$Q\bar{Q}$ static energy at N^{3}LL accuracy

Xavier Garcia i Tormo
University of Alberta
(work done with Nora Brambilla, Joan Soto and Antonio Vairo)

Phys. Rev. D **80**, 034016 (2009) [arXiv:0906.1390 [hep-ph]] + work in progress

Outline of the talk

Outline of the talk

- The $Q\bar{Q}$ static energy

Outline of the talk

- The $Q\bar{Q}$ static energy
- Lattice comparison at N^{3}LL accuracy

Outline of the talk

- The $Q\bar{Q}$ static energy
- Lattice comparison at N^{3}LL accuracy
- Determination of $r_0\Lambda_{\overline{\rm MS}}$

Outline of the talk

- The $Q\bar{Q}$ static energy
- Lattice comparison at N^{3}LL accuracy
- Determination of $r_0\Lambda_{\overline{\rm MS}}$
- Conclusions

$Q\bar{Q}$ static energy

Energy between a static quark and a static antiquark separated a distance r.

$E_0(r)$ at short distances can be written as

$$E_0(r) = V_s + \Lambda_s + \delta_{\mathrm{US}}$$

$Q\bar{Q}$ static energy

Energy between a static quark and a static antiquark separated a distance r.

$E_0(r)$ at short distances can be written as

$$E_0(r) = V_s + \Lambda_s + \delta_{\mathrm{US}}$$

V_s and Λ_s are matching coefficients in the effective theory *potential Non-Relativistic QCD* (pNRQCD):

-V_s: static potential

-Λ_s: inherits the residual mass term from HQET

$Q\bar{Q}$ static energy

Energy between a static quark and a static antiquark separated a distance r.

$E_0(r)$ at short distances can be written as

$$E_0(r) = V_s + \Lambda_s + \delta_{\mathrm{US}}$$

V_s and Λ_s are matching coefficients in the effective theory *potential Non-Relativistic QCD* (pNRQCD):

- V_s: static potential
- Λ_s: inherits the residual mass term from HQET

δ_{US} contains the contributions from ultrasoft gluons.

Virtual emission of ultrasoft gluons can change the color state of the pair from singlet to octet. Those effects first appear at three loop order $E_0 \sim \alpha_{\mathrm{s}}^4 \ln \alpha_{\mathrm{s}}$.

Nomenclature used in the talk:

Nomenclature used in the talk:
N^2LL accuracy $\rightarrow$ terms up to order $\alpha_s^{1+(2+n)}\ln^n\alpha_s$ with $n\geq 0$
N^3LL accuracy $\rightarrow$ terms up to order $\alpha_s^{1+(3+n)}\ln^n\alpha_s$ with $n\geq 0$

Nomenclature used in the talk:
N^2LL accuracy $\rightarrow$ terms up to order $\alpha_{\rm s}^{1+(2+n)} \ln^n \alpha_{\rm s}$ with $n \geq 0$
N^3LL accuracy $\rightarrow$ terms up to order $\alpha_{\rm s}^{1+(3+n)} \ln^n \alpha_{\rm s}$ with $n \geq 0$

Ultrasoft logs first appear in the static energy at

$$E_0 \sim -C_F \frac{\alpha_{\rm s}}{r} \left(\alpha_{\rm s}^3 \ln \alpha_{\rm s}\right)$$

Nomenclature used in the talk:
N^{2}LL accuracy $\rightarrow$ terms up to order $\alpha_{\rm s}^{1+(2+n)}\ln^n\alpha_{\rm s}$ with $n\geq 0$
N^{3}LL accuracy $\rightarrow$ terms up to order $\alpha_{\rm s}^{1+(3+n)}\ln^n\alpha_{\rm s}$ with $n\geq 0$

Ultrasoft logs first appear in the static energy at

$$E_0\sim -C_F\frac{\alpha_{\rm s}}{r}\left(\alpha_{\rm s}^3\ln\alpha_{\rm s}\right)$$

i.e. at N^{2}LL accuracy.

Nomenclature used in the talk:
N^2LL accuracy $\rightarrow$ terms up to order $\alpha_s^{1+(2+n)} \ln^n \alpha_s$ with $n \geq 0$
N^3LL accuracy $\rightarrow$ terms up to order $\alpha_s^{1+(3+n)} \ln^n \alpha_s$ with $n \geq 0$

Ultrasoft logs first appear in the static energy at

$$E_0 \sim -C_F \frac{\alpha_s}{r} \left(\alpha_s^3 \ln \alpha_s\right)$$

i.e. at N^2LL accuracy.

For simplicity, expressions without resummation of ultrasoft logs will be just referred to as : 1 loop, 2 loop...

US logs can be resummed using RG equations in the effective theory

US logs can be resummed using RG equations in the effective theory

$$\mu\frac{d}{d\mu}V_s \sim \gamma_s r^2 (V_o - V_s)^3$$

$$\mu\frac{d}{d\mu}\Lambda_s \sim \gamma_s' r^2 (\Lambda_o - \Lambda_s)^2$$

US logs can be resummed using RG equations in the effective theory

$$\mu\frac{d}{d\mu}V_s \sim \gamma_s r^2 (V_o - V_s)^3$$

$$\mu\frac{d}{d\mu}\Lambda_s \sim \gamma_s' r^2 (\Lambda_o - \Lambda_s)^2$$

US resummed expressions:

$$V_s(\mu) - V_s(1/r) \sim r^2 (V_o - V_s)^3 \ln\frac{\alpha_{\rm s}(\mu)}{\alpha_{\rm s}(1/r)}$$

US logs can be resummed using RG equations in the effective theory

$$\mu \frac{d}{d\mu} V_s \sim \gamma_s r^2 (V_o - V_s)^3$$

$$\mu \frac{d}{d\mu} \Lambda_s \sim \gamma_s' r^2 (\Lambda_o - \Lambda_s)^2$$

US resummed expressions:

$$V_s(\mu) - V_s(1/r) \sim r^2 (V_o - V_s)^3 \ln \frac{\alpha_{\rm s}(\mu)}{\alpha_{\rm s}(1/r)} \sim \mathcal{O}(\alpha_{\rm s}^3)$$

N^3LL accuracy thus includes:

N^{3}LL accuracy thus includes:
-V_s at three loop plus resummation of sub-leading ultrasoft logs

N^{3}LL accuracy thus includes:
-V_s at three loop plus resummation of sub-leading ultrasoft logs
-$\delta_{\mathrm{US}} \sim \alpha_{\mathrm{s}}^4$ at leading order

N^3LL accuracy thus includes:

-V_s at three loop plus resummation of sub-leading ultrasoft logs

-$\delta_{\mathrm{US}} \sim \alpha_{\mathrm{s}}^4$ at leading order

-Singlet-octet mixing induces $\Lambda_s \to K_1 + K_2 f(r)$.

N^3LL accuracy thus includes:
-V_s at three loop plus resummation of sub-leading ultrasoft logs
-$\delta_{\mathrm{US}} \sim \alpha_{\mathrm{s}}^4$ at leading order
-Singlet-octet mixing induces $\Lambda_s \to K_1 + K_2 f(r)$.

K_1 and K_2 are arbitrary constants, power counting tells us $K_i \sim \Lambda_{\overline{\mathrm{MS}}}$

N^{3}LL accuracy thus includes:
-V_s at three loop plus resummation of sub-leading ultrasoft logs
-$\delta_{\mathrm{US}} \sim \alpha_{\mathrm{s}}^4$ at leading order
-Singlet-octet mixing induces $\Lambda_s \to K_1 + K_2 f(r)$.

K_1 and K_2 are arbitrary constants, power counting tells us $K_i \sim \Lambda_{\overline{\mathrm{MS}}}$

The last missing ingredient was the three loop coefficient in the potential.

Static energy is now complete at N^{3}LL accuracy

To get a good convergent behavior of the perturbative series, it has been argued that it is necessary to use an scheme that cancels the renormalon singularity.

