PRISM WITH ADVANCED SCALING FFAG

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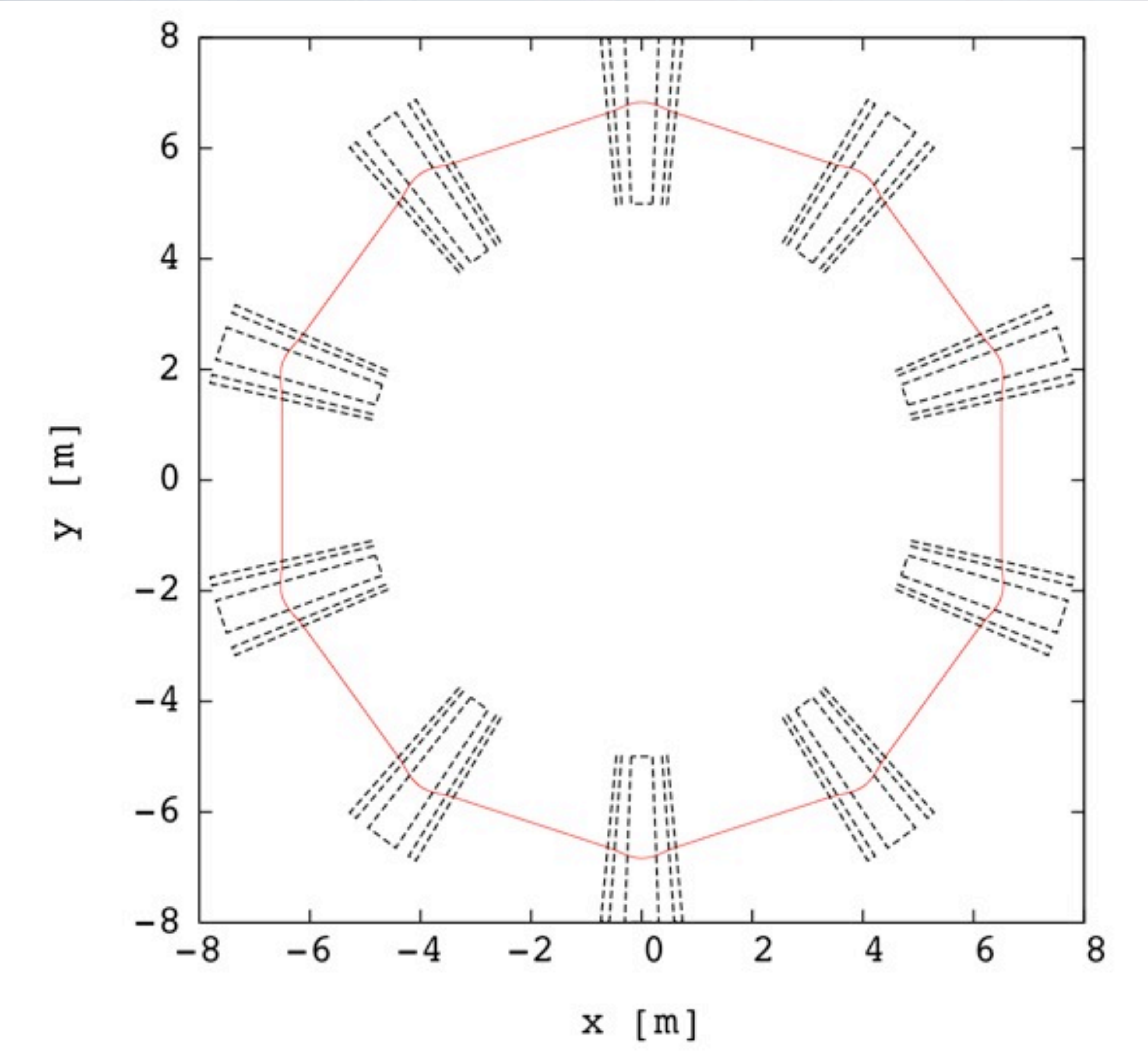
CONSTRAINTS

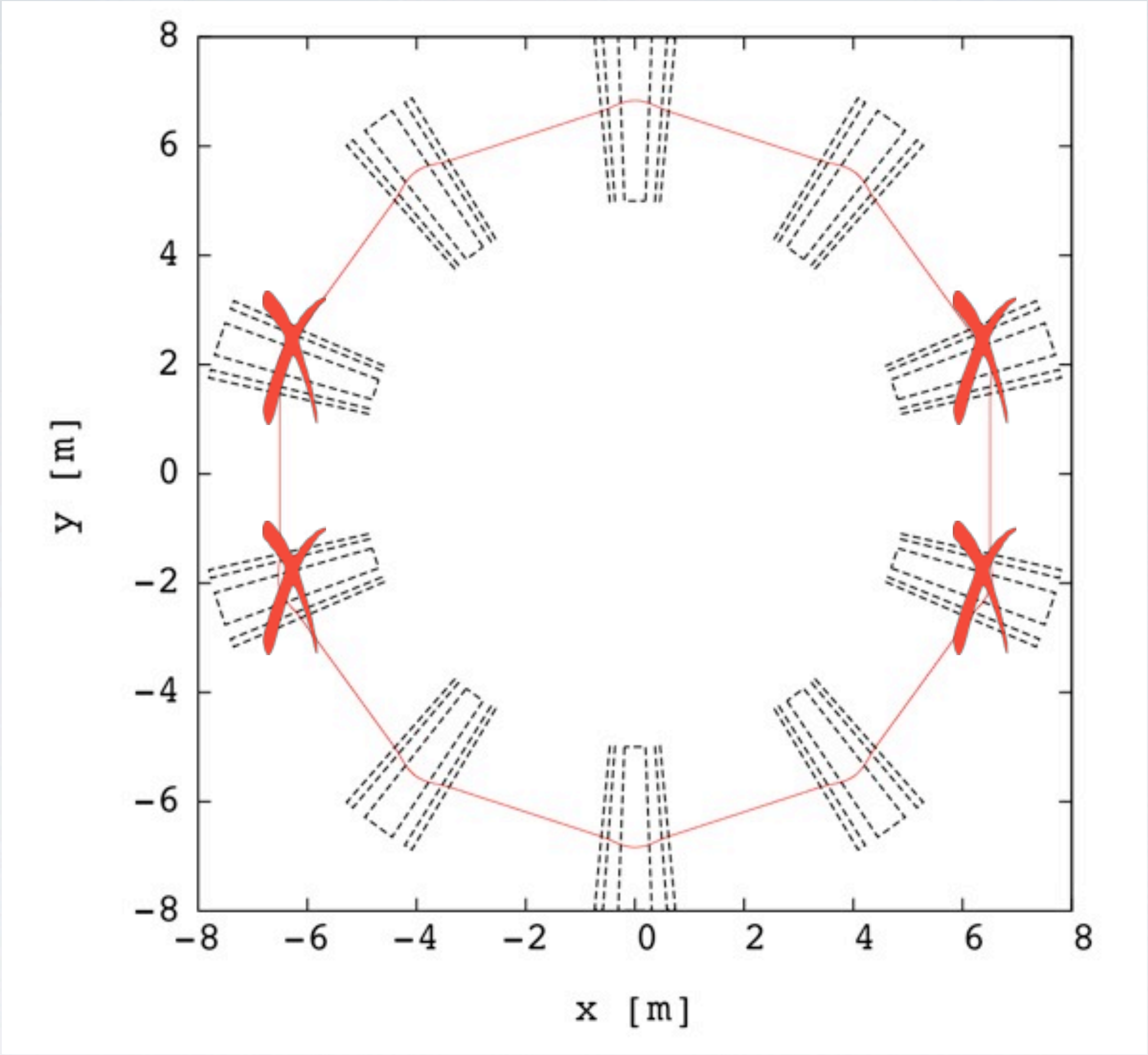
- Large transverse acceptance
  - horizontal: 20 000π mm.mrad
  - vertical: 3 000π mm.mrad
- Momentum acceptance: 68MeV ±20%



Original PRISM cell	
k	4.6
Average radius	6.5 m
Phase advances:	
horizontal μ_x	97 deg.
vertical μ_z	55 deg.
Dispersion	1.16 m

