The 2016 R-Matrix Workshop on Methods and Applications

Experimental application of the generalized R-matrix approach: $^{19}F(p,\alpha)^{16}O$ and $^{13}C(\alpha,n)^{16}O$

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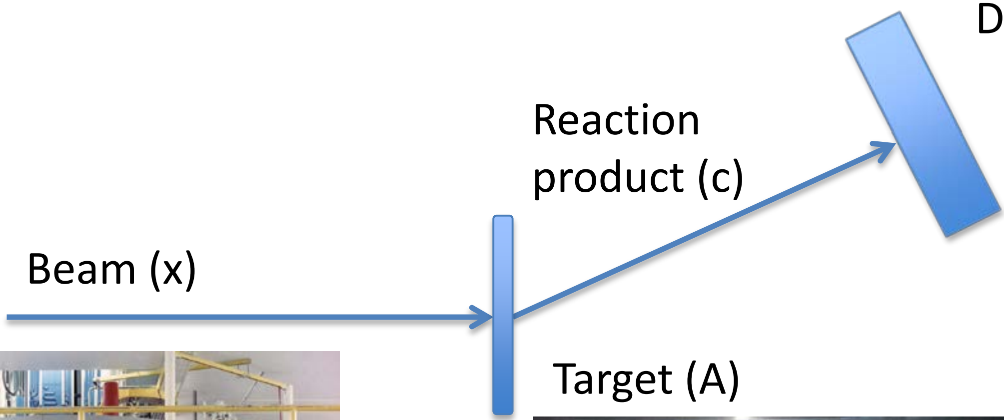
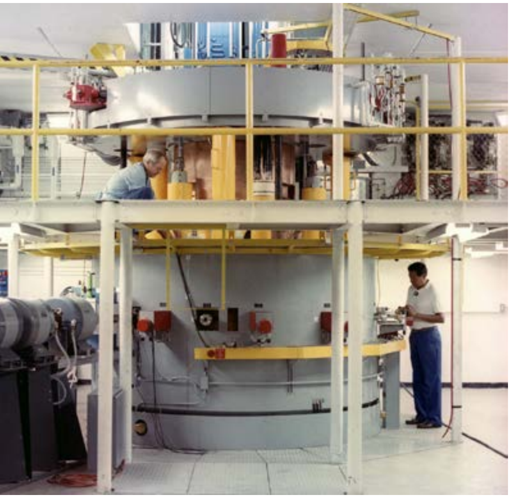
Ministero dell'Istruzione dell'Università e della Ricerca

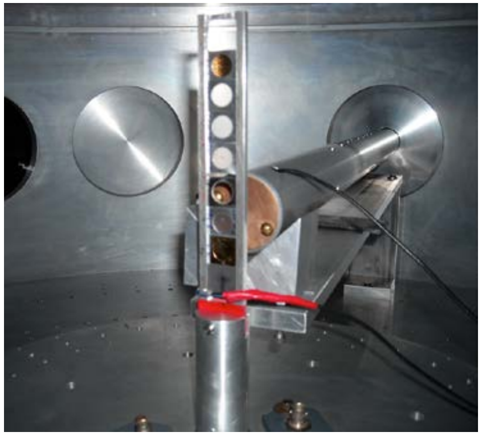
Recipe for direct measurements

How to measure the A+x→c+C reaction in a *direct* way?



Detector → kinematic observables
- Energy
- Emission angle
+ Particle identification





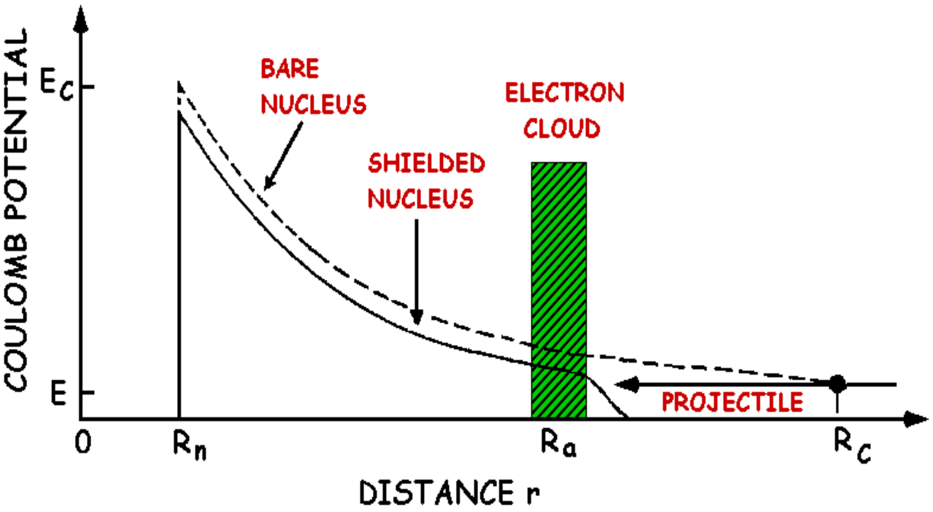
It looks *quite* simple!

The electron screening effect.

Thanks to recent experimental developments, measurements have been extended to the energies of interest for several reactions.

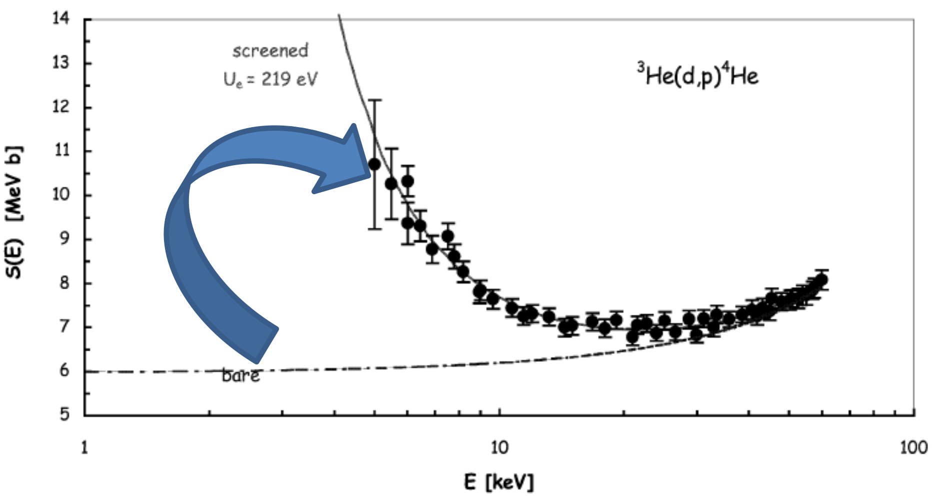
This lead to the discovery of electron screening: a such low energies atomic degrees of freedom cannot be neglected.

Pictorial view:



The electron cloud shields the nuclear charge thus the projectile meets a reduced Coulomb barrier → enhancement of the reaction probability as tunneling is more likely.

Experimentally:



Exponential enhancement!

$$\sigma_s(E)/\sigma_b(E) \propto \exp(\pi\eta U_e/E),$$

Bracci et al., NPA 513 (1990) 316
Rolfs & Rodney, Cauldrons in the Cosmos (Univ. Chicago Press) 1988
Strieder et al., Naturwissenschaften 88 (2001) 461

Direct vs. indirect measurements

Indirect measurements:

Complicated but rewarding

✓ High energy experiments: up to several hundreds MeV

→ no Coulomb barrier suppression

→ negligible straggling

→ no electron screening

Indirect measurements are the only ones allowing you to measure down to astrophysical energies with the present day facilities

Nuclear reaction theory required

→ cross checks of the methods needed

→ possible spurious contribution

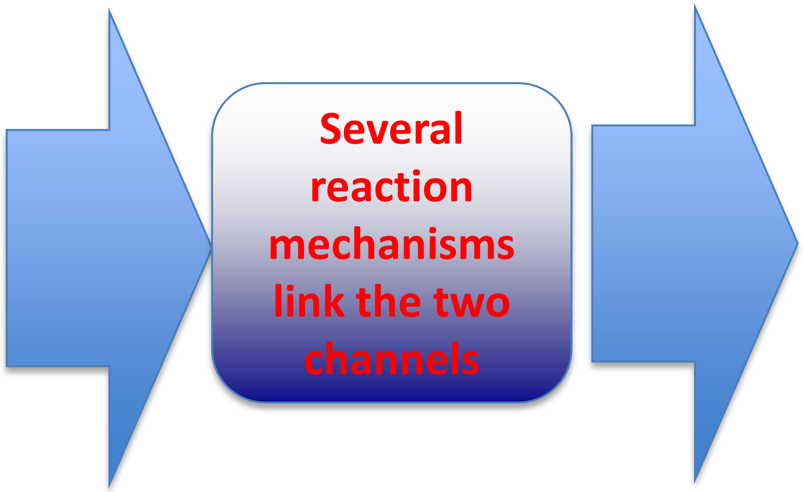
→ additional systematic errors (is the result model independent?)

