The Curvature of Higgs Field Space

Rodrigo Alonso
In collaboration with E.E. Jenkins & A.V. Manohar

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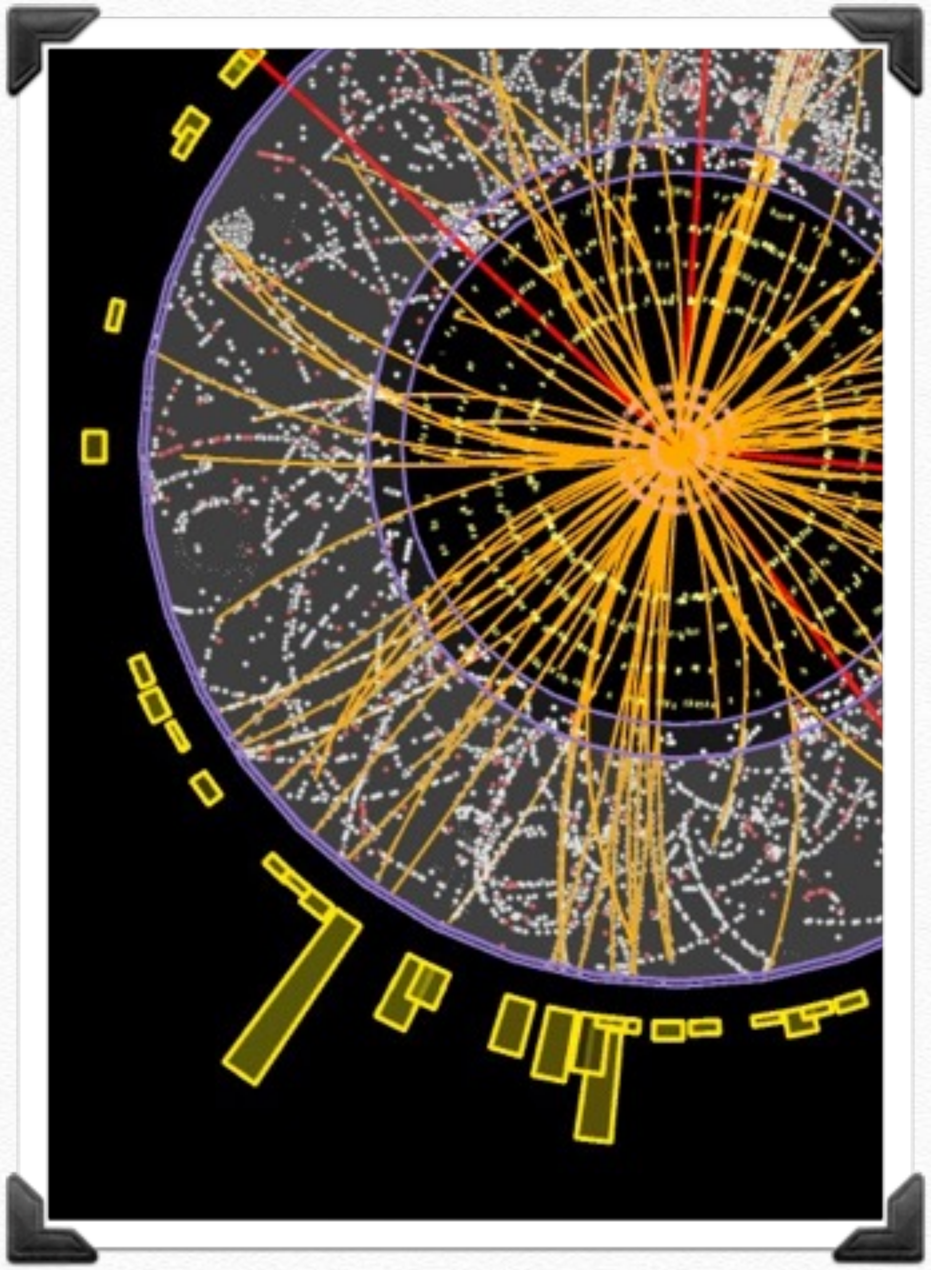
The Nature of the New Scalar

