Passive hybrid SiPM array

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Abstract

The development of the .

1 Introduction

This note analyzes the dynamics of a the so called Hybrid SiPM array. The hybrid array used external resistors and capacitors to separate the SiPM biasing from the fast AC signal. In that way, the DC bias voltage can be applied in parallel to all the SiPMs in the hybrid array; keeping the bias voltage equal to the single SiPM bias. Instead, the high frequency signal originated by photo electron activity travel through the SiPMs in a series connection, facilitated by low impedance capacitors. Figure 1a, 1b and 1c show how the DC bias parallel and fast signal series connection is facilitated by use of external R and C's.

To understand the dynamics of the hybrid model we need to understand first the dynamics of a single SiPM. The model used is the same model used in [1-2] and shown in Figure 1. A single photo electron (PE) signal is modeled by a pulsed current source of 50uA and 10ns. That is equivalent to the charge deposited by an avalanche with a typical gain of $3x10^6$. The R and C values of the SiPM modeled in 1 can be understood in the following way. The active cell (firing 1PE) is one in a large array. In this particular case we have used the example of a 6mm x 6mm Hamamatsu array with 14K microcells. The components R0, R1, C0, C1 are associated to the (firing) microcell. The capacitor C1 is very small, 3.3 fF. It adds a time constant of 50 pico seconds so their transient is much smaller than the duration of the pulse (i.e. 10ns) and can be ignored in this analysis. For the dynamic analysis we use the model in figure 2.

2 Dynamic model of a Single SiPM

For the analysis we use the Laplace transform

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt \tag{1}$$

where the complex variable $s = \sigma + i\omega$. The conversion to Fourier is straightforward replacing s by $i\omega$. For electrical circuits the impedances become R, $X_L = SL$ and $X_C = \frac{1}{SC}$. To analyze the dynamics we want to understand the transfer function $\frac{V_{M1}(s)}{IG1(s)}$ in the frequency domain and the $V_{M1}(t)$ in the time domain. In either case we need the transfer function in the complex variable s of the total impedance formed by R1, R2 and C0, C2, C3 and C4. That impedance is a combination of parallel and series. We define $Z_2 = R_2||C_2$; $Z_{23} = Z_2 + X_{C3}$; $Z_{234} = Z_{23}||X_{C4}$; $Z_{1234} = R_1 + Z_{234}$; and $Z_{total} = Z_{04} = C_0||Z_{1234}$ then:

$$Z_2 = \frac{R_2}{1 + SC_2R_2} \tag{2}$$

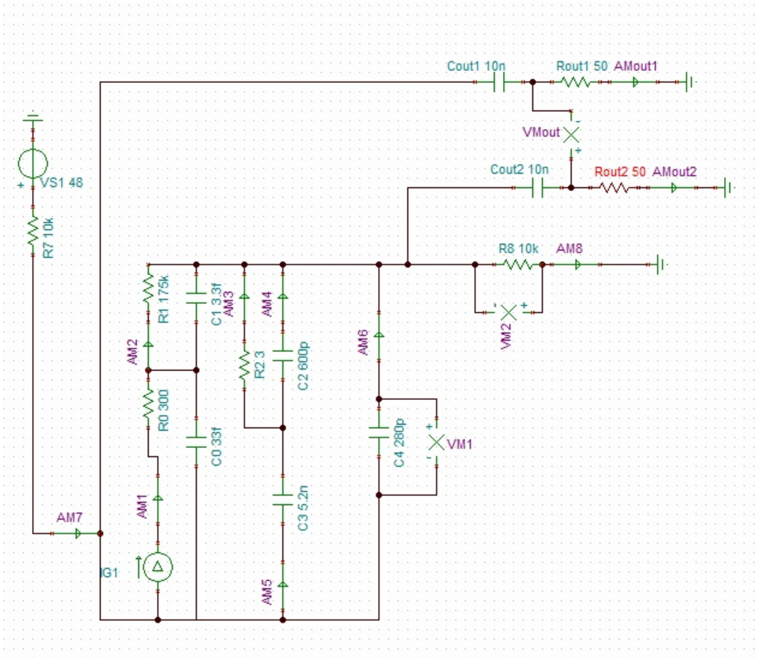


Figure 1: SIPM model including voltage bias source and differential load

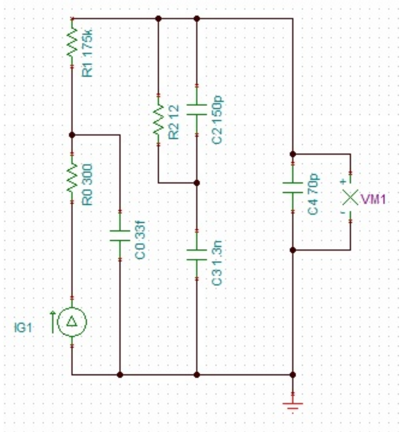


Figure 2: SIPM model for dynamic study

$$Z_{23} = \frac{R_2}{1 + SC_2R_2} + \frac{1}{SC_3} =$$
$$Z_{23} = \frac{1 + SR_2(C_2 + C_3)}{SC_3(1 + SC_2R_2} \tag{3}$$

$$Z_{234} = Z_{23}||\frac{1}{C_4} =$$
$$Z_{234} = \frac{1 + SR_2(C_2 + C_3)}{S(C_3 + C_4) + S^2R_2(C_2C_3 + C_2C_4 + C_3C_4)} = \tag{4}$$
$$Z_{234} = \frac{1 + SR_2(C_2 + C_3)}{S(C_3 + C_4)(1 + SR_2C_{eq})}$$

where
$$C_{eq} = \frac{C_2C_3 + C_2C_4 + C_3C_4}{C_3 + C_4} \tag{5}$$

We can define
$T_0 = R_2(C_2 + C_3)$
$T_1 = R_2C_{eq}$
$S_0 = T_0^{-1}$
$S_1 = T_1^{-1}$
then, Z_{234} can be written as a algebraic Laplace transform with one finite zero at S_0 and two poles at $S = 0$ and S_1

$$Z_{234} = Z_0\frac{S + S_0}{S(S + S_1)} \tag{6}$$

where $Z_0 = \frac{T_0}{T_1(C_3+C_4)}$

equivalently,
$$Z_{234} = \frac{1 + ST_0}{(C_3 + C_4)S(1 + ST_1)} \tag{7}$$

using the typical values for the 6mm x 6mm Hamamatsu S13360-6050VE the time constants and singularities are:
$T_0 = 17.4ns$
$T_1 = 2.6ns$
$S_0 = (2\pi)9.1MHz$
$S_1 = (2\pi)61.2MHz$
To form Z_{1234} we add R_1 in series with Z_{234}

$$Z_{1234} = \frac{SR_1(C_3 + C_4)(1 + ST_1) + 1 + ST_0}{(C_3 + C_4)S(1 + ST_1)} \tag{8}$$