To get a good convergent behavior of the perturbative series, it has been argued that it is necessary to use an scheme that cancels the renormalon singularity.

The use of this kind of schemes introduces an additional dimensionful scale ρ.

To get a good convergent behavior of the perturbative series, it has been argued that it is necessary to use an scheme that cancels the renormalon singularity.

The use of this kind of schemes introduces an additional dimensionful scale ρ.

We use the so-called RS scheme. Pineda'01

To get a good convergent behavior of the perturbative series, it has been argued that it is necessary to use an scheme that cancels the renormalon singularity.

The use of this kind of schemes introduces an additional dimensionful scale ρ.

We use the so-called RS scheme. Pineda'01

Normalization of the renormalon R_s needed, can only be determined approximately Lee'99. Now we can also use the 3-loop coefficient of the potential in that determination

$$R_s = -1.333 + 0.499 - 0.338 - 0.033 = -1.205$$

Lattice comparison at N^{3}LL accuracy

Lattice comparison at N^{3}LL accuracy

Calculation assumes the hierarchy

$$\frac{1}{r} \gg \frac{\alpha_{\rm s}}{r} \gg \Lambda_{\overline{\rm MS}} \sim \frac{\alpha_{\rm s}^2}{r}$$

Lattice comparison at N^{3}LL accuracy

Calculation assumes the hierarchy

$$\frac{1}{r} \gg \frac{\alpha_s}{r} \gg \Lambda_{\overline{MS}} \sim \frac{\alpha_s^2}{r}$$

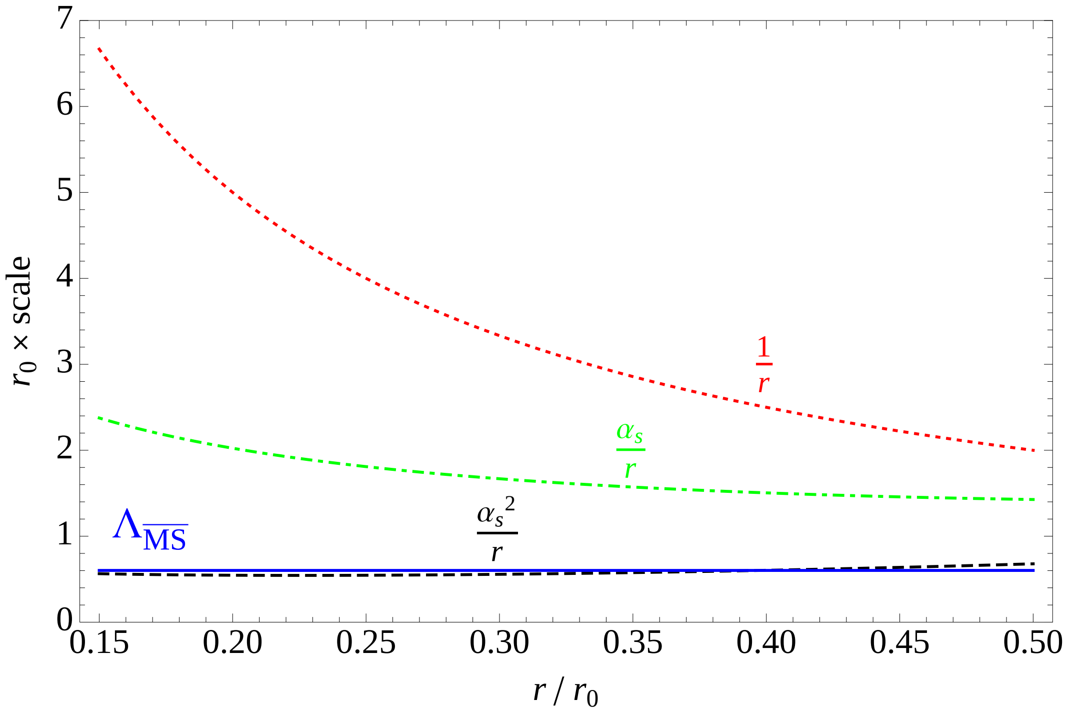
We compare with ($n_f = 0$) lattice data in the range $0.15 r_0 \leq r < 0.5 r_0$ (r_0 is the lattice reference scale)

Lattice comparison at N^{3}LL accuracy

Calculation assumes the hierarchy

$$\frac{1}{r} \gg \frac{\alpha_s}{r} \gg \Lambda_{\overline{MS}} \sim \frac{\alpha_s^2}{r}$$

We compare with ($n_f = 0$) lattice data in the range $0.15 r_0 \leq r < 0.5 r_0$ (r_0 is the lattice reference scale)



To compare with the lattice data (of Necco, Sommer'02):

To compare with the lattice data (of Necco, Sommer'02):

- Static energy expressed as a series in $\alpha_{\rm s}(\rho)$

To compare with the lattice data (of Necco, Sommer'02):

- Static energy expressed as a series in $\alpha_s(\rho)$
- We plot: $E_0(r) - E_0(r_{\min}) + E_0^{\mathrm{latt}}(r_{\min})$.

To compare with the lattice data (of Necco, Sommer'02):

- Static energy expressed as a series in $\alpha_s(\rho)$
- We plot: $E_0(r) - E_0(r_{\min}) + E_0^{\mathrm{latt}}(r_{\min})$. So, all the curves coincide at $r = r_{\min}$ by construction

To compare with the lattice data (of Necco, Sommer'02):

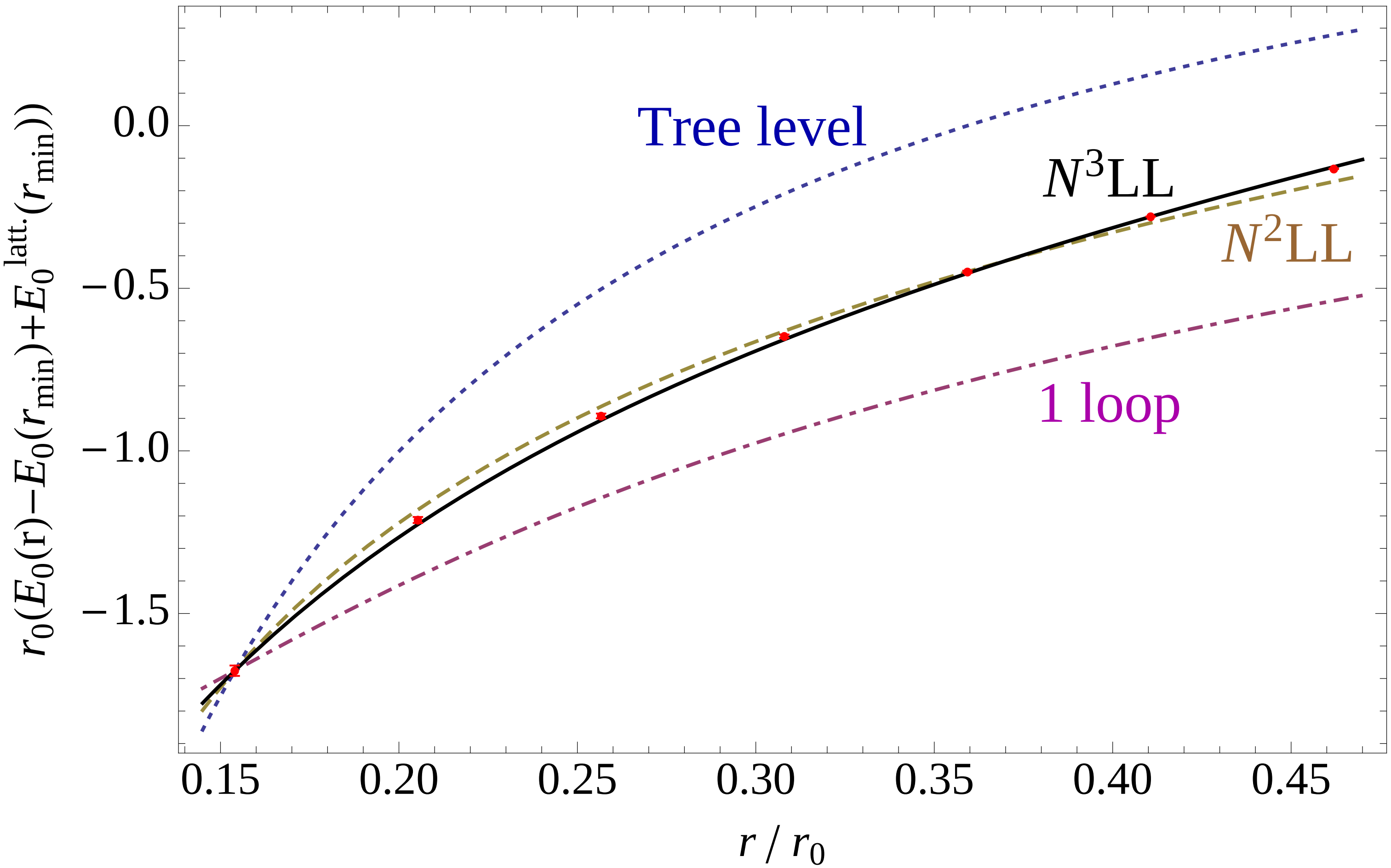
- Static energy expressed as a series in $\alpha_s(\rho)$
- We plot: $E_0(r) - E_0(r_{\min}) + E_0^{\text{latt}}(r_{\min})$. So, all the curves coincide at $r = r_{\min}$ by construction
- K_2 (only appears in the $\mathrm{N^3LL}$ curve) is fitted to the lattice data

