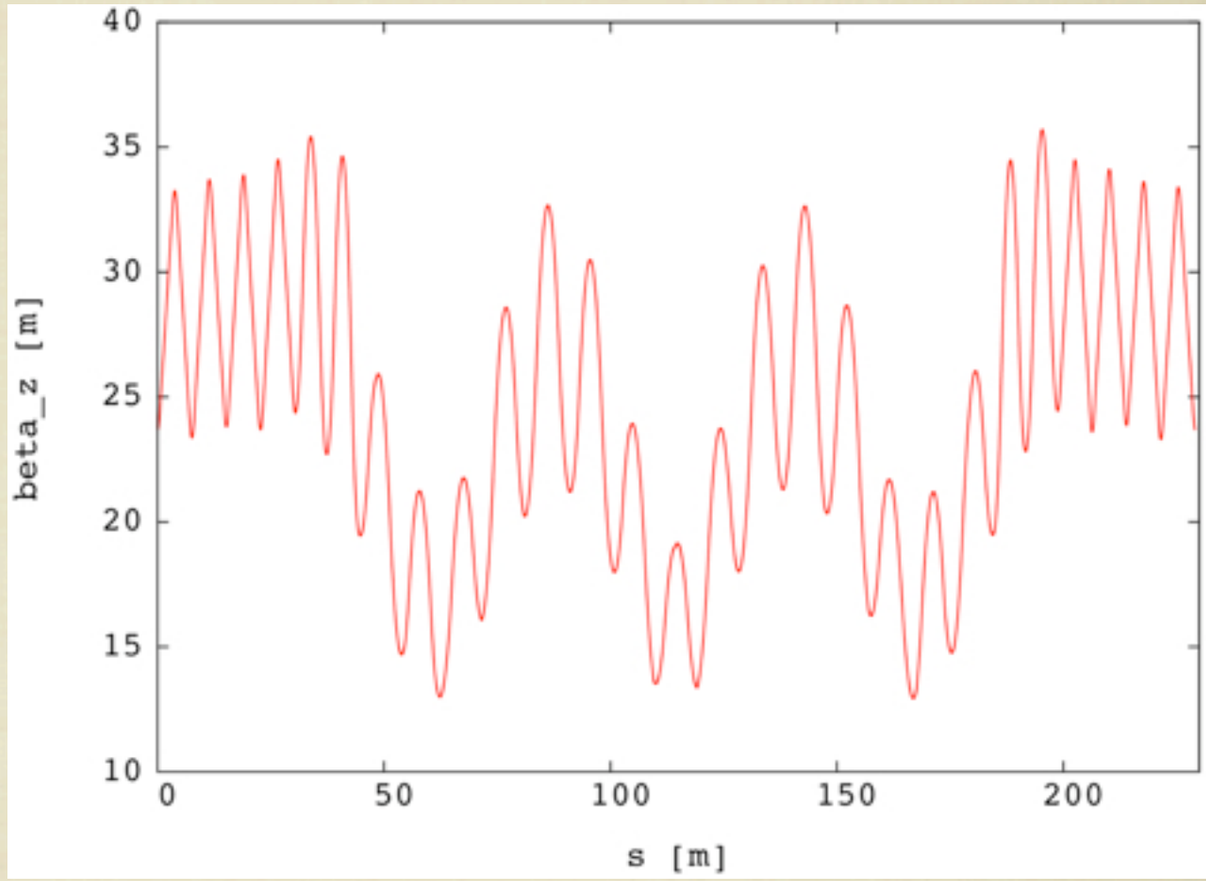


Figure 9 - μ- : Vertical beta function at 6 GeV (quarter a turn).

μ-: MATCHING DONE AT THE CENTER OF F MAGNETS

4D TRACKING RESULTS: RF FREQUENCY = 400 MHz, PEAK VOLTAGE 2GV/TURN.

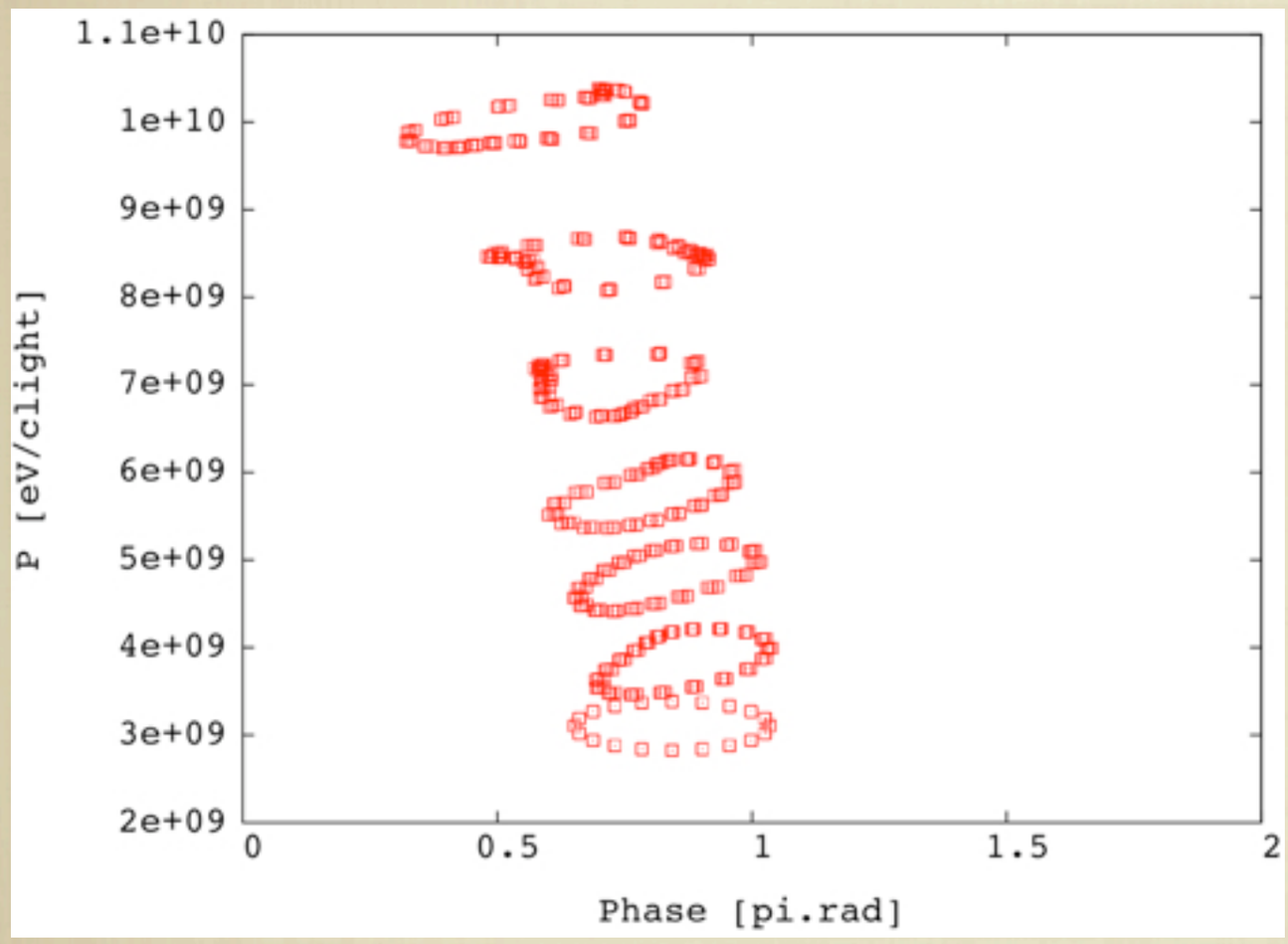


FIGURE 10 - μ- : **LONGITUDINAL PHASE SPACE** SHOWING A 6 TURNS ACCELERATION CYCLE FROM 3 TO 10 GEV WITH AN INITIAL BEAM 4D EMITTANCE OF 0.2 eV.sec × 30 000 π.mm.mrad.

