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R-MATRIX METHODS & APPLICATIONS WITH **EDA**

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Outline

- Overview
  - History at LANL
  - Existing analyses
- R-matrix formalism
  - Bloch formalism
  - Wolfenstein formalism
  - Fitting, errors, covariance
- Analyses (systems)
  - ^{7}Be, ^{17}O
- Resonance model (if there's time)
  - Particle spectra

Overview of multichannel reaction analysis

- History of **E**nergy **D**ependent **A**nalysis
  - Developers: D. Dodder, K. Witte, G. Hale, A. Sierk, MP
  - Some original motivation: hadronic analyses e.g. $\pi N \rightarrow \pi N$
  - EDA5 F77; EDA6 F90/95 (under development)
- Code overview
  - EDA5/6 implement Wigner/Eisenbud/Bloch phenomenological R matrix
  - Handles large number of two-body partitions & channels, including EM
  - Data: elastic, inelastic, reaction; diff'l, integrated, total, polarization
- Existing analyses to date…

EDA Existing Analyses

A	System	Channels	Energy Range (MeV)
2	N-N	p+p; n+p, γ+d	0-30 0-40
3	N-d	p+d; n+d	0-4
4	^{4}H ^{4}Li	n+t p+^{3}He	0-20
	^{4}He	p+t n+^{3}He d+d	0-11 0-10 0-10
5	^{5}He	n+α d+t ^{5}He+γ	0-28 0-10
	^{5}Li	p+α d+^{3}He	0-24 0-1.4

EDA Existing Analyses, Cont.

A	System (Channels)
6	^{6}He (^{5}He+n, t+t); ^{6}Li (d+^{4}He, t+^{3}He); ^{6}Be (^{5}Li+p, ^{3}He+^{3}He)
7	^{7}Li (t+^{4}He, n+^{6}Li); ^{7}Be (γ+^{7}Be, ^{3}He+^{4}He, p+^{6}Li)
8	^{8}Be (^{4}He+^{4}He, p+^{7}Li, n+^{7}Be, p+^{7}Li*, n+^{7}Be*, d+^{6}Li)
9	^{9}Be (^{8}Be+n, d+^{7}Li, t+^{6}Li); ^{9}B (γ+^{9}B, ^{8}Be+p, d+^{7}Be, ^{3}He+^{6}Li)
10	^{10}Be (n+^{9}Be, ^{6}He+α, ^{8}Be+nn, t+^{7}Li); ^{10}B (α+^{6}Li, p+^{9}Be, ^{3}He+^{7}Li)
11	^{11}B (α+^{7}Li, α+^{7}Li*, ^{8}Be+t, n+^{10}B); ^{11}C (α+^{7}Be, p+^{10}B)
12	^{12}C (^{8}Be+α, p+^{11}B)
13	^{13}C (n+^{12}C, n+^{12}C*)
14	^{14}C (n+^{13}C)
15	^{15}N (p+^{14}C, n+^{14}N, α+^{11}B)
16	^{16}O (γ+^{16}O, α+^{12}C)
17	^{17}O (n+^{16}O, α+^{13}C)
18	^{18}Ne (p+^{17}F, p+^{17}F*, α+^{14}O)

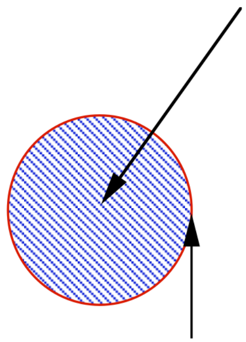
2→2 body R matrix formalism

$\mathscr{I}$

INTERIOR (Many-Body) REGION
(Microscopic Calculations)

$\underbrace{H + \mathcal{L}_B}$

compact, hermitian operator with real, discrete spectrum; eigenfunctions in Hilbert space



$$|\psi^+) = (H + \mathcal{L}_B - E)^{-1}\mathcal{L}_B|\psi^+)$$

SURFACE

$$\mathcal{L}_B = \sum_c |c)(c|\left(\frac{\partial}{\partial r_c} r_c - B_c\right),$$

$$(\mathbf{r}_c|c) = \frac{\hbar}{\sqrt{2\mu_c a_c}} \frac{\delta(r_c - a_c)}{r_c}[(\phi_{s_1}^{\mu_1} \otimes \phi_{s_2}^{\mu_2})_s^{\mu} \otimes Y_l^m(\hat{\mathbf{r}}_c)]_J^M$$

$$R_{c'c} = (c'|(H + \mathcal{L}_B - E)^{-1}|c) = \sum_\lambda \frac{(c'|\lambda)(\lambda|c)}{E_\lambda - E}$$

$\mathscr{E}$

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)

$$(r_{c'}|\psi_c^+\rangle = -I_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})S_{c'c}$$

or equivalently,

$$(r_{c'}|\psi_c^+\rangle = F_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})T_{c'c}$$

Measurements

Bloch/Green-function formalism: Hermiticity

UNE FORMULATION UNIFIÉE DE LA THÉORIE DES RÉACTIONS NUCLÉAIRES

CLAUDE BLOCH

Centre d'Études Nucléaires de Saclay, Gif-sur-Yvette (S. & O.)

Reçu le 13 avril 1957

A unified formulation of the theory of nuclear reactions

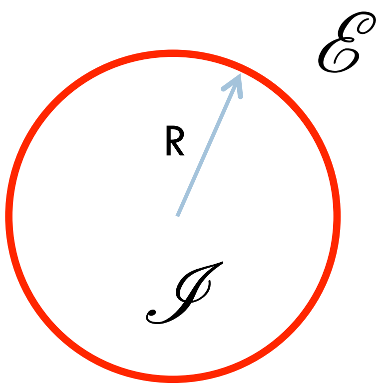
Claude Bloch[1]

Centre d'Études Nucléaires de Saclay, Gif-sur-Yvette (S. & O.)

$$\int_0^R r^2 dr [\psi_1^*(H\psi_2) - (H\psi_1)^*\psi_2] = -\frac{\hbar^2}{2M}\left[r\psi_1^*\frac{d(r\psi_2)}{dr} - \frac{d(r\psi_1^*)}{dr}r\psi_2\right]_R$$

$$\mathscr{H} = H + \frac{\hbar^2}{2MR}\delta(r-R)\frac{d}{dr}r,$$

$$\int_0^R r^2 dr\, [\psi_1^*(\mathscr{H}\psi_2) - (\mathscr{H}\psi_1)^*\psi_2] = 0.$$



Bloch/GF formalism: Generalized GF

$$[\mathscr{H} - E]\,\psi(r) = F(r), \qquad F(r) = f(r) + A\delta(r-R)$$

$$\mathscr{H} = H + \mathscr{L}_0, \qquad \mathscr{L}_0 = \frac{\hbar^2}{2M}\frac{\delta(r-R)}{R}\frac{d}{dr}r$$

Equivalent to:

$$[\mathscr{H} - E]\,\psi(r) = f(r), \qquad r < R$$

$$\frac{\hbar^2}{2MR}\left[\frac{d}{dr}\left(r\psi(r)\right)\right]_R = A, \qquad r = R$$

Bloch/GF formalism: representation independence

- The singularity Dirac delta function is only present in the position representation ('R' is the channel radius)

$$\mathscr{L}_B = \frac{\hbar^2}{2M}\frac{\delta(r-R)}{R}\left(\frac{d}{dr}r - B\right)$$

- Equivalent to

$$\hat{\mathscr{L}}_B = \frac{\hbar^2 R^2}{2M}|R\rangle\langle R|\left(i\hat{p}_r - B\right)$$

- Bloch operator as a projection operator

Bloch/GF formalism: multichannel case

- Solve Schrodinger knowing External solution ('a' chan. rad.)

$$[H-E]\Psi=0, \qquad \Psi=r^{-1}\Big[I-OS\Big], \qquad r\geq a$$

$$\Psi=G\mathscr{L}\Psi, \qquad G=[H-E+\mathscr{L}]^{-1}, \qquad \mathscr{L}=a^{-1}\left(\rho\frac{\partial}{\partial\rho}-B\right)$$

$$I-OS=R\left(\rho\frac{\partial}{\partial\rho}-B\right)[I-OS], \qquad R\equiv G\Big|_{\mathscr{S}}, \qquad \rho\frac{\partial}{\partial\rho}O=LO$$

$$S=O^{-1}I+2i\rho O^{-1}R_LO^{-1}, \qquad R_L=\Big[1+R(B-L)\Big]^{-1}R, \qquad \rho\frac{\partial}{\partial\rho}I=LI-2i\rho O^{-1}$$

- External Coulomb wave function relations

$$O=I^*=G+iF, \qquad 1=GF'-G'F,$$

$$L=\rho O^{-1}\frac{\partial}{\partial\rho}O\equiv\mathcal{S}+iP, \qquad \mathcal{S}=\rho\frac{GG'+FF'}{G^2+F^2}, \qquad P=\rho\frac{1}{G^2+F^2}$$

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Bloch/GF formalism: multichannel unitarity

$$S = O^{-1}I + 2i\rho O^{-1}R_L O^{-1}$$

$$S^\dagger = OI^{-1} - 2i\rho I^{-1}R_L^\dagger I^{-1}$$

$$\left(M^\dagger\right)^{-1} = \left(M^{-1}\right)^\dagger$$

$$S^\dagger S = 1 + 2i\rho I^{-1}R_L^\dagger\left[(R_L^{-1})^\dagger - R_L^{-1} + 2i\rho I^{-1}O^{-1}\right]R_L O^{-1}$$

$$R_L^{-1} = R^{-1} + B - L$$

$$B = B^* \implies L - L^* = 2i\rho I^{-1}O^{-1} \text{ or } P = \rho\frac{1}{G^2+F^2}$$

$$R_{c'c} = (c'|\left[H + \mathscr{L} - E\right]^{-1}|c) = \sum_\lambda \frac{\gamma_{c'\lambda}\gamma_{c\lambda}}{E_\lambda - E}$$

- Unitarity requires B real
- Energy independent level E_λ and reduced width $\gamma_{c\lambda}$ require B constant
- Unitarity is lost if $B = \mathcal{S}(E)$ with constant E_λ, $\gamma_{c\lambda}$

Unitarity constraint on T matrix

$$\left.\begin{aligned} \delta_{fi} &= \sum_n S^\dagger_{fn} S_{ni} \\ S_{fi} &= \delta_{fi} + 2i\rho_f T_{fi} \\ \rho_n &= \delta(H_0 - E_n) \end{aligned}\right\}$$

$$T_{fi} - T^\dagger_{fi} = 2i \sum_n T^\dagger_{fn} \rho_n T_{ni}$$

NB: unitarity implies optical theorem $\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im } f(0)$; but *not just* the O.T.