... Indirect techniques are complementary to direct measurements
Examples: Coulomb dissociation, ANC and Trojan horse method

Indirect Techniques: a cartoon

Entrance channel:

A+a

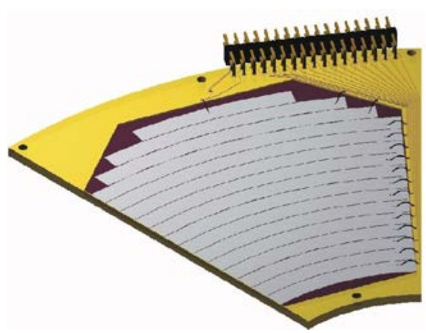


Reaction products

C+c+…

Only one reaction mechanism has to be selected, for which the reaction theory we have developed applies. A careful experimental & theoretical investigation is necessary

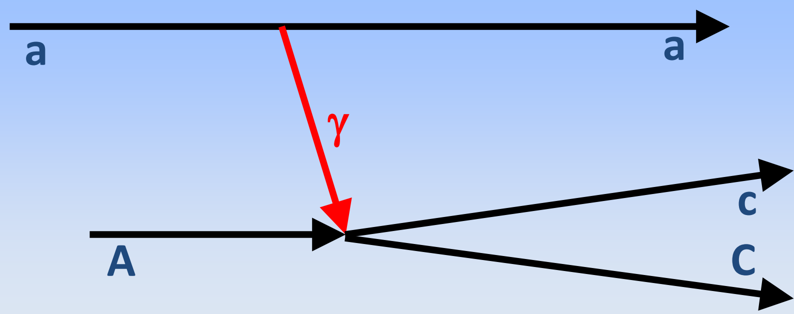
To recall the previous sketch:



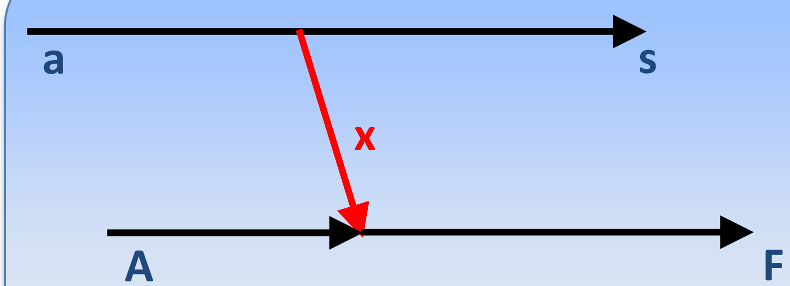
Nuclear reaction theory

R. Tribble et al., Rep. Prog. Phys. **77** (2014) 106901

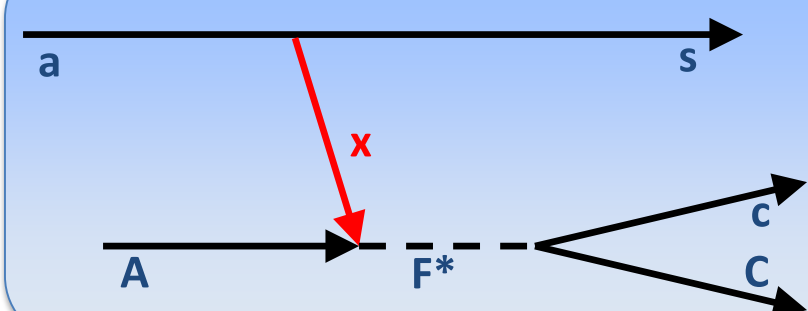
Indirect techniques: a comparison



In the **Coulomb dissociation**, a virtual photon beam is used to a photodisintegration reaction; the detailed balance principle is then used to recover the cross section of the relevant radiative capture reaction



In the **Asymptotic Normalization Coefficient (ANC) approach**, a transfer reaction to a bound state is measured to deduce the normalization constant of the bound state wave function, prop. to the A(x,γ)F c.s.

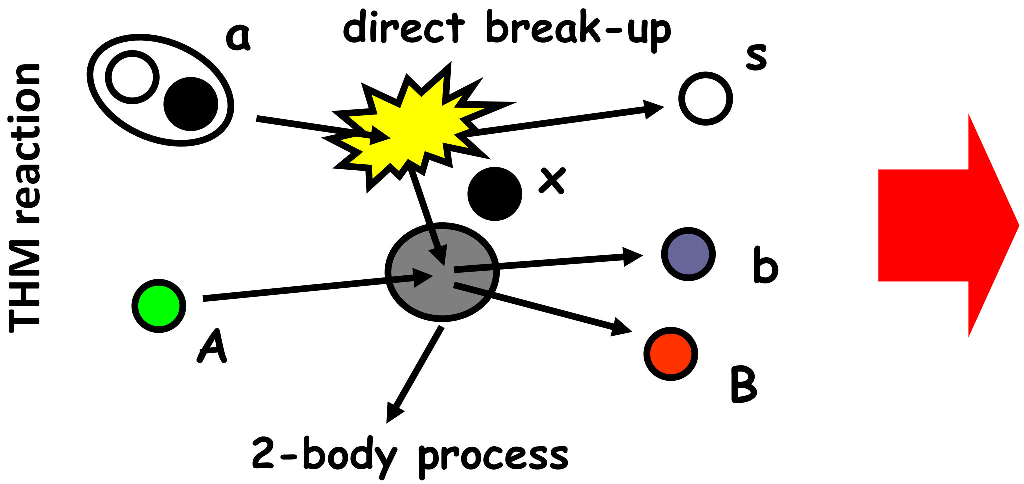


In the **Trojan Horse Method**, a transfer reaction to an unbound system is used to measure the c.s. of the A(x,c)C process. C and c can be charged or neutral particles (no photons)

The Trojan Horse Method: what you already know

Coulomb barrier → exponential suppression of the cross section at astrophysical energies
+ electron screening

→ low-energy, bare-nucleus cross section is experimentally available only through <u>extrapolation</u> <u>OR indirect measurements</u>



From

A+a(x⊕s) → b+B+s @ 10-60 MeV

$A + x \to b + B$ @ 5-20 keV

By selecting the QF contribution

Though $E_A >> V_{Coul}$ it is possible to measure at the Gamow peak since:

$$E_{c.m.}=E_{A-x}-Q_{x-s}$$

<u>Additional advantages:</u>

- reduced systematic errors due to straggling, background...
- magnifying glass effect

<u>But...</u>

- off-shell cross section deduced (x → virtual particle)
- no absolute units

Baur PLB 178 (1986) 135
Spitaleri, in *Problems of Fundamental Modern Physics II,* World Sci. (1991) 21
Spitaleri et al., PAN 74 (2011) 1763
Typel & Baur, Ann. Phys. 305 (2003) 228
Mukhamedzhanov et al., JPG 35 (2008) 014016

THM: Basic features

Plane Wave Impulse Approximation:

- **beam energy >> a = x ⊕ b breakup Q-value**
- **projectile wavelength k^{-1} << x – b intercluster distance**

  + plane waves in the entrance and exit channel

➔ **the 3-body cross section factorizes:**

$$\frac{d^3\sigma}{dE_C d\Omega_C d\Omega_c} \propto \mathrm{KF}\, |\phi(-\vec{p}_b)|^2 \cdot \left(\frac{d\sigma}{d\Omega_{c.m.}}\right)^{\text{off}}$$

From the experiment

Evaluated through a MC code

HOES 2-body cross section

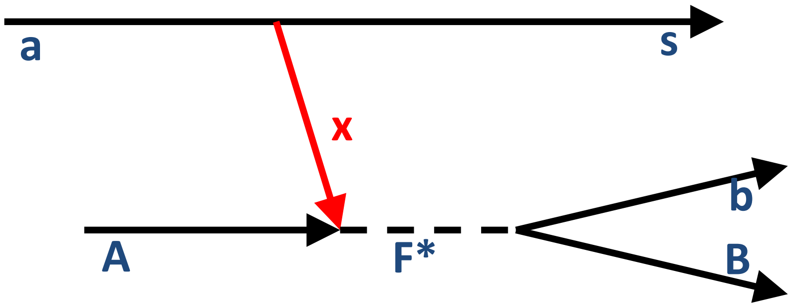
- **KF kinematic factor**
- **$\phi(p_b)^2$ spectator momentum distribution**
- **$d\sigma^{off}/d\Omega$ off-shell cross section or "nuclear" (N) cross section**

$d\sigma^{off}/d\Omega$ → $d\sigma/d\Omega$ (on shell)

The penetration factor P_l has to be introduced:

$$\frac{d\sigma}{d\Omega} = \sum_l P_l \frac{d\sigma_l^N}{d\Omega}$$

The full THM: the resonant case (A. Mukhamedzhanov)



R. Tribble et al., Rep. Prog. Phys. **77** (2014) 106901

In the latest years, large efforts were made to give a quantitative justification of THM, to estimate the uncertainties and improve the description of the 2→3 cross section

Amplitude of

Same R-matrix term as in OES cross section but for the appearence of the inverse penetration factor making it possible to observe suppressed resonances at low energies negligible at usual experimental precisions

$$
\begin{aligned}
M^{\mathrm{PWA(prior)}}(P,\mathbf{k}_{aA}) &= (2\pi)^2\sqrt{\frac{1}{\mu_{bB}k_{bB}}}\,\varphi_a(\mathbf{p}_{sx}) \\
&\times \sum_{J_F M_F j' l l' m_{j'} m_l m_{l'} M_n} i^{l+l'}\langle j m_j l m_l | J_F M_F\rangle\langle j' m_{j'} l' m_{l'} | J_F M_F\rangle \\
&\times \langle J_x M_x J_A M_A | j' m_{j'}\rangle\langle J_s M_s J_x M_x | J_a M_a\rangle\, \mathrm{e}^{-\mathrm{i}\delta^{hs}_{bBl}}\, Y_{l m_l}(-\hat{\mathbf{k}}_{bB}) \\
&\times \sum_{\nu,\tau=1}^{N}[\Gamma_{\nu bBjlJ_F}(E_{bB})]^{1/2}[\mathbf{A}^{-1}]_{\nu\tau}\, Y^{*}_{l' m_{l'}}(\hat{\mathbf{p}}_{xA}) \\
&\times \sqrt{\frac{R_{xA}}{\mu_{xA}}}[\Gamma_{\nu xAl'j'J_F}(E_{xA})]^{1/2} P_{l'}^{-1/2}(k_{xA},R_{xA})\big(j_{l'}(p_{xA}R_{xA}) \\
&\times [(B_{xAl'}(k_{xA},R_{xA})-1) - D_{xAl'}(p_{xA},R_{xA})] \\
&+ 2Z_x Z_A e^2 \mu_{xA}\int_{R_{xA}}^{\infty}\mathrm{d}r_{xA}\,\frac{O_{l'}(k_{xA},r_{xA})}{O_{l'}(k_{xA},R_{xA})}\, j_{l'}(p_{xA}r_{xA})\big).
\end{aligned}
$$

Fourier transform of the s-x relative motion wave function

solid sphere scattering phase shift

Inverse penetration

R-matrix-like boundary condition

Some considerations on the non resonant case

- **Surface integral formalism: TH reaction amplitude is expressed in terms of the OES level matrix elements for the binary sub-reaction.** The surface term contains the logarithmic derivative of the outgoing spherical wave resembling the R-matrix method.