- Composite?
- Elemental?
- Mass Stabilization?
- Effective Field Theories helps us stay agnostic
- A tool to frame the BEH scalar
- *This talk: a different view of (H)EFTs*

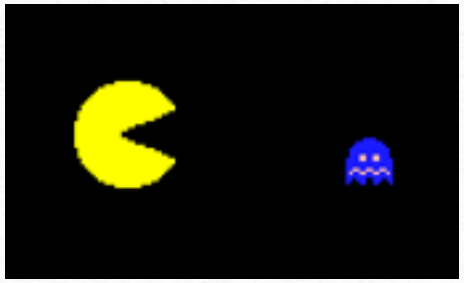


Part I: Curvature

Started from the bottom

W,Z Massive Gauge Bosons living in:

$SU(2)_L x U(1)_Y / U(1)_Q$



One needs fields that live in the broken group to be sacrificed in the gauge bosons altar: <u>a three sphere S^3</u> parametrized as:

$$u^i(\varphi) = \begin{bmatrix} u^1(\varphi) \\ u^2(\varphi) \\ u^3(\varphi) \\ u^4(\varphi) \end{bmatrix},$$

$$u(\varphi) \cdot u(\varphi) = 1,$$

$$u^i(0) = \delta^{i4},$$

Started from the bottom

W,Z Massive Gauge Bosons living in:
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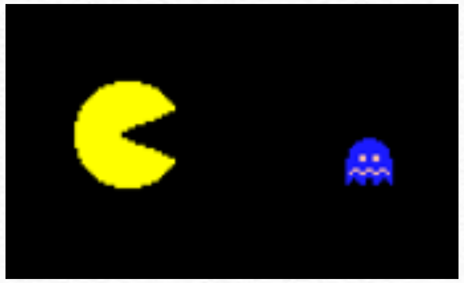


One needs fields that live in the broken group to be sacrificed in the gauge bosons altar: a three sphere S^3
parametrized as:

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} u^4 + iu^3 & u^2 + iu^1 \\ -u^2 + iu^1 & u^4 - iu^3 \end{pmatrix}$$

$$U \to L U R^\dagger$$

Started from the bottom

Three Massive Gauge Bosons living in:
$SU(2)_L x U(1)_Y / U(1)_Q$

a three sphere S^3

$$\varphi^a\,,\quad a = 1,2,3$$

$$\delta_A u(\varphi) = T^A \cdot u(\varphi), \qquad T^A = -(T^A)^T$$
$$A = 1,2,3,Y$$

$$\delta_A \varphi^a = \left(\frac{(\partial u)^2}{(\partial \varphi)^2}\right)^{-1} \frac{\partial u(\varphi)}{\partial \varphi} T^A \cdot u \equiv t^a_A$$

Defines the Gauge Covariant Derivative

Started from the bottom

Three Massive Gauge Bosons living in:
$SU(2)_L x U(1)_Y / U(1)_Q$

a three sphere S^3 $\varphi^a\,,\ a = 1,2,3$

$$\delta_A u(\varphi) = T^A \cdot u(\varphi), \qquad T^A = -(T^A)^T$$

$$A = 1,2,3,Y$$

$$\delta_\theta \varphi^a = \theta^A t^a_A(\varphi)$$

The most general transformation
is given by the Killing Vectors

Started from the bottom

Three Massive Gauge Bosons living in:

$SU(2)_L x U(1)_Y / U(1)_Q$

a three sphere S^3 $\varphi^a\,,\ a=1,2,3$

So that we have a Gauge Covariant Derivative:

$$D_\mu\varphi^a = \partial_\mu\varphi^a + gW^I_\mu t^a_I + g' B_\mu t^a_Y$$

and a Kinetic term:

$$\mathscr{L} = \frac{1}{2} D_\mu u(\varphi)\cdot D^\mu u(\varphi) = \frac{1}{2} g_{ab}\, D_\mu\varphi^a D^\mu\varphi^b$$