- **Implications of unitarity constraint on transition matrix**
  1. Doesn't uniquely determine T_{ij}; highly restrictive, however

     Elastic: $\text{Im } T_{11}^{-1} = -\rho_1, \; E < E_2$ (assuming T & P invariance)

     Multichannel: $\text{Im } \mathbf{T}^{-1} = -\boldsymbol{\rho}$
  2. Unitarity violating transformations
     - cannot scale ***any*** set: $T_{ij} \to \alpha_{ij} T_{ij} \qquad \alpha_{ij} \in \mathbb{R}$
     - cannot rotate ***any*** set: $T_{ij} \to e^{i\theta_{ij}} T_{ij} \qquad \theta_{ij} \in \mathbb{R}$

     ★ consequence of linear 'LHS' $\propto$ quadratic 'RHS'
  3. Unitary parametrizations of data provide constraints that experiment may violate

     ★ *normalization*, in particular → $\text{Observable} \propto \text{KF } |T_{fi}|^2$

Channel radius as ***regulator*** of the theory

- Simple example: single channel, s-wave, neutral

$$S = O^{-1}I + 2i\rho O^{-1}R_L O^{-1}, \qquad B = 0, \rho = ka$$

$$= e^{-2i\rho}\frac{1+i\rho R}{1-i\rho R}$$

$$\frac{\partial S}{\partial a} = 0 \implies 0 = \rho R'(\rho) + R(\rho) - \rho^2 R^2(\rho) - 1$$

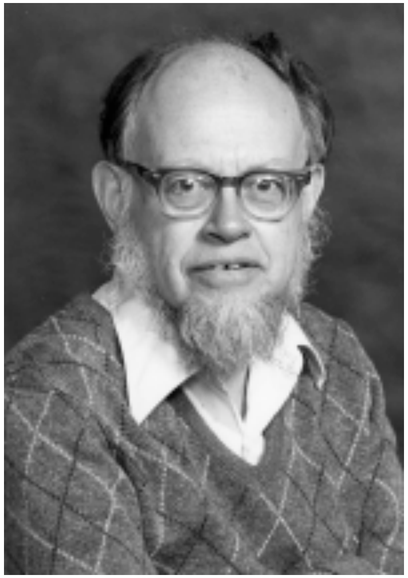
$$R(\rho) = \rho^{-1}\tan\left(\rho + f(k)\right)$$

- f(k) is a familiar function – What is it?

- A request: Does anyone have Teichmann's thesis?
  - Quoted in Lane & Thomas footnote #48, p. 275

Complete, polarization transition matrix

- Wolfenstein formalism



Lincoln Wolfenstein
1923-2015

$$\langle O_f \rangle = \frac{1}{\mathrm{Tr}(\rho_f)} \mathrm{Tr}(\rho_f O_f) = \frac{1}{\mathrm{Tr}(\rho_f)} \mathrm{Tr}(M \rho_i M^\dagger O_f),$$

$\rho = aa^\dagger$, and $a_f = M a_i$.

Using the expansion $\rho_i = \frac{1}{\mathrm{Tr}(\mathbb{1}_i)} \sum_i \langle O_i \rangle \, O_i$,

and defining $\mathrm{Tr}(\rho_f) = \sigma_0(\theta)$ gives finally

$$\sigma_0(\theta) \langle O_f \rangle = \frac{1}{\mathrm{Tr}(\mathbb{1}_i)} \sum_i \langle O_i \rangle \mathrm{Tr}(M O_i M^\dagger O_f), \quad \begin{cases} O_i = O_1 \otimes O_2 \\ O_f = O_3 \otimes O_4 \end{cases}$$

$$M_{fi} = \frac{4\pi}{k_i} \left\langle \phi_{s'}^{\mu'} \middle| \hat{T} \middle| \phi_s^{\mu} \right\rangle = \frac{4\pi}{k_i} \sum_{JMl'l} \left\langle \phi_{s'}^{\mu'} \middle| \mathcal{Y}_{Js'l'}^{M} \right\rangle T_{s'l',sl}^{J} \left\langle \mathcal{Y}_{Jsl}^{M} \middle| \phi_s^{\mu} \right\rangle.$$

Relativistic forms of EDA

$$R = \sum_\lambda \frac{\gamma_\lambda \gamma_\lambda^T}{E_\lambda(s) - E(s)},$$

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 = (\varepsilon_{\text{rel}} + M)^2.$$

Forms for $E_{(\lambda)}(s)$:

a) $\sqrt{s} - M = \varepsilon_{\text{rel}}$

b) $\dfrac{s - M^2}{2M} = \left(1 + \dfrac{\varepsilon_{\text{rel}}}{2M}\right)\varepsilon_{\text{rel}}$

c) $\dfrac{(s - M^2)(s - \Delta^2)}{8s\mu}$ (Layson)

d) ε_{nr} (norel=1)

$$\begin{cases} M = m_1 + m_2 \\ \Delta = m_1 - m_2 \\ \mu = \dfrac{m_1 m_2}{m_1 + m_2} \end{cases}$$

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EM Transitions and Photon Channels

Assume that in the one-photon sector of Fock space, a "wave function" is associated with the vector potential

$$\mathbf{A}_{\mathbf{k}}(\mathbf{r}) = \sqrt{\frac{2}{\pi\hbar c}} \sum_{jm} i^{j} \left[\alpha^{(e)}_{jm} \mathbf{A}^{(e)}_{jm}(\mathbf{r}) + \alpha^{(m)}_{jm} \mathbf{A}^{(m)}_{jm}(\mathbf{r}) \right],$$

$$\mathbf{A}^{(e)}_{jm}(\mathbf{r}) = \frac{1}{r}\left[u^{j}_{ee}(\underbrace{\rho}_{kr})\mathbf{Y}^{(e)}_{jm}(\hat{\mathbf{r}}) + u^{j}_{0e}(\rho)\mathbf{Y}^{(0)}_{jm}(\hat{\mathbf{r}}) \right], \text{ parity=}(-1)^{j},$$

$$\mathbf{A}^{(m)}_{jm}(\mathbf{r}) = \frac{1}{r} u^{j}_{mm}(\rho)\mathbf{Y}^{(m)}_{jm}(\hat{\mathbf{r}}), \text{ parity=}(-1)^{j+1}.$$

The physical radial functions have the asymptotic forms

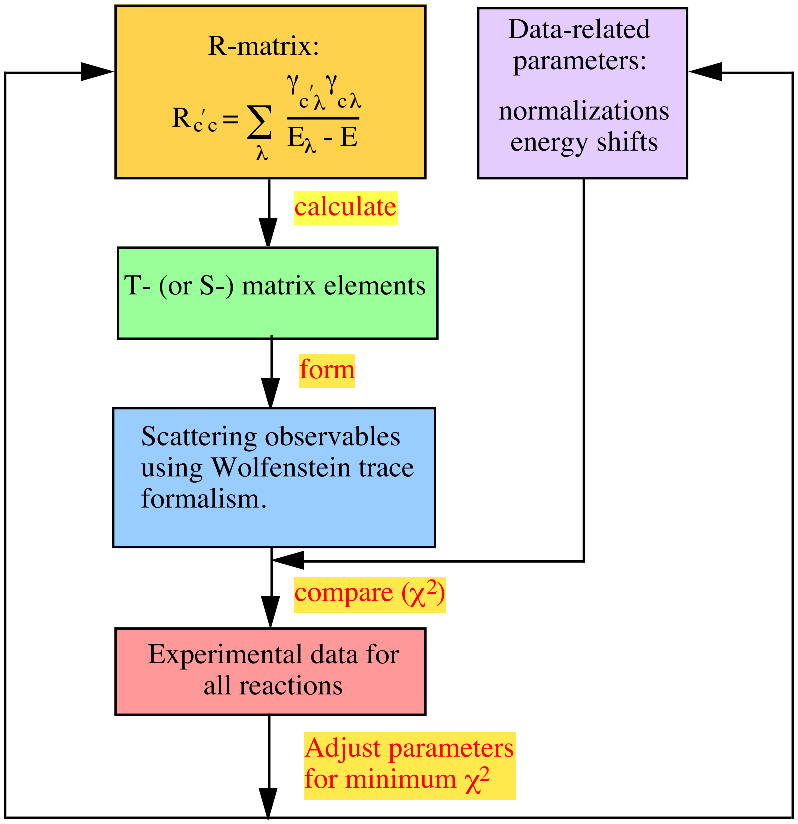
$$u^{j}_{ii}(\rho) = F^{(i)}_{j} + O^{(i)}_{j} t^{j}_{ii} \quad (i = e, m),$$

$$\text{with } O^{(m)}_{j} = h^{+}_{j}(\rho),\ O^{(e)}_{j} = -\partial_{\rho} h^{+}_{j}(\rho), \text{ and } F^{(i)}_{j} = \operatorname{Im} O^{(i)}_{j}.$$

