Quantum Structure of HEFT

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HEFT 2015 - Higgs Effective Field Theory

SISSA, Trieste

Work in collaboration with

J. R. Espinosa

A. Pomarol

based on

arXiv: 1412.7151

For closely related works

Alonso, Jenkins, Manohar 1409.0869

Cheung, Shen 1505.01844

Purpose of the talk:

Explain some surprising patterns of the quantum effects in the Higgs Effective Field theory (d=6, concretely).

This is interesting because operators mix, hence:

- Observables are related. One can learn about poorly measured quantities.

- If deviation are seen, it will be crucial in the future to unravel the UV model.

HEFT

Assuming a scale of new physics greater than M_W,
the SM EFT (SM + higher dimension operators) captures the dominant effect of possible BSM physics.

The scales Λ_B and Λ_L are large, dominant effects come from d=6 operators

$$\mathcal{O}\left(\frac{v^2}{\Lambda^2}, \frac{p^2}{\Lambda^2}\right)$$

$$\mathcal{L}_{\text{eff}} = \frac{\Lambda^4}{g_*^2}\mathcal{L}\left(\frac{D_\mu}{\Lambda}\,,\,\frac{g_H H}{\Lambda}\,,\,\frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}\,,\,\frac{g F_{\mu\nu}}{\Lambda^2}\right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

Operator mixing in the EFT

Λ

RGE

M_W

$$c_1 \mathcal{O}_1 \qquad\qquad c_2 \mathcal{O}_2$$

$$c_1\left(m_W\right) \sim \frac{c_2}{16\pi^2} \log\left(\frac{\Lambda}{m_W}\right)$$

log-enhancement

$$\log(3 \text{ TeV}/m_W)^2 \sim 7$$

This is very interesting:

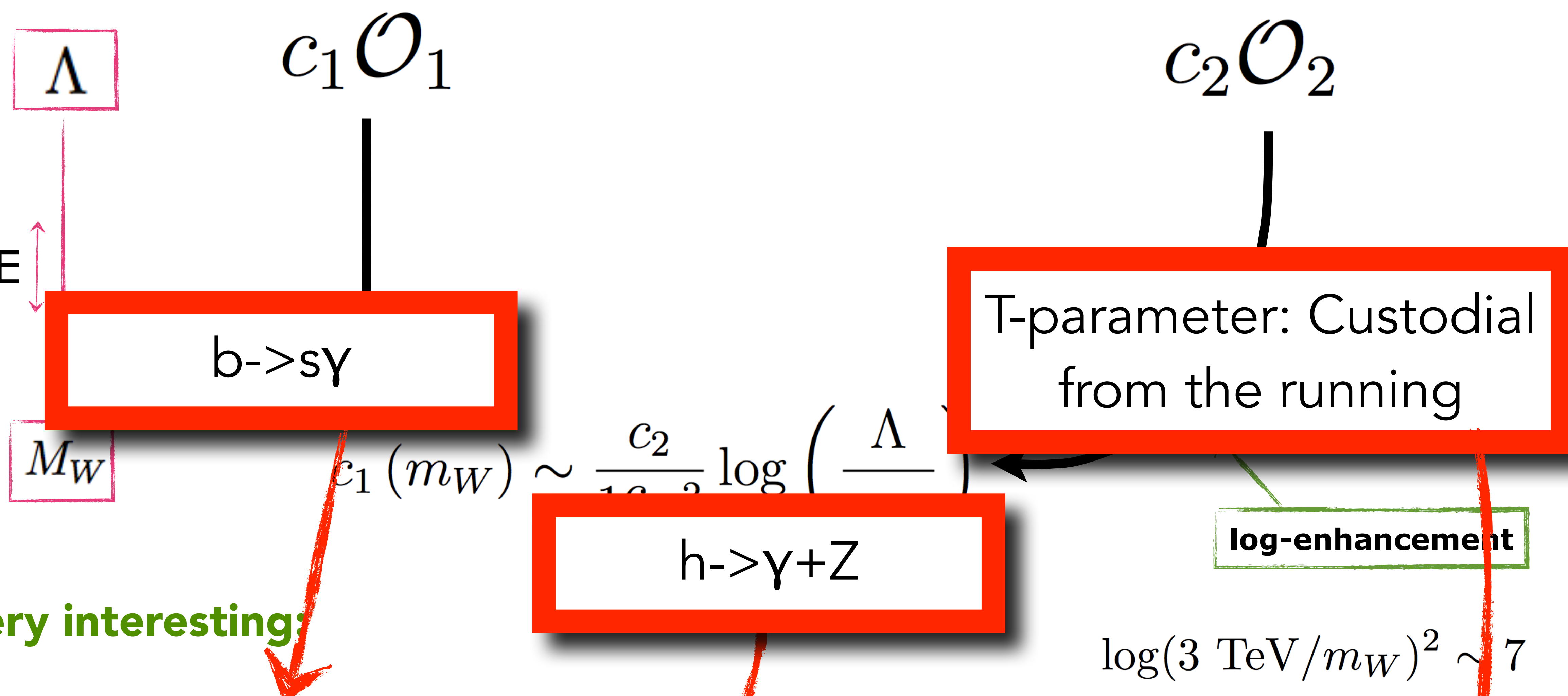
☞ possible big deviations, O(10%)!

☞ we can learn about observables that are otherwise poorly measured.

☞ possible deviations can be ascribed to operators that are not generated otherwise.

☞ A tree-level induced operator could be the leading contribution to a loop-suppressed SM process.

Operator mixing in the EFT



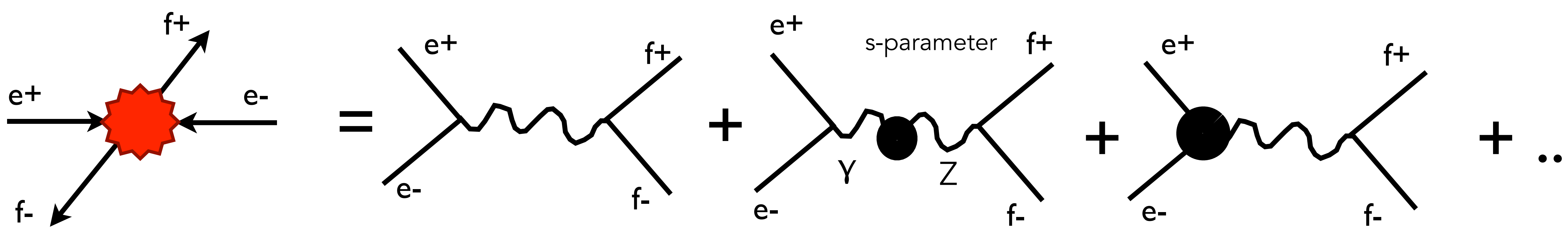
This is very interesting:

- possible big deviations, O(10%)!
- we can learn about observables that are otherwise poorly measured.
- possible deviations can be ascribed to operators that are not generated otherwise.
- A tree-level induced operator could be the leading contribution to a loop-suppressed SM process.

$W_{\mu\nu}^3$, dipoles, h->γγ

Example 1

Mixing between the Z-boson and the photon was very well measured (per-mille, LEP).