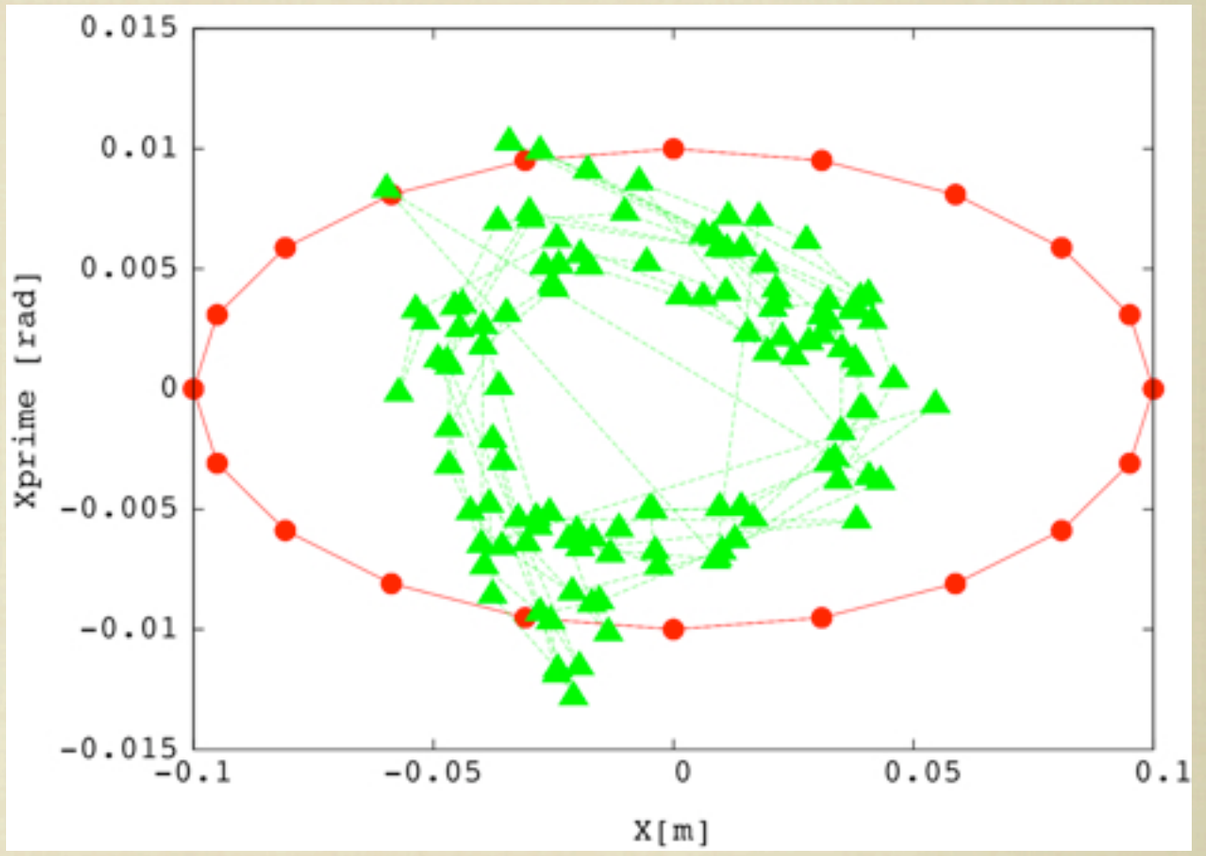


FIGURE 11 - μ- : **HORIZONTAL PHASE SPACE** SHOWING THE INJECTED BEAM PROFILE (RED) AND THE SAME BEAM AFTER A 6 TURNS ACCELERATION CYCLE (GREEN) WITH (4D EMITTANCE OF 0.2 eV.sec × 30 000 π.mm.mrad).

μ+: MATCHING DONE AT THE CENTER OF D MAGNETS

STUDY OF LINEAR PARAMETERS USING RUNGE-KUTTA STEPWISE TRACKING IN SOFT EDGE FIELD MODEL:

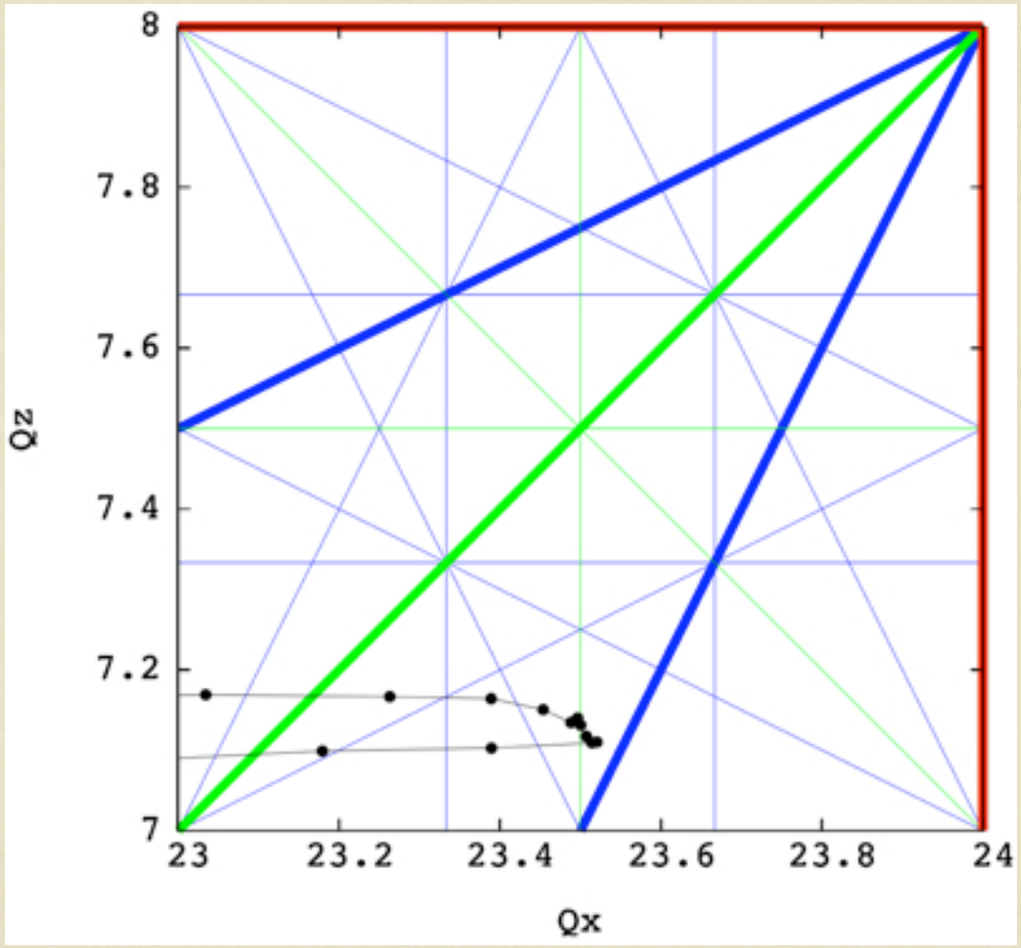


FIGURE 12 - μ+ TUNE VARIATION BETWEEN 3 AND 10 GEV IN THE LATTICE WITH INSERTIONS (FROM STEPWISE TRACKING IN A SOFT EDGE FIELD MODEL).