To compare with the lattice data (of Necco, Sommer'02):

- Static energy expressed as a series in $\alpha_s(\rho)$
- We plot: $E_0(r) - E_0(r_{\rm min}) + E_0^{\rm latt}(r_{\rm min})$. So, all the curves coincide at $r = r_{\rm min}$ by construction
- K_2 (only appears in the N^{3}LL curve) is fitted to the lattice data
- ρ is taken at the center of the r range, $\rho = 3.25 r_0^{-1}$

To compare with the lattice data (of Necco, Sommer'02):

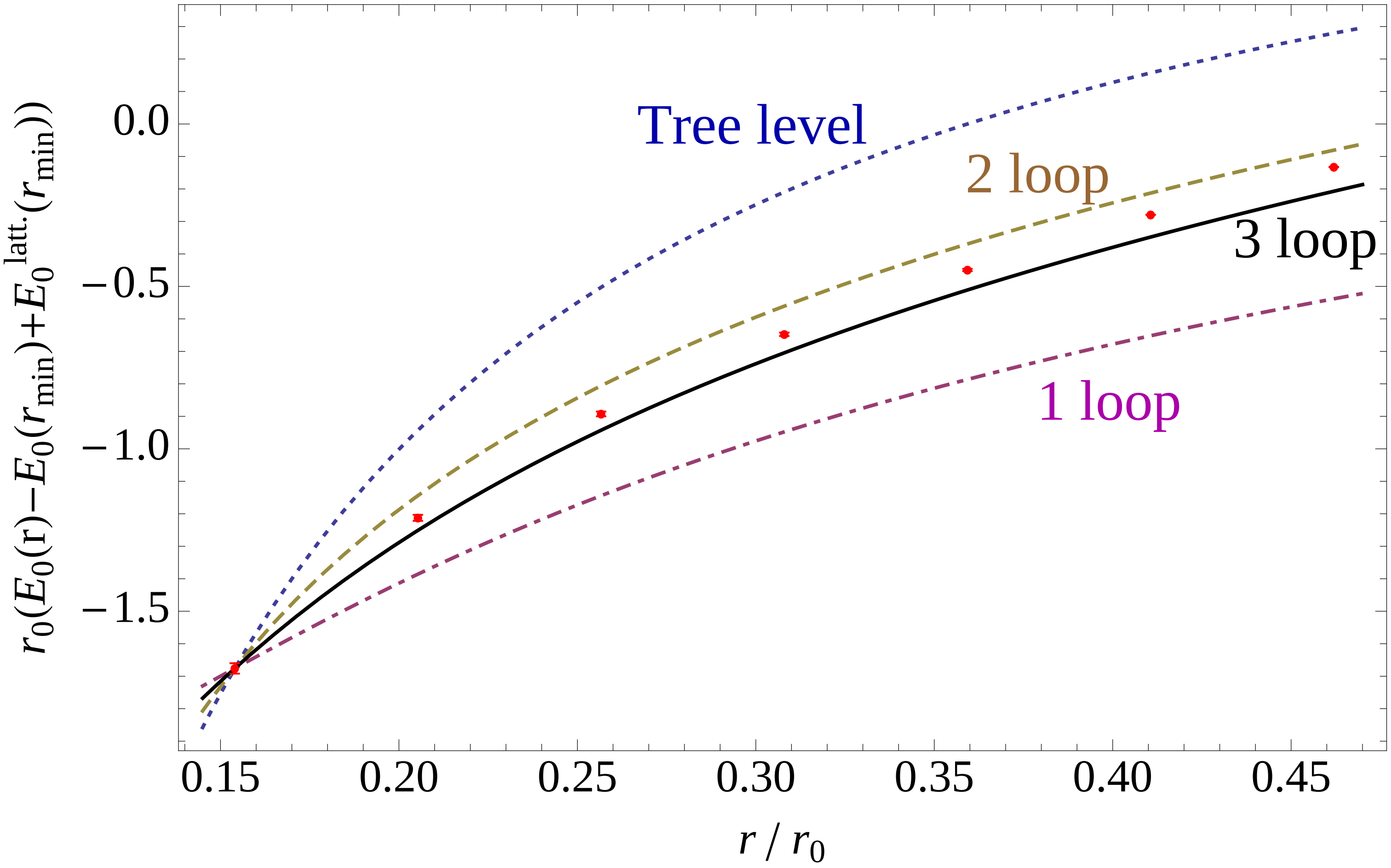
- Static energy expressed as a series in $\alpha_s(\rho)$
- We plot: $E_0(r) - E_0(r_{\rm min}) + E_0^{\rm latt}(r_{\rm min})$. So, all the curves coincide at $r = r_{\rm min}$ by construction
- K_2 (only appears in the N^{3}LL curve) is fitted to the lattice data
- ρ is taken at the center of the r range, $\rho = 3.25 r_0^{-1}$
- $r_0 \Lambda_{\overline{\rm MS}} = 0.602 \pm 0.048$ Capitani *et al.* [ALPHA Collaboration]'99



