Why μ to e conversion at $<10^{-18}$?

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The Project-X muon workshop

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Outline

- Introduction
- Constraints on new physics from lepton flavour violation
  - Model independent analysis
  - Extra-dimensional models
  - Little Higgs models
  - Supersymmetric models
  - See-saw models
- The connection LFV – cosmology:
  Testing the origin of the matter-antimatter asymmetry through μ to e conversion experiments at $<10^{-18}$.

Brief history of leptonic physics

- Leptonic Lagrangian (1967 - 1998)

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \text{h.c.}$$

$$U(3)_{e_R} \times U(3)_L \longrightarrow U(1)_e \times U(1)_\mu \times U(1)_\tau$$

Consistent with experiments searching for neutrinoless double beta decay and rare lepton decays, but not with neutrino oscillation experiments.

- Charged leptons massive
- neutrinos massless
- lepton flavour conserved
- total L number conserved

- Leptonic Lagrangians (1998 - ?)

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} + \text{h.c.}$$

$$U(3)_{e_R} \times U(3)_L \longrightarrow U(1)_{\rm lep}$$ Dirac Mass

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi} + \text{h.c.}$$

$$U(3)_{e_R} \times U(3)_L \longrightarrow \text{nothing}$$ Majorana Mass

- Charged leptons massive
- neutrinos massive
- lepton flavour violated – ν oscillations
- total L number conserved or violated

• Challenge: find evidences of the next term in the effective Lagrangian

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} + \sum_k \frac{\alpha_k}{\Lambda_k^2}\mathcal{O}_k^{D=6} + \text{h.c.}$$

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi} + \sum_k \frac{\alpha_k}{\Lambda_k^2}\mathcal{O}_k^{D=6} + \text{h.c.}$$

LFV in the charged lepton sector

Some dimension 6 operators are:

Two leptons, one gauge boson (+ one higgs)	$\bar{e}_{Ri}\sigma^{\mu\nu}L_j\phi B_{\mu\nu}$ $\bar{e}_{Ri}\sigma^{\mu\nu}\tau_I L_j\phi W^I_{\mu\nu}$	$\mu\to e\gamma$ $\tau\to\mu\gamma$ $Z\to\mu e$
Four leptons	$(\bar{L}_i\gamma^\mu L_j)(\bar{L}_k\gamma_\mu L_l)$ $(\bar{e}_i\gamma^\mu e_j)(\bar{e}_k\gamma_\mu e_l)$ $(\bar{L}_i\gamma^\mu e_j)(\bar{e}_k\gamma_\mu L_l)$	$\mu\to eee$ $\tau\to\mu\mu\mu$
Two leptons, two quarks	$(\bar{L}_i\gamma^\mu L_j)(\bar{Q}_k\gamma_\mu Q_l)$ $(\bar{e}_i\gamma^\mu e_j)(\bar{u}_k\gamma_\mu u_l)$ $(\bar{L}_i\gamma^\mu e_j)(\bar{u}_k\gamma_\mu Q_l)$	$\mu\, N\to e\, N$ $\tau\to\pi\,\mu$ $\tau\to\eta\,\mu$

⋮

(+ dim. 6 operators that violate total lepton number)

Neutrino masses violate flavour ⇒ they induce all these operators

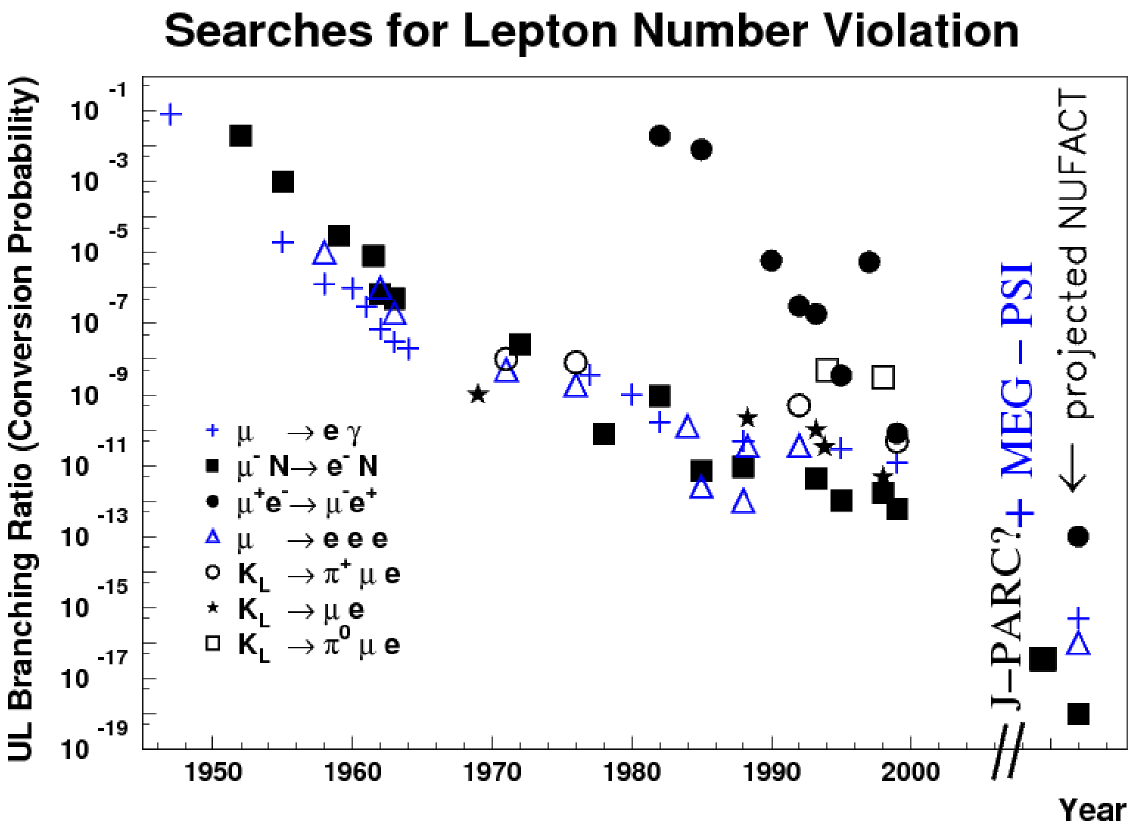
If the only source of LFV are neutrino masses, the dim-6 operators are very suppressed, giving

$$\mathrm{BR}(\mu \to e\gamma) \sim \frac{3\alpha}{32\pi}\left(\frac{\Delta m_\nu^2}{M_W^2}\right)^2 \sin^2\theta$$

The predictions for the rare lepton decays are

$\mathrm{BR}(\mu \to e\gamma) \simeq 10^{-57}$, $\mathrm{BR}(\tau \to \mu\gamma) \simeq 10^{-54}$, $\mathrm{BR}(\tau \to e\gamma) \simeq 10^{-57}$,

Well consistent with experiments searching for rare charged lepton decays.



Äystö et al.

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Well consistent with experiments searching for rare charged lepton decays.

However, there could be new sources of LFV apart from neutrino masses

$$-\mathcal{L}_{\mathrm{lep}} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi} + \sum_k \frac{\alpha_k}{\Lambda_k^2}\mathcal{O}_k^{D=6} + \mathrm{h.c.}$$

Scale of lepton number violation $\overset{?}{\neq}$ Scale of lepton flavour violation

[Also, for the same suppression Λ, the coefficient of the dimension 5 operator could be much smaller than the one of the dimension 6 operator]

Bounds on new physics from $\mu \to e\gamma$

Lowest dimension operator which induces $\mu \to e\gamma$

$$-\mathcal{L} = m_\mu \bar{\mu}(f^{\mu e}_{M1} + \gamma_5 f^{\mu e}_{E1})\sigma^{\mu\nu} e F_{\mu\nu} + \text{h.c.}$$

The rate for the rare muon decay is:

$$\text{BR}(\mu \to e\gamma) = \frac{96\pi^3\alpha}{G_F^2}(|f^{\mu e}_{E1}|^2 + |f^{\mu e}_{M1}|^2)$$

The present experimental bound $\text{BR}(\mu \to e\gamma) < 1.2\times10^{-11}$ gives:

$$|f^{\mu e}_{E1}|, |f^{\mu e}_{M1}| \lesssim 10^{-12}\text{GeV}^{-2}$$

Naively,

$$f^{\mu e} \sim \frac{1}{\Lambda^2} \longrightarrow \Lambda \gtrsim 300\text{TeV}$$

In most models the contact interaction arises as a result of quantum effects (new particles interacting with the muon and the electron circulating in loops).

$$f^{\mu e} \sim \frac{\theta_{\mu e}^2 \alpha}{\Lambda^2}$$

Then, the present bound on BR($\mu \to e\gamma$) requires

$$\Lambda \gtrsim 20\text{TeV} \quad \text{if} \quad \theta_{\mu e} \sim \frac{1}{\sqrt{2}}$$
$$\theta_{\mu e} \lesssim 0.01 \quad \text{if} \quad \Lambda \sim 300\text{GeV}$$

A large mass scale for the new particles and/or small coupling between the electron or muon with the new particles.