Figure 1: Original 10-cell PRISM ring

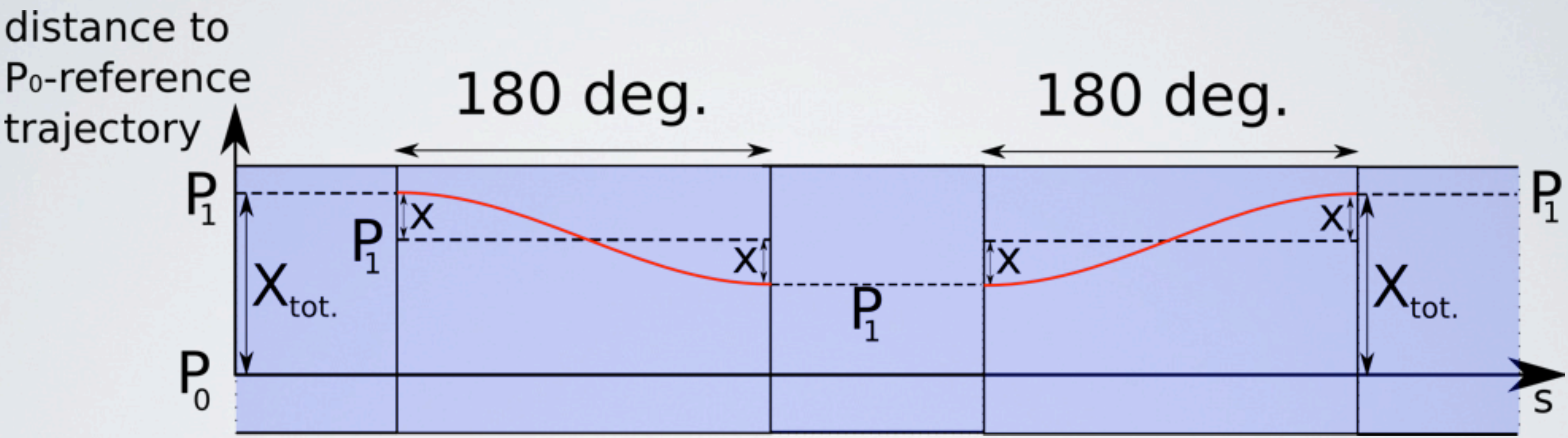
→ Problem of Injection/Extraction



Reduced-dispersion area wanted

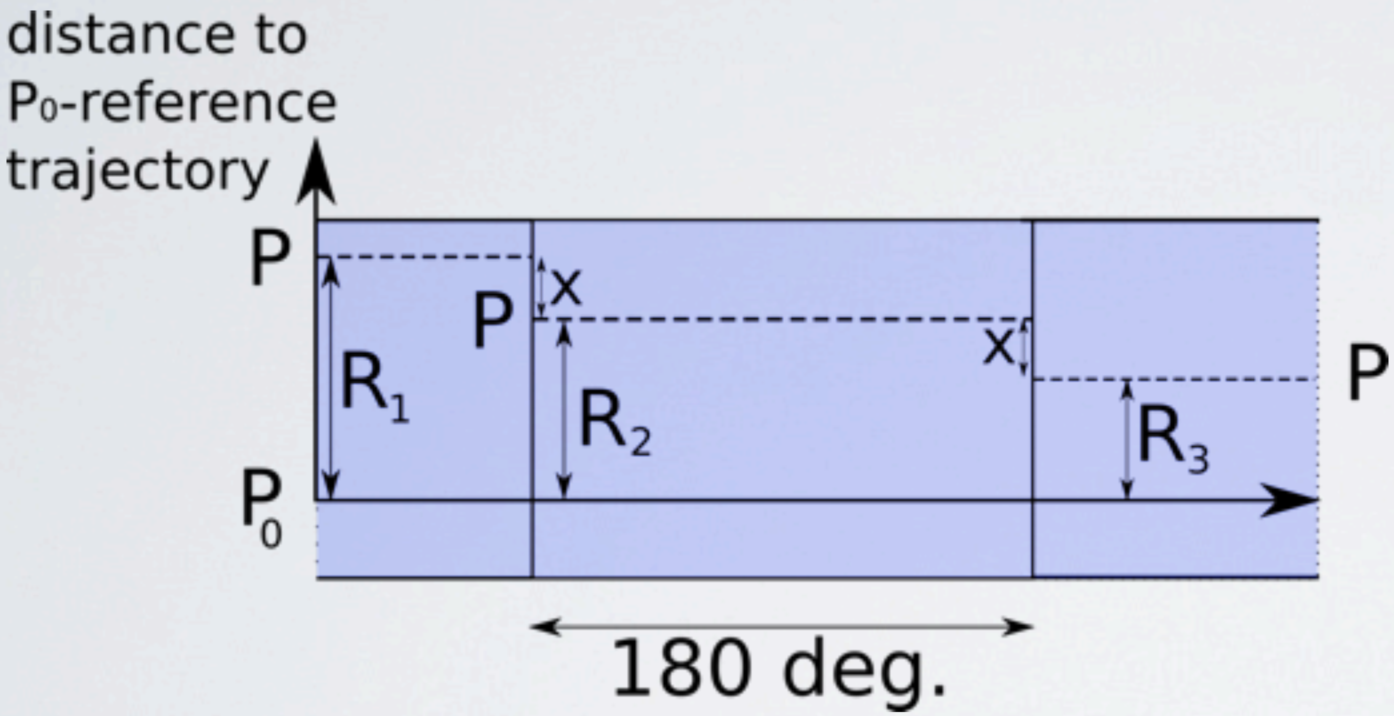
→ Dispersion suppressors

DISPERSION SUPPRESSOR



Bending:	k_1	k_2	k_3	k_2	k_1
Straight:	$\left(\frac{n}{\rho}\right)_1$	$\left(\frac{n}{\rho}\right)_2$	$\left(\frac{n}{\rho}\right)_3$	$\left(\frac{n}{\rho}\right)_2$	$\left(\frac{n}{\rho}\right)_1$

DISPERSION SUPPRESSOR IN BENDING LINES



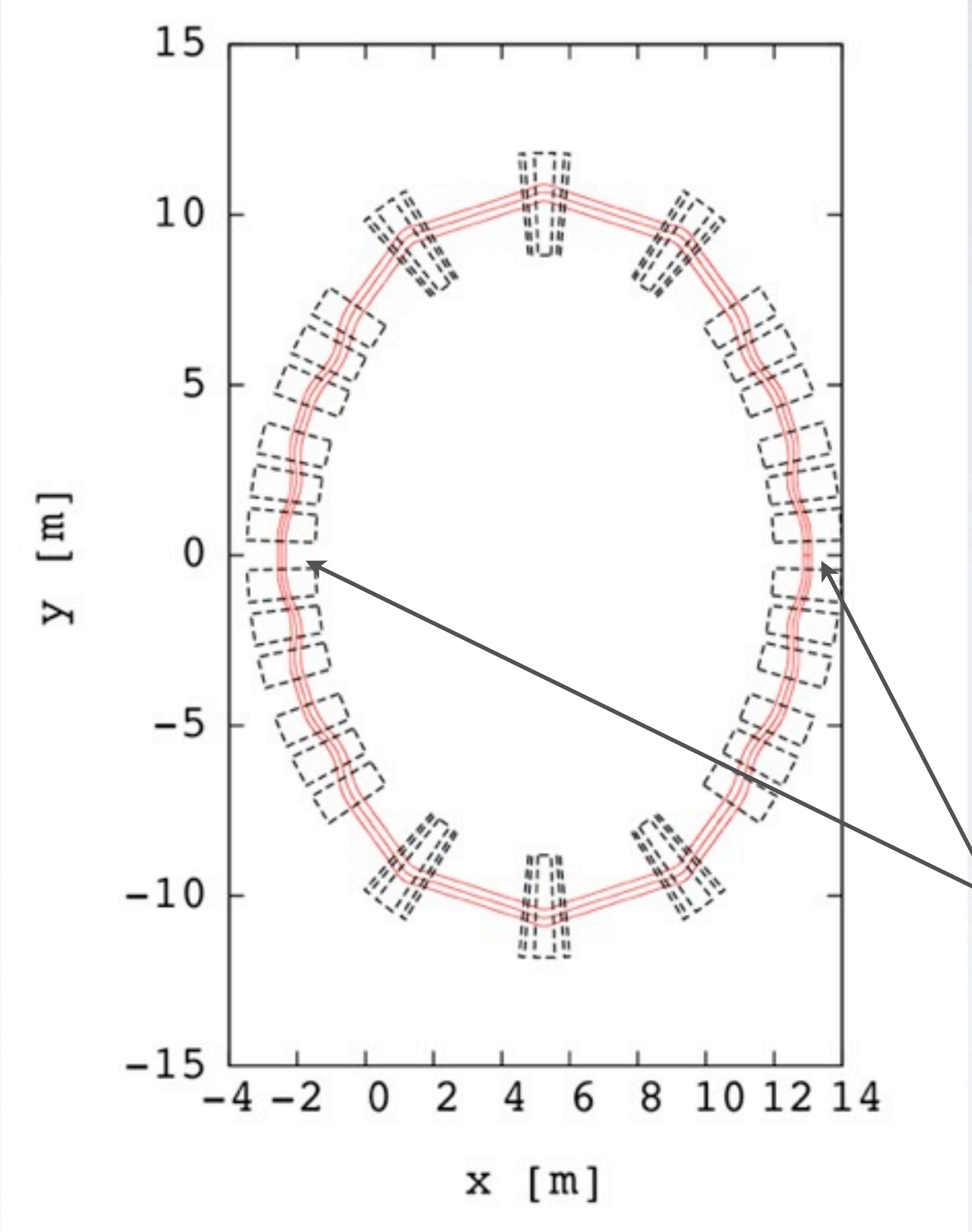
$$R_2 - (R_1 - R_2) = R_3$$

$$2R_2 = R_1 + R_3$$

$$R = R_0 \left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}$$

1st order

$$\Rightarrow \quad \frac{2}{k_2+1} = \frac{1}{k_1+1} + \frac{1}{k_3+1}$$



Dispersion suppressor cell FDF	
k	14.2
Average radius	13 m
Phase advances:	
horizontal μ_x	90 deg.
vertical μ_z	86 deg.
Dispersion	1.16 m to 0.58 m

Dispersion reduced areas

Figure 2: PRISM ring with 4 dispersion suppressors and 6 original PRISM magnets.