We see that the numerator of (15) is a second order polynomial in S, so it can be written as

$$Z_{1234} = \frac{R_1T_1(S + P_1)(S + P_2)}{S(1 + ST_1)} \tag{9}$$

where, P_1 and P_2 are the solution of the second order equation numerator of (15). Solving for P_1 and P_2 using the Hamamatsu S13360-6050VE values finds $P_1 \approx \frac{1}{T_1}$ and $P_2 \approx \frac{5.410^{-6}}{T_1} = \frac{a}{T_1}$. Then, equation (16) can also be expressed by

$$Z_{1234} = \frac{aR_1(1 + ST_1)(1 + ST_1/a)}{T_1S(1 + ST_1)} \tag{10}$$

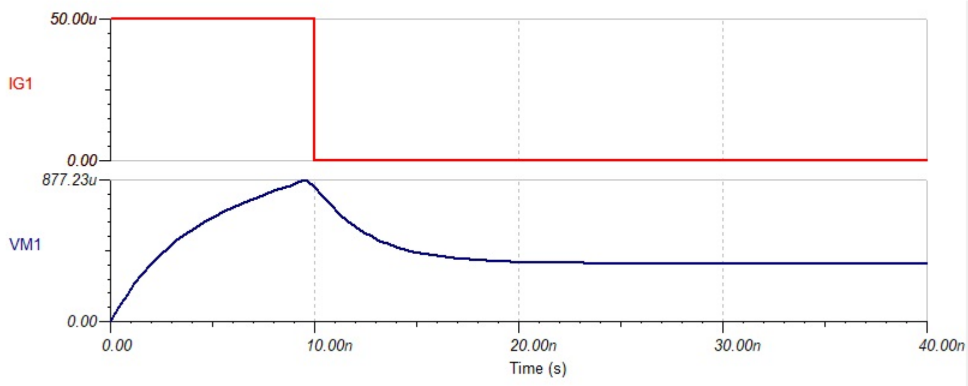


Figure 3: Single SiPM unloaded

Which means a zero and a pole cancel out and

$$Z_{1234} = \frac{aR_1(1 + ST_1/a)}{T_1 S} = \frac{R_1(S + P_2)}{S} \tag{11}$$

Hence, Z_{1234} behaves as a single zero at P_2 and a single pole at $S = 0$. Finally, Z_{01234} or Z_{04} for short is

$$Z_{1234} = \frac{(S + P_1)}{(S + P_T)} \tag{12}$$

where

$$P_T = \frac{1}{T_1} + \frac{1}{R_1 C_0} \tag{13}$$

again, using Hamamatsu S13360-6050VE values, $P_T = (2\pi)27.5MH_z$ or a time constant $T_T = 5.8ns$.
A single PE produces an avalanche of about 50uCoul that can be represented by a step function that generates a voltage V_{out} across Z_{04}

$$V_{out} = \frac{I_0(S)}{S} Z_{04} = V_0 \frac{S + P_1}{S^2(S + P_T)} \tag{14}$$

where $V_0 = \frac{I_0 T_0}{(C_3+C_4)T_1} = 650uV$

The output voltage transfer function in the time domain is found using the inverse Laplace transform from equation (16):

$$V_{out}(t) = V_0 \left[1 - e^{-S_1 t} + \frac{t}{T_0}\right] \tag{15}$$

Although the the dynamics has a term that grows to infinity, we have assumed a single step function in the positive direction, if we assume a 10ns current pulse, the input reverses sign at $t_p = 10ns$. when $\frac{t}{T_0}$ $0.6V_0$

$$V_{out}(t) = V_0 \left[\left[1 - e^{-S_1 t} + \frac{t}{T_0}\right] - \left[1 - e^{-S_1(t-t_0)} + \frac{t - t_0}{T_0}\right] u(t - t_p)\right] \tag{16}$$

The simulation shows that the V_{out} rises from 0 to $877uV$ at $t = 10ns$ and decays with the same time constants between $10ns < t < 20ns$. At $t = 20ns$ the output voltage stabilizes, since the ideal current source impedance is modeled as ∞, which is not a valid model in real life. If we include the biasing circuit (R7 and R8) $10K$ resistors the output discharges very slowly as sown in Figure 4. To

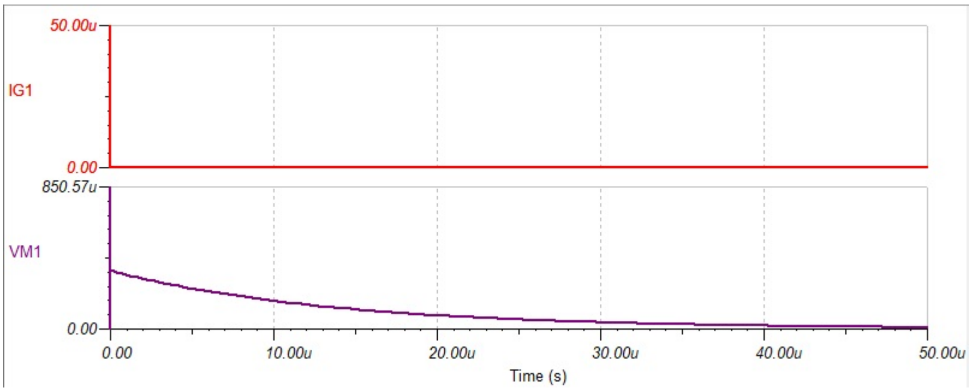


Figure 4: Single SiPM bias loaded

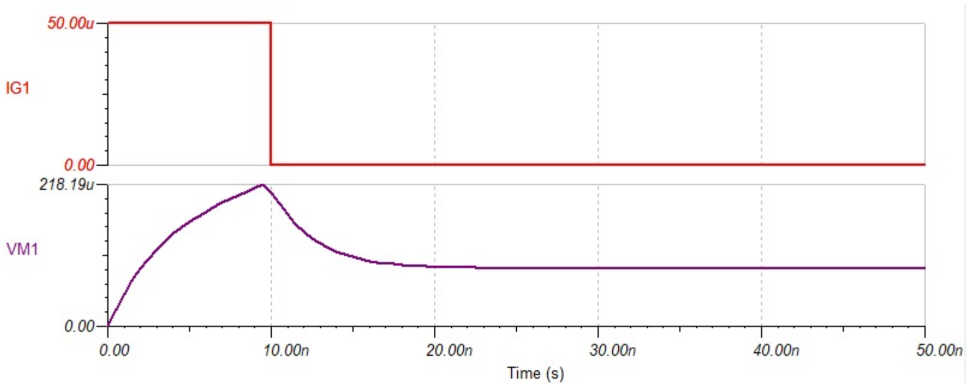


Figure 5: Four parallel SiPM bias loaded

avoid the long time discharge, another load (R$_{out1} and R_{out2} are provided (1). Furthermore, the$ V$_{out}$ is AC coupled using C$_{out1} and C_{out2} to eliminate the high DC bias voltage, typically$ 48$volts.$

Obviously, the loaded SiPM will have a different dynamics but before digging into that let's see what happens when we add SiPMs in parallel.