μ+: matching done at the center of D magnets

Study of linear parameters using Runge-Kutta stepwise tracking in soft edge field model:

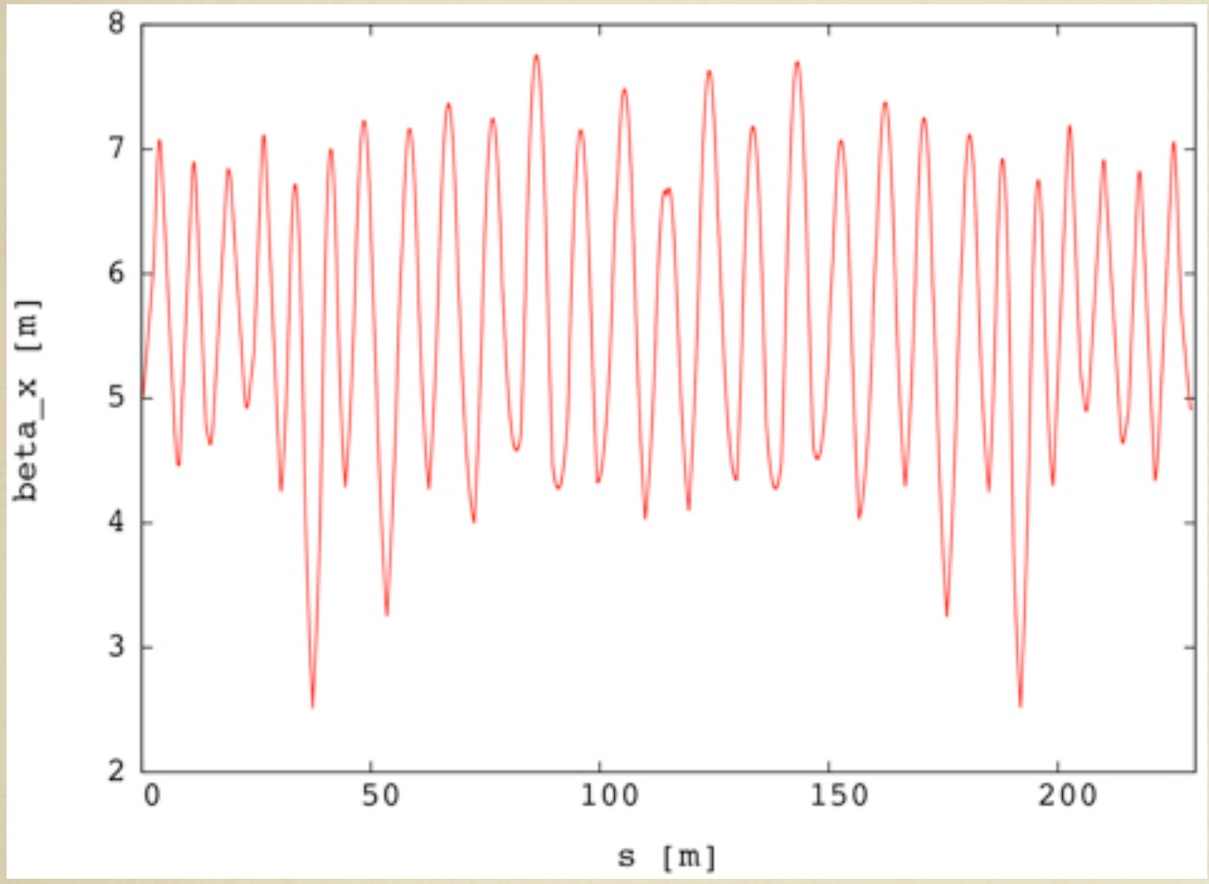


Figure 13 - μ+ : Horizontal beta function at 6 GeV (quarter of a turn).

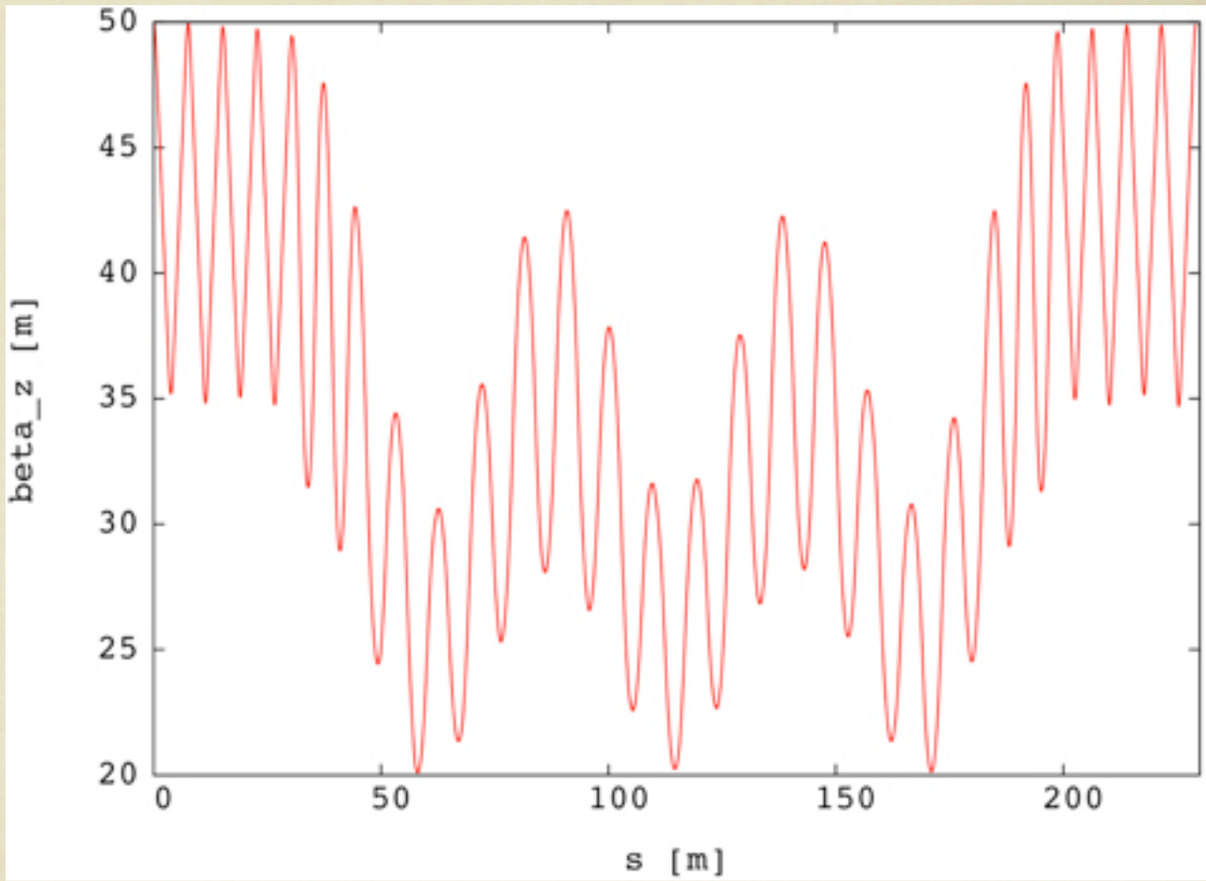


Figure 14 - μ+ : Vertical beta function at 6 GeV (quarter a turn).

μ+: matching done at the center of D magnets

Horizontal acceptance of about 40 000 π.*mm.mrad* 6 GeV.