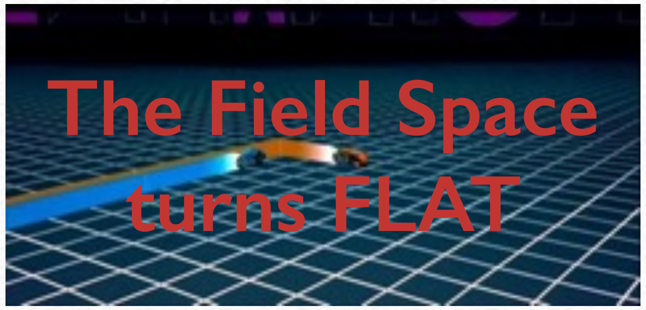
Now the Higgs is here

It is a singlet of the EW symmetry and appears where the NGB are, is it maybe the 'radius'?

$$\begin{bmatrix} \phi_H^1 \\ \phi_H^2 \\ \phi_H^3 \\ \phi_H^4 \end{bmatrix} \equiv (v+h) \begin{bmatrix} u^1(\varphi) \\ u^2(\varphi) \\ u^3(\varphi) \\ u^4(\varphi) \end{bmatrix},$$

Then the sphere S^3 gets expanded to R^4
we have the **SM Higgs doublet**

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_H^2 + i\phi_H^1 \\ \phi_H^4 - i\phi_H^3 \end{bmatrix}.$$



but u still transforms non-linearly!

What Higgs is it?

$$\phi^{\mathrm{i}} \equiv (\varphi^a, h)$$

$$D_\mu \phi^{\mathrm{i}} = \partial_\mu \phi^{\mathrm{i}} + g W_\mu^I t_I^{\mathrm{i}}(\phi) + g' B_\mu t_Y^{\mathrm{i}}(\phi),$$

$$= (D_\mu \varphi^a,\ \partial_\mu h)$$

Let's give him a kinetic term

$$\mathscr{L} = \frac{1}{2} g_{\mathrm{ij}}(\phi)\, D_\mu \phi^{\mathrm{i}} D^\mu \phi^{\mathrm{j}},$$

$$g_{\mathrm{ij}}(\phi) = \begin{bmatrix} F(h)^2 g_{ab}(\varphi) & 0 \\ 0 & 1 \end{bmatrix},$$

For Example:

$$F_{SM}(h) = 1 + \frac{h}{v}$$

Riemann Curvature

How is the space the 4 scalars live on?

1

$$R_{abcd}(\phi) = \left[\frac{1}{v^2} - (F'(h))^2\right] F(h)^2 \left(g_{ac}g_{bd} - g_{ad}g_{bc}\right),$$

$$R_{ahbh}(\phi) = -F(h)F''(h)g_{ab}$$

2

# of GB, 3

$$R(h) = \left[\frac{1}{v^2} - (F'(h))^2\right] \frac{N_\varphi(N_\varphi - 1)}{F(h)^2} - \frac{2N_\varphi F''(h)}{F(h)}.$$

$\mathfrak{R}(h)$

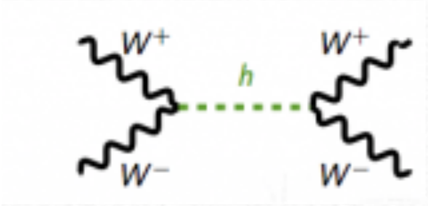
Functions of the singlet h characterizing Curvature

The Curvature and Physical Observables

E.g. in an HEFT longitudinal boson scattering is not fully unitarized:

[Barbieri, Bellazzini, Rychkov & Varagnolo; '07]

$$\mathcal{A}(W_L W_L \to W_L W_L) = \frac{s+t}{v^2}\mathfrak{R}(0)\,,$$



Which means new resonances are required at (NDA):

$$4\pi v/\sqrt{\mathfrak{R}}$$

Connection with Composite Models

take **O(5)/O(4)** and therefore a 4-sphere, S^4

[Agashe, Contino Pomarol; ‘05]

$$\hat{u} = \left(\cos\left(\frac{h + \langle h \rangle}{f}\right) , \ \sin\left(\frac{h + \langle h \rangle}{f}\right) u^i \right)$$

where the function of the singlet in the metric:

$$F(h) = \sin[(h + \langle h \rangle)/f]$$

and curvature:

$$\mathfrak{R}(h) = \sin^2\left(\frac{h + \langle h \rangle}{f}\right) \sim \frac{v^2}{f^2}$$

