# BPM Noise Characterization with SVD

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## Introduction and Motivation

In APS storage ring, the essential way to monitor and make real-time correction to the electron orbit is to use beam position monitors (BPM). Four Libera Brilliance+ BPMs were installed in sectors 27 and 28. These BPMs have a design resolution of 60 nm.

In this study, we aim to study the electronic noise of a set of four Libera Brilliance+ BPMs with singular value decomposition (SVD) with the hope that we can confirm the design resolution.

## Singular Value Decomposition

**Theorem:** Any $m \times n$ real or complex matrix $B$ can be factorized into a matrix product

$$B = USV^T,$$

where $U$ is a $m \times n$ column-orthogonal matrix, $V$ is a $n \times n$ orthogonal matrix, and $S$ is a $n \times n$ diagonal matrix with non-negative real singular values ranking from the largest to the smallest on the diagonal and zeroes everywhere else.

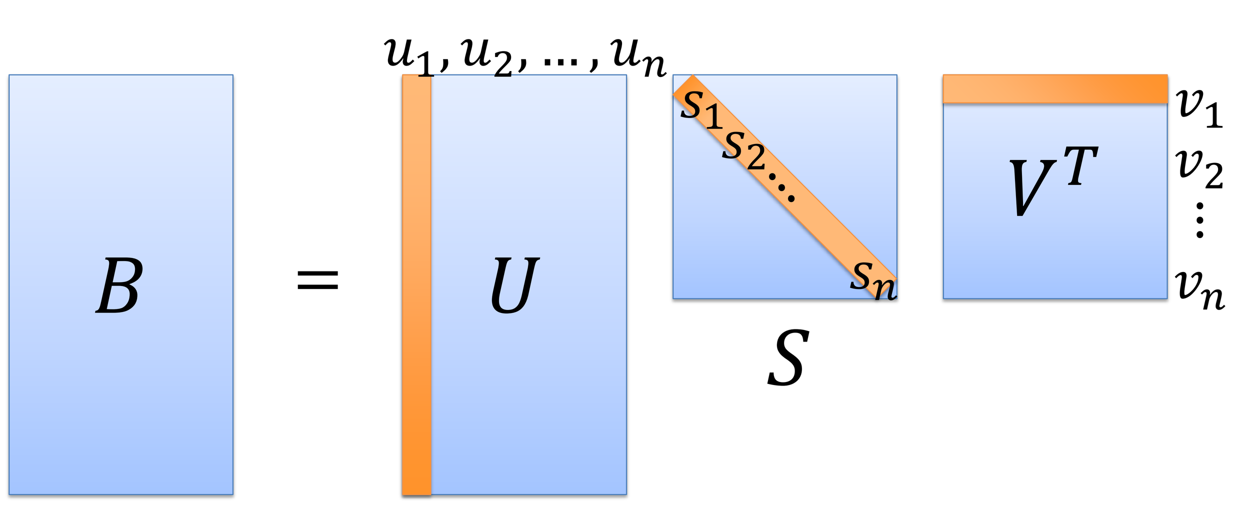


Fig.1 A block schematic of SVD. The $u_i$ vectors are time vectors and the $v_i$ vectors are space vectors.

### SVD in noise analysis:

The $B$ matrix in our study consists of $m$ columns of data, with $i$th column corresponding to the $i$th BPM. The number of rows is the number of readings from the BPMs.

Vectors $v_i$, $u_i$ make up orthogonal bases for $V$ and $U$ matrices respectively. The relative importance of these vectors are manifested by the singular values $s_i$. An example of singular value spectrum is shown in Figure 2, where only the first few singular values are outstanding, being due to actual beam motion. The rest of them are due to noise present in the signal.

By reconstructing the data matrix

$$B' = US'V^T,$$

where $S'$ is the lower diagonal matrix with large singular values eliminated, we obtain a matrix that is in principle primarily noise in nature.

## SVD on Simulated Data

BPM readings are simulated with APS SR lattice configuration in elegant[1].

### SVD analysis on 78 BPMs:

- Apparent noise floor in singular value spectrum that we could extract for noise analysis (Figure 2)
- Reconstructing data matrix $B$ with first 21 singular values (the ones above noise floor) eliminated from $S$ show a reduction of noise (Figure 3).

