Quark masses and strong coupling constant from Lattice QCD

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HPQCD collaboration

Quarkonium Working Group meeting,
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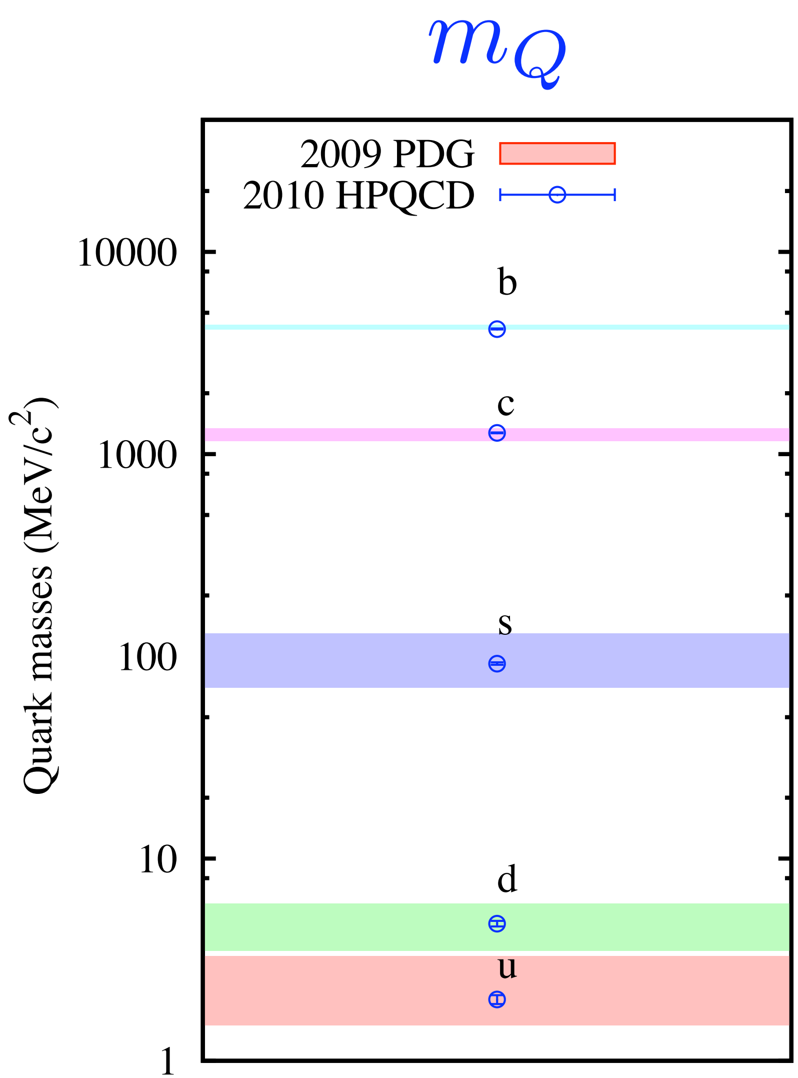
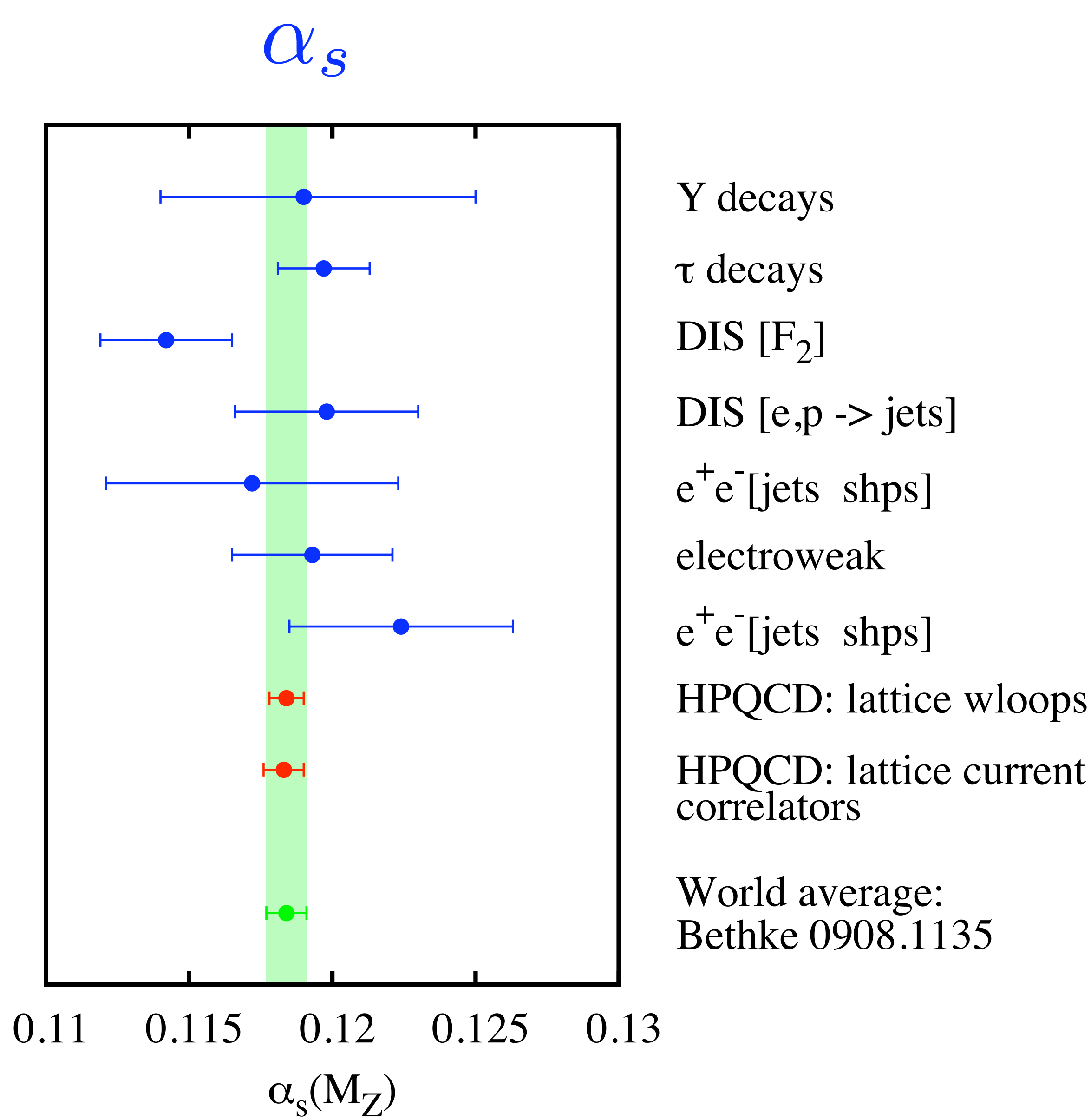
Preview of punchline - Lattice QCD is best method for extracting QCD parameters