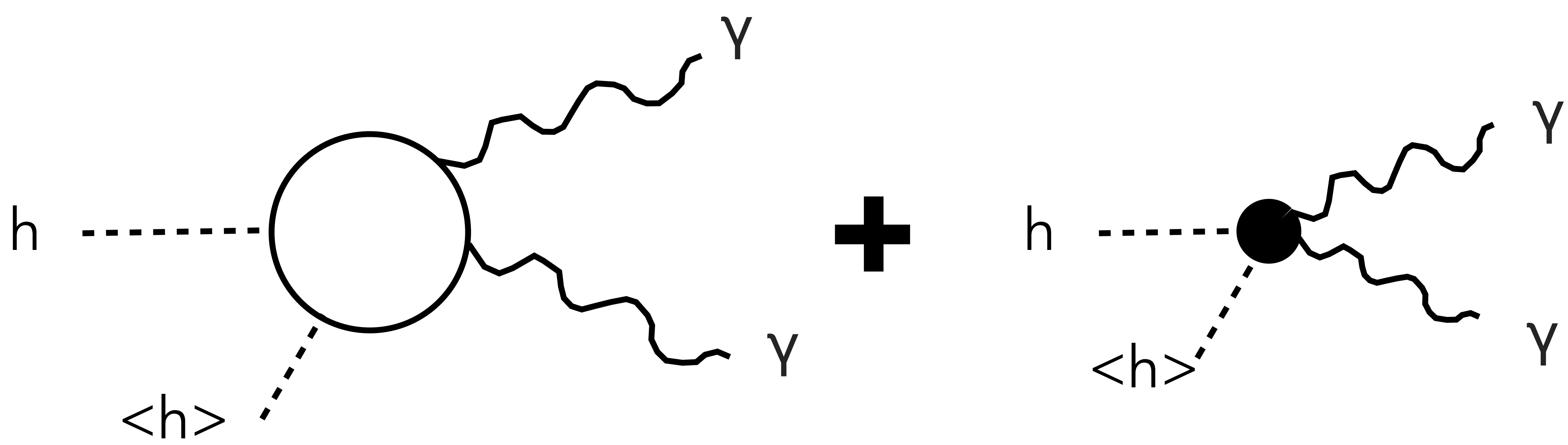


Precision measurements of SM phenomena are interpreted as limits on the scale suppressing higher dimensional operators.

$$s\, Z_{\mu\nu}A^{\mu\nu} \sim \frac{c_B}{\Lambda^2}(H^\dagger \overleftrightarrow{D}_\mu H)\partial_\nu B^{\mu\nu} + \frac{c_W}{\Lambda^2}(H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D_\nu W^{a\mu\nu} + \frac{c_{WB}}{\Lambda^2} H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu}$$

Example 1

h->γγ, clean at ATLAS/CMS.



The **loop** of SM particles + a point like interaction.
Dominant contribution from the **top-quark** and the **massive gauge bosons**.
Again, the measurement can be interpreted as limits on the operators

$$\delta\mathcal{L} = \frac{c_{BB}}{\Lambda^2}|H|^2 B_{\mu\nu}^2 + \frac{c_{WW}}{\Lambda^2}|H|^2 W_{\mu\nu}^{a\ 2}$$

Example 1

$$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu} \qquad \mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_{BB} = |H|^2 B^2_{\mu\nu} \qquad \mathcal{O}_{WB} = H^\dagger \sigma^a H B^{\mu\nu} W^a_{\mu\nu} \qquad \mathcal{O}_{WW} = |H|^2 W^a_{\mu\nu}{}^2$$

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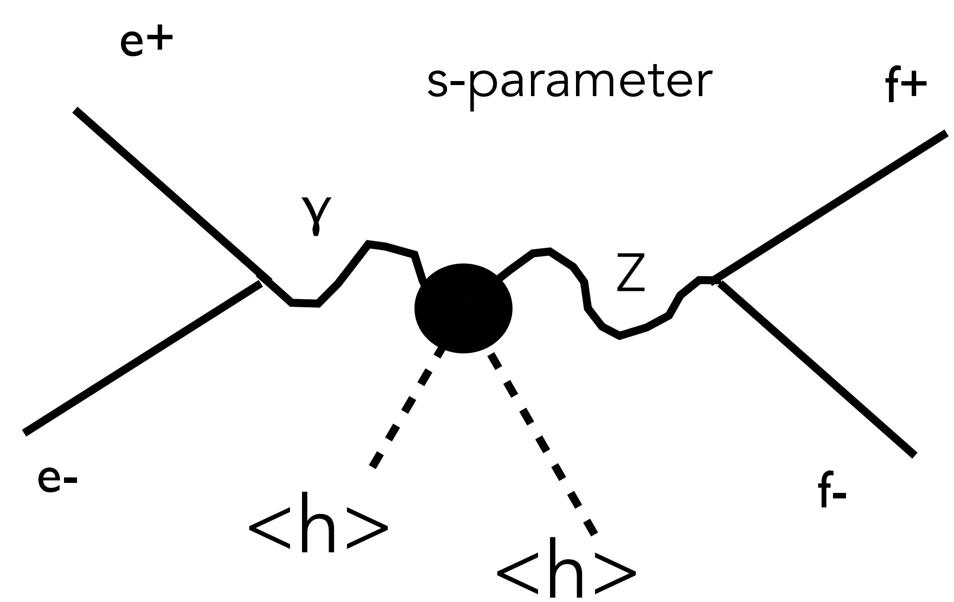

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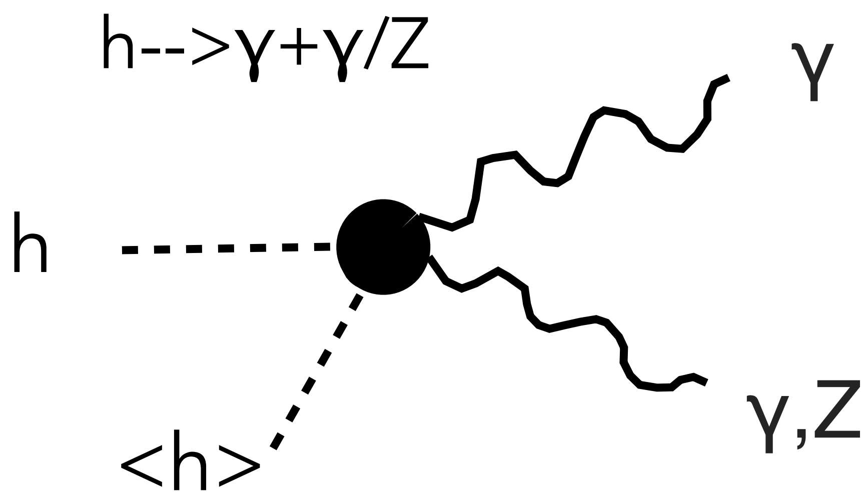

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We want to go one step further, and look for quantum effects on these operators, i.e. how do they mix under the RG flow.

Example 1

$$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu} \qquad \mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu}$$

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Block diagonal :-0!

$$\begin{pmatrix} \beta_{c_{BB}} \\ \beta_{c_{WW}} \\ \beta_{c_{WB}} \\ \beta_{c_B} \\ \beta_{c_W} \end{pmatrix} = \left(\begin{array}{ccc|cc} \bullet & \bullet & \bullet & \multicolumn{2}{c}{\multirow{3}{*}{$0_{3\times 2}$}} \\ \bullet & \bullet & \bullet & & \\ \bullet & \bullet & \bullet & & \\ \hline \multicolumn{3}{c|}{\multirow{2}{*}{$0_{2\times 3}$}} & \bullet & \bullet \\ & & & \bullet & \bullet \end{array}\right) \begin{pmatrix} c_{BB} \\ c_{WW} \\ c_{WB} \\ c_B \\ c_W \end{pmatrix} + \mathcal{O}(\hbar^2)$$

Example 2

SM after integrating out the W/Z bosons:

one-loop induced

$$\mathcal{O}_{loop} = \bar{s}_L \sigma^{\mu\mu} b_R F_{\mu\nu}$$

tree-level induced

$$\mathcal{O}_{tree} = (\bar{c}_L \gamma^\mu b_L)(\bar{s}_L \gamma^\mu c_L)$$

RGE

?

$$c_{\text{dipole}}(\mu) \sim \frac{c_{\text{4-fermion}}}{16\pi^2} \log\left(\frac{\mu}{m_W}\right)$$

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RGE

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c_{dipole} ... $\left(\frac{\mu}{m_W}\right)$

Explicit calculations showed no one-loop mixing!