In the usual approach, $O^{(e)}_{j} = O^{(m)}_{j} = h^{+}_{j}(\rho).$

Scheme and Properties of the EDA Code

Energy Dependent Analysis Code



- Accommodates general (spins, masses, charges) two-body channels
- Uses relativistic kinematics and R-matrix formulation
- Calculates general scattering observables for 2➔2 processes
- Has rather general data-handling capabilities (but not as general as, e.g., SAMMY)
- Uses modified variable-metric algorithm that gives parameter covariances at a solution

Uncertainties from Chi-Squared Minimization

$$\chi^2_{\mathrm{EDA}} = \sum_i \left[\frac{nX_i(\mathbf{p}) - R_i}{\Delta R_i}\right]^2 + \left[\frac{nS-1}{\Delta S/S}\right]^2 \quad \begin{cases} R_i,\ \Delta R_i = \text{relative measurement, uncertainty} \\ S,\ \Delta S = \text{experimental scale, uncertainty} \\ X_i(\mathbf{p}) = \text{observable calc. from res. pars. } \mathbf{p} \\ n = \text{normalization parameter} \end{cases}$$

Near a minimum of the chi-squared function at **p** = **p**$_0$,

$$\begin{aligned} \chi^2(\mathbf{p}) &= \chi_0^2 + (\mathbf{p}-\mathbf{p}_0)^{\mathrm{T}}\mathbf{g}_0 + \tfrac{1}{2}(\mathbf{p}-\mathbf{p}_0)^{\mathrm{T}}\mathbf{G}_0(\mathbf{p}-\mathbf{p}_0) \\ &= \chi_0^2 + \Delta\chi^2. \end{aligned} \qquad \begin{cases} \chi_0^2 = \chi^2(\mathbf{p}_0) \\ \mathbf{g}_0 = \nabla_{\mathbf{p}}\chi^2(\mathbf{p})\big|_{\mathbf{p}=\mathbf{p}_0} \approx 0 \\ \mathbf{G}_0 = \nabla_{\mathbf{p}}\mathbf{g}(\mathbf{p})\big|_{\mathbf{p}=\mathbf{p}_0} \end{cases}$$

The parameter covariance matrix is $\mathbf{C}_0 = 2\mathbf{G}_0^{-1}$, and so first-order error propagation gives for the cross-section covariances

$$\begin{aligned} \mathrm{cov}[\sigma_{\mathrm{i}}(E)\sigma_j(E')] &= \left[\nabla_{\mathbf{p}}\sigma_{\mathrm{i}}(E)\right]^{\mathrm{T}}\mathbf{C}_0\left[\nabla_{\mathbf{p}}\sigma_{\mathrm{j}}(E')\right]\Big|_{\mathbf{p}=\mathbf{p}_0} \\ &= \Delta\sigma_{\mathrm{i}}(E)\Delta\sigma_{\mathrm{j}}(E')\rho_{ij}(E,E'). \end{aligned}$$

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Parameter confidence intervals

It was proposed by Y. Avni [*Ap. J.* **210**, 642 (1976)] to define confidence intervals for the parameters of a fit by the condition

$$\Delta\chi^2 = \tfrac{1}{2}\Delta\mathbf{p}^{\mathrm{T}}\mathbf{G}_0\Delta\mathbf{p} \le \Delta\chi^2_{\max},$$

where $\Delta\chi^2_{\max}$ is chosen to give a particular confidence level (CL)

$$P(\Delta\chi^2|k) = \left[2^{\frac{k}{2}}\Gamma(\tfrac{k}{2})\right]^{-1}\int_0^{\Delta\chi^2_{\max}} t^{\frac{k}{2}-1}e^{-\frac{t}{2}}dt = \mathrm{CL}\ (\text{e.g.} \sim 0.68 \text{ for } 1\text{-}\sigma),\ 0.95 \text{ for } 2\text{-}\sigma, \text{etc.}$$

for a chi-squared distribution with *k* degrees of freedom. Many statistical analysis (not necessarily physical science) applications use this method to determine parameter uncertainties (usually with CL = 95%, or 2-σ). For CL = 68% (1-σ), $\Delta\chi^2_{\max} \approx k = \langle\Delta\chi^2\rangle$. This results in 1-σ parameter confidence intervals,*

$$\Delta p_i \le \sqrt{2\Delta\chi^2_{\max}H_{ii}} = \sqrt{\Delta\chi^2_{\max}C^0_{ii}} \approx \sqrt{kC^0_{ii}},$$

that are $\sim\sqrt{k}$ larger than the standard deviations (σ_{p_i}).

* when the remaining parameters are adjusted to obtain a new chi-square minimum

Analyses

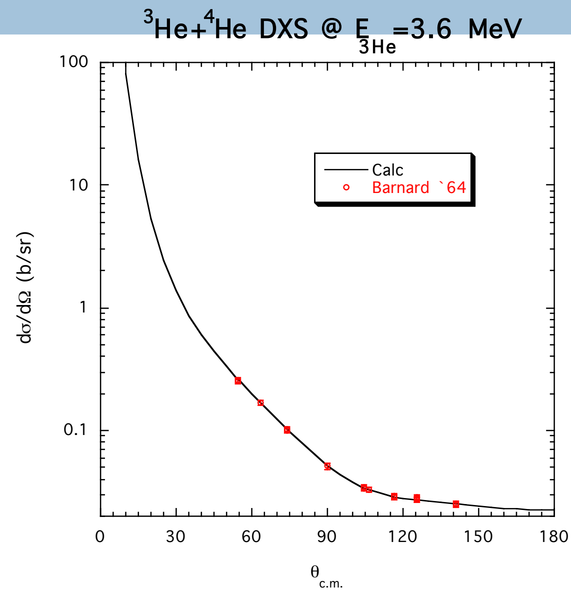
- ^{7}Be

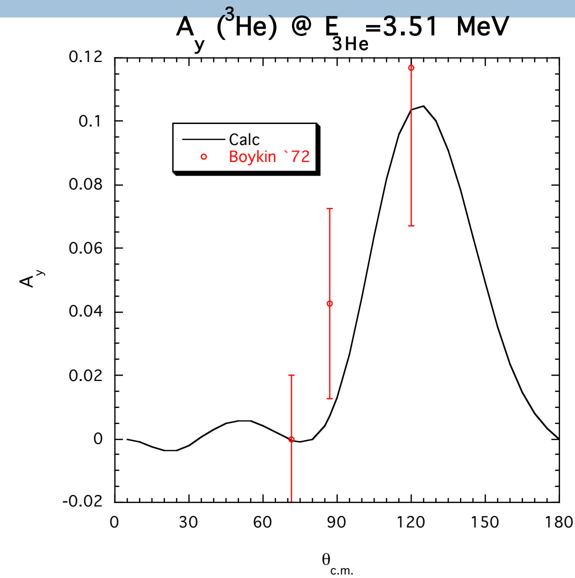
^{7}Be System Analysis

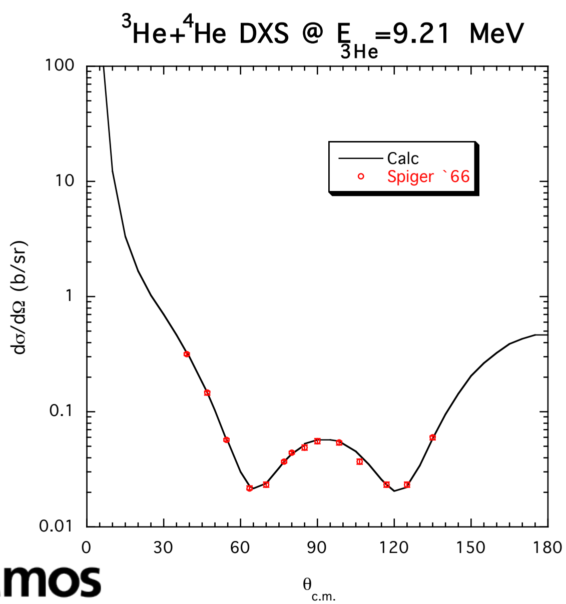
Channel	l_{max}	a_c (fm)
$^{3}He+^{4}He$	4	4.4
$p+^{6}Li$	1	3.1
$\gamma+^{7}Be$	1	50