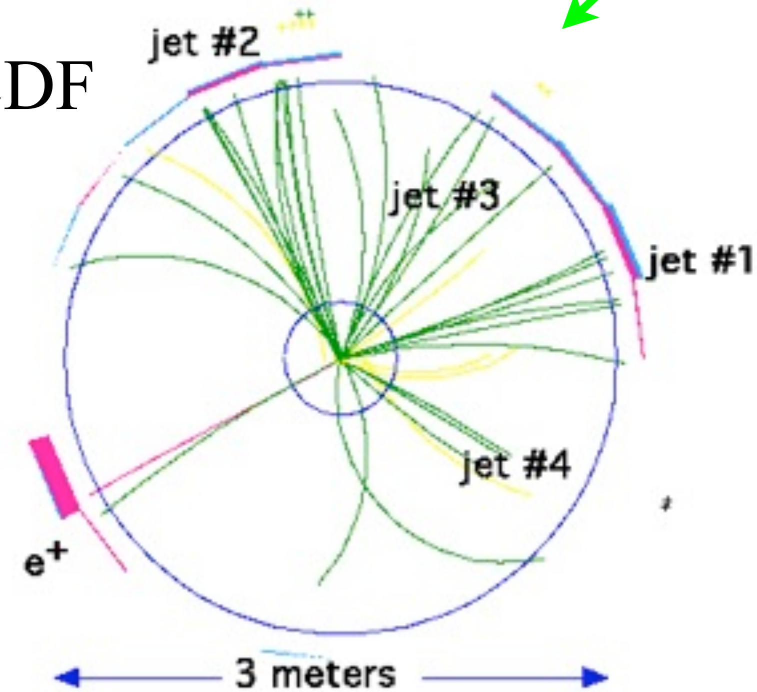


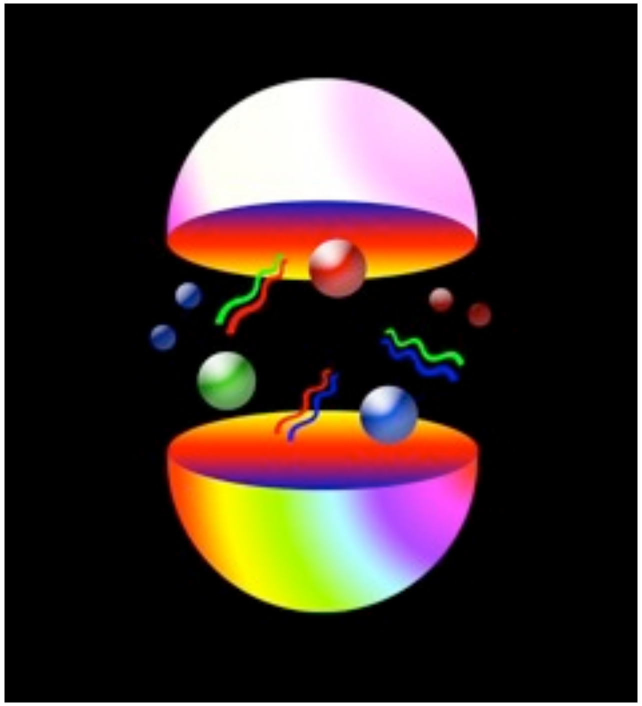


C. McNeile et al, HPQCD, 1004.4285

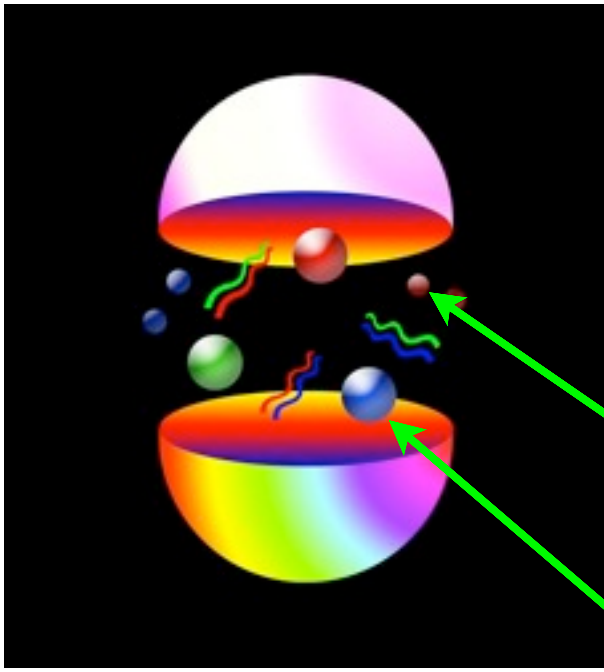
Quark confinement complicates the comparison of perturbative QCD to experiment for eg. jet shapes etc

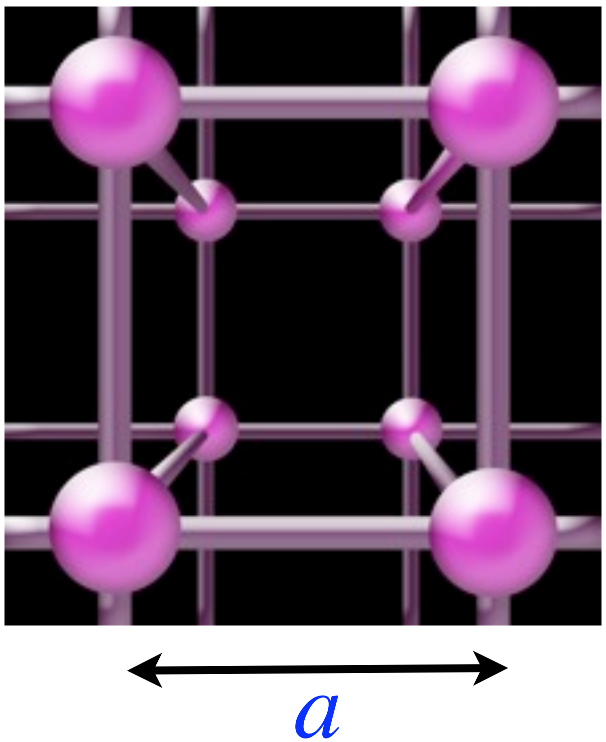
CDF





But simple properties of hadrons (e.g. masses) can now be accurately calculated in lattice QCD ⟶ QCD parameters in Lagrangian can be very accurately tuned. Issue is converting to non-lattice renormln schemes.





Lattice QCD = fully nonperturbative QCD calculation

RECIPE

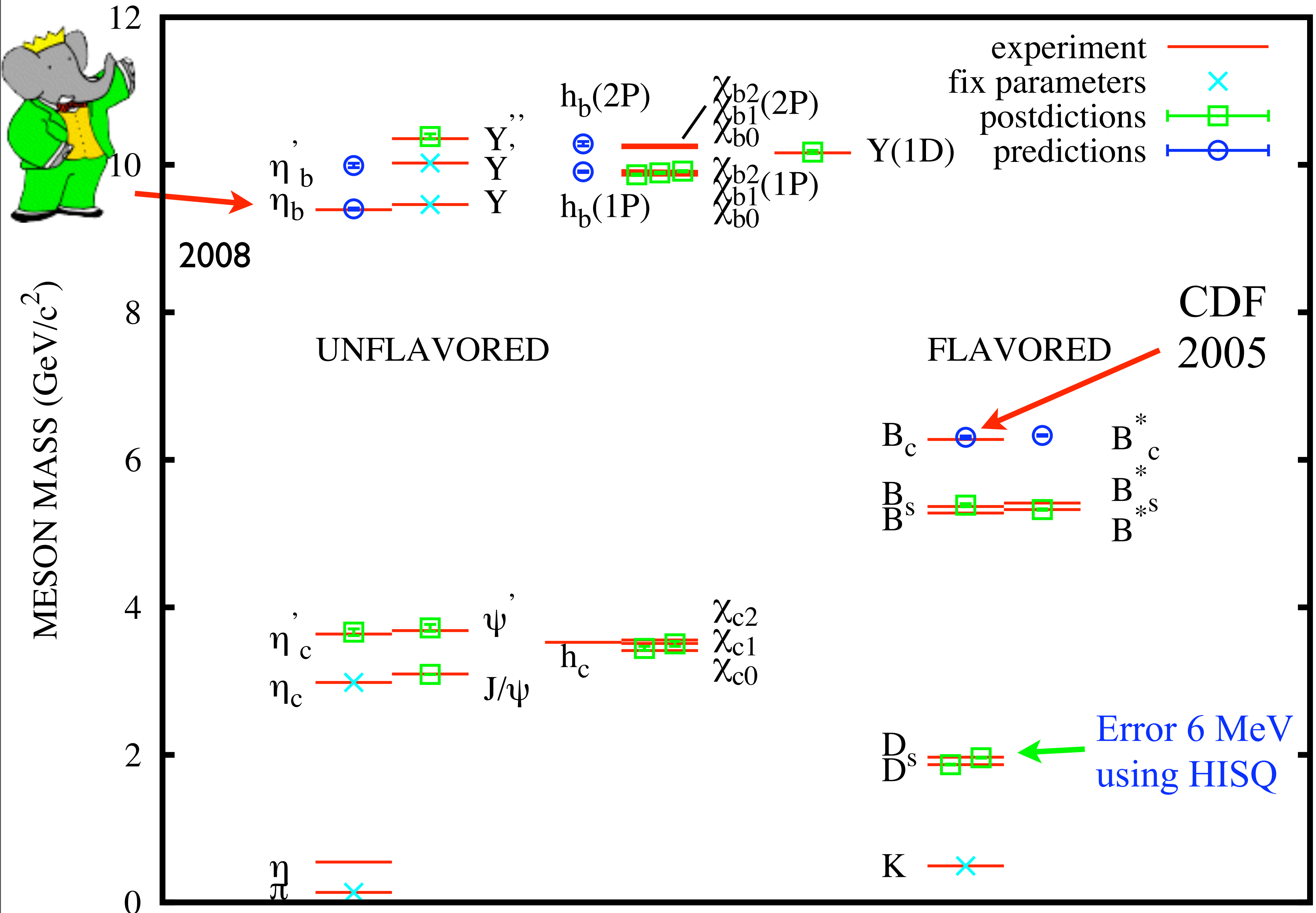
- Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of sea quarks)
- Calculate averaged "hadron correlators" from valence q props.

$$< 0|M^{\dagger}(0)M(t)|0 >$$

- Fit for hadron masses and decay constants.
- Fix m_q and a to get physical results

- extrapolate to real world

Overview: The gold-plated meson spectrum - HPQCD



Accuracy in spectrum requires accurate discretisation of quarks in lattice QCD...