3 Dynamic model of parallel SiPMs

When we gang many SiPMs in parallel, the capacitance C_2, C_3 and C_4 in the model (1) increase proportionally to the number of SiPMs. Since we are adding in parallel, the resistance R_2 divides by the same factor. Since the firing cell is still a single cell in the array, R_0, R_1, C_0 and C_1 remain unchanged. The singularities S_0 and S_1 in 6 also remain unchanged, as well as the time constants T_0 and T_1.

$$S'_0 = R'_2(C'_2 + C'_3) = \frac{R_2}{N}(NC_2 + NC_3) = R_2(C_2 + C_3) = S_0 \tag{17}$$

where N is the number of SiPMs in parallel. Same happens for S_1. However, the V_0 is modified. Since,

$$V'_0 = \frac{I_0 T'_0}{(C'_3 + C'_4)T'_1} = \frac{I_0 T_0}{(NC_3 + NC_4)T_1} = \frac{V_0}{N} \tag{18}$$

The output signal is inversely proportional to the number of SiPMs ganged in parallel. Figure 5 shows a maximum voltage of 219uV, four times lower than the maximum of a single SiPM in Figure 3

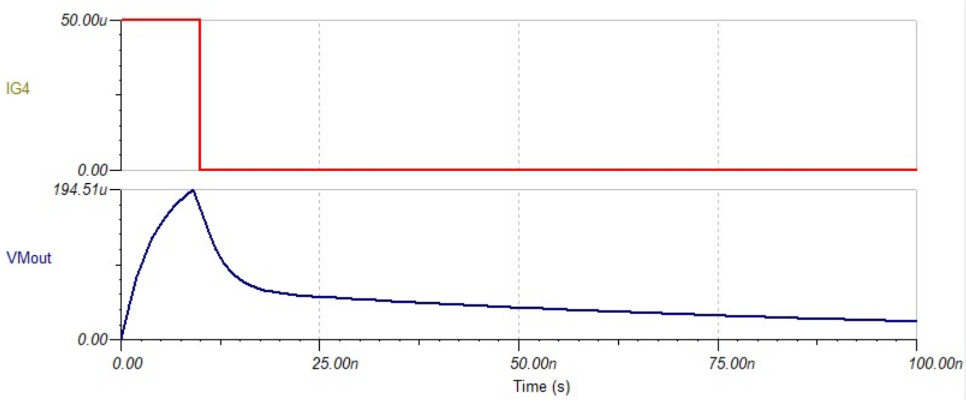


Figure 6: Four parallel SiPM with a load time constant $T_{load} = 100ns$

4 The effect of loading the circuit

Loading the SiPM parallel array with R_{out1}, R_{out2}, C_{out1}, and C_{out2}, modifies the circuit dynamics. Before deriving the analytical expression of the transfer function $V(S)$ and the temporal response $v(t)$ we can do a simple qualitative analysis. As shown in Figures 4 and 5after $t = 20ns$ the SiPMs, unless loaded, slowly discharge through the biasing circuit with a time constant $T_{bias} = C_{eq}\frac{10K}{2} = 1.1us$. During $10ns < t < 20ns$ the circuit discharges at T_1 and T_0, which are two orders of magnitude times faster. We can choose a load time constant that is fast enough to drain the C_{eq} charge fast but slow enough enough not to affect much the SiPM array dynamics. For instance, $R_{out} = 50ohm$, and $C_{out} = 2nF$ set a discharge time constant of $100ns$ 50 times larger than T_1. Figure 6 shows the SiPM array loaded with a load time constant $T_{load} = 100ns$. We can also see that the maximum SiPM array signal has decreased 10% from $218uV$ to $194uV$ to to the charge leaked by the load that increases the voltage across C_{out1} and C_{out2}.

A faster decay requires considering R_{out1}, R_{out2}, C_{out1}, and C_{out2} in the transfer function equations. Since they are in series we can simplify defining $R_L = R_{out1} + R_{out2}$ and $C_L = C_{out1}||C_{out2} = \frac{C_{out2}}{2}$. We can also define the load impedance $Z_L = R_L + \frac{1}{SC_L} = \frac{1+SC_LR_L}{SC_L}$. The load impedance adds a new zero and a new pole to equation 6.

$$Z_T = Z_{234}||Z_L = \frac{(1 + ST_0)(1 + ST_L)}{S^3C_3T_1T_L + S^2(C_LT_0 + C_3(T_1 + T_L)) + S(C_L + C_3)} \tag{19}$$

$$Z_T = \frac{1}{C_3T_1T_L}\frac{(1 + ST_0)(1 + ST_L)}{s(S^2 + S\omega_n/Q + \omega_n^2)} \tag{20}$$

where

$$\omega_n/Q = \frac{C_LT_0 + C_3(T_1 + T_L)}{C_3T_1T_L} \tag{21}$$

$$\omega_n^2 = \frac{C_L + C_3}{C_3T_1T_L} \tag{22}$$

Equation 20 has, then, a pole at $S = 0$ and two finite poles at $S = S_1$ and $S = S_2$. The location of the finite poles will depend on our choice for R_L and C_L (henceforth T_L). For instance, we desire $T_L > T_1$, so T_L and $T_1 >> T_0$, we can also make C_L similar in value to C_3. To have a critical (minimally) dumped system we can choose $S_1 \sim S_2 \sim \frac{\omega_n}{2Q}$. In that case $T_L = 5.8ns$, $T_1 = 15ns$, $C_L = 0.3nF$ and $R_L = 50\Omega$. Figure 7 shows a simulation of four parallel SiPMs loaded by and impedance with a 15ns time constant. It can be observed that even for a single PE excitation, the output voltage has a small undershoot. The undershoot becomes more pronounced for larger signals. At full dynamic range defined at 2000 PEs the undershoot is considerable. For that reason we choose loads with longer time constants.

Figure 8 load of $C_L = 3nF$ and $R_L = 50\Omega$ has a time constant of $T_L = 150ns$. We observe that $T_L = 150ns$ generates a longer return to baseline. The load time constant is also related to sampling frequency and signal to noise ratio.