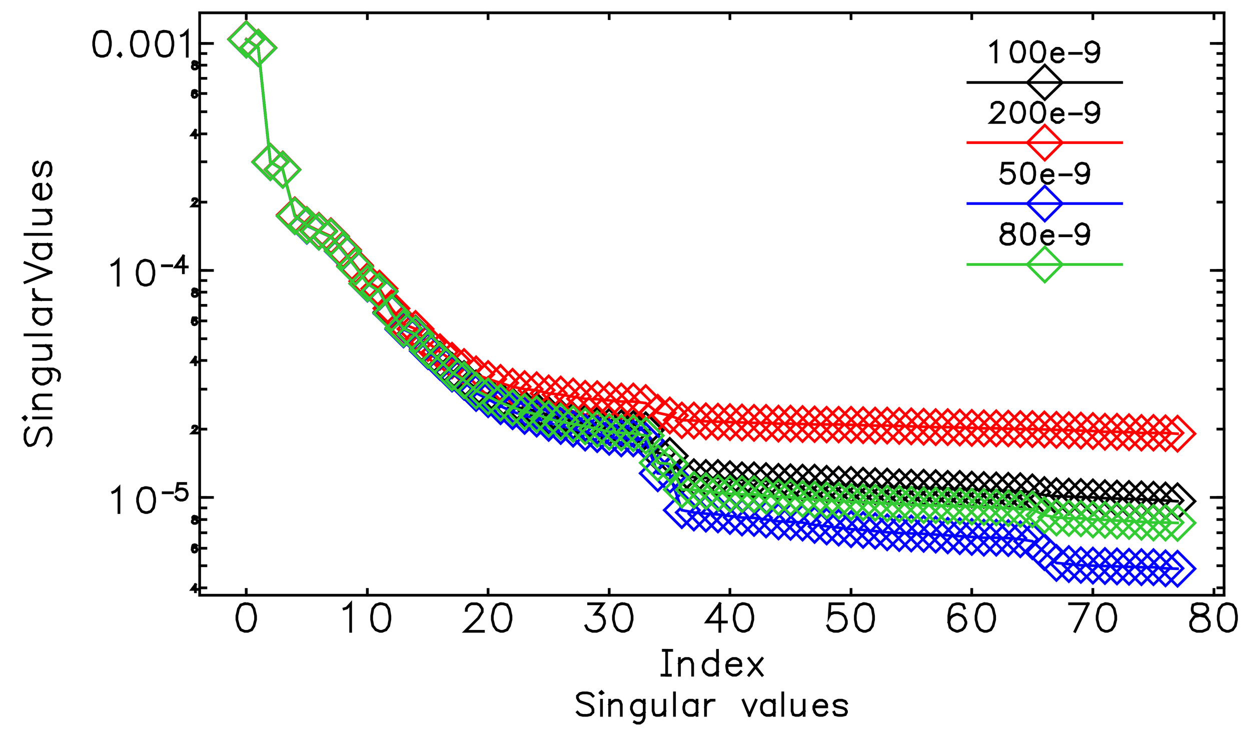


Fig.2 The singular value spectrum of 78 BPMs from simulated data. Known noise from 30 nm to 500 nm were injected. Note that increasing the sigma of noise does not affect the large singular values. (Axis in units of m)

### SVD analysis on 4 BPMs:

The readings from both $x$-, $y$-directions were combined so that we have more singular values to identify the noise floor.

- As the injected noise increases, the last four singular values are raised correspondingly
- The residual noise is as large as the injected noise after cutting the first 4 singular values, so a noise reduction was not achieved.