Charm quarks in lattice QCD - heavy or light?

Advantages of relativistic light quarks:

- $E_{sim} = m$
- PCAC relation (if enough chiral symmetry) gives $Z = 1$
- same action as for u, d, s, so cancellation in ratios

Key issue is discretisation errors:

$$m = m_{a=0}(1 + A(m_c a)^2 + B(m_c a)^4 + \ldots$$

$$m_c a \approx 0.4,\ (m_c a)^2 \approx 0.2,\ \alpha_s (m_c a)^2 \approx 0.06,\ (m_c a)^4 \approx 0.04$$

for $a \approx 0.1 fm$

Need to remove *all* of these errors for precision results

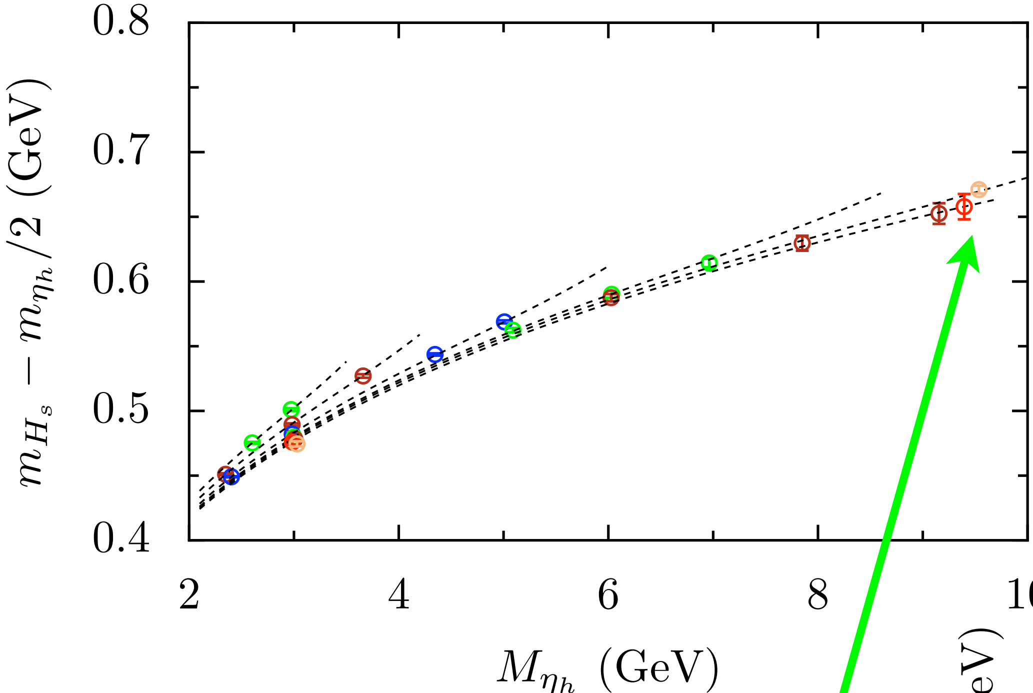
This is done in Highly Improved Staggered Quarks (HISQ) formalism, further improving Improved Staggered Quarks

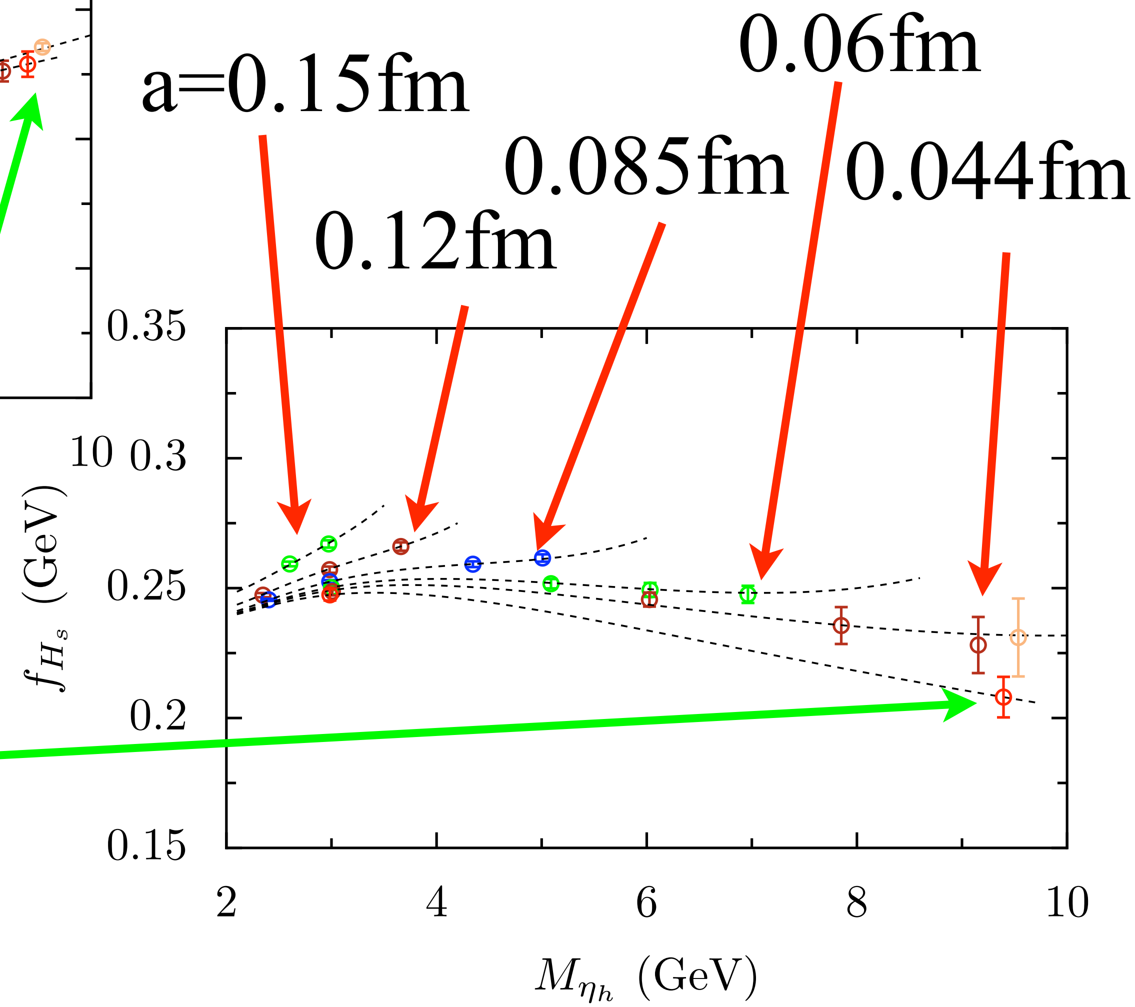
E. Follana et al, HPQCD, hep-lat/0610092

Has led to stunning accuracy in charm physics

e.g. 2% errors in f_{D_s} E. Follana et al, HPQCD, 0706.1726

Now using for even heavier quarks, up to bottom:





Extrapolate as a function of a and m_h to physical curve

Extrapolated HISQ result

HPQCD, in preparation

Determination of quark masses in $\overline{MS}$ scheme

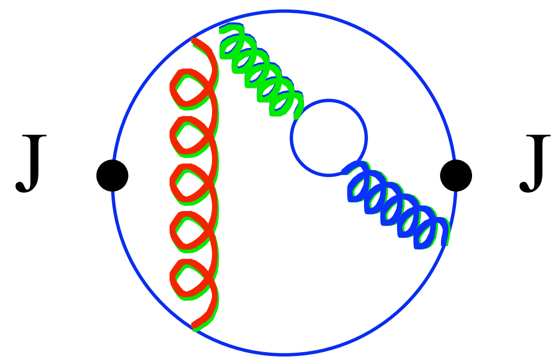
- Direct method: Determine $m_{q,latt}$ in lattice QCD.

$$m_{q,\overline{MS}} = Z m_{q,latt}$$

Calculate Z in lattice QCD pert. th. or use 'nonpert' lattice matching.
Best direct error on m_s 5% from α_s^2 pert. theory.

