Effective theories for heavy WIMP dark matter

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U. Chicago, TRIUMF, Perimeter Institute,

Higgs Effective Theory Workshop

4 November, 2015

based on work with M.P. Solon: (Sakurai thesis award)

Heavy WIMP Effective Theory 1111.0016, 1309.4092, PRL

Standard Model Anatomy of WIMP Direct Detection I, II 1401.3339, 1409.8290, PRD

and work with M. Bauer, T. Cohen and M.P. Solon

SCET for Heavy WIMP Annihilation 1409.7392, JHEP

Thanks to co-organizers and participants of MITP program "Effective Theories and Dark Matter", March 2015:

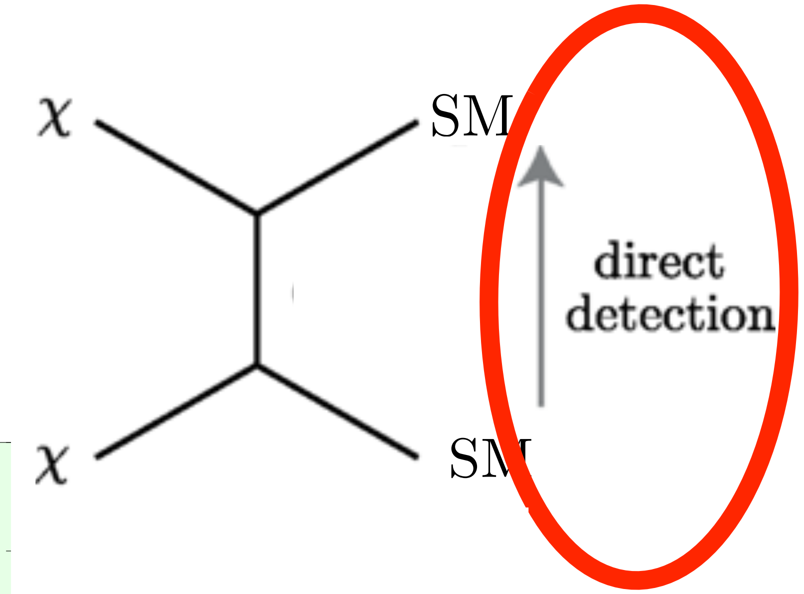
for many other applications of EFT and DM:

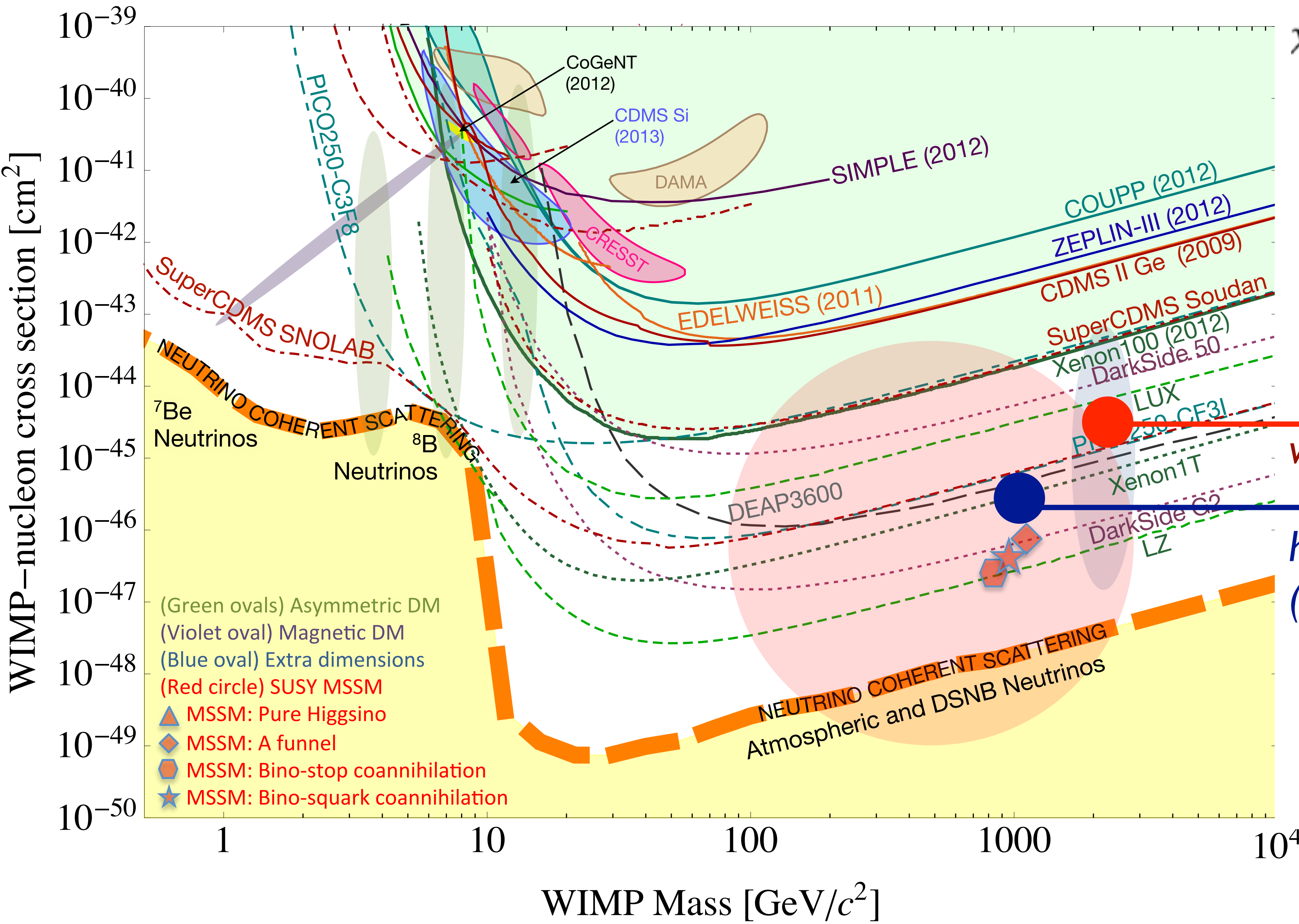
https://indico.mitp.uni-mainz.de/conferenceDisplay.py?ovw=True&confId=25

focus on 3 problems

- WIMP scattering + high-scale matching: *Heavy WIMP Effective Theory (HWET)*
- WIMP scattering + collider production, connecting weak scale to hadronic scale: *heavy quark decoupling*
- WIMP annihilation: *HWET+Soft Collinear Effective Theory*

Not quibbling about percents (example 1: heavy WIMP scattering)





wino: dimensional estimate

higgsino: Snowmass benchmark (2013)

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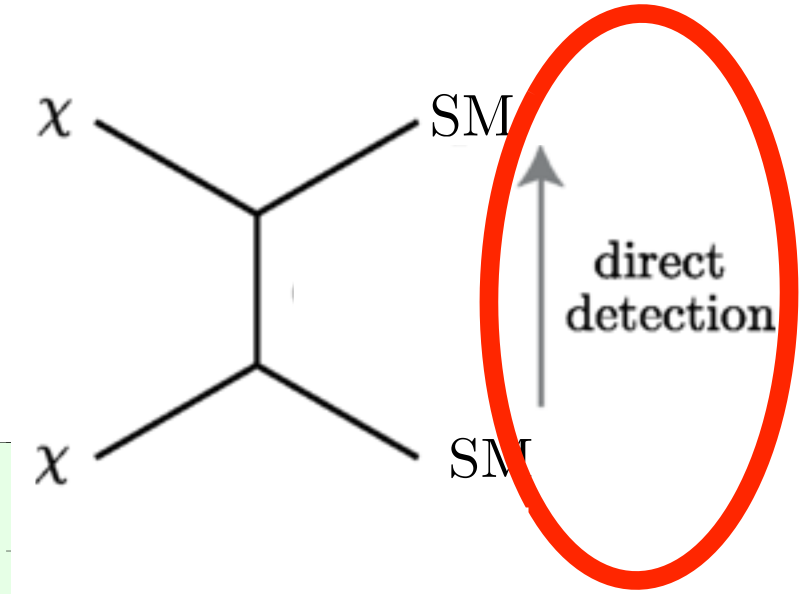

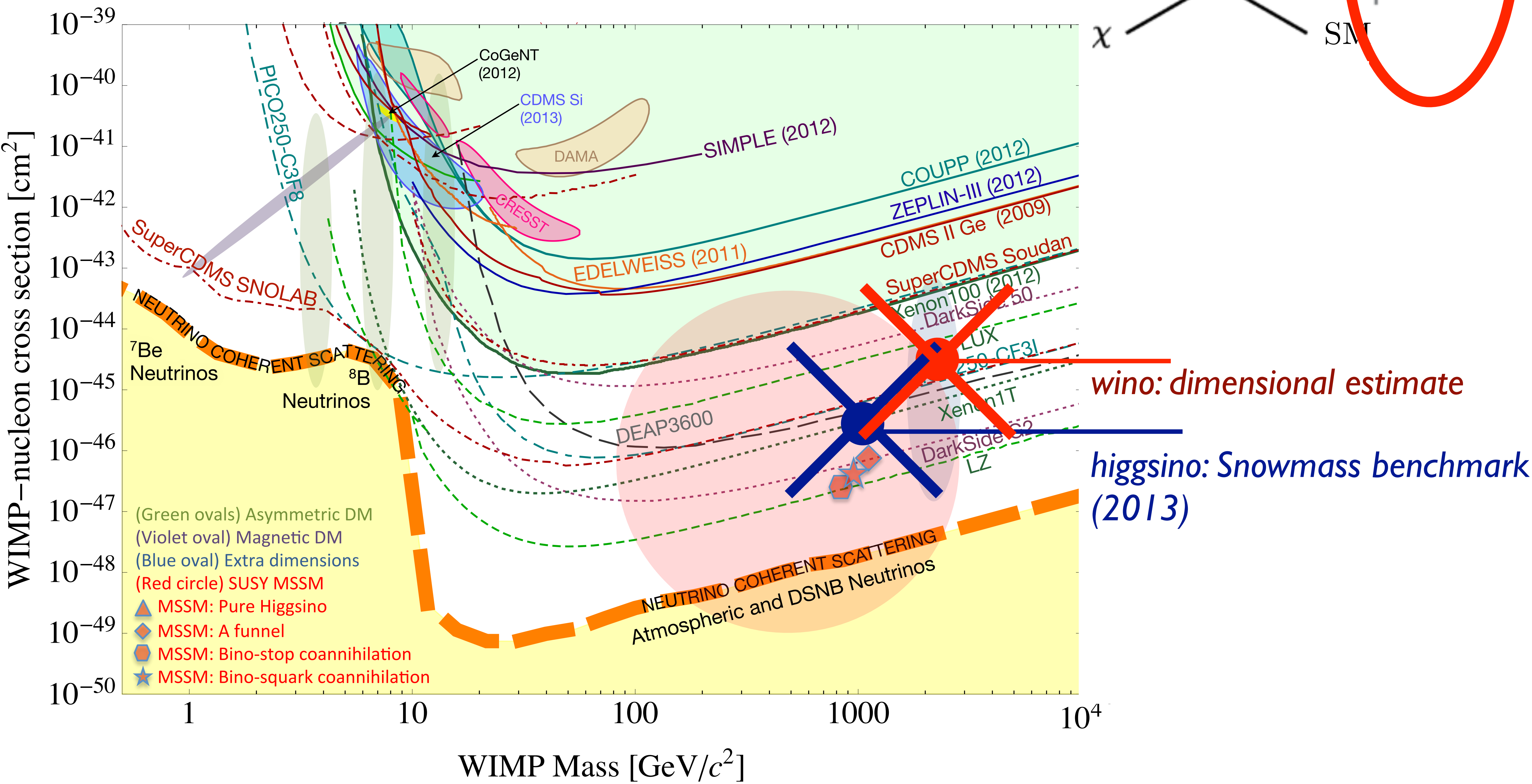

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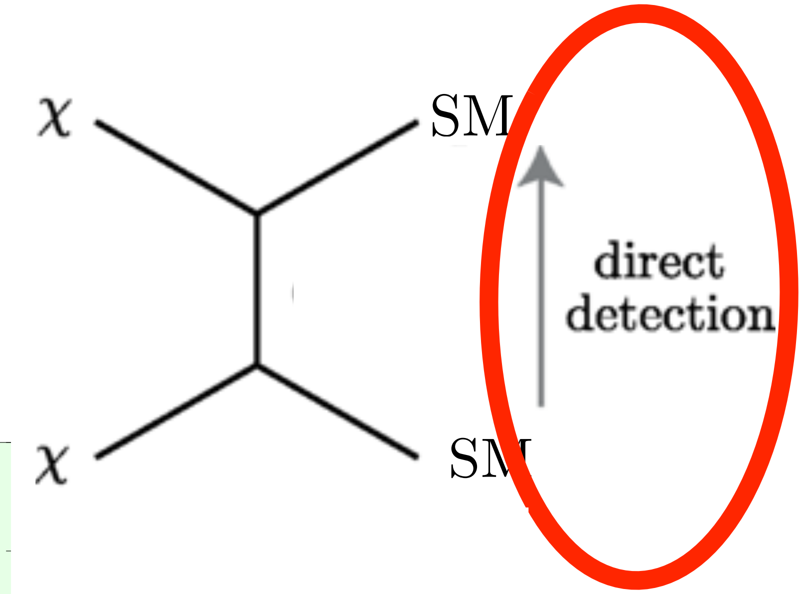

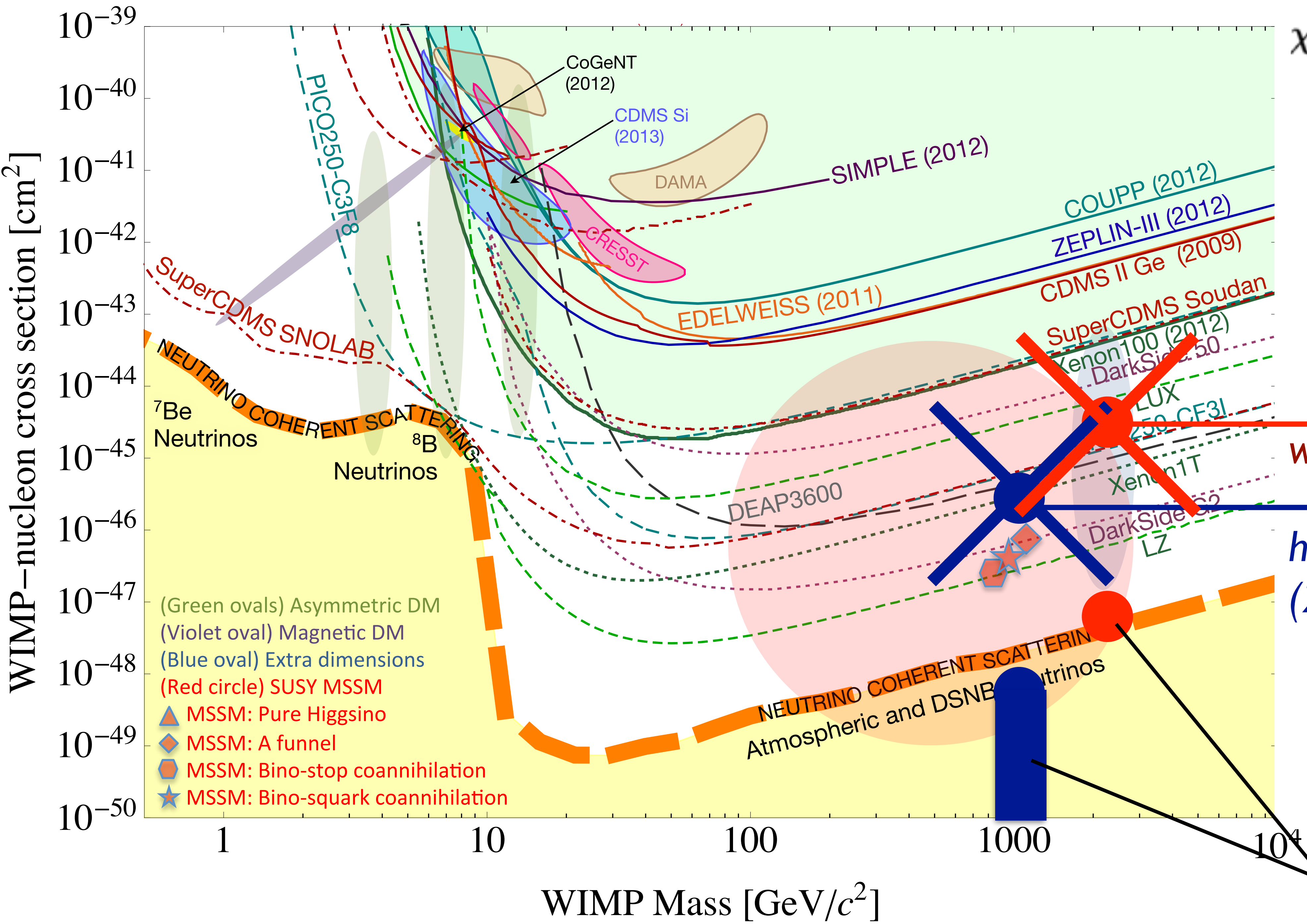

wino: dimensional estimate

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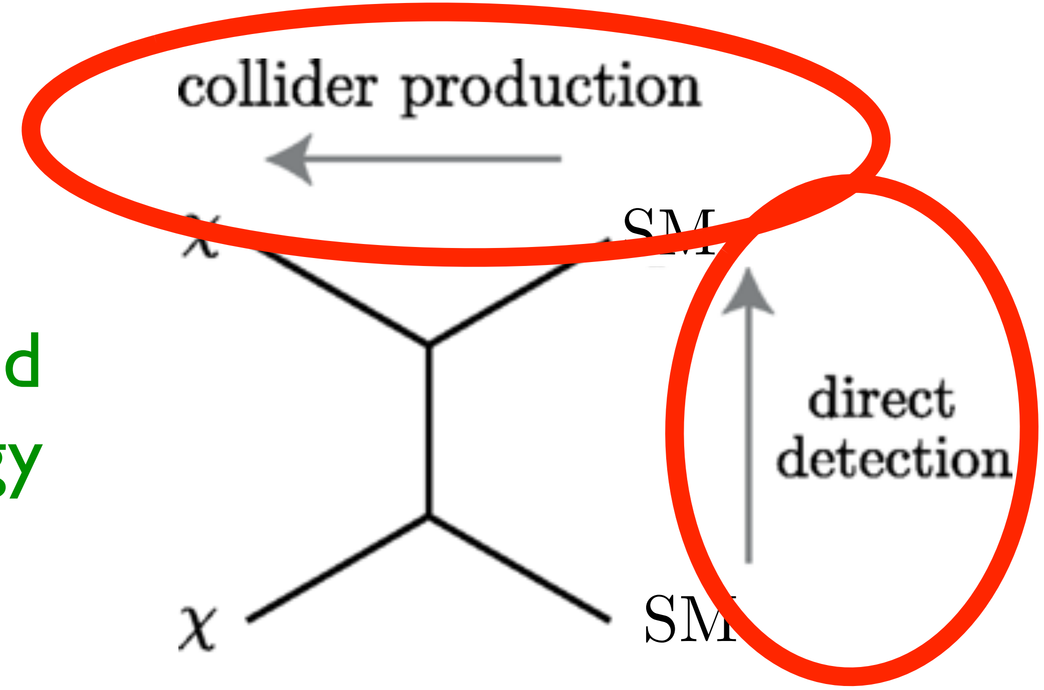
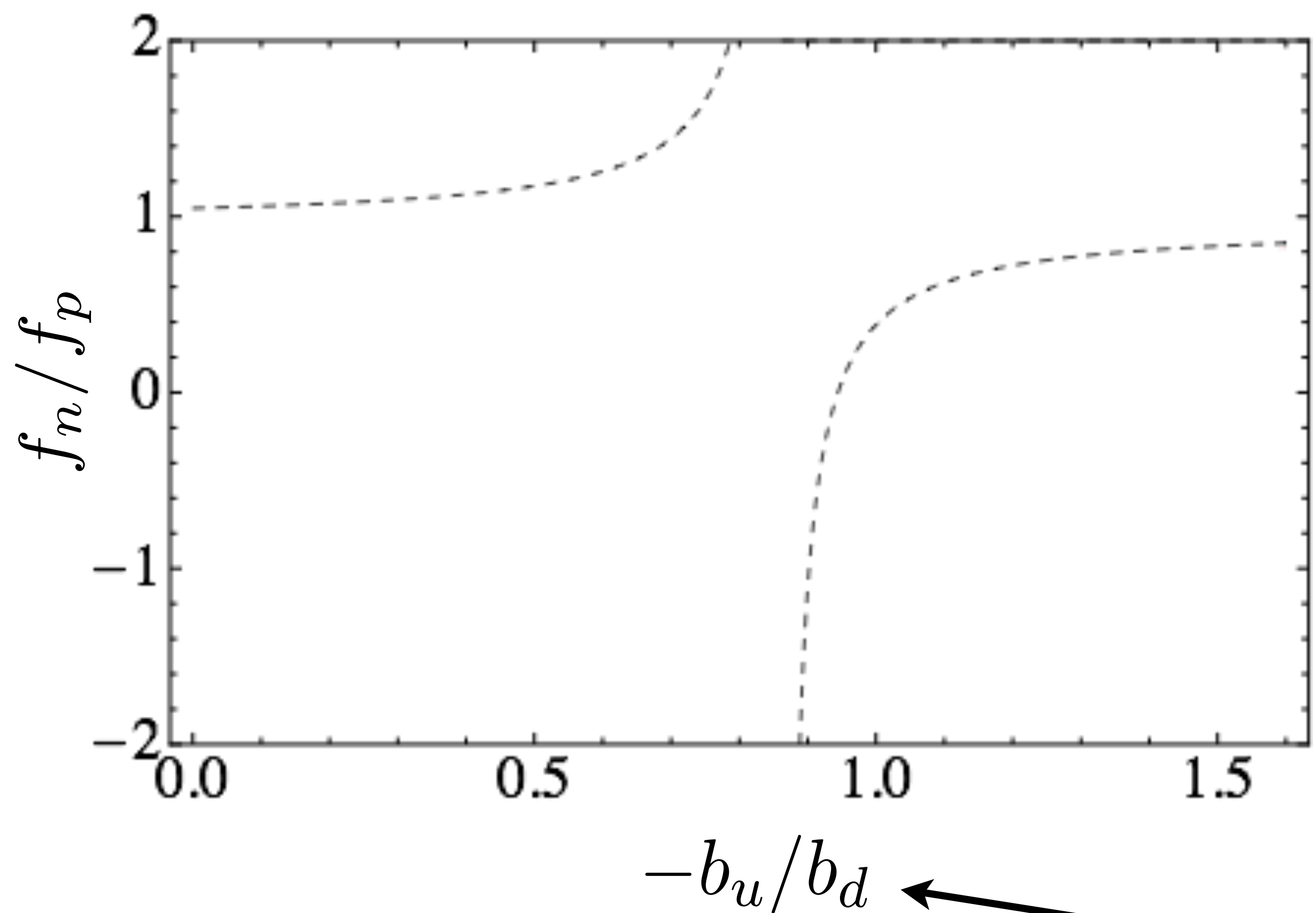
complete computation with in large mass limit with heavy WIMP effective theory