Grinstein, Springer, Wise 90'

Example 3

Any renormalizable BSM, e.g. MSSM

one-loop induced

$$\mathcal{O}_{loop} = |H|^2 B^{\mu\nu} B_{\mu\nu}$$

tree-level induced

$$\mathcal{O}_{tree} = (\partial_\mu |H|^2)^2$$

RGE

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$$c_{\rm loop}(\mu) \sim \frac{c_{\rm tree}}{16\pi^2} \log\left(\frac{\mu}{m_W}\right)$$

Hagiwara, Ishihara, Szalapski, Zeppenfeld 93' (in an other basis)

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$$c_{\rm loop} \left(\cdots \frac{\mu}{m_W} \right)$$

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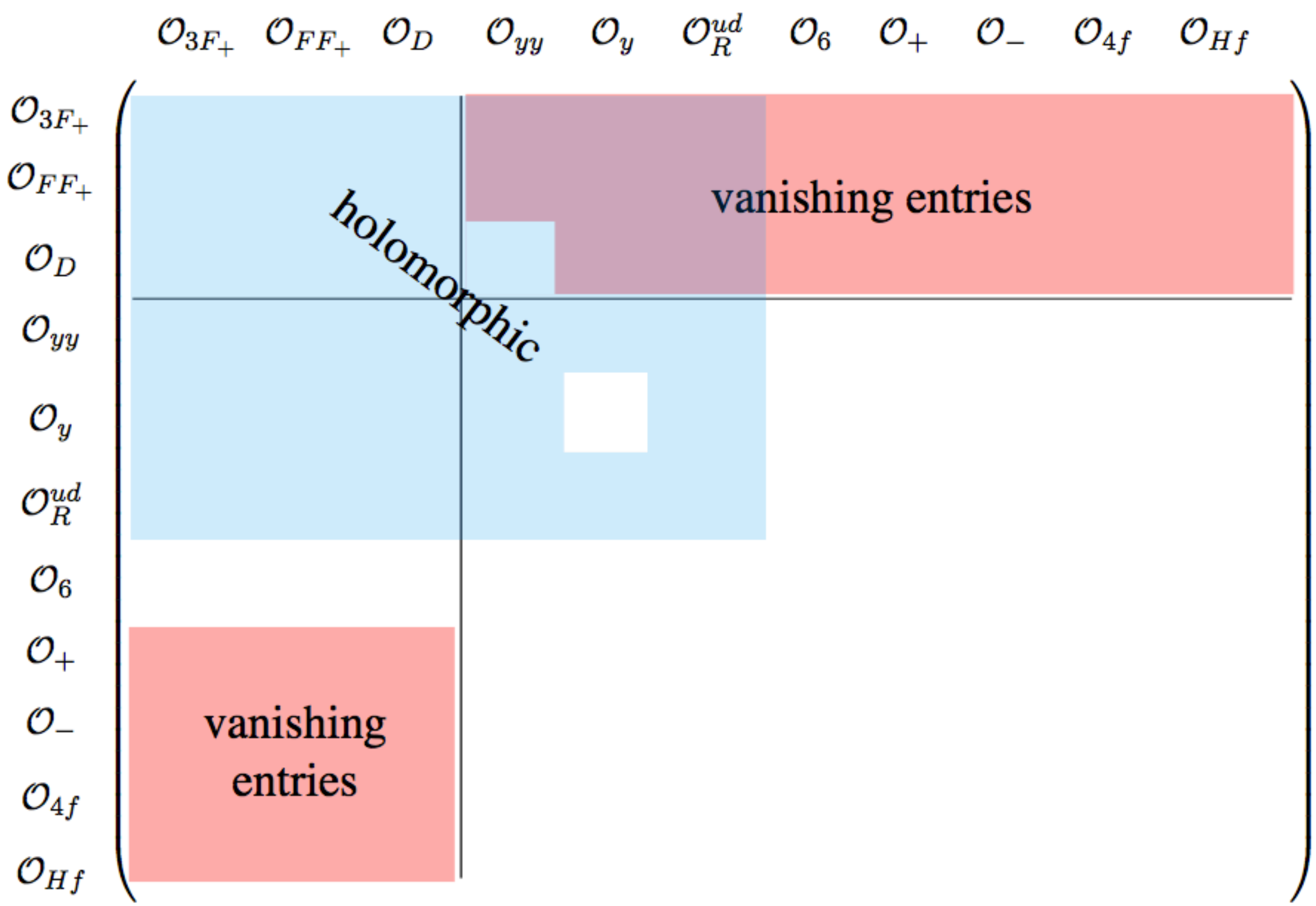
Pattern of zeroes in the one-loop anomalous dimension matrix.



explicit calculations were done in:

Jenkins, Manohar and Trott: **1308.2627**, **1312.2014**, **1310.4838** +Alonso **1312.2014**

Grojean, Jenkins, Manohar and Trott: **1301.2588**

EM, Espinosa, Pomarol and Masso: **1308.1879**, **1302.5661**

EM, Marzocca, Grojean and Gupta: **1312.2928**

see also:
C. Cheung and C-H. Shen: **1505.01844**

Patterns of operator mixing

"Loop" operators

Arise at one-loop
in renormalizable BSMs

$H^\dagger \bar{f}_R \sigma^{\mu\nu} t^a f_L \, F^a_{\mu\nu}$ $\longrightarrow$ **fermion dipoles**

$H^\dagger t^a t^b H F^a_{\mu\nu} \; F^{b\,\mu\nu}$ $\longrightarrow$ **hγγ, hZγ,hGG**

$f^{abc} F^{a\,\nu}_{\mu} F^{b\,\rho}_{\nu} \; F^{c\,\mu}_{\rho}$ $\longrightarrow$ **TGC**

+CP-violating

Patterns of operator mixing

"Loop" operators

$$H^\dagger \bar{f}_R \sigma^{\mu\nu} t^a f_L \, F^a_{\mu\nu}$$

$$H^\dagger t^a t^b H F^a_{\mu\nu} \;\; F^{b\,\mu\nu}$$

$$f^{abc} F^{a\,\nu}_{\mu} F^{b\,\rho}_{\nu} \;\; F^{c\,\mu}_{\rho}$$

+CP-violating

"Current-current " operators

$$J_i \cdot J_j$$

$$J^{a\,\mu}_H = H^\dagger t^a D^\mu H$$

$$J^{a\,\mu}_f = \bar{f} t^a \gamma^\mu f$$

$$, \ldots$$

I am only classifying the ops. into two classes. No assumptions of their relative importance, i.e. O(1) Wilson coefficients for all the d=6 SM ops.