Q. Mason et al, HPQCD, hep-lat/0511160

- Current-current correlator method: match time-moments of heavy-heavy meson correlators to energy-derivative moments at $q^2 = 0$ of heavy quark vac. pol. calculated in continuum QCD pert. th. (thru α_s^3)



HPQCD + Chetyrkin et al, 0805.2999

see C. Sturm talk

Lattice calculation: first pass

HPQCD + Chetyrkin et al, 0805.2999

- Fix m_q to m_c in correlators by getting m_η correct.
- Calculate time moments and ratio to tree level ('free') .

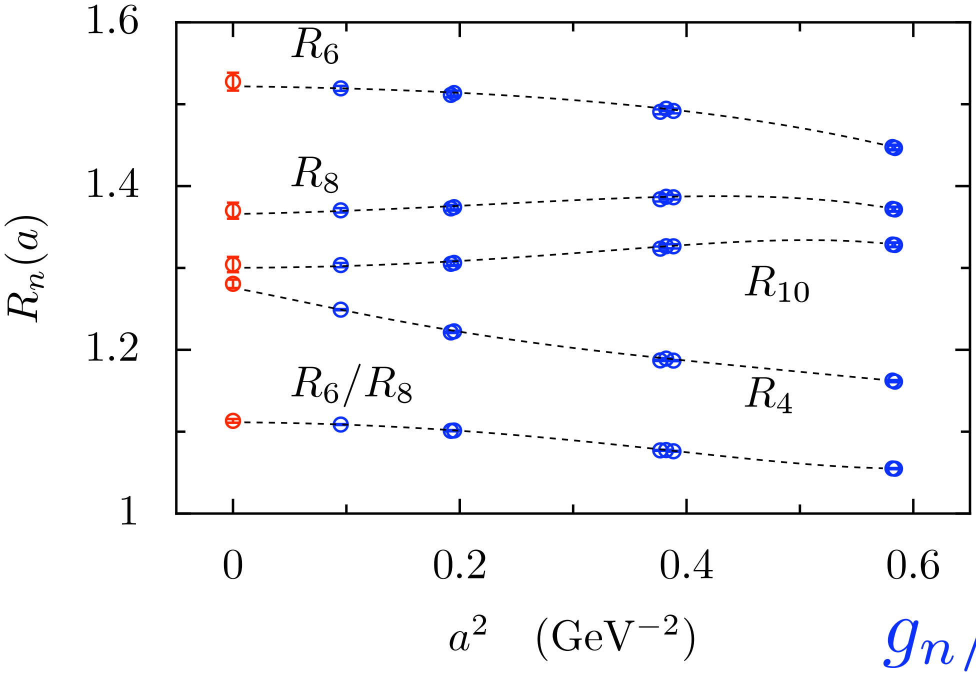
$$G(t) = a^6 \sum_{\vec{x}} (am_c)^2 < 0|j_5(\vec{x},t) j_5(0,0)|0 >$$

$$G_n = \sum_t (t/a)^n G(t)$$

$$R_{n,latt} = G_4/G_4^{(0)} \quad n = 4$$

$$= \frac{am_{\eta_c}}{2am_c} (G_n/G_n^{(0)})^{1/(n-4)} \quad n = 6, 8, 10 \ldots$$

- extrapolate to a=0 (and physical sea quark masses).

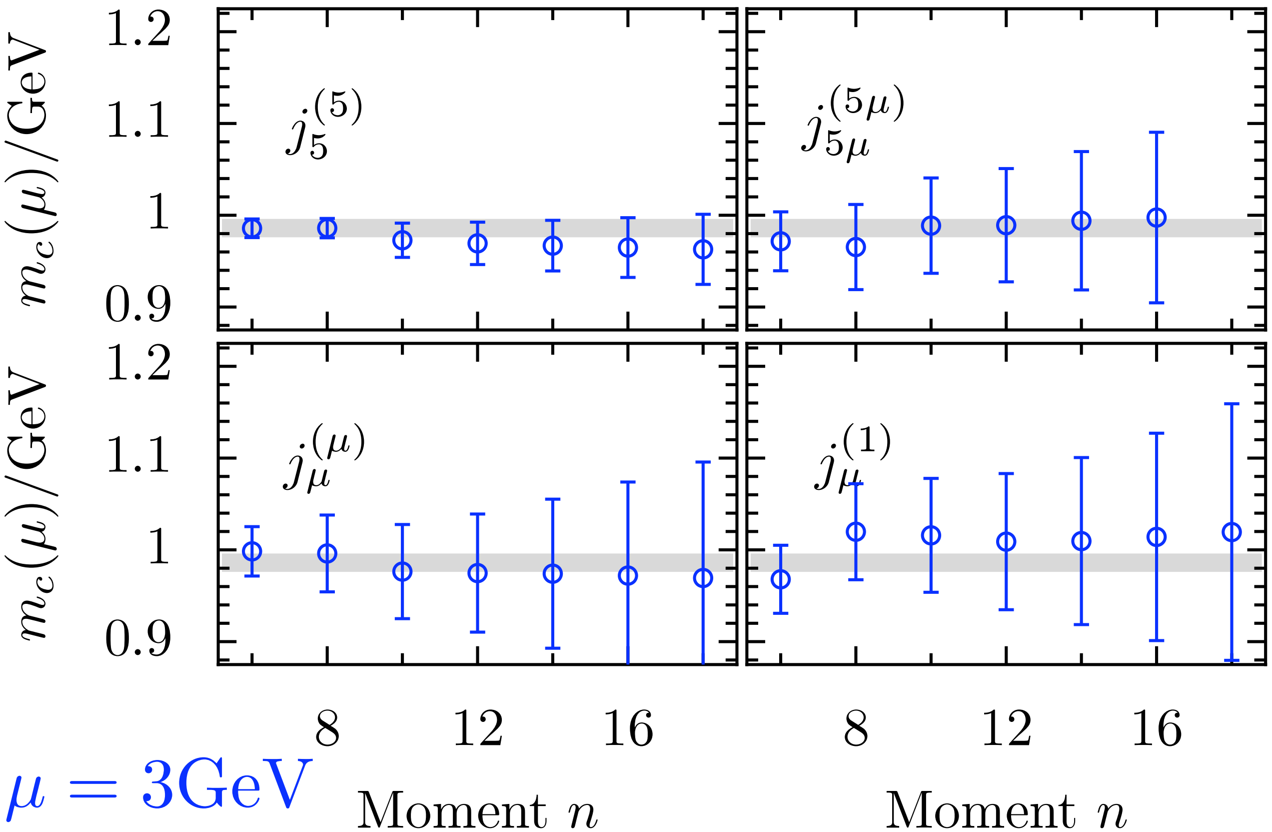


$$R_{n,cont} = g_4/g_4^0 \quad n = 4$$

$$= \frac{m_{\eta_c}}{2\overline{m}_c(\mu)} g_n/g_n^0$$

$$n = 6, 8, 10 \ldots$$

$$g_n/g_n^0 = 1 + \sum_i c_i(\mu/\overline{m}(\mu))\alpha_{\overline{MS}}(\mu)^i$$