Not quibbling about percents (example 2: light WIMPs)

DM complementarity: connect direct detection and collider phenomenology





f_n/f_p = ratio of SI nucleon amplitudes for WIMP-nucleon scattering

four-fermion interactions constrained by collider bounds on missing energy signatures

$$\mathcal{L}_{\chi,\mathrm{SM}} = \bar{\chi}\chi\left[b_u \bar{u}u + b_d \bar{d}d\right]$$

Not quibbling about percents (example 2: light WIMPs)

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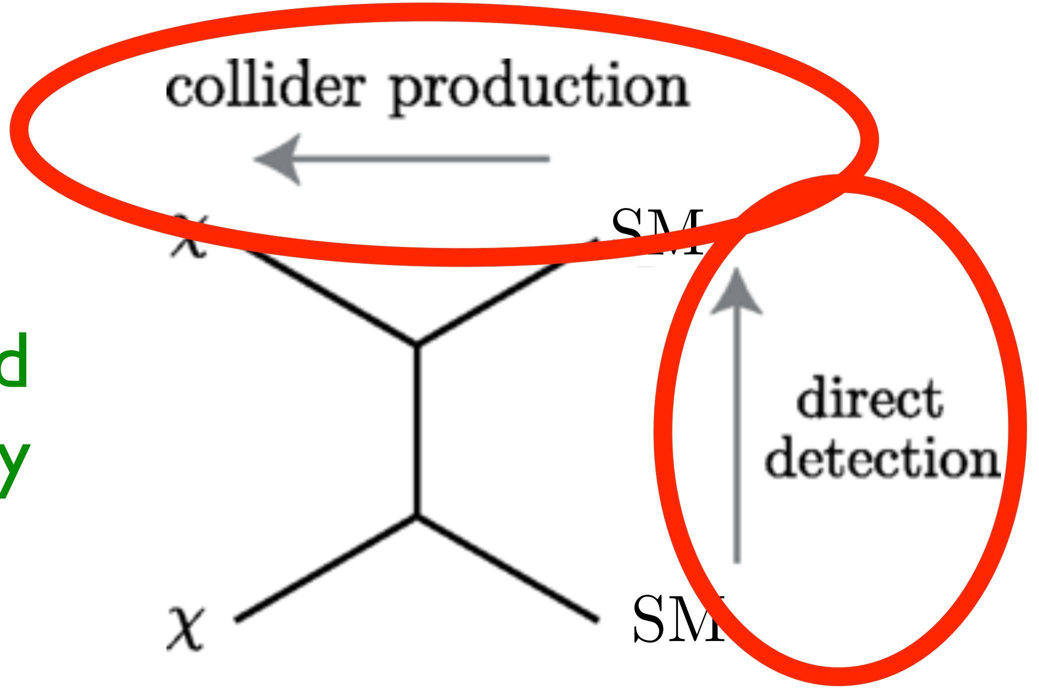

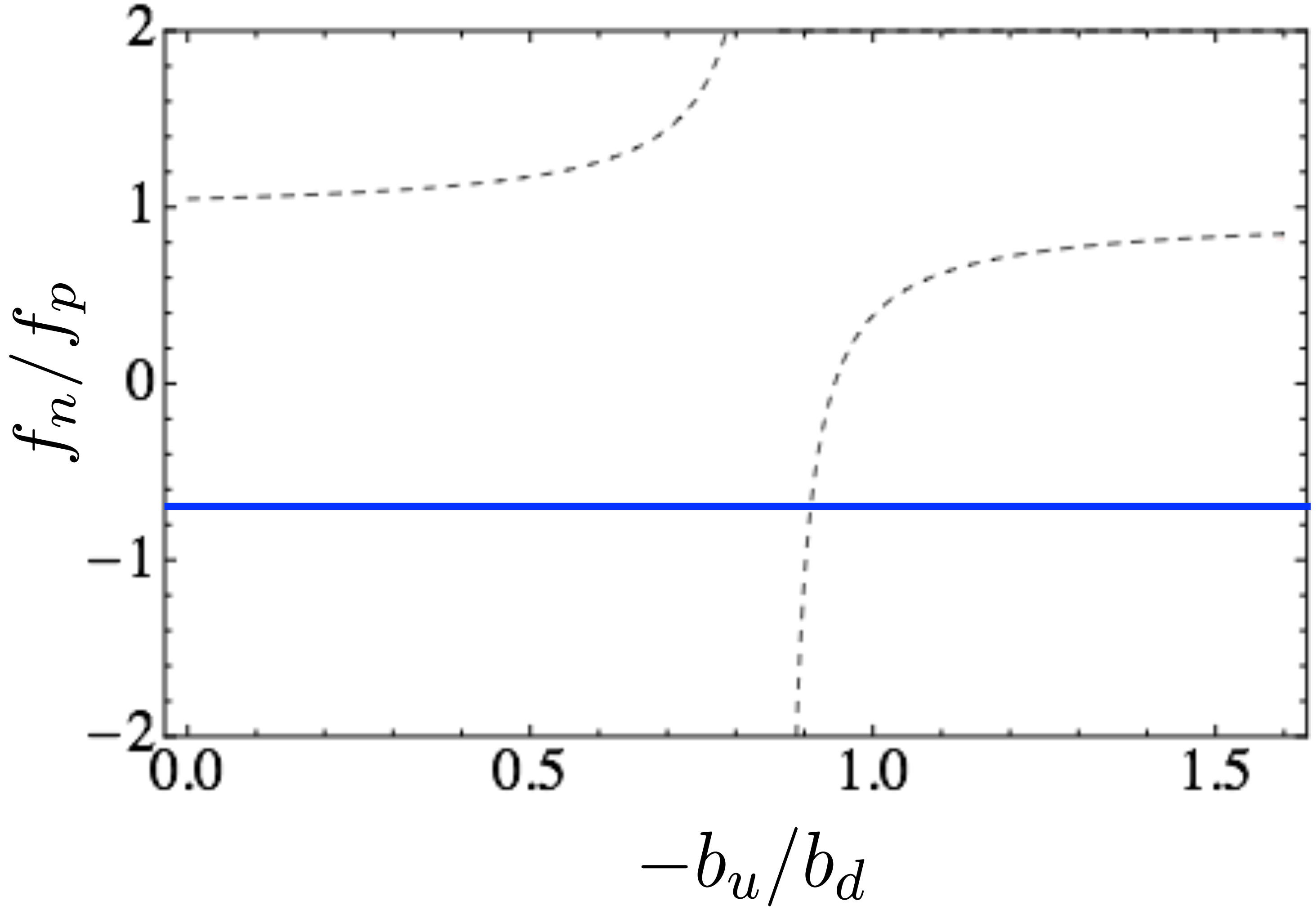

$f_n/f_p \approx -Z/(A-Z) \approx -0.7$ engineered to reconcile DAMA with results from Xe and other nuclei

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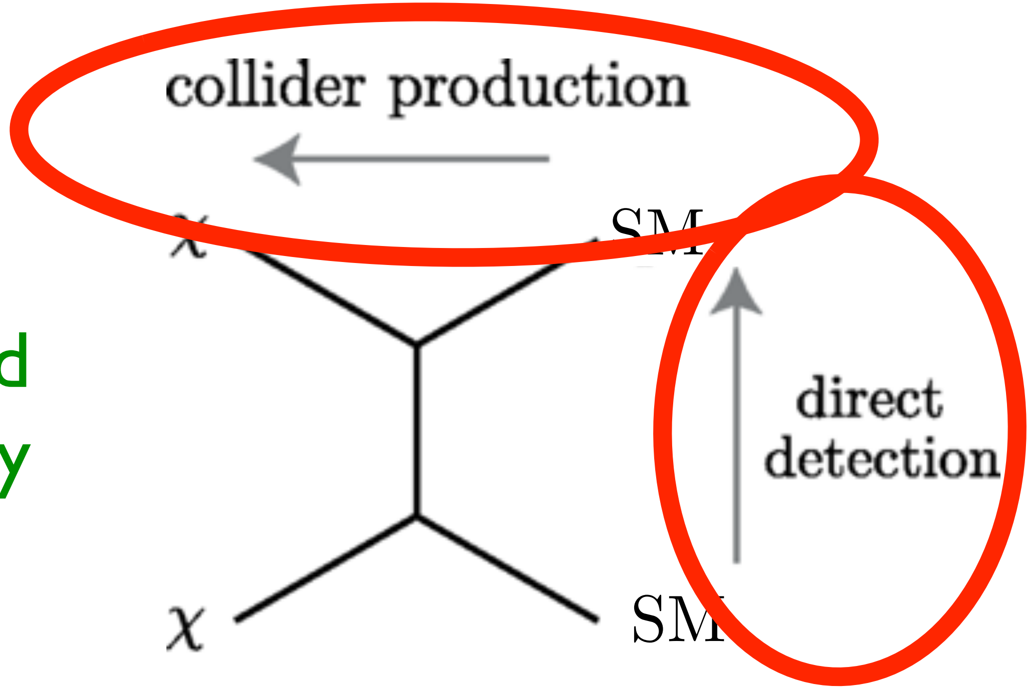

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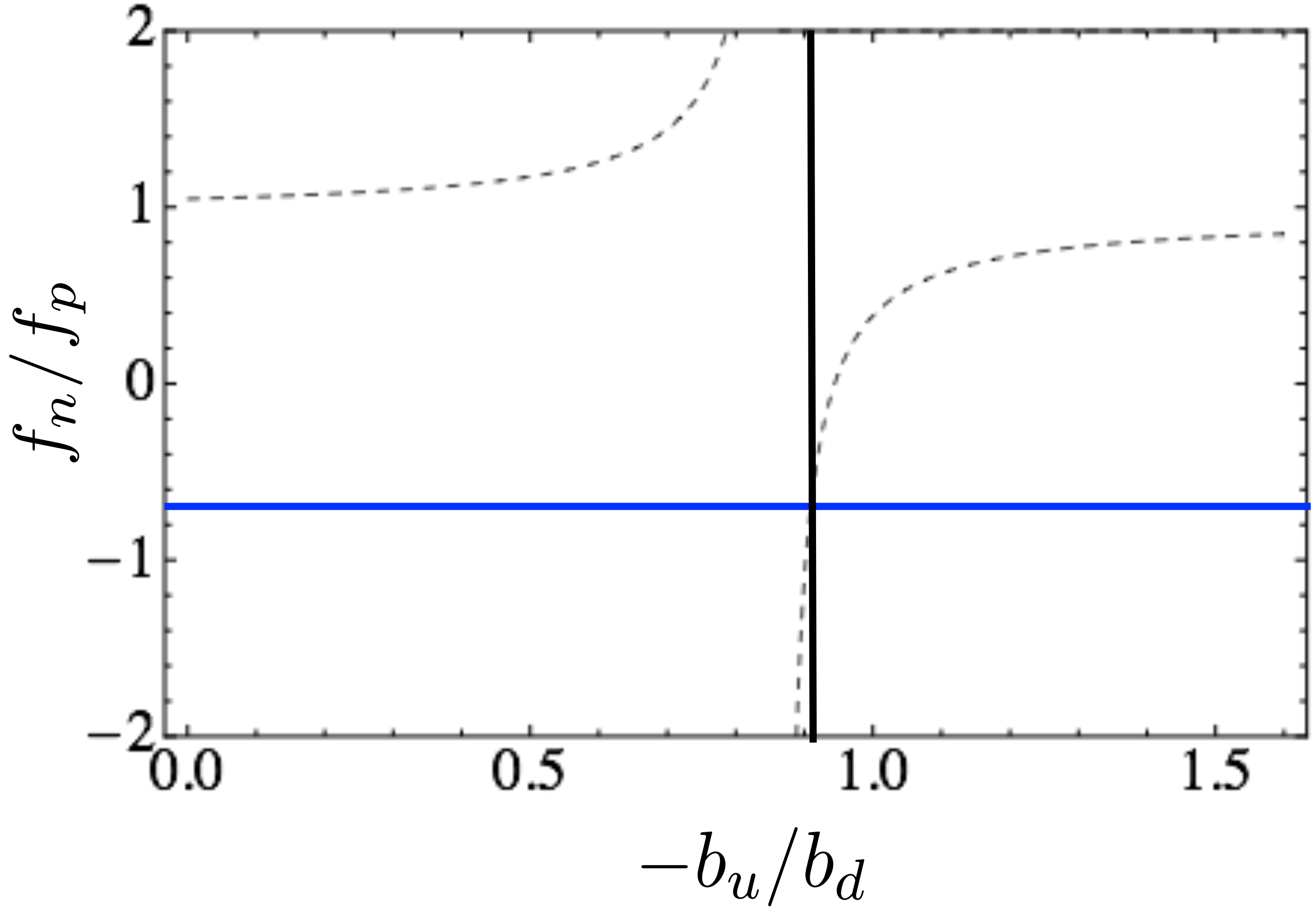

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Solution: $b_u/b_d=-0.9$

However, must account for uncertainties (hadronic and renormalization scale)

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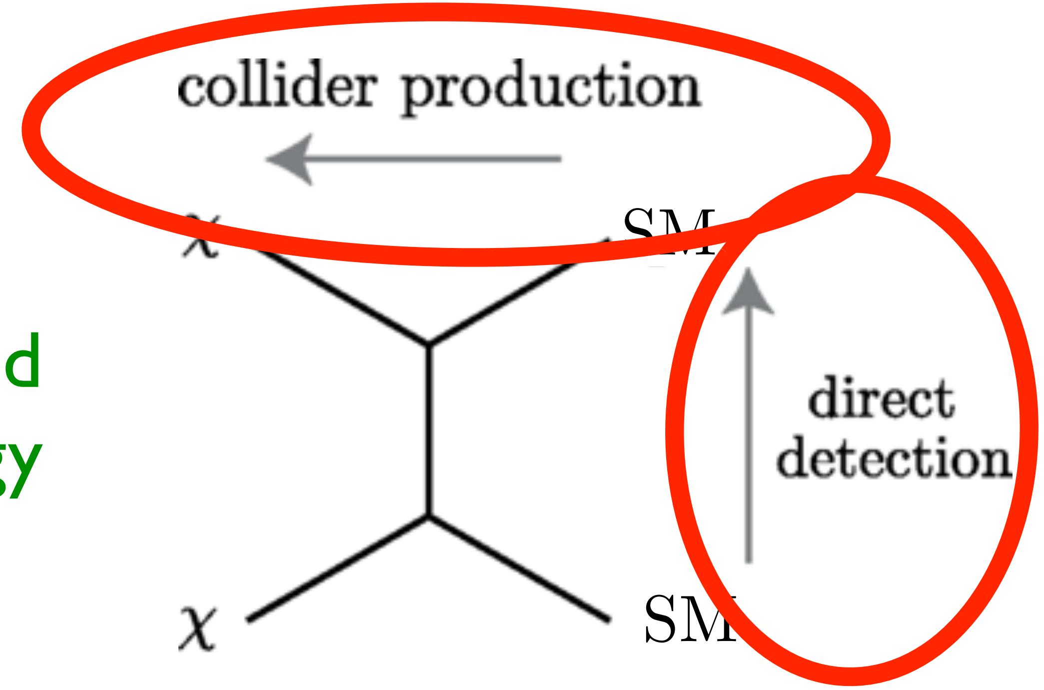

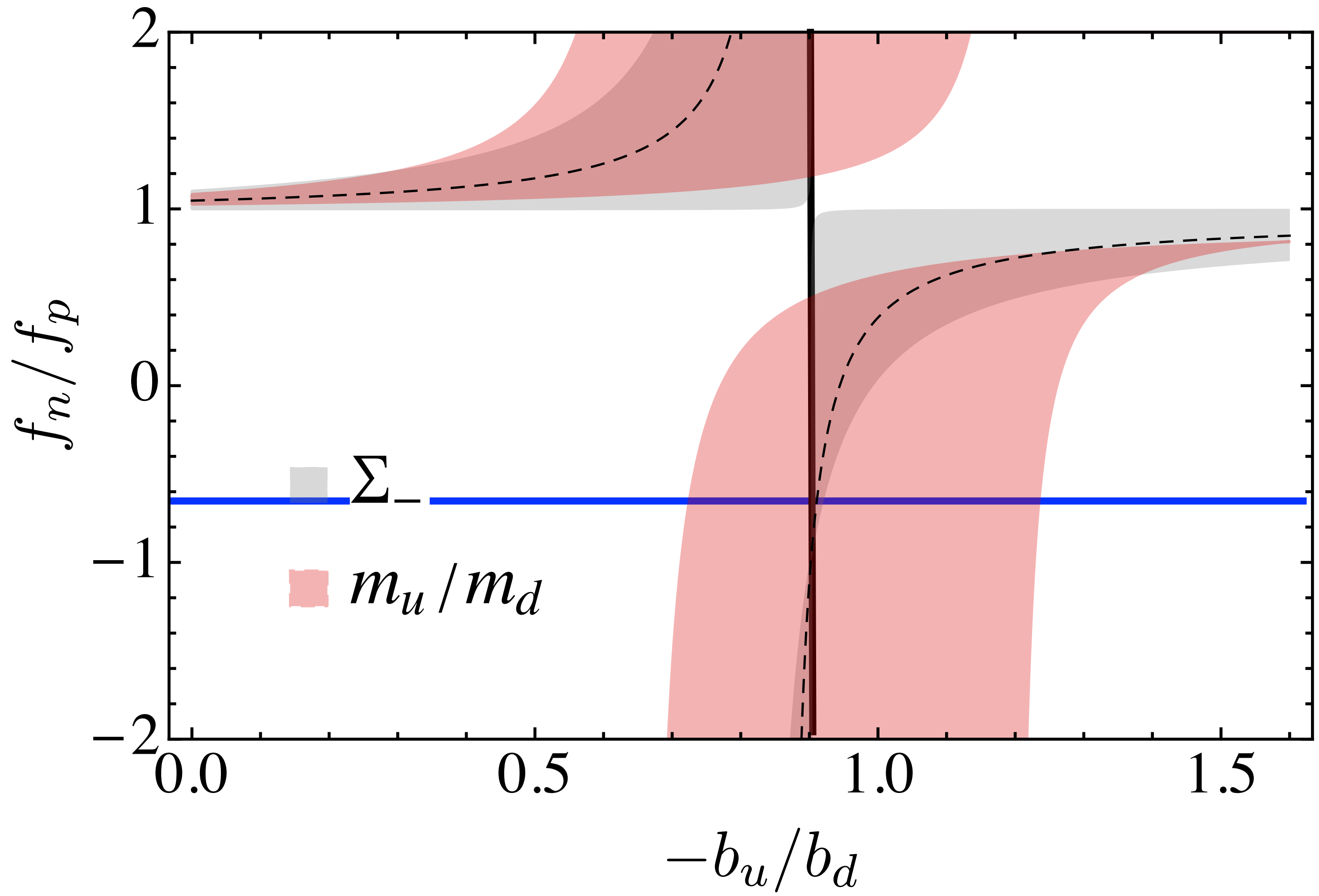

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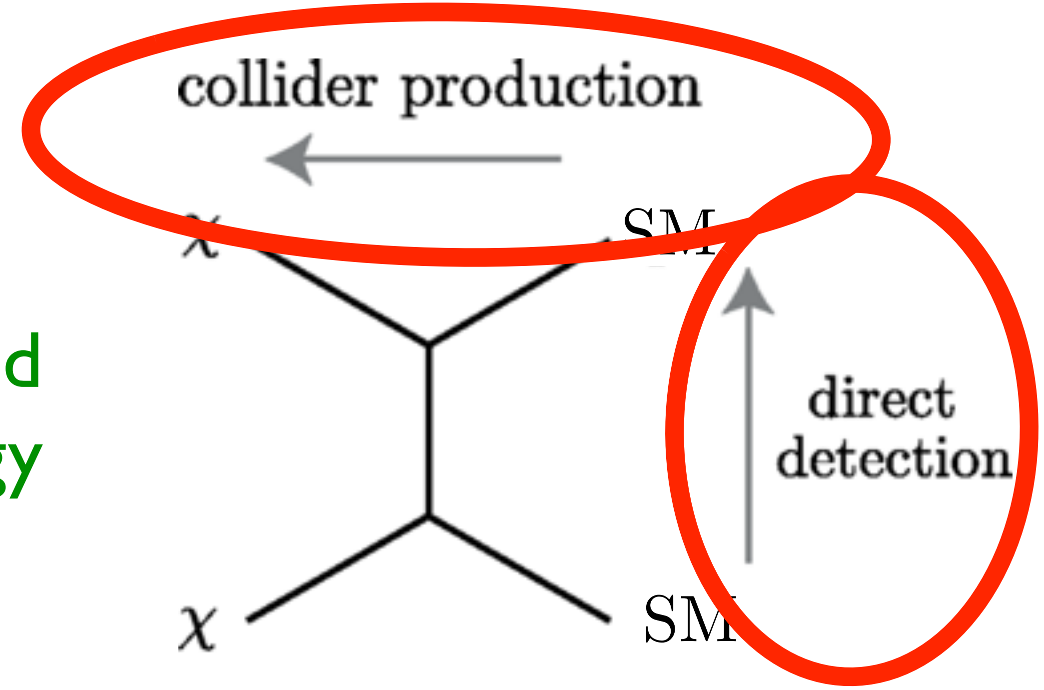