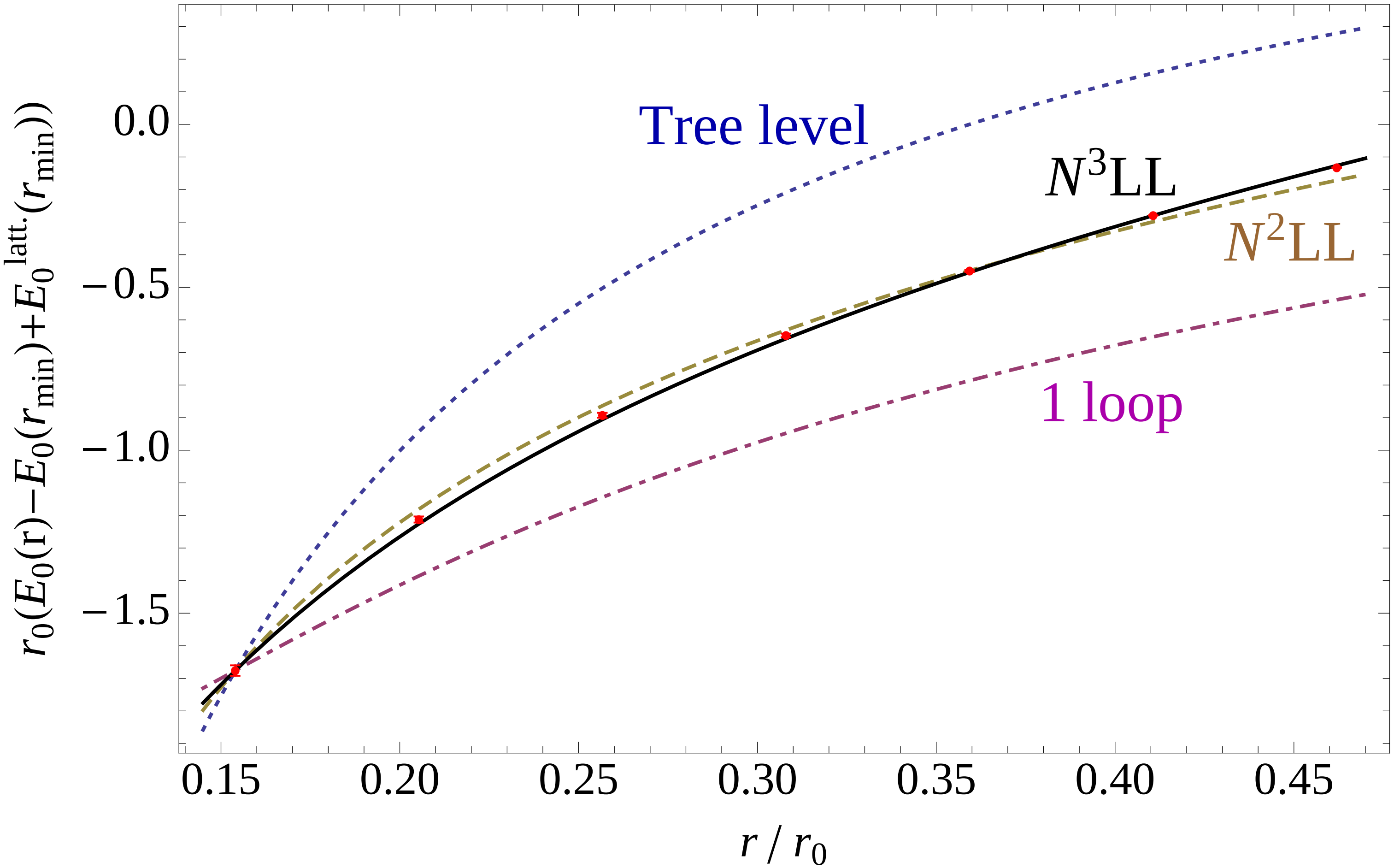
Tree level
N^3LL
N^2LL
1 loop
0.0
−0.5
−1.0
−1.5
0.15
0.20
0.25
0.30
0.35
0.40
0.45
r / r_0
$r_0(E_0(\mathrm{r})-E_0(r_{\mathrm{min}})+E_0^{\mathrm{latt.}}(r_{\mathrm{min}}))$

Impact of the resummation of ultrasoft logarithms

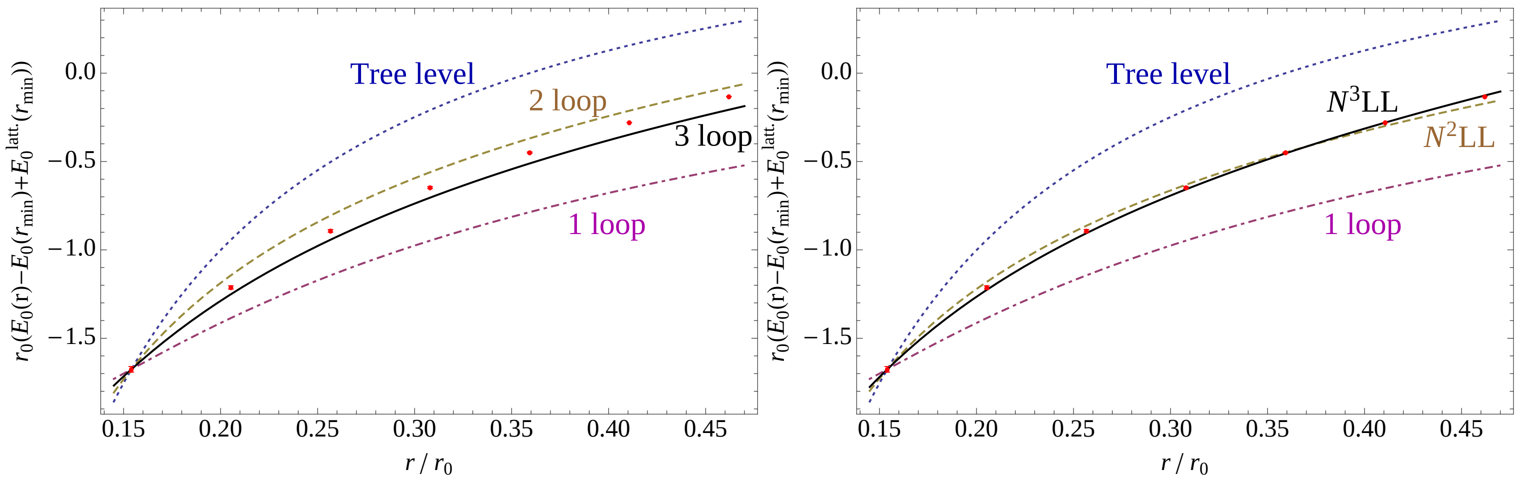
Impact of the resummation of ultrasoft logarithms



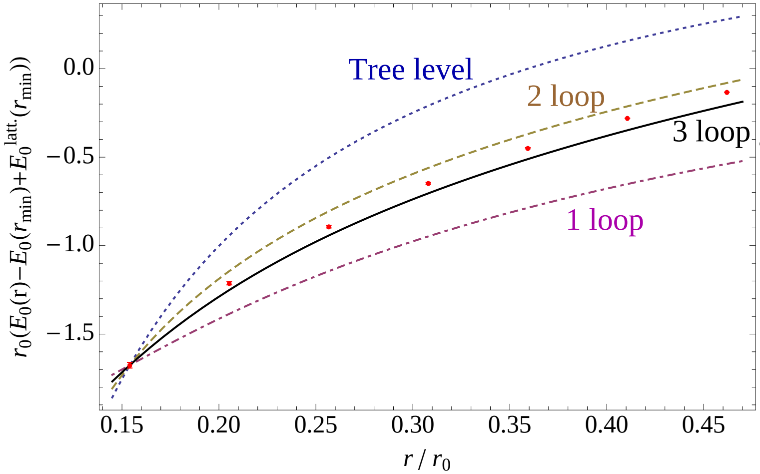
Impact of the resummation of ultrasoft logarithms