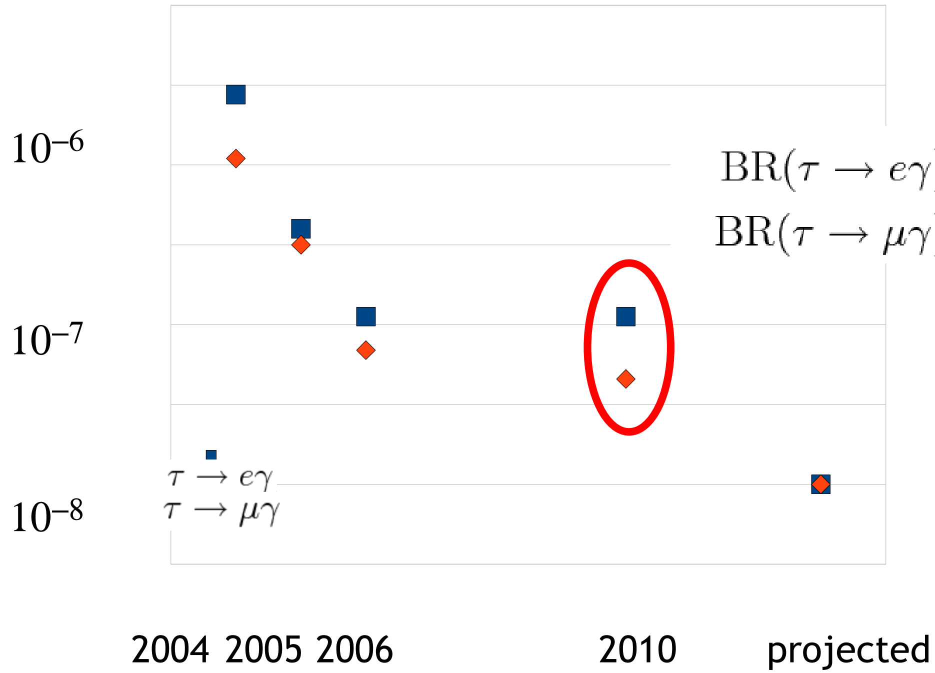
With an experiment searching for μ to e conversion at 10^{-18},

$$\Lambda \gtrsim 350 \text{ TeV} \quad \text{if} \quad \theta_{\mu e} \sim \frac{1}{\sqrt{2}}$$
$$\theta_{\mu e} \lesssim 5 \times 10^{-4} \quad \text{if} \quad \Lambda \sim 300 \text{ GeV}$$

Far beyond the reach of collider searches!

Rare tau decays

Complementary probe of lepton flavour violation.



$$\mathrm{BR}(\tau \to e\gamma) \le 1.1 \times 10^{-7} \qquad \mathrm{CL} = 90\%$$
$$\mathrm{BR}(\tau \to \mu\gamma) \le 4.5 \times 10^{-8} \qquad \mathrm{CL} = 90\%$$

The present experimental bounds on the rare tau decays yield:

From τ→eγ

$$\Lambda \gtrsim 1300\mathrm{GeV} \quad \text{if} \quad \theta_{\tau e} \sim \frac{1}{\sqrt{2}}$$
$$\theta_{\tau e} \lesssim 0.2 \quad \text{if} \quad \Lambda \sim 300\mathrm{GeV}$$

From τ→μγ

$$\Lambda \gtrsim 1700\mathrm{GeV} \quad \text{if} \quad \theta_{\tau\mu} \sim \frac{1}{\sqrt{2}}$$
$$\theta_{\tau\mu} \lesssim 0.1 \quad \text{if} \quad \Lambda \sim 300\mathrm{GeV}$$

fairly stringent constraints

Implications for Physics BSM

DRAMATIC! Many extensions of the Standard Model postulate new particles at the electroweak scale (hierarchy problem, "WIMP miracle", cosmic ray anomalies...)

Recall: the present bound on BR($\mu \to e\gamma$) requires

$$\Lambda \gtrsim 20\text{TeV} \quad \text{if} \quad \theta_{\mu e} \sim \frac{1}{\sqrt{2}}$$

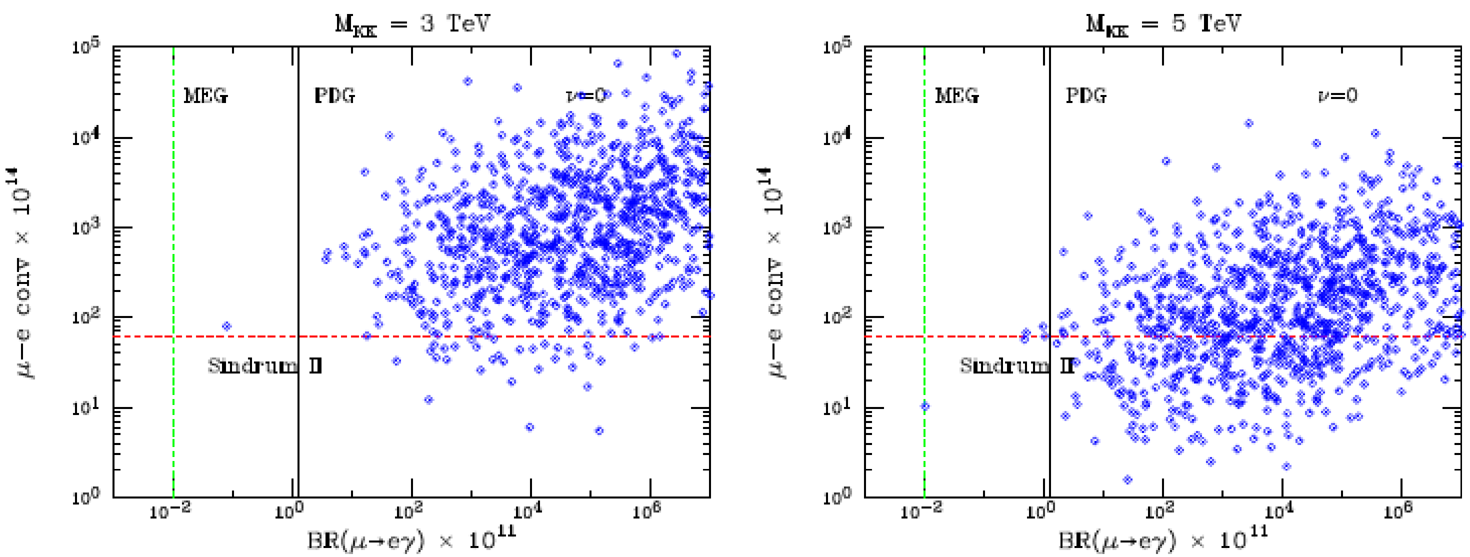
$$\theta_{\mu e} \lesssim 0.01 \quad \text{if} \quad \Lambda \sim 300\text{GeV}$$

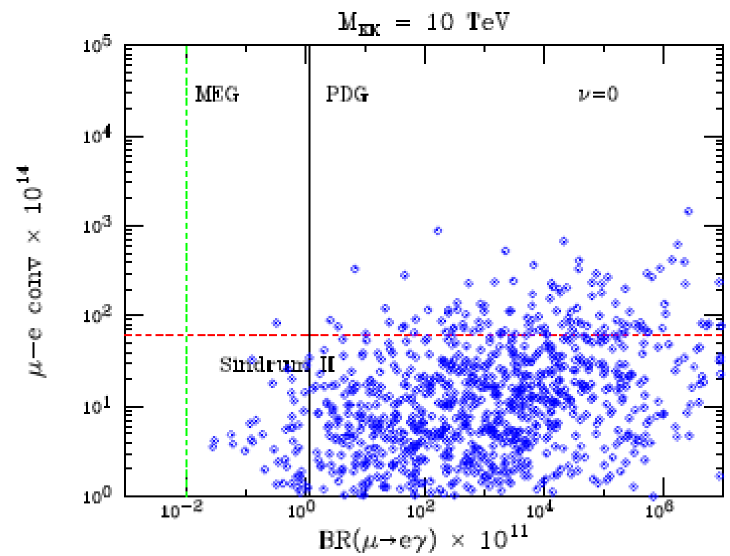
Very stringent constraints on models. Or on the positive side, detection might be around the corner.