- **The inverse penetration factors compensate for the steep decrease of the cross section due to the presence of the partial widths**

- **HOES effects can be accounted for** and are negligible in most of the cases (zero for neutrons! Pure pole mechanism, otherwise branching point singularity)

- **No need of normalization to direct data**: we can calculate the normalization constants (however, less model dependence is found in the case normalization to direct data is performed)

- The KF x $\varphi^2(p_s)$ x $d\sigma/d\Omega$ structure of the triple differential cross section is fully justified

- PWA makes calculations easier, though DWBA or CDCC can be used as well

- **QF mechanism has to enforced**, otherwise not a single term is present and the dependence of S-matrix is not straightforward

Resonant THM eqs. at work

Example of two interfering resonances: a simplified version of the theory

The TH triple differential cross section takes the form:

$$\frac{d^3\sigma}{dE_c\, d\Omega_c\, d\Omega_C} \propto |KF| \left| I^a_{sx}\left(\mathbf{k}_s - \frac{m_s}{m_a}\mathbf{k}_a\right)\right|^2 \times \left| \sum_{\tau=1,2} \frac{\Gamma^{1/2}_{Cc(\tau)}(E_{Cc}) \left[W^{F_\tau}_{Ax}\left(\mathbf{k}_A - \frac{m_A}{m_F}\mathbf{k}_F\right)\right]^*}{E_{Ax} - E_{R_\tau} + i\frac{\Gamma_\tau(E_{Ax})}{2}} \right|^2$$

$$M^0_\tau(\mathbf{k}_{sF}, \mathbf{k}_{aA}) = \left[W^{F_\tau}_{Ax}\left(\mathbf{k}_A - \frac{m_A}{m_F}\mathbf{k}_F\right)\right]^* \times I^a_{sx}\left(\mathbf{k}_s - \frac{m_s}{m_a}\mathbf{k}_a\right).$$

W^F_{sx} is the vertex form factor for $A + x \rightarrow F_\tau$:

$$W^{F_\tau}_{Ax}(\mathbf{k}_{Ax}) = \langle e^{i\mathbf{k}_{Ax}\cdot\mathbf{r}_{Ax}} | \langle V_{xA}\rangle_{Ax}(\mathbf{r}_{Ax}) | I^{F_\tau}_{Ax}(\mathbf{r}_{Ax})\rangle$$

$I^a_{sx}(\mathbf{p}_{sx})$ is the Fourier transform of the overlap function $I^a_{sx}(\mathbf{r}_{sx})$

in a strict approach the triple differential cross section is expressed in terms of the overlap function I^a_{sx} rather then the two-body bound state wave function φ_a. Note that I^a_{sx} and φ_a are related by

the vertex form factor W^F_{sx} contains tha ANC for the $A + x \rightarrow F_\tau$ system as it contains the overlap function I^F_{Ax}

$$I^a_{sx} = S^{1/2}_{sx}\, \varphi_a$$ S —> spectroscopic factor

THM vs. OES astrophysical factor

Direct data:

$$S(E_{xA}) = \frac{\mu_{cC}\,\mu_{xA}}{4\pi^2}\,\frac{k_{cC}}{k_{xA}}\,\frac{1}{\hat{J}_x\hat{J}_A}\,E_{xA}\,e^{2\pi\eta_{xA}} \times \left|\sum_{\nu,\tau=1}^{2}\tilde{V}_{\nu cC}(E_{cC})\,[\mathbf{D}^{-1}]_{\nu\tau}\,\tilde{V}_{\tau xA}(E_{xA})\right|^2$$

Remember that:

$$\tilde{\Gamma}_{\nu c}(E_c) = 2\pi\,|\tilde{V}_{\nu c}(E_c)|^2$$

is the formal partial resonance width for the decay of this level into channel c=x+A or c=b+B.

THM data:

$$S^{\mathrm{TH}}(E_{xA}) = \frac{\mu_{cC}\,\mu_{sF}\,\mu_{aA}}{2\pi^5}\,\frac{k_{cC}\,k_{sF}}{k_{aA}}\,\frac{1}{\hat{J}_a\hat{J}_A}\,E_{xA}\,e^{2\pi\eta_{xA}} \times \Gamma_{2xA}(E_{xA})\left|\sum_{\nu,\tau=1}^{2}\tilde{V}_{\nu cC}(E_{cC})\,[\mathbf{D}^{-1}]_{\nu\tau}\,L_{2\tau}(\mathbf{k}_{sF},\mathbf{k}_{aA})\right|^2$$

Where:

$$L_{2\tau}(\mathbf{k}_{sF},\mathbf{k}_{aA}) = \frac{M_\tau(\mathbf{k}_{sF},\mathbf{k}_{aA})}{M_2(\mathbf{k}_{sF},\mathbf{k}_{aA})}$$

It can be calculated (DWBA, CDCC, PWBA…) or taken from measurements

The matrix $\mathbf{D}^{-1}$ and $V_{\nu cC}(E_{cC})$ are the same in the TH and OES astrophysical factors.
The THM S-factor does not contain the penetration factor, which has to be inserted for comparison with direct data

Moreover: exploring negative energies with the THM

Using the kinematics of three body reactions:

$$E_{Ax} = \frac{m_x}{m_x + m_A} E_A - \frac{p_s^2}{2\mu_{sF}} + \frac{\mathbf{p}_s \cdot \mathbf{p}_A}{m_x + m_A} - \varepsilon_{sx}$$

s-x Fermi motion

s-x binding energy

It is possible to achieve negative energies in the A-x channel

How to deal with negative energies? what is their meaning?

Standard R-Matrix approach cannot be applied to extract the resonance parameters of the A(x,c)C reaction because x is virtual —> Modified R-Matrix is introduced instead (A. Mukhamedzhanov 2010)

$$\frac{d^2\sigma}{dE_{Cc}\, d\Omega_s} \propto \frac{\Gamma_{(Cc)_i}(E)\, |M_i(E)|^2}{(E - E_{R_i})^2 + \Gamma_i^2(E)/4}$$

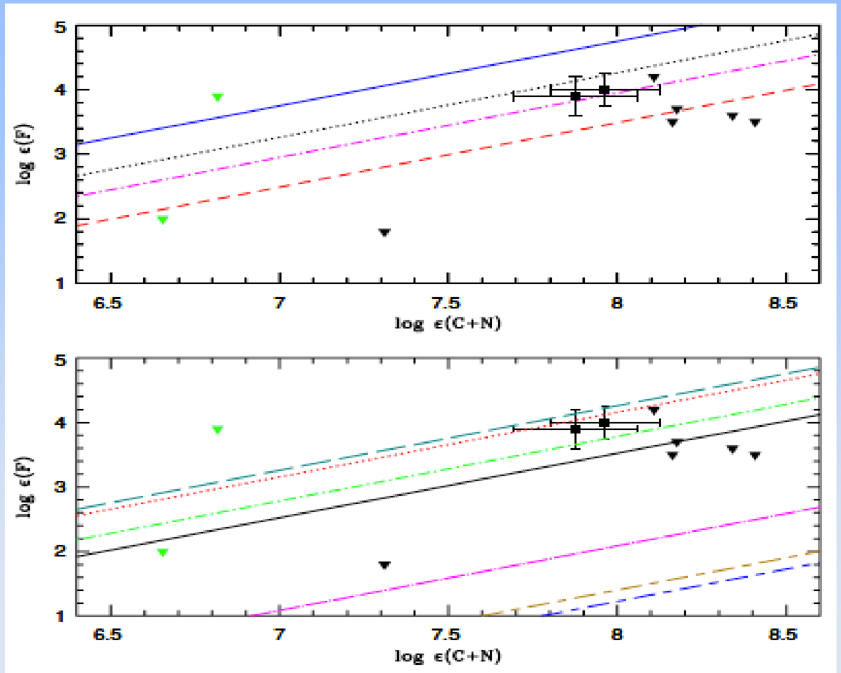
At negative energies M^2 is given by the product of the Whittaker function and the ANC of the F state populated in the transfer reaction

Merging together ANC and THM —> deep connection of these two indirect methods

$^{19}F(p,\alpha_0)^{16}O$

s-nuclei are produced and brought to the surface thanks to mixing phenomena, together with fluorine that is produced in the same region from the same n-source.

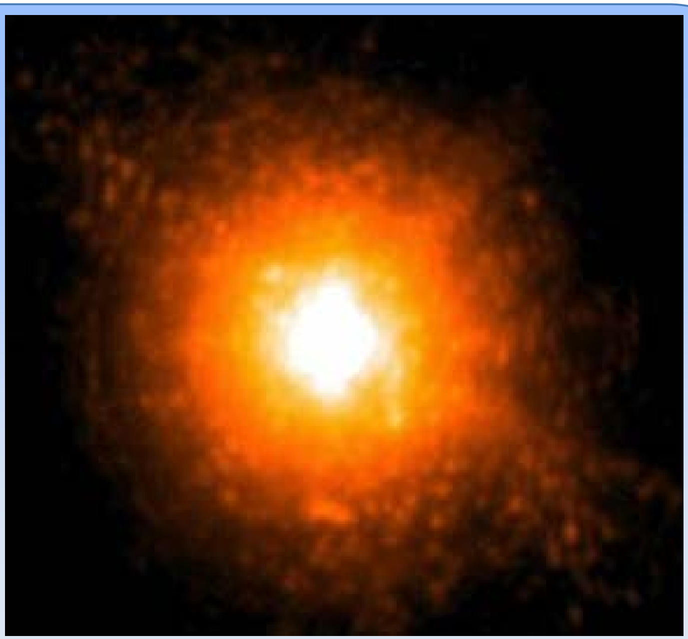
^{19}F is a key isotope as it can be used to probe AGB star mixing phenomena and nucleosynthesis. But its production is still uncertain!