Part II: Curvature is useful

Functional Methods

Partition Function and Effective Action:

$$Z[J] \equiv e^{iW[J]} = \int D\phi \exp\left[i\left(S[\phi] + \int \mathrm{d}x \; J\phi\right)\right]$$

$$\Gamma[\widetilde{\phi}] = W[J] - J\widetilde{\phi}, \qquad \widetilde{\phi} = \frac{\delta W}{\delta J}.$$

expanding the action around the classical solution

$$S = S(\tilde{\phi}) + \delta\phi \frac{\delta S}{\delta \phi} + \frac{1}{2}(\delta\phi)^2 \frac{\delta^2 S}{(\delta\phi)^2} + \cdots$$

1-loop result:

$$\Gamma[\widetilde{\phi}] = S[\widetilde{\phi}] + \frac{i}{2} \ln\det\left(\frac{\delta^2 S}{\delta\phi^{\mathrm{i}}\delta\phi^{\mathrm{j}}}\right)_{\phi=\widetilde{\phi}}.$$

Functional Methods

1-loop result:

$$\Gamma[\widetilde{\phi}] = S[\widetilde{\phi}] + \frac{i}{2}\ln\det\left(\frac{\delta^2 S}{\delta\phi^{\mathrm{i}}\delta\phi^{\mathrm{j}}}\right)_{\phi=\widetilde{\phi}} .$$

$$\delta^2 S = \frac{1}{2}\int dx(\mathscr{D}_\mu\delta\phi\mathscr{D}_\mu\delta\phi + X(\delta\phi)^2)$$

$$\frac{i}{2}\log\det\left(-\delta^2 S\right) = \frac{1}{32\pi^2\epsilon}\int dx\,\mathrm{Tr}\left\{\frac{[\mathscr{D}_\mu,\mathscr{D}_\nu]^2}{12} + \frac{X^2}{2}\right\},$$

[t'Hooft; '73]

Functional Methods: HEFT

Second variation of the action of HEFT

$$\delta^2 S = \frac{1}{2}\int \mathrm{d}x \Big[g_{\mathfrak{ij}} (\mathscr{D}_\mu \delta\phi)^{\mathfrak{i}} (\mathscr{D}^\mu \delta\phi)^{\mathfrak{j}} - R_{\mathfrak{ijkl}} D_\mu \widetilde{\phi}^{\mathfrak{i}} D^\mu \widetilde{\phi}^{\mathfrak{k}} \delta\phi^{\mathfrak{j}} \delta\phi^{\mathfrak{l}}$$

$$- g_{\mathfrak{il}} \Gamma^{\mathfrak{i}}_{\mathfrak{jk}} \delta\phi^{\mathfrak{j}} \delta\phi^{\mathfrak{k}} (\mathscr{D}^\mu D_\mu \widetilde{\phi})^{\mathfrak{i}} \Big].$$

Non-Invariant term!

$$X = D_\mu \phi D^\mu \phi \cdot R + \Gamma\, \delta S$$

Covariant Formalism

The problem is we do not know how to take derivatives

[Honerkamp; '72]
[Tataru; '75]

$$\tilde{\mathcal{D}}_{\mathfrak{i}} S = \frac{\delta S}{\delta \phi^{\mathfrak{i}}} , \quad \tilde{\mathcal{D}}_{\mathfrak{j}} \tilde{\mathcal{D}}_{\mathfrak{i}} S = \frac{\delta^2 S}{\delta \phi^{\mathfrak{j}} \delta \phi^{\mathfrak{i}}} - \Gamma^{\mathfrak{k}}_{\mathfrak{i}\mathfrak{j}} \frac{\delta S}{\delta \phi^{\mathfrak{k}}} .$$