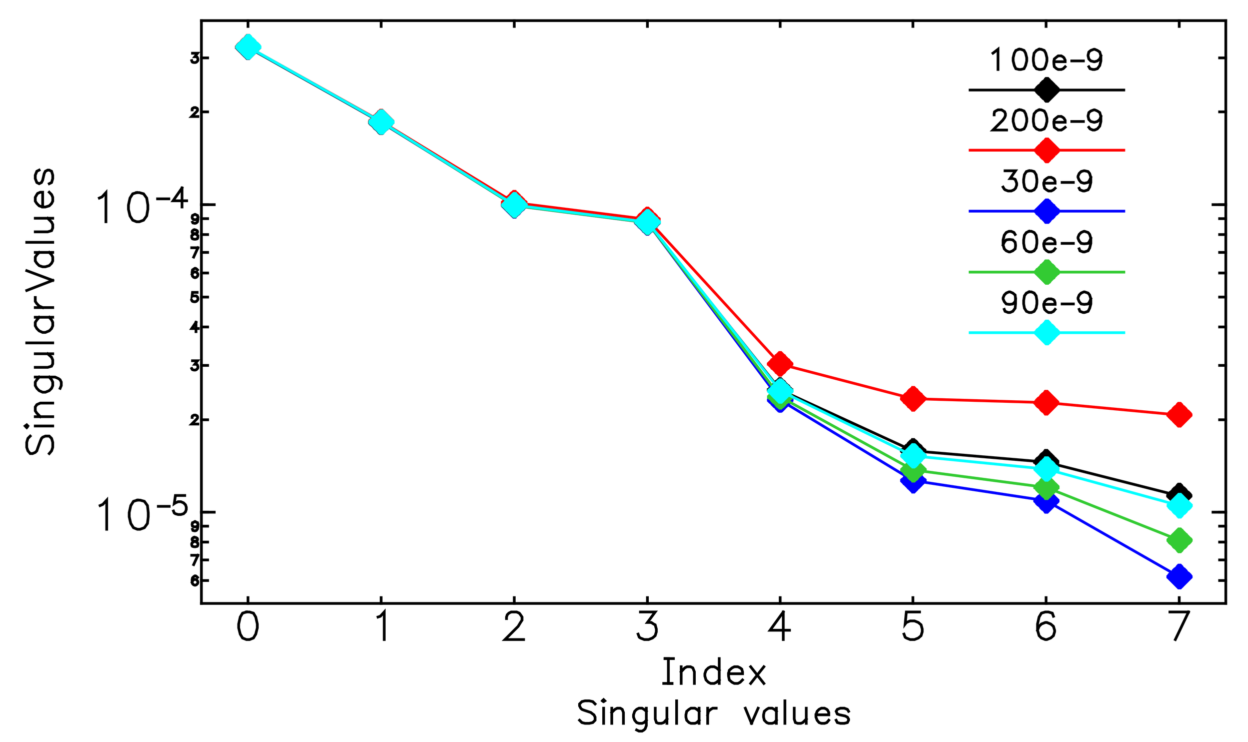


Fig.4 The singular value spectrum of 4 BPMs from simulated data. Known noise from 30 nm to 200 nm were injected. (In units of m)

## Acknowledgement

I would like to thank my supervisor Louis Emery who gave me a lot of guidance, Nicholas Sereno and Adam Brills who provided me the experimental data.

## SVD on Experimental Data

Four Libera Brilliance+ BPMs were used to obtain the data matrix. We performed a SVD analysis. As expected, the singular value spectrum did not show a well-defined noise floor.

Cutting the first four singular values reveal that the noise estimated by SVD is about 1 $\mu$m, which is most likely an overestimation since the smaller singular values still carries significant amounts of information about the real trajectory. Comparing the singular value spectra, we can conclude that the noise level increases as the beam current decreases.