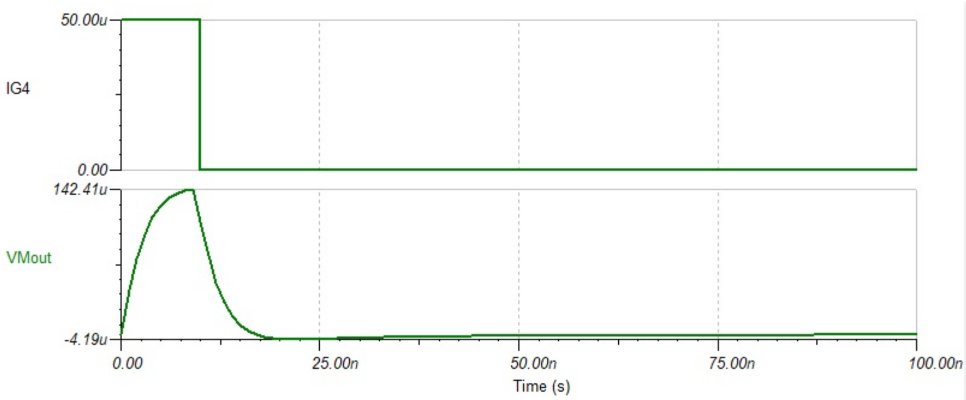


Figure 7: Four parallel SiPM with a load time constant $T_{load} = 15ns$

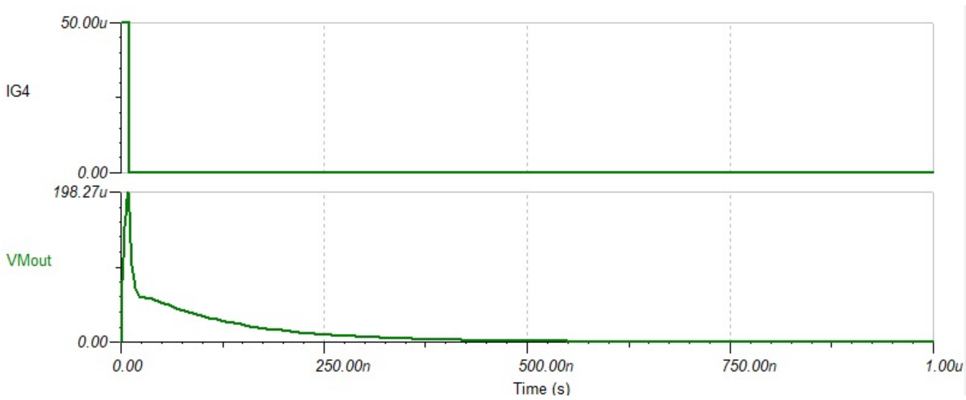


Figure 8: Four parallel SiPM with a load time constant $T_{load} = 150ns$

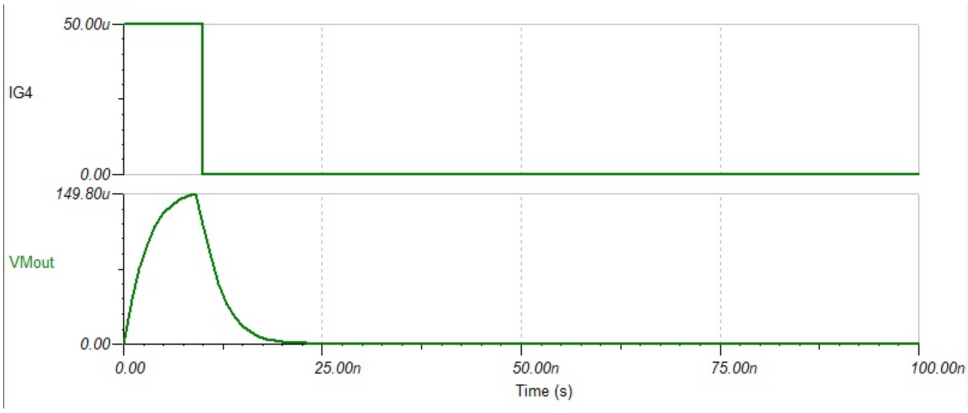


Figure 9: Four parallel SiPM with a load time constant $T_{load} = 17.5ns$

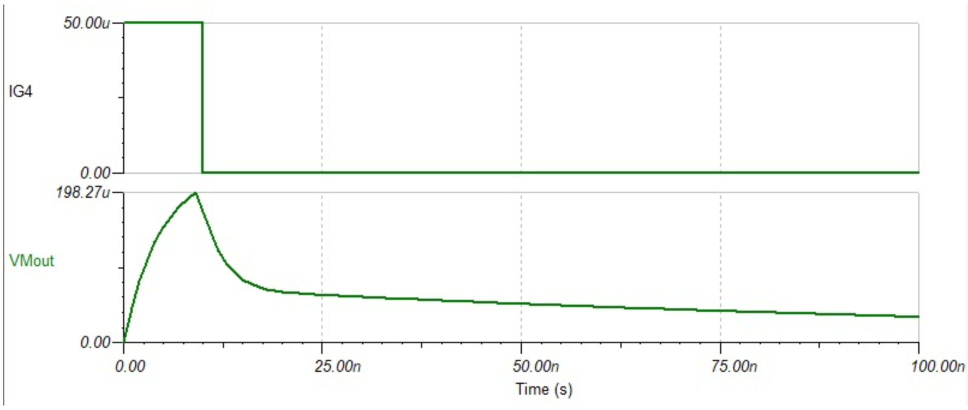


Figure 10: Four parallel SiPM with a load time constant $T_{load} = 150ns$

The equations that solve equation 12 is in Apendix A. I here mention two examples, the first one for a $C_L = 0.3nF$, and the second one for $C_L = 3nF$; $R_L = 50\Omega$ in both cases.

$$V_0\left(t\right) = 150\mu V\left[1 - 1.1E^{-t/0.95ns} + 0.076e^{-t/3.8ns} + 0.88\frac{t}{15.6ns}\right] \tag{23}$$

$$V_0\left(t\right) = 190\mu V\left[1 - 3.3E^{-t/3.7ns} + 2.7e^{-t/7.1ns} + 0.1\frac{t}{15.6ns}\right] \tag{24}$$

As we see in Figures 9 and 10 increasing the load capacitance C_L from $350pF$ to $3nF$ increases the time constants of the singularities in equations 23 and 23 from $T_1(C_L = 0.3nF) = 0.98ns$ to $T_1(C_L = 3nF) = 3.7ns$ and $T_2(C_L = 0.3nF) = 3.8ns$ to $T_1(C_L = 3nF) = 7.1ns$. The time constant of the term linear in t remains at $T_0 = 15.6ns$ because is independent of the location of poles at S_1 and S_2. The long tail in 10 is a combination of the slower decreasing exponential and the linear term discharging at T_0.