This is the case for:

- Supersymmetric models
- (SUSY) see-saw models
- Extra dimensional models
- Little Higgs models
- ...

• Extra-dimensional models

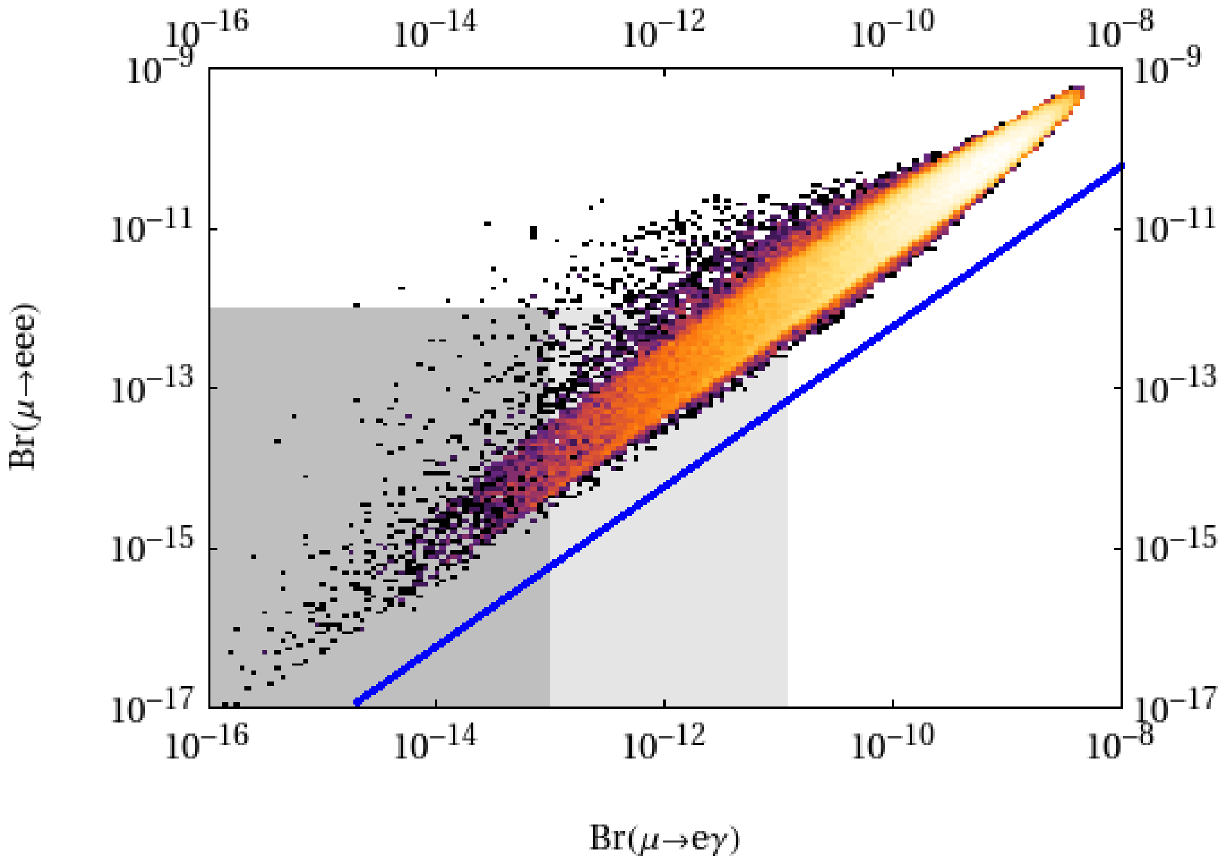




Agashe, Blechman, Petriello

"Anarchic" Randall-Sundrum model

• Little Higgs models (with T-parity)



Blanke et al.

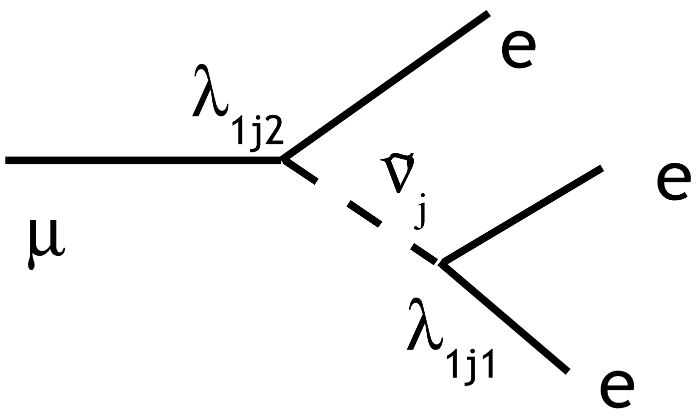
Mirror lepton masses between 300 GeV-1.5 TeV
Generic angles and phases

Supersymmetry

Many attractive features. However, SUSY has in flavour and CP its Achiles' heel. Even the minimal model introduces many new sources of flavour and CP violation.

1- Flavour and CP are badly violated at tree level:

$$W_{MSSM} = \mathbf{Y}^e_{ij} e^c_{Ri} L_j H_d + \mathbf{Y}^d_{ij} d^c_{Ri} Q_j H_d + \mathbf{Y}^u_{ij} u^c_{Ri} Q_j H_u + \mu H_u H_d + \frac{1}{2}\lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \frac{1}{2}\lambda''_{ijk} u^c_i d^c_j d^c_k + \mu'_i L_i H_u.$$



$\lvert\lambda_{1j1}\lambda_{1j2}\rvert < 7\times10^{-7}$	From $\mu\to 3e$
$\lvert\lambda_{231}\lambda_{131}\rvert < 7\times10^{-7}$	From $\mu\to 3e$
$\lvert\lambda_{231}\lambda_{232}\rvert < 5.3\times10^{-6}$	From μTi $\to e$Ti at one loop
$\lvert\lambda_{232}\lambda_{132}\rvert < 8.4\times10^{-6}$	From μTi $\to e$Ti at one loop
$\lvert\lambda_{233}\lambda_{133}\rvert < 1.7\times10^{-5}$	From μTi $\to e$Ti at one loop
$\lvert\lambda_{122}\lambda'_{211}\rvert < 4.0\times10^{-8}$	From μTi $\to e$Ti at tree level
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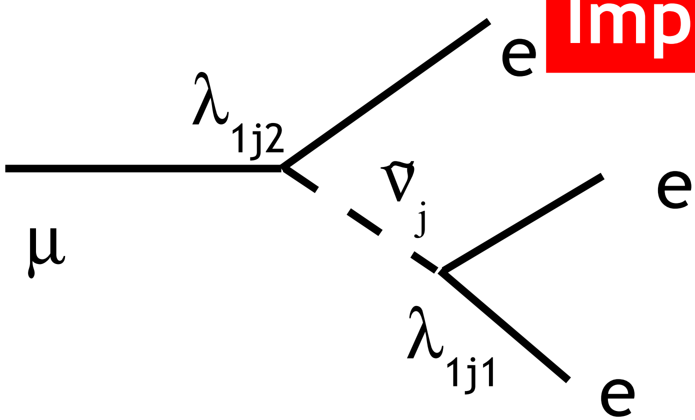
$$\frac{1}{2}\lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \frac{1}{2}\lambda''_{ijk} u^c_i d^c_j d^c_k + \mu'_i L_i H_u.$$