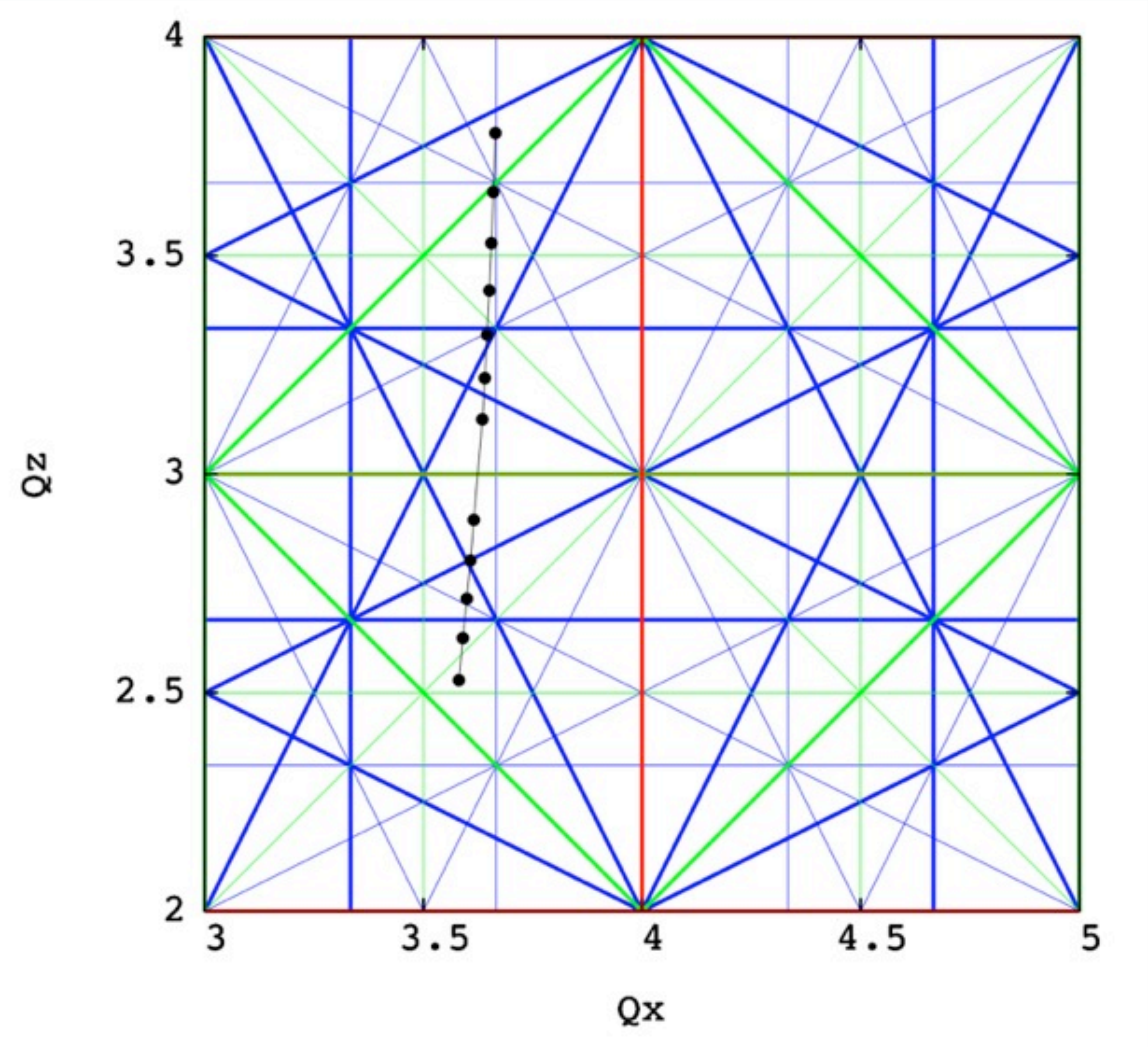
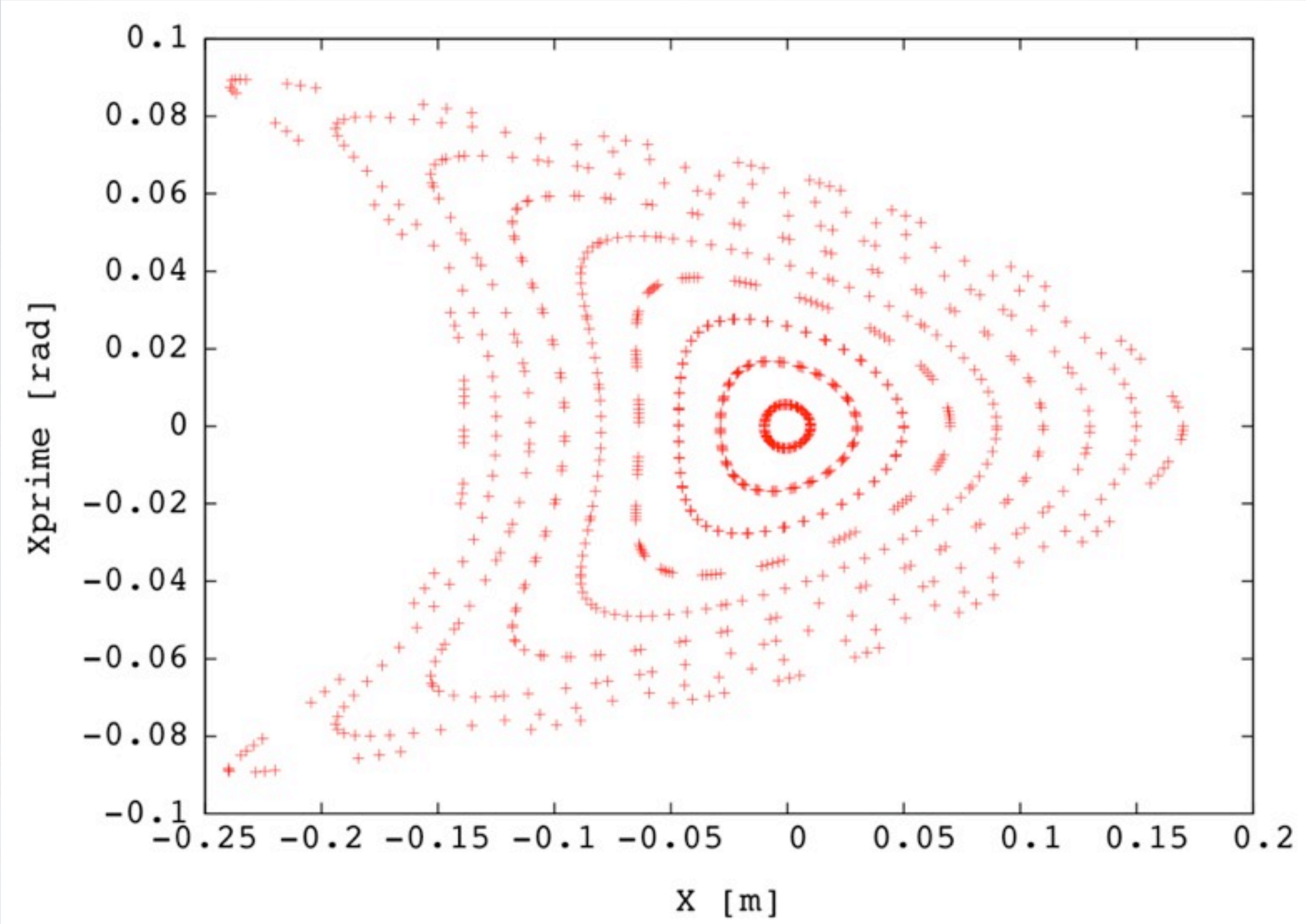
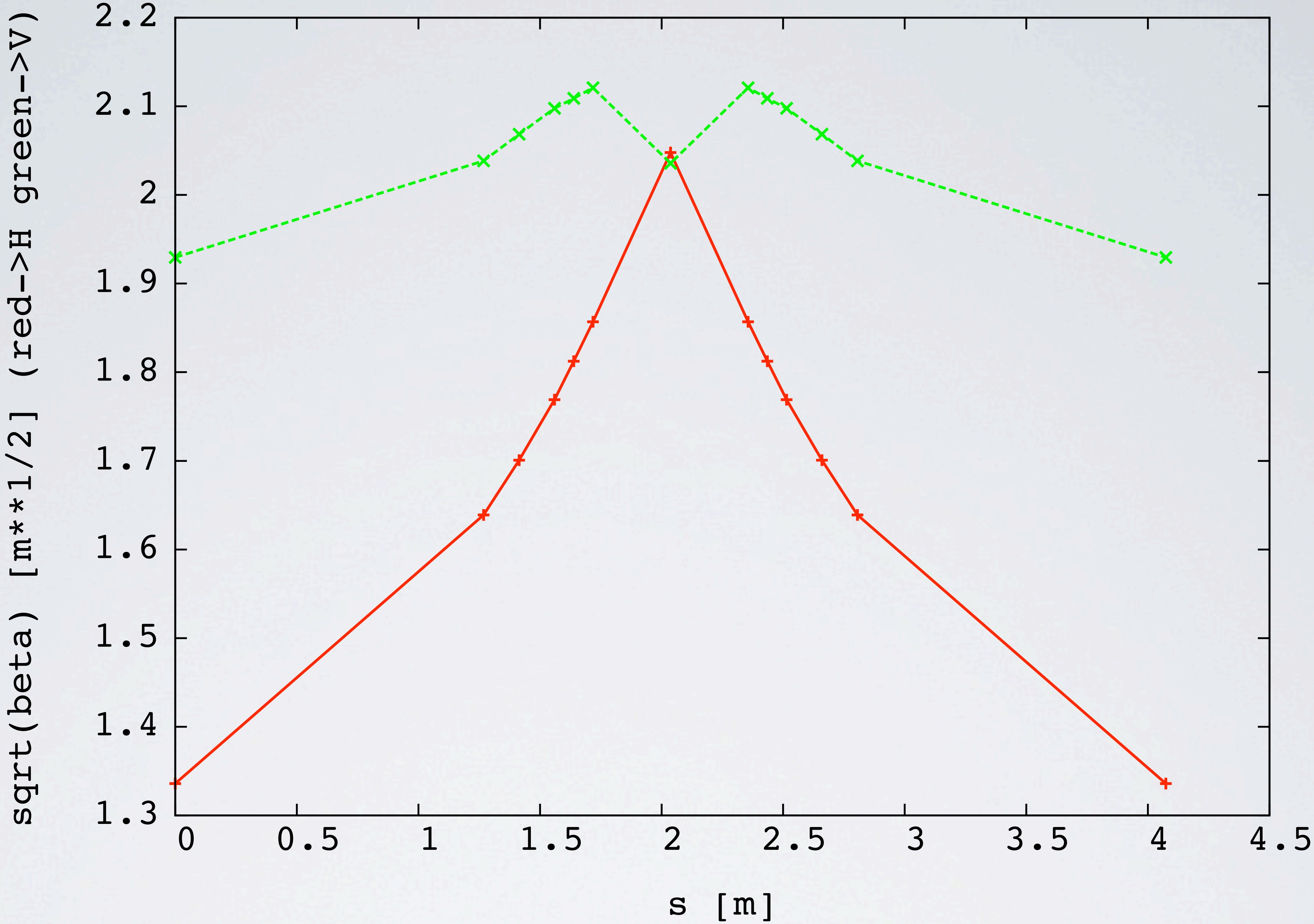