5 The Hybrid model

The hybrid model, as shown in Figure 11 combines the advantage of parallel biasing all the SiPMs in the array with the advantage of having a serial path for the high speed AC signal. In order to separate the DC currents and voltages from the AC (i.e. signal) a set of coupling R and C's are used in between the parallel stations. At a typical $V_{bias} = 48V$ two $10K\Omega$ resistors allow to negatively bias the SiPM array. When the signal fires in avalanche, the $10K\Omega$ resistors with the equivalent SiPM capacitance, of the order of few nF generate a time constant on the order of several $10's$ of ns, which is much longer than the signal measuring time, but also short enough to restore the biasing of the SiPM in due time for another incoming event.

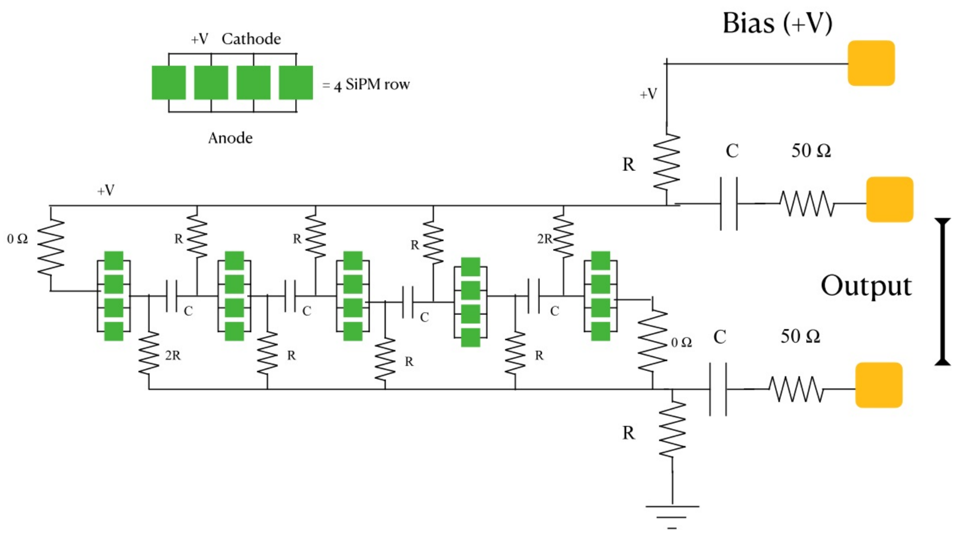


Figure 11: Hybrid model block diagram. Five blocks of 4 parallel SiPMs

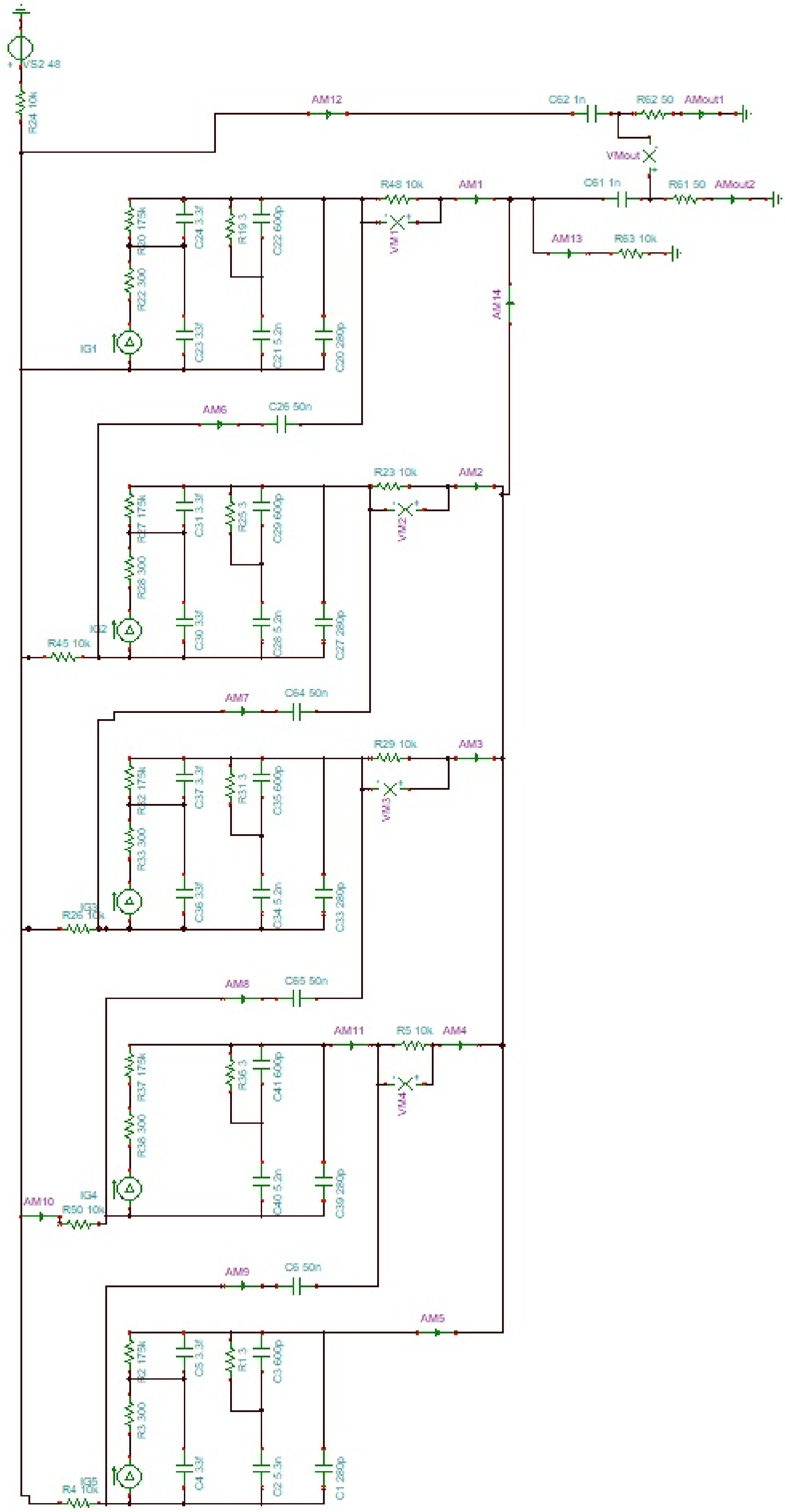


Figure 12: Hybrid simulation circuit model. Five blocks of 4 parallel SiPMs

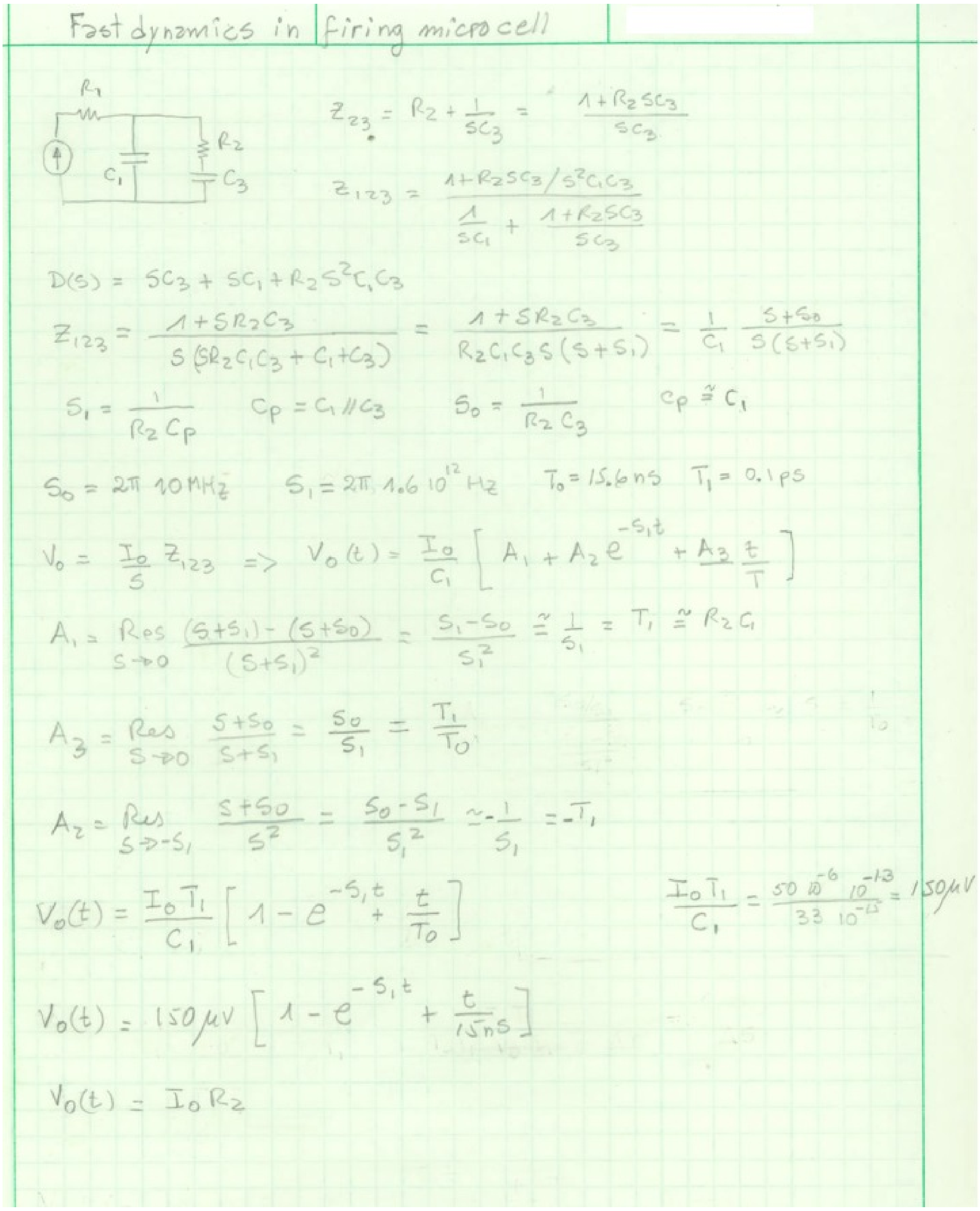
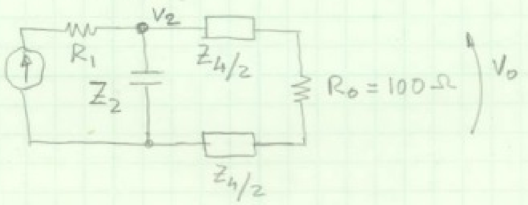