Patterns of operator mixing

"Loop" operators

$$H^\dagger \bar{f}_R \sigma^{\mu\nu} t^a f_L \, F^a_{\mu\nu}$$

$$H^\dagger t^a t^b H F^a_{\mu\nu} \;\; F^{b\,\mu\nu}$$

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+CP-violating

No mixing found by explicit calculations

Mixing

"Current-current " operators

$$J_i \cdot J_j$$

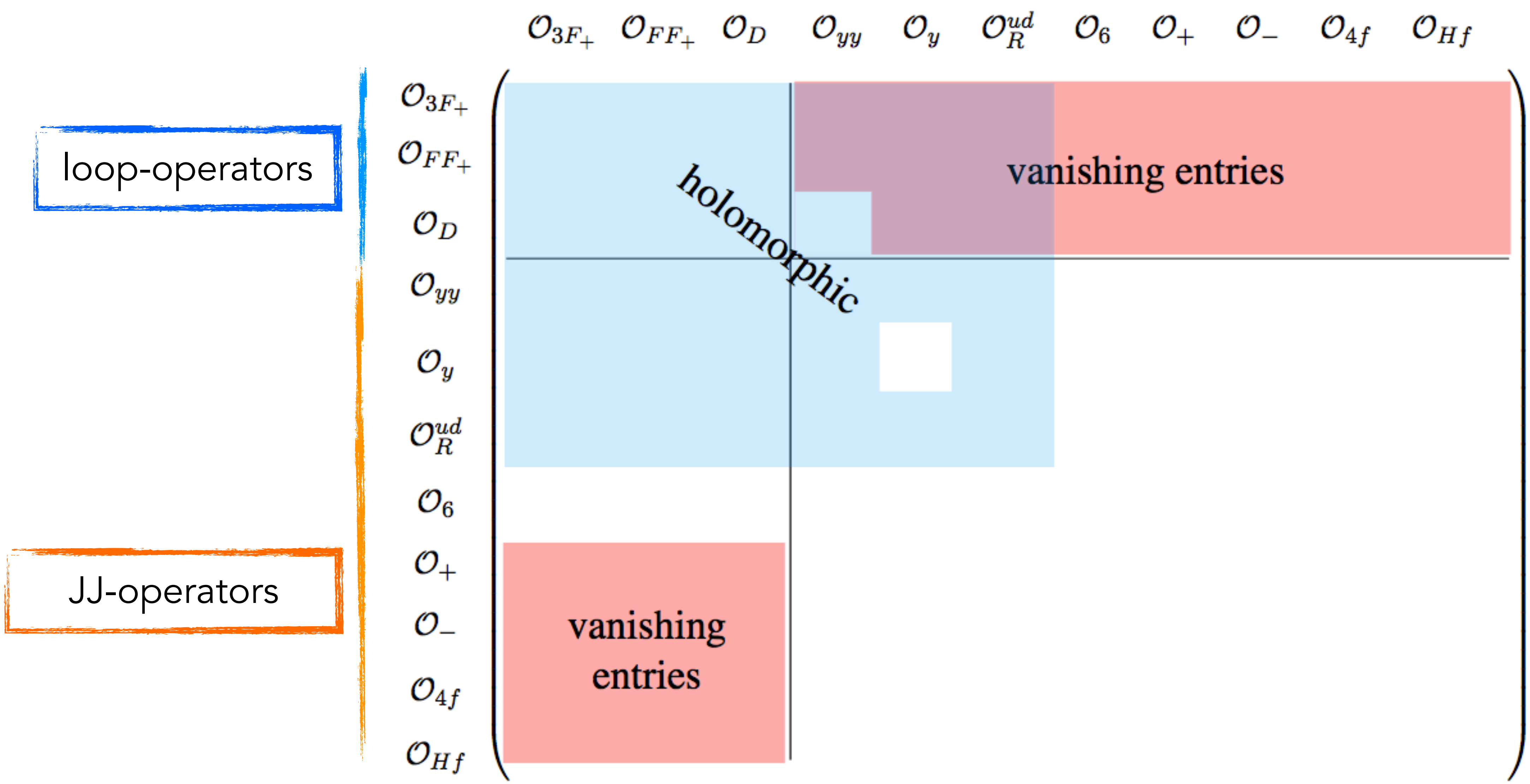
$$J^{a\,\mu}_H = H^\dagger t^a D^\mu H$$

$$J^{a\,\mu}_f = \bar{f} t^a \gamma^\mu f$$

$$, \ldots$$

Only one exception to this rule: $\mathcal{O}_{yy} = (\bar{f}_R t^a f_L)\,(\bar{f}_R t^a f_L) \sim \psi^4$

In fact, the full anomalous dimension matrix of the SM exhibits an analogous structure



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SUSY tool

The JJ-operators are in the Kähler while loop-operators are either absent or can be embedded in the superpotential

+

strong non-renormalization results in SUSY is suggestive.

$$\delta\mathcal{L} = A_\mu J^\mu + \cdots \qquad \partial_\mu J^\mu = 0$$

SUSY

$$\delta\mathcal{L} = \int d^4\theta \mathcal{J} V + \cdots \qquad D^2\mathcal{J} = \bar{D}^2\mathcal{J} = 0$$

e.g. $\mathcal{J} = \Phi^\dagger\Phi$

supersymmetrization

supersymmetrization

Recall:

$$\Phi(y) \sim \phi + \theta\psi + \theta^2 F$$

$$\mathcal{W}_\alpha(y) \sim \lambda + D\theta + \theta F_{\mu\nu} + i\theta^2 \partial_\mu \lambda^\dagger$$

$$x = y + i\theta\sigma\bar{\theta}$$

supersymmetrization

F-terms of non-chiral superfields:

$$\mathcal{O}_{FF} = |\phi|^2 F_{\mu\nu} F^{\mu\nu} \longrightarrow \Phi^\dagger e^{V_\Phi} \Phi \, \mathcal{W}^\alpha \mathcal{W}_\alpha = -\frac{1}{2}\theta^2 \mathcal{O}_{FF} + \cdots$$

$$\mathcal{O}_D = \phi(q\sigma^{\mu\nu}u)F_{\mu\nu} \longrightarrow \Phi\,(Q\overleftrightarrow{\mathcal{D}}_\alpha U)\,\mathcal{W}^\alpha = -\theta^2 \mathcal{O}_D + \cdots$$

$$\mathcal{O}_{3F} = f^{abc} F_\mu^{a\,\nu} F_\nu^{b\,\rho} F_\rho^{c\,\mu} \longrightarrow f^{abc} \mathcal{D}^\beta \mathcal{W}^{a\,\alpha} \mathcal{W}^b_\beta \mathcal{W}^c_\alpha = i\theta^2 \mathcal{O}_{3F} + \cdots$$

They can only be embedded upon introducing a spurion $\eta = \theta^2$

e.g.

$$\int d^4\theta \, \Phi^\dagger e^{V_\Phi} \Phi \, \mathcal{W}_\alpha \mathcal{W}^\alpha \, \eta^\dagger = c_{FF} |H|^2 F_\mu F^{\mu\nu} + \cdots$$

supersymmetrization

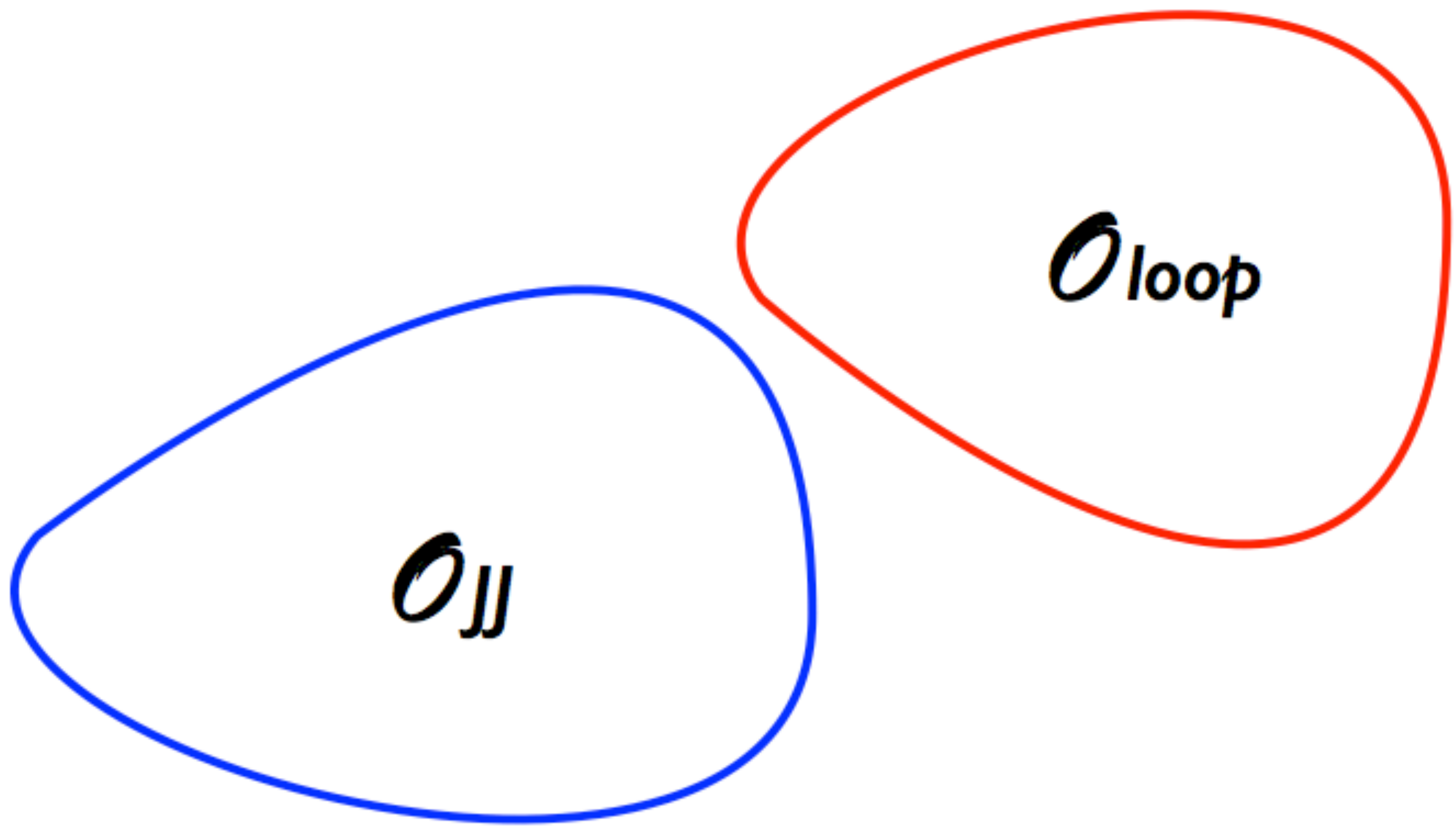
There are two "current-current" operators that
also arise from F-terms of non-chiral superfields:
(i.e. one spurion η power)

$$\mathcal{O}_{y_u} = |\phi|^2 \phi q u \longrightarrow \left(\Phi^\dagger e^{V_\Phi} \Phi\right) \Phi Q U = \theta^2 \mathcal{O}_{y_u} + \cdots$$

$$\mathcal{O}_{y_u y_d} = q u q d \longrightarrow (QU)\, \mathcal{D}^2\, (QD) \;=\; -4\theta^2 \mathcal{O}_{y_u y_d} + \cdots$$