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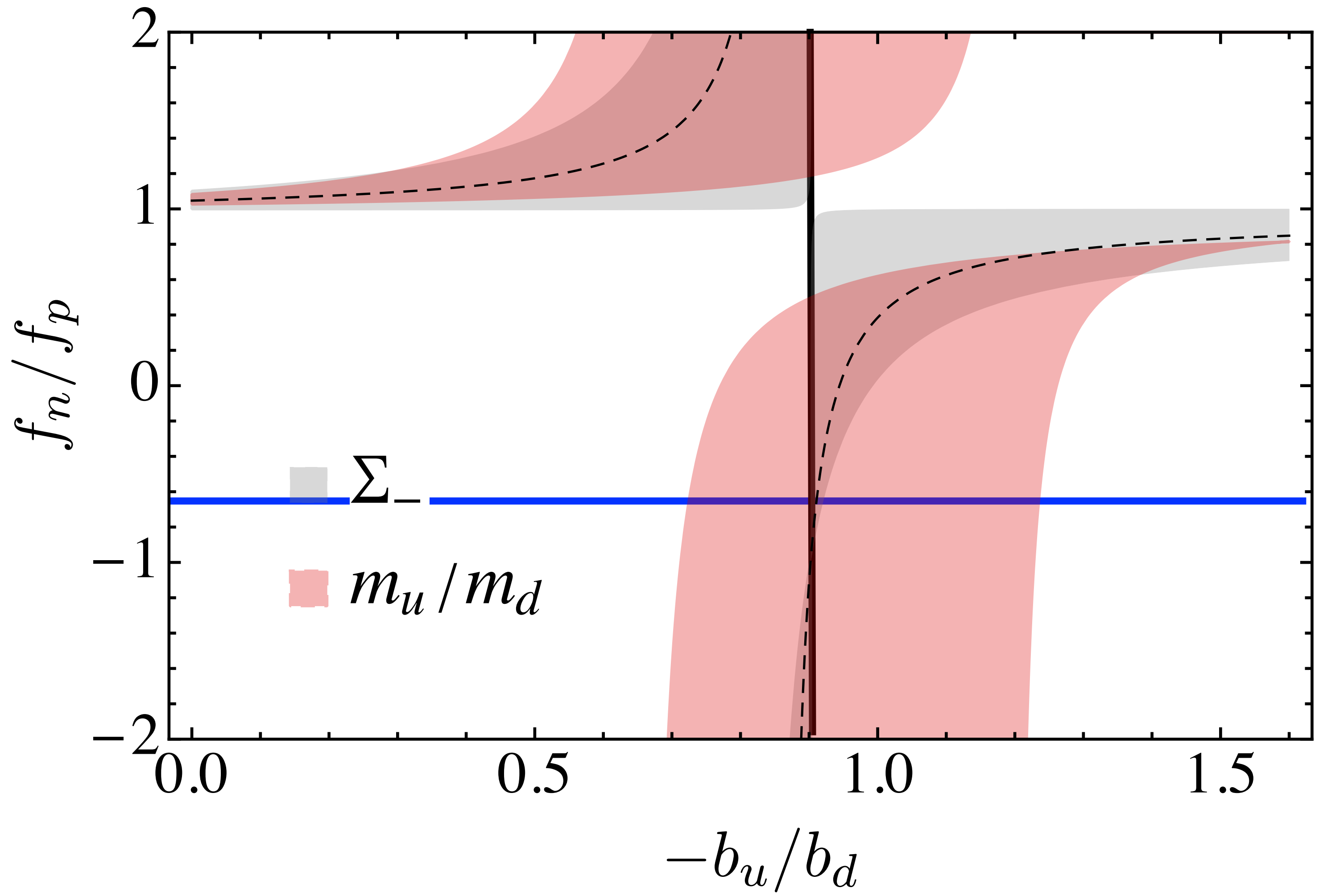

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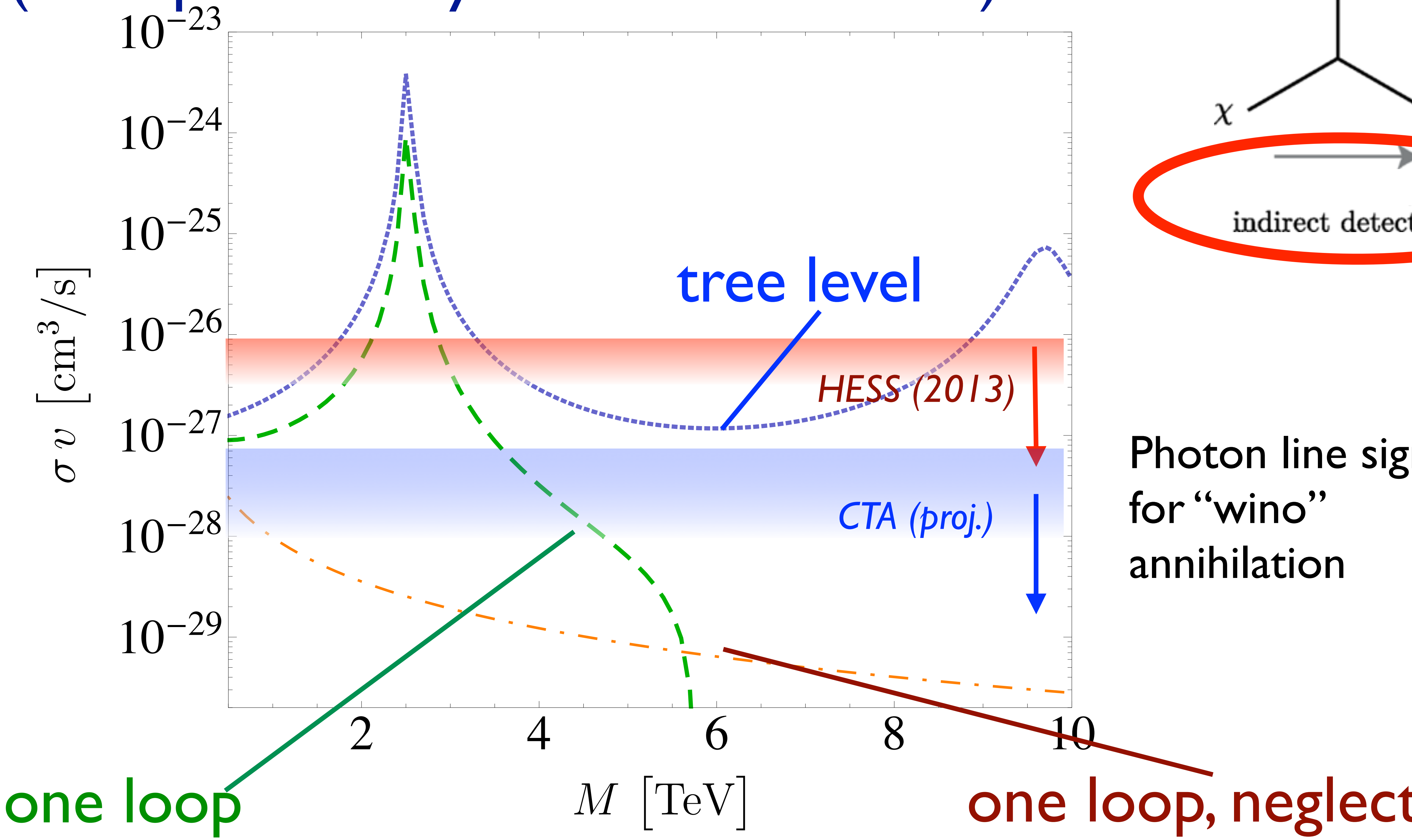
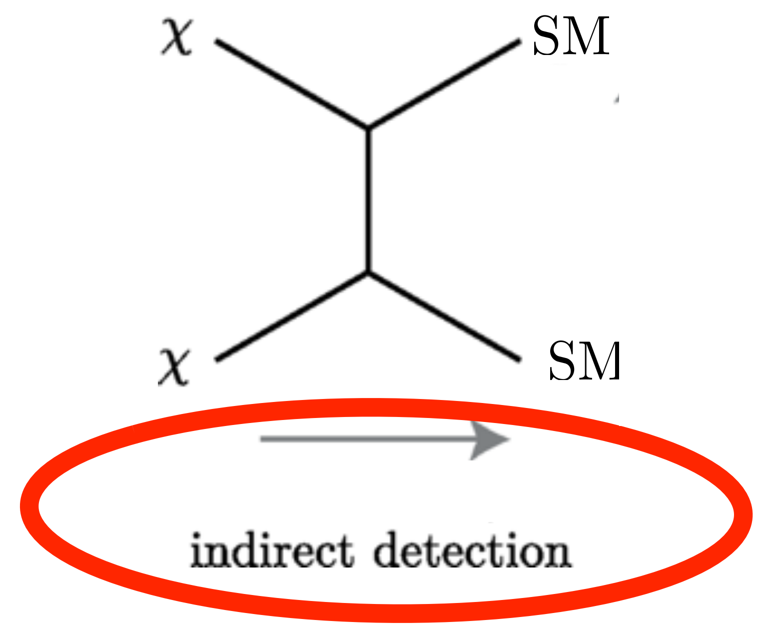
cf. b_u/b_d=-1.08 from “isospin-violating” DM

Assumed one-to-one mapping between b_u/b_d and f_n/f_p invalid

Nontrivial mapping from colliders to direct detection

Not quibbling about percents (example 3: heavy WIMP annihilation)

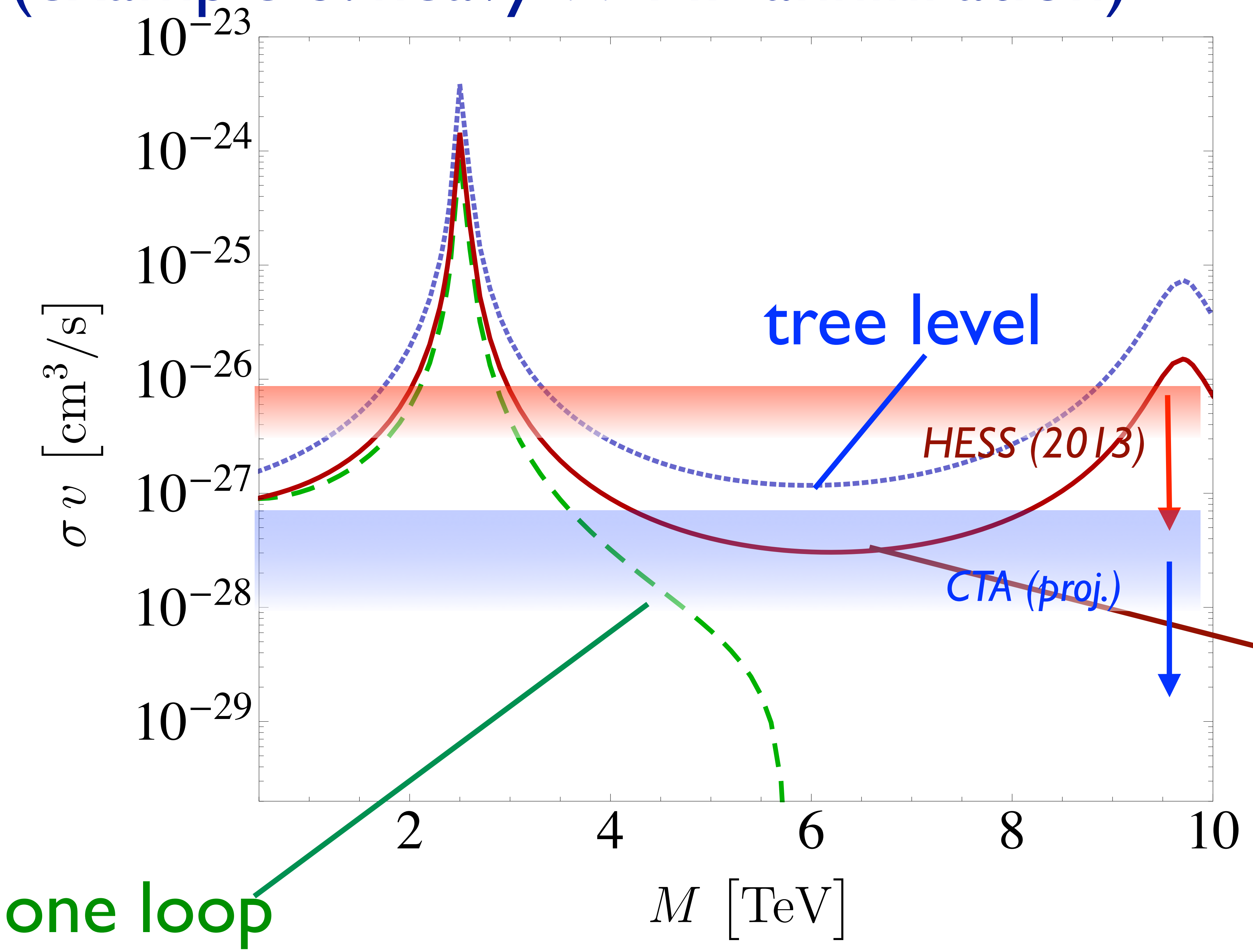


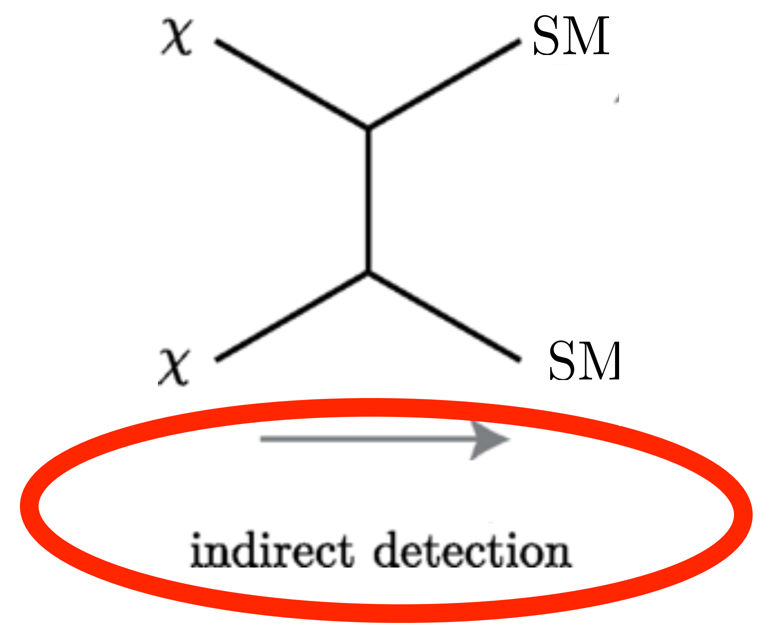


Photon line signal for “wino” annihilation

Multi-scale field theory problem, breakdown of naive perturbation theory

Not quibbling about percents (example 3: heavy WIMP annihilation)





Multi-scale field theory problem, breakdown of naive perturbation theory

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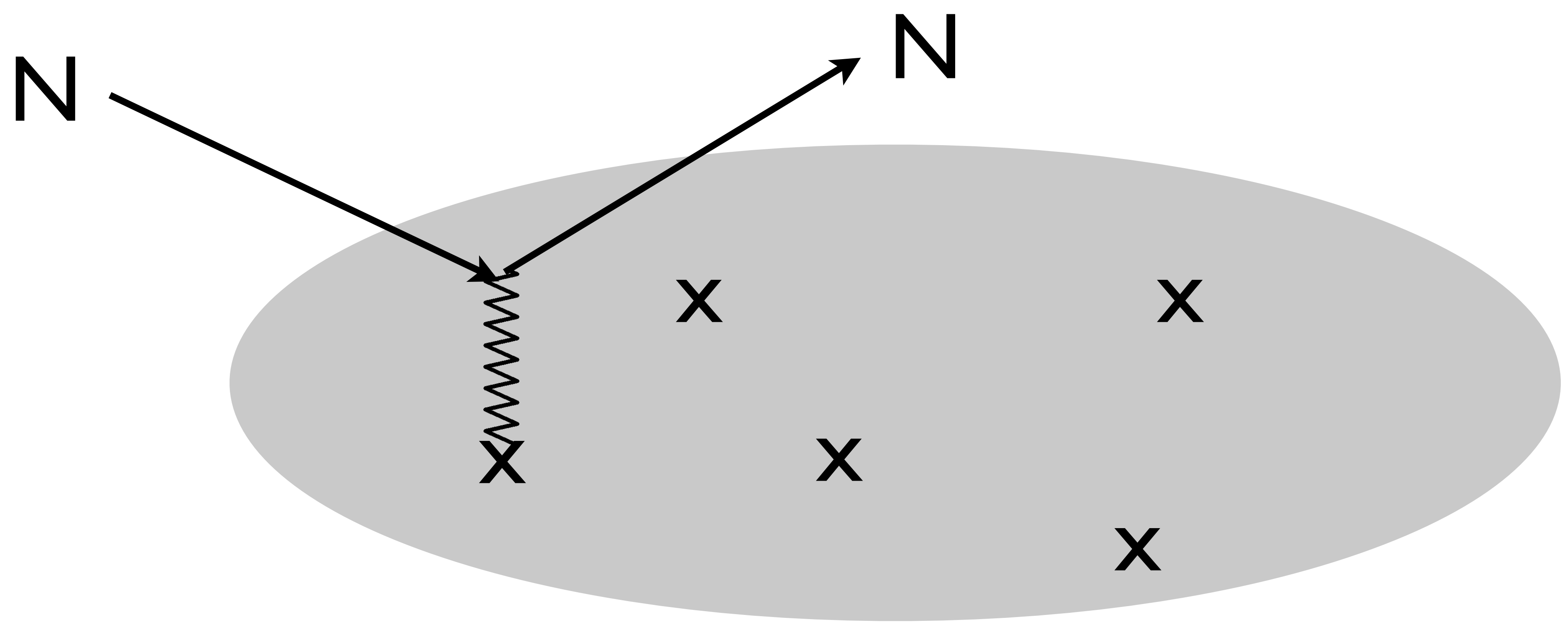
Mechanisms versus models

Electroweak charged WIMP Mechanism versus WIMP Model

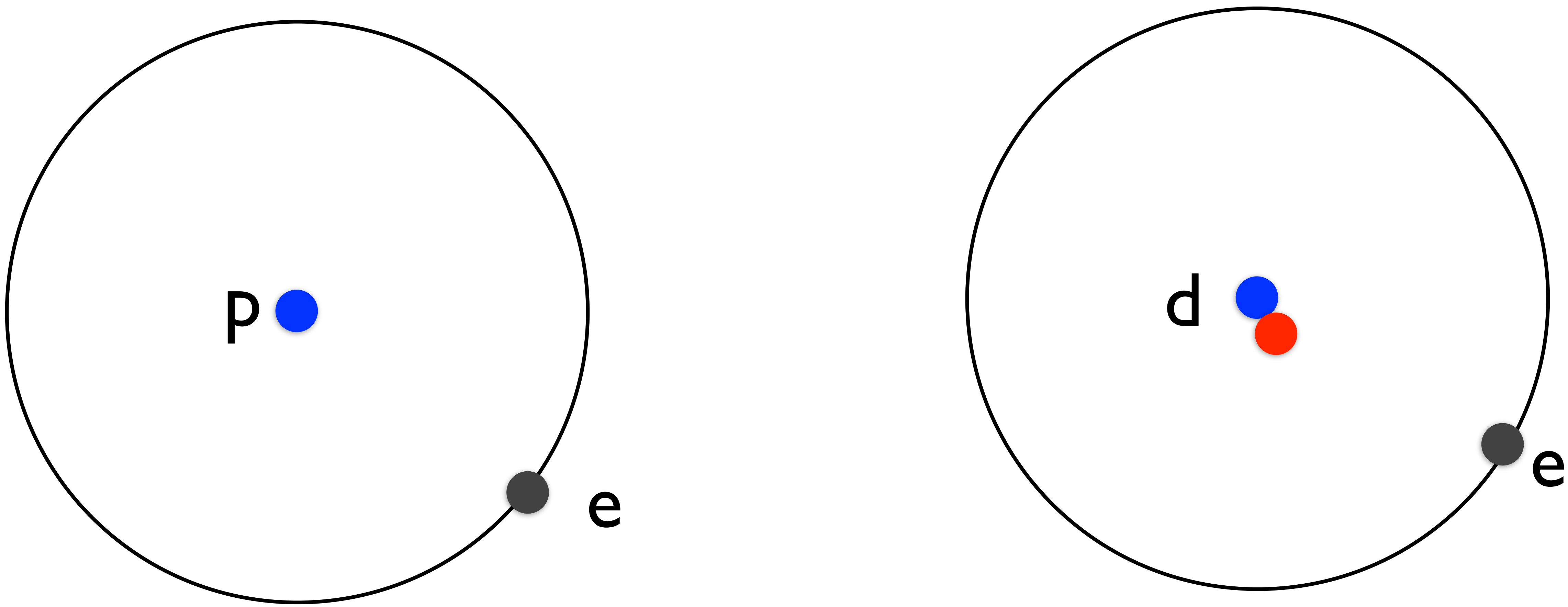


Focus on self-conjugate SU(2) triplet. Could be:

- SUSY wino
- Weakly Interacting Stable Pion
- Minimal Dark Matter
- ...