So the second variation of the action is:

$$\frac{1}{2} \eta^{\mathfrak{i}} \left(\tilde{D}_{\mathfrak{j}} \tilde{D}_{\mathfrak{i}} S \right) \eta^{\mathfrak{j}} = \frac{1}{2} \int \mathrm{d}x \left[g_{\mathfrak{i}\mathfrak{j}} (\mathscr{D}_\mu \eta)^{\mathfrak{i}} (\mathscr{D}^\mu \eta)^{\mathfrak{j}} - R_{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}} D_\mu \widetilde{\phi}^{\mathfrak{i}} D^\mu \widetilde{\phi}^{\mathfrak{k}} \eta^{\mathfrak{j}} \eta^{\mathfrak{l}} \right] .$$

Or equivalently we must use <u>geodesics</u> to deviate from the background field

HEFT at one-loop

$$\mathscr{L} = \frac{1}{2} g_{\mathfrak{ij}}(\phi)\, D_\mu \phi^{\mathfrak{i}} D^\mu \phi^{\mathfrak{j}} + \mathcal{I}(\phi),$$

$$= \frac{1}{2}\partial_\mu h\, \partial^\mu h + \frac{1}{2} F(h)^2 g_{ab}(\varphi) D_\mu \varphi^a D_\mu \varphi^b$$

$$- V(h) + K(h)\, w^{\mathfrak{i}} u^{\mathfrak{i}}(\varphi),$$

$$w^{\mathfrak{i}} = \bar{q}_L \sigma^{\mathfrak{i}} Y_q\, q_R + \bar{\ell}_L \sigma^{\mathfrak{i}} Y_\ell\, \ell_R + \text{h.c.},$$

Introduce:

$$[\mathscr{D}_\mu\,, \mathscr{D}_\nu] = R^{\mathfrak{i}}{}_{\mathfrak{jkl}} D^\mu \phi^{\mathfrak{k}} D^\nu \phi^{\mathfrak{l}} + (\mathscr{A}^{\mu\nu})^{\mathfrak{i}}_{\mathfrak{j}}\,,$$

$$X^{\mathfrak{i}}{}_{\mathfrak{j}} = \mathcal{D}^{\mathfrak{i}} \mathcal{D}_{\mathfrak{j}} \mathcal{I} - R^{\mathfrak{i}}{}_{\mathfrak{kjl}} D^\mu \phi^{\mathfrak{k}} D_\mu \phi^{\mathfrak{l}},$$

$$(\mathscr{A}^{\mu\nu})^{\mathfrak{i}}_{\mathfrak{j}} = \left(\partial_{[\mu} A^B_{\nu]} + f^B_{CD} A^C_\mu A^D_\nu\right) \mathcal{D}_{\mathfrak{j}} t^{\mathfrak{i}}_B,$$

HEFT at one-loop

$\frac{1}{32\pi^2\epsilon}$ times

$$
\begin{aligned}
&\frac{1}{2}\left(V''-K''w\cdot u\right)^2+\left((K/(vF))'\right)^2\left[w\cdot w-(w\cdot u)^2\right]+\frac{1}{2}N_\varphi\left[\left(\frac{F''}{F}\right)(\partial_\mu h\partial^\mu h)-\frac{V'F'}{F}+(w\cdot u)\left(\frac{F'K'}{F}-\frac{K}{v^2F^2}\right)\right]^2\\
&-\left[(v^2FF'')\,(V''-K''u\cdot w)+(N_\varphi-1)\left[1-(vF')^2\right]\left\{-\frac{V'F'}{F}+(w\cdot u)\left(\frac{F'K'}{F}-\frac{K}{v^2F^2}\right)\right\}\right](D_\mu u\cdot D^\mu u)\\
&-\left[\frac{1}{3}(vF'')^2+(N_\varphi-1)\left[1-(vF')^2\right]\frac{F''}{F}\right](\partial_\nu h\partial^\nu h)(D_\mu u\cdot D^\mu u)+\frac{2}{3}\left[1-(vF')^2\right]^2(D_\mu u\cdot D_\nu u)^2\\
&+\left[\frac{1}{2}(v^2FF'')^2+\frac{3N_\varphi-7}{6}\left[1-(vF')^2\right]^2\right](D_\mu u\cdot D^\mu u)^2+\frac{4}{3}(vF'')^2(\partial^\mu h\partial^\nu h)(D_\mu u\cdot D_\nu u)-2F''(\partial^\mu h)\,(K/F)'\,(w\cdot D_\mu u)\\
&-\frac{1}{3}\left[1-(vF')^2\right](D^\mu u)^T A_{\mu\nu}(D^\nu u)-\frac{2}{3}(vF')(vF'')(\partial_\mu h)(D_\nu u)^T A^{\mu\nu}u+\frac{1}{12}\mathrm{tr}(A_{\mu\nu}A^{\mu\nu})+\frac{1}{6}\left[(vF')^2-1\right]u^T(A_{\mu\nu}A^{\mu\nu})u.
\end{aligned}
$$