The rest of the operators are SUSY-preserving or embedded with other spurion power.

Groups of dim-6 operators



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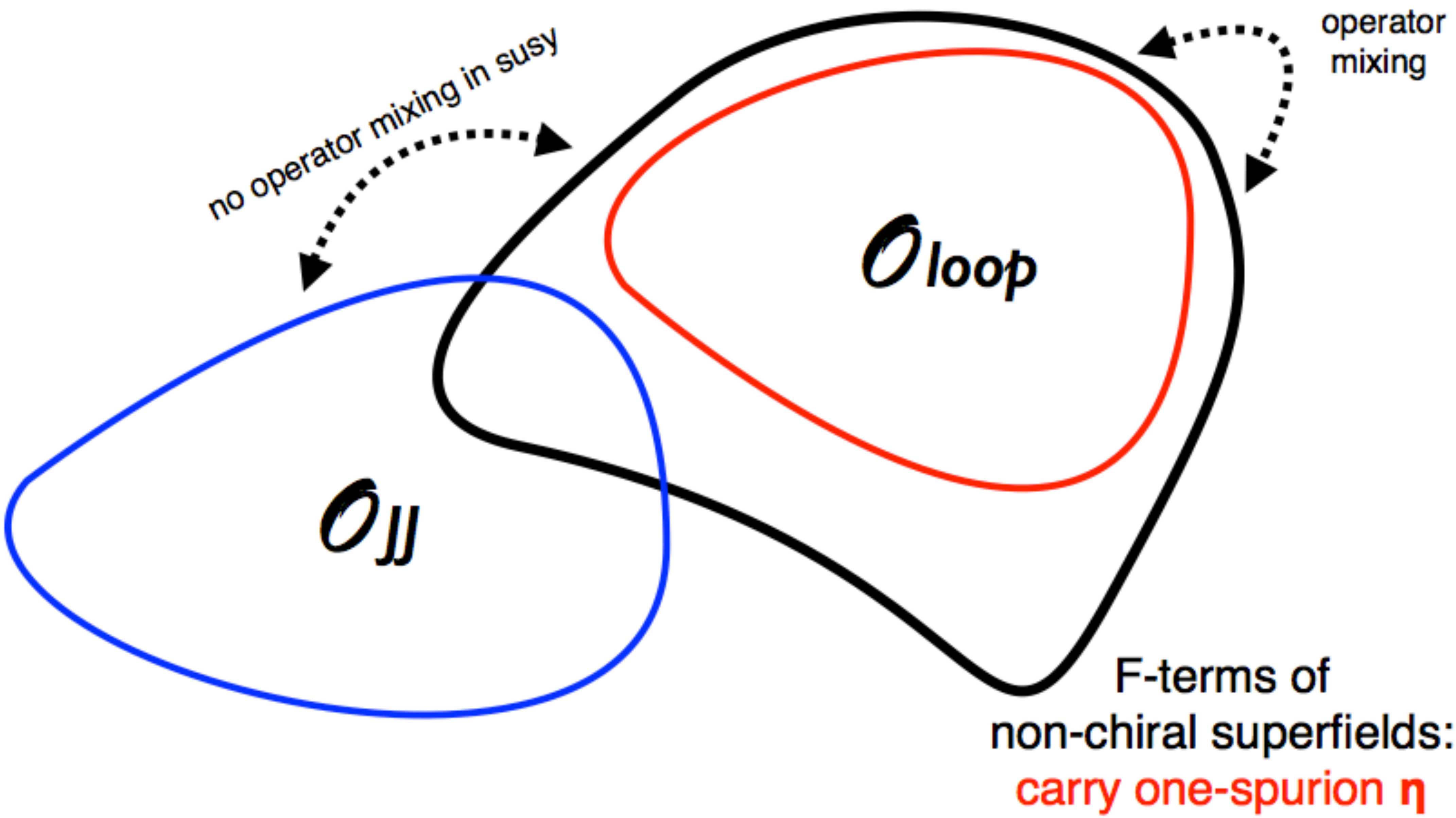