Figure 13: Hybrid model transfer function page 1

Hybrid model

Summing N blocks $Z_{234}^{(1)}+\dots+Z_{234}^{(N)}$ (identical)

$$\frac{N}{sC_3}\frac{(1+sT_0)}{(1+sT_1)} = Z_4$$



$$V_0(s) = V_2(s)\frac{R_0}{R_0+Z_4} = \frac{R_0}{R_0+\frac{N(1+sT_0)}{sC_3(1+sT_1)}} = \frac{sR_0C_3(1+sT_1)}{sR_0C_3(1+sT_1)+N(1+sT_0)}$$

$$D(s) = s^2 R_0C_3T_1 + s(T_0+R_0C_3)+4$$

if $R_0 = 100\,\Omega$ $R_0C_3 = 530\,ns$ $T_0 = 15.6\,ns$ $T_1 = 2.65\,ns$

$s_0 = 10.3\,MHz\,(2\pi)$ $s_1 = (2\pi)\,61.2\,MHz$ $s_{03} = (R_0C_3)^{-1} = 2\pi(0.3)\,MHz$

$$V_0(s) = \frac{s(1+sT_1)\,V_2(s)}{T_1(s^2+2\omega_n s+\omega_n^2)} \qquad 2\omega_n = \frac{T_0+R_0C_3}{R_0C_3T_1} \simeq \frac{1}{T_1}$$

$$\omega_n^2 = \frac{4}{R_0C_3T_1}$$

$$P_1, P_2 = -\frac{1}{2T_1}\left(1 \pm \sqrt{1-\frac{4q}{p^2}}\right)$$

$$\frac{4q}{p^2} = \frac{4T_1^2}{R_0C_3T_1} = \frac{4T_1}{R_0C_3} << 1 \Rightarrow P_2 = -\frac{1}{2T_1}\left(1-\sqrt{1-\frac{4T_1}{R_0C_3}}\right)$$

$$P_2 \simeq -\frac{1}{2T_1}\left(1-\left(1-\frac{4T_1}{2R_0C_3}\right)\right) = -\frac{1}{R_0C_3}$$ (slow dynamics)

$P_1 \simeq -\frac{1}{T_1} = s_1 \Rightarrow$ fast dynamics.

