PARAMETER SPACE OPTIMIZATION FOR THE THIRD INTEGER RESONANT EXTRACTION.*

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Abstract

In modern storage rings with slow resonant extraction one of the main parameters for optimization is the step size, which largely determines the extraction inefficiency, or the fraction of beam lost due to hitting the septum plane. For given parameters of machine acceptance, beam emittance and lattice functions at the extraction septum, the optimization constrains all the geometry and determines the best achievable extraction efficiency. This paper presents a technique of calculating the machine performance.

INTRODUCTION

Third order resonant slow extraction is a widely used technique for delivering a good quality continuous beam during a finite time period (spill). The range of possible applications of this technique is wide, as well as the range of requirements that are imposed by each project. Due to the high complexity, the performance optimization at the design stage is best be determined in the detailed tracking simulations that may include machine optics specific details, beam properties, space charge effects, RF feedback loops and so on. However it is often desirable to determine main limitations of the design without a rigorous simulation effort, in particular evaluate the maximum possible extraction efficiency. This possibility is offered by simple calculations based on the perturbation theory and a few reasonable assumptions. Estimating the orbits' stepsizes at the septum and the probability of hitting its wires are among the handful of important calculations on resonant extraction that can be done in quadrature. This is done by approximating the orbits of extracted particles as though they were exactly on the outgoing branches of the separatrix. The approximation is reasonable provided extraction is adiabatic.

PERTURBATION THEORY

Figure 1 illustrates the idealized separatrix for the third-integer resonance. It is drawn in a complexified, normalized, horizontal phase space with coordinates as

$$a = \sqrt{I} \cdot e^{i\varphi} = \frac{x + i(\alpha x + \beta x')}{\sqrt{2\beta}} \qquad \{1\},$$

where β and α are the usual Courant-Snyder lattice functions. If (x, x') are canonically conjugate, then so are (φ, I) and (a, a^*) as can be verified by their Poincaré invariants. The Hamiltonian associated with the third-integer resonance model which, written in angle-action, (φ, I) coordinates, is **Error! Reference source not found.**,

$$\begin{aligned} H &= \Delta\nu\, a^* a - ig\, a^3 + ig^* a^{*3} + \ldots \\ &= \Delta\nu\, I - (g e^{-i3\phi} + g^* e^{i3\phi})\, I^{3/2} + \ldots \quad . \end{aligned} \qquad \{2\}$$

Here, $\Delta\nu \equiv \nu_x - 29/3 \approx 0$ is the difference between the linear (small amplitude) horizontal tune and the resonant tune and is presumed to be small; the "resonance coupling constant," g, is a linear functional of the sextupole field strength distribution.

$$g = \frac{i}{6\sqrt{2}} \frac{1}{4\pi} \sum \frac{B''l}{B\rho} \beta_x^{3/2}(\theta)\, e^{-i3(\psi_x(\theta) - \Delta\nu\,\theta)} \qquad \{3\}$$

where the sum is carried out over the locations of the sextupoles. The phase of the complex parameter g determines the orientation of the third-integer separatrix, which is bound by the equilateral triangle with vertices

$$|a_0| = \sqrt{I_0} = \left|\frac{\Delta\nu}{3g}\right| \qquad \{4\}$$

as shown in Figure 1. In the beginning of extraction all beam should be included in the starting separatix – this determines initial Δν and g. As Δν reduces during extraction, the separatrix boundary squeezes leaving less and less stable beam in the machine. Particles outside the separatrix are streaming away along the outcoming rays of the triangle like shown by arrows in Figure 1. The direction of streaming is dependent on the sign of the Δν.

STEP SIZE

The step size is the measure of the speed of particles streaming away and it's determined as an increase of its horizontal projection after 3 consecutive turns. In order to evaluate the step size we assume that extraction process is adiabatic and unstable particle motion occurs in close vicinity of the separatrix lines [LM]:

$$a = a_0 (1 + re^{\,\mathrm{sgn}(\Delta\nu) i\pi/6}) \qquad \{5\}$$

Transformation to the x-coordinate can be written then as

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$$x = \sqrt{2\beta}\cdot \mathrm{Re}\left\{a_0\left(1 + re^{\mathrm{sgn}(\Delta\nu)i\pi/6}\right)\right\} \qquad \{6\}$$

and the step size is determined from $\frac{da}{d\theta} = -i\frac{\partial H}{\partial a^*}$, and yields

$$\frac{dr}{d\theta} = \Delta\nu(\sqrt{3}r + r^2)\ , \qquad \{7\}$$

which integrates to

$$\Delta r = \frac{r(r+\sqrt{3})}{\left[\sqrt{3}/(\exp(6\pi\sqrt{3}\Delta\nu)-1)\right]-r} \qquad \{8\}$$