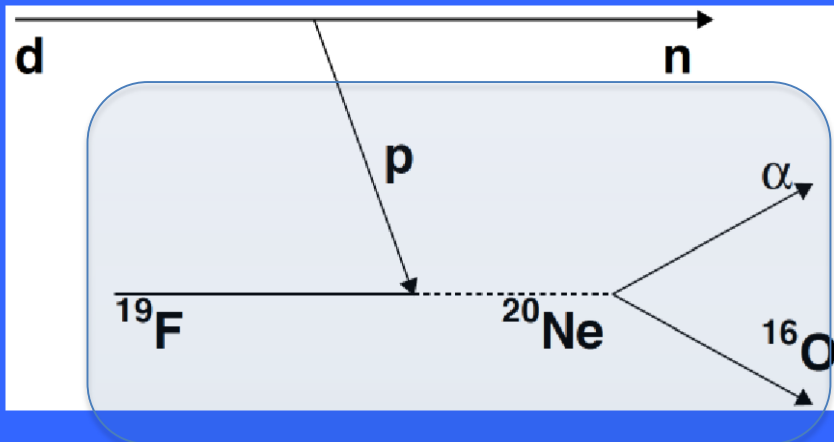
$^{13}C(\alpha,n)^{16}O$

The $^{13}C(\alpha,n)^{16}O$ reaction is the main neutron source in low mass AGB stars at temperatures between 0.8 and 1 x 10^8 K in radiative conditions.

^{13}C is produced starting from ^{12}C present in the instershell region when protons squeeze in during the third dredge-up.



The experiment



d: Trojan horse nucleus p+n

B=2.2 MeV $|p_s|$=0 MeV/c

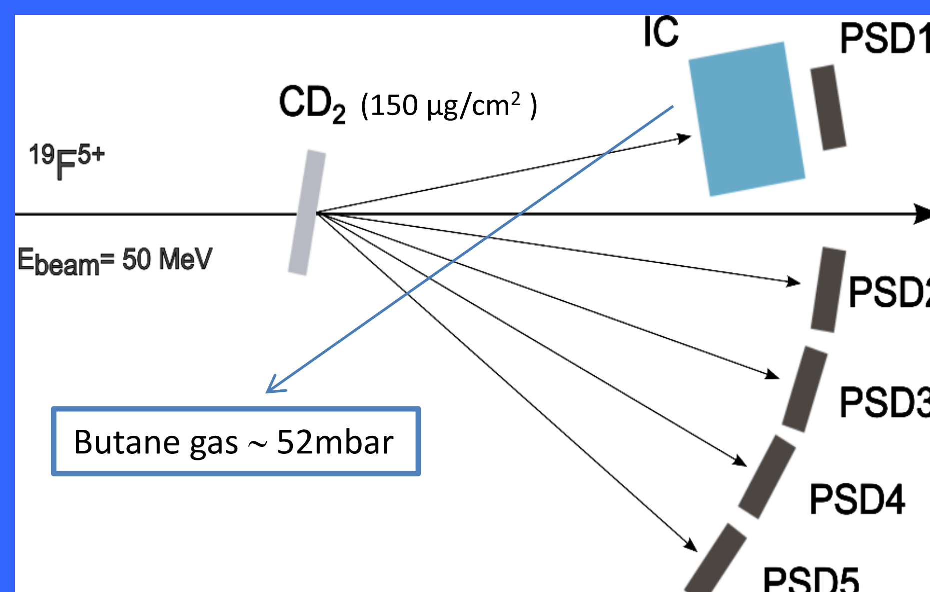
n: spectator

p: participant

$d(^{19}F,\alpha\,^{16}O)n$ Q_{3b}=5.889 MeV

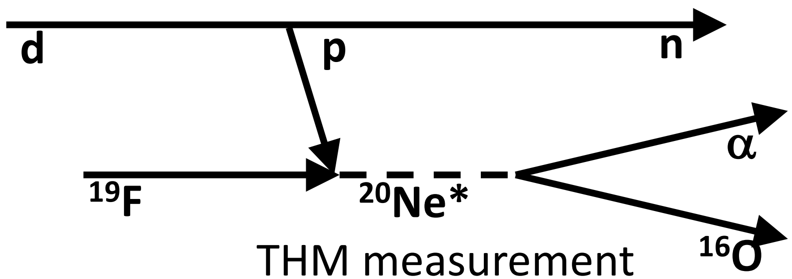
$^{19}F(p,\alpha)\,^{16}O$ Q_{2b}=8.113 MeV

15 MV Tandem at INFN-LNS Catania

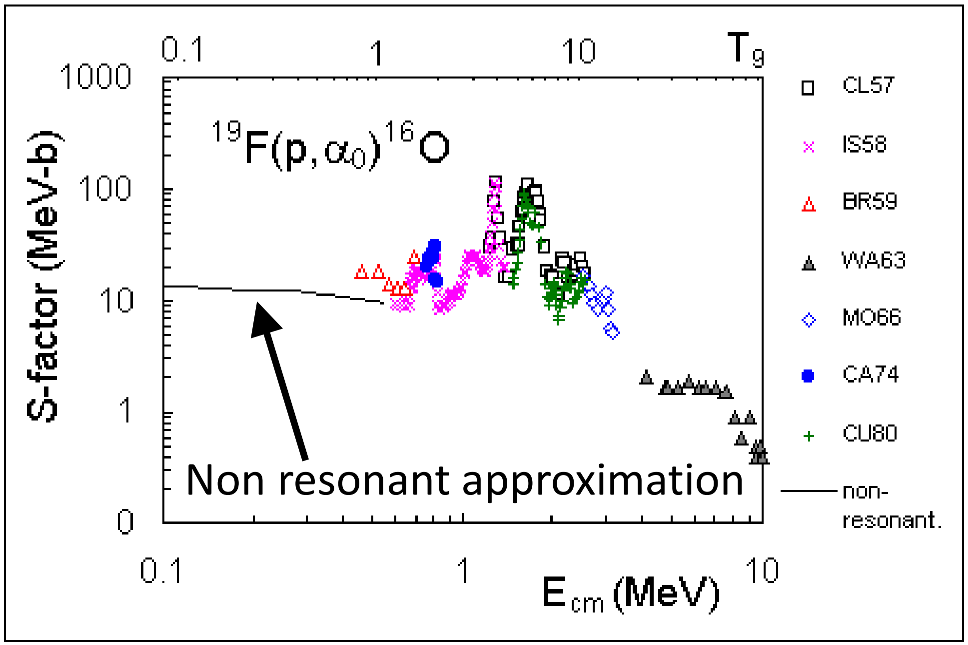


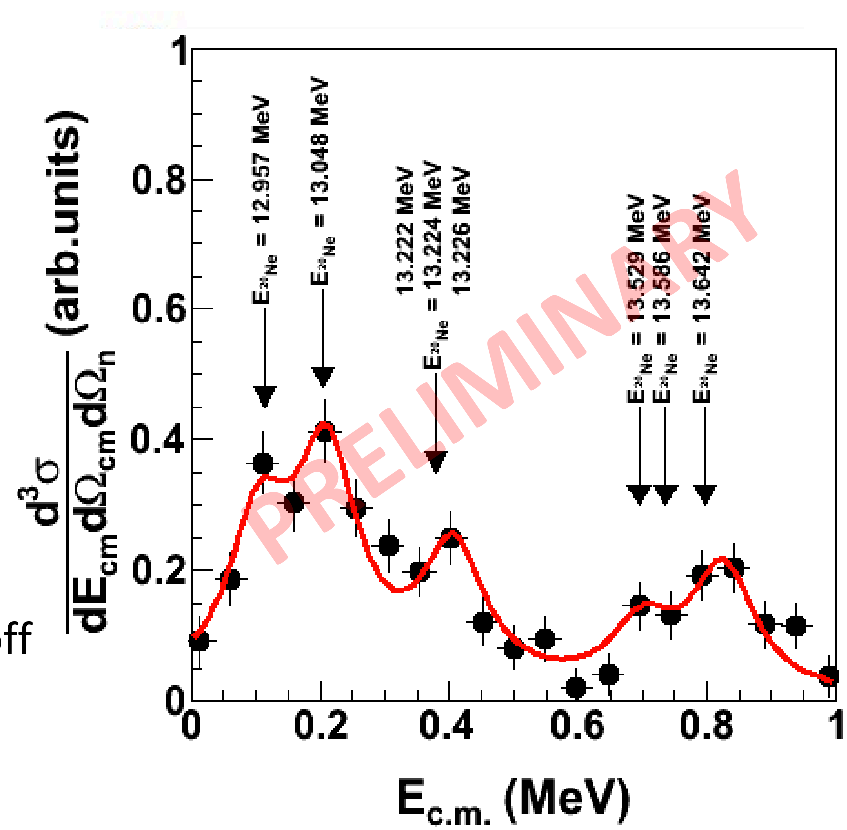
Measurement of the $^{19}F(p,\alpha_0)^{16}O$ reaction using the Trojan Horse Method (THM)

The $^{19}F(p,\alpha_0)^{16}O$ reaction is the main fluorine destruction channel but no data are available around ~100 keV (energy of astrophysical interest)