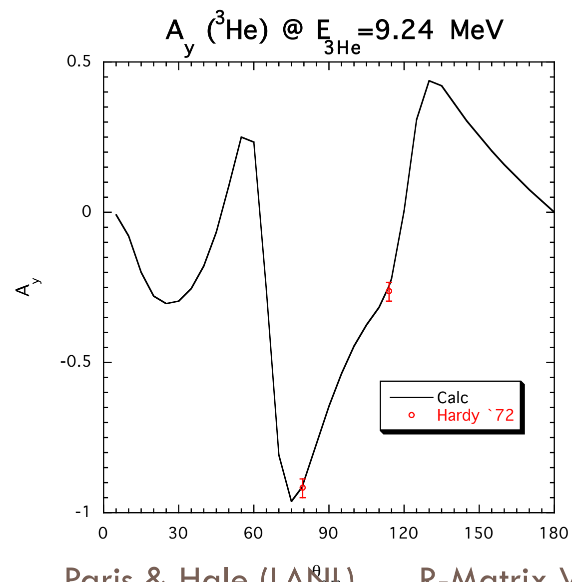
Reaction	Energy range (MeV)	# obs. types	# data points
$^{4}He(^{3}He,^{3}He)^{4}He$	E_{3He} = 1.7-10.8	2	1487
$^{4}He(^{3}He,p)^{6}Li$	E_{3He} = 8.2-10.8	1	130
$^{4}He(^{3}He,\gamma)^{7}Be$	E_{3He} = 0-2.2	1	40
$^{6}Li(p,\ ^{3}He)^{4}He$	E_p = 0-2.7	2	488
$^{6}Li(p,p)^{6}Li$	E_p = 1.2-2.5	1	187
$^{6}Li(p,\gamma)^{7}Be$	E_p = 0-1.2	1	28
Totals		8	2360

Example: $^3He+^4He$ Scattering

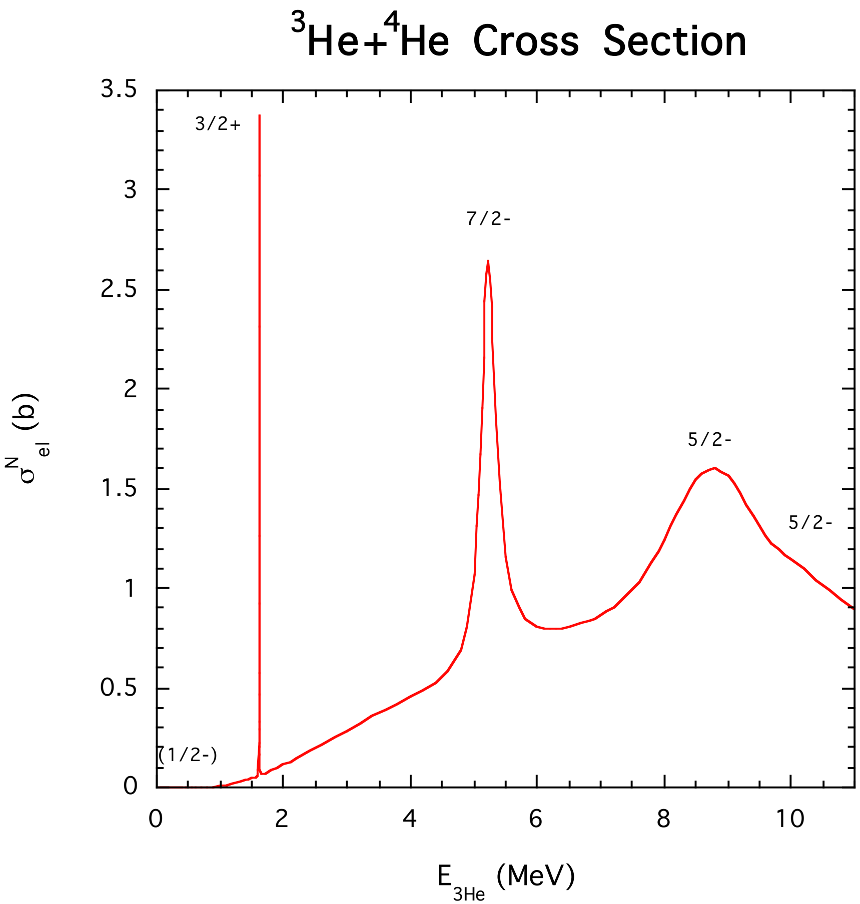


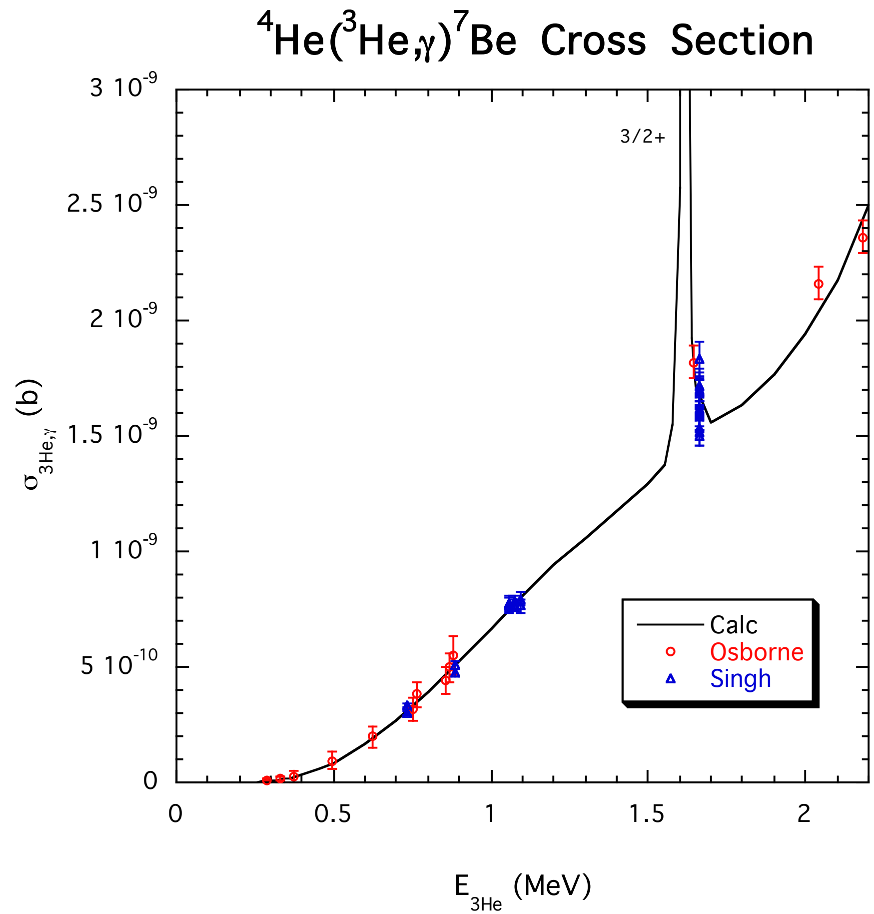






Resonances in the Cross Sections





^{17}O System Analysis

Channel	a_c (fm)	l_{max}
n+^{16}O	4.3	4
α+^{13}C	5.4	5

Reaction	Energies (MeV)	# data points	Data types
^{16}O(n,n)^{16}O	E_n = 0 – 7	2718	σ_T, $\sigma(\theta)$, $P_n(\theta)$
^{16}O(n,α)^{13}C	E_n = 2.35 – 5	850	σ_{int}, $\sigma(\theta)$, $A_n(\theta)$
^{13}C(α,n)^{16}O	E_α = 0 – 5.4	874	σ_{int}
^{13}C(α,α)^{13}C	E_α = 2 – 5.7	1296	$\sigma(\theta)$
total		5738	8

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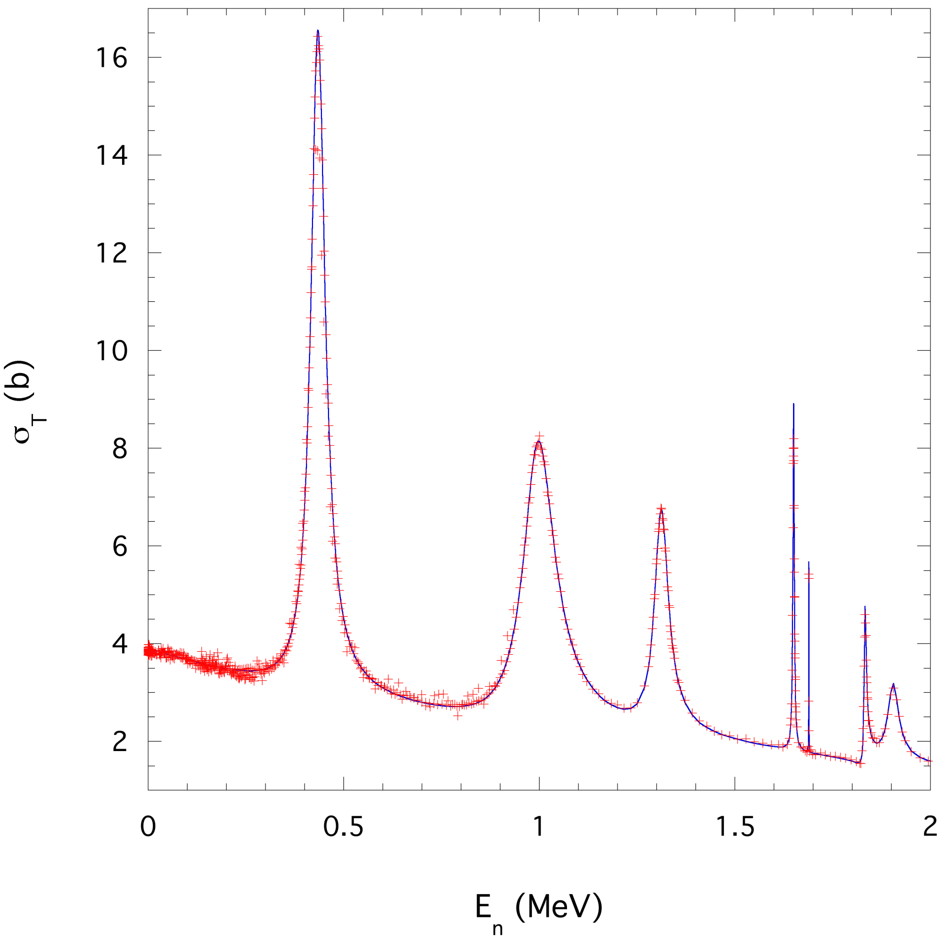
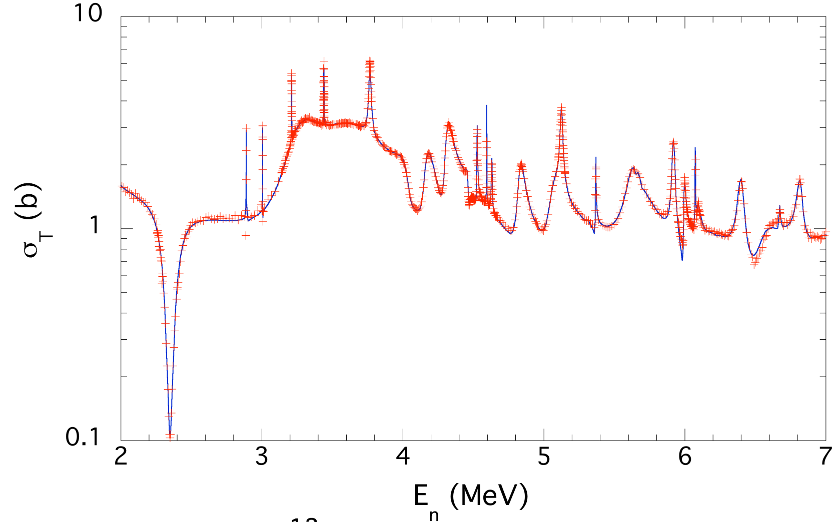
^{17}O System: comparison with data