$\mu = 3\mathrm{GeV}$

extract m_c from ratio to m_{η_c}

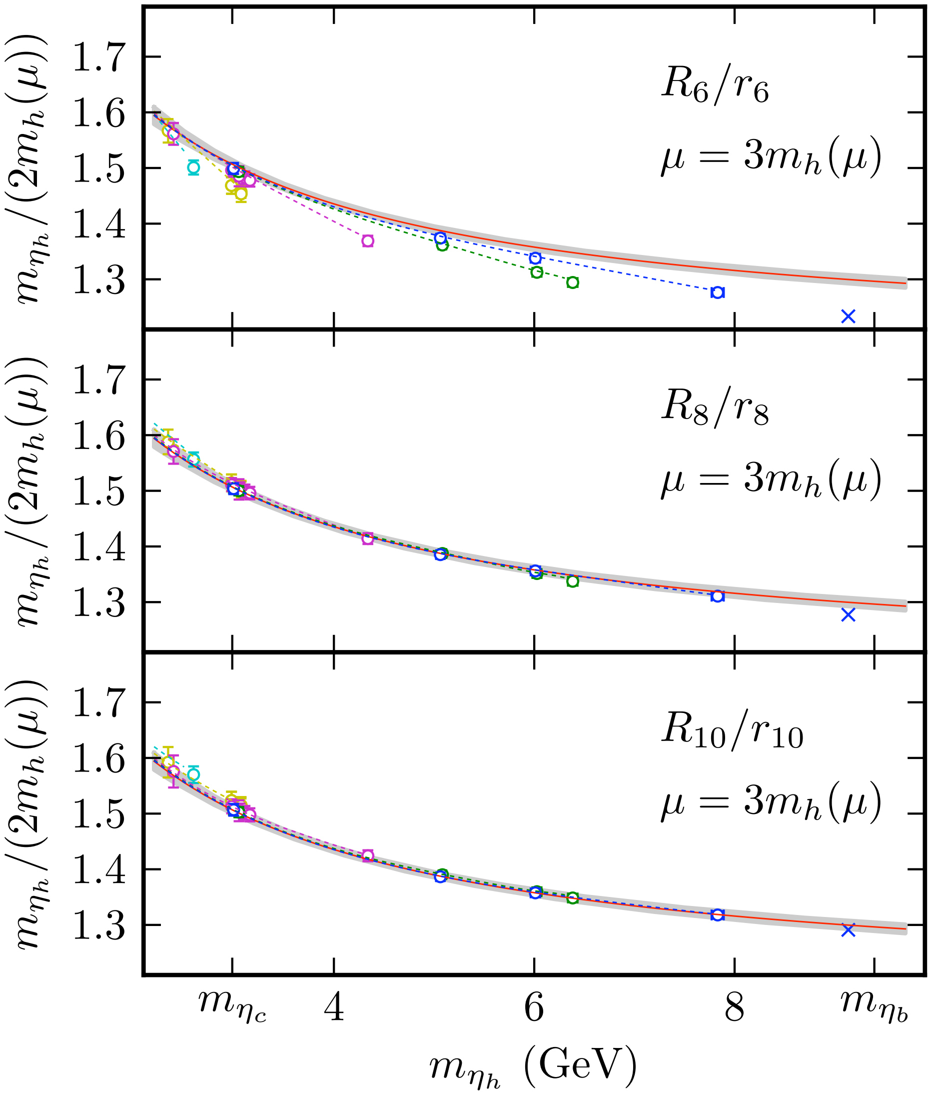
Different j agree, but pseudoscalar best.

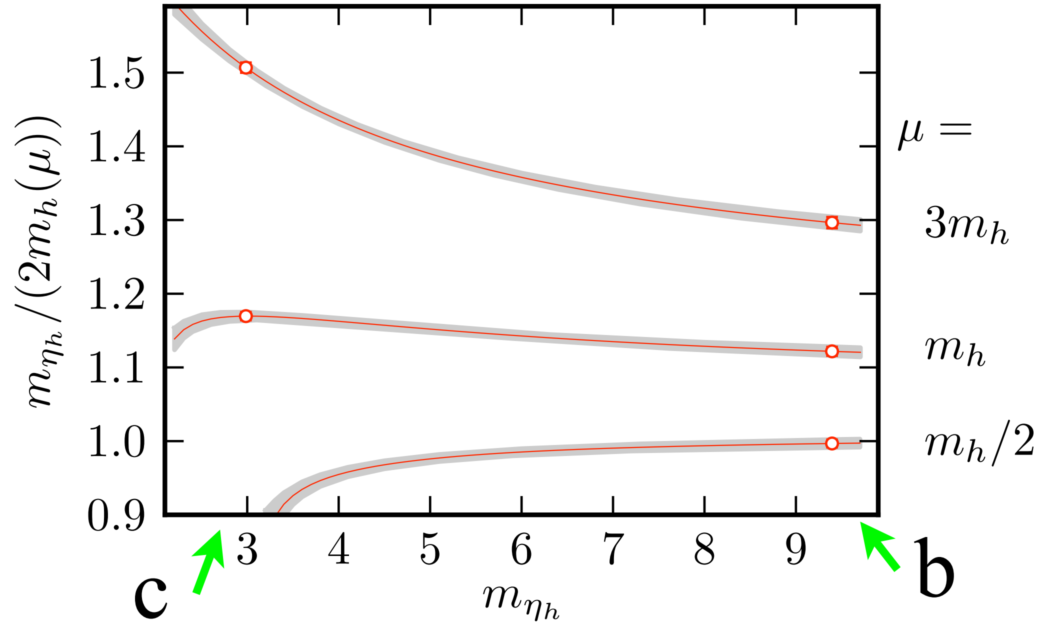
Can also determine α_s

Lattice calculation: second pass

HPQCD, 1004.4285

- Repeat calcln for $m_q \geq m_c$ inc. ultrafine lattices



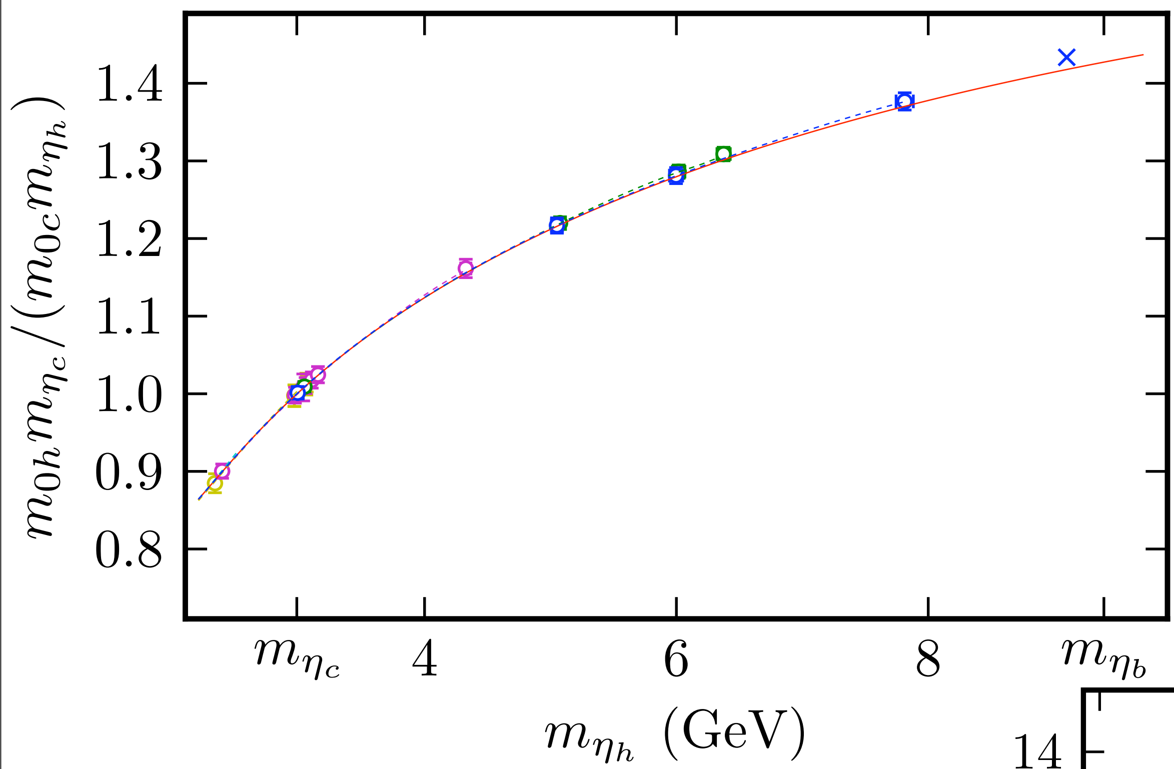


Can determine m_h/m_{η_h} for heavy quarks - extrapolate (slightly) to b.

$$m_c^{n_f=4}(3\text{GeV}) = 0.986(6)\text{GeV}$$
$$m_b^{n_f=5}(10\text{GeV}) = 3.618(25)\text{GeV}$$

Agree well with results using $R_{e^+e^-}$ (C. Sturm talk)

Quark mass ratios from lattice QCD



$$\left(\frac{m_{q1,latt}}{m_{q2,latt}} \right)_{a=0} = \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

completely nonpert.