Basic idea:

We are all familiar with Heavy Particle Symmetry

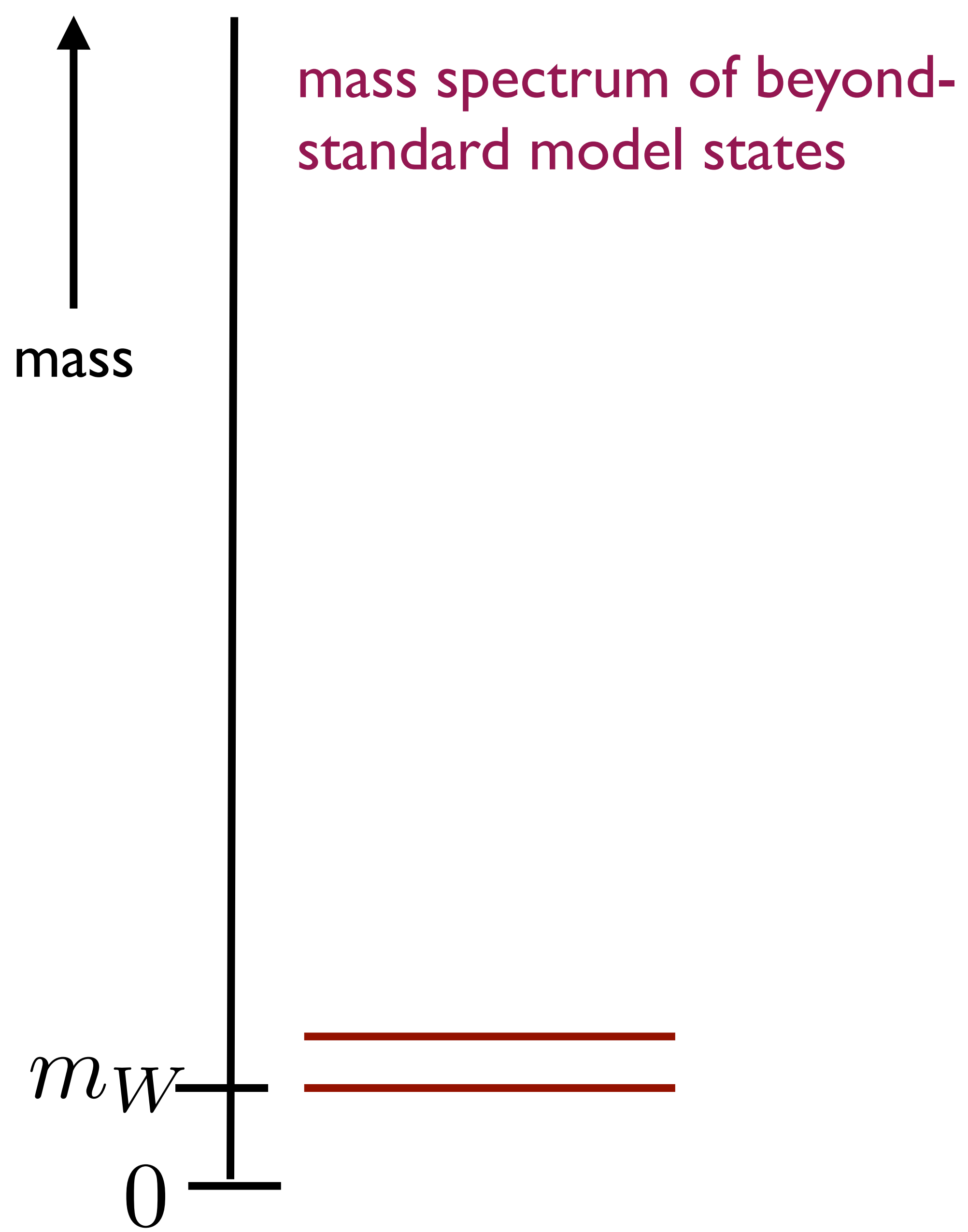


To leading order in p/M_{proton} the electron doesn't know about details of the nucleus beyond its charge

$$H_{\text{Hydrogen}} = H_{\text{Deuterium}} = \frac{p^2}{2m_e} - \frac{\alpha}{r}$$

Apply Heavy WIMP Symmetry to provide absolute predictions for dark matter observables

Present null results of direct detection and collider searches may indicate large WIMP mass scale



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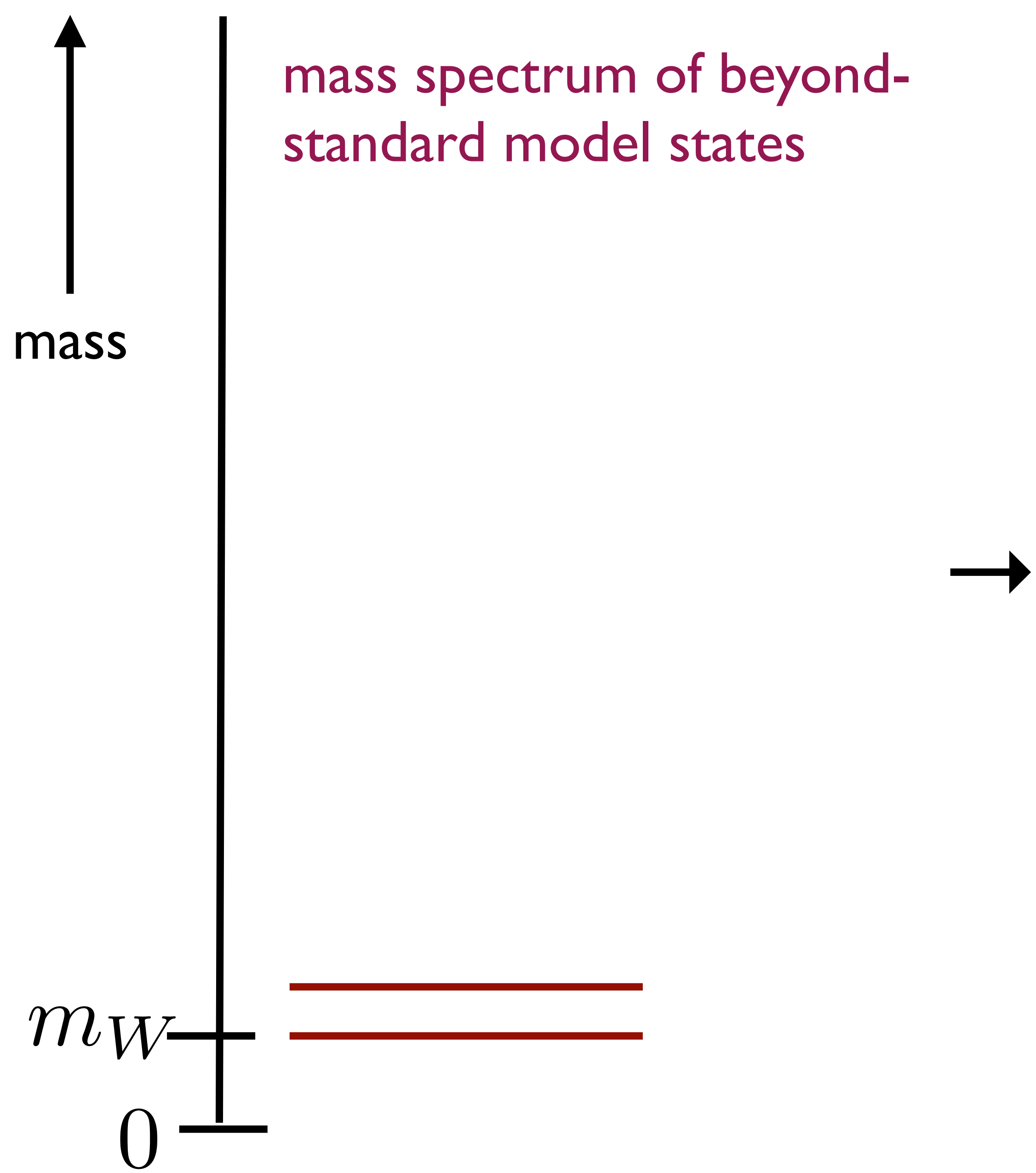
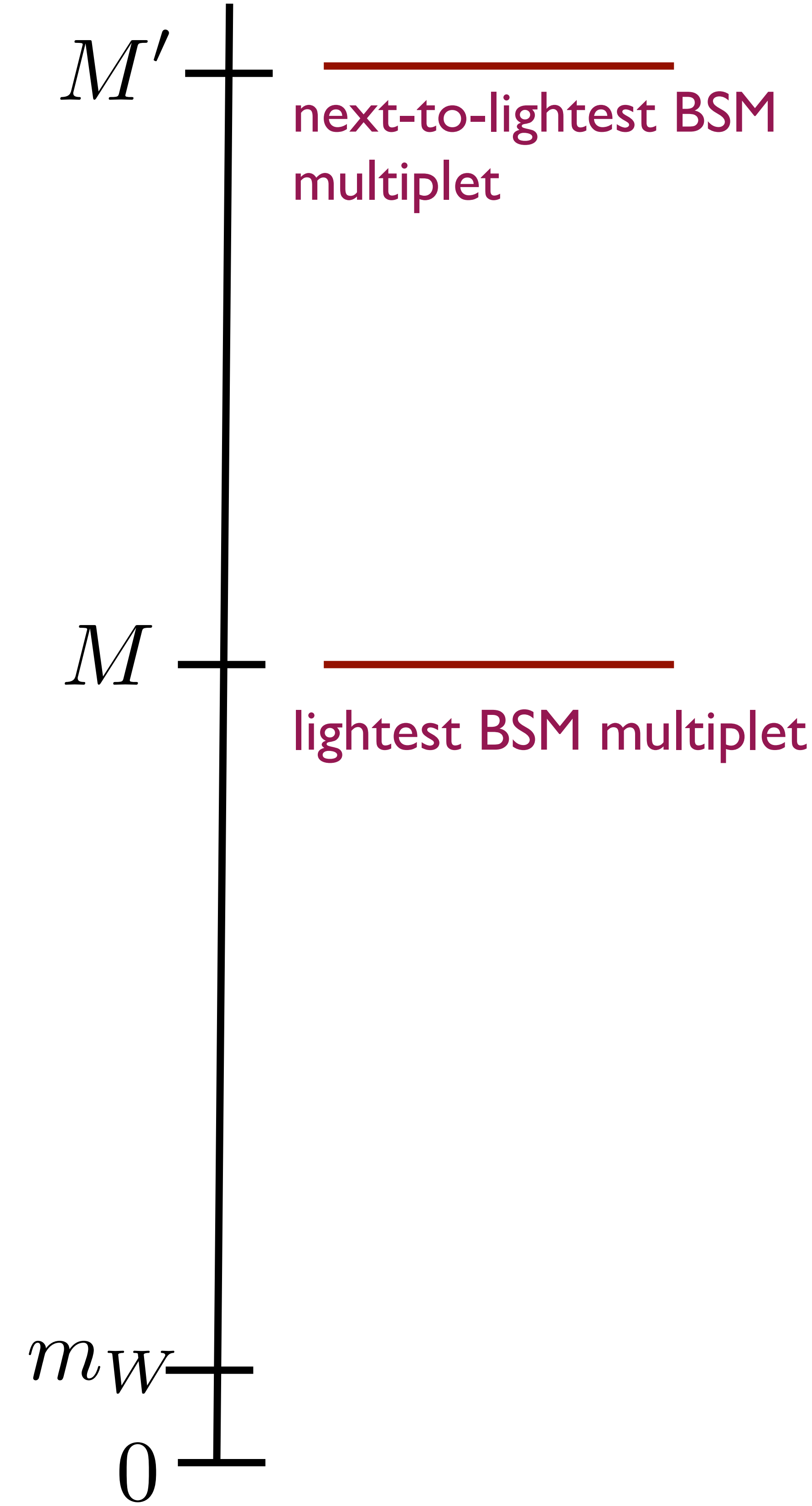
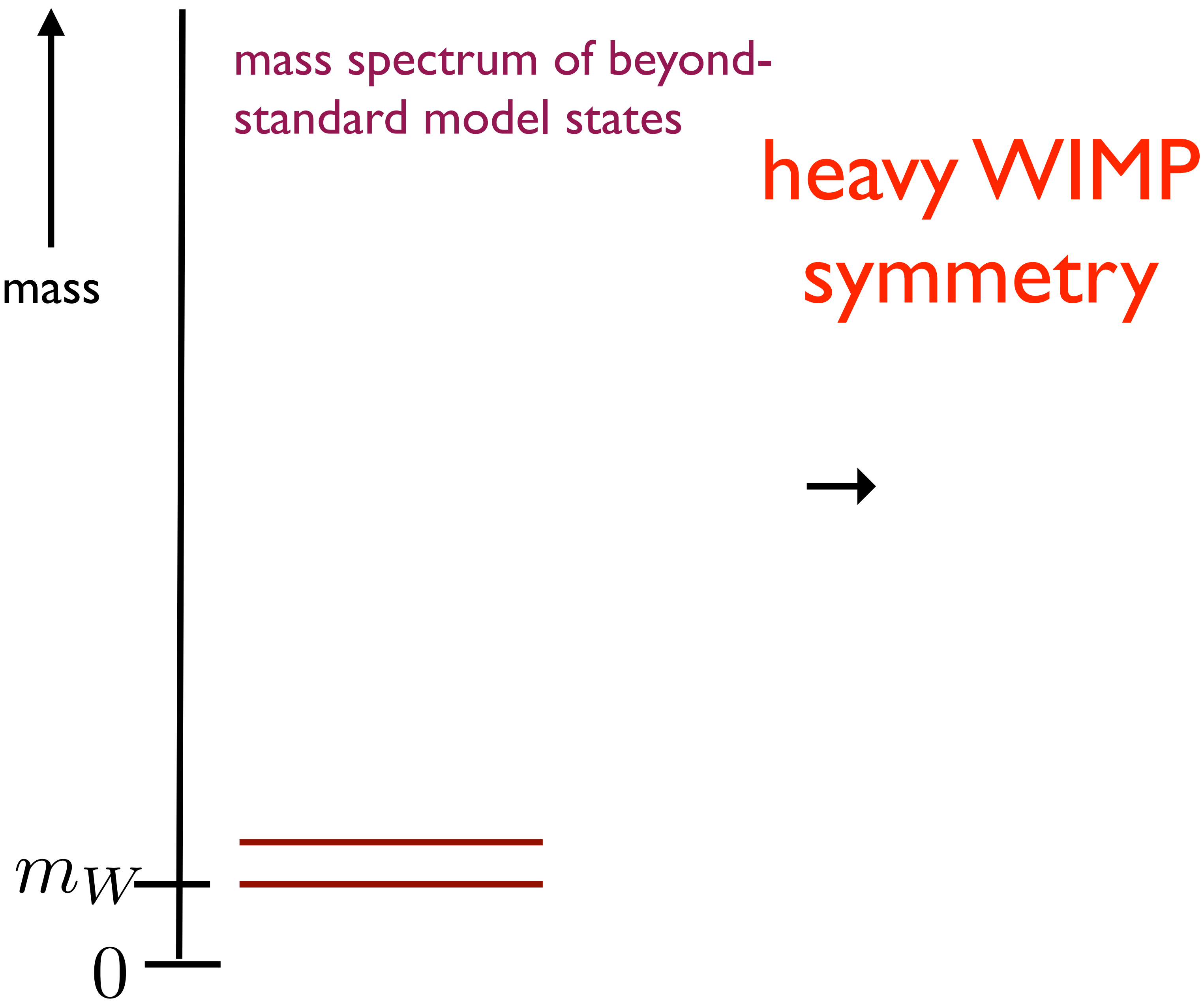


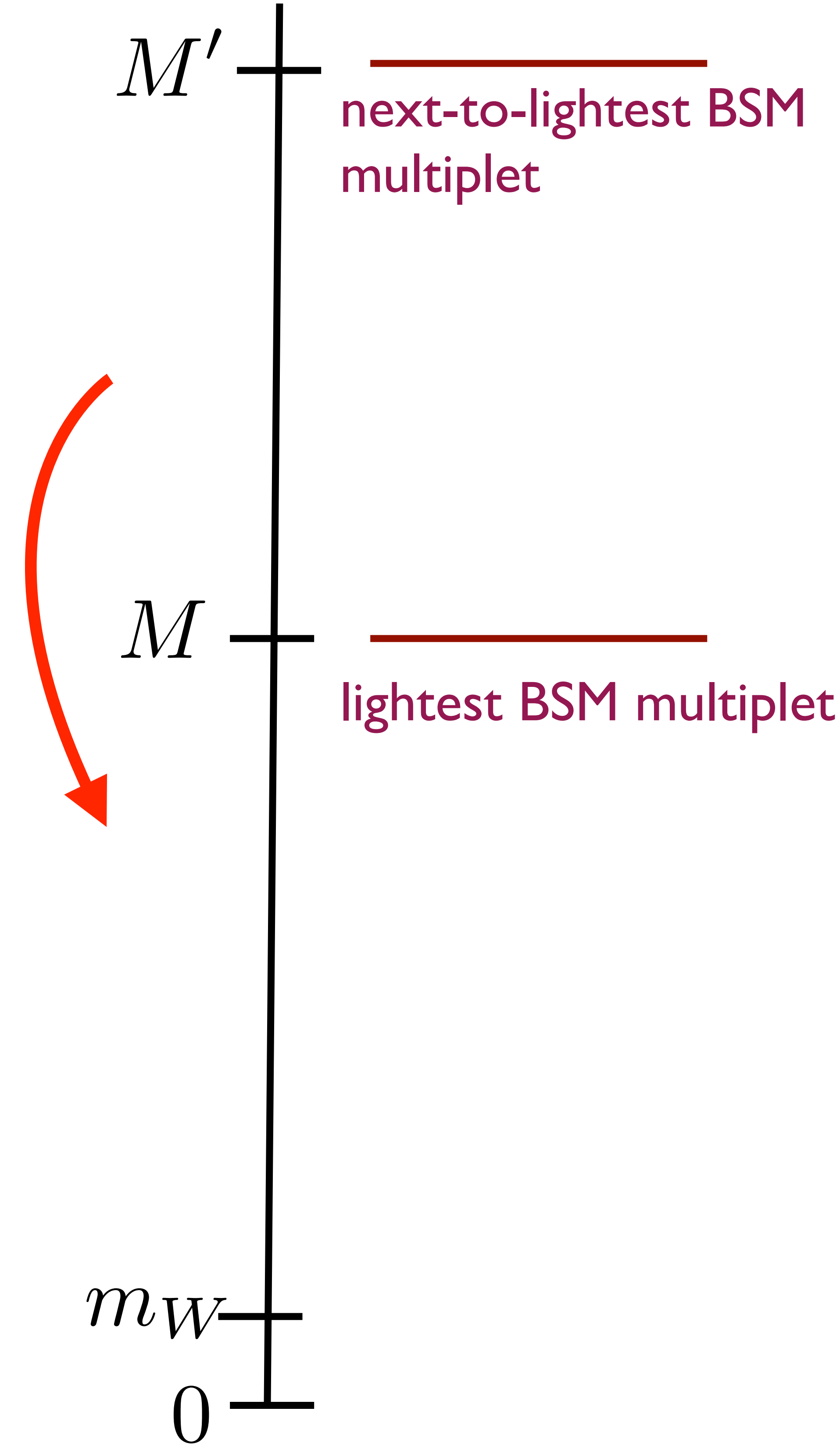


If WIMP mass M >> m_W , isolation (M'-M >> m_W) becomes generic. Expand in m_W/M, m_W/(M'-M)

Large WIMP mass regime is a focus of future experiments in direct, indirect and collider probes

Present null results of direct detection and collider searches may indicate large WIMP mass scale





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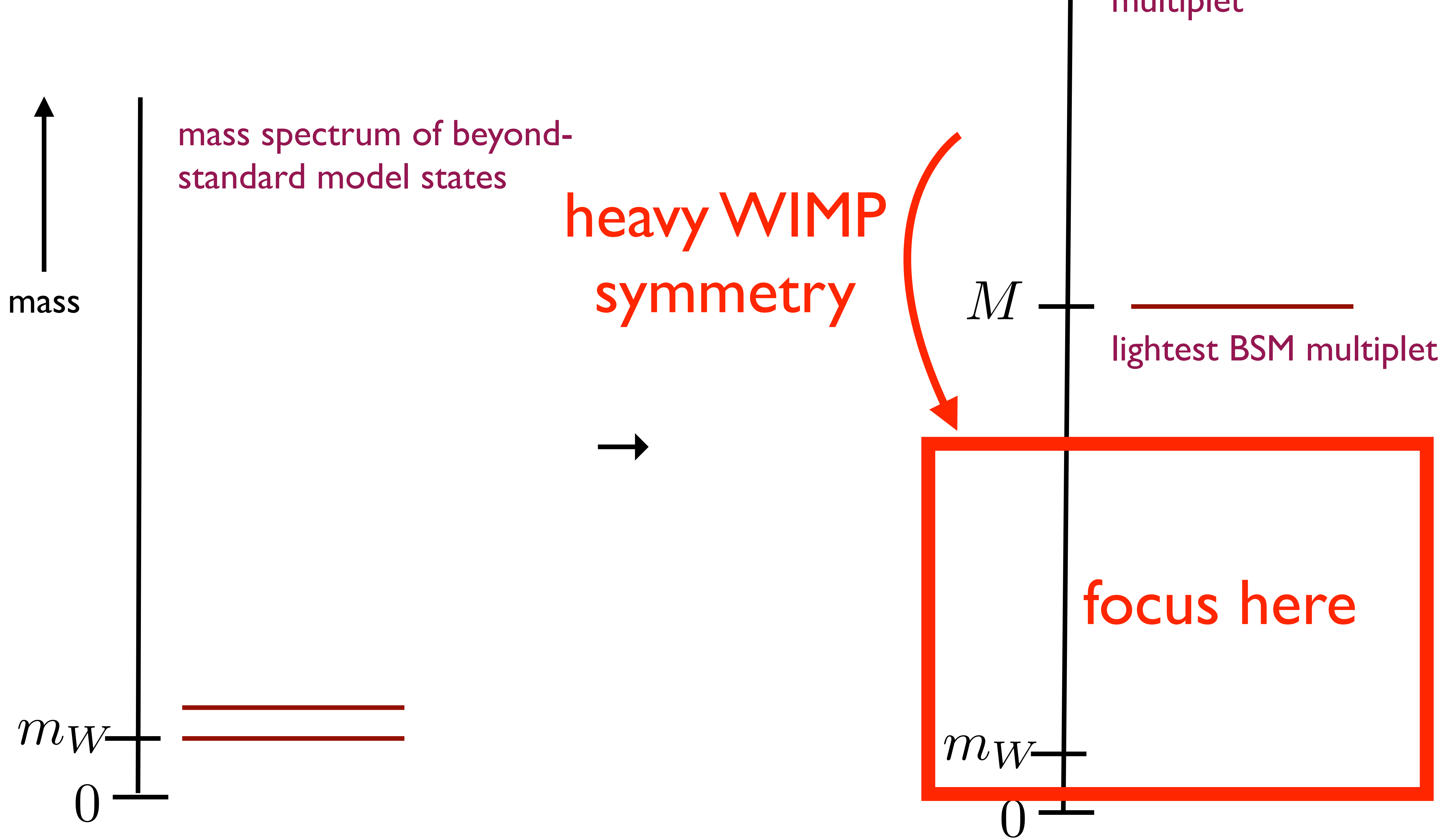

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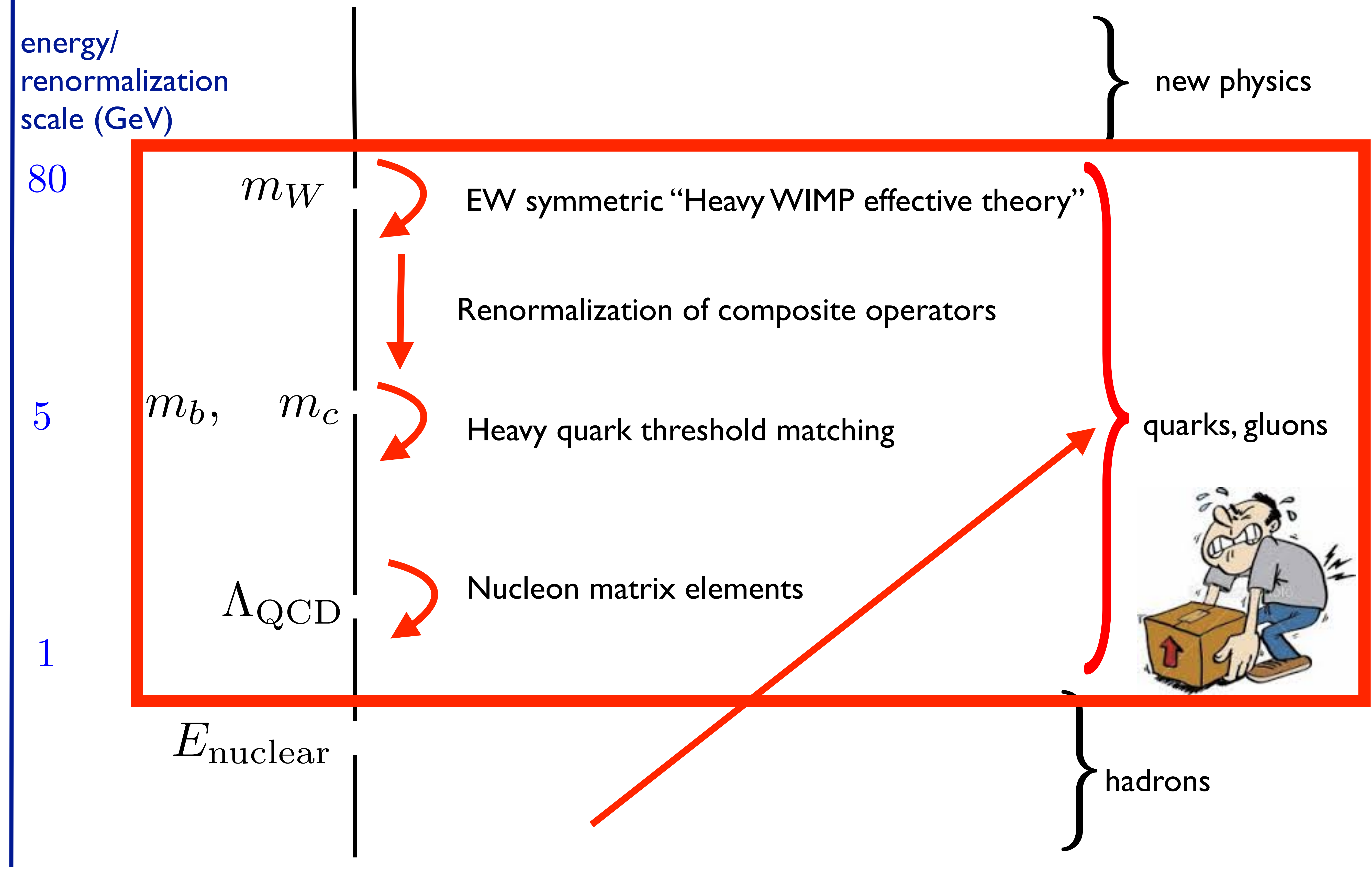
Five distinct regimes relevant for scattering on nuclear targets