Figure 3: Change of working point in tune diagram.

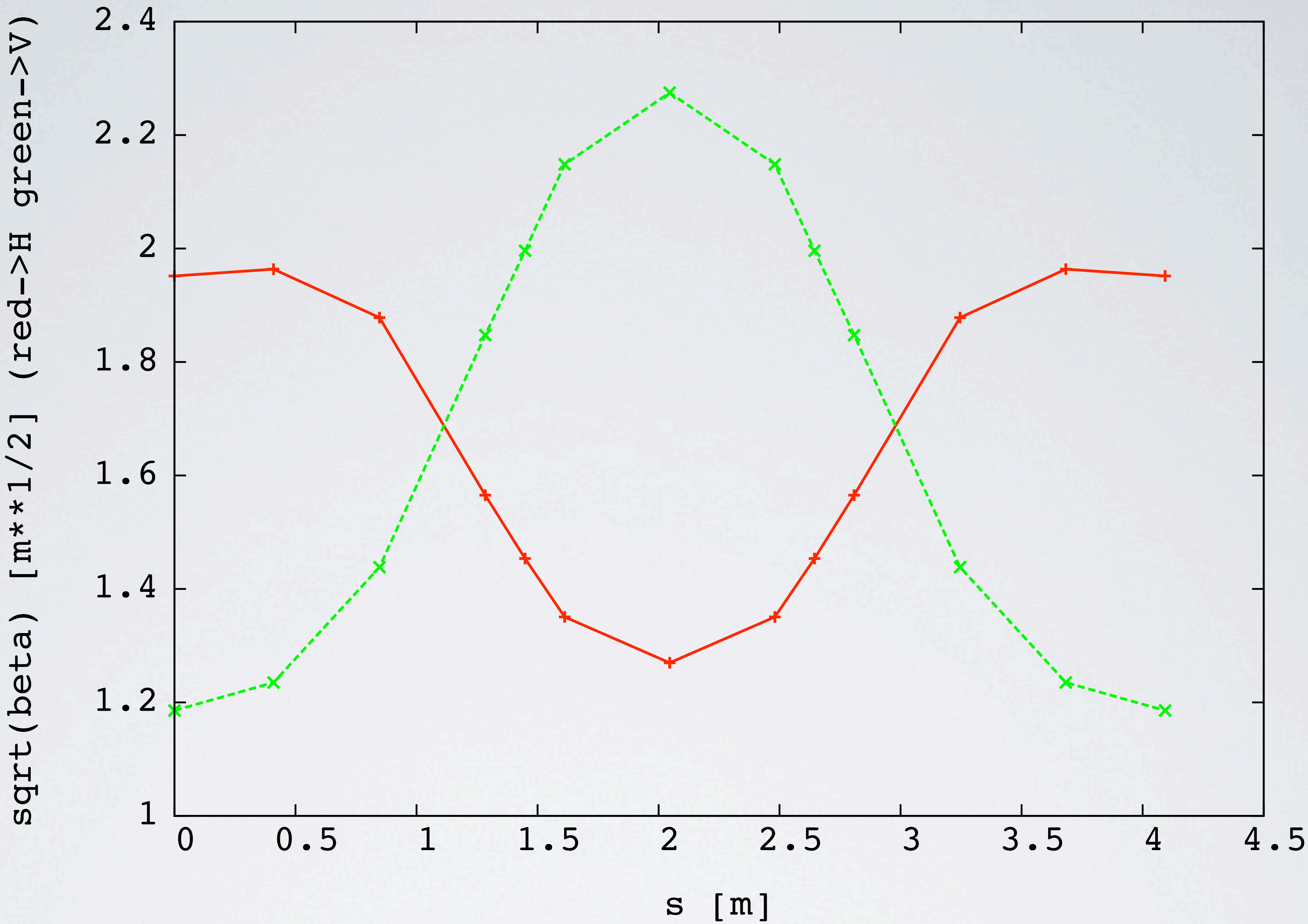


Horizontal Poincarre map

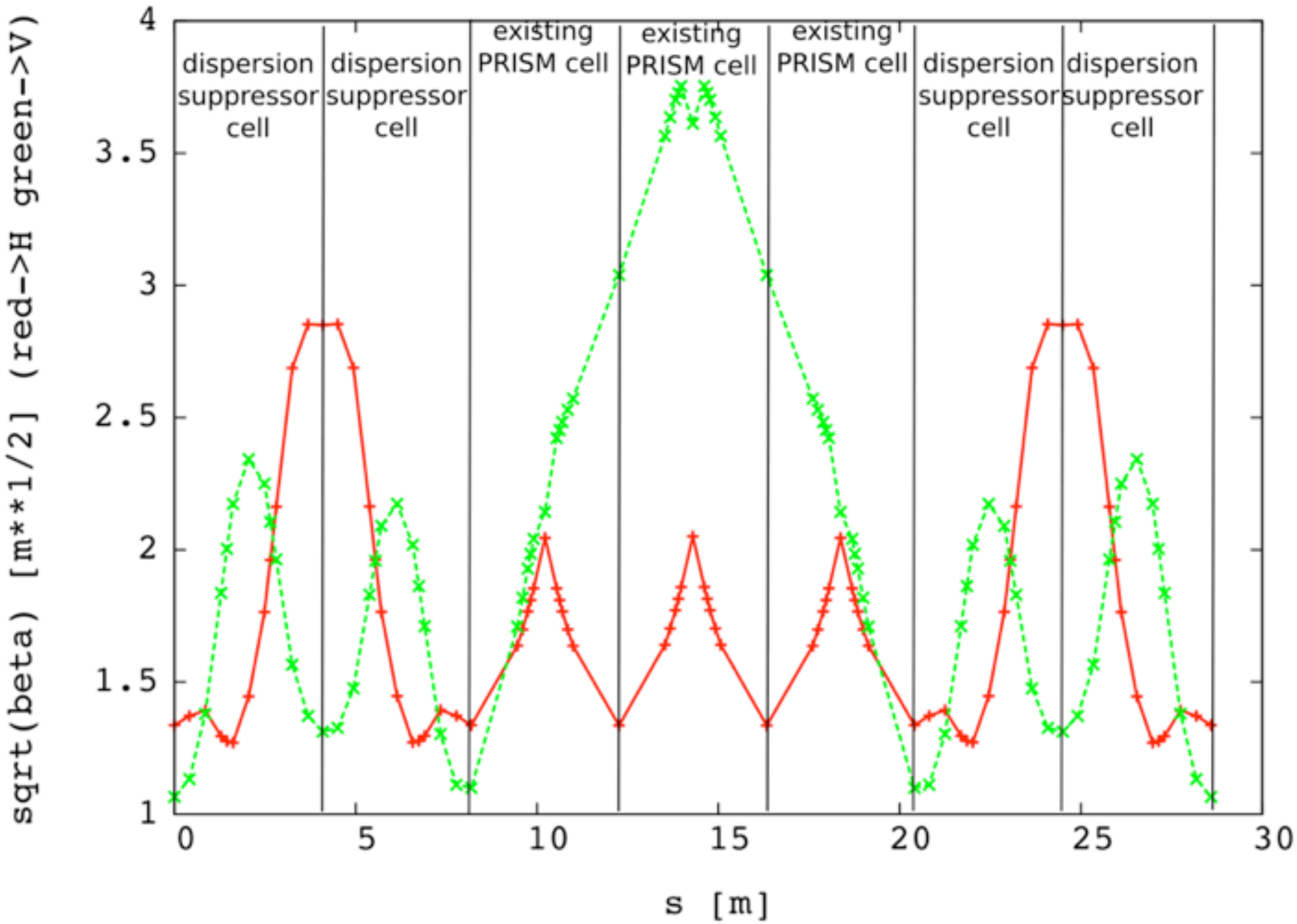
Qx = 3.65, Qz = 3.54



Betafunctions of original PRISM cell.
(red: horizontal, green: vertical)



Betafunctions of a dispersion-suppressor cell (90 deg.)
(red: horizontal, green: vertical)



Betafunctions of PRISM ring with dispersion suppressor.
(red: horizontal, green: vertical)

SCALING STRAIGHT LINES

Straight section = Bending section with infinite radius