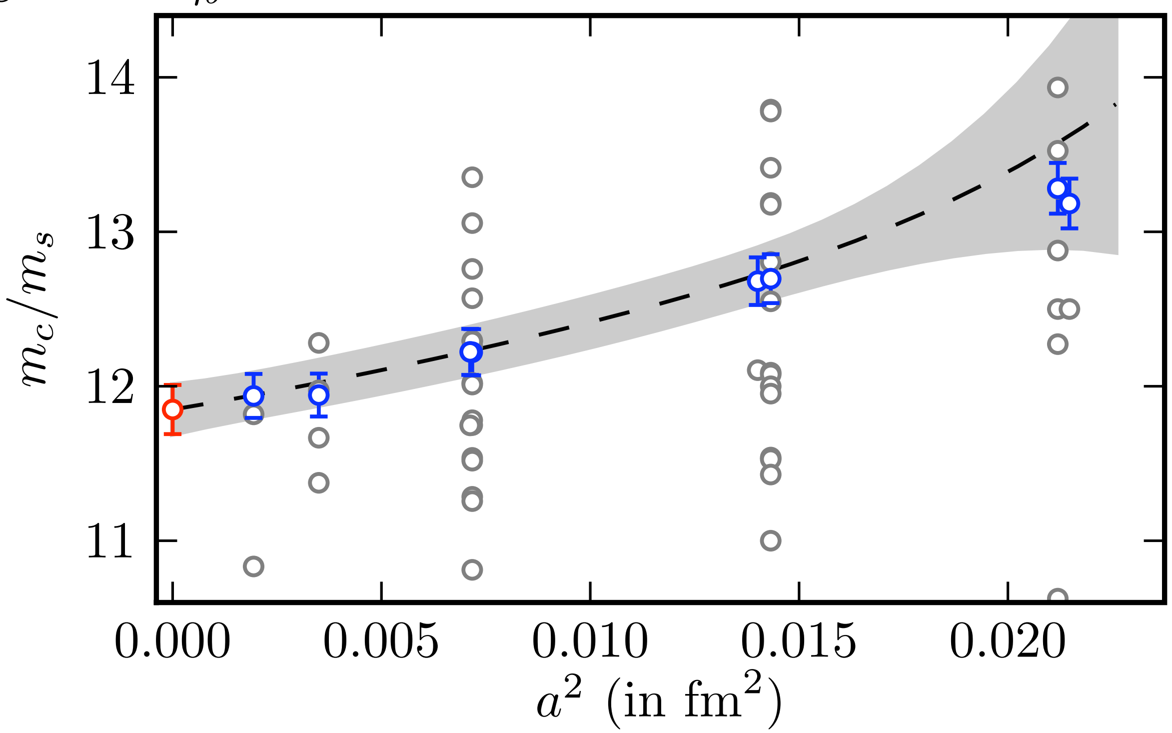
$$\frac{m_b}{m_c} = 4.49(4)$$

strong test of c.c. method.

Determine m_c/m_s using HISQ for both - allows connection from heavy to light for first time