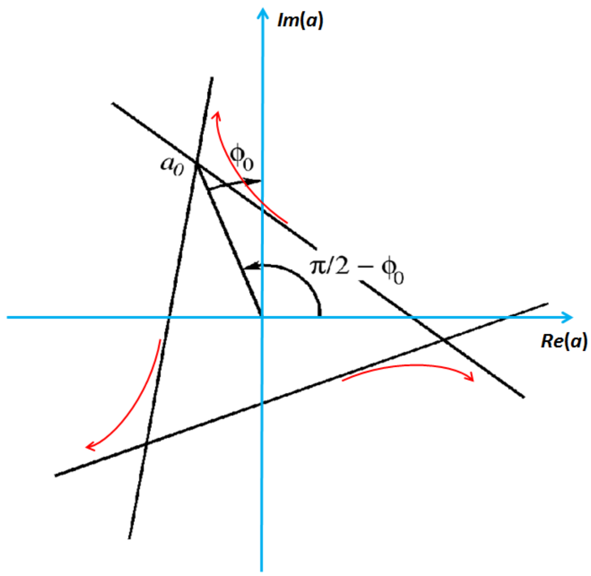

Figure 1. Separatrix geometry

OPTIMAL CONDITIONS FOR EXTRACTION EFFICIENCY

Figure 2 shows $X_1(r)$, the horizontal position as function of the dimensionless parameter r, and $X_2(r)$, the horizontal position on the next step, or after next 3 turns.

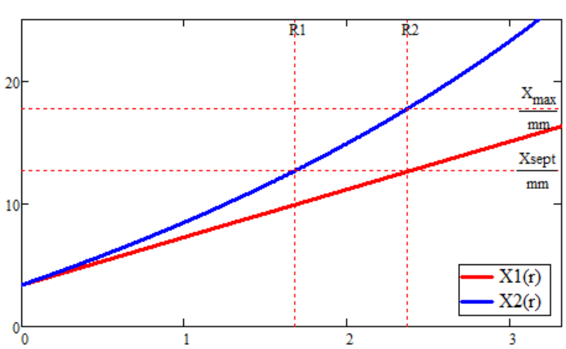

Figure 2. Current x-position X_1 and x-position after 3 turns, X_2 as functions of parameter r.

The horizontal marker lines on the plot are showing the X_{max}, the boundary corresponding to the machine acceptance and the position of the septum wire plane, $X_{sept.}$ The vertical marker R_1 indicates the point of extraction beginning which happens when on the next step a particle would get into the septum field region. The next vertical marker R_2 indicates the end of extraction where X_1 no longer belongs to the circulating beam. Optimum condition of the step size requires that the blue curve crosses the upper boundary at the same time. Although normally no physical aperture is present at the location of the septum, crossing this imaginary line earlier would mean that the particle have reached the amplitude bigger than the machine acceptance and is likely to be lost elsewhere before reaching the septum. If the blue line crosses the upper boundary substantially later, it would mean that there is still room to increase the step size. This can be acceptable if the machine acceptance is very large and the step size is limited by other factors, like the maximum strength of the sextupole magnets, for example. But in many cases, including FNAL Debuncher ring, machine aperture is a limiting factor. The starting point of the curves, X_0 and their slopes are determined by the size of the initial separatrix, which is in turn, determined by the initial beam emittance. Assuming that the sextupole strength g is constant during extraction and the $\Delta\nu$ is known at the beginning of the spill, all parameters including the septum wire position turn out to be constrained at this moment. There is some room for trade-off between g and $\Delta\nu_0$ according to {4}, which is resolved with other practical considerations.

The fraction of beam losses on the septum can now be calculated as a ratio

$$R_L = \frac{\left[\int n(x)dx\right]_{wire}}{\int_{Xsep}^{X\max} n(x)dx} \qquad \{9\}$$