$$\lim_{r_0\to\infty}\left(\frac{r}{r_0}\right)^k = \lim_{r_0\to\infty}\left[\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{x}{r_0}k} = \left[\lim_{r_0\to\infty}\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{n}{\rho}x} = e^{\frac{n}{\rho}x}$$

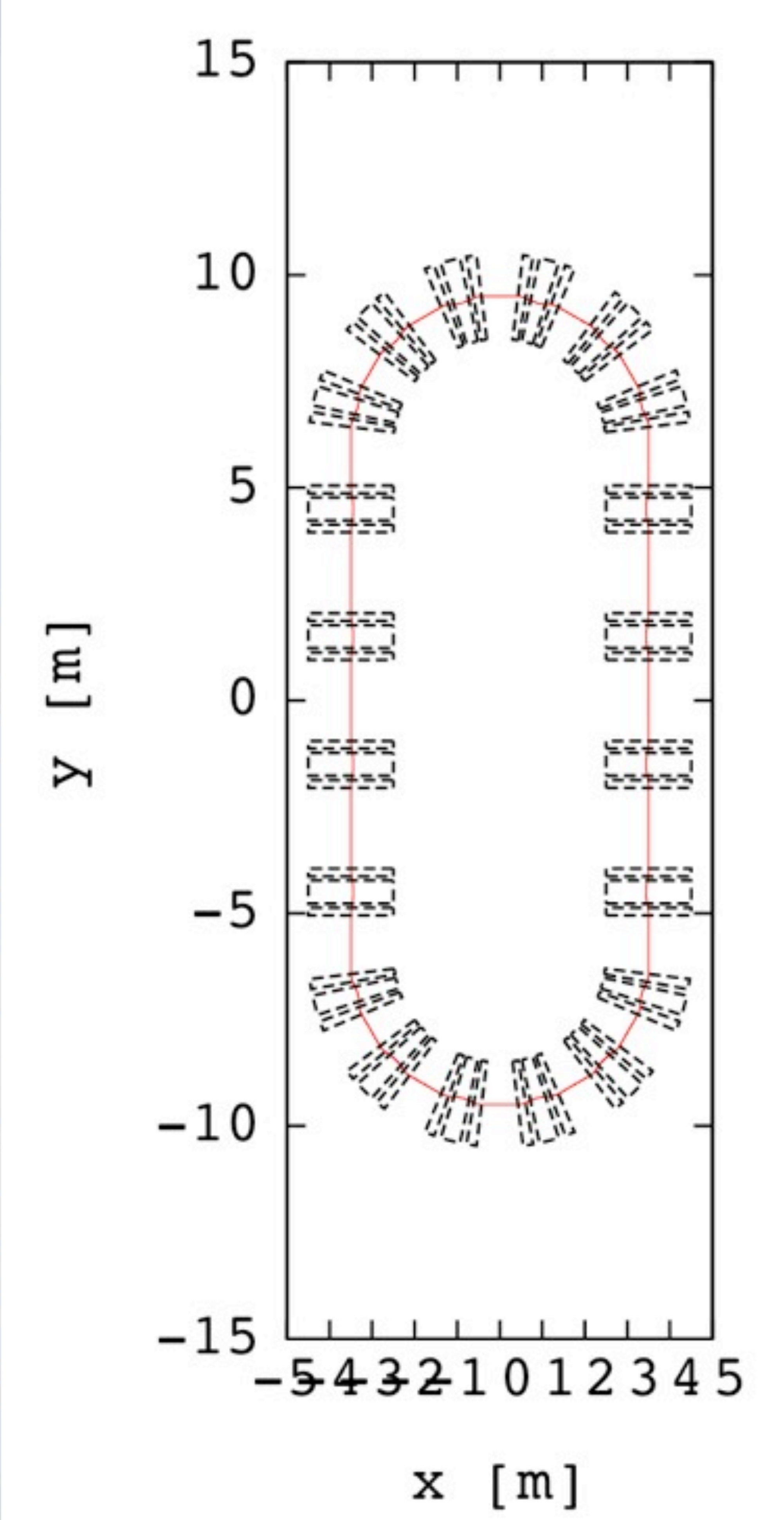
with $r = x + r_0$

$$k = \frac{r_0}{\rho}n$$

$$n = \frac{\rho}{B}\left(\frac{dB}{dx}\right)_{z=0}$$

⇒

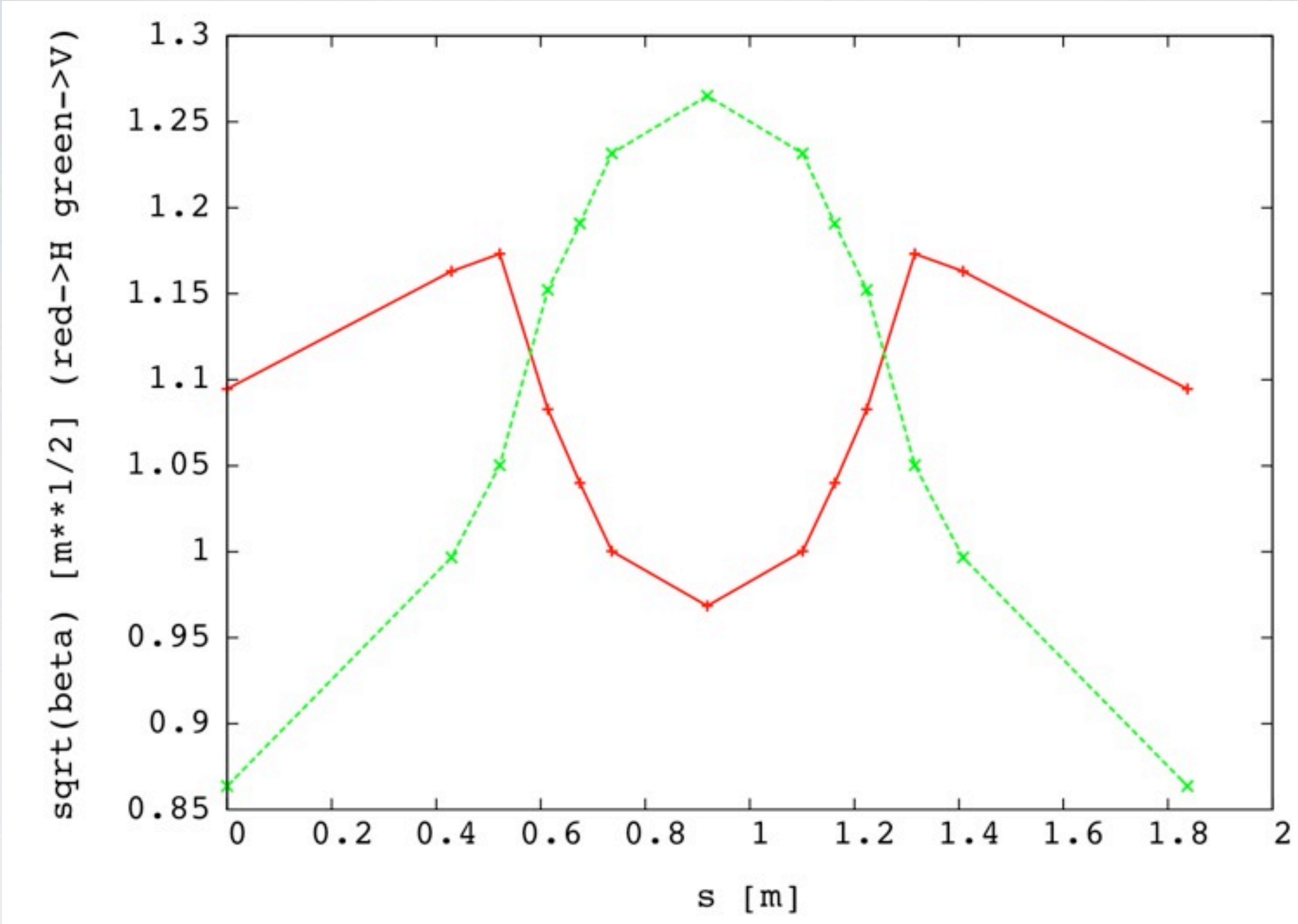
$$\boxed{B_z = B_0 e^{\frac{n}{\rho}(X-X_0)}}$$



