Noise Estimation of Libera Brilliance+ BPM with Singular Value Decomposition

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INTRODUCTION

In sector 27 and 28 of the APS storage ring (SR) at Argonne National Laboratory, four Libera Brilliance + beam position monitors (BPMs) were installed near the injection device. These BPMs are designed to have resolution at the order of 10 nm. We aim to confirm the noise originated from electronics with principal axes analysis (PCA) and singular value decomposition (SVD). The identification of noise could potentially be helpful for noise reduction in the future.

Using SVD to perform a principal component analysis (PCA) on BPMs usually requires greater number of BPMs than the number of physical variables governing the beam motion, similar work has been done with more than 100 BPMs in SLAC by Chun-xi[1]. Another SVD analysis on BPMs was carried out in LCLS with three BPMs separated only by drift space[2]. The four Libera BPMs in APS SR, however, are separated by active elements such as quadrupole magnets and correctors. In order to validate the SVD results on four BPMs, we first simulated beam trajectories with elegant [3], injected known noise to the system, and attempted to recover the noise with SVD.

SINGULAR VALUE DECOMPOSITION ON BPM DATA

A real rectangular $m \times n$ matrix $\mathbf{B}$ can be decomposed into

$$\mathbf{B} = \mathbf{U}\mathbf{S}\mathbf{V}^T, \tag{1}$$

where $\mathbf{U}$ is a $m \times m$ orthogonal matrix, $\mathbf{S}$ is a $m \times n$ rectangular diagonal matrix, and $\mathbf{V}$ is a $n \times n$ orthogonal matrix. For practical purposes, we use a more efficient numerical computation called the economy mode that results in $\mathbf{U}$ being a $m \times n$ column-wise orthogonal matrix, $\mathbf{S}$ being a $n \times n$ diagonal matrix, and $\mathbf{V}$ staying the same. The diagonal entries in $\mathbf{S}$ are called the singular values, which are the non-negative square roots of the eigenvalues of $\mathbf{B}\mathbf{B}^T$. Each singular value corresponds to a column in $\mathbf{U}$ or $\mathbf{V}$.

We took m samples of p synchronized BPMs and arranged readings of BPMs into columns of a matrix,

$$\mathbf{B} = \begin{pmatrix} b_1^1 & b_1^2 & \dots & b_1^p \\ b_2^1 & b_2^2 & \dots & b_2^p \\ b_3^1 & b_3^2 & \dots & b_3^p \\ \vdots & \vdots & \ddots & \vdots \\ b_m^1 & b_m^2 & \dots & b_m^p \end{pmatrix}. \tag{2}$$

The first several vectors $\mathbf{v}_i$ $(i \in \{0, 1, 2, \dots, p\})$ in V from eq.1 are spatial mode vectors that correspond to prominent physical beam motions registered by the BPMs **CITATION**. The singular values indicate the importance or the contribution of the space vectors. We can extract information about the beam motion by examining the $\mathbf{v}_i$ vectors.

SIMULATION

We simulated closed orbit beam trajectories in elegant with APS SR lattice definition. Uniformly distributed random noise of 18 nm were applied on correctors in both vertical and horizontal directions. After the beam trajectories were generated, we extracted the BPM readings and injected Gaussian noise of different sigmas ranging from 10 nm to 1 μm. Then we decomposed the $\mathbf{B}$ data matrix with SVD and studied the singular value spectrum. We expect the first few largest singular values to be due to physical beam motions, which we can identify from the spatial vectors $\mathbf{v}_i$s. The rest of the singular values are primarily due to noise.

We set the few largest singular values in $\mathbf{S}$ due to physical beam motions to zeroes and obtained a trimmed singular value matrix S'. Taking the product of $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{S}'$, we obtained a reconstructed matrix,

$$\mathbf{B}' = \mathbf{U}\mathbf{S}'\mathbf{V}^T, \tag{3}$$

which, in principle, is close to the noise matrix $\mathbf{N}$ we injected. In order to better quantify the results, we define noise residuals to be the difference between the noise and the reconstructed noise, or

$$\mathbf{R} = \mathbf{N} - \mathbf{B}'. \tag{4}$$

SIMULATION RESULTS

From the simulations, we first used the readings of 78 BPMs to verify the noise reconstruction on large number of BPMs. Figure 1 shows the full singular value spectra with different amounts of injected noise. We could see clearly that the small singular values increased as the injected noise increased, and that a floor of singular values due to noise could be found.

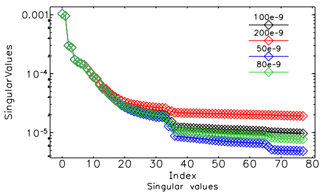

FIG. 1: The singular value spectra of 78 BPMs from simulation with different amounts of noise injected. Note that a floor of singular values due to noise could be observed and that the small singular values increase as the injected noise increase. The y-axis and legends are in units of meters.

Figure 2 compares the noise residuals (in red) and the injected noise (in black) with $\sigma = 100$ nm. The standard deviation of the noise residual is at the order of 10 nm, so the noise can be reduced by a factor of about 10.