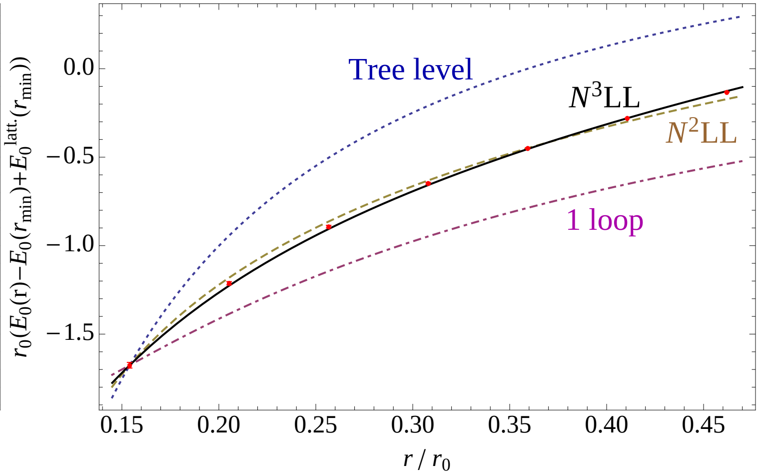


Impact of the resummation of ultrasoft logarithms



Impact of the resummation of ultrasoft logarithms



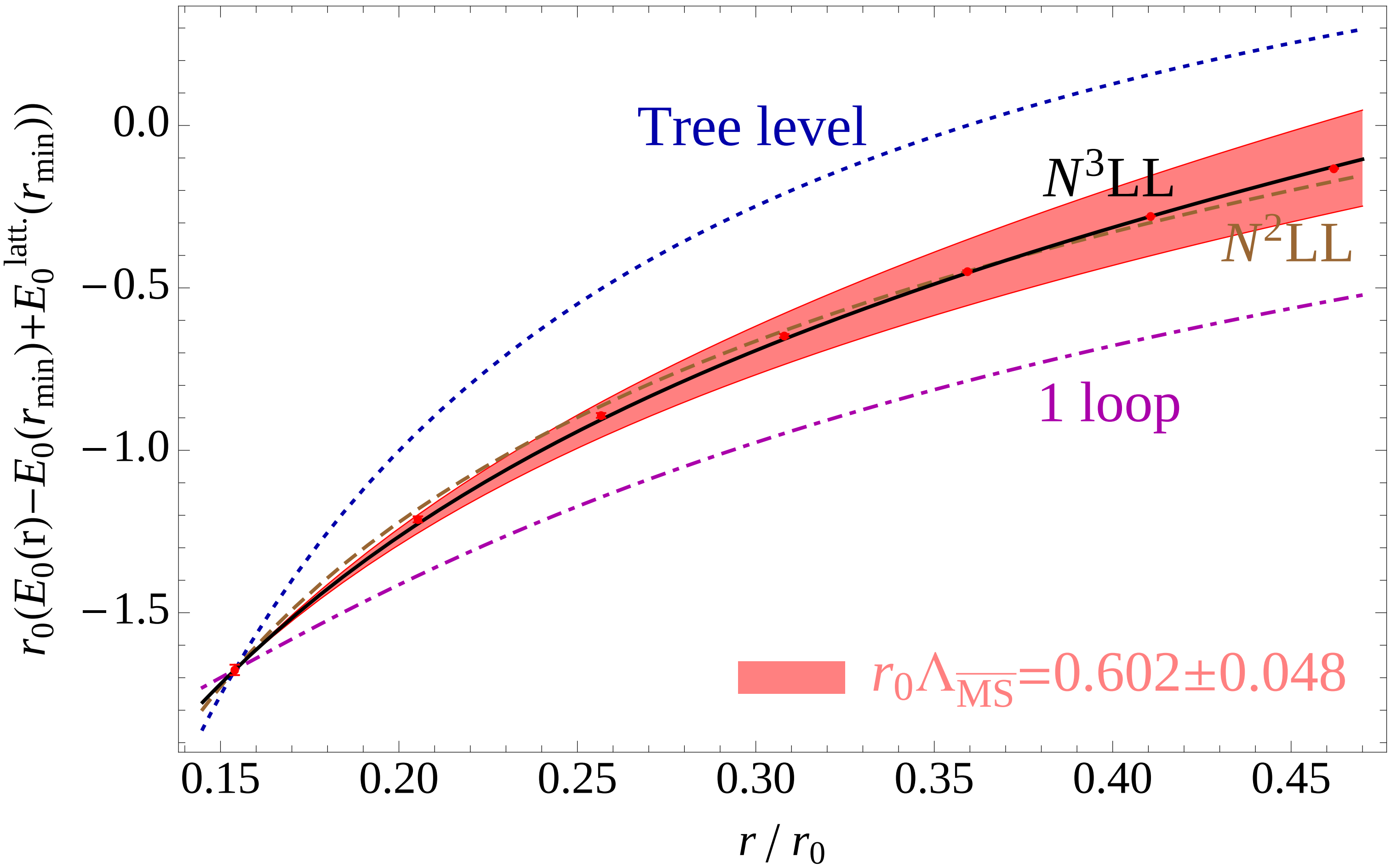


The resummation improves the agreement with lattice.

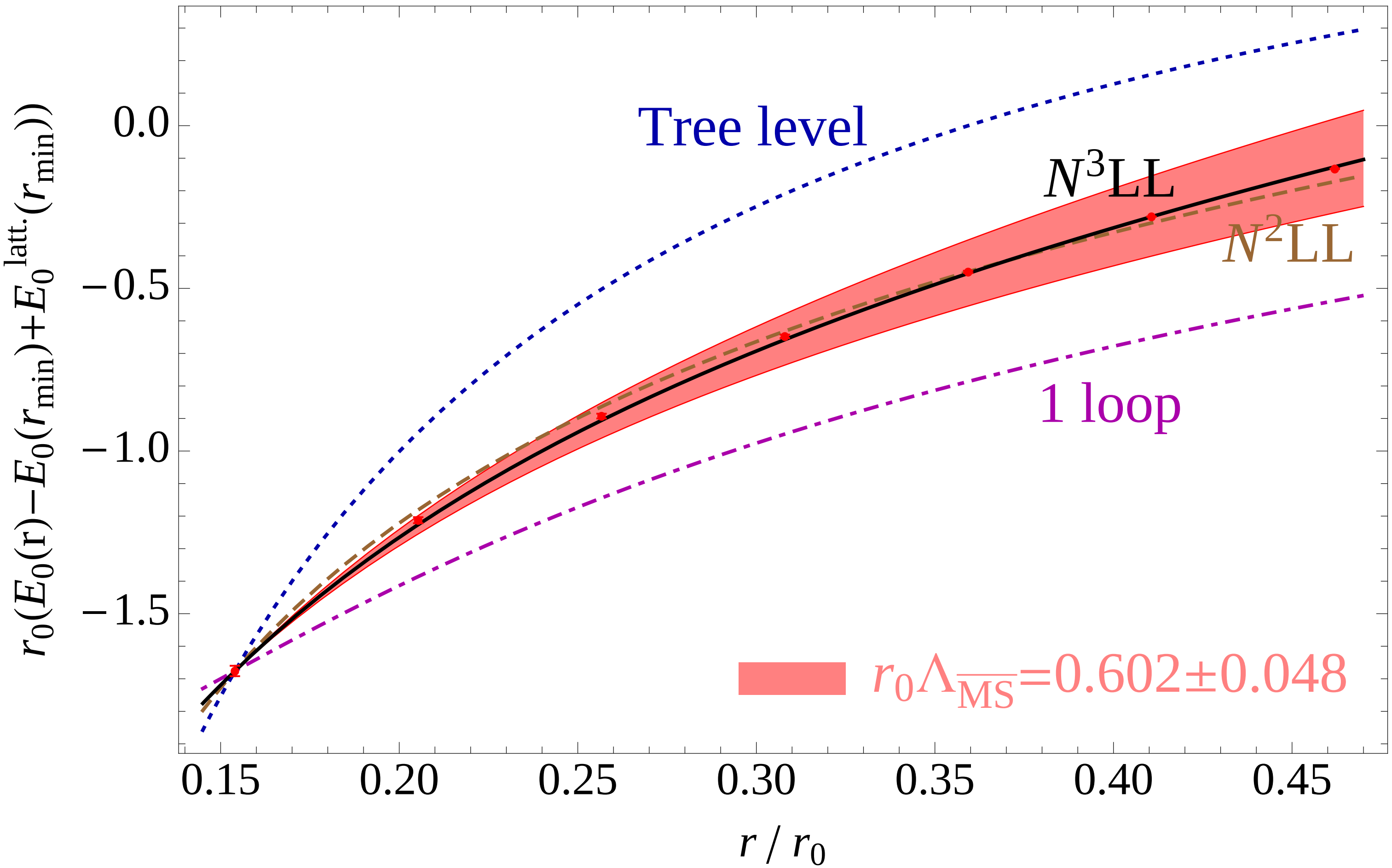
Uncertainties associated with the value of $r_0\Lambda_{\overline{\mathrm{MS}}}$