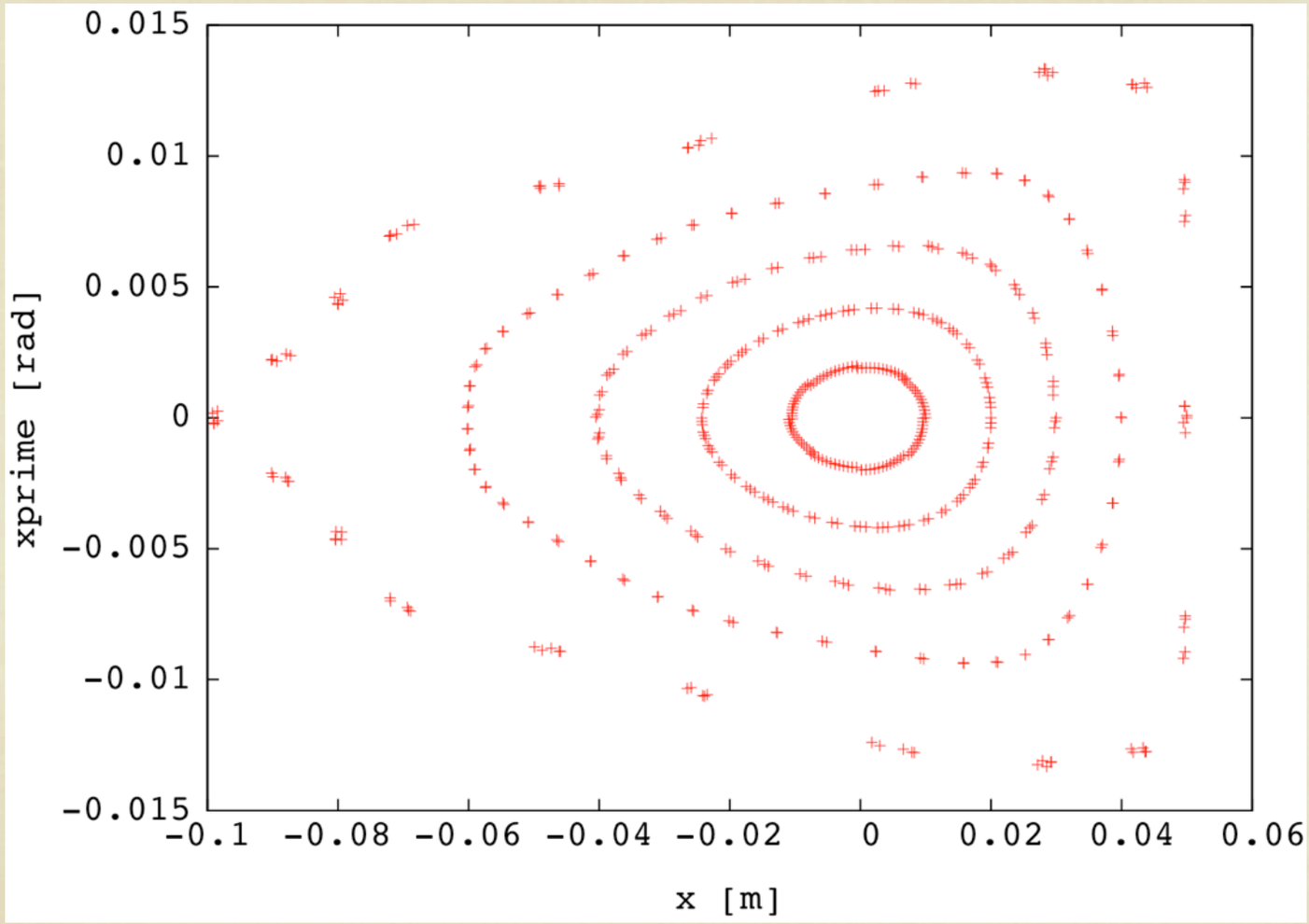


Figure 15 - μ+ : horizontal phase space showing 5 particles tracked over 100 turns (a fixed energy = 6 GeV).

μ+: matching done at the center of D magnets

4D tracking results:

Tried 3 to 10 GeV acceleration cycle

(with RF frequency = 400 Hz, peak voltage 2GV/turn)

Particle lost on collimator even for small transverse emittance...

Summary on harmonic number jump

Works well...

* Large transverse acceptance.
* Large longitudinal acceptance, and no emittance degradation during acceleration.
* Excursion reduction of a factor 3 is already possible.
* Possible with RF frequency in the 200 MHz to 400 MHz range.

...But not yet for both charge in the same time.

Part II

Scaling FFAG lattices for Stationary bucket acceleration

STATIONARY BUCKET ACCELERATION

Stationary bucket acceleration principle:

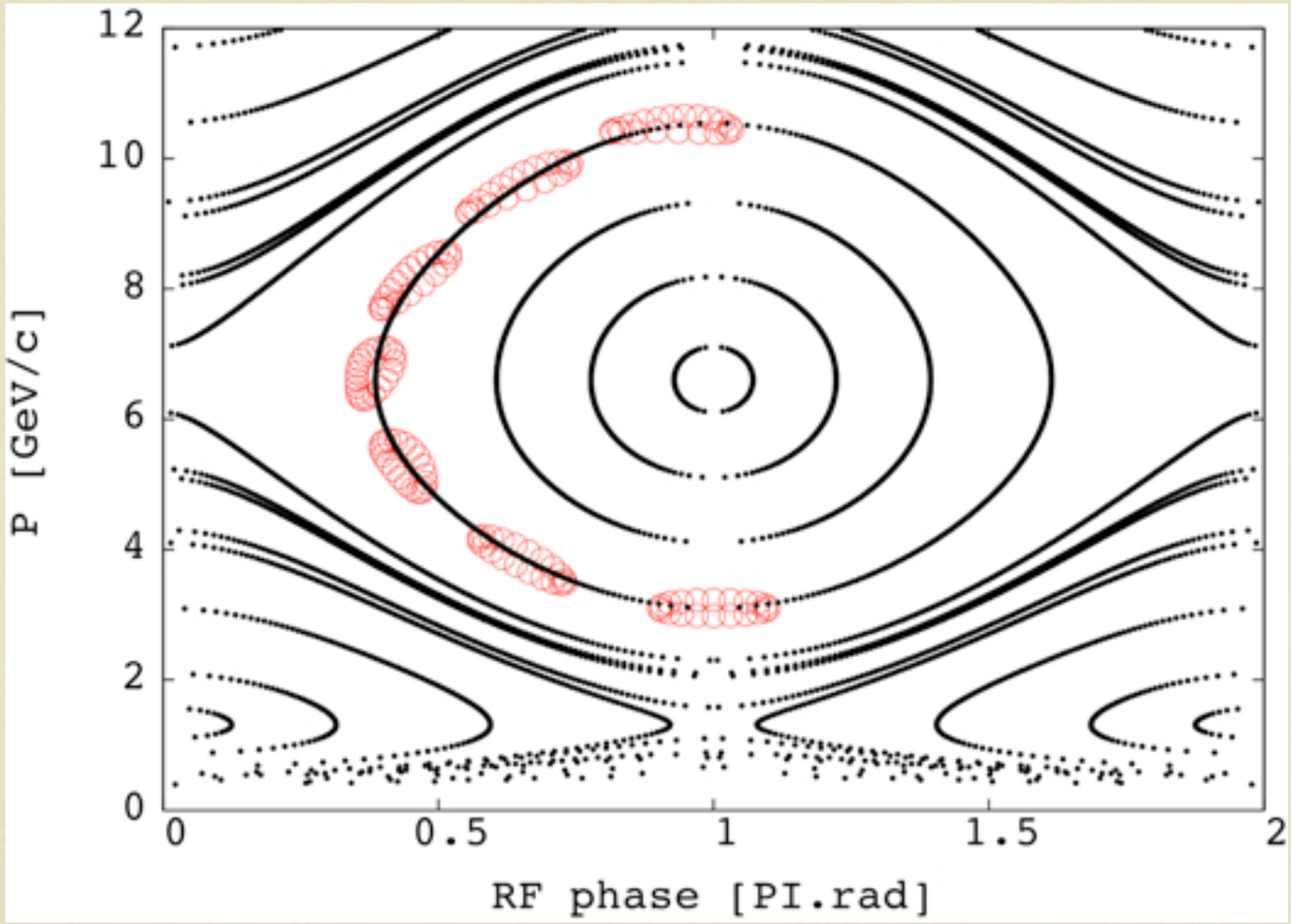


Figure 16 - Longitudinal phase space showing a 6 turns acceleration cycle (in red) as well as the equi-hamiltonian lines (in black).

STATIONARY BUCKET ACCELERATION

Lattice example with super-periodicity of 6:

B_{max}	3 T
Horizontal tune	36.8
Vertical tune	11.02

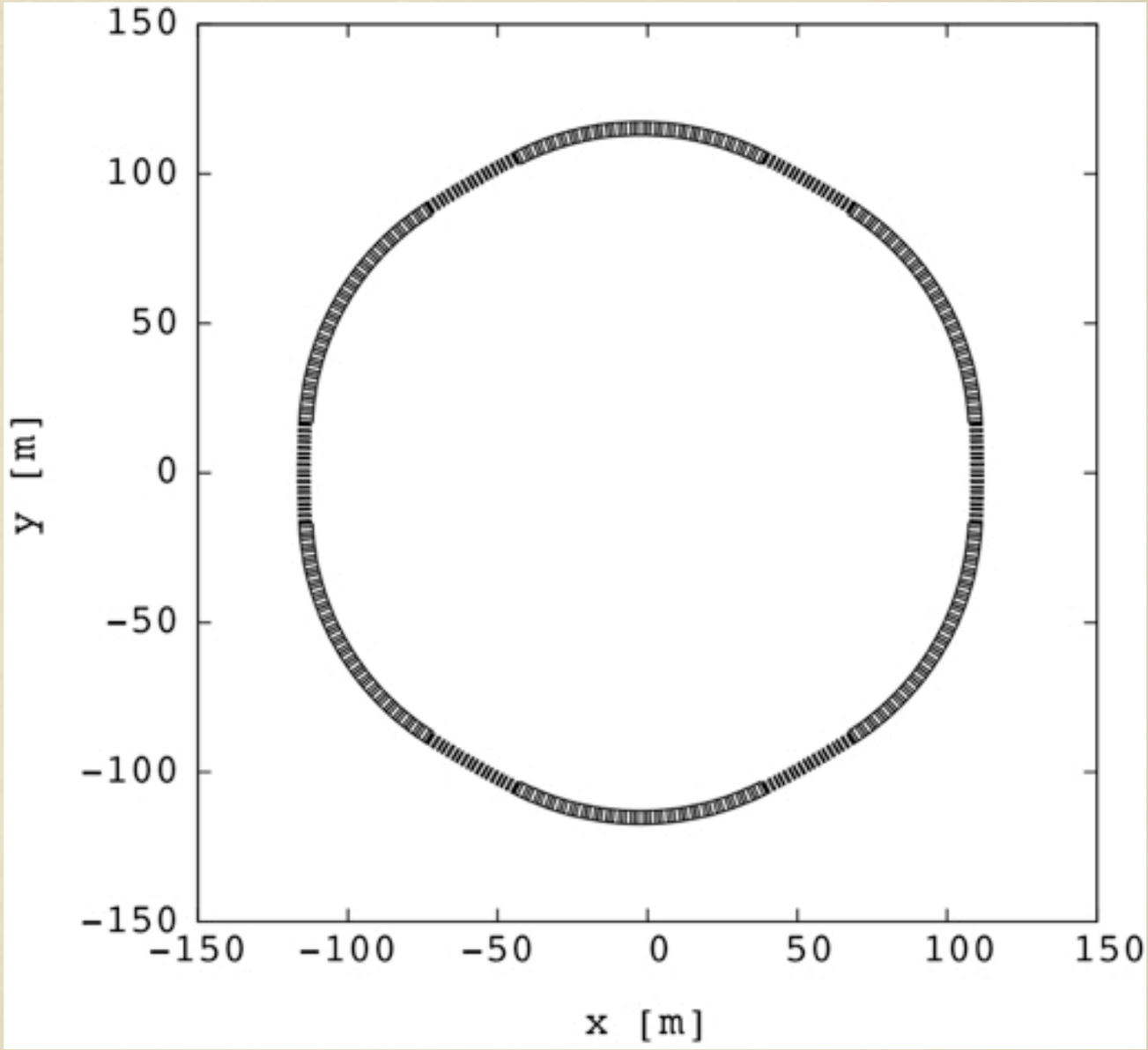
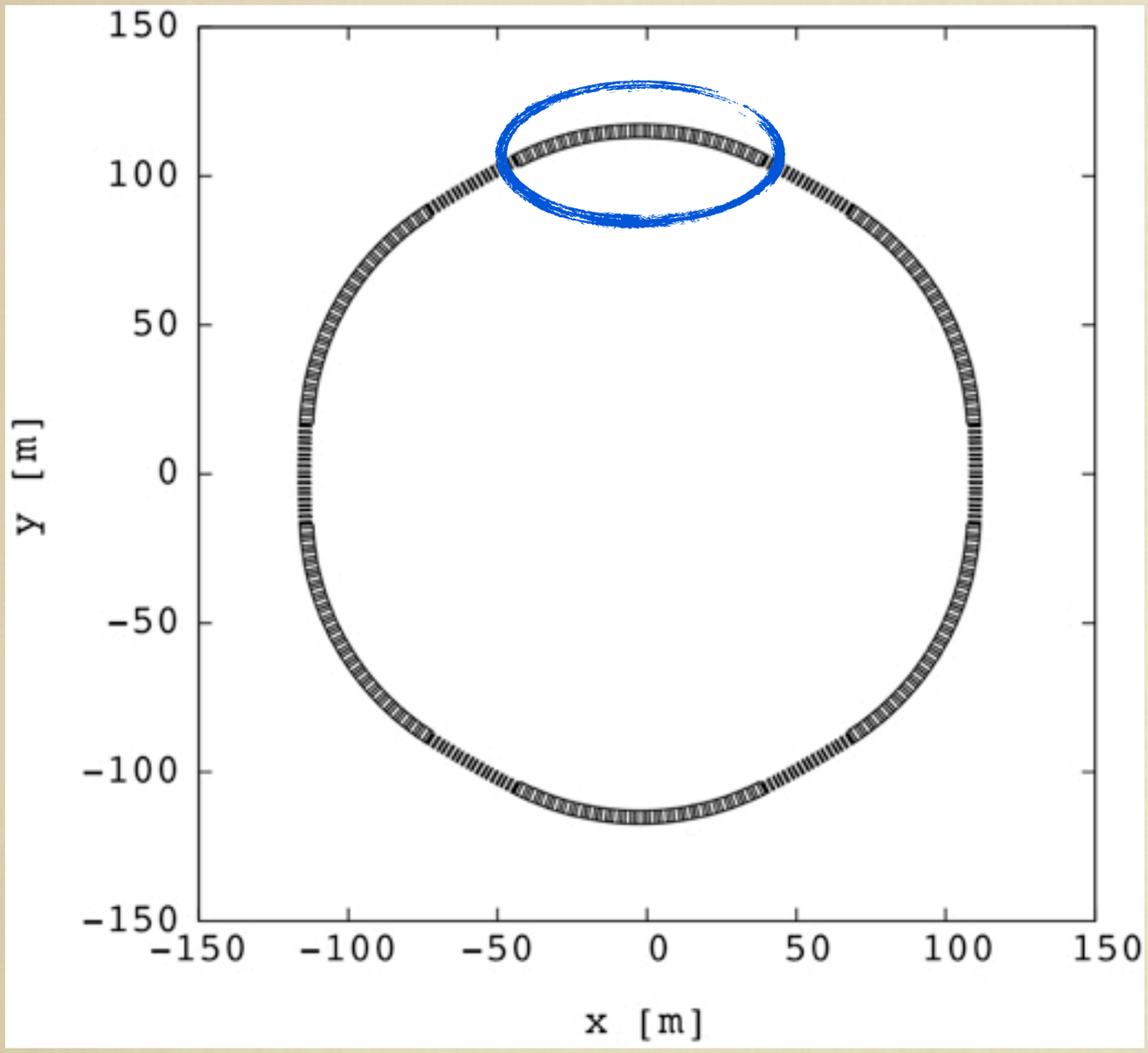


Figure 17 - Schematic view of a 3 to 10 GeV muon FFAG ring with 6 "almost-straight" insertions.