Impose R-parity conservation



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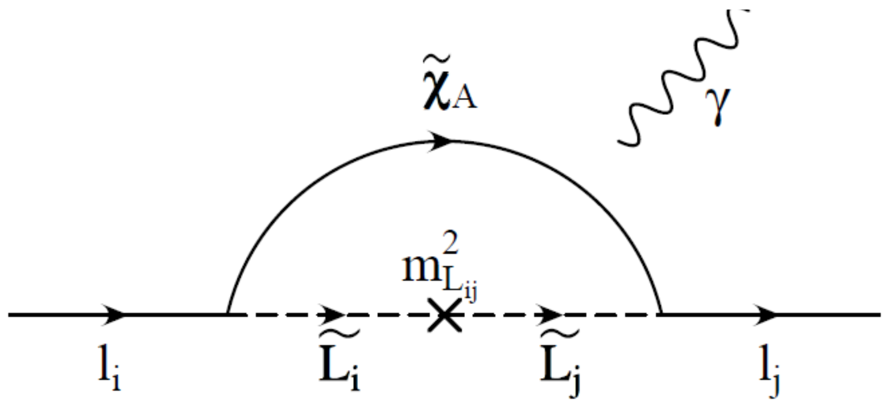
2- Soft SUSY breaking terms in general violate flavour

$$-\mathcal{L}_{\rm soft}^{\rm lep} = (\mathbf{m}_{\tilde L}^2)_{ij}\tilde{L}_i^*\tilde{L}_j + (\mathbf{m}_{\tilde e}^2)_{ij}\tilde{e}_{Ri}^*\tilde{e}_{Rj} + \left(\mathbf{A}_{eij}\tilde{e}_{Ri}^*\tilde{L}_j H_d + \text{h.c.}\right)$$



Back of the envelope calculation of BR($l_i \to l_j \gamma$):

$$\mathrm{BR}(\ell_j \to \ell_i\gamma) \simeq \frac{\alpha^3}{G_F^2}\frac{|(\mathbf{m}_{\tilde L}^2)_{ij}|^2}{m_S^8}\tan^2\beta\,\mathrm{BR}(\ell_j \to \ell_i\nu_j\bar\nu_i)$$

$$\begin{aligned}(\mathbf{m}_{\tilde L}^2)_{12}/m_S^2 &< 3\times 10^{-4}\\ (\mathbf{m}_{\tilde L}^2)_{13}/m_S^2 &< 0.09\\ (\mathbf{m}_{\tilde L}^2)_{23}/m_S^2 &< 0.09\end{aligned}$$

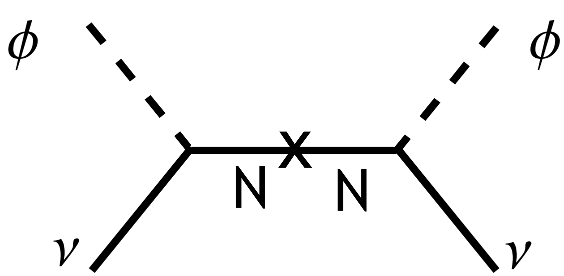
(for m_S=400 GeV and tanβ=10)

Possible explanation: messenger sector does not distinguish among flavours (gravity mediation, gauge mediation, gaugino mediation)

See-saw models

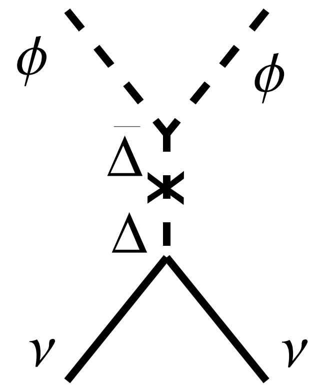
The smallness of neutrino masses can be very elegantly explained introducing new heavy degrees of freedom:

Type I



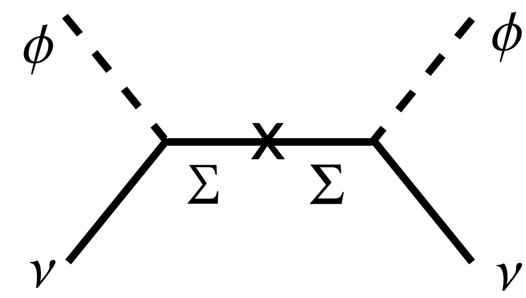
(fermion singlets)

Type II



(scalar triplets)

Type III



(fermion triplets)

Effective theory

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi}$$

After the EW symmetry breaking, generates tiny Majorana neutrino masses, if the scale of new physics Λ is large.

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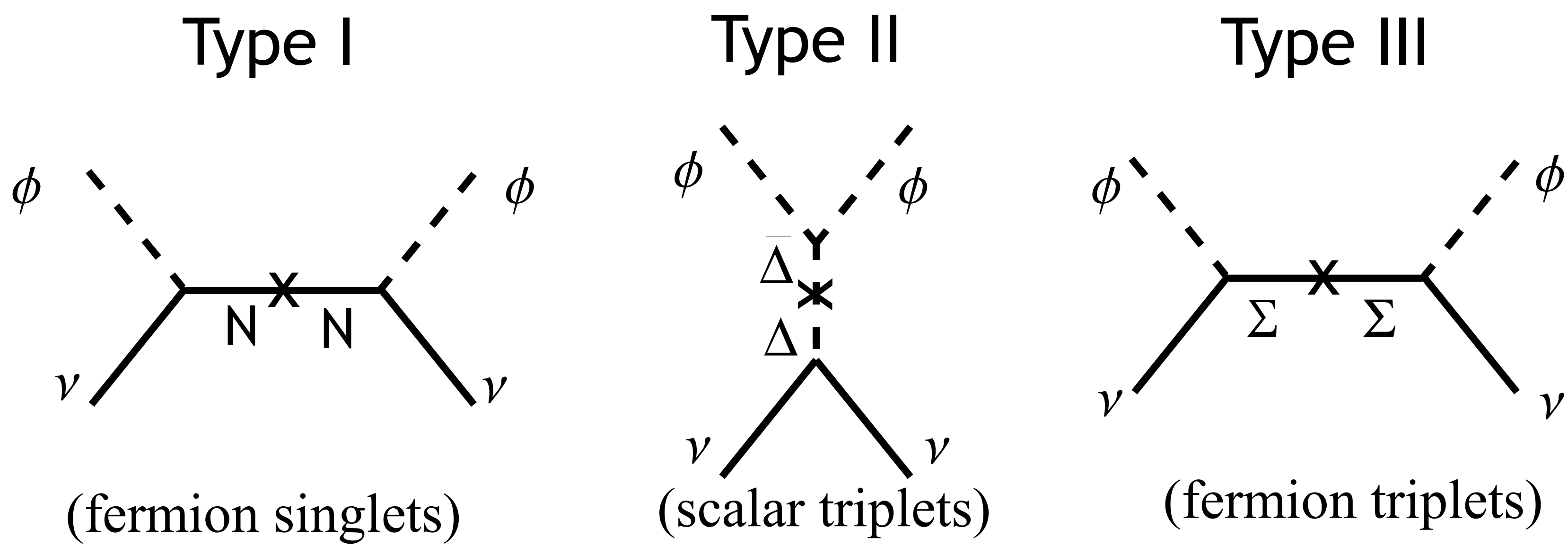

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Same suppression

The new degrees of freedom induce LFV processes, with rates suppressed by the large mass scale of the new particles.

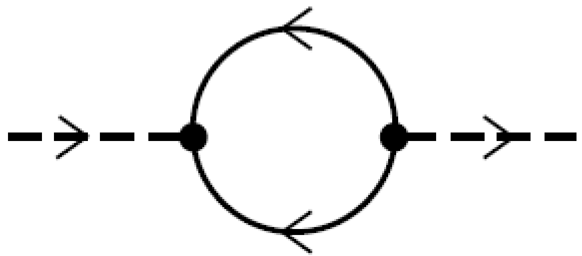
Good agreement with experiments, but the model is unnatural...