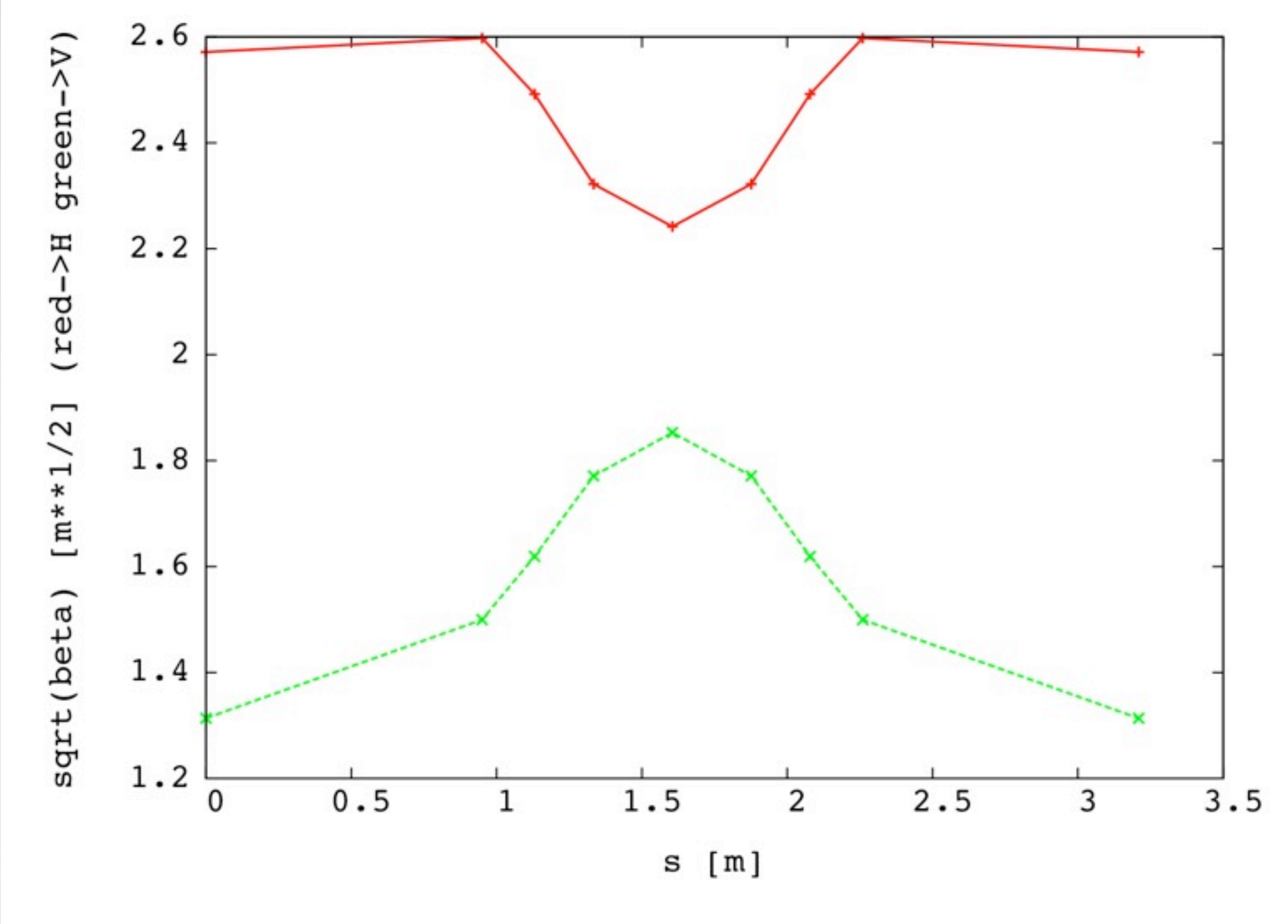
ANOTHER LATTICE

Bending cell	
k	6.5
Average radius	3.5 m
Phase advances:	
horizontal μ_x	90 deg.
vertical μ_z	87 deg.
Dispersion	0.47 m

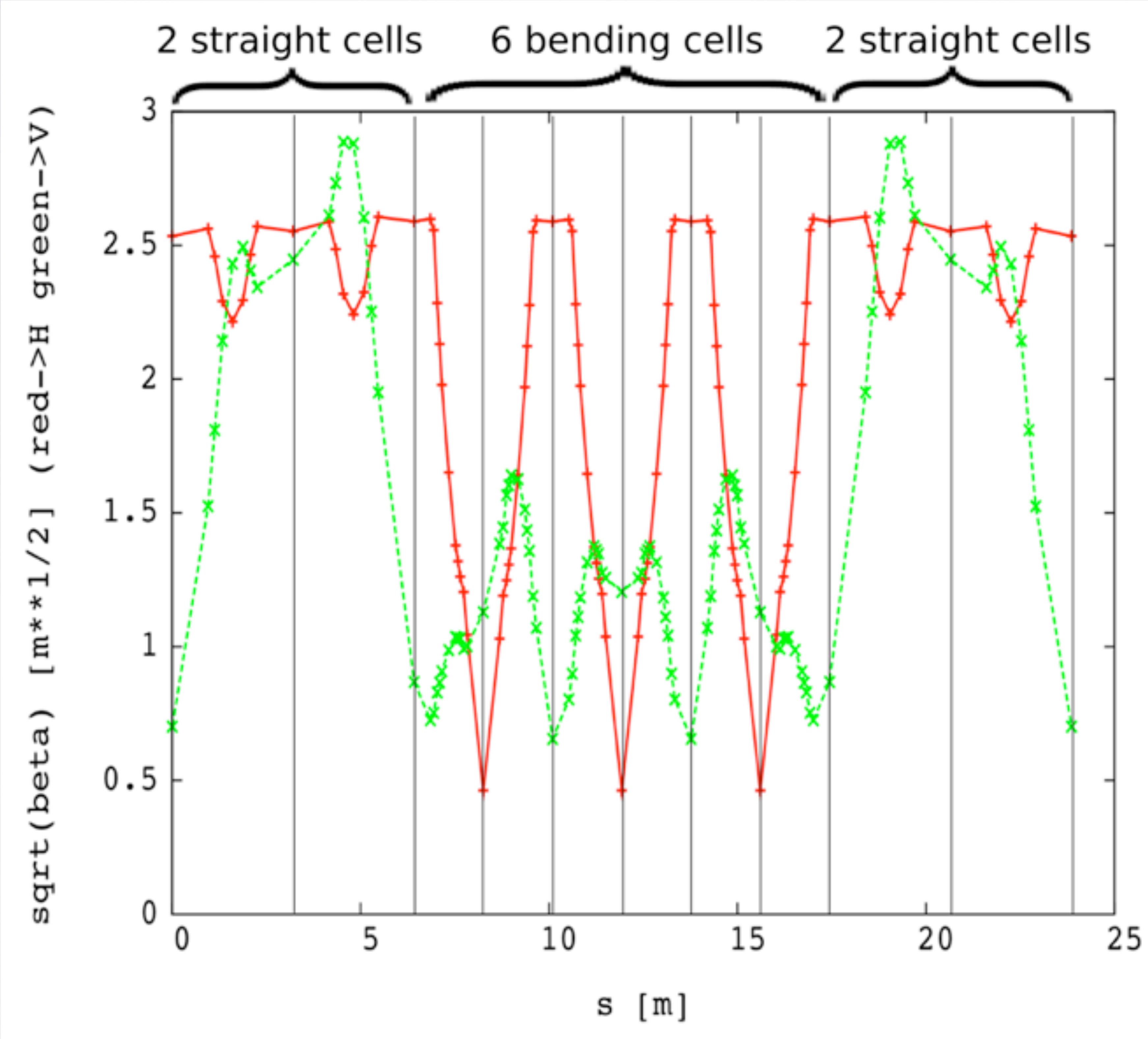
Straight cell	
n/ρ	$2.14\,m^{-1}$
Length	$3\,m$
Phase advances:	
horizontal μ_x	24 deg.
vertical μ_z	87 deg.