UNIVERSITY OF ALBERTA

Uncertainties associated with the value of $r_0\Lambda_{\overline{\rm MS}}$

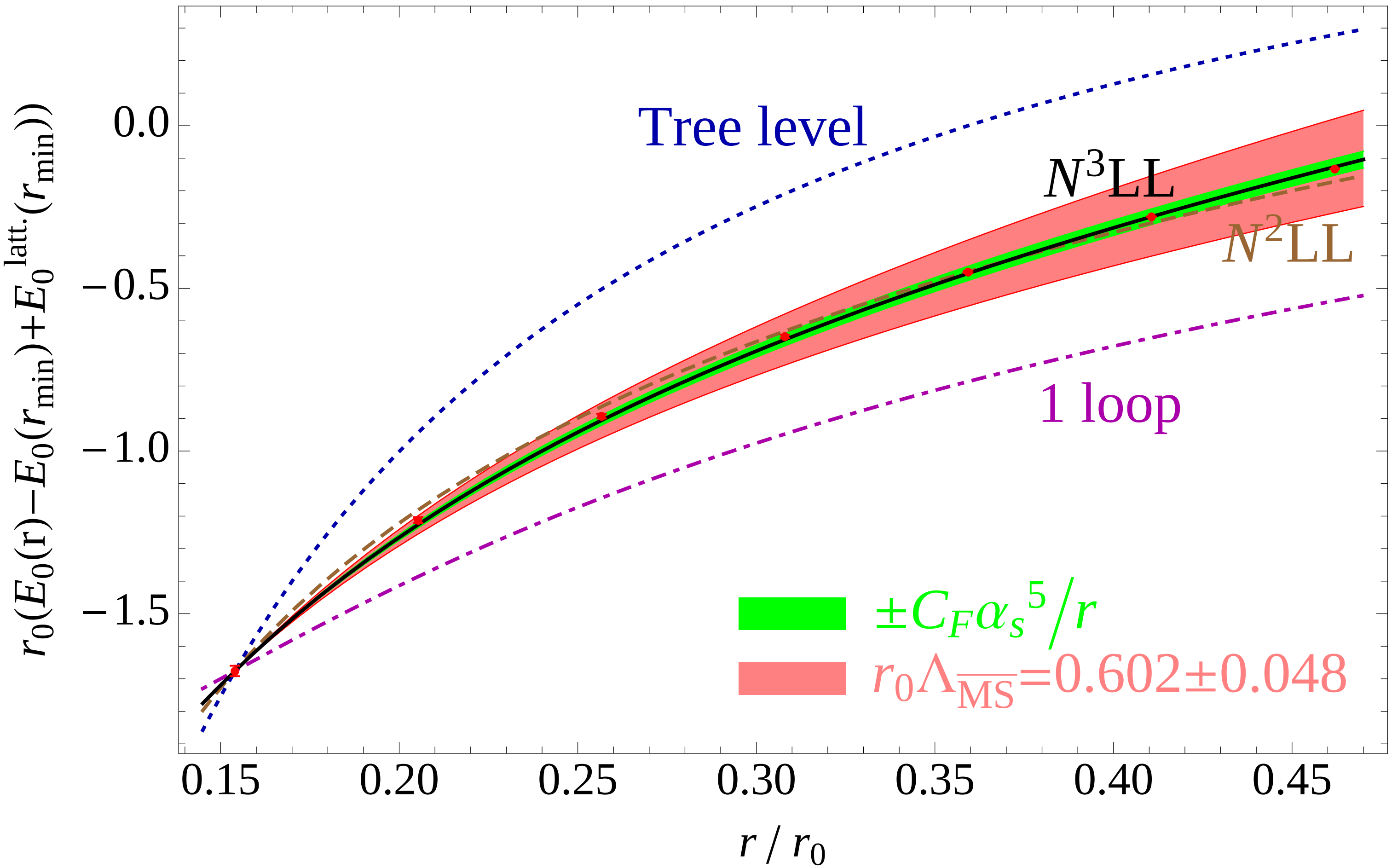


Uncertainties associated with the value of $r_0\Lambda_{\overline{\rm MS}}$



Seems to be larger than the ones due to neglected higher order terms

Uncertainties associated with the value of $r_0\Lambda_{\overline{\rm MS}}$



Seems to be larger than the ones due to neglected higher order terms

Determination of $r_0\Lambda_{\overline{\mathrm{MS}}}$

Assume non-perturbative effects are small and neglect them.
Use lattice data to determine $r_0\Lambda_{\overline{\mathrm{MS}}}$. Brambilla, XGT, Soto, Vairo in preparation

Determination of $r_0\Lambda_{\overline{\rm MS}}$

Assume non-perturbative effects are small and neglect them.
Use lattice data to determine $r_0\Lambda_{\overline{\rm MS}}$. Brambilla, XGT, Soto, Vairo in preparation

Comparison with lattice provided prediction for three loop coefficient c_0 (Brambilla, XGT, Soto, Vairo'09)

$$c_0^{\rm Pred} \to (215, 350) \quad c_0^{\rm True} = 222.703...$$

Determination of $r_0\Lambda_{\overline{\mathrm{MS}}}$

Assume non-perturbative effects are small and neglect them.

Use lattice data to determine $r_0\Lambda_{\overline{\mathrm{MS}}}$. Brambilla, XGT, Soto, Vairo in preparation

Comparison with lattice provided prediction for three loop coefficient c_0 (Brambilla, XGT, Soto, Vairo'09)

$$c_0^{\mathrm{Pred}} \to (215, 350) \quad c_0^{\mathrm{True}} = 222.703...$$

Any ρ (RS-scheme scale) should cancel the renormalon.

Determination of $r_0\Lambda_{\overline{\rm MS}}$

Assume non-perturbative effects are small and neglect them.
Use lattice data to determine $r_0\Lambda_{\overline{\rm MS}}$. Brambilla, XGT, Soto, Vairo in preparation

Comparison with lattice provided prediction for three loop coefficient c_0 (Brambilla, XGT, Soto, Vairo'09)

$$c_0^{\rm Pred} \to (215, 350) \quad c_0^{\rm True} = 222.703...$$

Any ρ (RS-scheme scale) should cancel the renormalon. Search for a set of ρ values which are optimal for the determination of $r_0\Lambda_{\overline{\rm MS}}$.

Determination of $r_0\Lambda_{\overline{\rm MS}}$

Assume non-perturbative effects are small and neglect them.
Use lattice data to determine $r_0\Lambda_{\overline{\rm MS}}$. Brambilla, XGT, Soto, Vairo in preparation

Comparison with lattice provided prediction for three loop coefficient c_0 (Brambilla, XGT, Soto, Vairo'09)

$$c_0^{\rm Pred} \to (215, 350) \quad c_0^{\rm True} = 222.703...$$

Any ρ (RS-scheme scale) should cancel the renormalon. Search for a set of ρ values which are optimal for the determination of $r_0\Lambda_{\overline{\rm MS}}$. Procedure:

Determination of $r_0\Lambda_{\overline{\mathrm{MS}}}$

Assume non-perturbative effects are small and neglect them.
Use lattice data to determine $r_0\Lambda_{\overline{\mathrm{MS}}}$. Brambilla, XGT, Soto, Vairo in preparation

Comparison with lattice provided prediction for three loop coefficient c_0 (Brambilla, XGT, Soto, Vairo'09)

$$c_0^{\mathrm{Pred}} \to (215, 350) \quad c_0^{\mathrm{True}} = 222.703...$$

Any ρ (RS-scheme scale) should cancel the renormalon. Search for a set of ρ values which are optimal for the determination of $r_0\Lambda_{\overline{\mathrm{MS}}}$. Procedure:

- Vary ρ by $\pm 25\%$ around $\rho = 3.25 r_0^{-1}$

Determination of $r_0\Lambda_{\overline{\rm MS}}$

Assume non-perturbative effects are small and neglect them.
Use lattice data to determine $r_0\Lambda_{\overline{\rm MS}}$. Brambilla, XGT, Soto, Vairo in preparation

Comparison with lattice provided prediction for three loop coefficient c_0 (Brambilla, XGT, Soto, Vairo'09)

$$c_0^{\rm Pred} \to (215, 350) \quad c_0^{\rm True} = 222.703...$$

Any ρ (RS-scheme scale) should cancel the renormalon. Search for a set of ρ values which are optimal for the determination of $r_0\Lambda_{\overline{\rm MS}}$. Procedure:

- Vary ρ by $\pm 25\%$ around $\rho = 3.25 r_0^{-1}$
- Perform fit for each value of ρ and at each order in p.t.