$$\frac{m_c}{m_s} = 11.85(16)$$

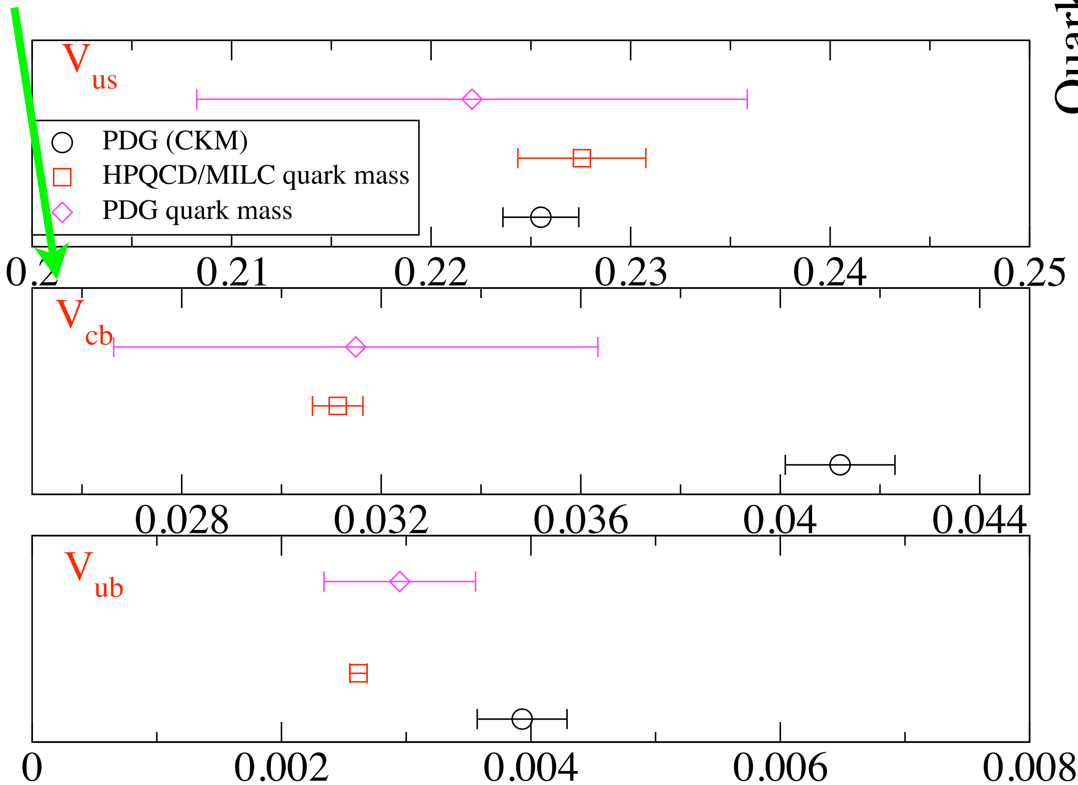
HPQCD, 0910.3102

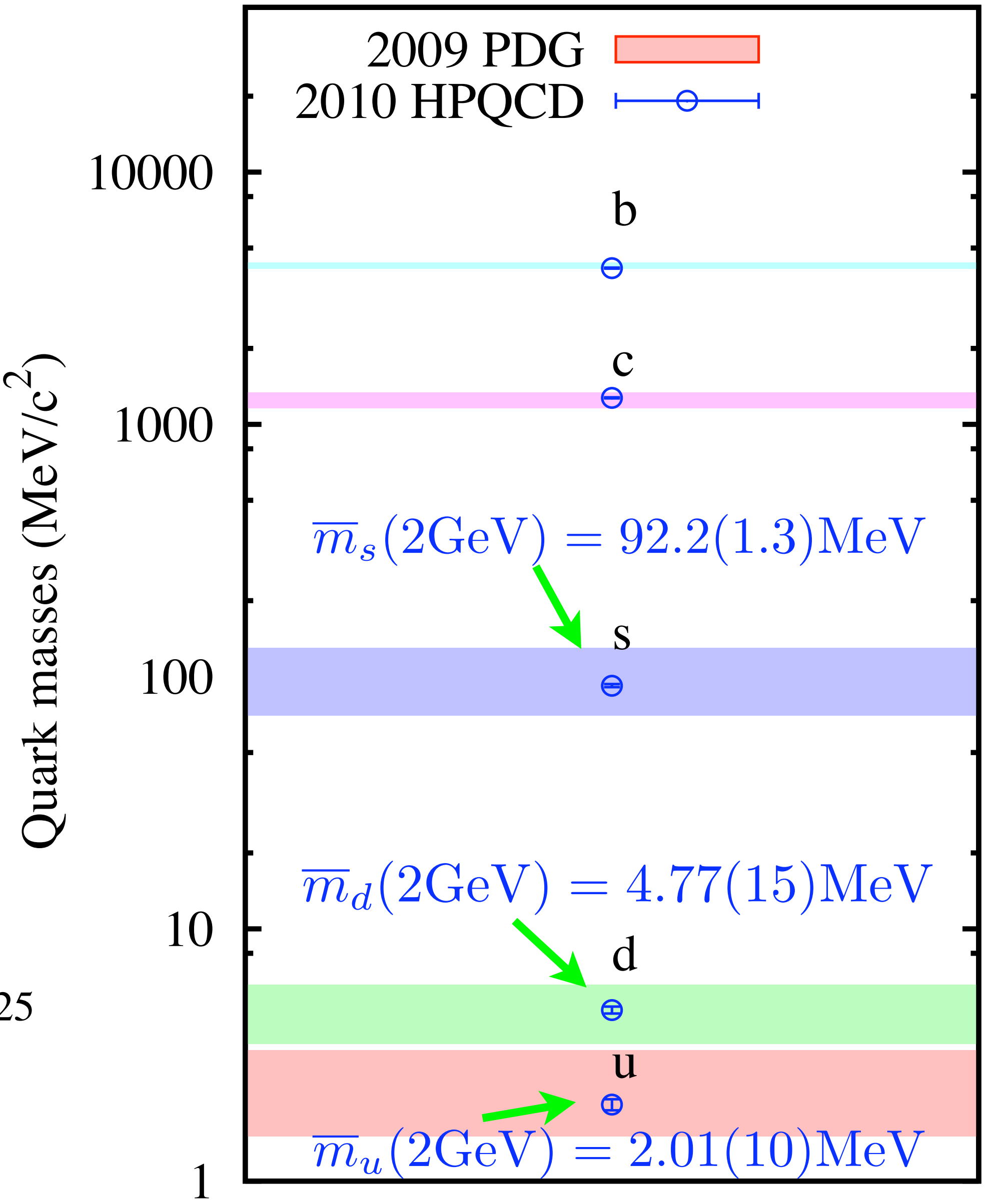


Summary - quark masses

Using heavy quark leverage, m_s error down to 1.5% (PDG has 30%!)

Accurate ratios rule out some quark mass matrix models based on textures.





HPQCD, 1004.4285

C. McNeile, 1004.4985

Determination of α_s in $\overline{MS}$ scheme

1) current-current correlator method

Comparing R_n to contnm pert. theory allows $\alpha_{\overline{MS}}(\mu)$ to be determined along with masses.

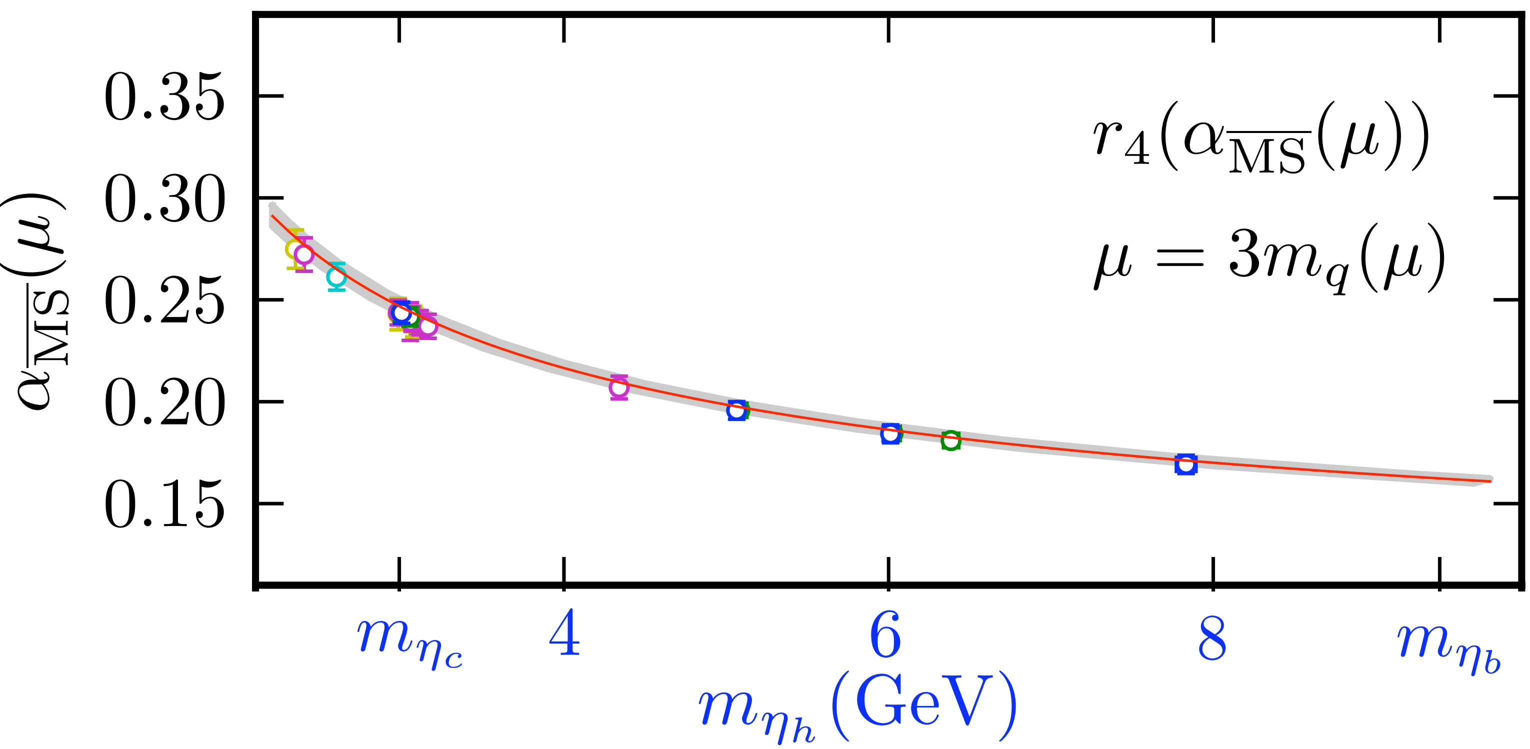
An NNNLO determn, but higher orders also inc. in fit.

$\alpha_{\overline{MS}}^{n_f=3}(5\mathrm{GeV})$

$= 0.2034(20)$

$\alpha_{\overline{MS}}^{n_f=5}(m_Z)$

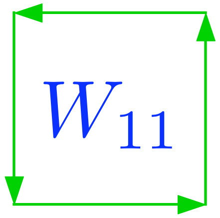
$= 0.1183(7)$



Determination of α_s in $\overline{MS}$ scheme

1) Wilson loops method

$$X = \sum_n c_n \alpha_s^n \left(\frac{d}{a}\right)$$



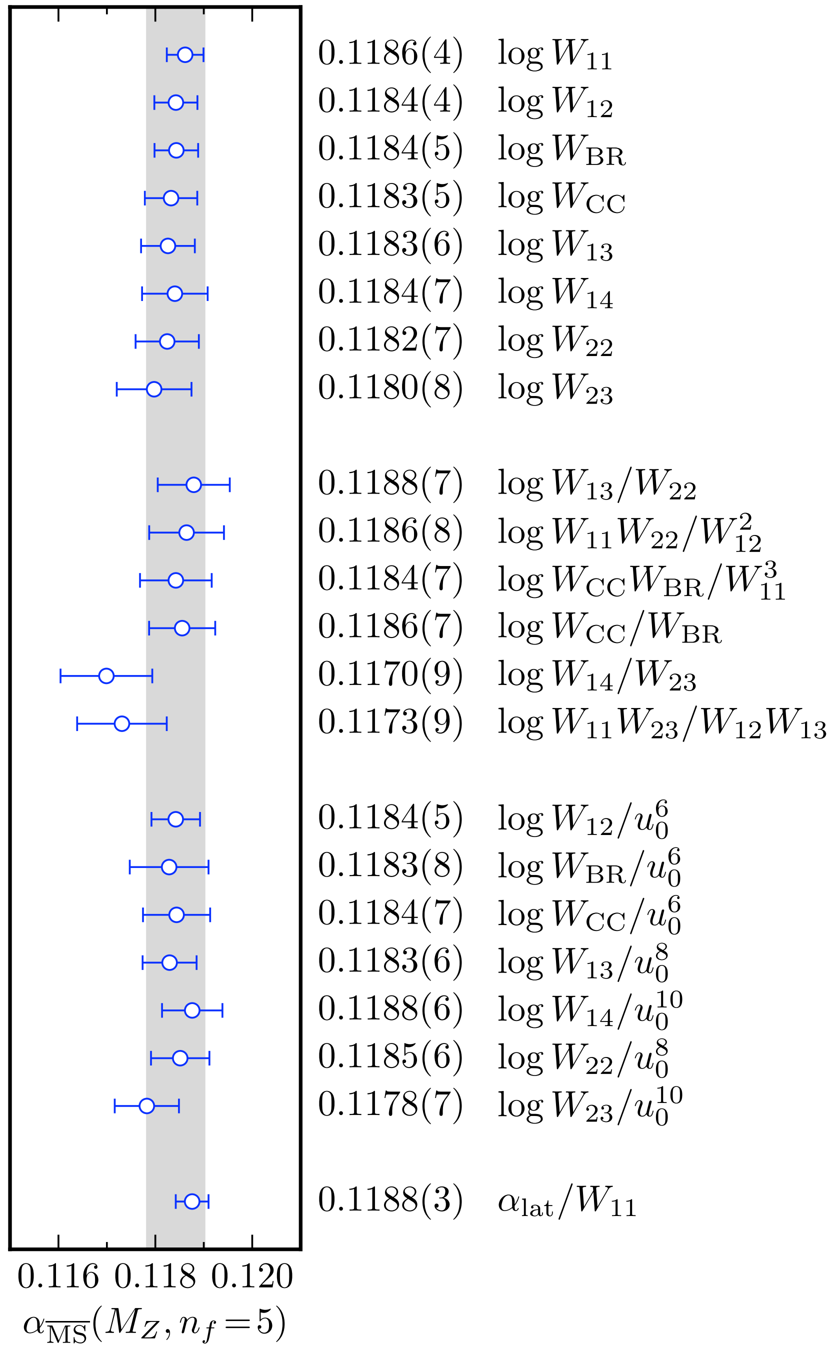
Calculate for 22 Wilson loops in lattice QCD pert. th. inc. α_s^3