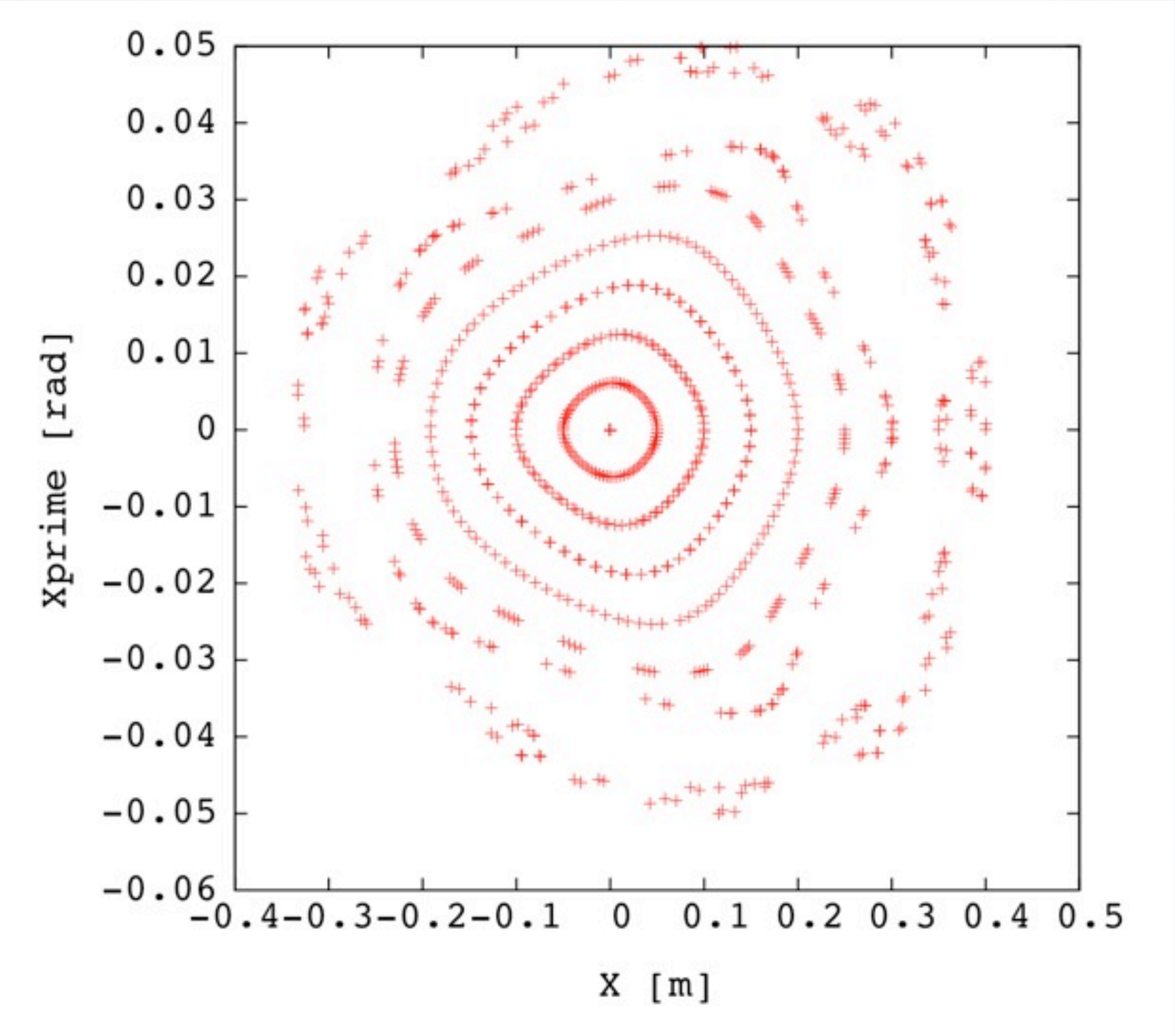
Betafunctions of bending cell.
(red: horizontal, green: vertical)



Betafunctions of straight cell.
(red: horizontal, green: vertical)



Betafunctions of bending and straight cells (half ring)
(red: horizontal, green: vertical)