THM measurement



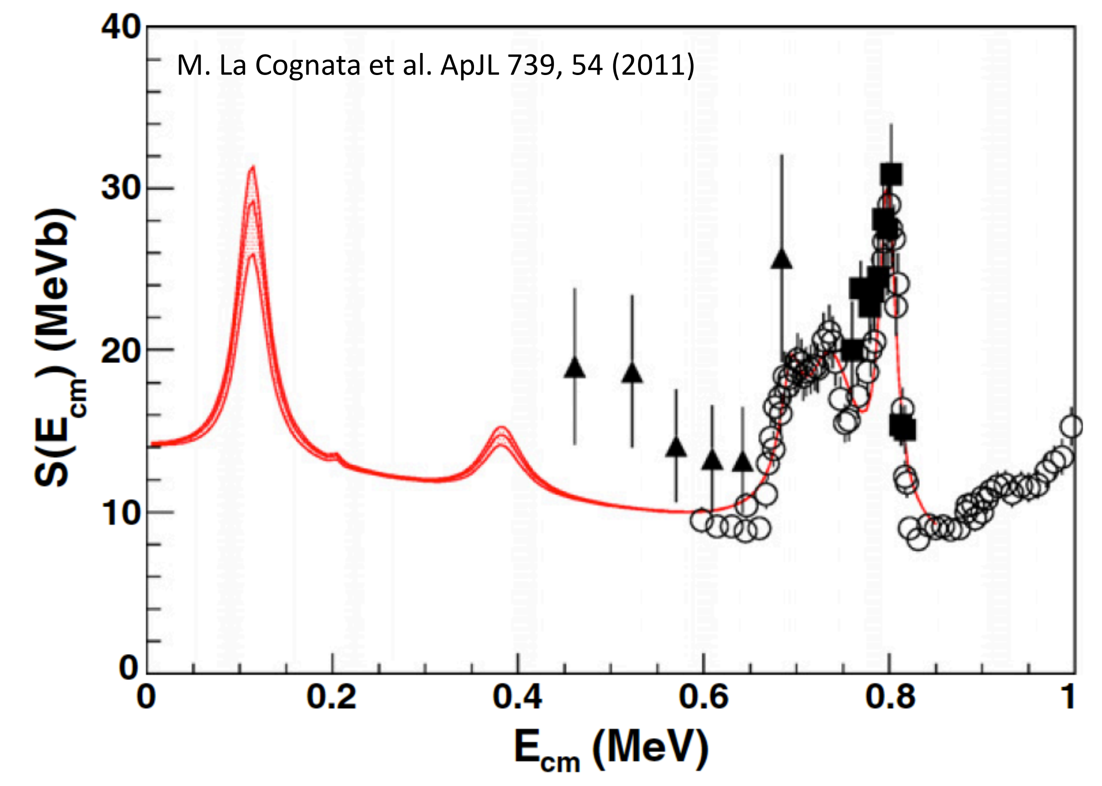


THM: ^{20}Ne states are populated through p-transfer off deuteron and resonance parameters are deduced

→ **THM x-section shows a rich resonant pattern!**

Investigation of the α_π channel @ LNS under analysis

The $^{19}F(p,\alpha)^{16}O$ cross section



R-matrix parameterization of the $^{19}F(p,\alpha_0)^{16}O$ astrophysical factor.

Above 0.6 MeV, the reduced partial widths were obtained through an *R*-matrix fit of direct data

Below 0.6 MeV, the resonance parameters were obtained from the modified *R*-matrix fit

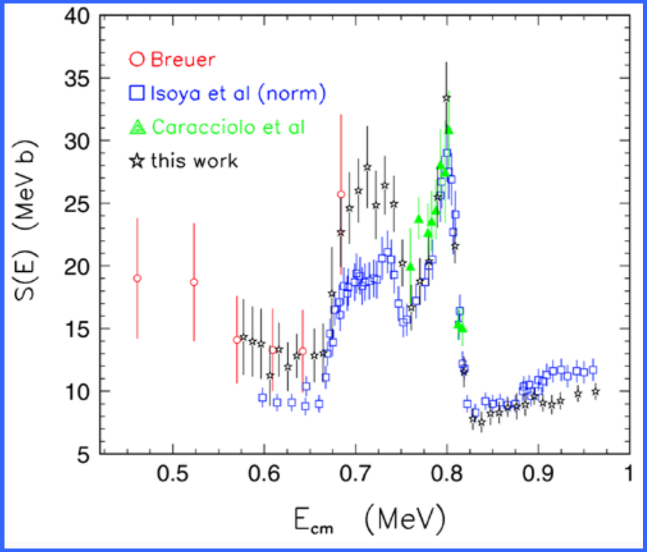
The non-resonant contribution is taken from NACRE (1999).

The non-resonance contribution has been calculated for s-wave (see NACRE compilation), so no significant interference is expected.

Because of spin-parity, only the resonance at 12.957 MeV provide a significant contribution

Gamow window: 27-94 keV → this level lies right at edge of the Gamow window for extra-mixing in AGB stars

Robustness of the THM approach. Effect of normalization



- Larger S-factor below 0.75 MeV with respect to Isoya et al. (1958) data

- Better agreement with Breuer (1959) → Rising S(E) with decreasing energies

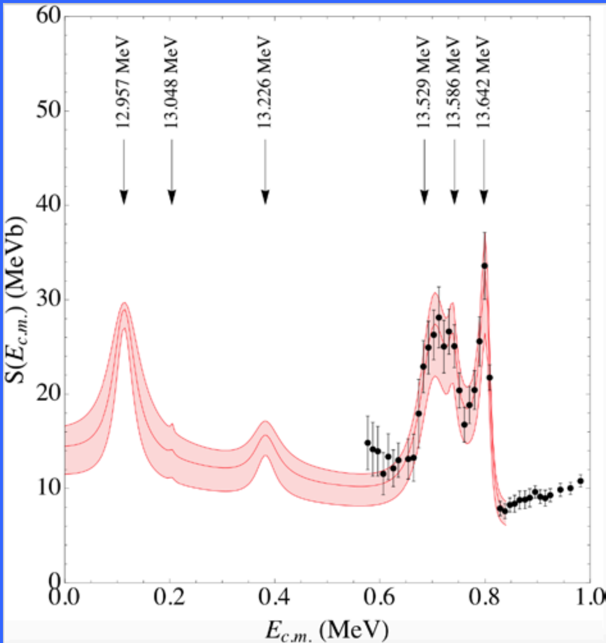
- 0^+ assignment to the 800 keV resonance

A check of the change of the 113 keV resonance strength is mandatory

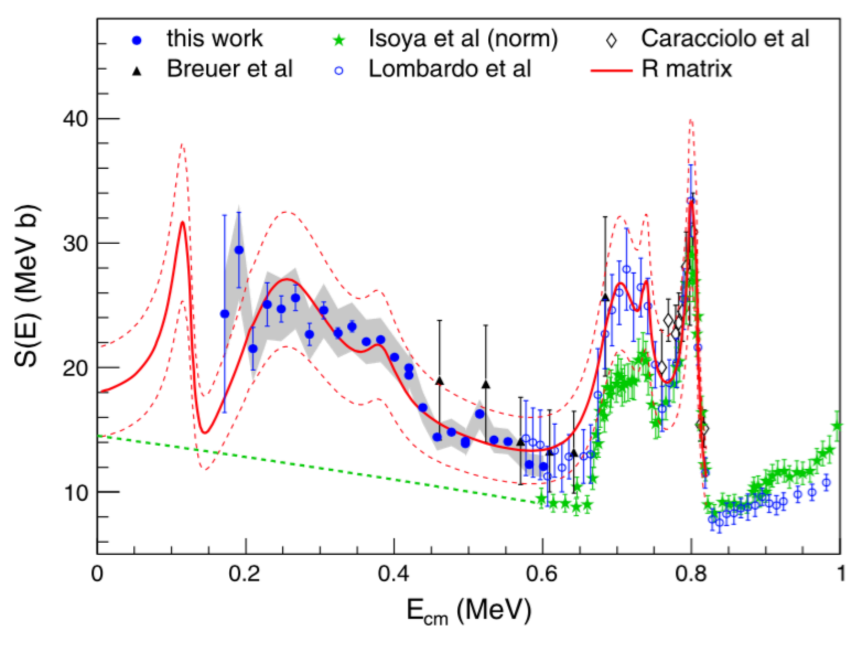
I. Lombardo et al., J. Phys. G: Nucl. Part. Phys. **40** (2013) 125102

New measurement by I. Lombardo et al. in 2013 challenges the accepted $^{19}F(p,\alpha_0)^{16}O$ astrophysical factor in the 0.6-1 MeV energy region.

The new THM analysis has lead to a S(E) factor in agreement with the one previously published



New direct measurement at low energies



Blue points: new data with statistical error only

Gray band: systematic uncertainty

I. Lombardo et al., Phys. Lett. B 748 (2015) 178

New measurement down to about 200 keV

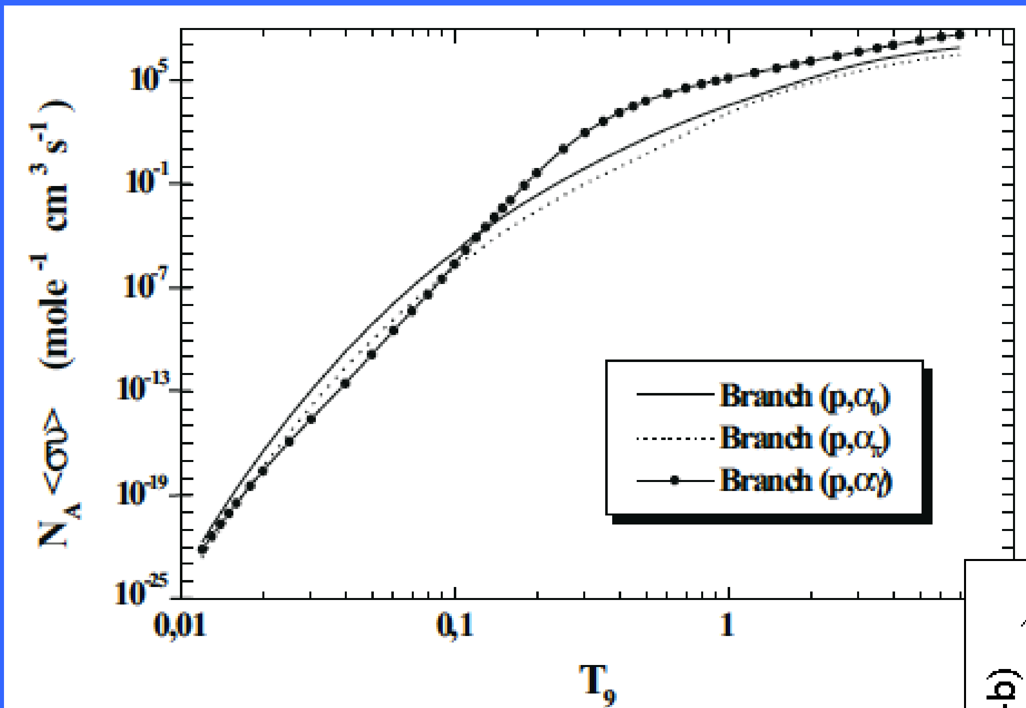
R-matrix calculation: same levels as in the THM measurement, but the 251 keV broad 2^+ state, probably missed because of the poor energy resolution in the 2010 experiment