After confirming the SVD model with data from 78 BPMs, we moved on to analyzing the data matrix with only 4 BPMs in order to compare with experimental results. Preliminary analysis indicated that four singular values limited our ability to identify the noise floor so we merged the BPM readings in both x- and y- directions. The singular value spectra of 8 columns of readings are shown in Figure 3. We noticed an increase in the last four singular values as we injected more noise. However, the reconstructed $\mathbf{B}'$ matrix was not close to the injected noise matrix when we removed the first four singular values. Attempts to reconstruct reasonably approximating noise matrix with the smallest 3 and 5 singular values were also failed.

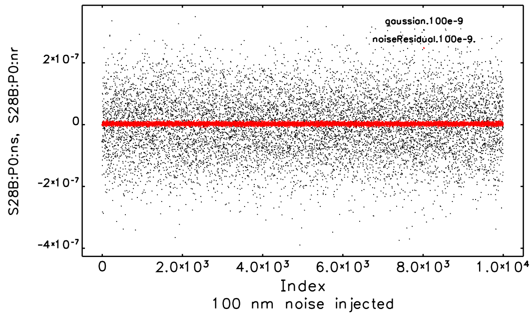

FIG. 2: A comparison between the noise residuals and the injected noise in S28B:P0 BPM when a Gaussian noise with $\sigma = 100$ nm was injected. Dots in black were injected noise and those in red were noise residuals. A noise reduction is clearly achieved here.

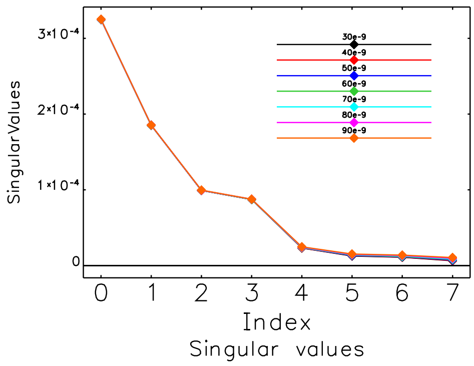

FIG. 3: The singular value spectra of 8 BPM readings (4 readings from x-direction and another 4 from y-direction). Different amounts of noise were injected. Note that a floor of singular values due to noise could be observed and that the small singular values increase as the injected noise increase. The y-axis and legends are in units of meters.

ANALYSIS ON EXPERIMENTAL DATA

Experimental data were taken with four Libera Brilliance+ BPMs with different currents of in the SR. Same SVD analysis procedure was applied and we obtained the singular value spectra in Figure 4. From the singular value spectra, we were not able to identify the noise floor singular values. By attempting to reconstruct the noise matrix with the smallest 3, 4, and 5 singular values respectively, we found the noise sigma to be at the order of 1 μm, which is much greater than the designed performance. The space vectors $\mathbf{v}_i$s seem to still capture some physical beam dynamics.

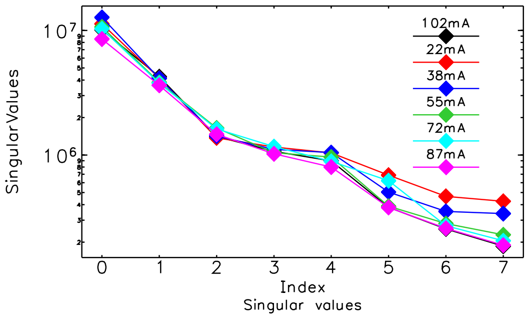

FIG. 4: The singular value spectra of 8 BPM readings (4 readings from x-direction and another 4 from y-direction) from experimental data. The y-axis are in units of nano-meters.

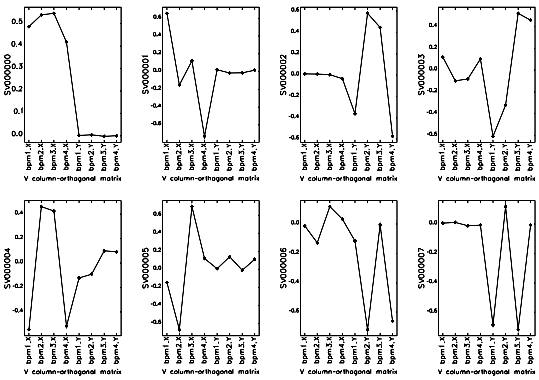

FIG. 5: The plot of column vectors in $\mathbf{V}$. Note that SVD was able to separate the physical beam motion in x- and y-dimension

While applying SVD to 8 columns of BPM readings to confirm the noise level, we found that SVD was able to separate the beam motions in x- and y-directions. The first two plots in Figure 5 showed essentially zeroes in the y dimension, and the third plot showed essentially zeroes in the x dimension.

CONCLUSION

From the analysis on the simulation results, we confirmed that SVD works well with large number of BPMs. However, SVD is not ideal in terms of identifying the noise in only a few BPMs even with data in two dimensions combined.

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[2] N. S. Sereno, "Comparison of Model Independent Analysis (MIA) and Least Squares Fitting of Monte-Carlo BPM Data with Application to LCLS BPM Resolution Analysis," Advanced Photon Source, OAG-TN-2007-033, October 2007

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