Studies of Li-rod based muon ionization cooling channel

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28 January

The presentation covers results prepared to publication[a]

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[a] The article preprint will be sent to the *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* in joint authorship with Skrinsky A.N. (BINP, Russia) as “Studies of Li-rod based muon ionization cooling channel”

Excerpts from abstract:

“... Features of muon beam motion are discussed, namely the influence of non-paraxiality of motion and transverse-longitudinal coupling. The inclusion of an emittance exchanger to the cooling channel can result in the cooling of all degrees of freedom. The appropriate beam parameters for emittance exchange procedure and their dependence on transverse emittance and beam longitudinal parameters are discussed...”
“ ... a comparison between simulations results and the predictions of a linear model serves both to examine the simulation code and to determine the contribution of non-paraxiality to the beam motion...”

1.1 Period of the cooling channel

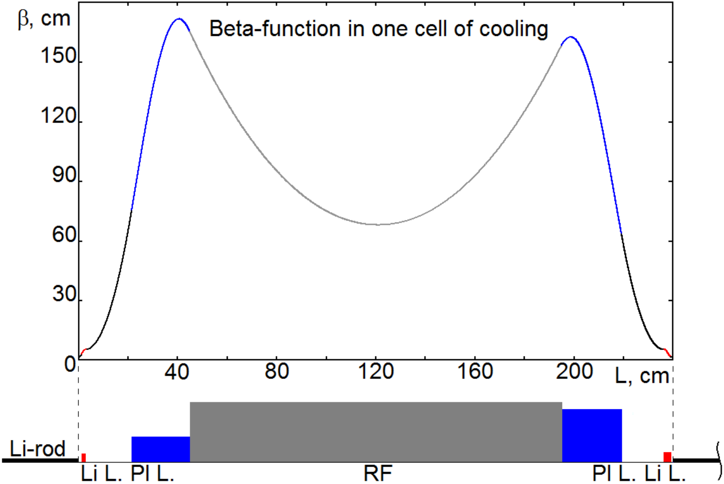
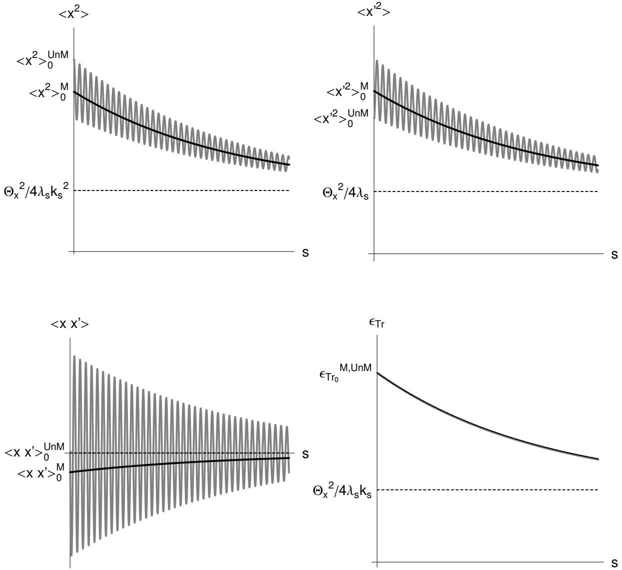


Figure: Example of the β-function behavior in one period of the cooling channel.

2.1 Linear model

The transverse motion is determined by the following processes:

- Friction force — ionization losses;
- “Diffusion” — Coulomb scattering;
- Focusing.


$<x^2>$
$<x^2>_0^{UnM}$
$<x^2>_0^{M}$
$\Theta_x^2/4\lambda_s k_s^2$
s
$<x'^2>$
$<x'^2>_0^{M}$
$<x'^2>_0^{UnM}$
$\Theta_x^2/4\lambda_s$
s
$<x\,x'>$
$<x\,x'>_0^{UnM}$
$<x\,x'>_0^{M}$
s
ϵ_{Tr}
$\epsilon_{Tr_0}^{M,UnM}$
$\Theta_x^2/4\lambda_s k_s$
s

2.2 Results of simulation

Non-paraxiality of motion:

Relatively simple analytical model agrees with simulation to high precision, with a discrepancy of less than 5%

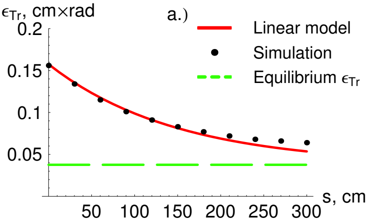


Figure: Linear model of transverse motion compared to simulation in the cases of a) large initial transverse emittance and b) small one.