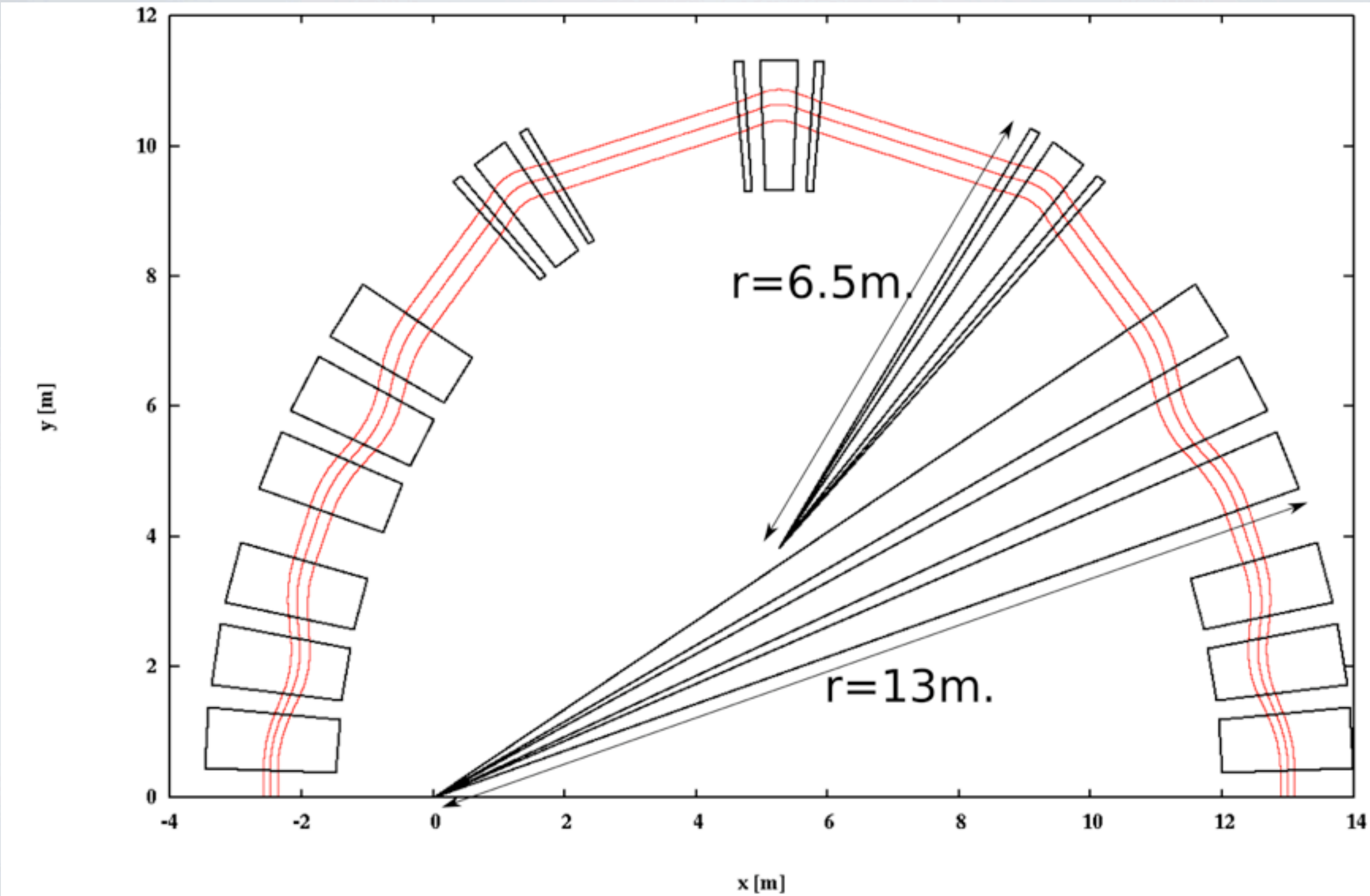


Horizontal Poincarre map

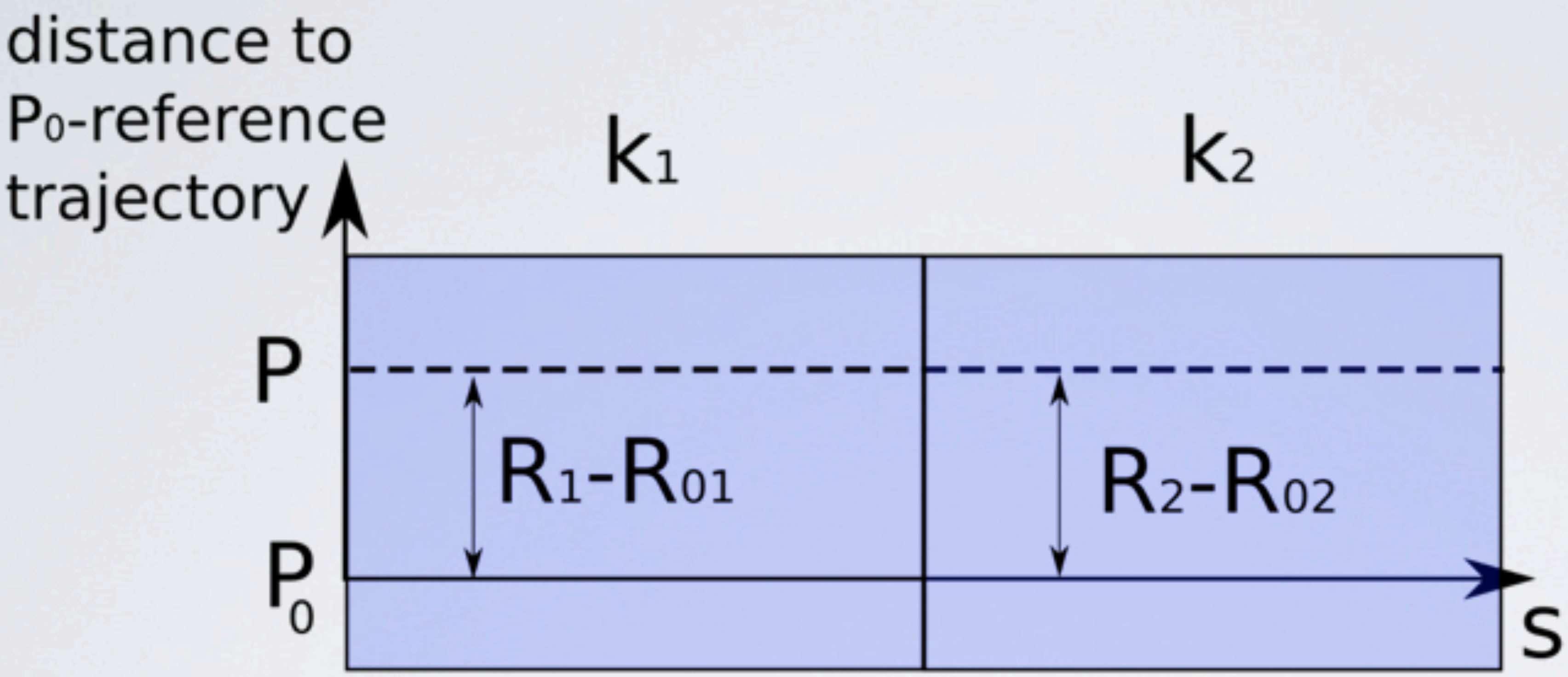
STILL A LOT OF WORK TO DO...

THANK YOU FOR YOUR ATTENTION

APPLICATION: PRISM



CHANGE RADIUS



$$R_1 - R_{01} = R_2 - R_{02}$$

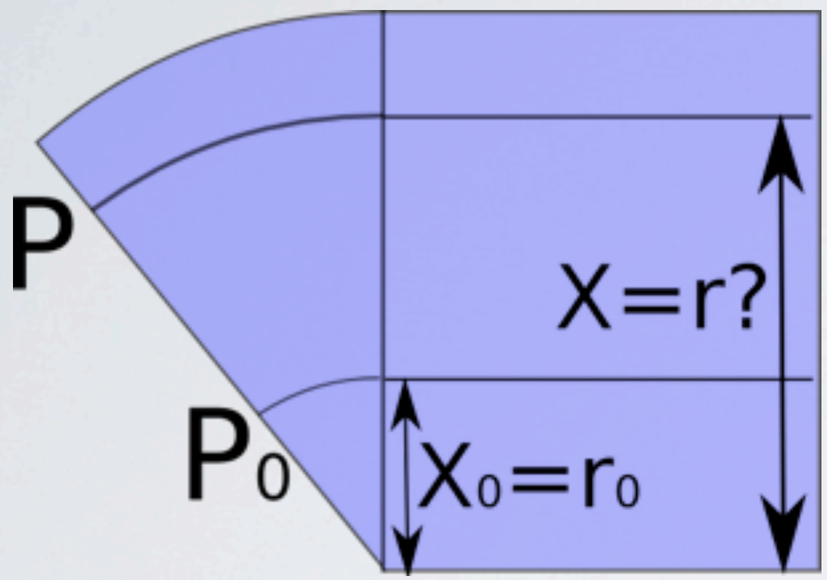
$$R = R_0 \left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}$$

1st order

$$\frac{R_{01}}{R_{02}} = \frac{k_1 + 1}{k_2 + 1}$$

MISMATCH BEND-STRAIGHT

k_b, r_0 $\quad n/\rho, X_0$



Straight cell: $B_z = B_{0s} e^{\frac{n}{\rho_s}(X - X_0)}$

Bending cell: $B_z = B_{0b} \left(\frac{r}{r_0} \right)^{k_b}$

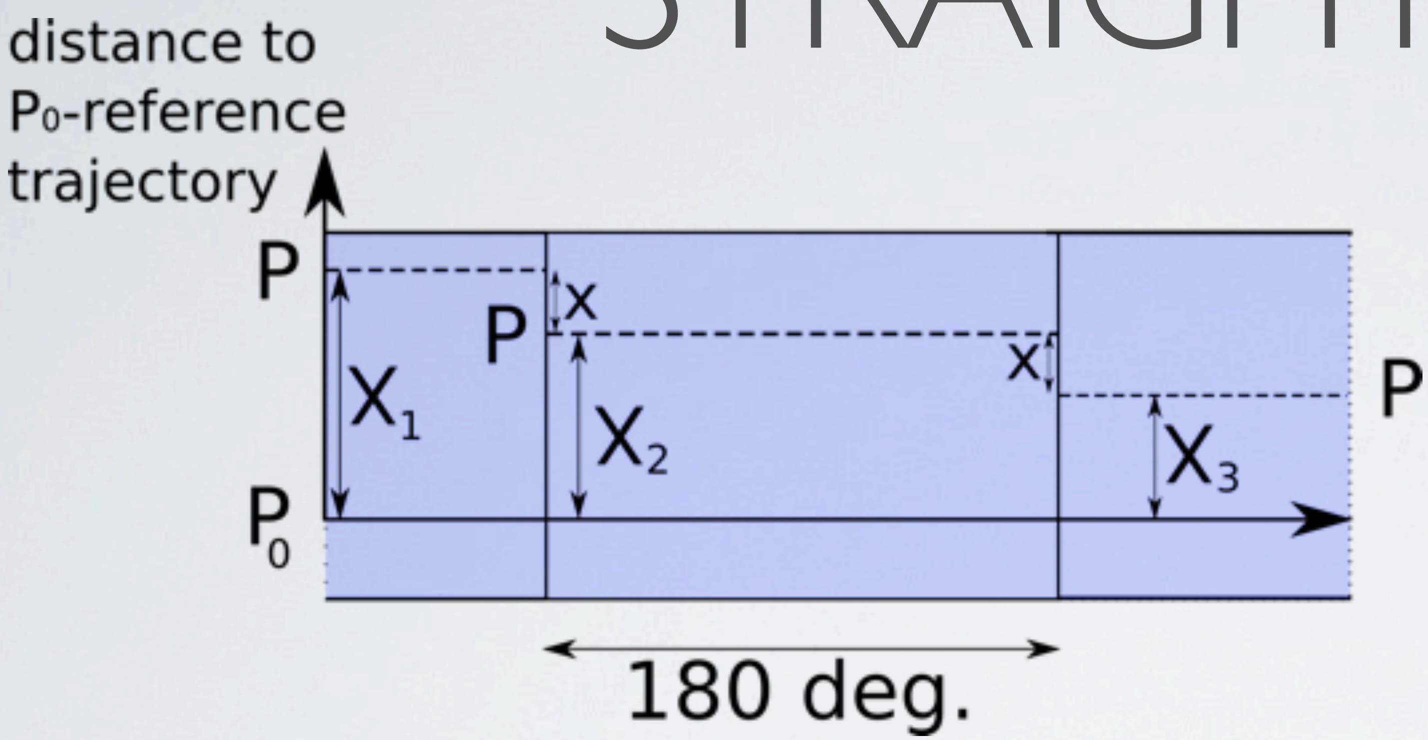
Matching of P_0: $B_{0s}\rho_s = B_{0b}\rho_b$

Matching of P: $B_{0s}\rho_s e^{\frac{n}{\rho}(X - X_0)} = B_{0b}\rho_b \left(\frac{r}{r_0} \right)^{k_b+1}$

1st order

$$\Rightarrow \quad 1 + (k_b + 1)\left(\frac{r - r_0}{r_0} \right) = 1 + \frac{n}{\rho_s}(X - X_0) \quad \Rightarrow \quad \frac{n}{\rho_s} = \frac{k_b + 1}{r_0}$$

DISPERSION SUPPRESSOR IN STRAIGHT LINES



$$X_2 - (X_1 - X_2) = X_3$$

$$2X_2 = X_1 + X_3$$

$$X = \frac{\rho}{n}\ln(\frac{P}{P_0})$$

$$\Rightarrow \quad 2\frac{\rho_2}{n_2} = \frac{\rho_1}{n_1} + \frac{\rho_3}{n_3}$$