[Guo, Ruiz-Femenia & Sanz-Cillero; ’15]
[RA,Jenkins, Manohar]

Non-invariant terms

They are calculable with functional methods:

$$\delta\mathscr{L} = \frac{1}{32\pi^2\epsilon}\left(X^{\mathfrak{ij}}\Gamma^{\mathfrak{k}}_{\mathfrak{ij}}\frac{\delta S}{\delta\phi^{\mathfrak{k}}} + \frac{1}{2}\Gamma^{\mathfrak{k}}_{\mathfrak{ij}}\frac{\delta S}{\delta\phi^{\mathfrak{k}}}\Gamma^{\mathfrak{l}\mathfrak{ij}}\frac{\delta S}{\delta\phi^{\mathfrak{l}}}\right)$$

and we can compare with the literature:

$$\frac{1}{32\pi^2\epsilon}\left\{\left(\frac{3}{2}+10\eta+18\eta^2\right)\frac{(\varphi\Box\varphi)^2}{v^4} - c_1\,(3+10\eta)\,\frac{\varphi\Box\varphi\Box h}{v^3}\right\}$$

[Appelquist & Bernard; ‘81]

[Gavela, Machado, Kanshin, Saa; ‘14]

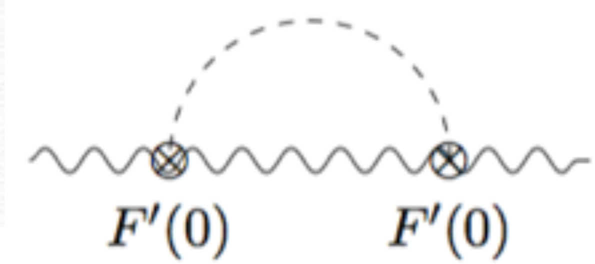
The Limits of HEFT

- SM Higgs; Flat
- Technicolor; Curved (=1/v)
- Composite Higgs; Curved (tunable)
- Dilaton; Flat to first order

Measure the Curvature

Two curvature magnitudes:

1 $$(1-(vF')^2) = \left(1-\left(v\frac{dF}{dh}\right)^2\right)$$

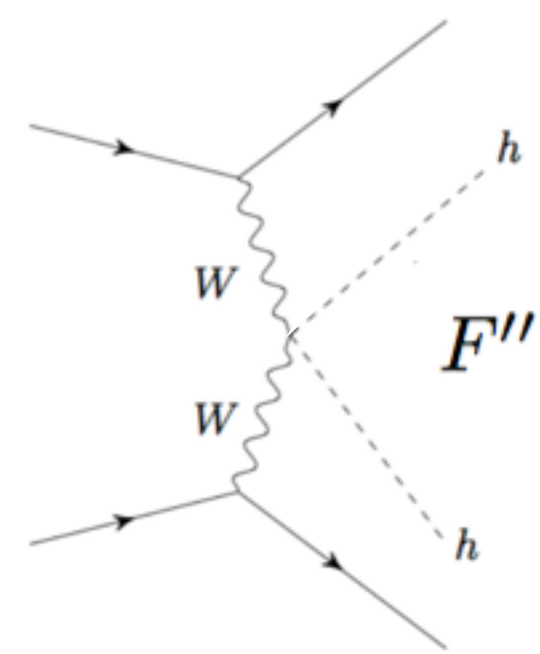


quite constrained e.g.