An explicit hierarchy problem

The see-saw Lagrangian is:

$$-\mathcal{L}_{\text{lep}} = (\mathbf{Y}_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} - \frac{1}{2}M_{ij}\bar{\nu}_{Ri}\nu^c_{Rj} + \text{h.c.}$$

The Higgs doublet interacts with heavy degrees of freedom



$$\delta m_\phi^2 \sim \frac{1}{16\pi^2}Y_\nu^2 M^2$$

Quadratic divergence!

$$m_\nu \sim \frac{Y_\nu^2 \langle\phi^0\rangle^2}{M}$$

$$\delta m_\phi^2 \sim \frac{1}{16\pi^2}\frac{m_\nu M^3}{\langle\phi^0\rangle^2}$$

$$\delta m_\phi^2 \lesssim m_\phi^2 \Rightarrow M \lesssim 10^7\ \text{GeV}$$

$$(\Rightarrow Y_\nu \lesssim 10^{-4})$$

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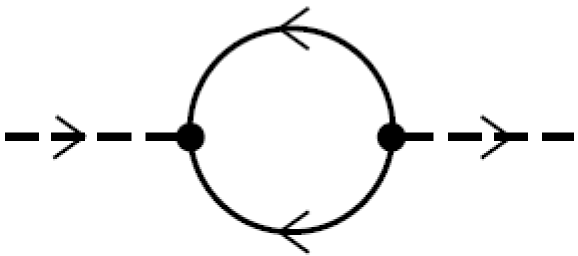

$$\delta m^2_\phi \sim \frac{1}{16\pi^2}Y_\nu^2 M^2$$

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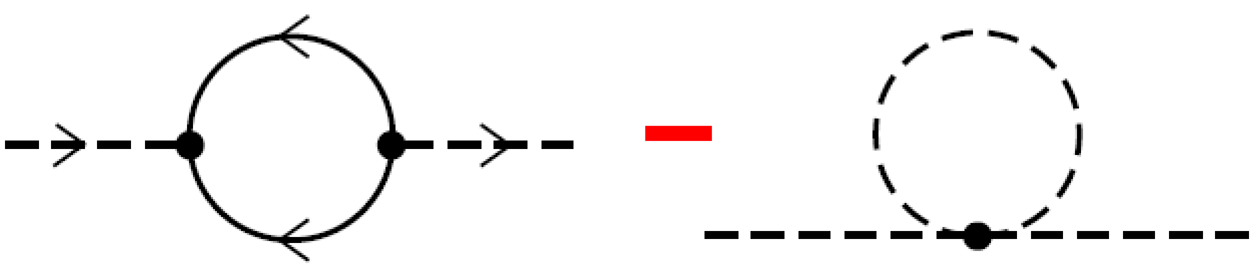
In the SUSY version of the see-saw



$$\delta m^2_H \sim \frac{1}{16\pi^2}Y^2 \log\frac{M^2}{\Lambda^2}$$

SUSY is the natural framework to implement the (high-scale) see-saw mechanism

New opportunities to test the see-saw mechanism!

Supersymmetric (type I) see-saw model

Consider the scenario with least number of new sources of LFV:

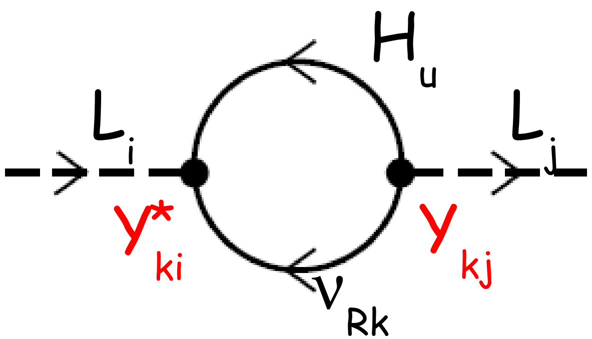
- R-parity conserved:

$$W_{\rm lep} = e^c_{Ri}\mathbf{Y}_{eij}L_jH_d + \nu^c_{Ri}\mathbf{Y}_{\nu ij}L_jH_u - \frac{1}{2}\nu^c_{Ri}\mathbf{M}_{ij}\nu^c_{Rj}$$

$$W^{\rm eff}_{\rm lep} = e^c_{Ri}\mathbf{Y}_{eij}L_jH_d + \frac{1}{2}\left(\mathbf{Y}_\nu^T\mathbf{M}^{-1}\mathbf{Y}_\nu\right)_{ij}(L_iH_u)(L_jH_u)$$

- Flavour blind mediation mechanism: no LFV in the soft terms at the cut-off scale.

If the particles responsible for neutrino masses are lighter than the mediation scale, quantum corrections will necessarily generate flavour violating terms in the slepton sector: Borzumati, Masiero



$$\left(\delta\mathbf{m}^2_L\right)_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2+|A_0|^2)(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{ij}\log\left(\frac{\Lambda}{M_{\rm maj}}\right)$$

$$\left(\delta\mathbf{m}^2_e\right)_{ij} \simeq 0\ ,$$

$$(\delta\mathbf{A}_e)_{ij} \simeq \frac{-3}{8\pi^2}A_0\mathbf{Y}_e(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{ij}\log\left(\frac{\Lambda}{M_{\rm maj}}\right)\ ,$$

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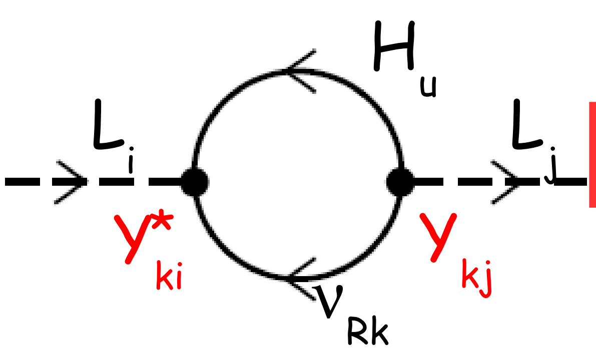

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$$(\delta\mathbf{m}^2_e)_{ij} \simeq 0\,,$$

$$(\delta\mathbf{A}_e)_{ij} \simeq \frac{-3}{8\pi^2}A_0\mathbf{Y}_e(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)_{ij}\log\left(\frac{\Lambda}{M_{\rm maj}}\right).$$

Logarithmic dependence with M

The calculation of the rate is, however, full of uncertainties

Back of the envelope calculation of BR($l_i \rightarrow l_j \gamma$):

$$\mathrm{BR}(\ell_j \to \ell_i \gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|(\mathbf{m}_L^2)_{ij}|^2}{m_S^8} \tan^2\beta \, \mathrm{BR}(\ell_j \to \ell_i \nu_j \bar{\nu}_i)$$

$$(\mathbf{m}_L^2)_{ij} \simeq \mathbf{m}_L^2(\Lambda)_{ij} - \frac{1}{8\pi^2}(3m_0^2 + |A_0|^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{\Lambda}{M_{\mathrm{maj}}}\right)$$

Cut-off scale?
Flavour structure of the soft terms at the cut-off scale?
soft-SUSY parameters?
$\tan\beta$?
Size and flavour structure of the Yukawa couplings?
Right-handed neutrino masses?

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$$\mathrm{BR}(\ell_j \to \ell_i \gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|(\mathbf{m}_L^2)_{ij}|^2}{m_S^8} \tan^2\beta \, \mathrm{BR}(\ell_j \to \ell_i \nu_j \bar{\nu}_i)$$

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Flavour structure of the soft terms at the cut-off scale?
soft-SUSY parameters?
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$$\mathbf{Y}_\nu, M$$

See-saw parameters

$$\mathcal{M}_\nu = \mathbf{Y}_\nu^T M^{-1} \mathbf{Y}_\nu \langle \phi^0 \rangle^2$$

Neutrino masses and mixing angles

LFV

$$\left(\mathbf{m}_L^2\right)_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2 + |A_0|^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{M_X}{M}\right)$$