"SM anatomy" of interactions between weak and hadronic scales

Start here: (e.g. fermion or composite boson UV completion)

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\bar{\tilde{w}}(i\not{D} - M)\tilde{w} \qquad \mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{4}(\hat{A}^a_{\mu\nu})^2 + \bar{\psi}(i\not{\partial} + \hat{g}\hat{\not{A}} + g_2\not{W})\psi$$

End up here

$$\mathcal{L} = N^\dagger\left(i\partial_t + \frac{\boldsymbol{\partial}^2}{2m_N}\right)N + \chi^\dagger\left(i\partial_t + \frac{\boldsymbol{\partial}^2}{2M}\right)\chi + c_{\rm SI}N^\dagger N\chi^\dagger\chi + \ldots$$

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Fill in here

End up here

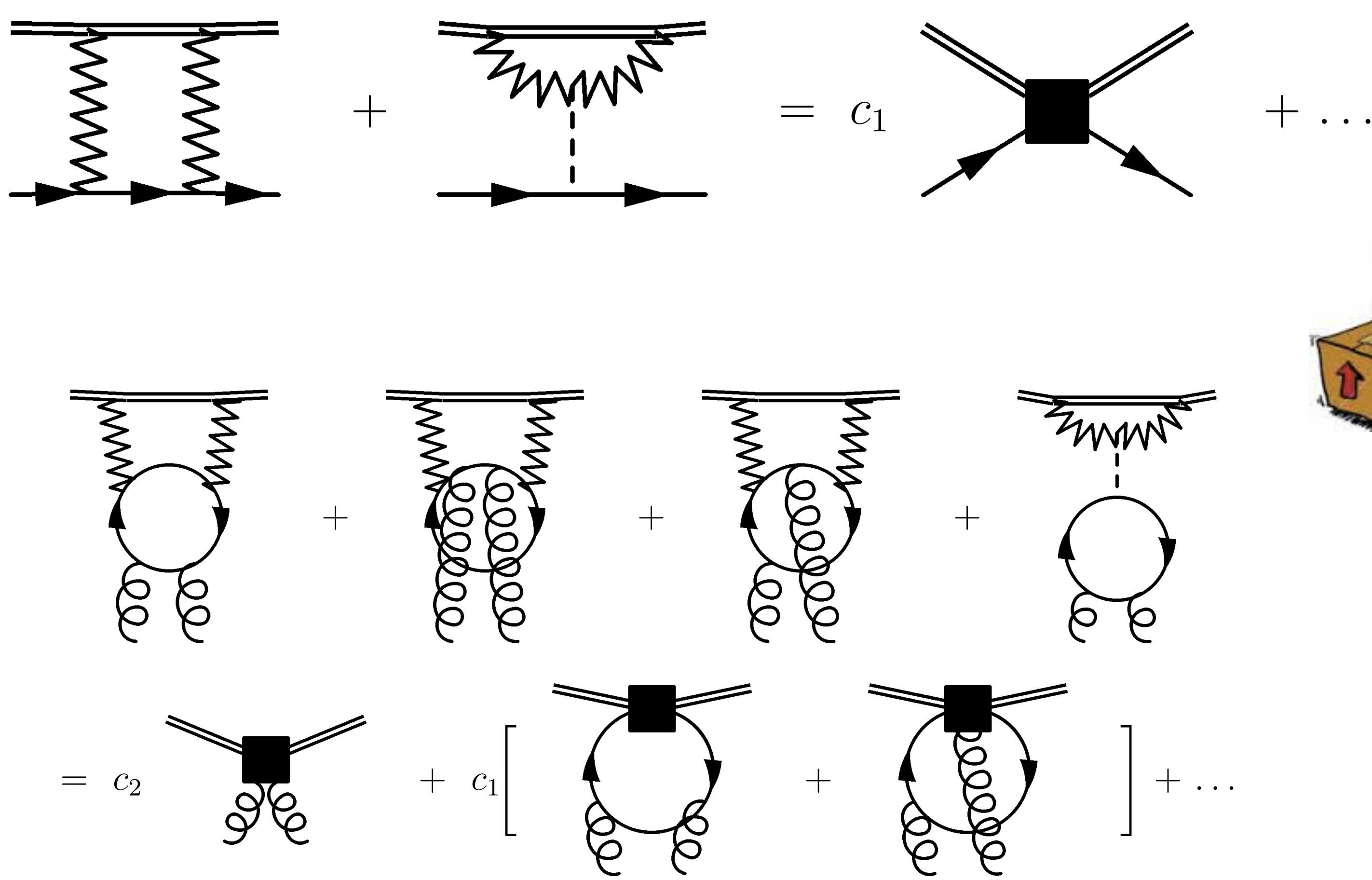
$$\mathcal{L} = N^\dagger\left(i\partial_t + \frac{\boldsymbol{\partial}^2}{2m_N}\right)N + \chi^\dagger\left(i\partial_t + \frac{\boldsymbol{\partial}^2}{2M}\right)\chi + c_{\rm SI}N^\dagger N\chi^\dagger\chi + \dots$$

"SM anatomy" of interactions between weak and hadronic scales

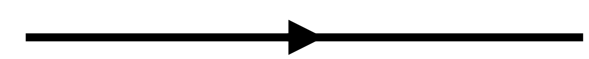
Heavy particle symmetry and weak-scale matching

12 operators (classified as spin-0 and spin-2) and 12 coefficients

$$\mathcal{L}_{\phi_0,\mathrm{SM}} = \frac{1}{m_W^3}\phi_v^*\phi_v\left\{\sum_q\left[c_{1q}^{(0)}O_{1q}^{(0)} + c_{1q}^{(2)}v_\mu v_\nu O_{1q}^{(2)\mu\nu}\right] + c_2^{(0)}O_2^{(0)} + c_2^{(2)}v_\mu v_\nu O_2^{(2)\mu\nu}\right\} + \dots$$

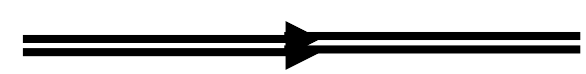


Besides universality,
Heavy WIMP Effective Theory
Feynman rules drastically
simplifiy integrals:



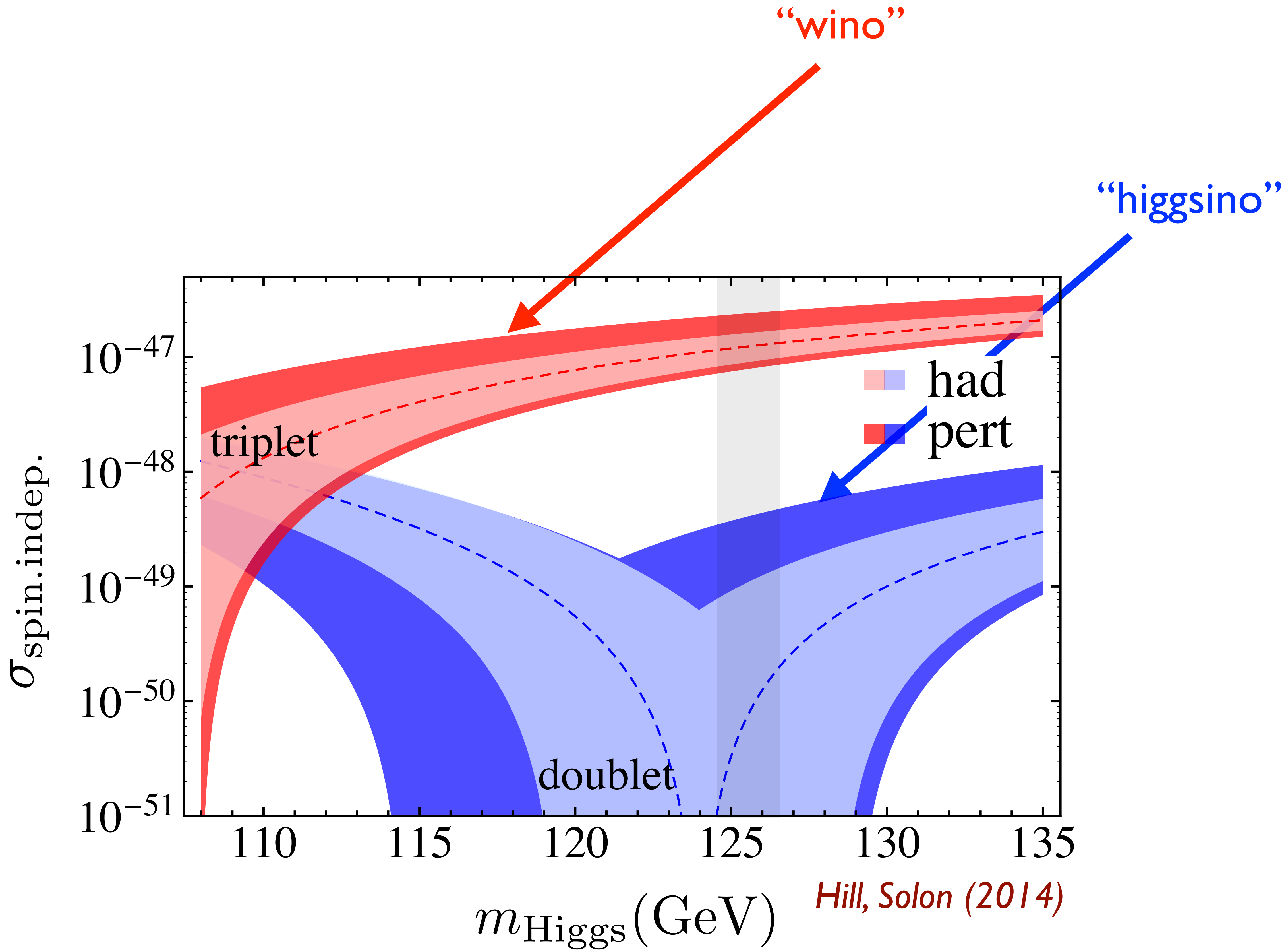
$$\frac{i}{L^2 - M^2 + i\epsilon}$$

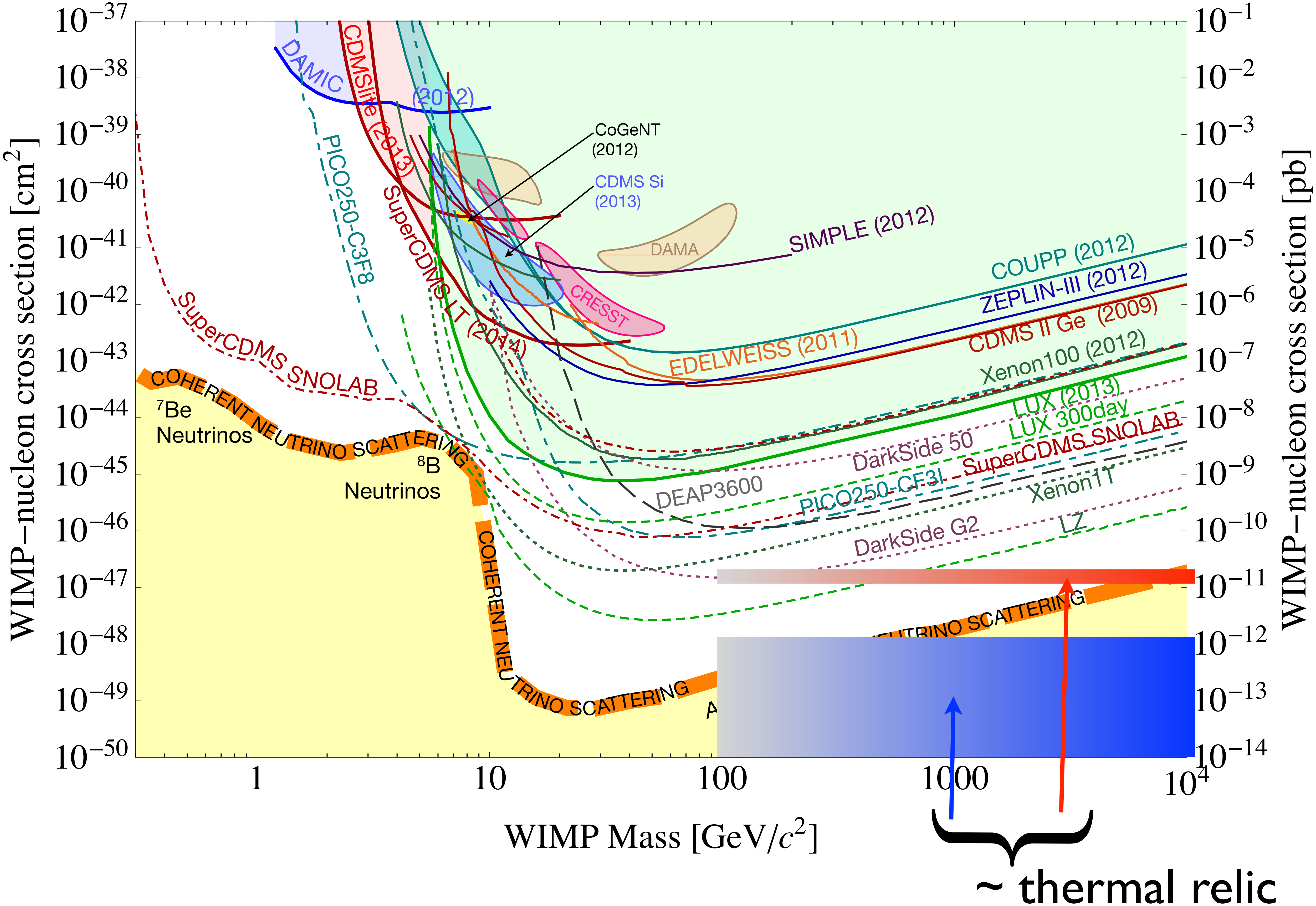
vs.:



$$\frac{i}{v \cdot L + i\epsilon}$$

Benchmarks: pure states





WIMP–nucleon cross section [cm²]
WIMP–nucleon cross section [pb]
WIMP Mass [GeV/c²]
DAMIC
CDMSlite (2013)
(2012)
CoGeNT (2012)
CDMS Si (2013)
PICO250-C3F8
SuperCDMS LT (2014)
SIMPLE (2012)
DAMA
CRESST
COUPP (2012)
ZEPLIN-III (2012)
CDMS II Ge (2009)
EDELWEISS (2011)
Xenon100 (2012)
LUX (2013)
LUX 300day
SuperCDMS SNOLAB
DarkSide 50
DEAP3600
PICO250-CF3I
Xenon1T
DarkSide G2
LZ
SuperCDMS SNOLAB
COHERENT NEUTRINO SCATTERING
⁷Be Neutrinos
⁸B Neutrinos
~ thermal relic

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Dark matter - Standard Model interactions

$$\mathcal{L} = \frac{1}{\Lambda^n} O_{\rm DM} \times O_{\rm SM}$$

d	Fermion
3	$\bar\psi[1\,,\ i\gamma_5\,,\ \gamma^\mu\gamma_5\,,\ \{\gamma^\mu\,,\ \sigma^{\mu\nu}\}]\psi$
4	$\bar\psi[\{1\,,\ i\gamma_5\,,\ \gamma^\mu\gamma_5\}\,,\ \gamma^\mu\,,\ \sigma^{\mu\nu}]i\partial^\rho_-\psi$

d	Scalar
2	$\lvert\phi\rvert^2$
3	$\{\phi^* i\partial^\mu_-\phi\}$

d	Heavy particle
3	$\bar\chi_v[1\,,\ \{\sigma^{\mu\nu}_\perp\}]\chi_v$
4	$\bar\chi_v[\{1\}\,,\ \sigma^{\mu\nu}_\perp]i\partial^\rho_{\perp -}\chi_v$

d	QCD operator basis
3	$V^\mu_q = \bar q\gamma^\mu q$
	$A^\mu_q = \bar q\gamma^\mu\gamma_5 q$
4	$T^{\mu\nu}_q = im_q\bar q\sigma^{\mu\nu}\gamma_5 q$
	$O^{(0)}_q = m_q\bar q q\,,\quad O^{(0)}_g = G^A_{\mu\nu}G^{A\mu\nu}$
	$O^{(0)}_{5q} = m_q\bar q i\gamma_5 q\,,\quad O^{(0)}_{5g} = \epsilon^{\mu\nu\rho\sigma}G^A_{\mu\nu}G^A_{\rho\sigma}$
	$O^{(2)\mu\nu}_q = \frac{1}{2}\bar q\left(\gamma^{\{\mu}iD^{\nu\}}_- - \frac{g^{\mu\nu}}{4}i\not{D}_-\right)q\,,\quad O^{(2)\mu\nu}_g = -G^{A\mu\lambda}G^{A\nu}{}_\lambda + \frac{g^{\mu\nu}}{4}(G^A_{\alpha\beta})^2$
	$O^{(2)\mu\nu}_{5q} = \frac{1}{2}\bar q\gamma^{\{\mu}iD^{\nu\}}_-\gamma_5 q$

complete QCD basis for d≤7

Renormalization: (focus on ops relevant to heavy WIMPs)

$$\mathcal{L}_{\phi_0,\mathrm{SM}} = \frac{1}{m_W^3}\phi_v^*\phi_v\left\{\sum_q\left[c_{1q}^{(0)}O_{1q}^{(0)} + c_{1q}^{(2)}v_\mu v_\nu O_{1q}^{(2)\mu\nu}\right] + c_2^{(0)}O_2^{(0)} + c_2^{(2)}v_\mu v_\nu O_2^{(2)\mu\nu}\right\} + \dots$$

$m_q\bar{q}q$ → $O_{1q}^{(0)}$; $G^A_{\mu\nu}G^{A\mu\nu}$ → $O_2^{(0)}$

$$c_i(\mu_l) = R_{ij}(\mu_l,\mu_h)c_j(\mu_h)$$

focus on spin-0 (evaluate spin-2 at weak scale)

$$R(\mu,\mu_h) = \left(\begin{array}{ccc|c} 1 & & & R_{qg} \\ & \ddots & & \vdots \\ & & 1 & R_{qg} \\ \hline 0 & \cdots & 0 & R_{gg} \end{array}\right)$$

$$\langle\theta^\mu_\mu\rangle = m_N = (1-\gamma_m)\sum_{q=u,d,s,\dots}^{n_f}\langle O_q^{(0)}\rangle + \frac{\tilde\beta}{2}\langle O_g^{(0)}\rangle$$

$$\langle O_i'^{(S)}\rangle(\mu_h) = R_{ji}^{(S)}(\mu,\mu_h)\langle O_j^{(S)}\rangle(\mu)$$

$$\frac{2}{\tilde\beta(\mu)}R_{gg} = \frac{2}{\tilde\beta(\mu_h)}\,, \qquad R_{qg} - \frac{2}{\tilde\beta(\mu)}\left[1-\gamma_m(\mu)\right]R_{gg} = -\frac{2}{\tilde\beta(\mu_h)}\left[1-\gamma_m(\mu_h)\right]$$

Quark threshold matching:

$$c_i(\mu_Q) = M_{ij}(\mu_Q) c'_j(\mu_Q) \, .$$

$$M(\mu_Q) = \left(\begin{array}{ccc|c|c} & & & M_{qQ} & M_{qg} \\ & \mathbb{1}(M_{qq} - M_{qq'}) + \mathbb{J} M_{qq'} & & \vdots & \vdots \\ & & & M_{qQ} & M_{qg} \\ \hline M_{gq} & \cdots & M_{gq} & M_{gQ} & M_{gg} \end{array} \right)$$

$$\langle \theta^\mu_\mu \rangle = m_N = (1 - \gamma_m) \sum_{q=u,d,s,\ldots}^{n_f} \langle O_q^{(0)} \rangle + \frac{\tilde{\beta}}{2} \langle O_g^{(0)} \rangle$$

$$\langle O_i'^{(S)} \rangle (\mu_b) = M_{ji}^{(S)}(\mu_b) \langle O_j^{(S)} \rangle (\mu_b) + \mathcal{O}(1/m_b) \, .$$

$$0 = \tilde{\beta}^{(n_f)} - \tilde{\beta}^{(n_f+1)} M_{gg} - 2\left[1 - \gamma_m^{(n_f+1)}\right](M_{gQ} + n_f M_{gq}) \, ,$$

$$0 = 2\left\{1 - \gamma_m^{(n_f)} - \left[1 - \gamma_m^{(n_f+1)}\right](M_{qQ} + M_{qq} + (n_f - 1) M_{qq'})\right\} - \tilde{\beta}^{(n_f+1)} M_{qg}$$