2.2 Results of simulation

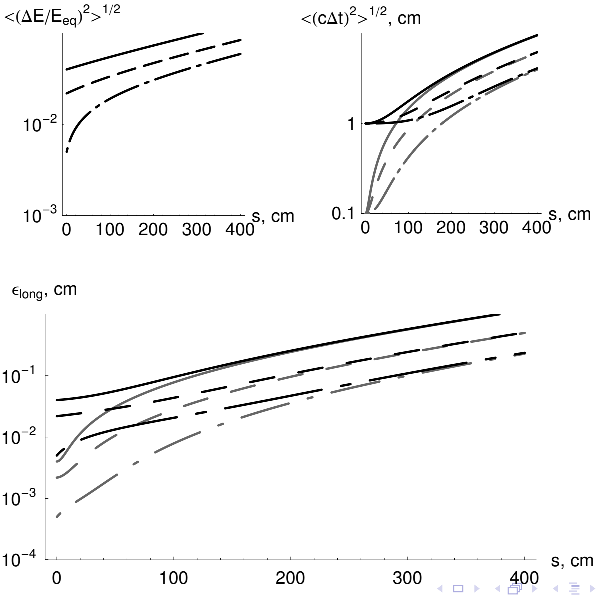
Transverse–longitudinal coupling:

It has been found that only in cases of relatively small transverse emittance close to the equilibrium value or substantial initial energy spread (more than 10%) is there a small decrease in the rate of cooling compared (around 5%) to the linear model.

3.1 Linear model

The longitudinal motion is determined by the concurrence of:

- “Anit–damping” — negative derivative of ionization friction force;
- “Diffusion” — ionization losses fluctuations.


$<(\Delta E/E_{eq})^2>^{1/2}$
10^{-2}
10^{-3}
0
100
200
300
400
s, cm
$<(c\Delta t)^2>^{1/2}$, cm
1
0.1
0
100
200
300
400
s, cm
ϵ_{long}, cm
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
100
200
300
400
s, cm

3.2 Results of simulation

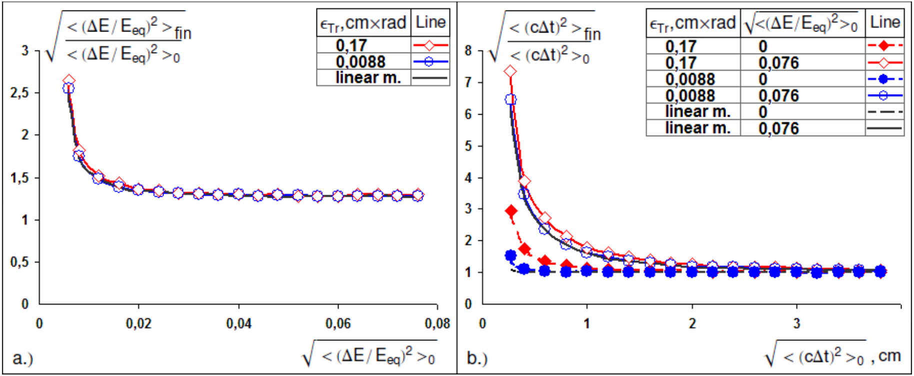


Figure: Growth of beam longitudinal parameters after single rod passage under different initial conditions in comparison with a linear model (denoted by "linear m.").

3.2 Results of simulation

Longitudinal–transverse coupling:

- The ratio of the final root-mean-square energy spread upon the exit of a lithium rod to that at the entrance) is independent of the transverse beam parameters, even for large values (though undoubtedly these parameters are limited from above to values reasonable for this cooling scheme);
- In contrast, the growth of $\langle (c\Delta t)^2 \rangle$ conforms to the linear model prediction with confidence only in the case of small transverse emittance; the increment of the spread in the arrival time upon the exit of the rod grows with the transverse emittance (at fixed initial parameters).

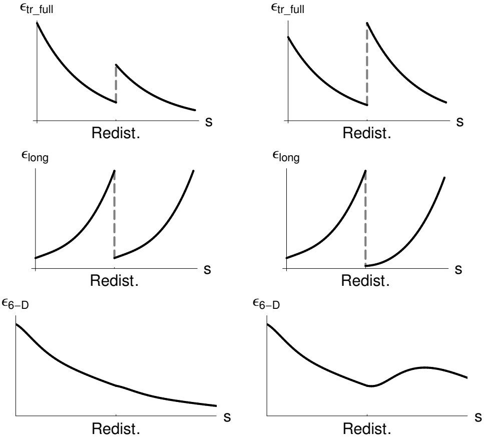


Figure: Behavior of full transverse, longitudinal and 6-D emittances as functions of the length passed in matter during cooling with the optimal redistribution from the longitudinal to transverse degrees of freedom (left column) and excessive redistribution with undesirable full emittance increment (right column).