$$V_0(s) \simeq V(s)\frac{s(s+s_1)}{(s+P_1)(s+P_2)} \simeq \frac{s}{s+P_2}$$

if $V(s) = \frac{V_0}{s}$ $V_0(s) = \frac{1}{s+P_2} \Rightarrow V_0(t) = A\,e^{-P_2 t}$

Figure 14: Hybrid model transfer function page 2

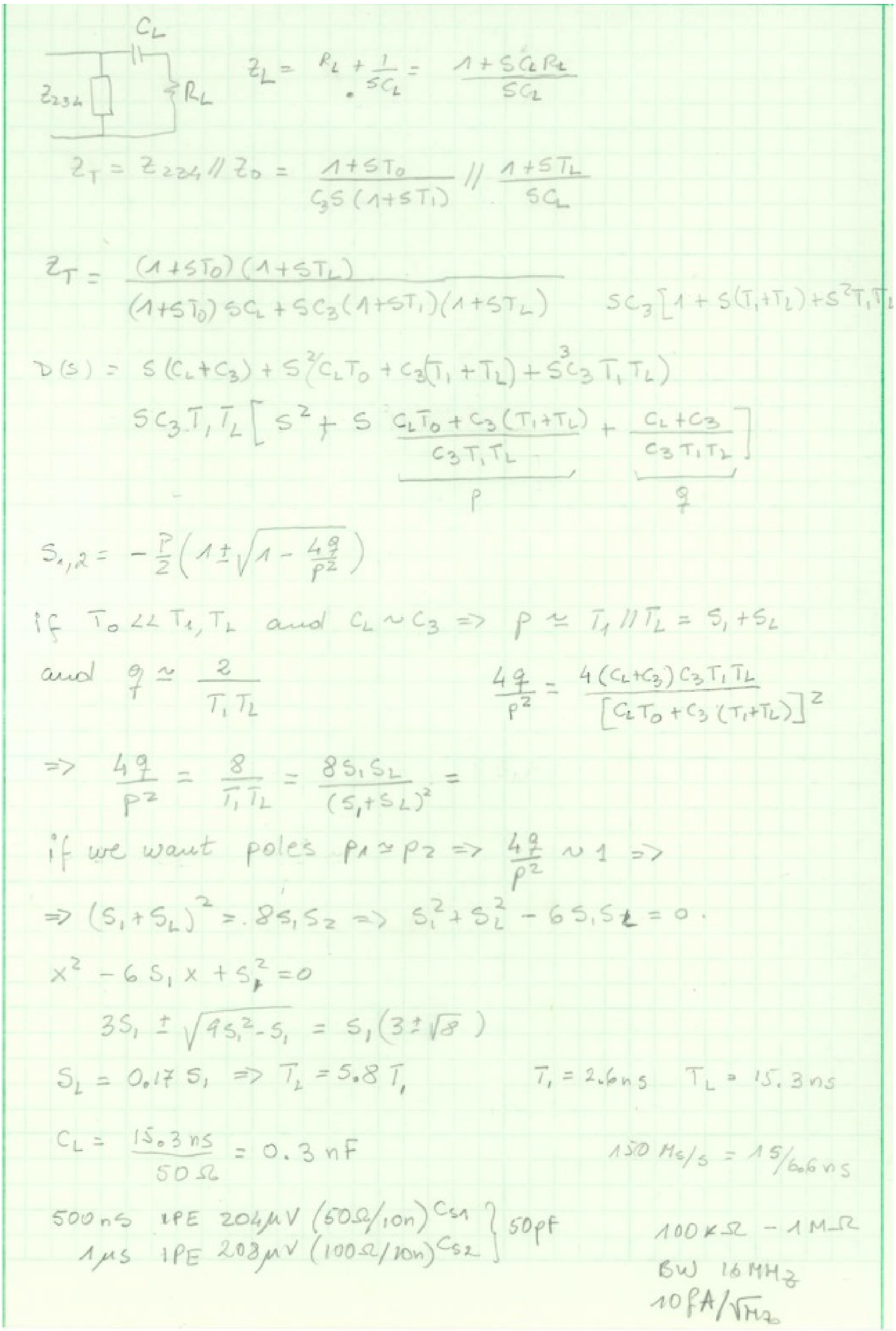


Figure 15: Apendix page 1

$$Z_T = \frac{(1+sT_0)(1+sT_L)}{sC_3T_1T_L\left[s^2+ps+q\right]} = \frac{(1+sT_0)(1+sT_L)}{sC_3T_1T_L(s+s_1)(s+s_2)} = \frac{T_0}{C_3T_1}\,\frac{(s+s_0)(s+s_L)}{s(s+s_1)(s+s_2)}$$

$$V_0 = \frac{I_0T_0}{C_3T_1}\,\frac{(s+s_0)(s+s_L)}{s^2(s+s_1)(s+s_2)} = V_0\left[\frac{A_1}{s}+\frac{A_2}{s^2}+\frac{A_3}{s+s_1}+\frac{A_4}{s+s_2}\right]$$

$$A_1 = \operatorname*{Res}_{s\to 0}\left[\frac{\left[(s+s_L)+(s+s_0)\right](s+s_1)(s+s_2)-\left[(s+s_2)+(s+s_1)\right](s+s_0)(s+s_L)}{(s+s_1)^2(s+s_2)^2}\right]$$

$$A_1 = \frac{(s_L+s_0)s_1s_2-(s_1+s_2)s_0s_L}{s_1^2s_2^2}$$

$T_L >> T_0$

$s_L << s_0$

$$A_2 = \operatorname*{Res}_{s\to 0}\frac{(s+s_0)(s+s_L)}{(s+s_1)(s+s_2)} = \frac{s_0s_L}{s_1s_2}$$

$$A_3 = \operatorname*{Res}_{s\to -s_1}\frac{(s+s_0)(s+s_L)}{s^2(s+s_2)} = \frac{(s_0-s_1)(s_L-s_1)}{s_1^2(s_2-s_1)}$$

$$A_4 = \operatorname*{Res}_{s\to -s_2}\frac{(s+s_0)(s+s_L)}{s^2(s+s_1)} = \frac{(s_0-s_2)(s_L-s_2)}{s_2^2(s_1-s_2)}$$