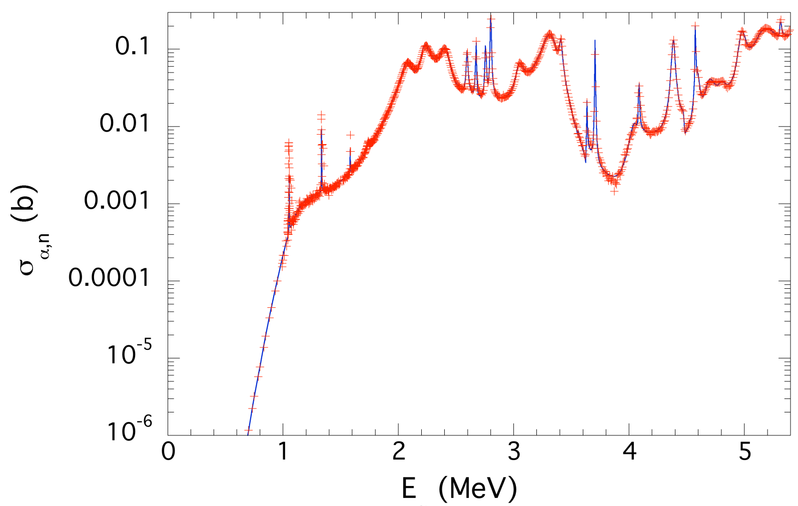
n+^{16}O Total Cross Section



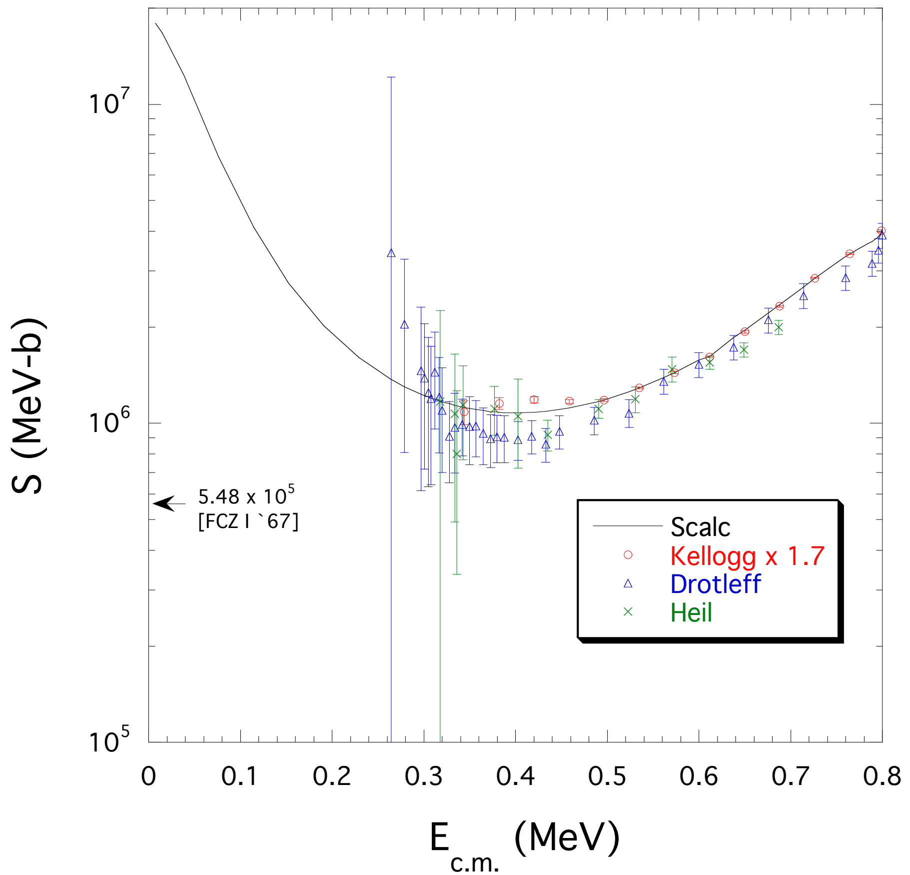
n+^{16}O Total Cross Section



$^{13}C(\alpha,n)$ Cross Section



^{17}O System: $^{13}C(a,n)^{16}O$ S-factor



Spectra

- 3-body final states are handled by the LANL auxiliary code SPECT

Motivation

- Study A=6 compound systems (R*)

R* → ^{6}He	^{6}Li	^{6}Be
$T(t,n)n\alpha$	$T(^3\text{He},p)n\alpha$	$^3\text{He}(^3\text{He},p)p\alpha$

- Stellar astrophysics: pp chain
- Cosmology: big bang nucleosynthesis '^{7}Li problem'
  - more neutrons from T(t,n)nα can destroy mass-7 via eg. ^{7}Be(n,p)
- Inertial confinement fusion
  - OMEGA & NIF facilities are being used to study reactions of light nuclei

Collaborators

LANL: Hale, Herrmann, Kim, McEvoy, Zylstra

OU: Brune

MIT: Frenje, Gatu-Johnson, Li, Petrasso

Rochester: Forrest, Knauer, Stoeckl

LLNL: Hohensee, McNabb, Pino, Sayre

Resonance model assumptions

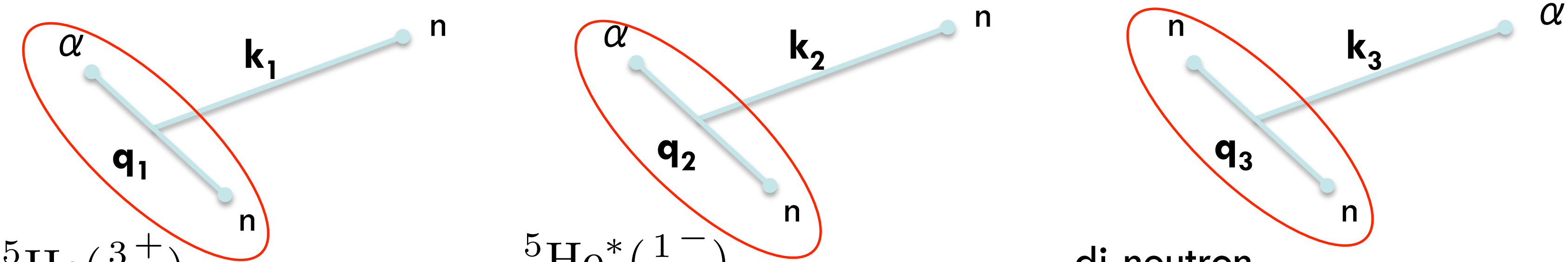
- Objective: use two-body data to describe three-body final states
- Approximate the three-body transition matrix as sum of sequential two-body decays

$$T^{(3)} = \sum_{i}^{N_c} c_i \tilde{T}_i^{(2)}$$

  - Limit $N_c \rightarrow \infty$ this is exact; we approximate ∞ by 3(!): 'Faddeev-like'
- Single level R-matrix description for $T_i^{(2)}$
- Interference between configurations ignored

Resonance model for 3-body final state (I)

- Configurations (T(t,n)n α)