Non resonant contribution taken from NACRE. As a cross check, the NRC from Yamashita & Kudo was also tested → negligible difference

Direct data quality is still quite poor at low energies → More work is necessary

New THM experiments performed in INFN-LNL (Legnaro) aimed at covering the low energy region with better resolution. Analysis is ongoing (only preliminary spectra)

Measurement of the $^{19}F(p,\alpha_{\square})^{16}O$ channel

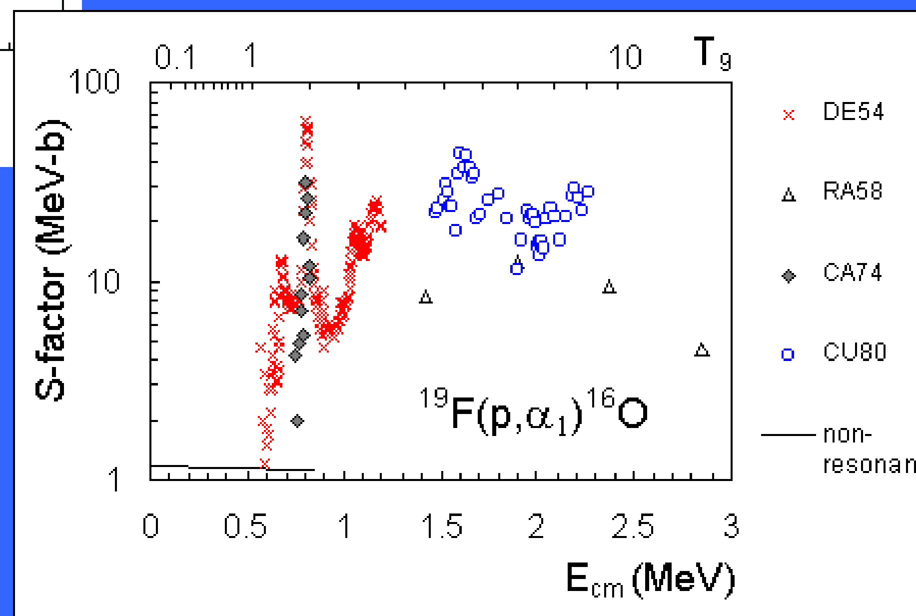


The contribution of the α_0 channel only has been addressed since this is currently regarded as the dominant one at temperatures relevant for AGB stars

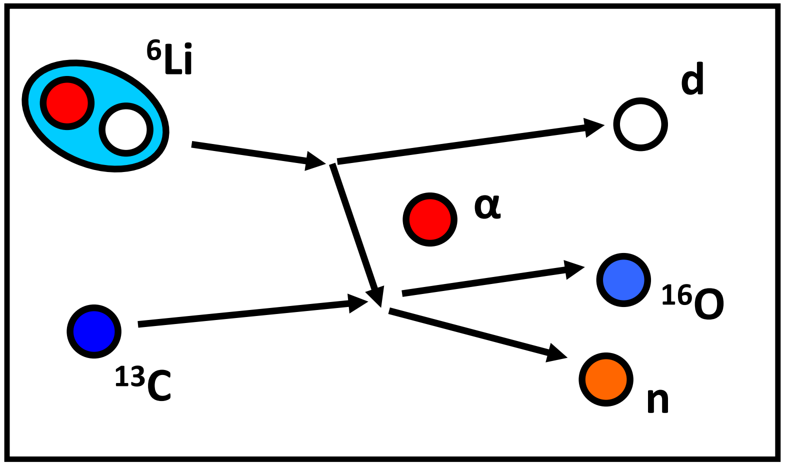
← Spyrou et al. (2000)

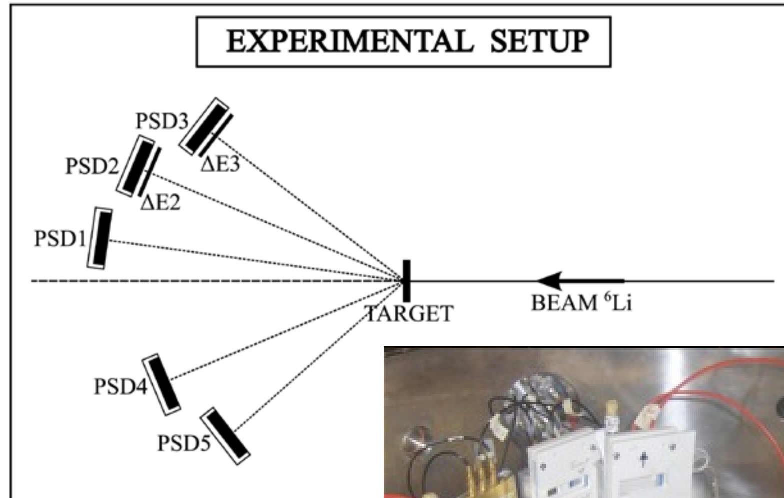
$\alpha_{\square}$ channel is even more uncertain!

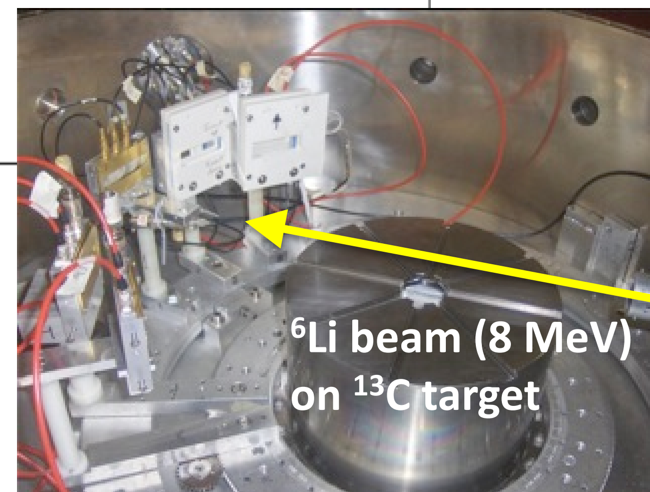
An experiment has been performed @ INFN-LNS (Catania, Italy) to extract the cross section for the $\alpha_{\square}$ channel and to improve the spectroscopy of the resonances discussed here.



Measurement of the $^{13}C(\alpha,n)^{16}O$ reaction through the THM







The experiment was performed at the Florida State University applying the indirect THM. Our experiment was performed by measuring the sub-Coulomb $^{13}C(\alpha,n)^{16}O$ reaction through the $^{13}C(^{6}Li,n^{16}O)d$ reaction in the quasi-free kinematics regime.

FLORIDA STATE UNIVERSITY

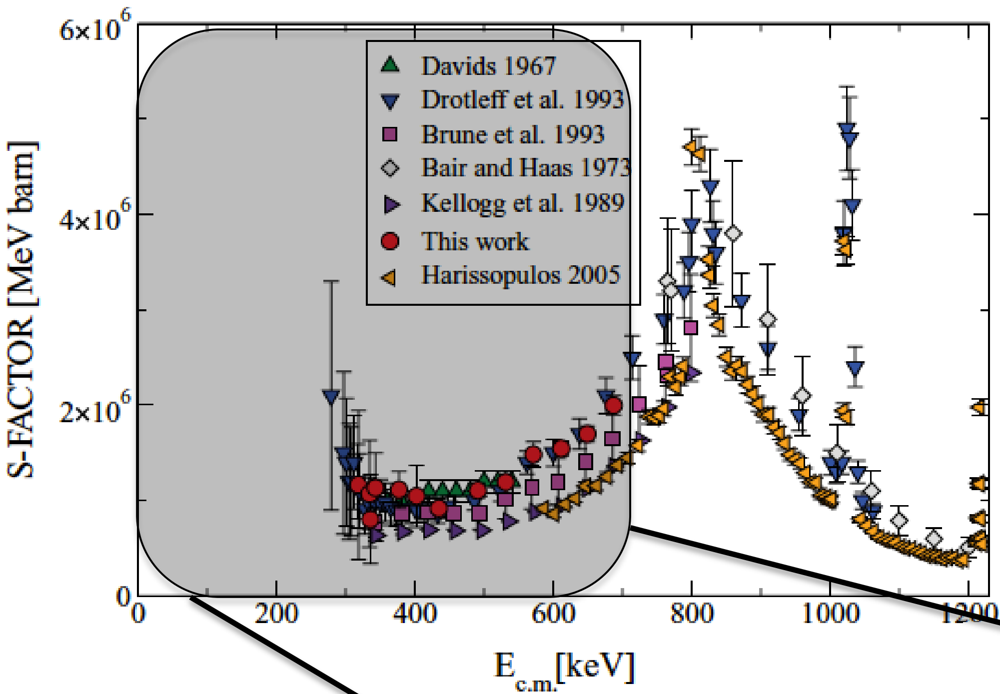
Pros and cons of the experimental approach:

Deuteron detection in PSD 1-2-3 → No need for neutron detectors

Better detection efficiency and lower chances of systematic errors (see direct measurements!)