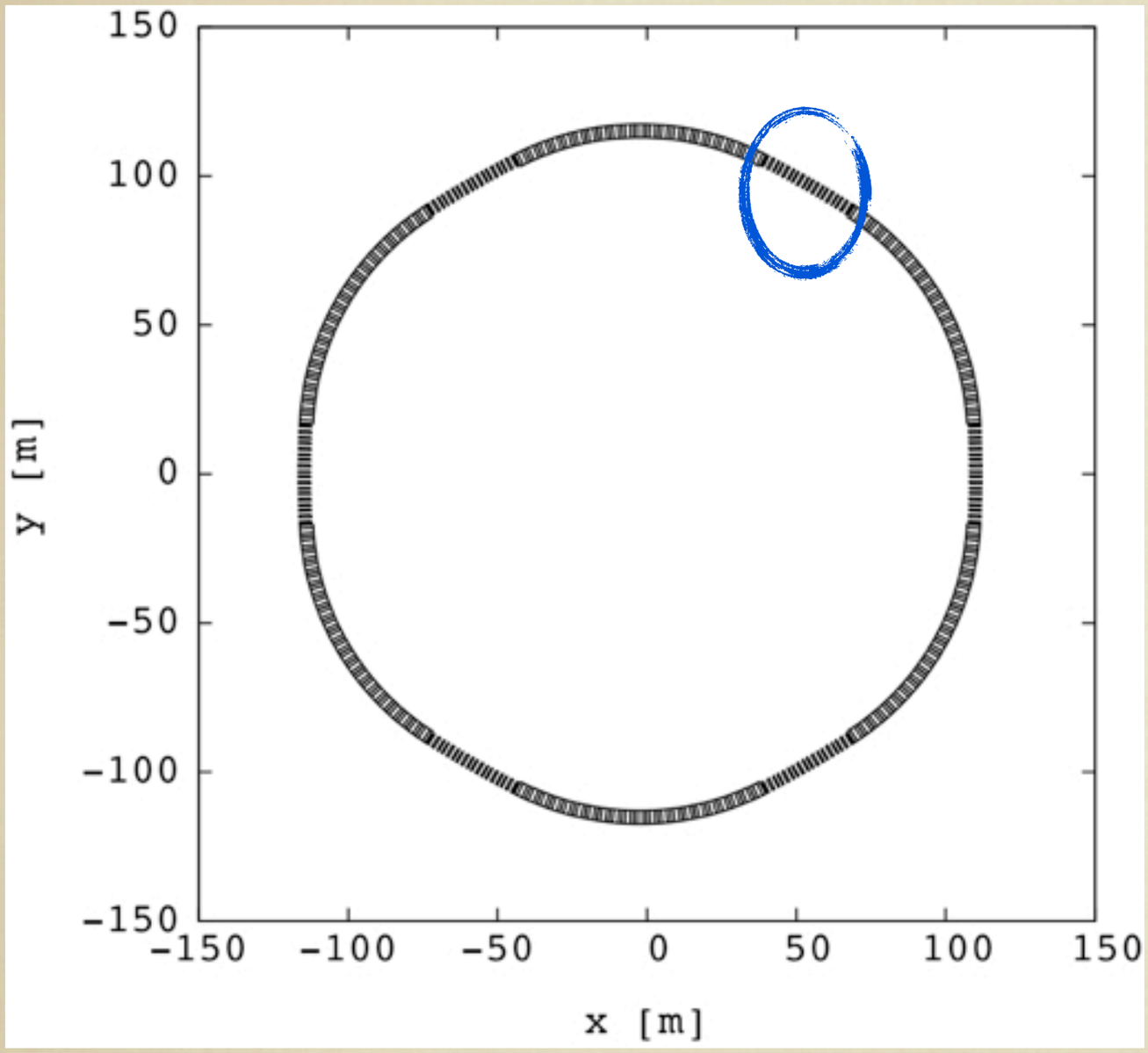
Lattice for stationary bucket acceleration



Mean radius	90 m
Number of cells	6×21
Cell opening angle	2.6 deg.
Field index k	500
B_{max}	3 T
Horiz. phase adv. per cell	85.9 deg.
Vert. phase adv. per cell	19.3 deg.

Figure 17 - Schematic view of a 3 to 10 GeV muon FFAG ring with 6 "almost-straight" insertions.

Lattice for stationary bucket acceleration



Mean radius	360 m
Number of cells	6×6
Cell opening angle	0.9 deg.
Field index k	2003
B_{max}	3 T
Horiz. phase adv. per cell	67.7 deg.
Vert. phase adv. per cell	42.2 deg.

Figure 17 - Schematic view of a 3 to 10 GeV muon FFAG ring with 6 "almost-straight" insertions.

Lattice for stationary bucket acceleration

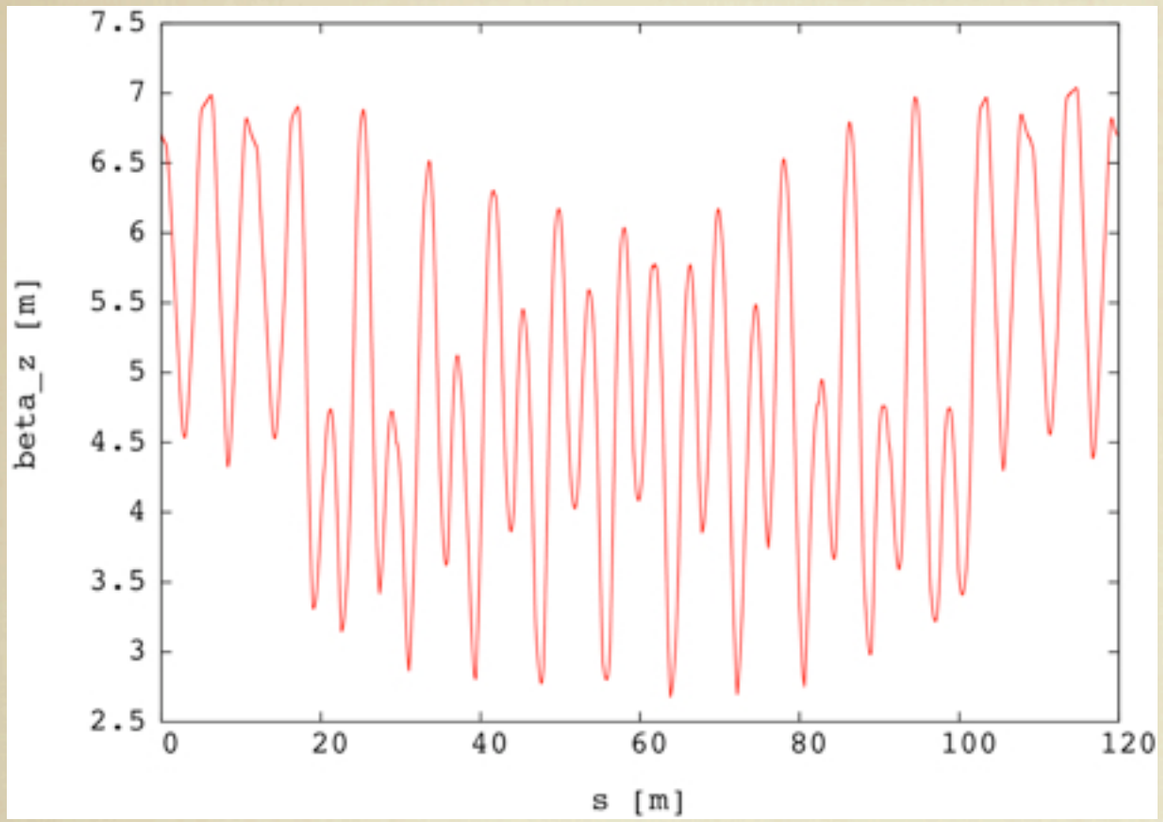


Figure 18 - **Horizontal** beta function at 6 GeV (1/6 of a turn).

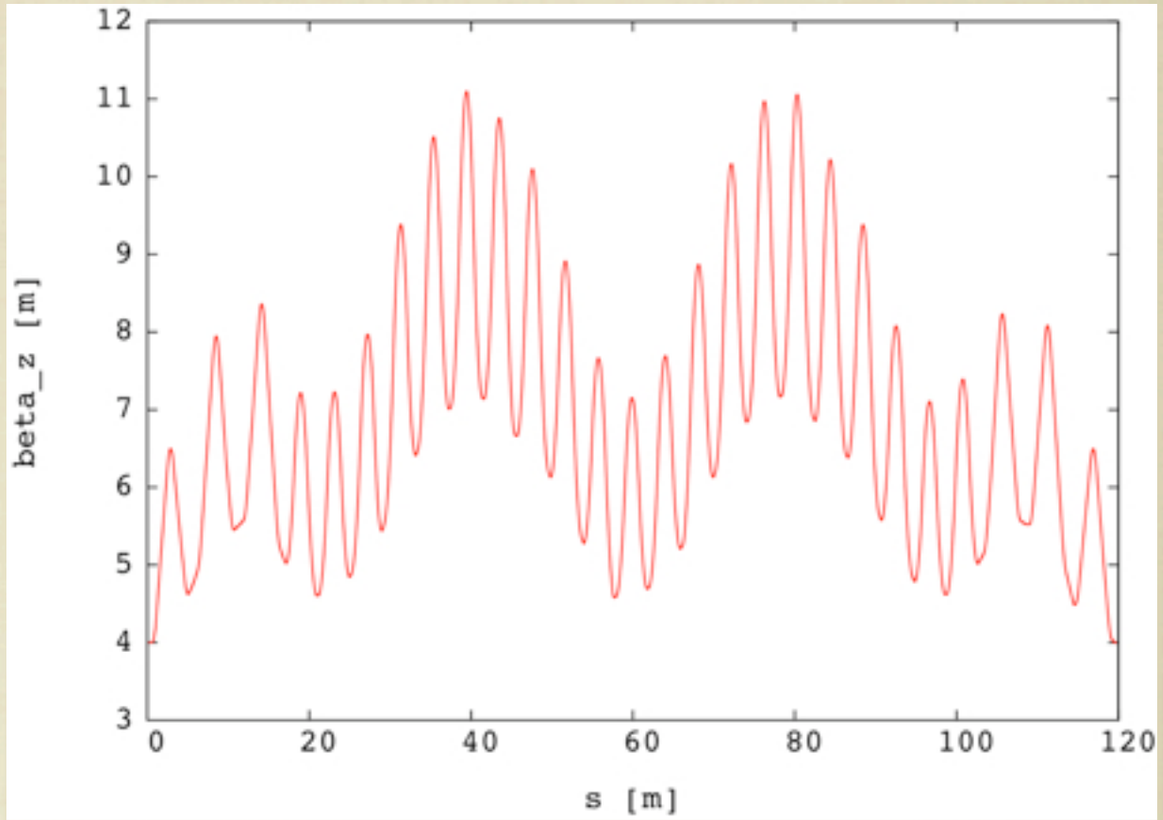


Figure 19 - **Vertical** beta function at 6 GeV (1/6 of a turn).