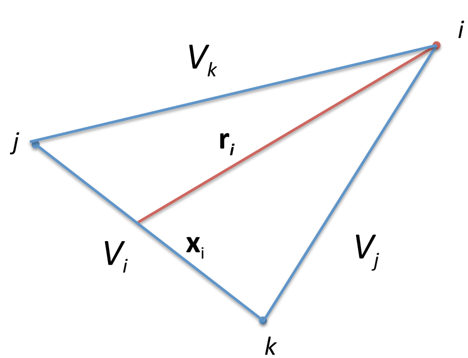
$^5\mathrm{He}(\frac{3}{2}^+)$ $^5\mathrm{He}^*(\frac{1}{2}^-)$ di-neutron

$$\frac{d^3\sigma}{d\mathbf{k}_1'} = \frac{(2\pi)^4}{v_{\rm rel}} \int d^3k_2' d^3k_3' \left| T_{fi}^{(3)} \right|^2 \delta(\epsilon_i + Q - \epsilon_f)$$

$$T_{fi}^{(3)} = -\pi \langle \chi_f^{(3)} | \sum_i V_i | \psi_{\mathbf{k}_0}^{(+)} \rangle = -\pi \sum_i \langle \phi_{\mathbf{q}_i}^{(-)} \chi_{\mathbf{k}_i} | \hat{T}_i | \psi_{\mathbf{k}_0}^{(+)} \rangle$$

$$\phi^{(\pm)}(\mathbf{q}_i) = \chi_{\mathbf{q}_i} + G_{0,i}^{(\pm)}(\epsilon_{\mathbf{q}_i}) V_i \phi^{(\pm)}(\mathbf{q}_i)$$

$$\hat{T}_i = V_i + V_i G_{0i}^+ V_i$$



'Faddeev-like': two-body subsystems are not bound

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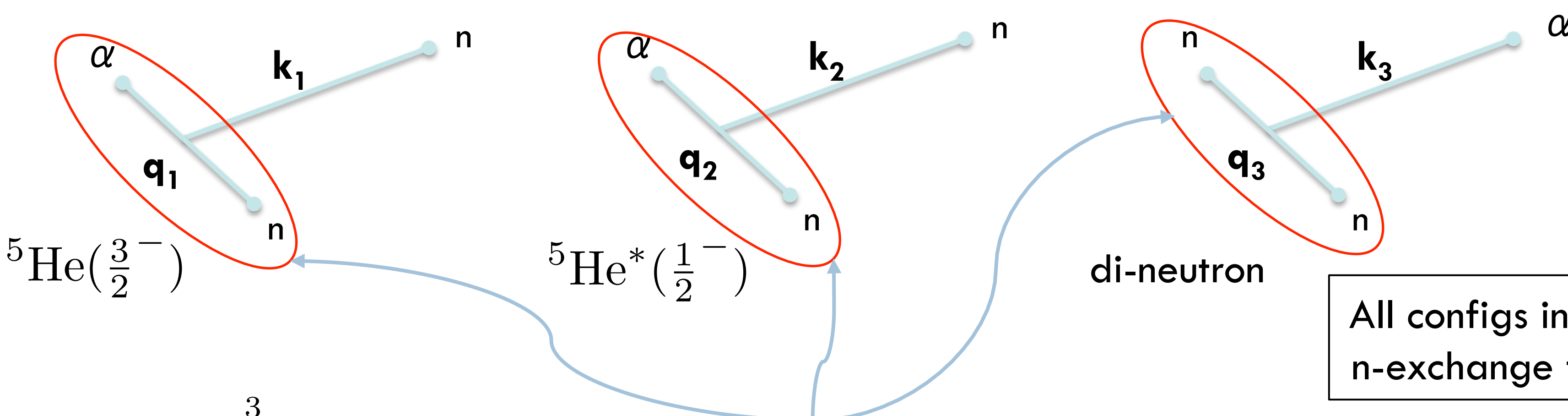
Off-shell distorted wave

Resonance model for 3-body final state (II)

- Configurations (T(t,n)n α)



All configs include n-exchange terms

$$T^{(3)}_{fi} = -\pi \sum_{i=1}^{3} \langle \phi^{(-)}_{\mathbf{q}_i} \chi_{\mathbf{k}_i} | \hat{T}_i | \psi^{(+)}_{\mathbf{k}_0} \rangle$$

$$= \sum_{i=1}^{3} \sum_{\lambda_i} c^{(i)}_{\lambda_i}(\mathbf{q}_i) \tilde{T}^{(2)}_{\lambda_i \mathbf{k}_i \mathbf{k}_0}$$

$$\tilde{T}^{(2)}_{\lambda \mathbf{k} \mathbf{k}_0} = \sum_{Js'\ell' s\ell} \mathcal{Y}_{Js'\ell'}(\hat{\mathbf{k}})\, O^{-1}_{\ell'}(k)$$

$$\times R^{\overline{L}}_{\lambda s'\ell' s\ell}(\epsilon) O^{-1}_{\ell'}(k)\, \mathcal{Y}_{Js'\ell'}(\hat{\mathbf{k}}_0)$$

$$\phi^{(-)}_{\mathbf{q}} = \sum_{\lambda} c^{(i)}_{\lambda}(\mathbf{q}) \phi^{(i)}_{\lambda}(\mathbf{x})$$

$$\mathscr{L}_a |\phi^{(\pm)}\rangle = (H + \mathscr{L}_a - E) |\phi^{(\pm)}\rangle$$

$$c^{(i)}_{\lambda}(\mathbf{q}) = \frac{1}{\sqrt{\pi}} \sum_{\lambda'} A_{\lambda\lambda'} \gamma_{\lambda' i} \mathcal{O}^{-1}_{i}(\mathbf{q}) \mathcal{Y}^{j\pi *}_{i}(\hat{\mathbf{q}})$$

$$\sim \frac{1}{E_R - \frac{i}{2}\Gamma_R - \epsilon_q} e^{-i\phi_{\tilde{\ell}}} Y^0_{\tilde{\ell}}(\hat{\mathbf{q}})$$

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Data fitting

- Livermore '65 accelerator T(t,n)nα
  - Recommended resolution broadening 0.5 MeV FWHM (determined from 14 MeV n peak)
  - Difficult to get realistic fit with 0.5 MeV at large E_n; used 0.250 MeV & more narrow 'extended' R-matrix parameters
- OMEGA ^{3}He^{3}He data (courtesy A. Zylstra, LANL)
  - Scaled data (arbitrarily) to theory
    - should use total cross section
  - Gaussian thermal broadening 0.230 MeV
  - neglects small instrument broadening
- Least squares fit
  - parameters (next slide) $R_L^{(i)}$

Model parameters

- $T(t,n)n\alpha$

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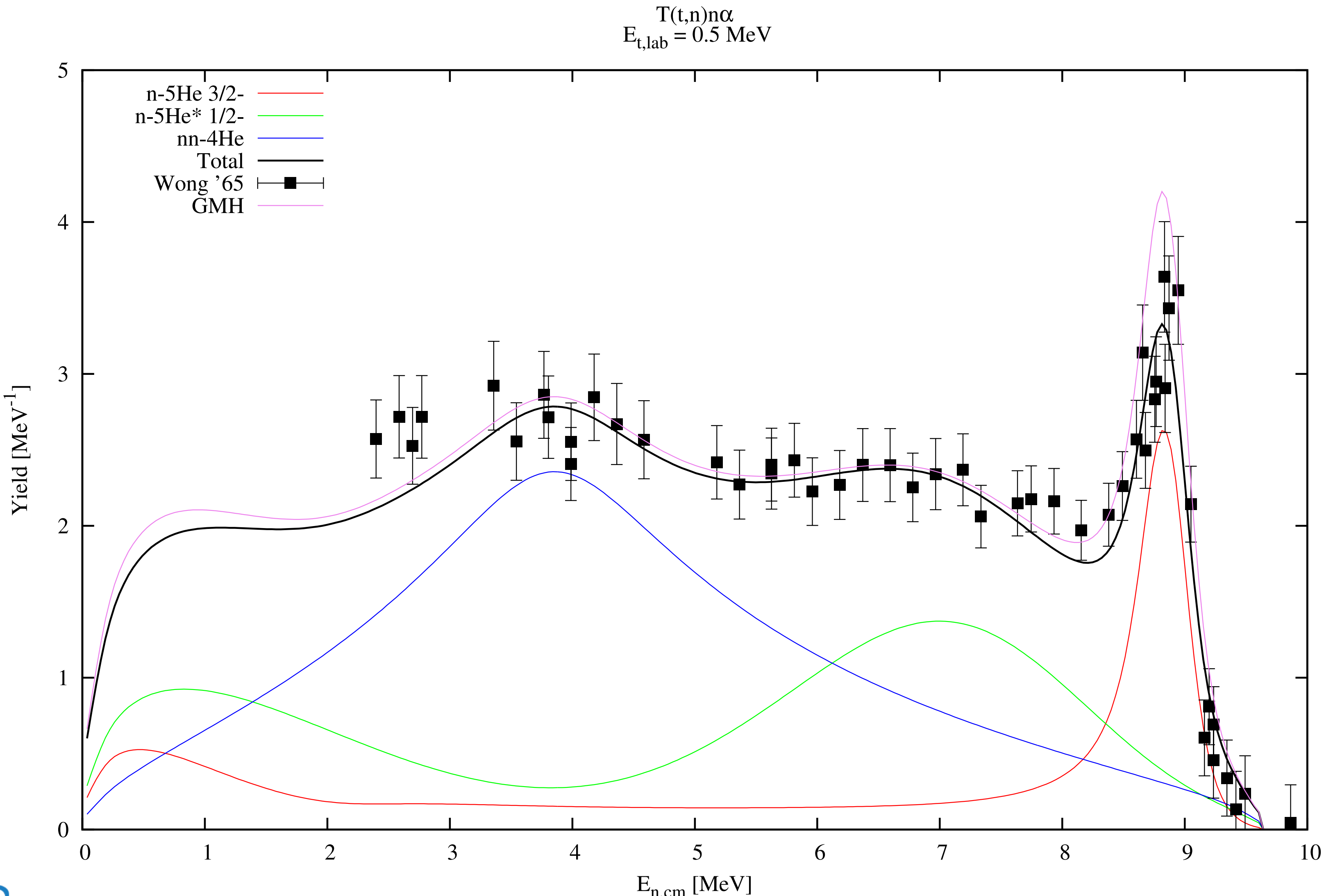
Channel (resonance)	J^π	E_R (MeV)	Γ (MeV)	$\lvert R^L \rvert$
^5He+n	$3/2^-$	0.99	0.65	0.157
$^5He^*+n$	$1/2^-$	6.66	20.6	0.547
$^4He+(nn)$	0^+	-0.07	0.	0.338