However d is emitter at zero degrees → the QF peak cannot be accessed in the experiment

State of the art

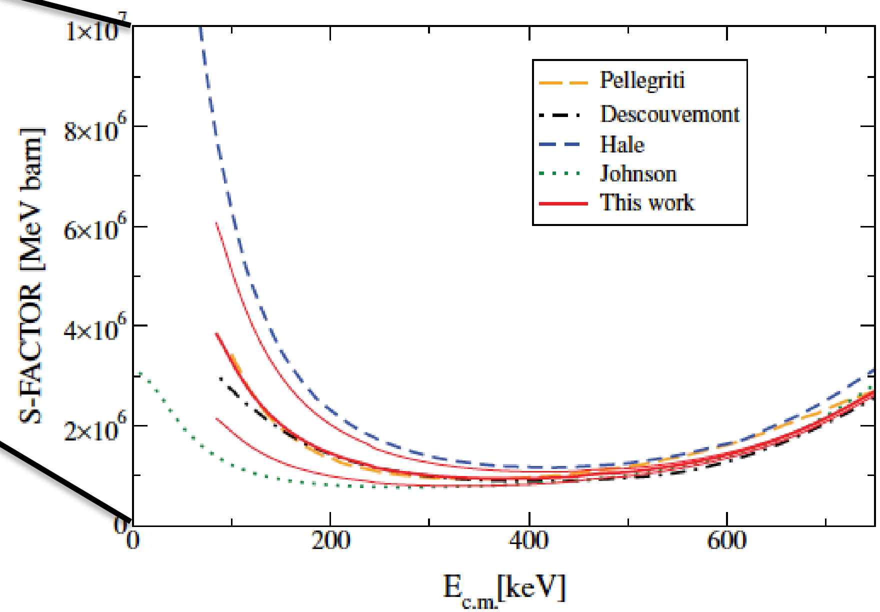


Direct data → several open issues

- **Normalization especially at low energies**
- **Extrapolation to astrophysical energies**
- **Correction for the electron screening enhancement**

Why?

Low energies, neutron detection, theory



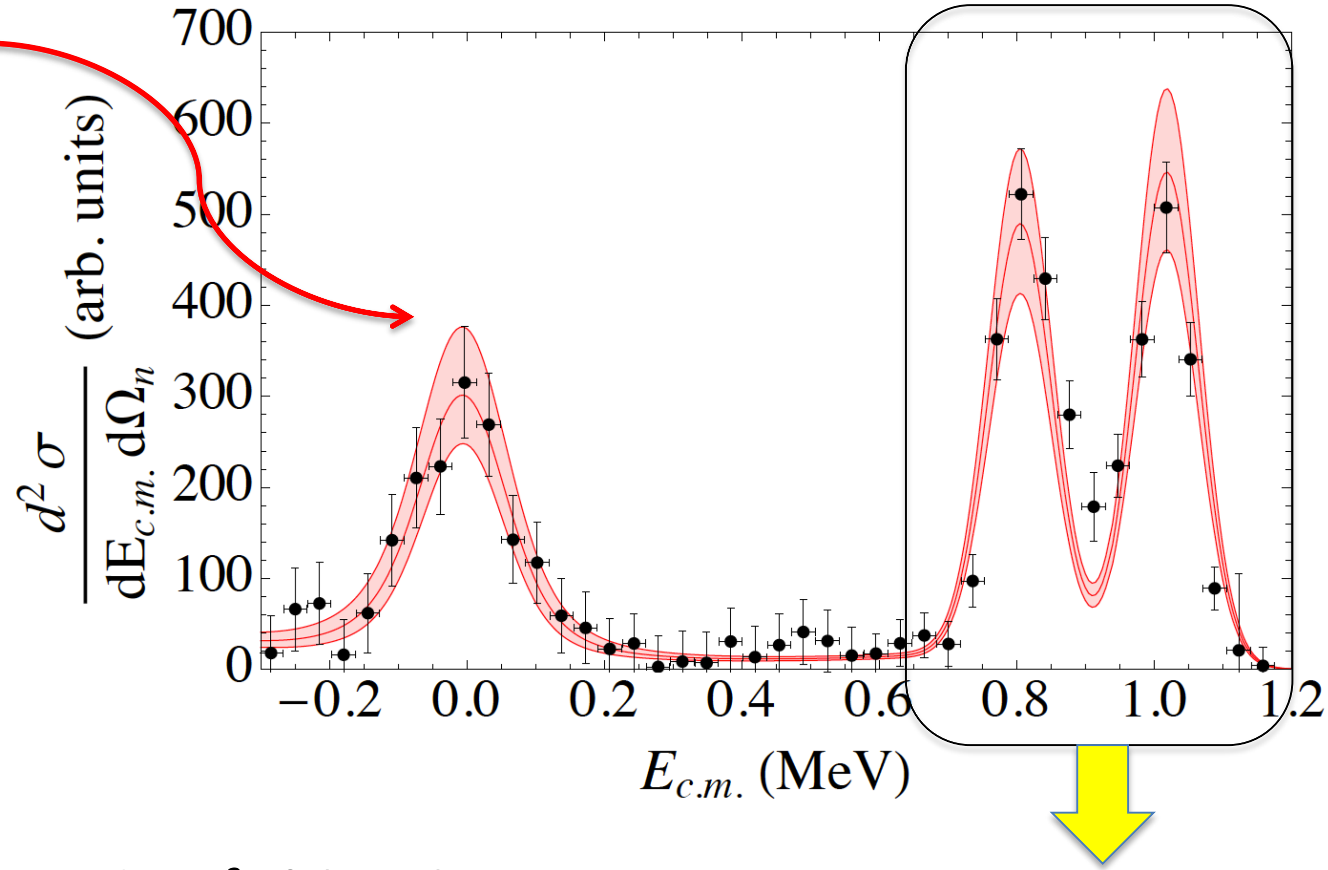
Extrapolation to astrophysical energies Heil et al. 2008

At astrophysical energies an error as large as a factor of 2 is obtained

Change on the cross section influences the neutron abundance and so the yield of some elements like Rb and Sr.

Fitting THM data with the HOES R-matrix

Fitted HOES cross section: 2 parameters, the reduced n- and α-widths. Channel radii fixed at the Heil et al. ones [5.2 and 4 fm for the α- and n-channels]



Coulomb corrected ANC^2 of the -3 keV resonance is: $7.7^{+1.6}_{-1.5}$ fm^{-1} (maximum error)

Γ_n of the -3 keV resonance is: $0.107^{+0.010}_{-0.007}$ MeV

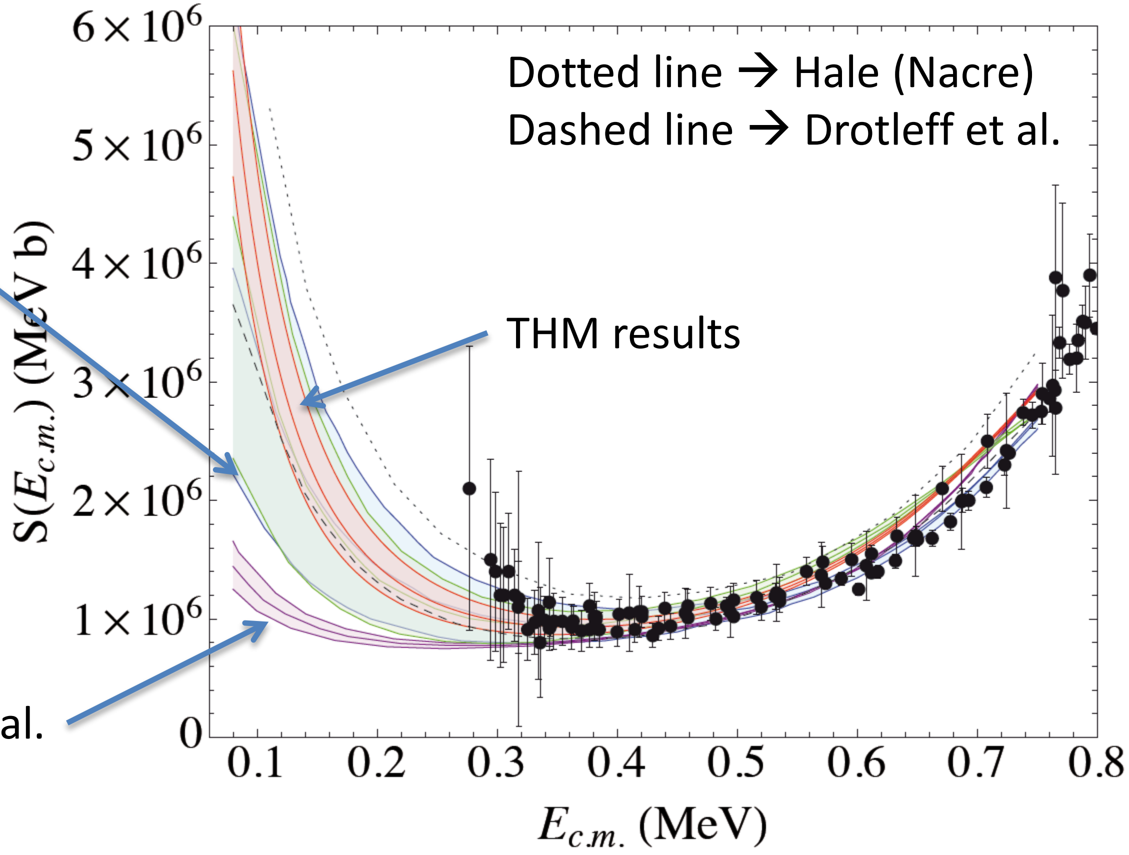
Normalization region: Scaling factor and energy resolution obtained (this one in agreement with the calculated one, 46 keV)

Effect of DW: 9.5%, included into the normalization error as it modifies the 2-peak relative height

Comparison of our low-energy S-factor with some of the others present in the literature

Green and blue bands (barely distinguishable) are Pellegriti et al. and Heil et al.

Johnson et al.



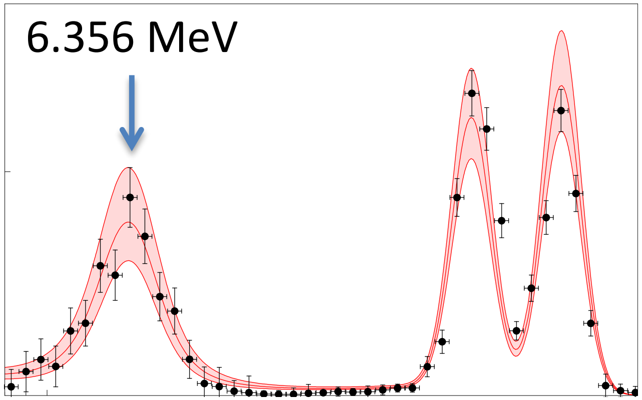
M. La Cognata et al. PRL 109, 232701 (2012)

M. La Cognata et al. ApJ 777, 143 (2013)

Several extrapolations are available, we show the most recent ones or those commonly in astrophysical modeling → Nacre (essentially the R-matrix by Hale) and Drotleff et al. (the type of calculation is not disclosed).