$$\Delta S = \frac{1}{12\pi}\Re(0)\log\left(\frac{\Lambda^2}{M_Z^2}\right).$$

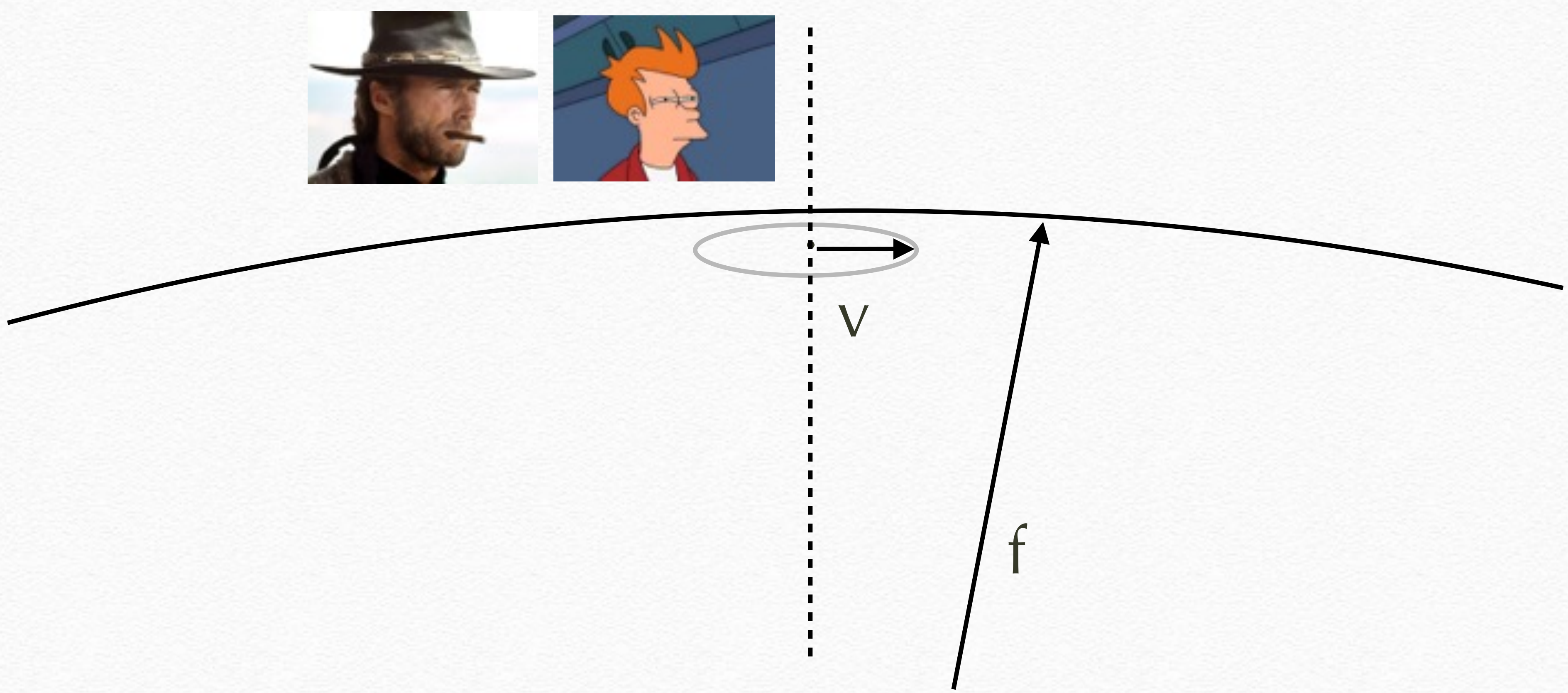
2 $$F'' = \frac{d^2F}{dh^2}$$

The second is hard to measure



$F''(0)$

Is the ~~Earth~~ Higgs field round?