Determination of $r_0\Lambda_{\overline{\mathrm{MS}}}$

Assume non-perturbative effects are small and neglect them.
Use lattice data to determine $r_0\Lambda_{\overline{\mathrm{MS}}}$. Brambilla, XGT, Soto, Vairo in preparation

Comparison with lattice provided prediction for three loop coefficient c_0 (Brambilla, XGT, Soto, Vairo'09)

$$c_0^{\mathrm{Pred}} \to (215, 350) \quad c_0^{\mathrm{True}} = 222.703...$$

Any ρ (RS-scheme scale) should cancel the renormalon. Search for a set of ρ values which are optimal for the determination of $r_0\Lambda_{\overline{\mathrm{MS}}}$. Procedure:

- Vary ρ by $\pm 25\%$ around $\rho = 3.25 r_0^{-1}$
- Perform fit for each value of ρ and at each order in p.t.
- Select ρ's for which χ^2 decreases when increasing pert. ord

Determination of $r_0\Lambda_{\overline{\rm MS}}$

Assume non-perturbative effects are small and neglect them.
Use lattice data to determine $r_0\Lambda_{\overline{\rm MS}}$. Brambilla, XGT, Soto, Vairo in preparation
Comparison with lattice provided prediction for three loop coefficient c_0 (Brambilla, XGT, Soto, Vairo'09)

$$c_0^{\rm Pred} \to (215, 350) \quad c_0^{\rm True} = 222.703...$$

Any ρ (RS-scheme scale) should cancel the renormalon. Search for a set of ρ values which are optimal for the determination of $r_0\Lambda_{\overline{\rm MS}}$. Procedure:

- Vary ρ by $\pm 25\%$ around $\rho = 3.25 r_0^{-1}$
- Perform fit for each value of ρ and at each order in p.t.
- Select ρ's for which χ^2 decreases when increasing pert. ord
- Select the values that respect power counting for K_2

Consider the fitted values of $r_0\Lambda_{\overline{\rm MS}}$ at N^{3}LL

Consider the fitted values of $r_0\Lambda_{\overline{\rm MS}}$ at N^{3}LL, weight them by the inverse of the χ^2 and use the weighted average as central value

Consider the fitted values of $r_0\Lambda_{\overline{\rm MS}}$ at $\mathrm{N^3LL}$, weight them by the inverse of the χ^2 and use the weighted average as central value

$$r_0\Lambda_{\overline{\rm MS}} = 0.622$$

Consider the fitted values of $r_0\Lambda_{\overline{\mathrm{MS}}}$ at N^{3}LL, weight them by the inverse of the χ^2 and use the weighted average as central value

$$r_0\Lambda_{\overline{\mathrm{MS}}} = 0.622$$

For the error consider

Consider the fitted values of $r_0\Lambda_{\overline{\mathrm{MS}}}$ at N^{3}LL, weight them by the inverse of the χ^2 and use the weighted average as central value

$$r_0\Lambda_{\overline{\mathrm{MS}}} = 0.622$$

For the error consider

- Weighted standard deviation: 0.009

Consider the fitted values of $r_0\Lambda_{\overline{\rm MS}}$ at N^{3}LL, weight them by the inverse of the χ^2 and use the weighted average as central value

$$r_0\Lambda_{\overline{\rm MS}} = 0.622$$

For the error consider

- Weighted standard deviation: 0.009

- Difference with average at N^{2}LL: 0.003

Consider the fitted values of $r_0\Lambda_{\overline{\rm MS}}$ at N^{3}LL, weight them by the inverse of the χ^2 and use the weighted average as central value

$$r_0\Lambda_{\overline{\rm MS}} = 0.622$$

For the error consider

- Weighted standard deviation: 0.009
- Difference with average at N^{2}LL: 0.003

Add in quadrature. That should account for uncertainties due to neglected higher order terms.

Consider the fitted values of $r_0\Lambda_{\overline{\rm MS}}$ at N^{3}LL, weight them by the inverse of the χ^2 and use the weighted average as central value

$$r_0\Lambda_{\overline{\rm MS}} = 0.622$$

For the error consider

- Weighted standard deviation: 0.009
- Difference with average at N^{2}LL: 0.003

Add in quadrature. That should account for uncertainties due to neglected higher order terms. We obtain the final result

Consider the fitted values of $r_0\Lambda_{\overline{\rm MS}}$ at N^{3}LL, weight them by the inverse of the χ^2 and use the weighted average as central value

$$r_0\Lambda_{\overline{\rm MS}} = 0.622$$

For the error consider

- Weighted standard deviation: 0.009
- Difference with average at N^{2}LL: 0.003

Add in quadrature. That should account for uncertainties due to neglected higher order terms. We obtain the final result

$$r_0\Lambda_{\overline{\rm MS}} = 0.622 \pm 0.009 \quad (\mathit{Preliminary}!)$$

Consider the fitted values of $r_0\Lambda_{\overline{\rm MS}}$ at N^{3}LL, weight them by the inverse of the χ^2 and use the weighted average as central value

$$r_0\Lambda_{\overline{\rm MS}} = 0.622$$

For the error consider

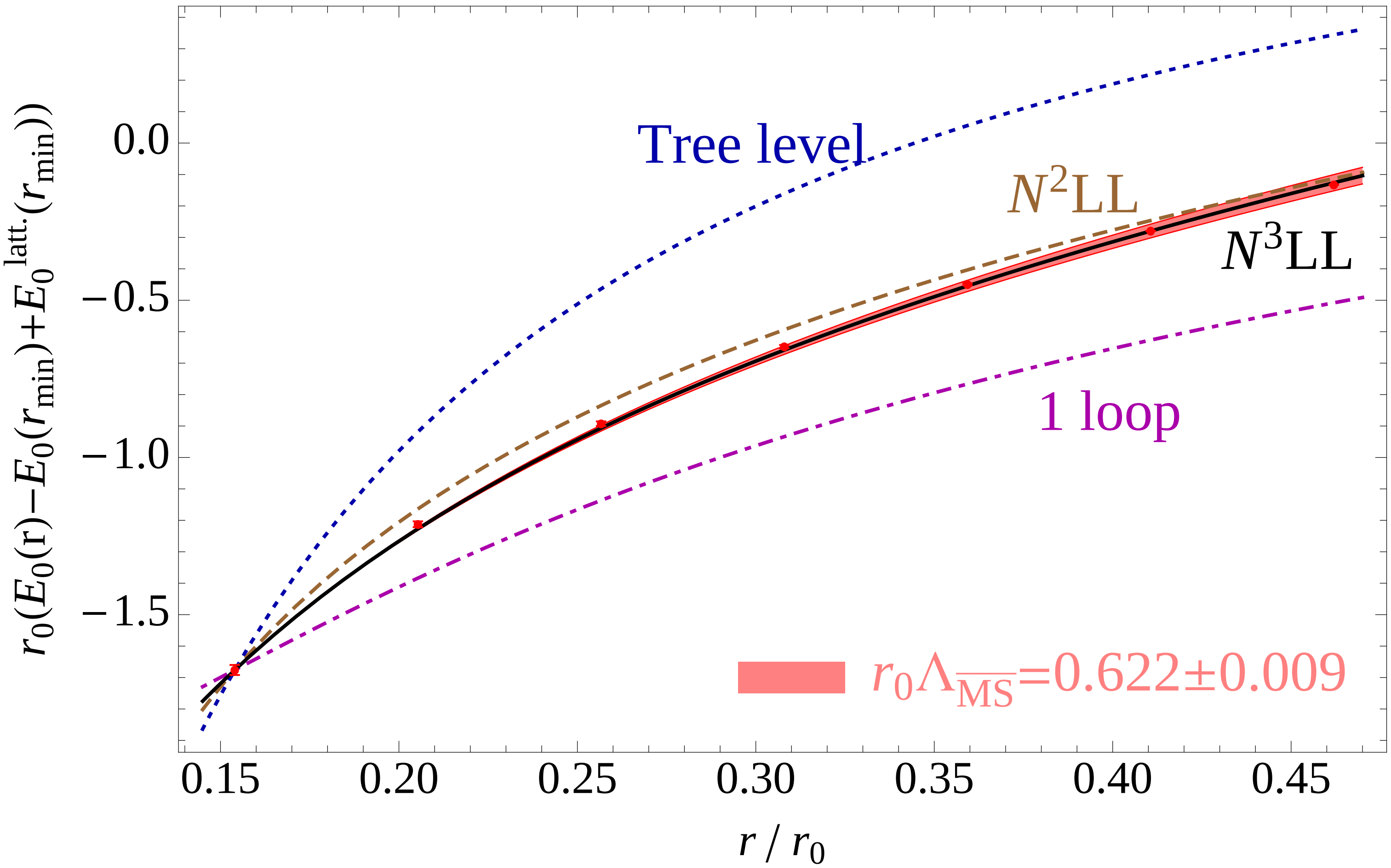
- Weighted standard deviation: 0.009
- Difference with average at N^{2}LL: 0.003