Electron screening? Included in Drotleff et al. calculation, not included by Heil et al.

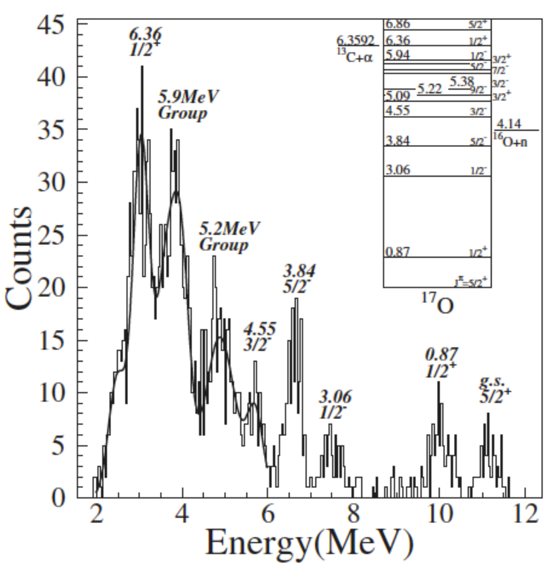
Comparison with other indirect approaches

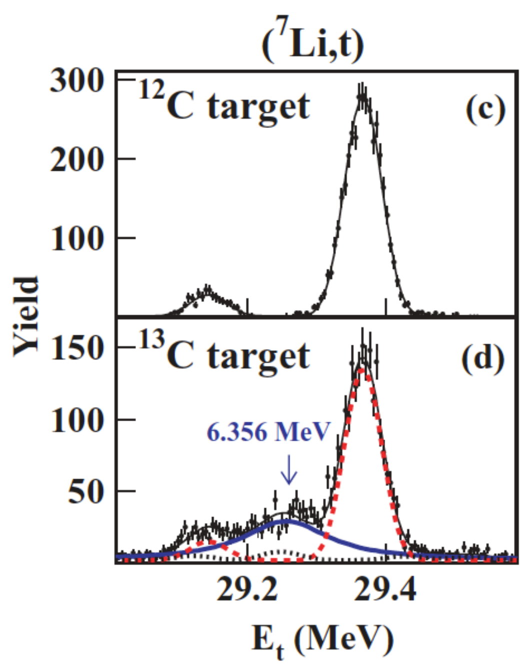


Clear signature of the presence of the sub threshold 6.356 MeV state in ^{17}O responsible of the occurrence of the -3 keV resonance
Also normalization is straightforward thanks to the clearly visible high energy states

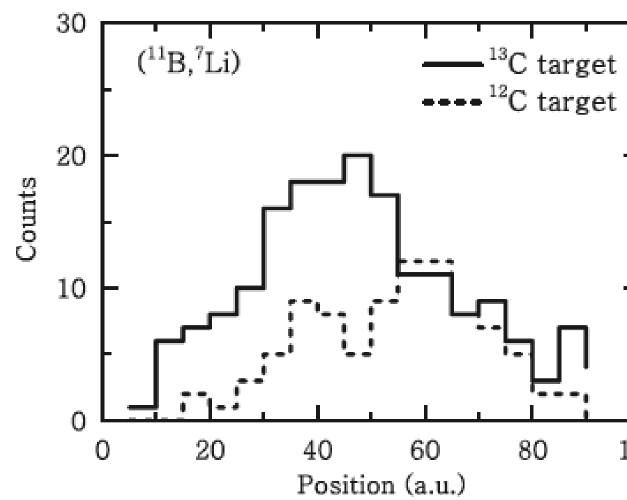
Pellegriti et al.

Johnson et al.





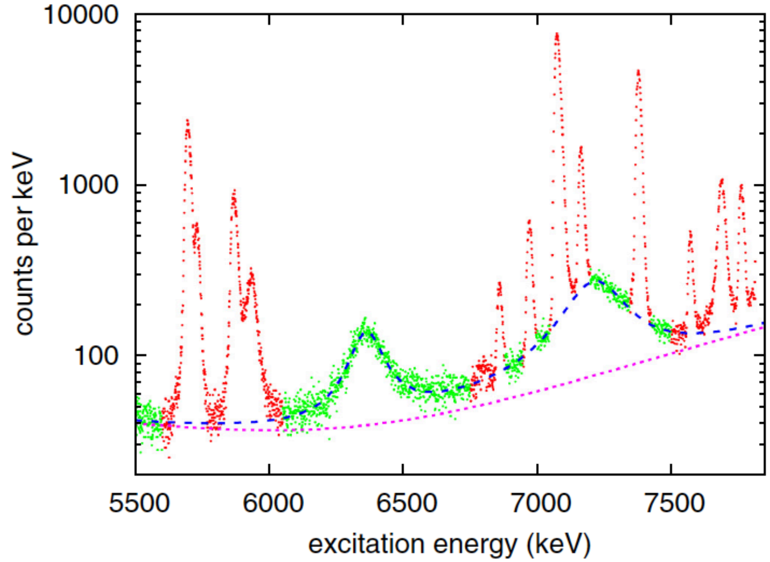
Guo et al.



Comparison with Faestermann et al 2015 & Avila et al. 2015

High energy spectrum for the $^{19}F(d,\alpha)^{17}O$ reaction at Θ_{lab} = 10°. The simultaneously fitted background (dotted) and the total fit function (dashed) are shown.

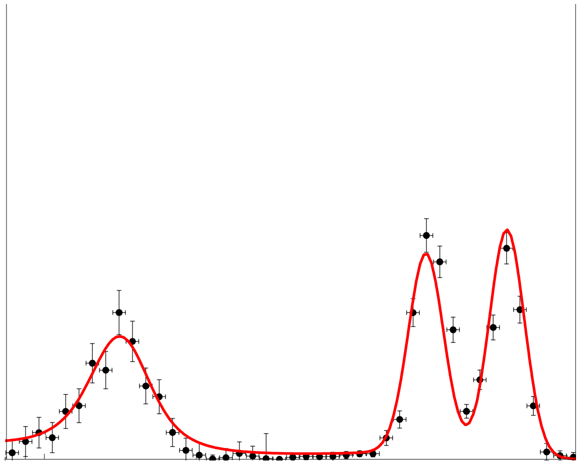
The strongest peak at 7075 keV is the only background line from the $^{16}O(d,\alpha_0)^{14}N$ reaction.



E∗ = 6363.4±3 .1keV   Γ= 136±5 keV.

→ 4.7±3 keV above threshold!

THM: Γ= 107±9 keV



How good is the agreement with THM data?

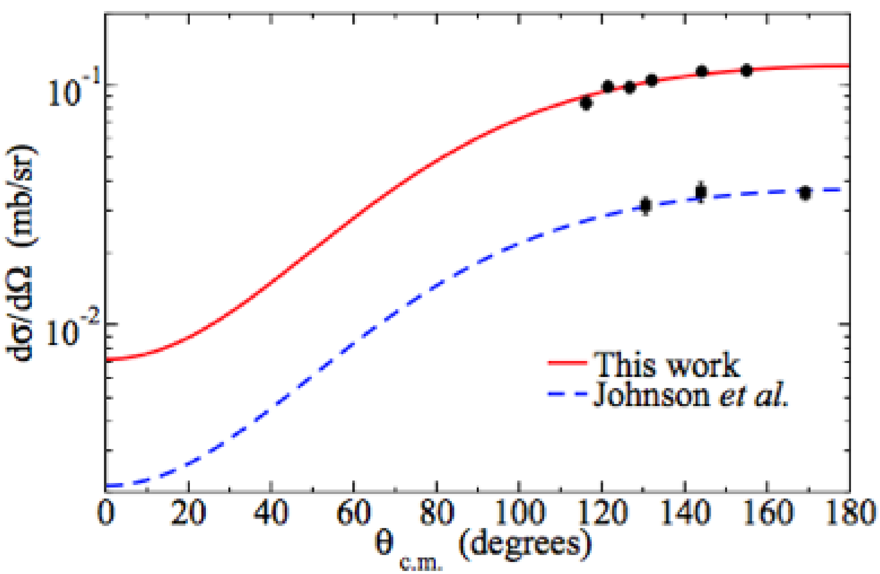
→ Within 5%, still fine:

(increasing the ANC within the uncertainty)

Comparison with Faestermann et al 2015 & Avila et al. 2015

TABLE I. Summary of the previous and current results for the squared Coulomb-modified ANC and spectroscopic factor S_α for the $1/2^+$ subthreshold resonance at an excitation energy of 6.356 MeV in ^{17}O.

$(\tilde{C}_{\alpha-^{13}C}^{^{17}O(1/2+)})^2 fm^{-1}$	S_α	Ref.
	0.01	[3]
0.89 ± 0.23		[5]
	0.36–0.40	[4]
4.5 ± 2.2	0.29 ± 0.11	[6]
$7.7 \pm 0.3_{stat}{}^{+1.6}_{-1.5\,norm}$		[9]
4.0 ± 1.1	0.37 ± 0.12	[8]
3.6 ± 0.7		This work



The ANC for the 6.356 MeV state is deduced from the fitting of the angular distribution of the sub-Coulomb alpha transfer off ^{6}Li to ^{13}C.

Agreement with Guo et al.

The deduced ANC is half of the value obtained with THM.

Model dependence?
Error underestimate?

Pellegriti et al. find a large error coming from the energy dependence of the spectroscopic factor.

Guo et al. and Avila et al. → single energy measurements

Summary

- Resonant reactions play a key role in astrophysics and in nuclear physics. The presence of a resonance can have dramatic consequences on nucleosynthesis

- Indirect measurements are precious tools to investigate reactions or energy ranges difficult to study otherwise

- Fields of interest: astrophysics, applied physics, nuclear structure, fundamental interactions

- However, guidance by direct data and normalization to them is necessary not to incur in systematic errors

- A synergic application of direct and indirect approaches is the best guarantee of accurate reaction rates for astrophysical application

Thanks for your attention