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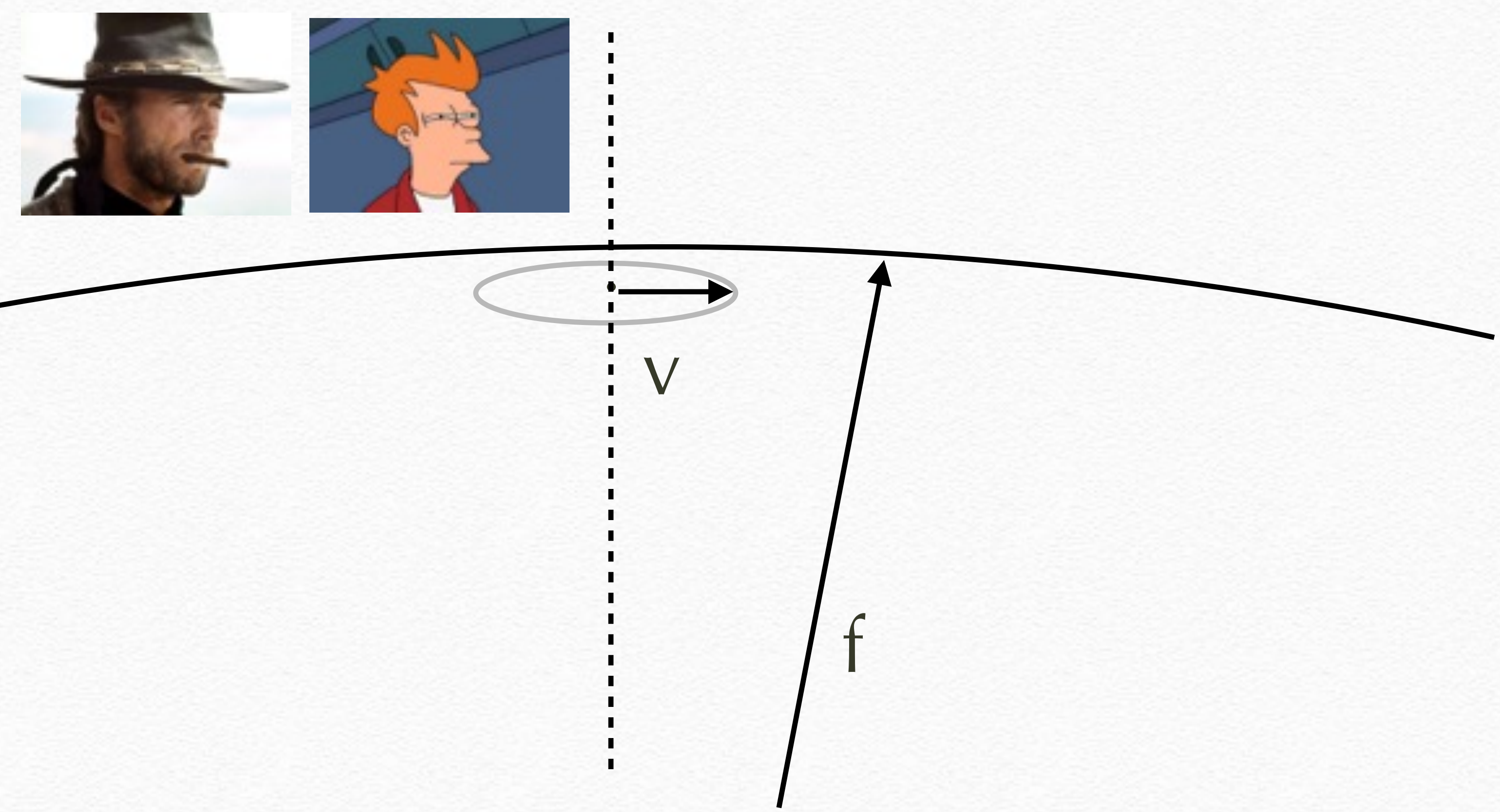

–Thank You