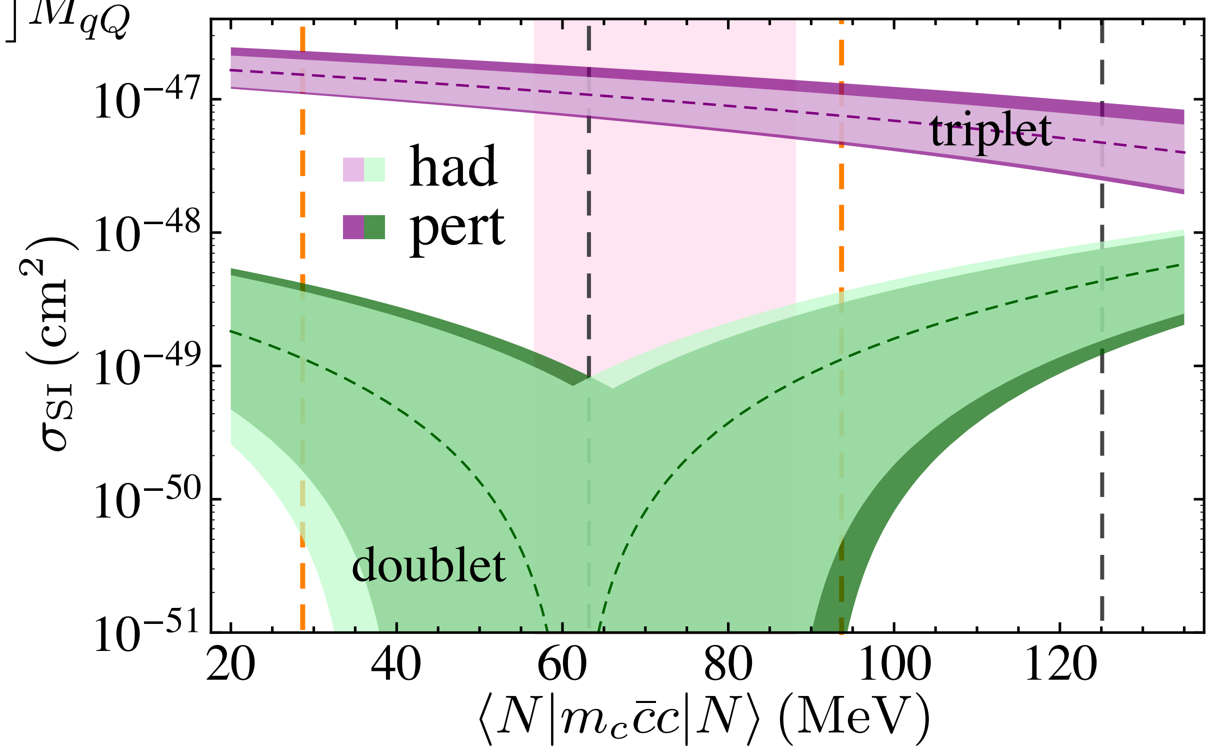
Notice that:

$$M_{qq} \equiv 1 \, , \quad M_{qq'} \equiv 0 \, , \quad M_{gq} \equiv 0$$

Remaining relations are determined by sum rule in terms of M_{gQ} and M_{qQ}

$$M_{gg} = \frac{\tilde\beta^{(n_f)}}{\tilde\beta^{(n_f+1)}} - \frac{2}{\tilde\beta^{(n_f+1)}}\left[1-\gamma_m^{(n_f+1)}\right] M_{gQ}\,,$$

$$M_{gq} = \frac{2}{\tilde\beta^{(n_f+1)}}\left[\gamma_m^{(n_f+1)} - \gamma_m^{(n_f)}\right] - \frac{2}{\tilde\beta^{(n_f+1)}}\left[1-\gamma_m^{(n_f+1)}\right] M_{qQ}$$



M_{gQ} and M_{qQ} known through 3 loops:

Chetyrkin et al. (1997)

New results for gluon-induced decoupling relations

$$M_{gg}^{(2)} = \frac{11}{36} - \frac{11}{6}\log\frac{\mu_Q}{m_Q} + \frac{1}{9}\log^2\frac{\mu_Q}{m_Q}$$

$$M_{gg}^{(3)} = \frac{564731}{41472} - \frac{2821}{288}\log\frac{\mu_Q}{m_Q} + \frac{3}{16}\log^2\frac{\mu_Q}{m_Q} - \frac{1}{27}\log^3\frac{\mu_Q}{m_Q} - \frac{82043}{9216}\zeta(3)$$
$$+ n_f\left[-\frac{2633}{10368} + \frac{67}{96}\log\frac{\mu_Q}{m_Q} - \frac{1}{3}\log^2\frac{\mu_Q}{m_Q}\right],$$

$$M_{qg}^{(2)} = -\frac{89}{54} + \frac{20}{9}\log\frac{\mu_Q}{m_Q} - \frac{8}{3}\log^2\frac{\mu_Q}{m_Q}\,.$$

Hill, Solon (2014)

New result for heavy quark scalar matrix element of nucleon:

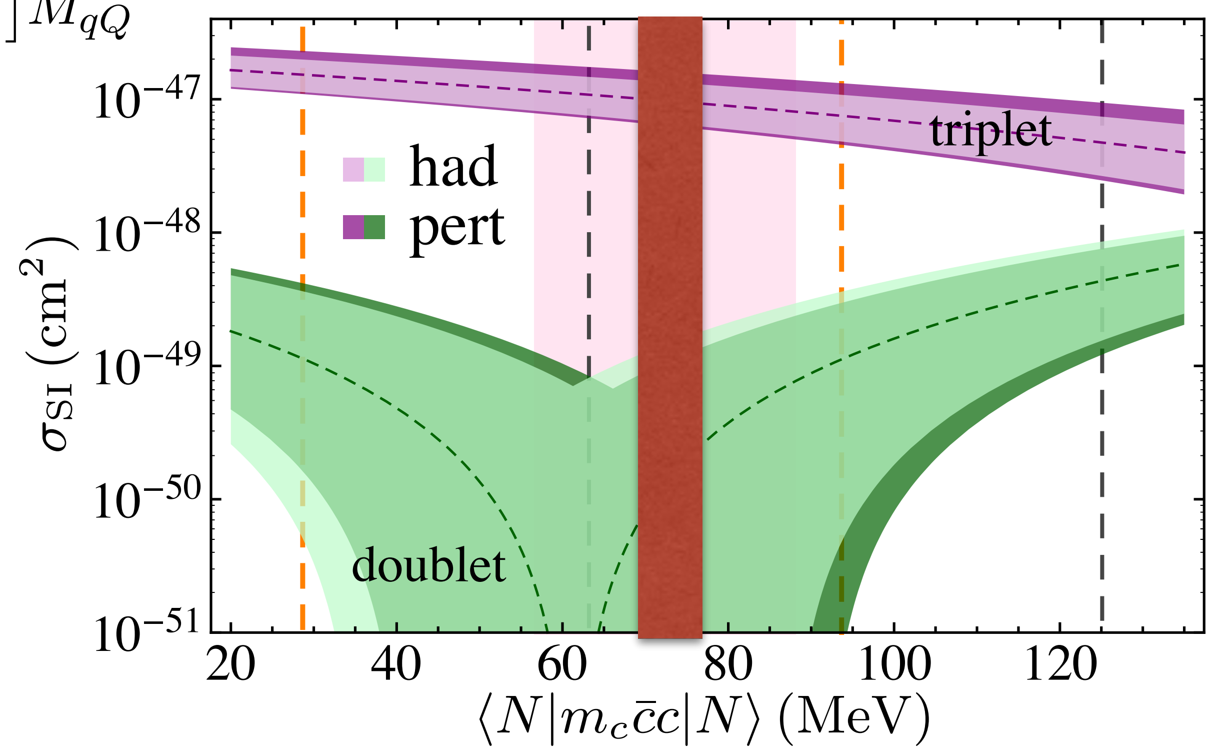
$$f_{c,N}^{(0)\prime} = 0.083 - 0.103\lambda + \mathcal{O}(\alpha_s^4, 1/m_c) = 0.073(3) + \mathcal{O}(\alpha_s^4, 1/m_c)$$

$$f_{q,N}^{(0)\prime} = f_{q,N}^{(0)} + \mathcal{O}(1/m_c)\,,$$

Remaining relations are determined by sum rule in terms of M_{gQ} and M_{qQ}

$$M_{gg} = \frac{\tilde{\beta}^{(n_f)}}{\tilde{\beta}^{(n_f+1)}} - \frac{2}{\tilde{\beta}^{(n_f+1)}}\left[1-\gamma_m^{(n_f+1)}\right] M_{gQ}\,,$$

$$M_{gq} = \frac{2}{\tilde{\beta}^{(n_f+1)}}\left[\gamma_m^{(n_f+1)} - \gamma_m^{(n_f)}\right] - \frac{2}{\tilde{\beta}^{(n_f+1)}}\left[1-\gamma_m^{(n_f+1)}\right] M_{qQ}$$



M_{gQ} and M_{qQ} known through 3 loops:

Chetyrkin et al. (1997)

New results for gluon-induced decoupling relations

$$M_{gg}^{(2)} = \frac{11}{36} - \frac{11}{6}\log\frac{\mu_Q}{m_Q} + \frac{1}{9}\log^2\frac{\mu_Q}{m_Q}$$

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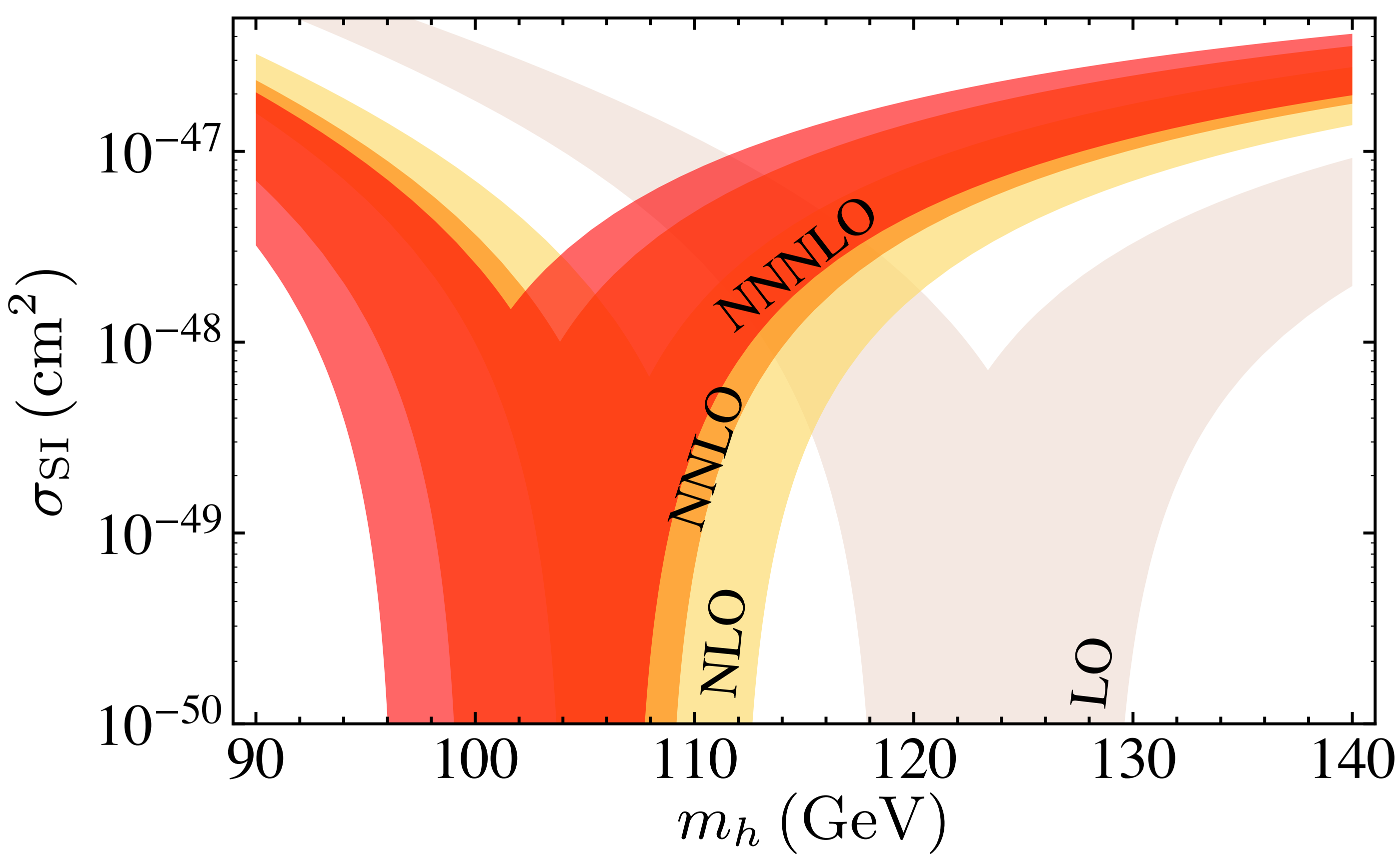
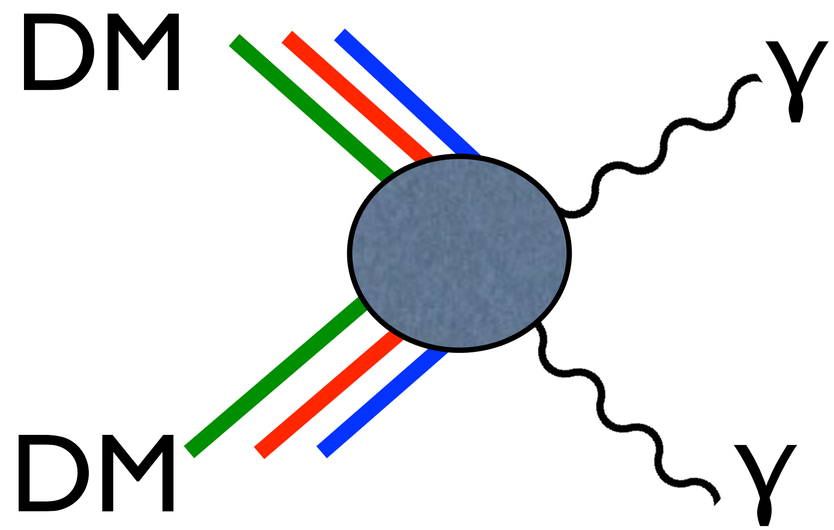
$$f_{q,N}^{(0)\prime} = f_{q,N}^{(0)} + \mathcal{O}(1/m_c)\,,$$

Impact of NLO corrections on wino-like direct detection cross section:



- WIMP scattering + high-scale matching: *Heavy WIMP Effective Theory (HWET)*
- WIMP scattering + collider production, connecting weak scale to hadronic scale: *heavy quark decoupling*
- WIMP annihilation: *HWET+Soft Collinear Effective Theory*

Consider heavy neutral wino/WISP/heavy triplet WIMP annihilating to neutral gauge bosons

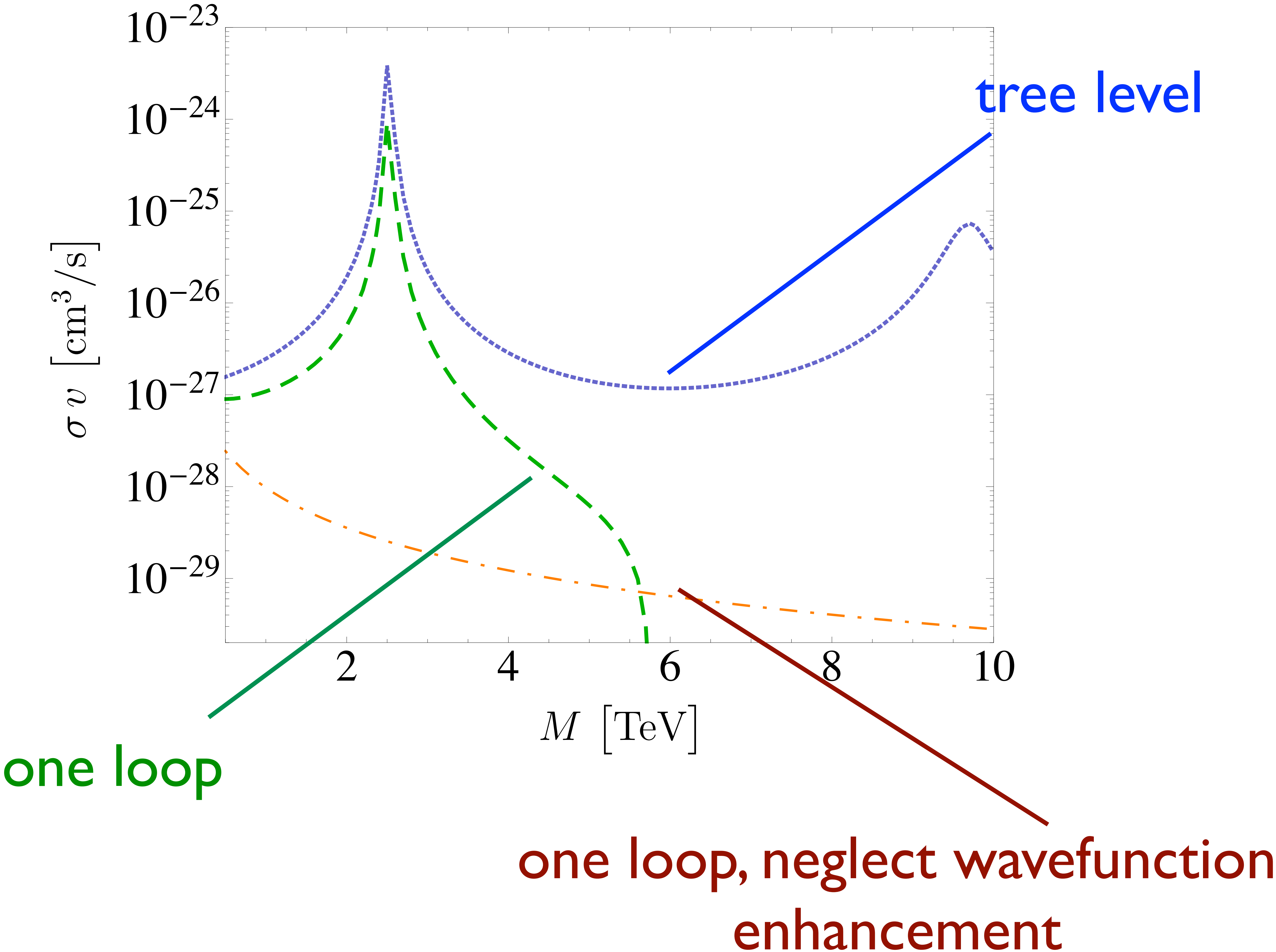


Intricate process: loop induced, and interplay of 4 effects:

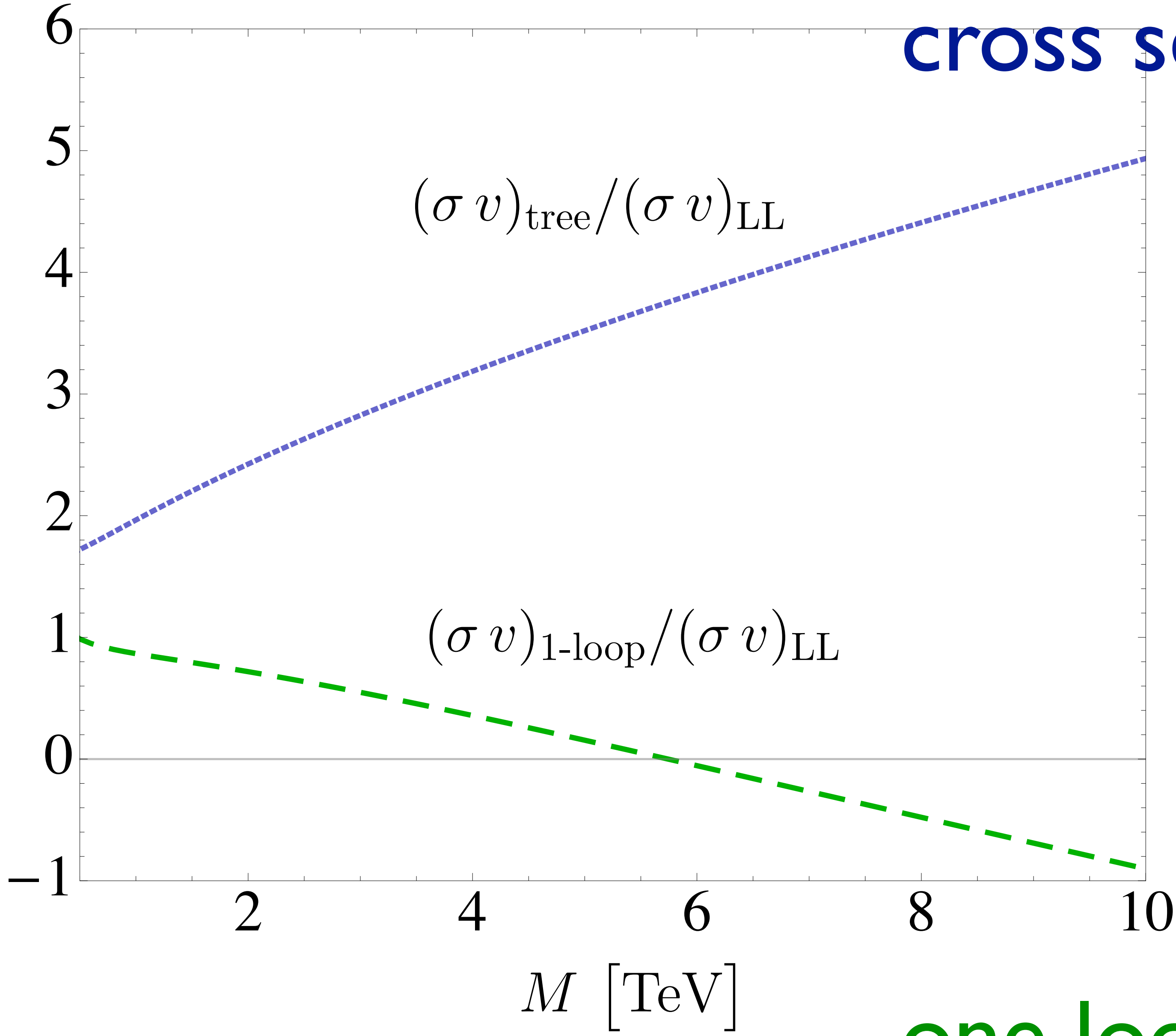
- hard annihilation (high scale matching)
- Sudakov suppression (RG evolution)
- Collinear anomaly (low scale matching)
- Sommerfeld enhancement (nonperturbative wavefunction solution)