Fit to data from lattices at 6 a values - 0.15 fm to 0.045 fm, allowing for higher orders.

$$\alpha_{\overline{MS}}^{n_f=5}(m_Z) = 0.1184(6)$$

Improved a reduces error 8 to 6.

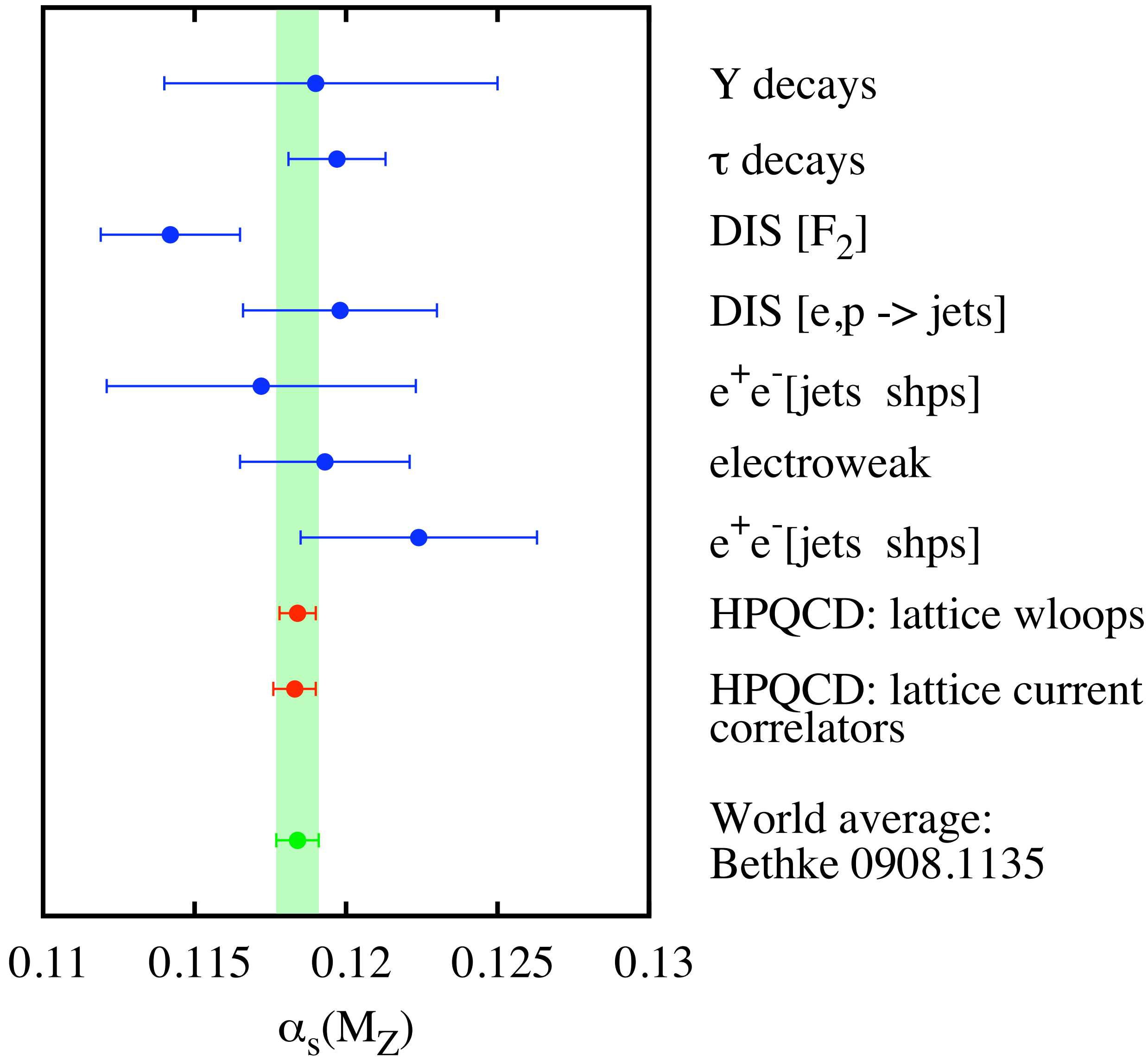
HPQCD, 1004.4285 and 0807.1687



Summary - α_s

Lattice QCD now has 2 determns of α_s to better-than-1%.

Key points:
- high statistical precision
- high order pert. th. exists and can estimate higher orders
- higher twist not a significant issue
- two approaches very different - good test



Conclusions

• Lattice QCD is unbeatable at accurate determination of QCD parameters. This is not surprising.

• Keys were a highly improved quark formalism (HISQ) on very fine lattices + high order pert. th. Would be good to get results from other quark formalisms, but ...

Further calculations in future

• α_s from light meson correlators at large space-like

JLQCD with 1 lattice spacing get $\alpha_{\overline{MS}}^{n_f=5} = 0.1183(13)$

JLQCD, 1002.0371

• Generating ensembles with $a = 0.03\text{fm}$ now possible with improved staggered formalism. Would allow relativistic b quarks with no extrapolation.

Error budgets

current-current correlator method

	$\alpha_{\overline{\rm MS}}(M_Z)$	$m_b(10)$	m_b/m_c	$m_c(3)$
a^2 extrapolation	0.2%	0.6%	0.5%	0.2%
perturbation theory	0.5	0.1	0.5	0.4
statistical errors	0.1	0.3	0.3	0.2
m_h extrapolation	0.1	0.1	0.2	0.0
errors in r_1	0.2	0.1	0.1	0.1
errors in r_1/a	0.1	0.3	0.2	0.1
errors in m_{η_c},m_{η_b}	0.2	0.1	0.2	0.0
α_0 prior	0.1	0.1	0.1	0.1
gluon condensate	0.0	0.0	0.0	0.2
Total	0.6%	0.7%	0.8%	0.6%

HPQCD, 1004.4285
and 0807.1687

wilson loops method

	$\log W_{11}$	$\log W_{12}$	$\log W_{22}$	$\log W_{11}W_{22}/W_{12}^2$	$\log W_{12}/u_0^6$	$\log W_{22}/u_0^8$	$\alpha_{\rm lat}/W_{11}$
$c_1 \ldots c_3$	0.1%	0.1%	0.1%	0.3%	0.1%	0.1%	0.1%
c_n for $n \geq 4$	0.2	0.3	0.3	0.4	0.3	0.4	0.3
am_q, $r_1 m_q$ extrapolation	0.1	0.1	0.0	0.1	0.1	0.1	0.0
$(a/r_1)^2$ extrapolation	0.2	0.3	0.4	0.3	0.2	0.2	0.0
$(r_1/a)_i$ errors	0.4	0.4	0.4	0.3	0.3	0.3	0.3
r_1 errors	0.3	0.3	0.3	0.3	0.3	0.3	0.3
gluon condensate	0.1	0.1	0.1	0.2	0.1	0.1	0.1
statistical errors	0.0	0.0	0.0	0.1	0.0	0.0	0.0
$V \to \overline{\rm MS} \to M_Z$	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Total	0.6%	0.6%	0.7%	0.7%	0.6%	0.6%	0.5%

now halved by improved scale determn