- $^3He(^3He,p)p\alpha$

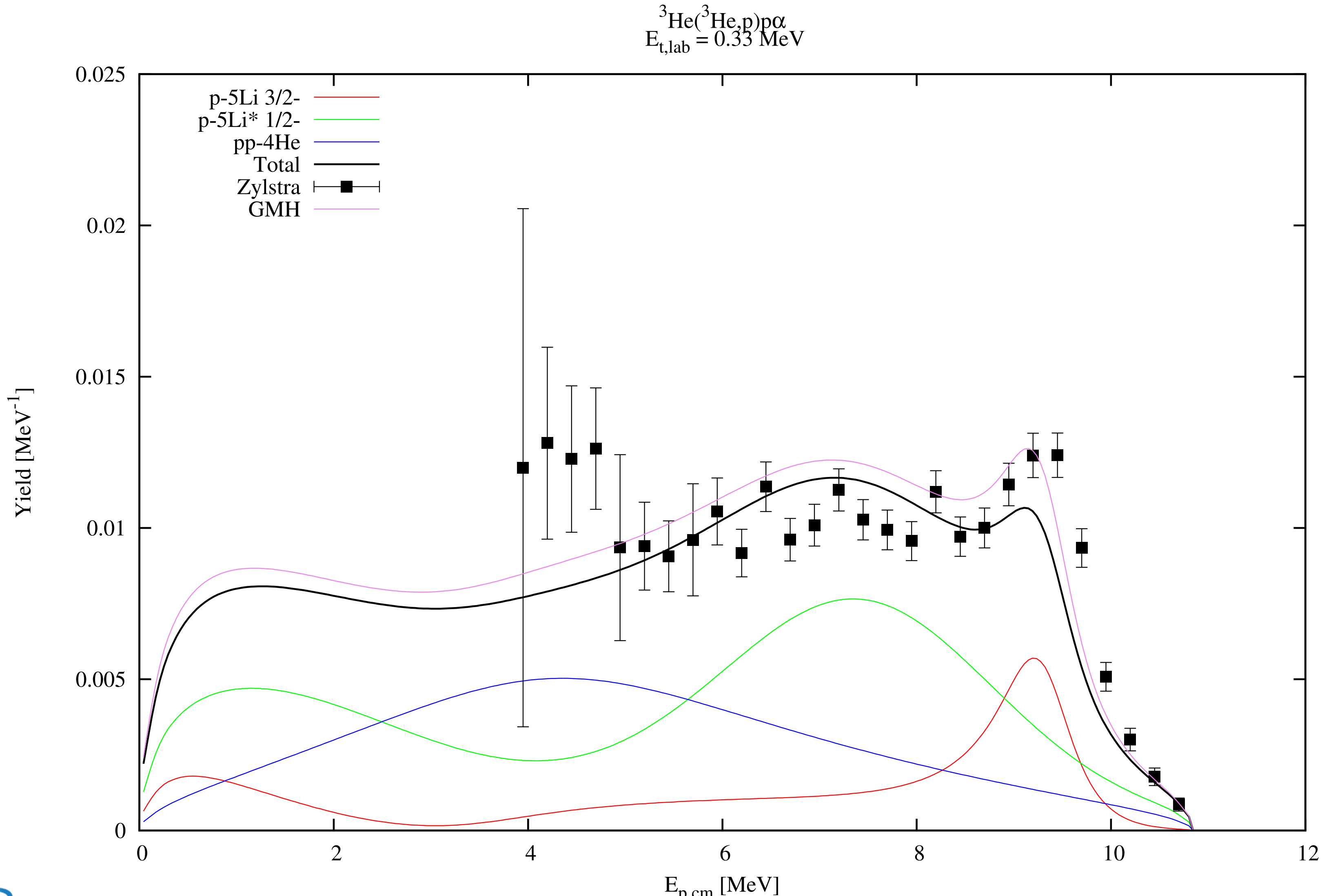
Fixed by 2-body data

Channel (resonance)	J^π	E_R (MeV)	Γ (MeV)	$\lvert R^L \rvert$
^5Li+p	$3/2^-$	2.08	2.11	0.157
$^5Li^*+p$	$1/2^-$	8.26	19.8	0.547
$^4He+(pp)$	0^+	1.56	0.	0.234

$T(t,n)n\alpha$


T(t,n)nα
E_t,lab = 0.5 MeV
n-5He 3/2-
n-5He* 1/2-
nn-4He
Total
Wong '65
GMH
Yield [MeV^-1]
E_n,cm [MeV]
0
1
2
3
4
5
6
7
8
9
10

$^{3}\mathrm{He}(^{3}\mathrm{He}, p)p\alpha$



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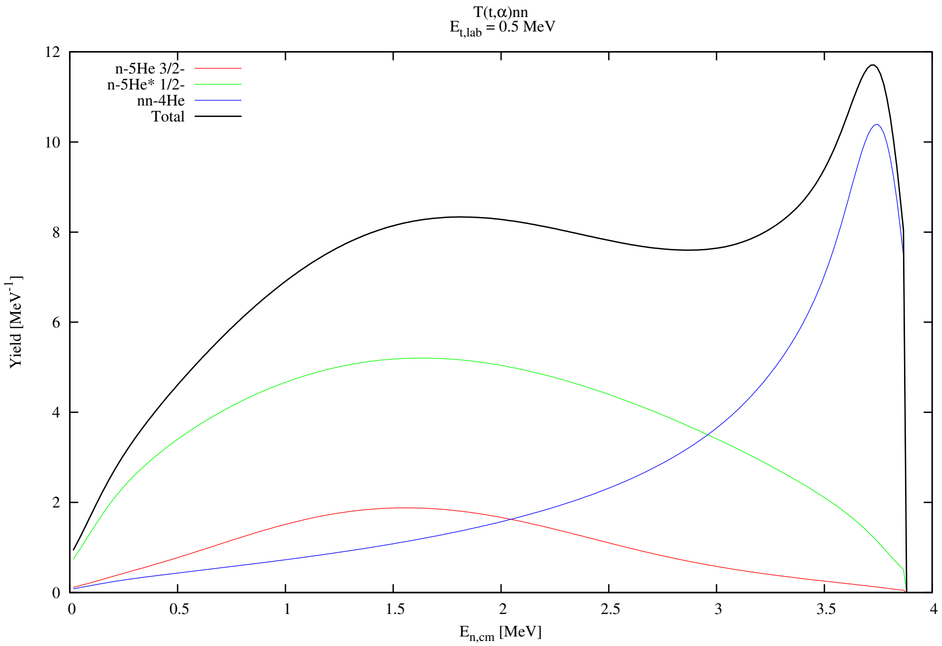
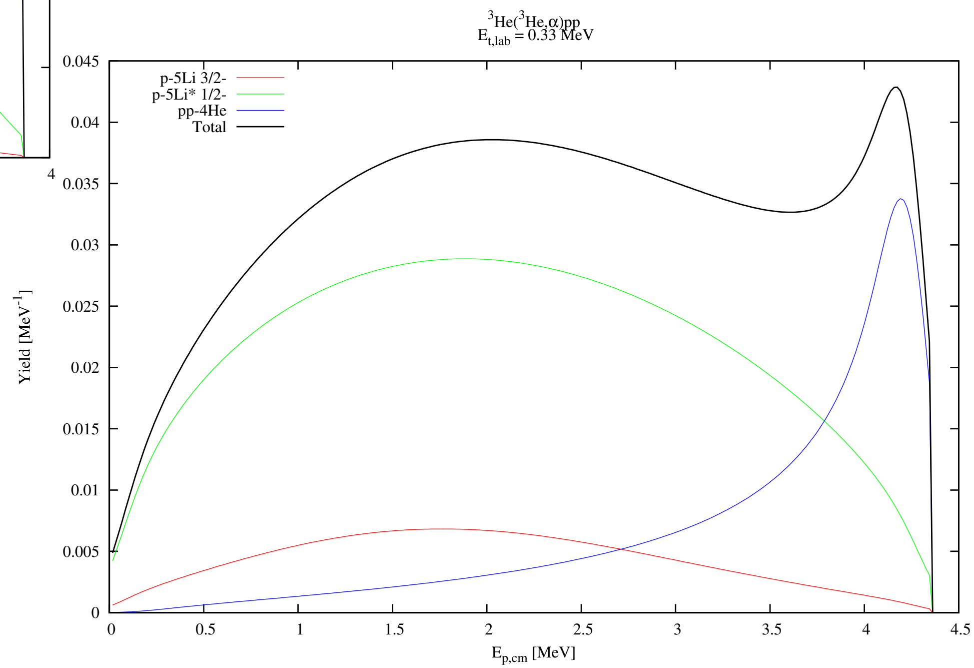
Alpha spectra