Treated systematically in a sequence of matching+running in EFT

A systematic treatment is not optional, especially for large mass

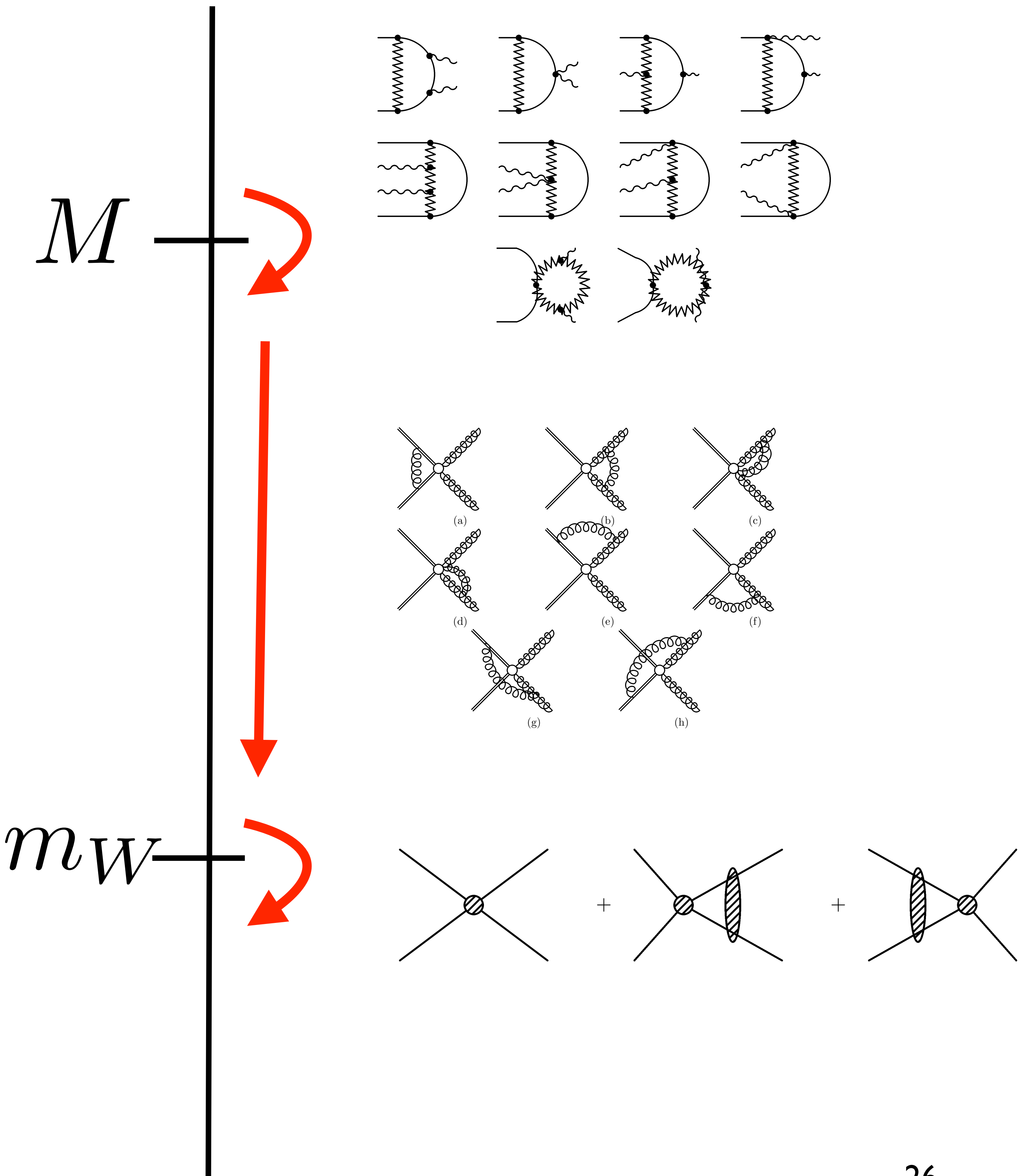


tree level severely overestimates cross section



one loop severely underestimates cross section

Scales of heavy WIMP annihilation



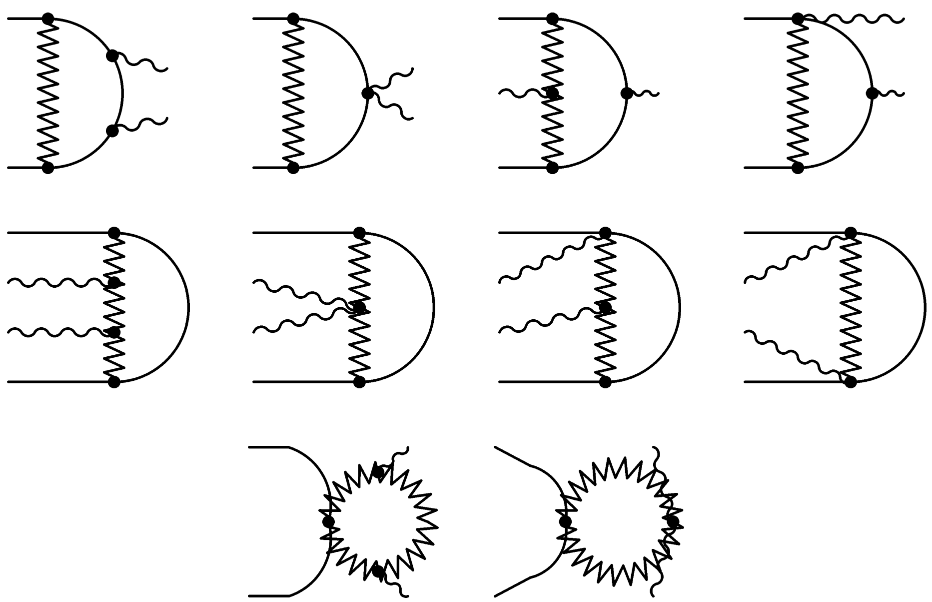
hard annihilation
(makes it happen)

Sudakov suppression
(makes it slower)

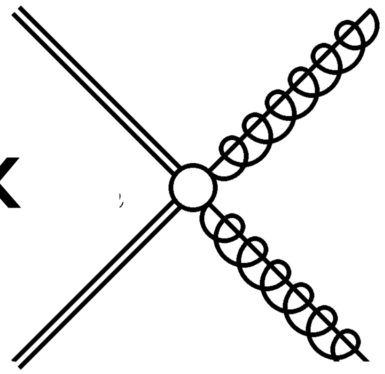
Collinear anomaly:
remant of nonfactorization

Sommerfeld enhancement
(makes it faster)

Match onto SCET at hard scale μ~2M:



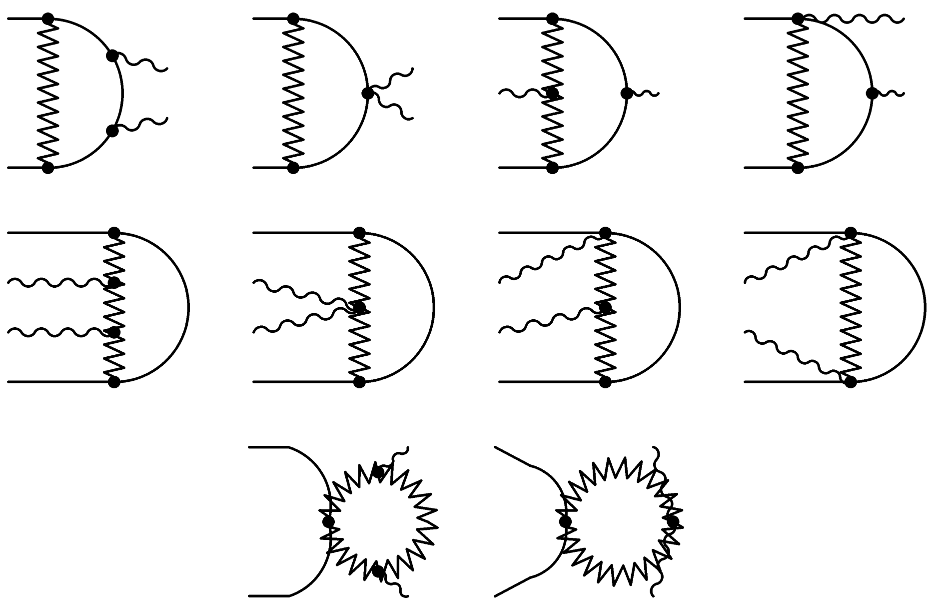
$$= \; c(\mu^2 \sim 4M^2) \; \times$$



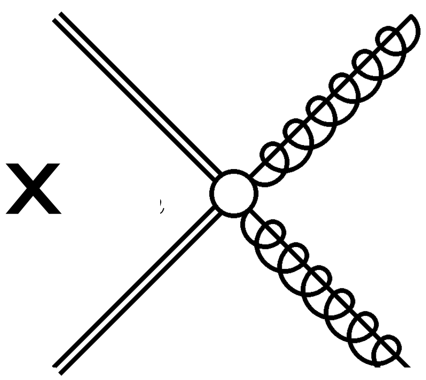
Resummation governed by cusp:

$$\Gamma(R) = \frac{1}{2}\gamma_{\rm cusp}\left[\underbrace{(C_2(r) + C_2(r'))}_{\text{group theory}}\left(\log\frac{4M^2}{\mu^2} - i\pi\right) + i\pi C_2(R)\right] + \gamma^r + \gamma^{r'} + \gamma^R - 2\frac{\beta(g)}{g}$$

Match onto SCET at hard scale μ~2M:



$$= c(\mu^2 \sim 4M^2) \times$$



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group theory

Becher, Hill, Lange, Neubert (2004)

Becher, Neubert (2009)

Beneke, Falgari, Schwinn (2009)

Annihilation of nonrelativistic particles described by QM:

e.g.

$$H = \frac{p^2}{2m} + V + iW$$

$$V = -\frac{\alpha}{r} \qquad W = w\delta^3(\mathbf{r})$$

Bound state annihilation:

$$\Gamma = -2\langle\psi|W\psi\rangle = -2w|\psi(0)|^2$$

$$\langle\psi|\psi\rangle = 1$$

$$|\psi(0)|^2 = \frac{(m\alpha)^3}{\pi n^3}$$

Asymptotic plane wave annihilation:

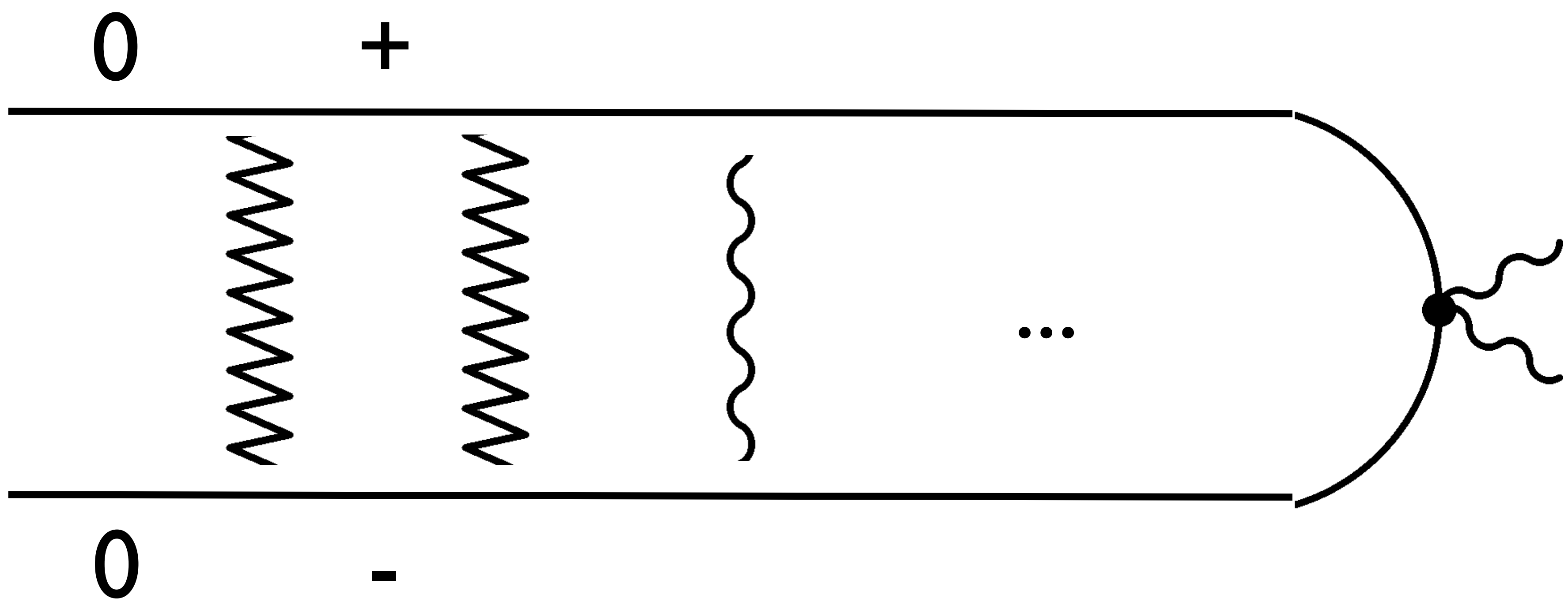
$$\sigma v = -2\langle\psi|W\psi\rangle = -2w|\psi(0)|^2 \qquad \psi \to e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$$

$$|\psi(0)|^2 = \frac{\frac{2\pi\alpha}{v}}{1 - \exp\left[-\frac{2\pi\alpha}{v}\right]}$$

Heavy SU(2) triplet: multi-channel annihilation process:

charged states lifted by EWSB effects:

$$M_{(Q)} - M_{(Q=0)} = \alpha_2 Q^2 m_W \sin^2 \frac{\theta_W}{2} + \mathcal{O}(1/M) \approx (170\,\mathrm{MeV}) Q^2$$



asymptotic neutral channel, but leading hard annihilation through charged channel

Below electroweak scale, match to QM

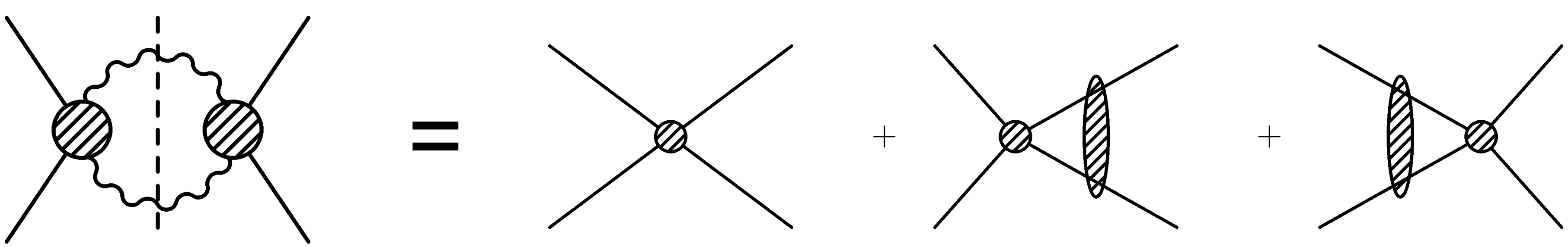
$$V^{S\text{-wave}} = \begin{pmatrix} 0 & -\sqrt{2}\frac{\alpha_2}{r}e^{-m_W r} \\ -\sqrt{2}\frac{\alpha_2}{r}e^{-m_W r} & -\frac{\alpha}{r} - \frac{\alpha_2 c_W^2}{r}e^{-m_Z r} \end{pmatrix}$$

$$\left\langle \boldsymbol{k}' \middle| W^{(\gamma)} \middle| \boldsymbol{k} \right\rangle \equiv \begin{pmatrix} w_{00}^{(\gamma)} & w_{00;\pm}^{(\gamma)} \\ w_{\pm;00}^{(\gamma)} & w_{\pm}^{(\gamma)} \end{pmatrix}$$

$$H = \frac{p^2}{2M_r} + \Delta + V + iW$$

$$p^2 \begin{pmatrix} \frac{1}{M_0} & 0 \\ 0 & \frac{1}{M_\pm} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 2\delta \end{pmatrix}$$

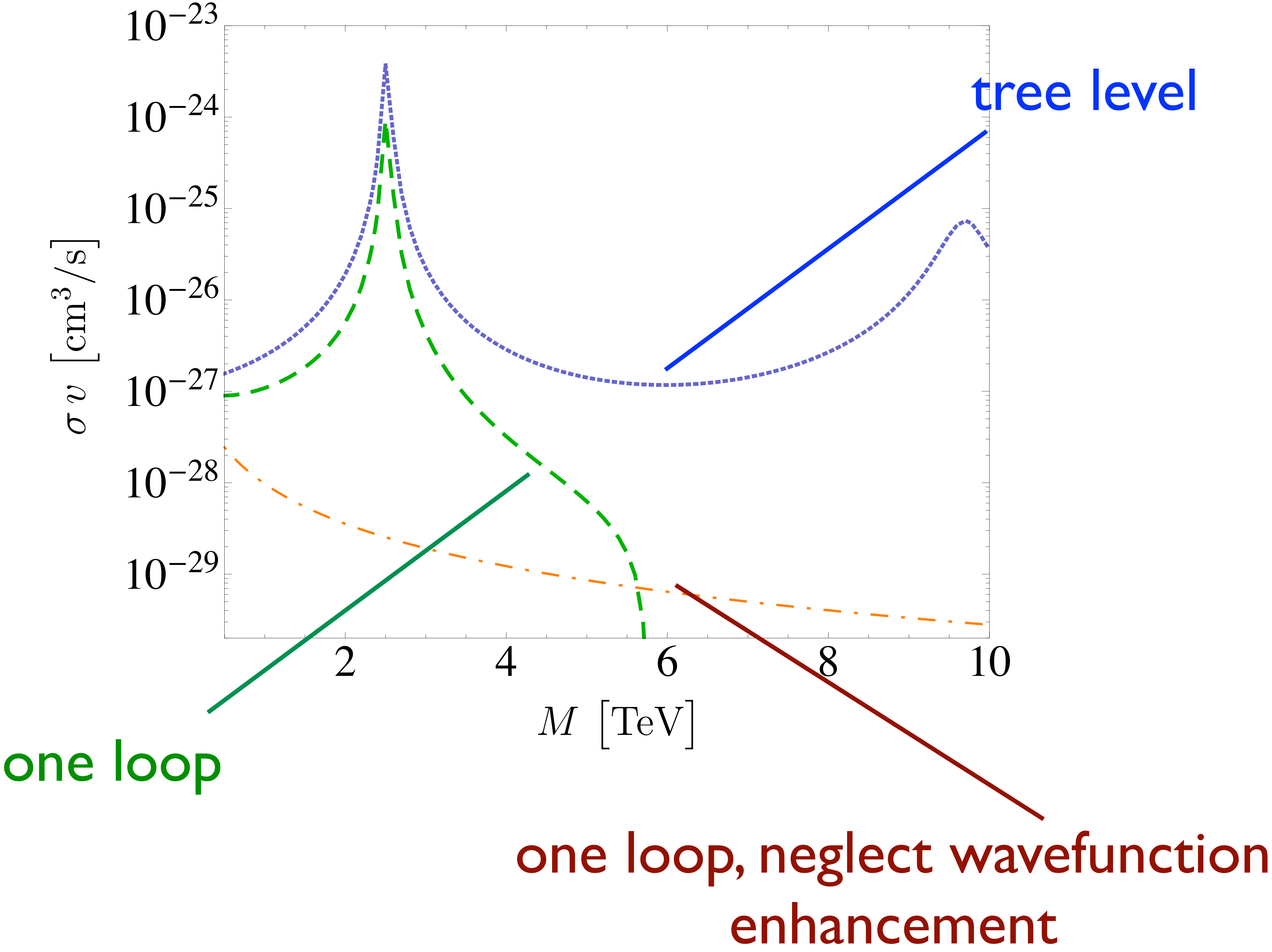
SCET = QM



Annihilation rate given by

$$\sigma v = -2\langle \psi | W \psi \rangle = -2\psi^*(0)_i W_{ij} \psi(0)_j$$

Nontrivial wavefunction effects:



Recall that the messenger modes introduce a new scale

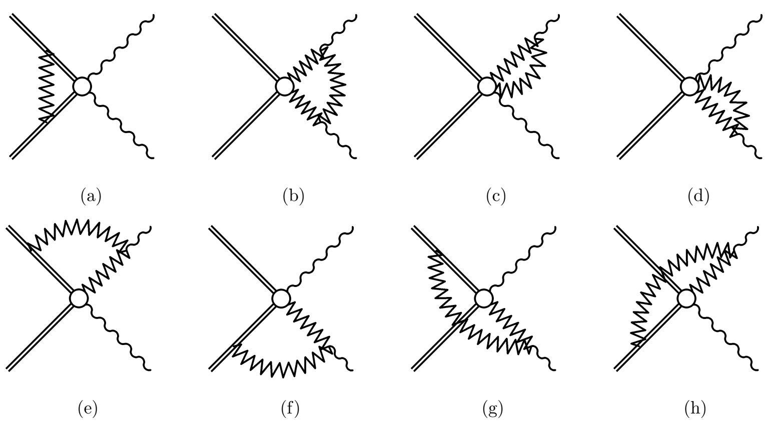
$$\text{collinear}: \quad p^\mu \sim Q(\lambda^2, 1, \lambda)$$

$$\text{collinear}': \quad p^\mu \sim Q(1, \lambda^2, \lambda)$$

$$\text{messenger}: \quad p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$$

$$p^2_{\text{messenger}} \sim \frac{p^2 p'^2}{Q^2} \ll p^2$$

This allows large logarithms to sneak in the back door



$$i\mathcal{M}^{+-\to\gamma\gamma} =$$

$$c_1(\mu)\left[-\frac{4\pi^2}{3} + 32\log\frac{2M}{\mu}\log\frac{m_W}{\mu} - 16i\pi\log\frac{m_W}{\mu} - 16\log^2\frac{m_W}{\mu}\right]$$

$$+\, c_2(\mu)\left[-\frac{4\pi^2}{3} + 32\log\frac{2M}{\mu}\log\frac{m_W}{\mu} - 8i\pi\log\frac{m_W}{\mu} - 16\log^2\frac{m_W}{\mu} - 8\log\frac{m_W}{\mu}\right]$$