Using $n(r) \propto \left(\frac{dr}{d\theta}\right)^{-1}$ and {7}, we derive

$$R_L = \frac{d_w}{X_S^2 - X_0^2}\frac{2X_0}{\ln\left(\frac{X_{\max}-X_0}{X_{\max}+X_0}\frac{X_S+X_0}{X_S-X_0}\right)} \qquad \{10\}$$

d_w here is the effective wire plane thickness. R_L reaches its minimum in the end of spill, when $X_0 \rightarrow 0$:

$$R_L = \frac{d_w}{X_S}\frac{X_S X_{\max}}{X_{\max} - X_S} \qquad \{11\}$$

PARAMETRIZATION IN TIME

Now we note that the plot in Figure 2 is made at a particular moment – at the beginning of the squeeze. As the tune moves to the resonance, the shape of the separatrix changes and so do the curves on this plot. The optimum conditions are changing, and we can parametrize these conditions with a single parameter $\Delta\nu$ if only this parameter is ramped during the spill. First, note from {6} that only a_0 determines curve $X_1(r)$. According to {4} we can write

$$|a_0(t)| = \frac{\Delta\nu(t)}{\Delta\nu(0)}|a_0(0)| \qquad \{12\}$$

And $a_0(0)$ is a function of initial beam emittance. Definition of function $X_2(r)$ also includes dependence {8} on $\Delta\nu(t)$. Finally we arrive at analytical expressions of optimal X_S and R_L as functions of initial beam emittance, $\Delta\nu(t)$, machine acceptance and b-function at the location of the septum. However it is not practically possible to adjust the wire position X_S during the spill. This can be compensated to some extent by the dynamic position bump. If not, the septum plane position will have to be set at some acceptable location which would not be optimal at some part of the spill. In this case this constant value should be used in expression {10}. We do not reproduce here exact formulae, because they are bulky but quite simple to reproduce from the above considerations. Instead, we show the fractional loss R_L plotted in the conditions close to those in the FNAL Debuncher ring. Figure 3 shows the septum losses in optimum vs β-function at septum at three different values of tune distance to resonance. Machine acceptance is assumed 35π mm-mr (unnormalized) and initial beam emittance is 16 π mm-mr (normalized) at 95%. More than 15m β-function is needed to reduce losses to the level of 2%.

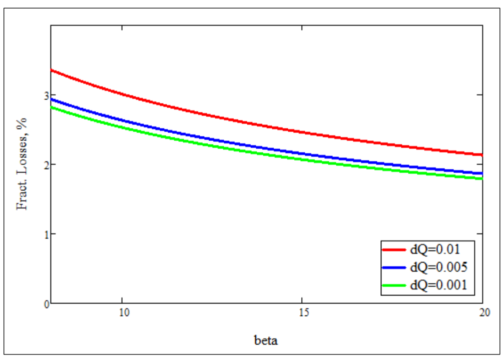

Figure 3. Septum wire losses vs the β-function

Figure 4 explores how the machine acceptance helps reducing the septum losses. Again 3 moments in the ramp are shown, assuming that ramp starts at DQ=0.01 and initial beam emittance is 16 π mm-mr (normalized), β-function is 15m.

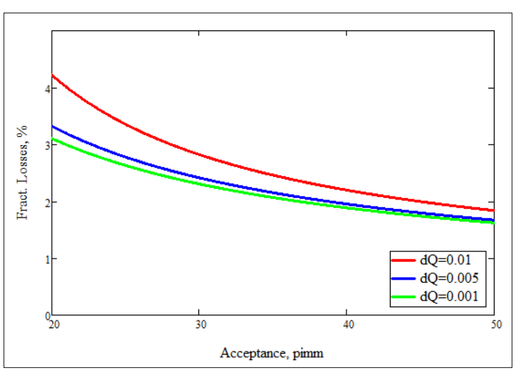

Figure 4. Septum wire losses vs machine acceptance

Extraction efficiency improves through the ramp for the obvious reason of reducing the beam size. The fractional losses curve vs the tune difference is shown in Figure 5.

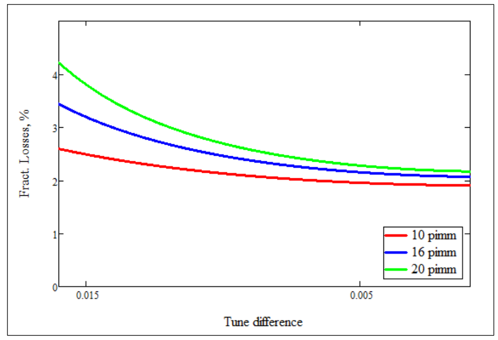

Figure 5. Septum wire losses vs tune difference

SUMMARY

We described here an semi-analytic approach to calculate the limits of fractional beam losses during the 3rd integer resonance extraction. The parametric calculations are offered for arbitrary machine and beam parameters.

REFERENCES

[1] Leo Michelotti, *Intermediate Classical Dynamics with Applications to Beam Physics*. John Wiley & Sons, Inc., New York, 1995.

[2]