Tracking results

4D tracking results: RF frequency = **200 MHz**, peak voltage **2GV/turn**. Acceleration within only 4 turns.

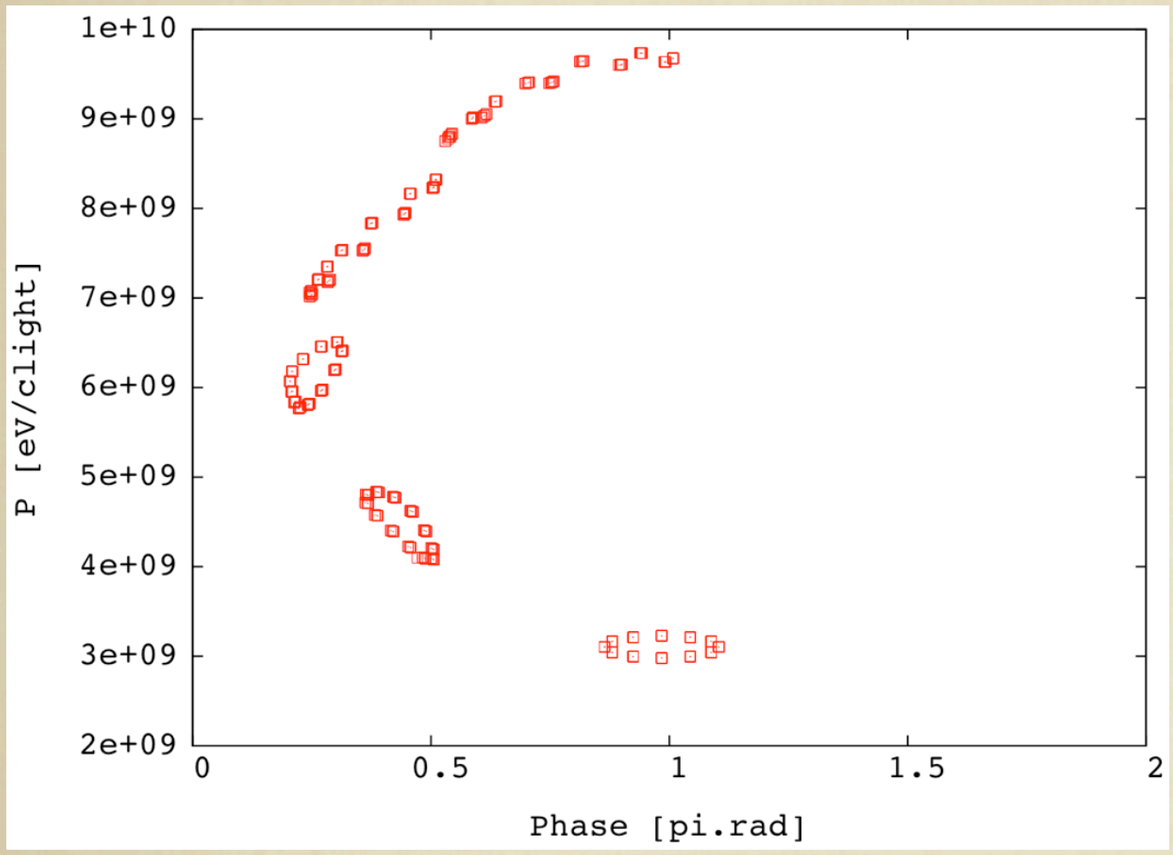


Figure 20 - **Longitudinal phase space showing a 4 turns acceleration cycle from 3 to 10 GeV with an initial beam 4D emittance of 0.1** eV.sec **x 24 000** π.mm.mrad.

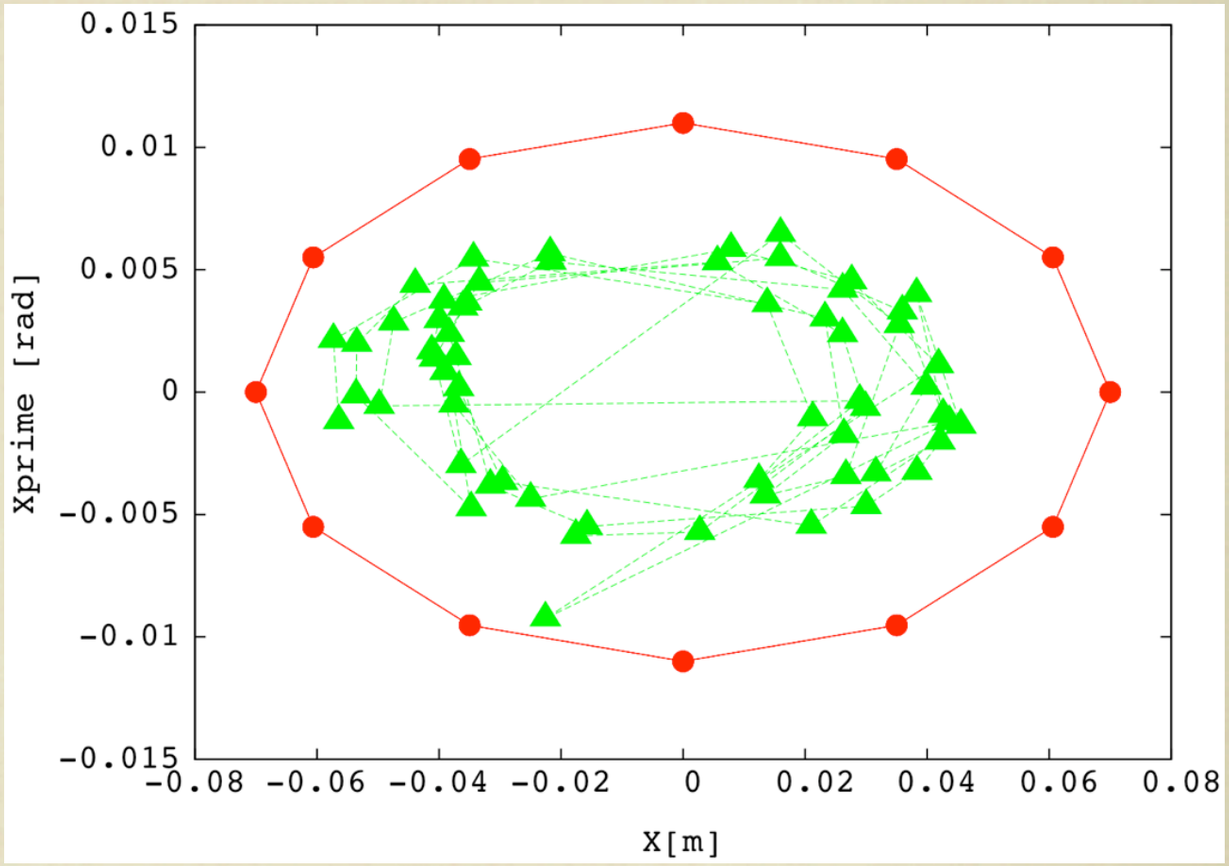


Figure 21 - **Horizontal phase space showing the injected beam profile (red) and the same beam after a 4 turns acceleration cycle (green) with (4D emittance of 0.1** eV.sec **x 24 000** π.mm.mrad**).**

Tracking results

2D tracking results: RF frequency = **100 MHz**, peak voltage **2GV/turn**. Acceleration within only 4 turns.