Happily, the dependence on the large scale may be resummed

Basic idea:

$$\frac{d}{d\log\mu}[\text{observable}] = 0$$

$$\frac{d}{d\log\mu}\log^2\frac{\mu^2}{M^2} = 4\log\frac{\mu}{M}$$

The only thing whose variation can cancel this dependence is

$$\log\frac{\mu^2}{M^2}\log\frac{\mu^2}{m_W^2}$$

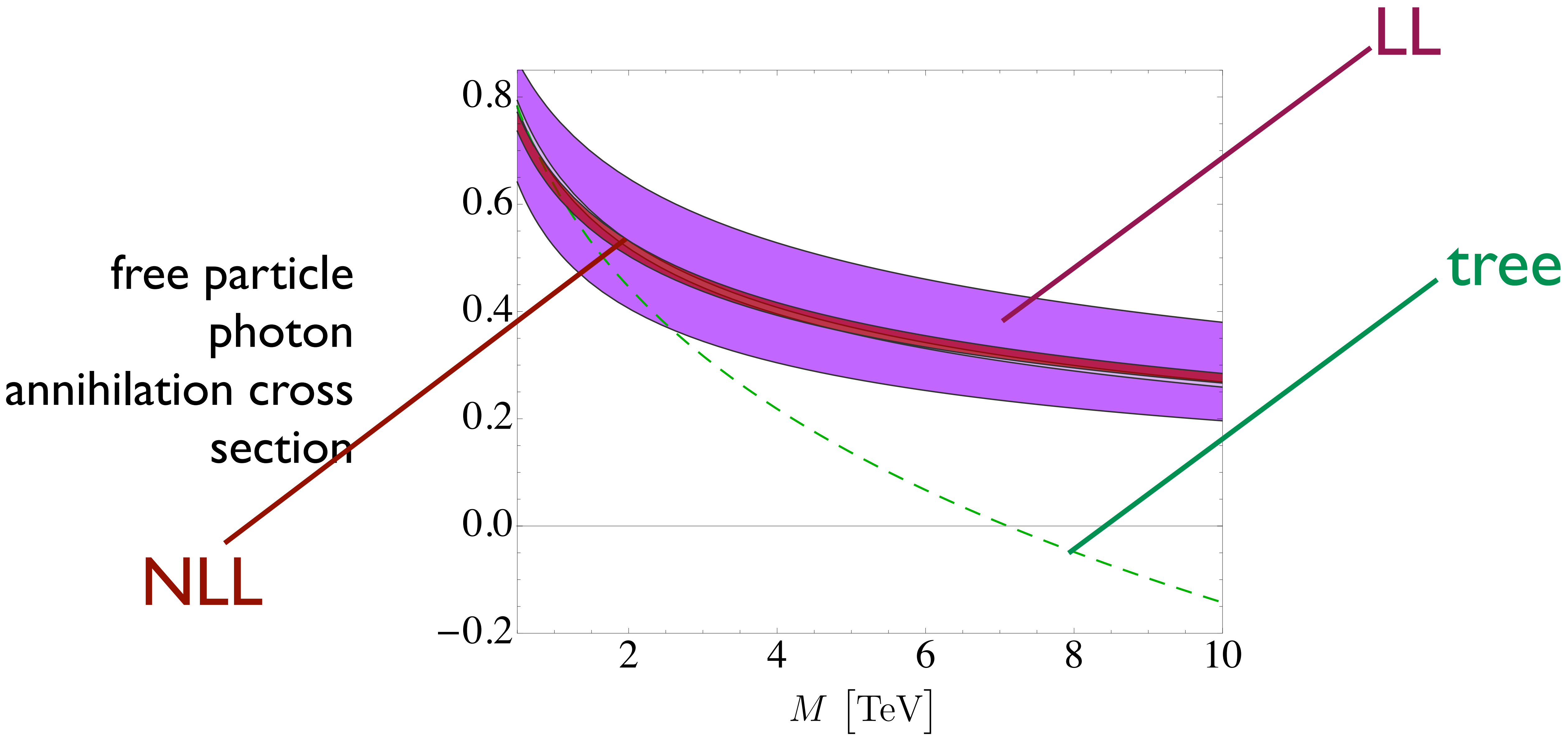
And so the coefficient is tied to the universal cusp structure

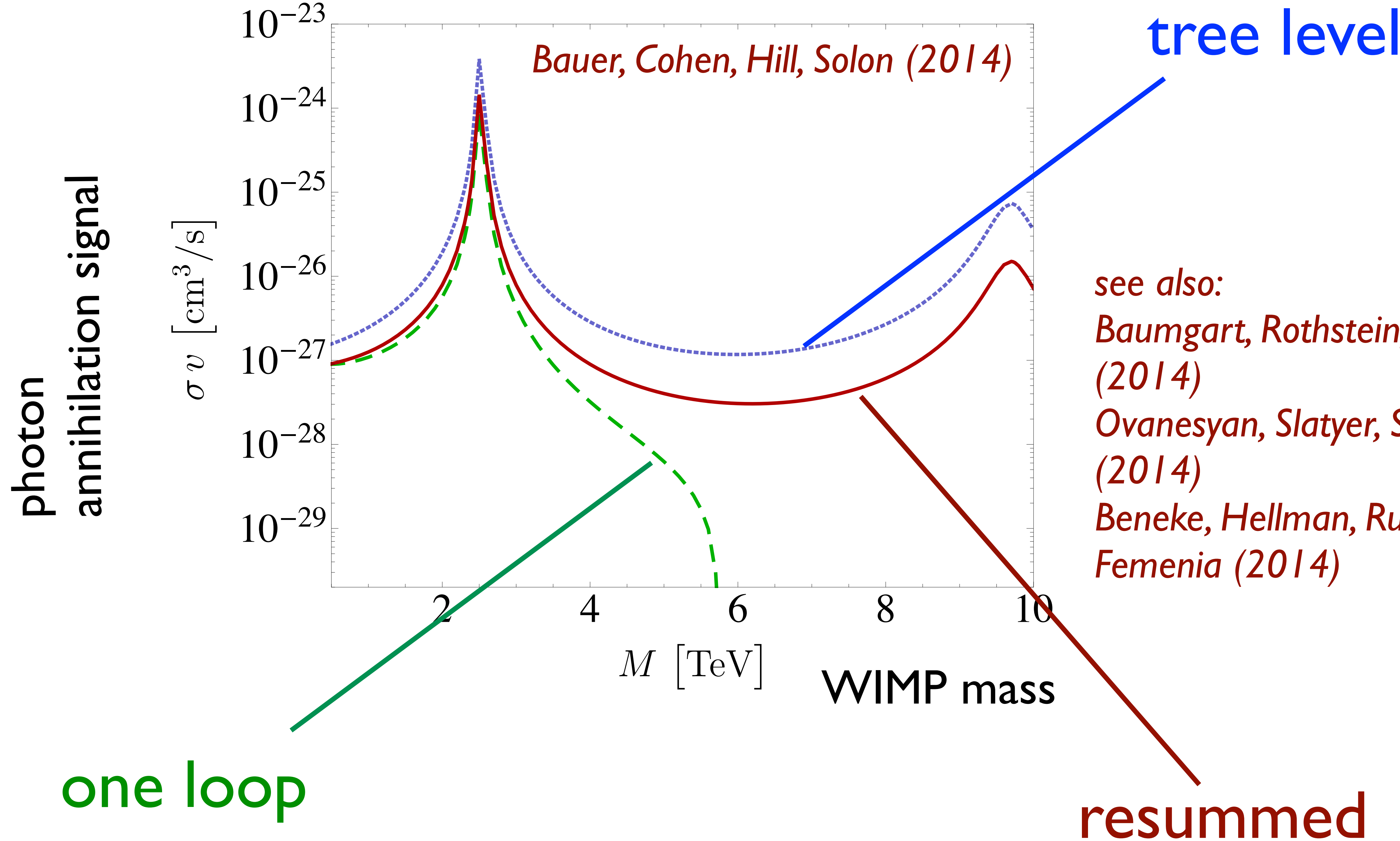
Can now resum these subleading logs:

$$c_i(\mu) \to c_i(\mu)\left(\frac{4M^2}{\mu^2}\right)^{-\frac{1}{2}F(m_W,\mu)}$$

determined by cusp structure

Next-to-leading log, versus leading-log resummation:





see also:
Baumgart, Rothstein, Vaidya (2014)
Ovanesyan, Slatyer, Stewart (2014)
Beneke, Hellman, Ruiz-Femenia (2014)

General framework in which to reliably compute annihilation signals for heavy WIMPs.

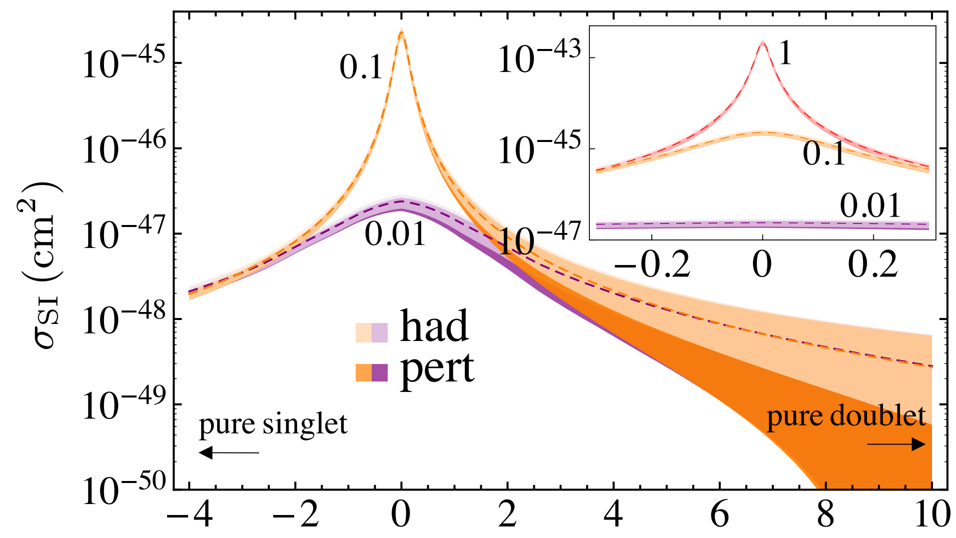
- QCD corrections are important to dark matter searches
    - determine discovery potential (e.g. heavy pure states)
    - determine compatibility of potential signals between experiments
- interplay with perturbative and nonperturbative QCD
    - lattice matrix elements
    - high-order decoupling relations
    - novel nuclear responses

- EFT developments
  - matching and renormalization in HPET
  - Lorentz invariance in HPET
  - high-order decoupling relations
  - interplay of collinear anomaly and EWSB
- work to do:
  - 1/M HWET
  - $1/m_c$ corrections to decoupling (lattice QCD)
  - nuclear responses (identical at 1-body level)

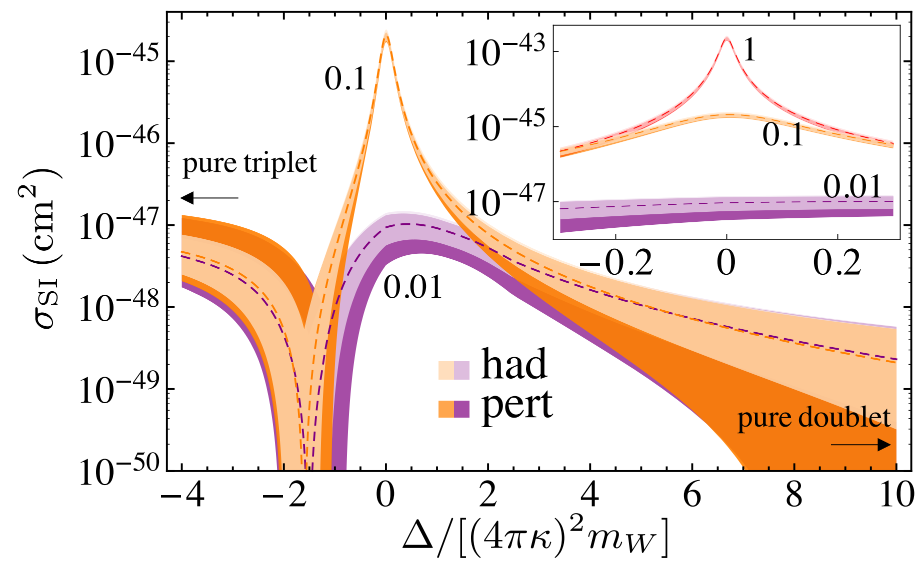
extra slides

Additional states in the dark sector

singlet-doublet (e.g., bino-higgsino)



triplet-doublet (e.g., wino-higgsino)



Δ: mass splitting of multiplets, in units where tree/loop crossover occurs at ~1

interplay of mass-suppressed (tree level) and loop suppressed contributions

Single-nucleon operators

$$\mathcal{L}_{N\chi,PT} = \frac{1}{m_N^2}\Big\{d_1 N^\dagger\sigma^i N\ \chi^\dagger\sigma^i\chi + d_2 N^\dagger N\ \chi^\dagger\chi\Big\} + \frac{1}{m_N^4}\Big\{d_3 N^\dagger\partial_+^i N\ \chi^\dagger\partial_+^i\chi + d_4 N^\dagger\partial_-^i N\ \chi^\dagger\partial_-^i\chi$$

$$+ d_5 N^\dagger(\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2)N\ \chi^\dagger\chi + d_6 N^\dagger N\ \chi^\dagger(\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2)\chi + id_8\epsilon^{ijk}N^\dagger\sigma^i\partial_-^j N\ \chi^\dagger\partial_+^k\chi$$

$$+ id_9\epsilon^{ijk}N^\dagger\sigma^i\partial_+^j N\ \chi^\dagger\partial_-^k\chi + id_{11}\epsilon^{ijk}N^\dagger\partial_+^k N\ \chi^\dagger\sigma^i\partial_-^j\chi + id_{12}\epsilon^{ijk}N^\dagger\partial_-^k N\ \chi^\dagger\sigma^i\partial_+^j\chi$$

$$+ d_{13}N^\dagger\sigma^i\partial_+^j N\ \chi^\dagger\sigma^i\partial_+^j\chi + d_{14}N^\dagger\sigma^i\partial_-^j N\ \chi^\dagger\sigma^i\partial_-^j\chi + d_{15}N^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\partial}_+ N\ \chi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\partial}_+\chi$$

$$+ d_{16}N^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\partial}_- N\ \chi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\partial}_-\chi + d_{17}N^\dagger\sigma^i\partial_-^j N\ \chi^\dagger\sigma^j\partial_-^i\chi$$

$$+ d_{18}N^\dagger\sigma^i(\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2)N\ \chi^\dagger\sigma^i\chi + d_{19}N^\dagger\sigma^i(\partial^i\partial^j + \overleftarrow{\partial}^j\overleftarrow{\partial}^i)N\ \chi^\dagger\sigma^j\chi$$

$$+ d_{20}N^\dagger\sigma^i N\ \chi^\dagger\sigma^i(\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2)\chi + d_{21}N^\dagger\sigma^i N\ \chi^\dagger\sigma^j(\partial^i\partial^j + \overleftarrow{\partial}^j\overleftarrow{\partial}^i)\chi\Big\} + \mathcal{O}(1/m_N^6)\,, \quad ($$

Lorentz invariance:

$$rd_4 + d_5 = \frac{d_2}{4}\,, \quad d_5 = r^2 d_6\,, \quad 8r(d_8 + rd_9) = -rd_2 + d_1\,, \quad 8r(rd_{11} + d_{12}) = -d_2 + rd_1$$

$$rd_{14} + d_{18} = \frac{d_1}{4}\,, \quad d_{18} = r^2 d_{20}\,, \quad 2rd_{16} + d_{19} = \frac{d_1}{4}\,, \quad r(d_{16} + d_{17}) + d_{19} = 0\,, \quad d_{19} = r^2 d_{21}\,,$$

Light WIMP+ SM

$$\mathcal{L}_{\psi,\mathrm{SM}} = \frac{c_{\psi 1}}{m_W}\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} + \frac{c_{\psi 2}}{m_W}\bar{\psi}\sigma^{\mu\nu}\psi \tilde{F}_{\mu\nu} + \sum_{q=u,d,s,c,b} \left\{ \frac{c_{\psi 3,q}}{m_W^2}\bar{\psi}\gamma^\mu\gamma_5\psi\bar{q}\gamma_\mu q + \frac{c_{\psi 4,q}}{m_W^2}\bar{\psi}\gamma^\mu\gamma_5\psi\bar{q}\gamma_\mu\gamma_5 q \right.$$

$$+ \frac{c_{\psi 5,q}}{m_W^2}\bar{\psi}\gamma^\mu\psi\bar{q}\gamma_\mu q + \frac{c_{\psi 6,q}}{m_W^2}\bar{\psi}\gamma^\mu\psi\bar{q}\gamma_\mu\gamma_5 q + \frac{c_{\psi 7,q}}{m_W^3}\bar{\psi}\psi m_q\bar{q}q + \frac{c_{\psi 8,q}}{m_W^3}\bar{\psi}i\gamma_5\psi m_q\bar{q}q$$

$$+ \frac{c_{\psi 9,q}}{m_W^3}\bar{\psi}\psi m_q\bar{q}i\gamma_5 q + \frac{c_{\psi 10,q}}{m_W^3}\bar{\psi}i\gamma_5\psi m_q\bar{q}i\gamma_5 q + \frac{c_{\psi 11,q}}{m_W^3}\bar{\psi}i\partial_-^\mu\psi\bar{q}\gamma_\mu q$$

$$+ \frac{c_{\psi 12,q}}{m_W^3}\bar{\psi}\gamma_5\partial_-^\mu\psi\bar{q}\gamma_\mu q + \frac{c_{\psi 13,q}}{m_W^3}\bar{\psi}i\partial_-^\mu\psi\bar{q}\gamma_\mu\gamma_5 q + \frac{c_{\psi 14,q}}{m_W^3}\bar{\psi}\gamma_5\partial_-^\mu\psi\bar{q}\gamma_\mu\gamma_5 q$$

$$\left. + \frac{c_{\psi 15,q}}{m_W^3}\bar{\psi}\sigma_{\mu\nu}\psi m_q\bar{q}\sigma^{\mu\nu}q + \frac{c_{\psi 16,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar{\psi}\sigma^{\mu\nu}\psi m_q\bar{q}\sigma^{\rho\sigma}q \right\} + \frac{c_{\psi 17}}{m_W^3}\bar{\psi}\psi G^A_{\alpha\beta}G^{A\alpha\beta}$$

$$+ \frac{c_{\psi 18}}{m_W^3}\bar{\psi}i\gamma_5\psi G^A_{\alpha\beta}G^{A\alpha\beta} + \frac{c_{\psi 19}}{m_W^3}\bar{\psi}\psi G^A_{\alpha\beta}\tilde{G}^{A\alpha\beta} + \frac{c_{\psi 20}}{m_W^3}\bar{\psi}i\gamma_5\psi G^A_{\alpha\beta}\tilde{G}^{A\alpha\beta} + \dots,$$

Majorana:

$c_{\psi n}$ with $n = 1, 2, 5, 6, 11, 12, 13, 14, 15, 16$ vanish,

Heavy WIMP + SM

$$
\begin{aligned}
\mathcal{L}_{\chi_v,\mathrm{SM}} = {} & \frac{c_{\chi 1}}{m_W}\bar\chi_v \sigma_\perp^{\mu\nu}\chi_v F_{\mu\nu} + \frac{c_{\chi 2}}{m_W}\bar\chi_v \sigma_\perp^{\mu\nu}\chi_v \tilde F_{\mu\nu} + \sum_{q=u,d,s,c,b}\Bigg\{ \frac{c_{\chi 3,q}}{m_W^2}\epsilon_{\mu\nu\rho\sigma} v^\mu \bar\chi_v \sigma_\perp^{\nu\rho}\chi_v \bar q \gamma^\sigma q \\
& + \frac{c_{\chi 4,q}}{m_W^2}\epsilon_{\mu\nu\rho\sigma} v^\mu \bar\chi_v \sigma_\perp^{\nu\rho}\chi_v \bar q \gamma^\sigma \gamma_5 q + \frac{c_{\chi 5,q}}{m_W^2}\bar\chi_v\chi_v \bar q \not v\, q + \frac{c_{\chi 6,q}}{m_W^2}\bar\chi_v\chi_v \bar q \not v\, \gamma_5 q + \frac{c_{\chi 7,q}}{m_W^3}\bar\chi_v\chi_v m_q \bar q q \\
& + \frac{c_{\chi 8,q}}{m_W^3}\bar\chi_v\chi_v \bar q \not v\, iv\cdot D_- q + \frac{c_{\chi 9,q}}{m_W^3}\bar\chi_v\chi_v m_q \bar q i\gamma_5 q + \frac{c_{\chi 10,q}}{m_W^3}\bar\chi_v\chi_v \bar q \not v\, \gamma_5 iv\cdot D_- q \\
& + \frac{c_{\chi 11,q}}{m_W^3}\bar\chi_v \sigma_\perp^{\mu\nu} i\partial_{-\mu}^\perp \chi_v \bar q \gamma_\nu q + \frac{c_{\chi 12,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v \sigma_\perp^{\mu\nu} i\partial_-^{\perp\rho}\chi_v \bar q\gamma^\sigma q + \frac{c_{\chi 13,q}}{m_W^3}\bar\chi_v \sigma_\perp^{\mu\nu} i\partial_{-\mu}^\perp \chi_v \bar q\gamma_\nu\gamma_5 q \\
& + \frac{c_{\chi 14,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v \sigma_\perp^{\mu\nu} i\partial_-^{\perp\rho}\chi_v \bar q\gamma^\sigma\gamma_5 q + \frac{c_{\chi 15,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma} v^\mu \bar\chi_v \sigma_\perp^{\nu\rho}\chi_v\, \bar q(\not v\, iD_-^\sigma + \gamma^\sigma iv\cdot D_-) q \\
& + \frac{c_{\chi 16,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma} v^\mu \bar\chi_v \sigma_\perp^{\nu\rho}\chi_v \bar q(\not v\, iD_-^\sigma + \gamma^\sigma iv\cdot D_-)\gamma_5 q + \frac{c_{\chi 17,q}}{m_W^3}\bar\chi_v i\partial_-^{\perp\mu}\chi_v \bar q\gamma_\mu q \\
& + \frac{c_{\chi 18,q}}{m_W^3}\bar\chi_v \sigma_\perp^{\mu\nu}\partial_{+\mu}^\perp \chi_v \bar q\gamma_\nu q + \frac{c_{\chi 18,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v \sigma_\perp^{\mu\nu}\partial_+^{\perp\rho}\chi_v \bar q\gamma^\sigma q + \frac{c_{\chi 20,q}}{m_W^3}\bar\chi_v i\partial_-^{\perp\mu}\chi_v \bar q\gamma_\mu\gamma_5 q \\
& + \frac{c_{\chi 21,q}}{m_W^3}\bar\chi_v \sigma_\perp^{\mu\nu}\partial_{+\mu}^\perp \chi_v \bar q\gamma_\nu\gamma_5 q + \frac{c_{\chi 22,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v \sigma_\perp^{\mu\nu}\partial_+^{\perp\rho}\chi_v \bar q\gamma^\sigma\gamma_5 q + \frac{c_{\chi 23,q}}{m_W^3}\bar\chi_v \sigma_\perp^{\mu\nu}\chi_v m_q \bar q\sigma_{\mu\nu} q \\
& + \frac{c_{\chi 24,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v \sigma_\perp^{\mu\nu}\chi_v m_q \bar q\sigma^{\rho\sigma} q \Bigg\} + \frac{c_{\chi 25}}{m_W^3}\bar\chi_v\chi_v G^A_{\alpha\beta}G^{A\alpha\beta} + \frac{c_{\chi 26}}{m_W^3}\bar\chi_v\chi_v G^A_{\alpha\beta}\tilde G^{A\alpha\beta} \\
& + \frac{c_{\chi 27}}{m_W^3}\bar\chi_v\chi_v v_\mu v_\nu G^{A\mu}{}_\alpha G^{A\nu\alpha} + \frac{c_{\chi 28}}{m_W^3}\bar\chi_v\sigma_\perp^{\mu\nu}\chi_v \epsilon_{\mu\nu\alpha\beta} v^\alpha v^\gamma G^{A\beta\delta}G^A_{\gamma\delta} + \dots\,, \qquad (7
\end{aligned}
$$

Lorentz:

$$\frac{m_W}{M}c_{\chi 3} + 2c_{\chi 12} = \frac{m_W}{M}c_{\chi 4} + 2c_{\chi 14} = \frac{m_W}{M}c_{\chi 5} - 2c_{\chi 17} = \frac{m_W}{M}c_{\chi 6} - 2c_{\chi 20} = c_{\chi 11} = c_{\chi 13} = 0\,,$$

Majorana: $c_{\chi n}$ vanish for n=1, 2, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.