T(t, α)nn

^{3}He(^{3}He, α)pp



Planned improvements

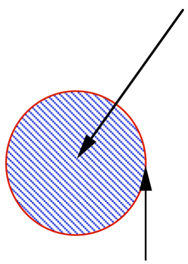
- Data
  - Incorporate all OMEGA ^{3}He^{3}He & ^{3}HeT
  - Include NIF TT
  - α spectra
- Fitting
  - Non-linear least squares
    - Marquardt-Levenberg; gradient; etc.
- Model
  - Angular dependence nn wave function
  - Incident energy dependence

Follow-on material

2→2 body R matrix formalism

INTERIOR (Many-Body) REGION
(Microscopic Calculations)

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)



$\underbrace{H + \mathcal{L}_B}$ compact, hermitian operator with real, discrete spectrum; eigenfunctions in Hilbert space

$$|\psi^+) = (H + \mathcal{L}_B - E)^{-1}\mathcal{L}_B|\psi^+)$$

SURFACE

$$\mathcal{L}_B = \sum_c |c)(c| \left(\frac{\partial}{\partial r_c} r_c - B_c\right),$$

$$(\mathbf{r}_c|c) = \frac{\hbar}{\sqrt{2\mu_c a_c}}\frac{\delta(r_c - a_c)}{r_c}[(\phi_{s_1}^{\mu_1} \otimes \phi_{s_2}^{\mu_2})_s^{\mu} \otimes Y_l^m(\hat{\mathbf{r}}_c)]_J^M$$

$$R_{c'c} = (c'|(H + \mathcal{L}_B - E)^{-1}|c) = \sum_\lambda \frac{(c'|\lambda)(\lambda|c)}{E_\lambda - E}$$

$$(r_{c'}|\psi_c^+\rangle = -I_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})S_{c'c}$$

or equivalently,

$$(r_{c'}|\psi_c^+\rangle = F_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})T_{c'c}$$

Measurements

- Channels: $r_c = a_c$
  - not nec. nucl. surface; regulator
  - $\exists F : R_B(a_c) \xrightarrow{F} R_{B'}(a'_c)$

    and $T \xrightarrow{F} T$
- Sturm-Liouville eigenvalue prob.
  - BC at *finite* a_c
  - complete set $|\lambda\rangle$
- Hermiticity: $(H + \mathcal{L}_B)^\dagger = H + \mathcal{L}_B$
  - Bloch operator

$$\mathcal{L}_{BC}\psi_c^{(+)} = 0 \Leftrightarrow \frac{1}{r_c\psi_c^{(+)}}\frac{\partial}{\partial r_c} r_c\psi_c^{(+)}\Big|_{a_c} = B_c$$

- T matrix

$$T = O^{-1}R_L O^{-1} - O^{-1}F$$

$$B_c = L_c = \frac{r_c}{O_c}\frac{\partial O_c}{\partial r_c}\Big|_{r_c = a_c}$$

$$R_B^{-1} + B = R_{B'}^{-1} + B'$$

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Analyses

- ^{5}He

R-matrix analysis of ^{5}He system

^{5}He Analysis

Channel	lmax	ac (fm)
d-t	5	5.1
n-^{4}He	5	3.0
n-^{4}He*	1	5.0
γ-^{5}He	1	60.

Reaction	Energy Range	# Obs. Types	# Data Points	χ^2
T(d,d)T	E_d=0-8.6 MeV	6	700	1231
T(d,n)^{4}He	E_d=0-11 MeV	14	1185	1523
T(d,n)^{4}He*	E_d=4.8-8.3 MeV	1	10	18
T(d,γ)^{5}He	E_d=0-8.6 MeV	2	17	29
^{4}He(n,n)^{4}He	E_n=0-28 MeV	2	817	1117
Totals		25	2729	3918

parameters = 117 $\Rightarrow$ χ^2 per degree of freedom = 1.50

[109 phase parameters are necessary to describe the S matrix at a single energy]

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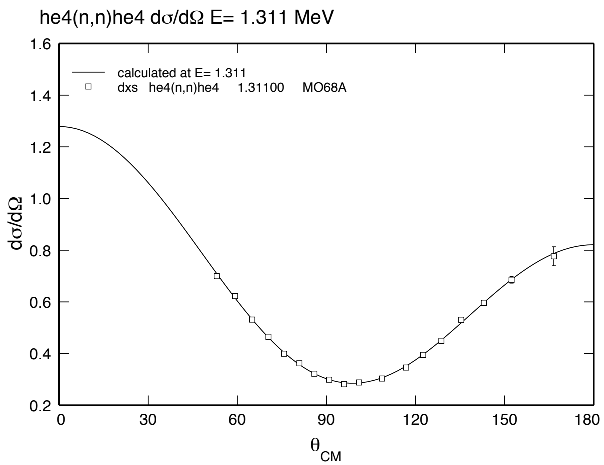
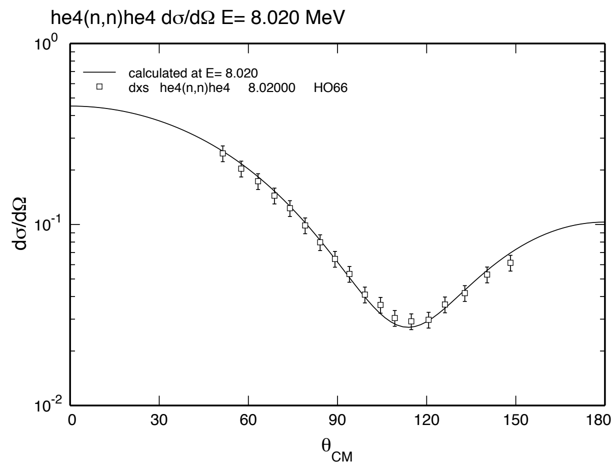
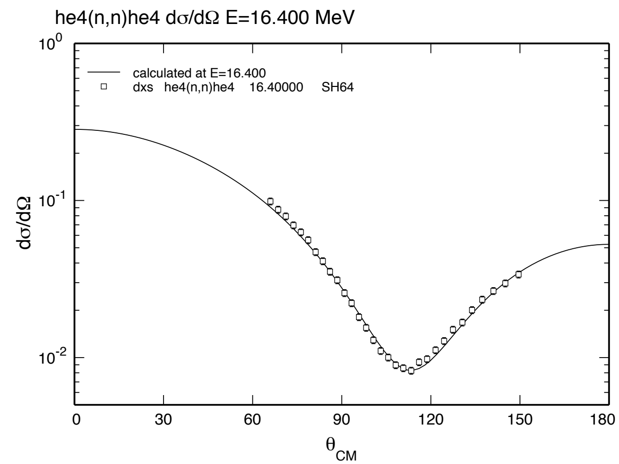
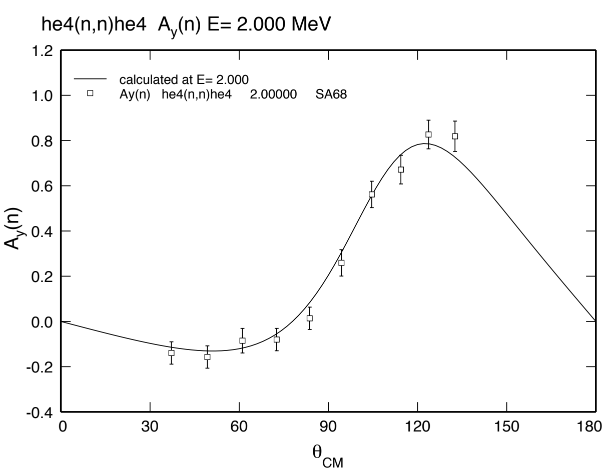
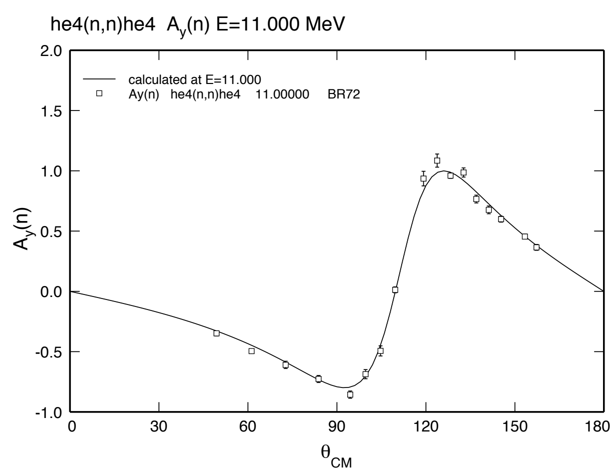
^{4}He(n,n)^{4}He differential cross section

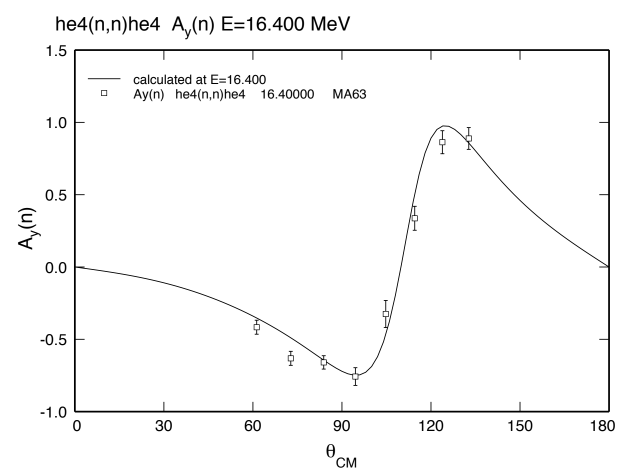












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Integrated cross section