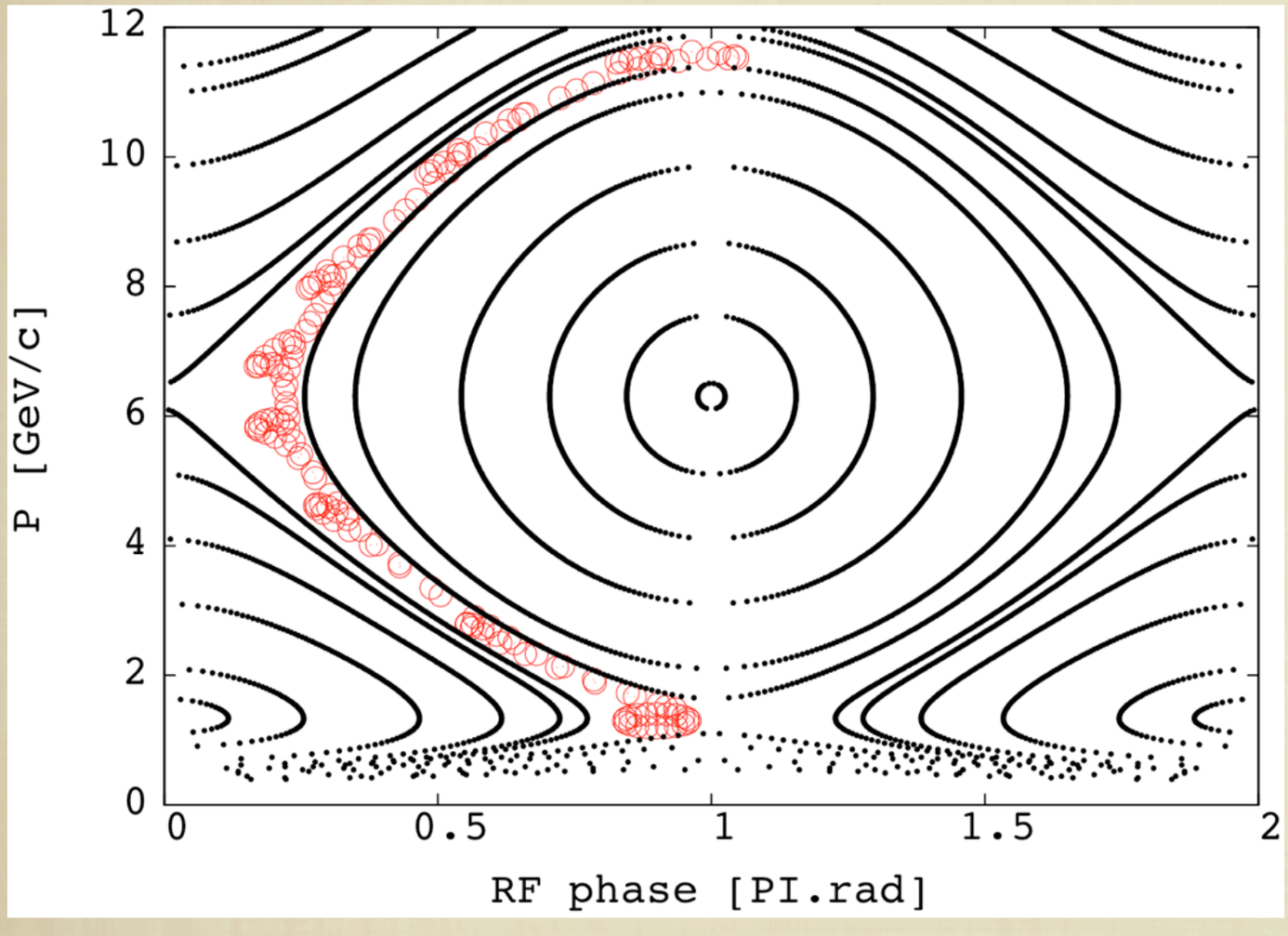
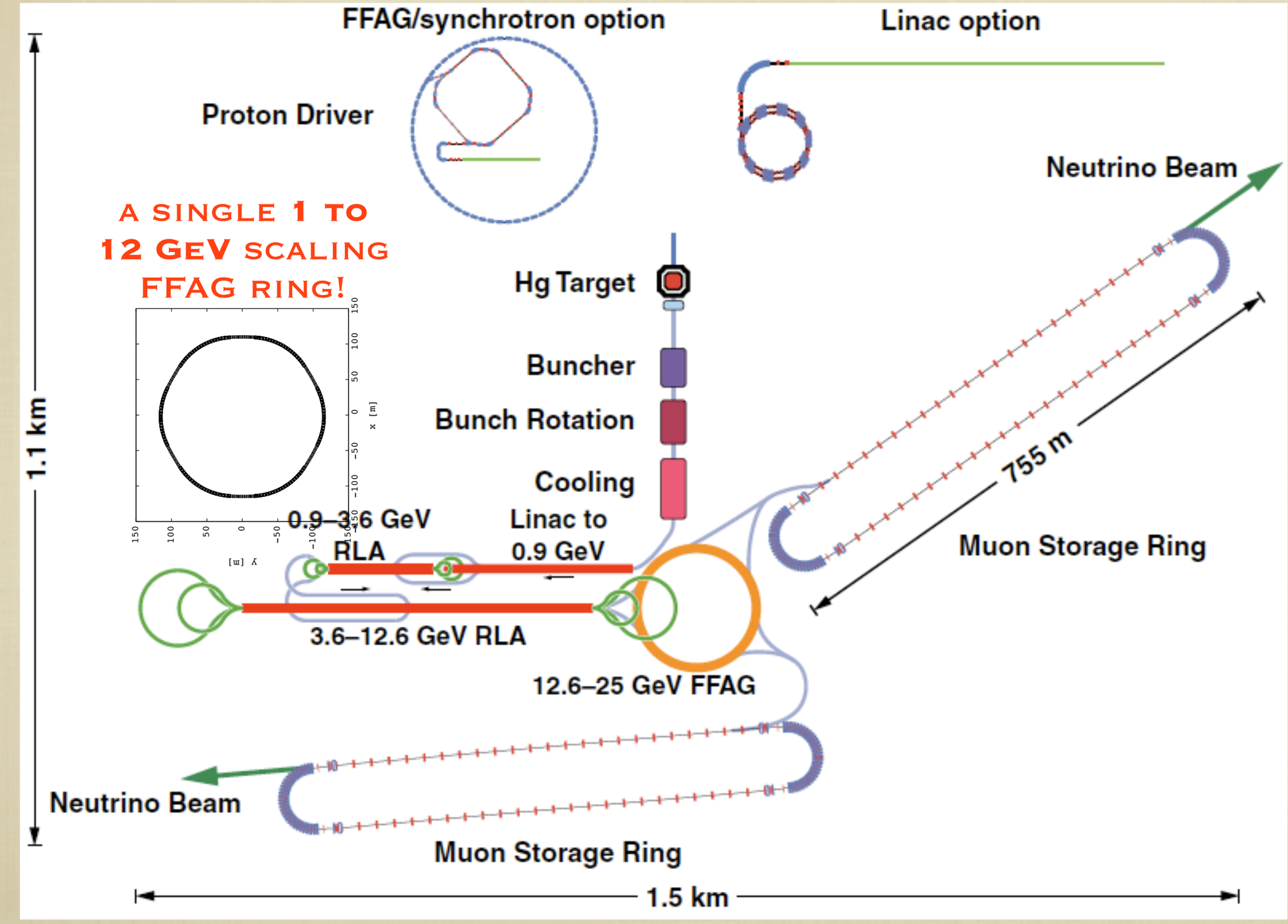


Figure 22 - **Longitudinal phase space** showing a 7 turns acceleration cycle (2D longitudinal tracking) from 3 to 10 GeV with an initial beam 2D emittance of 0.2 eV.sec.

Open a way for 1 to 12 GeV acceleration at 100 MHz in a single FFAG ring!

Summary on stationary bucket acceleration



Summary

Advanced scaling FFAG lattices can be used for both:

Harmonic number jump acceleration of muons

Stationary bucket acceleration of muons

Thank you!

Additional material...