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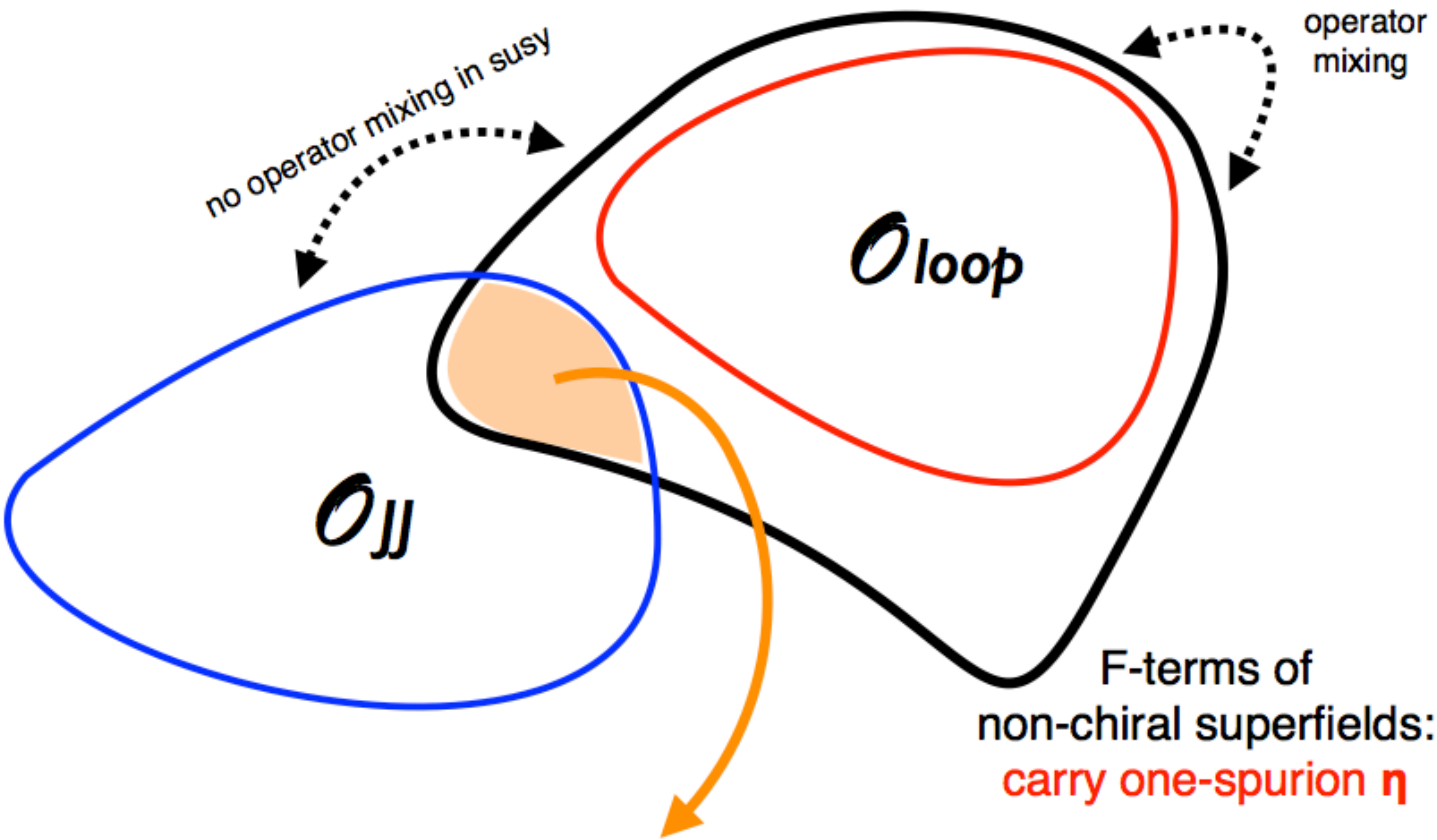
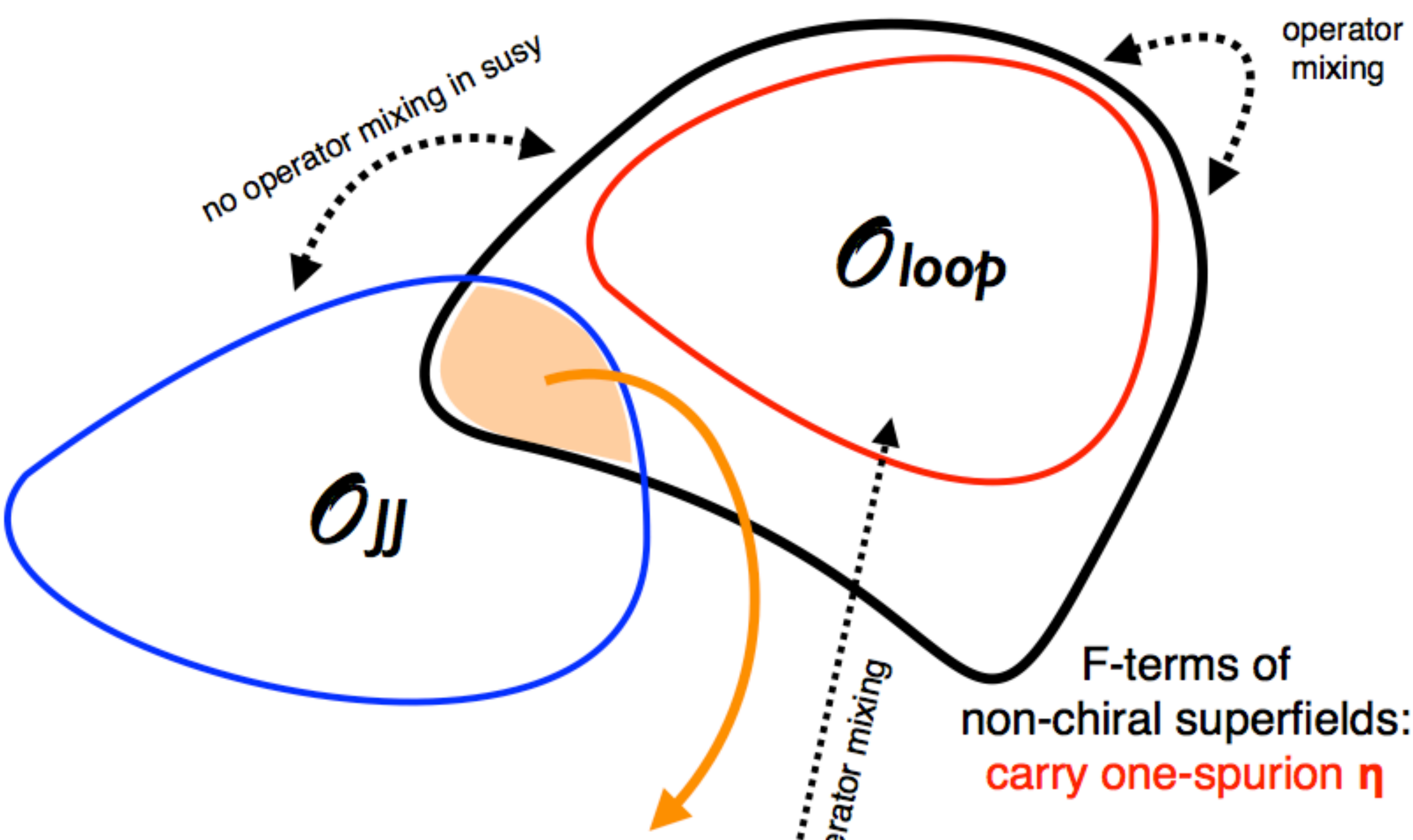


$$\mathcal{O}_y = |H|^2 H \bar{f}_R f_L$$

$$\mathcal{O}_{yy} = \left(\bar{f}_R t^a f_L\right)\left(\bar{f}_R t^a f_L\right)$$

Groups of dim-6 operators



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$$\mathcal{O}_{yy} = \left(\bar{f}_R t^a f_L\right)\left(\bar{f}_R t^a f_L\right)$$

trivially cannot contribute

the only "tree-level" operator affecting "loop" one: Dipole of *f*

(as explicit calculations show)

$\mathcal{O}_{yy}$ the only "current-current" operator that renormalized a loop operator, the dipole

From integrating out

(1,2) $_{1/2}$ — $\mathcal{O}_{y_u y_d} = y_u y_d (\bar{Q}^r_L u_R)\epsilon_{rs}(\bar{Q}^s_L d_R)$

(8,2)$_{1/2}$ — $\mathcal{O}^{(8)}_{y_u y_d} = y_u y_d (\bar{Q}^r_L T^A u_R)\epsilon_{rs}(\bar{Q}^s_L T^A d_R)$

$\mathcal{O}_{y_u y_e} = y_u y_e (\bar{Q}^r_L u_R)\epsilon_{rs}(\bar{L}^s_L e_R)$ → Trivially can't mix

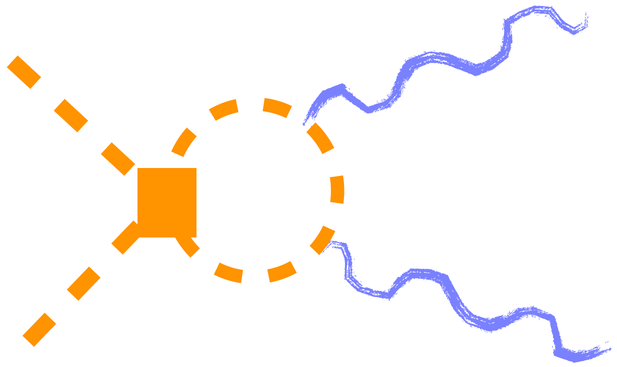
(3,2)$_{-7/6}$ — $\mathcal{O}'_{y_u y_e} = y_u y_e (\bar{Q}^{r\,\alpha}_L e_R)\epsilon_{rs}(\bar{L}^s_L u^\alpha_R)$

$\mathcal{O}_{y_e y_d} = y_e y_d^\dagger (\bar{L}_L e_R)(\bar{d}_R Q_L)$ → Trivially can't mix

All tree-level integrations of scalars done in
Blas, Chala, Perez-Victoria, Santiago **1412.8480**

At the component level, take the easiest!