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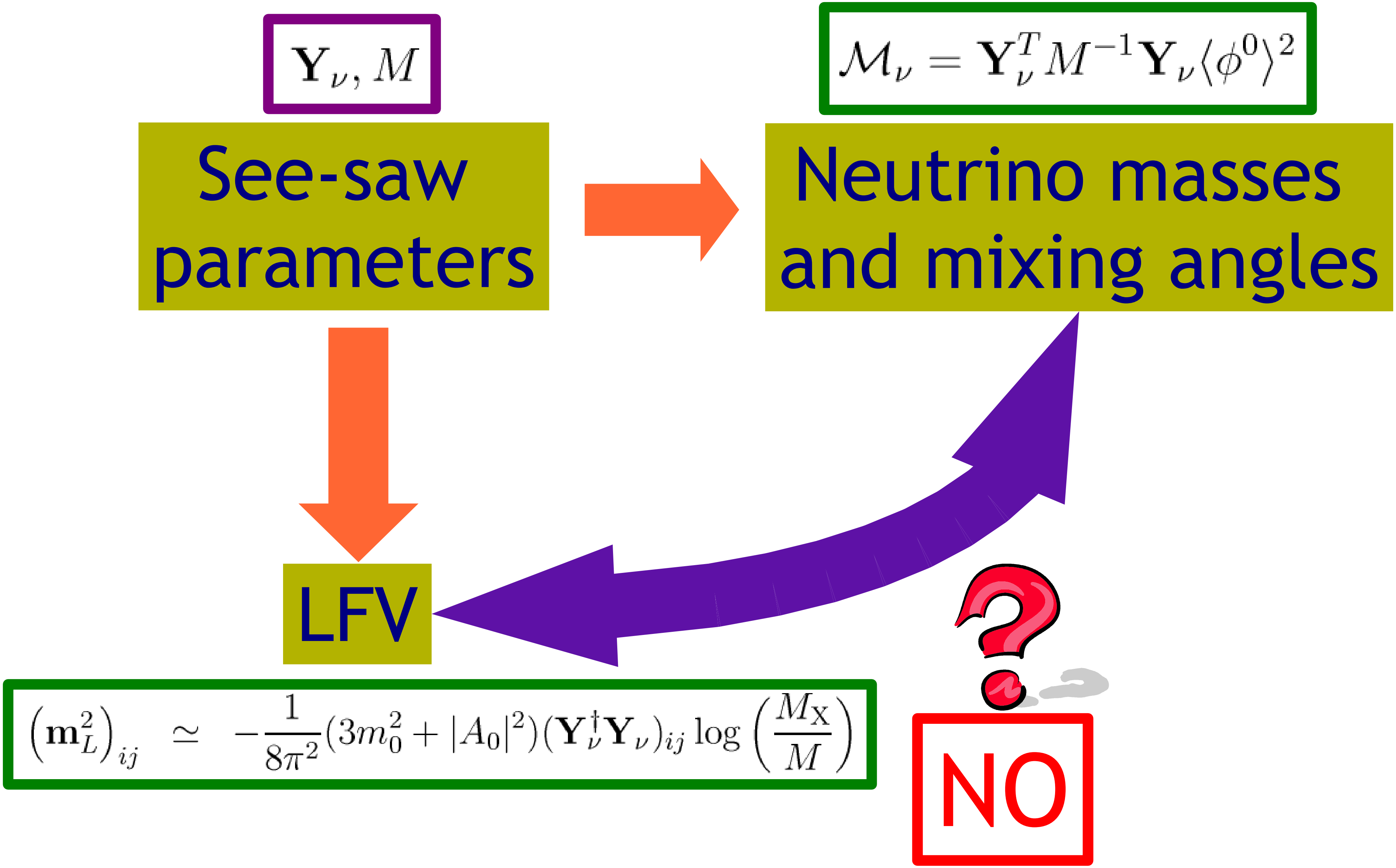

The see-saw Lagrangian has 12+6 new parameters.
Neutrino observations at most can fix 6+3 parameters.
Still, there are 6+3 free parameters.

There are, compatible with the observed neutrino parameters, an infinite set of Yukawa couplings! Casas, AI

$$\mathbf{Y}_\nu = \frac{1}{\langle H_u^0 \rangle} \sqrt{D_M} R \sqrt{D_m} U_{\text{lep}}^\dagger$$

Right-handed neutrino masses

Complex orthogonal matrix

"Fixed" by experiments

Changing R and the right-handed neutrino masses, any $Y^\dagger Y$ can be obtained.

In fact, there is a one-to-one correspondence between

$$\{\mathbf{Y}_\nu, M\} \leftrightarrow \left\{\mathcal{M}, \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right\}$$

Davidson, AI

High-energy parameters of the see-saw Lagrangian $\longleftrightarrow$ Low energy observables: neutrino mass matrix, BR($l_i \to l_j \gamma$), EDMs

From a *model independent* perspective, the type-I see-saw can accommodate anything at low energies!! **No predictions**

Is this a dead-end? Is it impossible to test the SUSY see-saw?

Remarkably, under some well motivated assumptions, it is possible to derive predictions for the LFV processes, in the form of lower bounds.

Procedure:

- Consider the worst case scenario to detect LFV $\hookrightarrow$ Lower bounds
- Assume absence of tunings.
- Assume hierarchical neutrino Yukawa couplings.
- Make the calculations carefully!

The worst case scenario for the detection of LFV in the SUSY see-saw is:

- R-parity conserved
- $(\mathbf{m}_L^2)_{ij}$, $(\mathbf{m}_e^2)_{ij}$, $\mathbf{A}_{eij}$, $i \neq j$ vanish at high energies
  (no LFV in the soft terms at the cut-off scale)
- $(Y_\nu^\dagger Y_\nu)$ diagonal
  (compatible with neutrino masses)

The back of the envelope calculation gives $\mathrm{BR}(l_i \to l_j \gamma)=0$

$$\mathrm{BR}(\ell_j \to \ell_i \gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|(\mathbf{m}_L^2)_{ij}|^2}{m_S^8} \tan^2\beta \, \mathrm{BR}(\ell_j \to \ell_i \nu_j \bar{\nu}_i)$$

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However, this calculation implicitely assumes that all the right-handed neutrinos decouple at the same scale M_{maj}

Strictly speaking $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log\left(\frac{\Lambda}{M_{\mathrm{maj}}}\right) \longrightarrow \sum_k \mathbf{Y}_{\nu ki}^* \log\left(\frac{\Lambda}{M_k}\right) \mathbf{Y}_{\nu kj}$

which is necessarily different from zero (unless cancellations take place)

Assume:

- No cancellations
- Hierarchical neutrino Yukawa eigenvalues: $y_1 \ll y_2 \ll y_3$
  (as in the rest of known Yukawa matrices)
- Cut-off scale at very high energies.

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 1.2 \times 10^{-11} \left(\frac{y_1}{4 \times 10^{-2}}\right)^4 \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2 ,$$

Where the smallest Yukawa coupling is related to the lightest right-handed neutrino mass through: $M_1 \lesssim \frac{y_1^2 \langle H_u^0 \rangle^2}{\sqrt{\Delta m^2_{\mathrm{sol}}}}$

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 1.2 \times 10^{-11} \left(\frac{M_1}{5 \times 10^{12}\mathrm{GeV}}\right)^2 \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2$$

AI, Simonetto

Experiments on rare decays provide upper bounds on see-saw parameters:

$\mathrm{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}$

$y_1 \lesssim 4 \times 10^{-2}$

$M_1 \lesssim 5 \times 10^{12}$ GeV

→

$R(\mu\, N \to e\, N) < 10^{-18}$

$y_1 \lesssim 2 \times 10^{-3}$

$M_1 \lesssim 2 \times 10^{10}$ GeV

For m_S=200 GeV
$\tan\beta$=10

Relevant for baryogenesis through leptogenesis: $M_1 \gtrsim 10^9$ GeV

Baryogenesis through leptogenesis

After the discovery of neutrino oscillations, leptogenesis stands as a very attractive explanation for the observed matter-antimatter asymmetry.

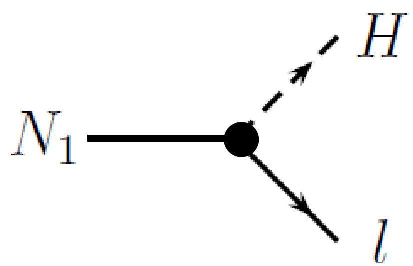
The simplest leptogenesis scenario consists in the out of equilibrium decay of the lightest right-handed neutrino. Then, the three Sakharov conditions are automatically fulfilled.

- Violation of B-L. Guaranteed if neutrinos are Majorana particles.
- C and CP violation. Guaranteed if the neutrino Yukawa couplings contain physical phases.
- Departure from thermal equilibrium. Guaranteed, due to the expansion of the Universe.

The generation of a baryon asymmetry is guaranteed in the leptogenesis mechanism. But, can leptogenesis generate the *observed* baryon asymmetry?

Roughly speaking, the generation of a BAU through leptogenesis proceeds in three steps:

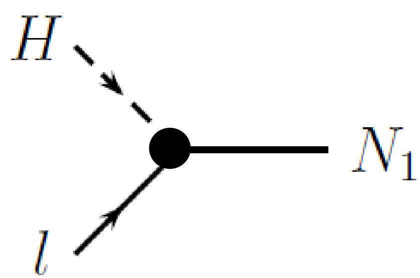
1- Generation of a lepton asymmetry in the decay of the lightest right-handed neutrino.



CP- asymmetry:

$$\epsilon_1 \lesssim \frac{3}{8\pi} \frac{M_1 \sqrt{\Delta m^2_{\rm atm}}}{v^2}$$

2- Washout of the lepton asymmetry.