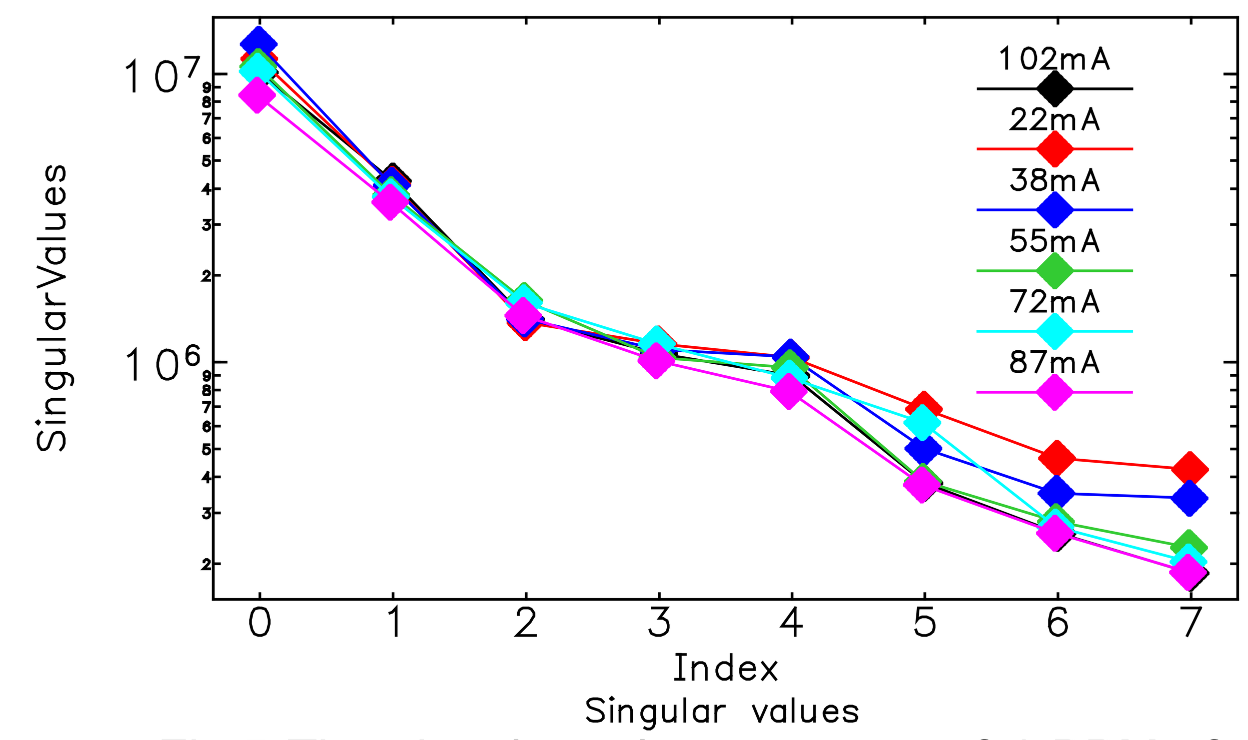


Fig.5 The singular value spectrum of 4 BPMs from experimental data of different beam currents. (Axis in units of nm)

The first four column vectors in the $V$ matrix indicate that the physical beam dynamics were captured by SVD. Little coupling between $x$- and $y$-directions occurred.

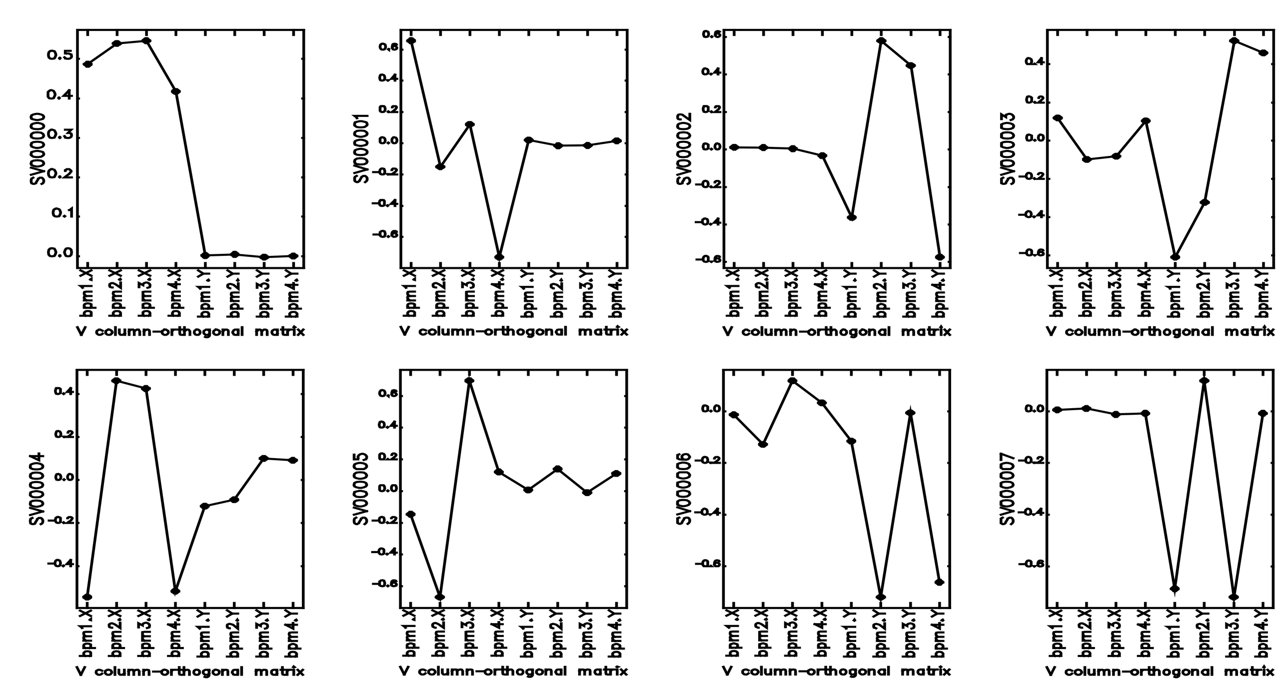

Fig.6 Plots of space vectors $v_i$ in $V$ matrix. SVD was able to separate the dynamics in $x$ and $y$ planes.

## Conclusion

SVD method works well for noise identification and reduction when we have a decent number of BPMs. We would need more Libera Brilliance+ BPMs in order to apply the same method.

## Reference

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