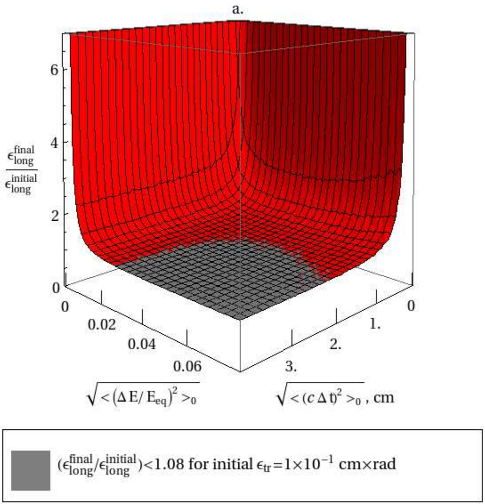


Figure: Growth of the longitudinal emittance ($\varepsilon_{long_{fin}}/\varepsilon_{long_{ini}}$) as a function of the initial root-mean-square energy and arrival time spreads after passage through one lithium rod for a fixed value of initial ε_{tr}.

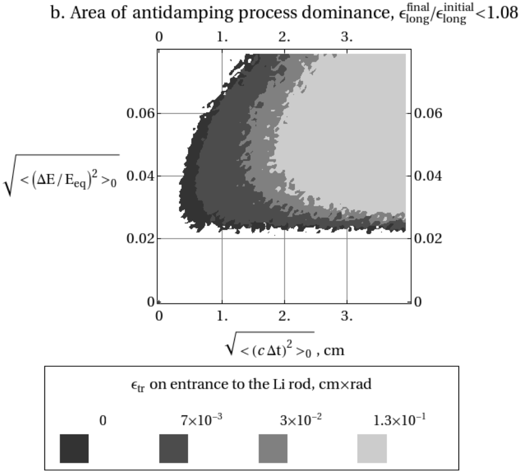


Figure: The region in which the antidamping process dominates over the fluctuation of ionization losses (the region where the growth of the ε_{long} is below a preassigned value defined from the asymptotics).

4. Summary

Main conclusions:

- The selected linear model is able to describe the transverse motion with high precision independent of beam longitudinal parameters.
- The evolution of the relative energy deviation also converges with the linear model for a wide range of transverse emittances and is independent of the deviation in the arrival time. In contrast, the spread of the arrival time agrees with the linear model only in the case of small transverse emittance, which is a direct consequence of non-paraxiality.
- The additional increase of the longitudinal emittance in comparison with the linear model prediction have been described in more detail. The statistical approach gives cogent evidence of the efficiency of the "head-tail" rotation procedure for suppression of non-paraxiality related effects.

Thank you for your attention!

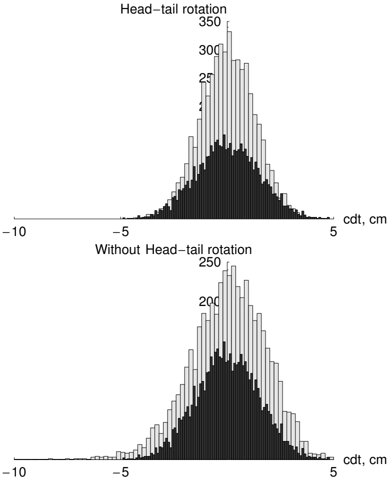


Figure: The evolution of the distribution of the arrival time spread after passing through 5 rods in cases of (top) “head-tail” rotation usage and (bottom) without it; ■ — initial distribution, □ — distribution after passing 5 rods.

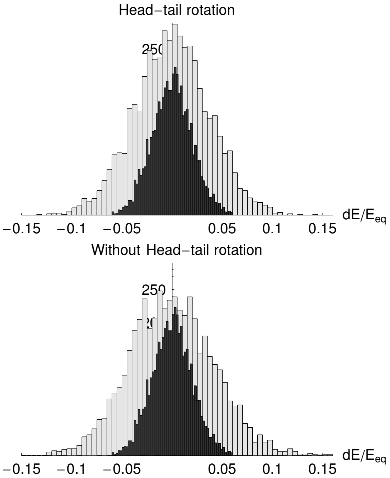


Figure: The evolution of the distribution of the relative energy spread after passing through 5 rods in cases of (top) “head-tail” rotation usage and (bottom) without it; ■ — initial distribution, □ — distribution after passing 5 rods.