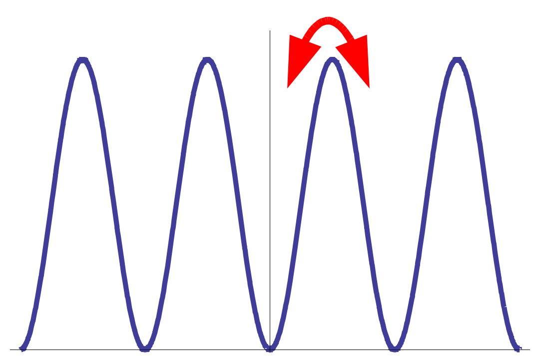


"Wash-out parameter":

$$\widetilde{m}_1 = (\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)_{11} \frac{v^2}{M_1}$$

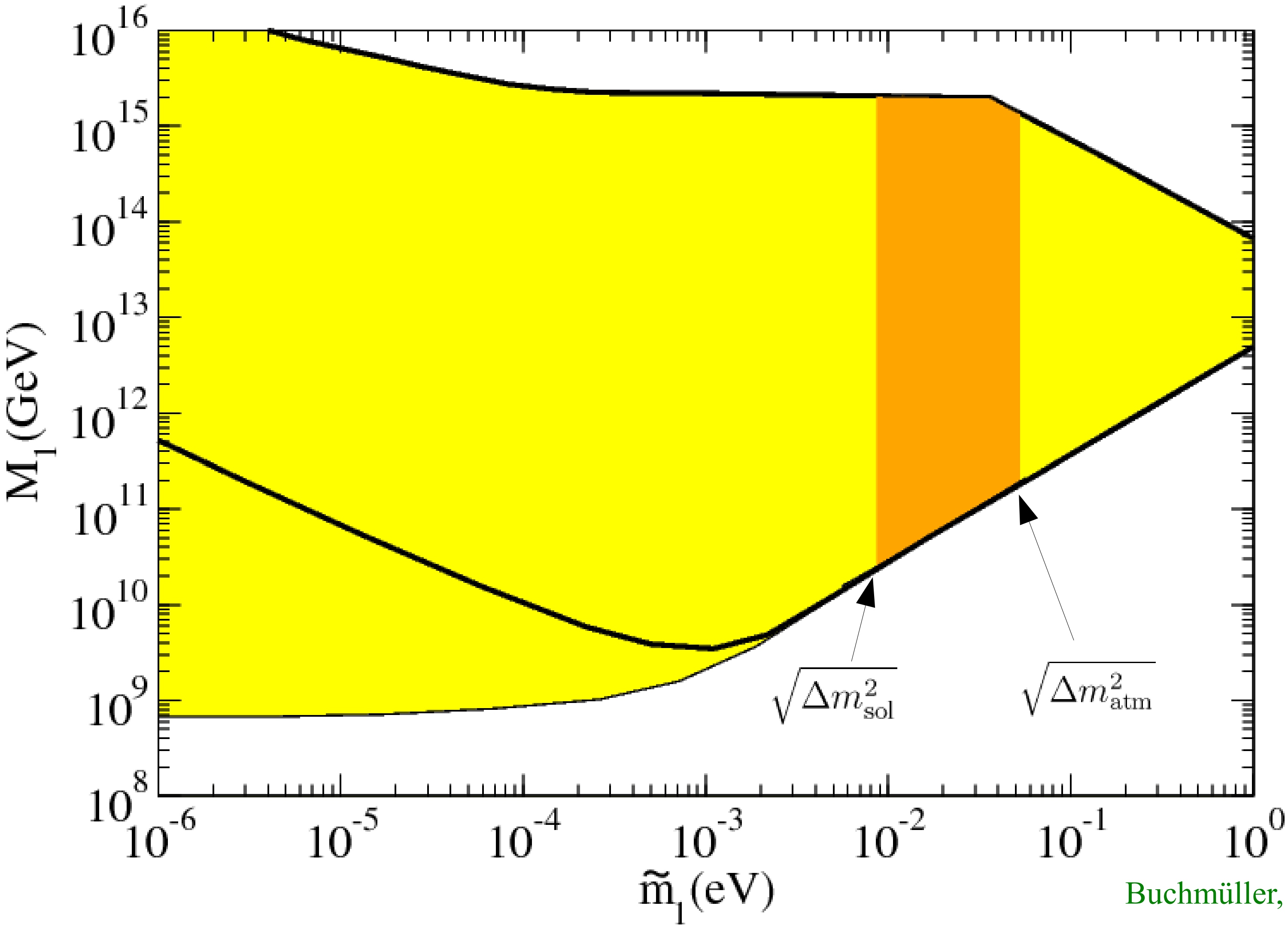
typically $\sqrt{\Delta m^2_{\rm sol}} \lesssim \widetilde{m}_1 \lesssim \sqrt{\Delta m^2_{\rm atm}}$

3- Conversion of the lepton asymmetry into a baryon asymmetry.



$$\eta_{\rm B} \approx \eta_{\rm L}/2$$

Leptogenesis parameter space



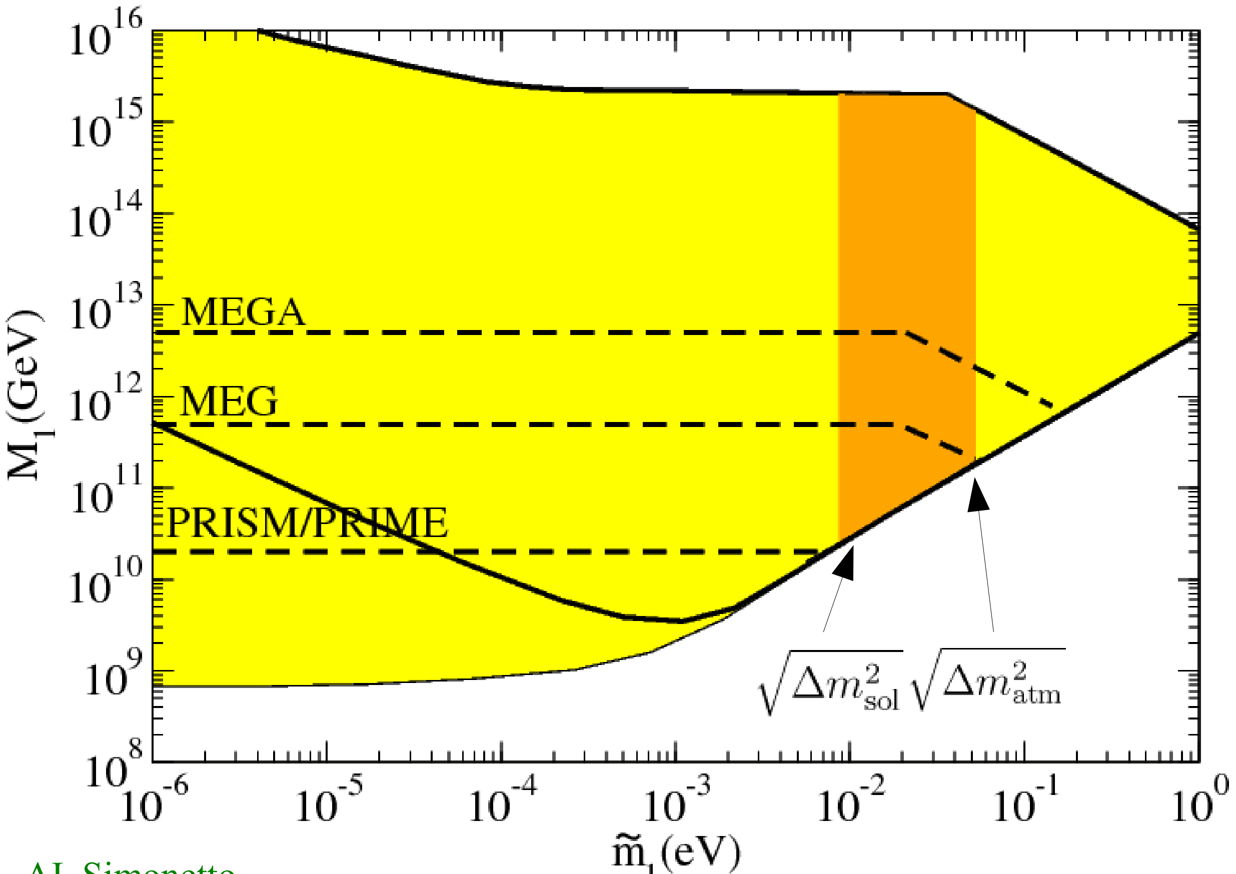
Buchmüller, Di Bari, Plümacher

Probing SUSY leptogenesis with LFV

Assumptions:

- No cancellations
- hierarchical neutrino Yukawa eigenvalues: $y_1 \ll y_2 \ll y_3$

(Leptogenesis requires $\Lambda > 10^{16}$ GeV, so no need to assume a large cut-off)