Add in quadrature. That should account for uncertainties due to neglected higher order terms. We obtain the final result

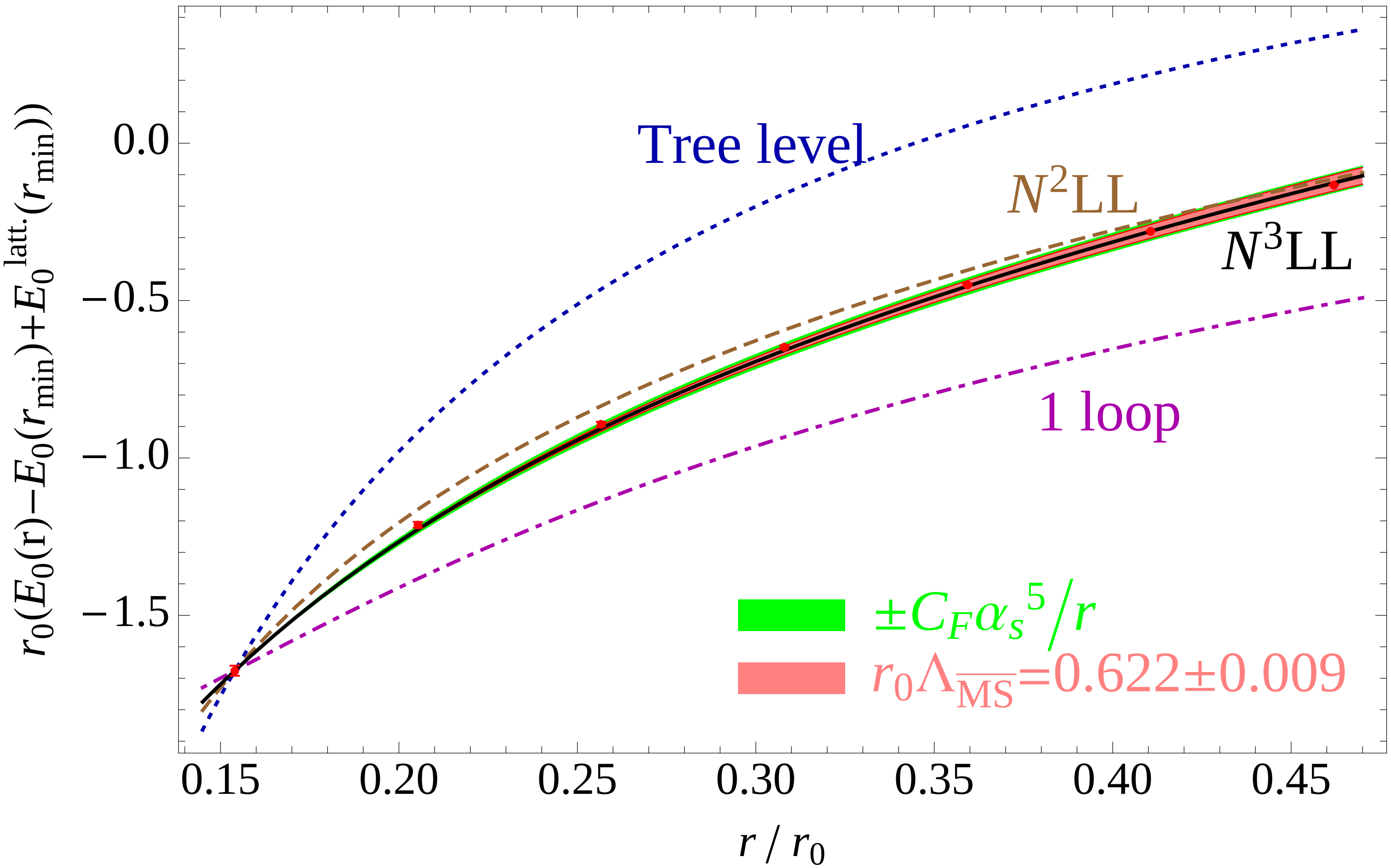
$$r_0\Lambda_{\overline{\rm MS}} = 0.622 \pm 0.009 \quad (\mathit{Preliminary}!)$$

Compatible but more precise than the number we used previously

$$r_0\Lambda_{\overline{\rm MS}} = 0.602 \pm 0.048$$



$r_0(E_0(\mathrm{r})-E_0(r_{\min})+E_0^{\mathrm{latt.}}(r_{\min}))$
0.0
−0.5
−1.0
−1.5
0.15
0.20
0.25
0.30
0.35
0.40
0.45
$r\,/\,r_0$
Tree level
N^2LL
N^3LL
1 loop
$r_0\Lambda_{\overline{\mathrm{MS}}}$=0.622±0.009



Tree level
N^2LL
N^3LL
1 loop
$\pm C_F \alpha_s{}^5/r$
$r_0\Lambda_{\overline{MS}}$=0.622±0.009
0.0
−0.5
−1.0
−1.5
0.15
0.20
0.25
0.30
0.35
0.40
0.45
$r\,/\,r_0$
$r_0(E_0(\mathrm{r})-E_0(r_{\min})+E_0^{\mathrm{latt.}}(r_{\min}))$

Conclusions

Conclusions

- Comparison of the static energy with lattice at unprecedented N^3LL accuracy

Conclusions

- Comparison of the static energy with lattice at unprecedented N^3LL accuracy
  - Excellent agreement

Conclusions

- Comparison of the static energy with lattice at unprecedented N^3LL accuracy
  - Excellent agreement
  - Resummation of ultrasoft logs helps in improving agreement

Conclusions

- Comparison of the static energy with lattice at unprecedented N^3LL accuracy
  - Excellent agreement
  - Resummation of ultrasoft logs helps in improving agreement
- Determination of $r_0\Lambda_{\overline{\rm MS}}^{n_f=0} = 0.622 \pm 0.009$

Conclusions

- Comparison of the static energy with lattice at unprecedented N^{3}LL accuracy
  - Excellent agreement
  - Resummation of ultrasoft logs helps in improving agreement
- Determination of $r_0\Lambda_{\overline{\rm MS}}^{n_f=0} = 0.622 \pm 0.009$
  - Same procedure to determine $r_0\Lambda_{\overline{\rm MS}}$ could be used with unquenched lattice data for the static energy at short distances, when available

Conclusions

- Comparison of the static energy with lattice at unprecedented N^{3}LL accuracy
  - Excellent agreement
  - Resummation of ultrasoft logs helps in improving agreement
- Determination of $r_0\Lambda_{\overline{\rm MS}}^{n_f=0} = 0.622 \pm 0.009$
  - Same procedure to determine $r_0\Lambda_{\overline{\rm MS}}$ could be used with unquenched lattice data for the static energy at short distances, when available

Thank you