For $T_1 = 2.5\,ns$ $T_0 = 15.4\,ns$

$$\frac{4q}{p^2} = \frac{4(C_L+C_3)C_3T_1T_L}{\left[C_LT_0+C_3(T_1+T_L)\right]^2} = 1$$

$$4(C_L+C_3)C_3T_1T_L = \left[C_LT_0+C_3(T_1+T_L)\right]^2$$

$$\underbrace{4(C_L+C_3)C_3T_1\,50\,C_L}_{3.24\,10^{-34}} = C_L^2T_0^2+2C_LT_0C_3(T_1+50C_L)+C_3^2(T_1+50C_L)^2$$

$$\boxed{C_L = 300\,pF}$$

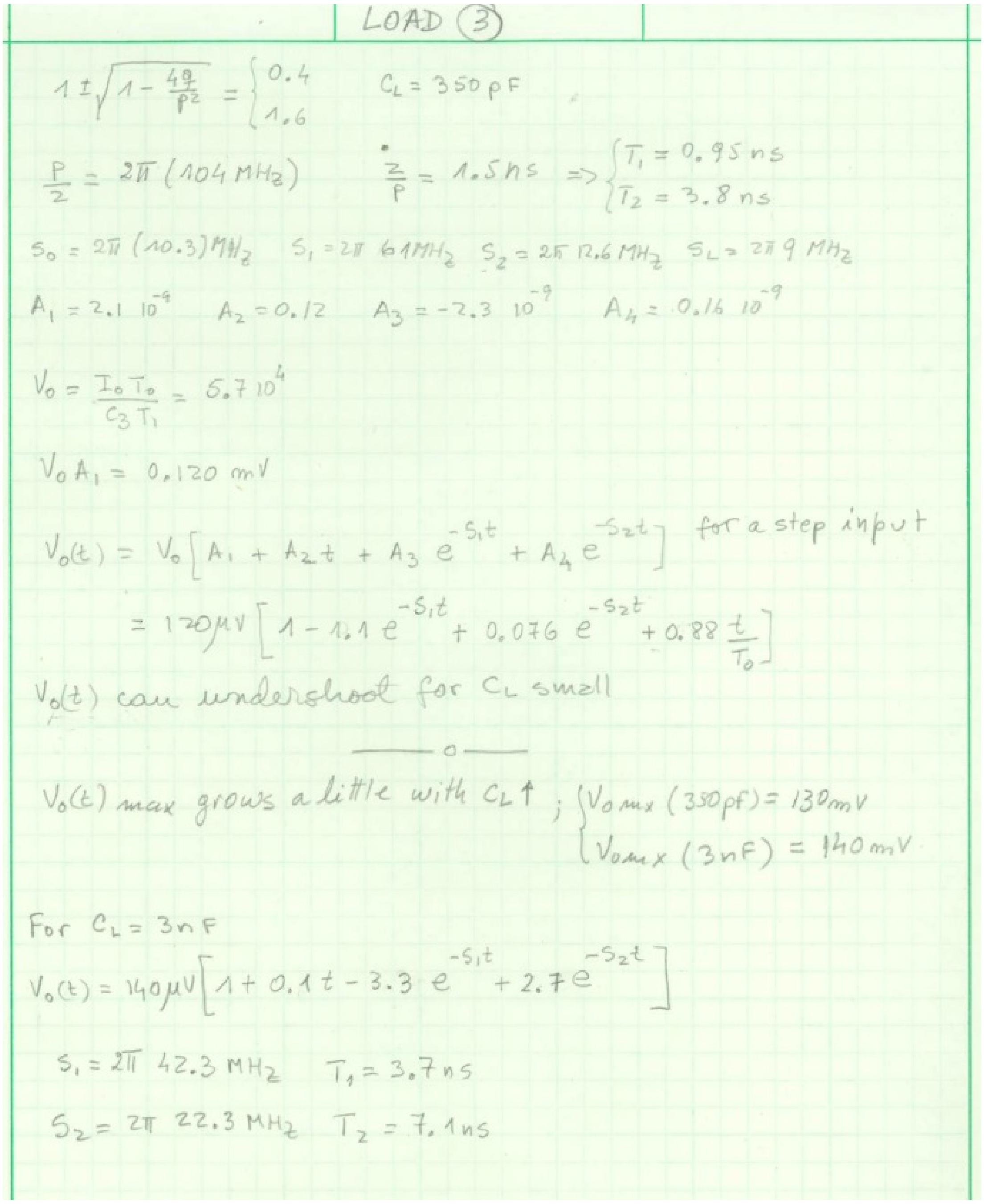
$$\frac{p}{2} = \frac{C_LT_0+C_3(T_1+T_L)}{C_3T_1T_L}$$

$C_L = C_3$ $R_L = 50\,\Omega$

p.ej $C_L = C_1 = 1.3\,nF$; $T_L = 65\,ns$; $s_L = 2\pi(2.45)\,MHz$

$T_1 = 2.5\,n$ $C_{eq} = 0.86\,nF$ $R = 3\Omega$ $T_L = 2.5$ $R = 50\Omega \Rightarrow C_L = 50\,pF$

Figure 16: Apendix page 2



LOAD (3)

$1 \pm \sqrt{1 - \frac{4q}{p^2}} = \begin{cases} 0.4 \\ 1.6 \end{cases}$ $C_L = 350\,pF$

$\frac{p}{2} = 2\pi\,(104\,MHz)$ $\frac{\dot{z}}{p} = 1.5\,ns \Rightarrow \begin{cases} T_1 = 0.95\,ns \\ T_2 = 3.8\,ns \end{cases}$

$s_0 = 2\pi\,(10.3)\,MHz$ $s_1 = 2\pi\,61\,MHz$ $s_2 = 2\pi\,12.6\,MHz$ $s_L = 2\pi\,9\,MHz$

$A_1 = 2.1\,10^{-9}$ $A_2 = 0.12$ $A_3 = -2.3\,10^{-9}$ $A_4 = 0.16\,10^{-9}$

$V_0 = \frac{I_0 T_0}{C_3 T_1} = 5.7\,10^4$

$V_0 A_1 = 0.120\,mV$

$V_0(t) = V_0\left[A_1 + A_2 t + A_3 e^{-s_1 t} + A_4 e^{-s_2 t}\right]$ for a step input

$= 120\,\mu V\left[1 - 1.1\,e^{-s_1 t} + 0.076\,e^{-s_2 t} + 0.88\,\frac{t}{T_0}\right]$

$V_0(t)$ can undershoot for C_L small

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$V_0(t)$ max grows a little with $C_L \uparrow$; $\begin{cases} V_{0\,max}(350pF) = 130\,mV \\ V_{0\,max}(3nF) = 140\,mV \end{cases}$

For $C_L = 3\,nF$

$V_0(t) = 140\,\mu V\left[1 + 0.1\,t - 3.3\,e^{-s_1 t} + 2.7\,e^{-s_2 t}\right]$

$s_1 = 2\pi\,42.3\,MHz$ $T_1 = 3.7\,ns$

$s_2 = 2\pi\,22.3\,MHz$ $T_2 = 7.1\,ns$

Figure 17: Apendix page 3