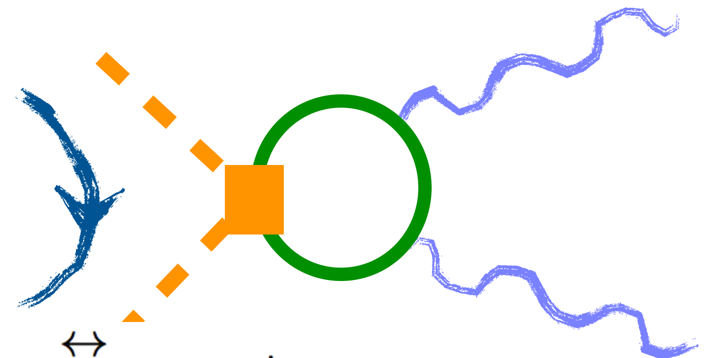
SM



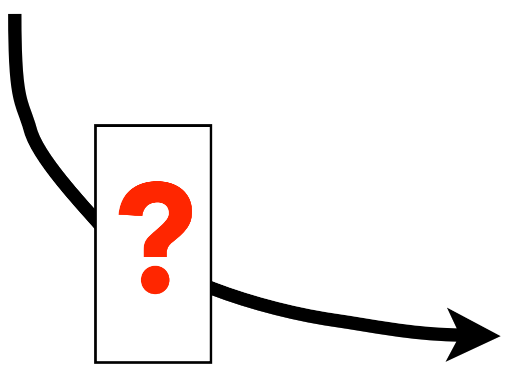
$$|\phi|^2|D_\mu\phi|^2$$

Spartners

$$J_\mu = \phi^* \overleftrightarrow{D}_\mu \phi$$

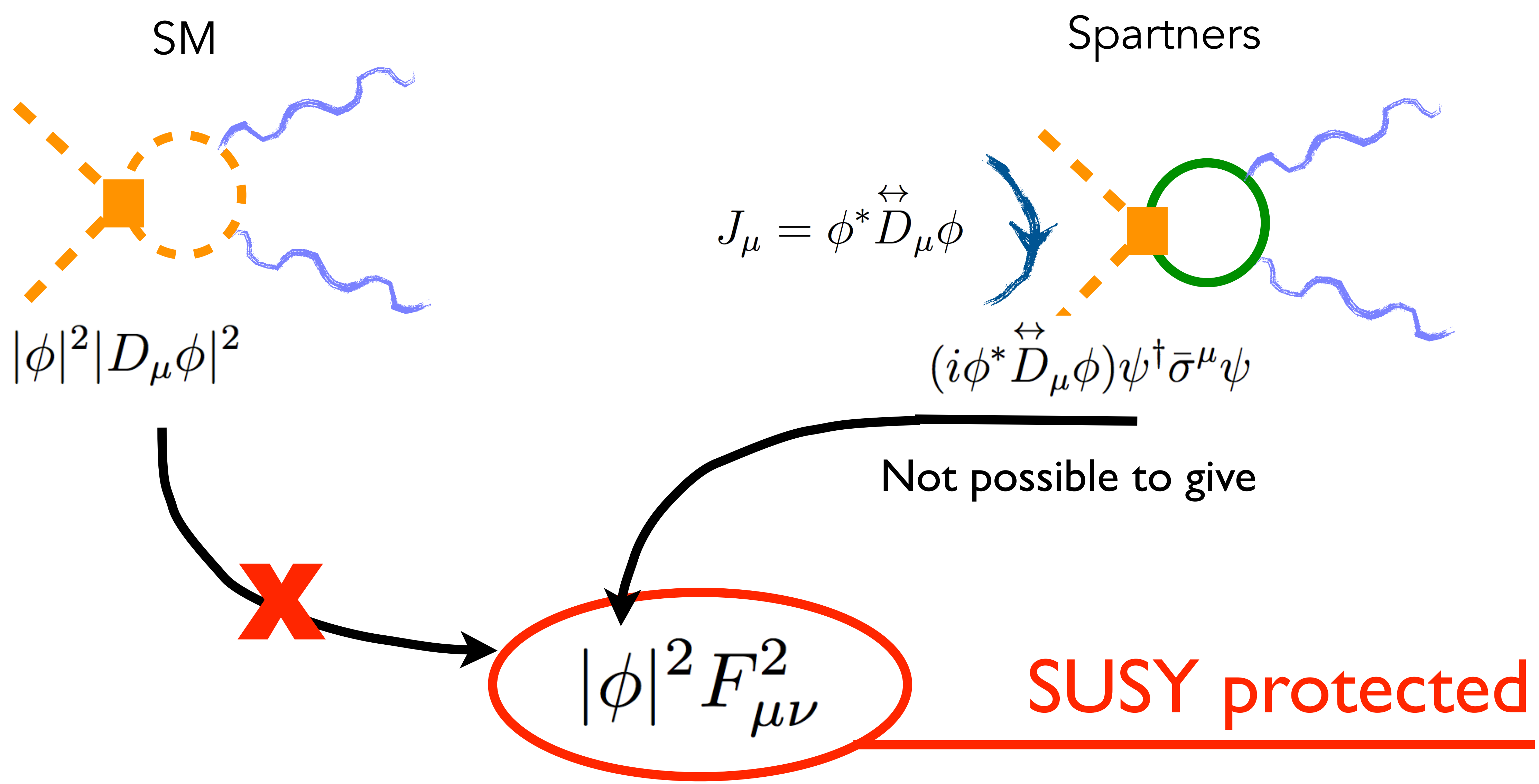


$$(i\phi^* \overleftrightarrow{D}_\mu \phi)\psi^\dagger \bar{\sigma}^\mu \psi$$



$$|\phi|^2 F_{\mu\nu}^2$$

At the component level, take the easiest!



Of course, the real reason is not SUSY. Only the Lorentz structure of the vertices matters. But SUSY is a useful tool to organize the calculation.

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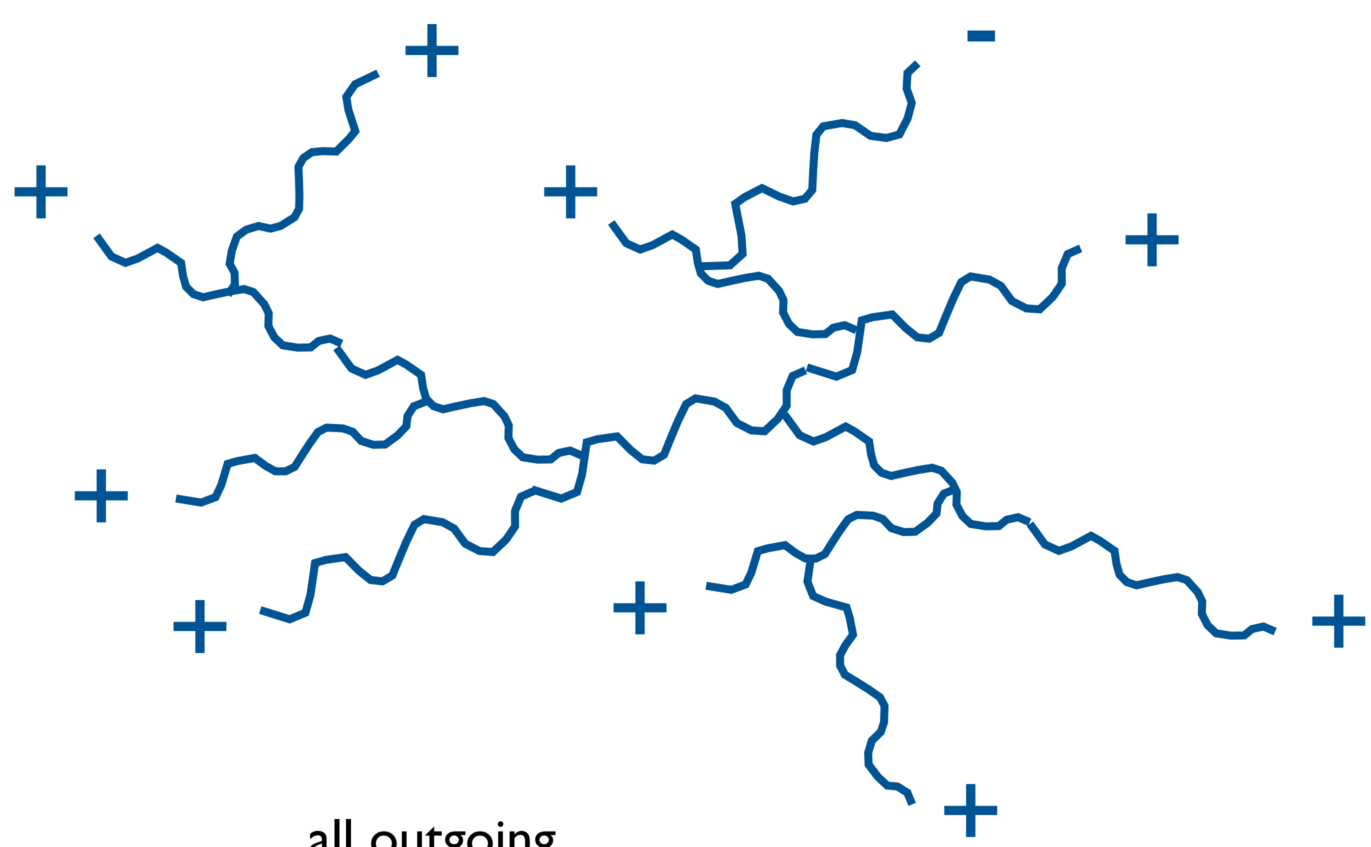
A logic analogy