AI, Simonetto

m_S=200 GeV, tanβ=10

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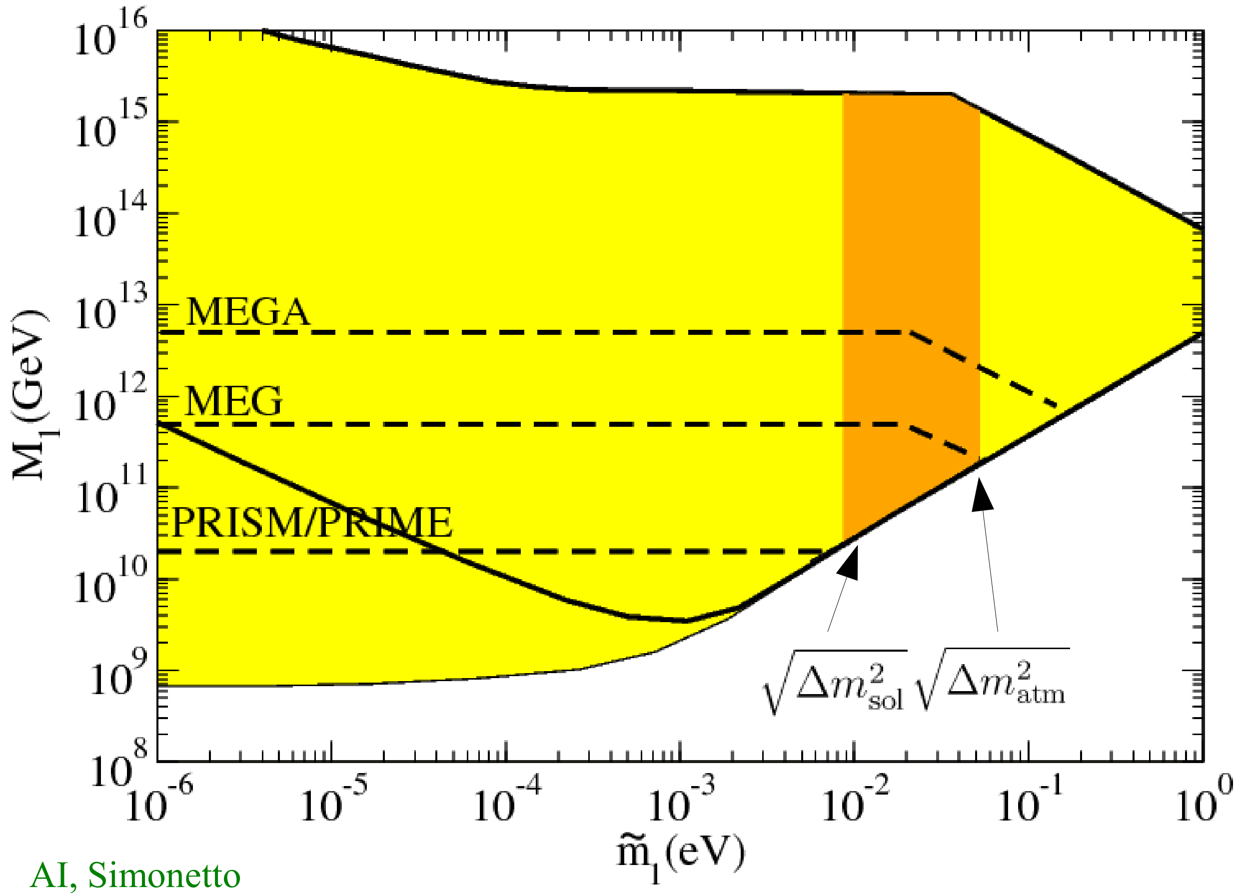
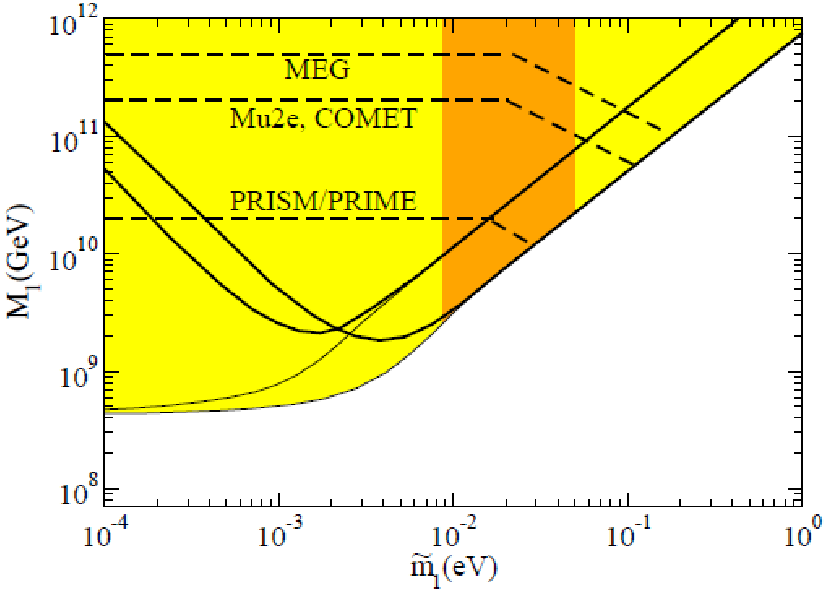


AI, Simonetto

m_S=200 GeV, tanβ=10

Including flavour effects in leptogenesis



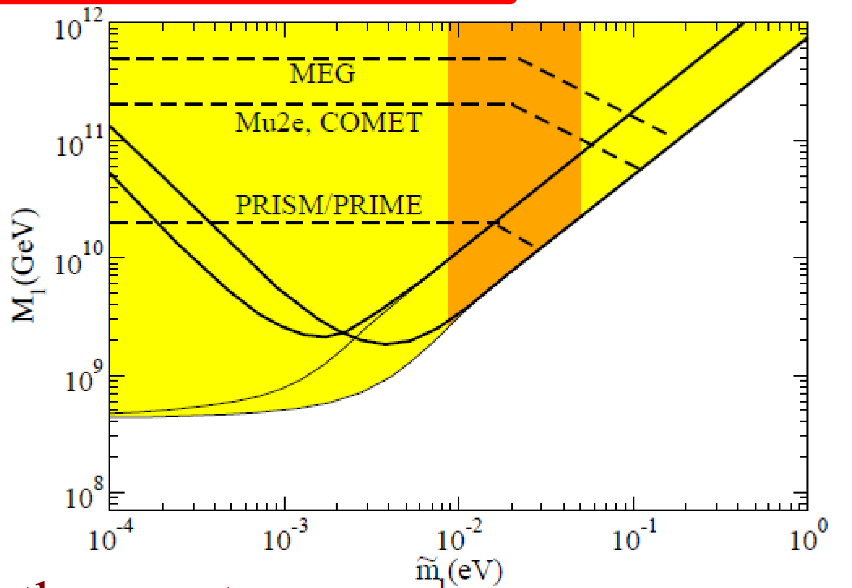
μ-e conversion at 10^{-18}??

$$R(\mu \mathrm{Ti} \to e \mathrm{Ti}) \gtrsim 10^{-18} \left(\frac{M_1}{2 \times 10^{10}\ \mathrm{GeV}}\right)^2 \left(\frac{m_S}{200\ \mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10\ \mathrm{GeV}}\right)^2$$

Expectations from leptogenesis,
(for m_S=200 GeV, tanβ=10)

Natural region for $\tilde{m}_1$:

No flavour effects: $R \gtrsim 10^{-18}$
"typical" flavour effects: $R \gtrsim 2\times10^{-19}$
"extreme" flavour effects: $R \gtrsim 2\times10^{-20}$



Note that in deriving this result we have assumed the worst case scenario for the detection of μ – e flavour violation:

- R-parity conserved.
- Universal soft terms at the cut-off scale.
- Yukawa textures that minimize the flavour violation: $(Y_\nu^\dagger Y_\nu)$ diagonal.
- Also, it is unlikely that M_1 saturates the lower bound (this requires optimal CP phases).

→ in general, much larger rates expected

Conclusions

Why μ to e conversion at $<10^{-18}$?

• Many models of new physics can be probed, even at energy scales much larger than the ones reachable by the Tevatron/LHC.

• A positive signal is in general expected if (SUSY) leptogenesis the correct mechanism to explain the observed matter-antimatter asymmetry.

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Thank you for your attention