In QCD

$$A_n^{tree}[g^- g^+ g^+ \cdots g^+] = A_n^{tree}[g^+ g^+ \cdots g^+] = 0$$



all outgoing

A logic analogy

In QCD

$$A_n^{tree}[g^- g^+ g^+ \cdots g^+] = A_n^{tree}[g^+ g^+ \cdots g^+] = 0$$

Easiest way to prove it: consider SQCD and recall that the Ward identity reads

$$\begin{aligned} 0 &= < [Q^\dagger, \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n)] > \\ &= \sum_i^n (-1)^{\sum_{i<j} |\mathcal{O}_i|} < \mathcal{O}_1(x_1) \cdots [Q^\dagger, \mathcal{O}_i(p_i)] \cdots \mathcal{O}_n(x_n) > \end{aligned}$$

Now, for SQCD

$$[Q^\dagger, a_\lambda] = a_g \quad , \qquad [Q^\dagger, a_g] = 0$$

So, applying the ward identity one finds

$$0 = \langle [Q^\dagger, a_\lambda a_g \cdots a_g] \rangle \sim \langle a_g \cdots a_g \rangle$$

Therefore, in SQCD

$$A_n^{L-loop}[g^- g^+ g^+ \cdots g^+] = A_n^{L-loop}[g^+ g^+ \cdots g^+] = 0$$

easy!

Lastly, one notices that the SQCD tree-level diagrams with *n* external gluons only contains gluons, hence is QCD

$$A_n^{tree}[g^- g^+ g^+ \cdots g^+] = A_n^{tree}[g^+ g^+ \cdots g^+] = 0$$

In short, tree-level pure QCD *is accidentally SUSY*.

Many more examples used to compute scattering amplitudes.

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Implications for the Chiral Lagrangian

Recall that...

$$\mathcal{L}_2 = \frac{f^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle$$

$$\mathcal{L}_4 = -iL_9\langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\rangle + L_{10}\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle + \cdots$$

$$\mathcal{L}_2 \text{ @ 1-loop} \sim \mathcal{L}_4 \text{ @ tree-level}$$

Explicit computations show

$$\gamma_{L_9+L_{10}} = \frac{1}{4} - \frac{1}{4} = 0$$ where $$\gamma_{L_i} = (4\pi)^2 dL_i/d\log\mu$$

Now we know why, rotate the original Chiral Lagrangian

$$\mathcal{L}_4 \;=\; -iL_9\langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\rangle + L_{10}\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle$$

To the more convenient basis

$$\mathcal{L}_4 = iL_{JJ}\langle D_\mu F_L^{\mu\nu}(U^\dagger \overleftrightarrow{D}_\nu U) + (U\overleftrightarrow{D}_\nu U^\dagger) D_\mu F_R^{\mu\nu}\rangle + L_{loop}\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle$$

where $L_{JJ} = L_9/2$ and $L_{loop} = L_9 + L_{10}$.

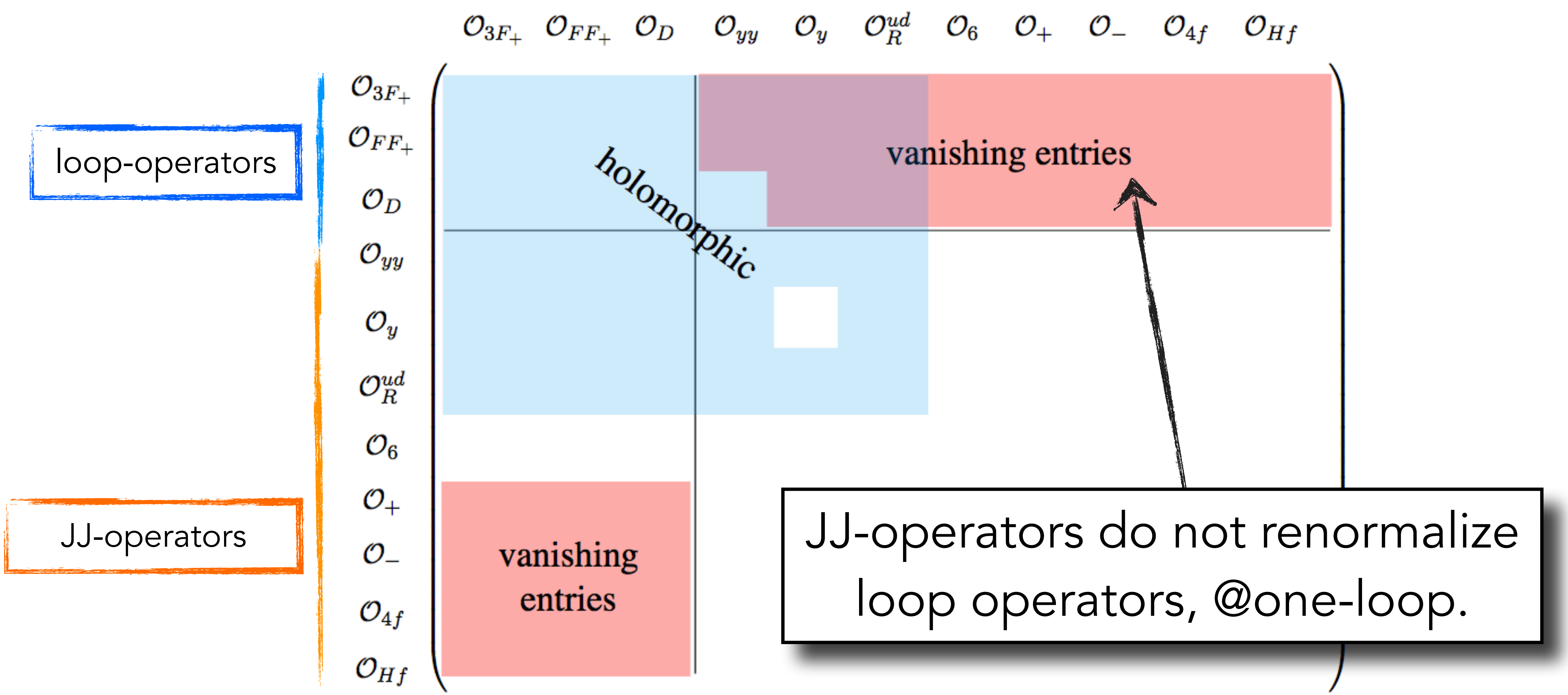
Now, the loop operator can only be embedded in the θ^2 term of the operator

$\langle \mathcal{U}^\dagger \mathcal{W}_R^\alpha \mathcal{U} \mathcal{W}_{\alpha L}\rangle$ $\mathcal{U} \equiv e^{i\Phi}$, with Φ being a chiral superfield

Therefore it can't be renormalized by $\mathcal{L}_2$ in the SUSY limit. Contributions from spartners are easily seen to vanish and hence L_{loop} is zero at one loop.

Summary and outlook

The structure is not due to the SM internal or accidental symmetries.



Various physical phenomena can be read form here.

Summary and outlook

- Dissection of the one-loop anomalous dimension matrix. SUSY as tool.

- Loop-operators not renormalized by JJ-operators up to the holomorphic 4-fermion.

- I haven't covered the holomorphy of the anomalous dim.

$$\frac{\partial \gamma_{c_i}}{\partial c_j^*} = 0$$

see 1412.7151.

- Chiral Lagrangian anomalous dimension matrix. I just did one example...

- Possible applications to other EFTs. The same procedure might be a good starting point for other analysis.

- Interesting to understand the concrete connection with the approach taken by Cheung and Shen.