Design and Calculation of a Magnetic Field Probe for Half-Wave Resonators

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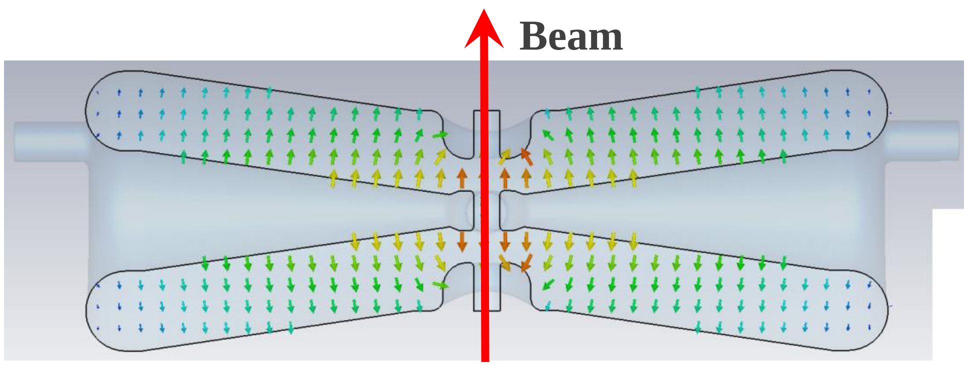
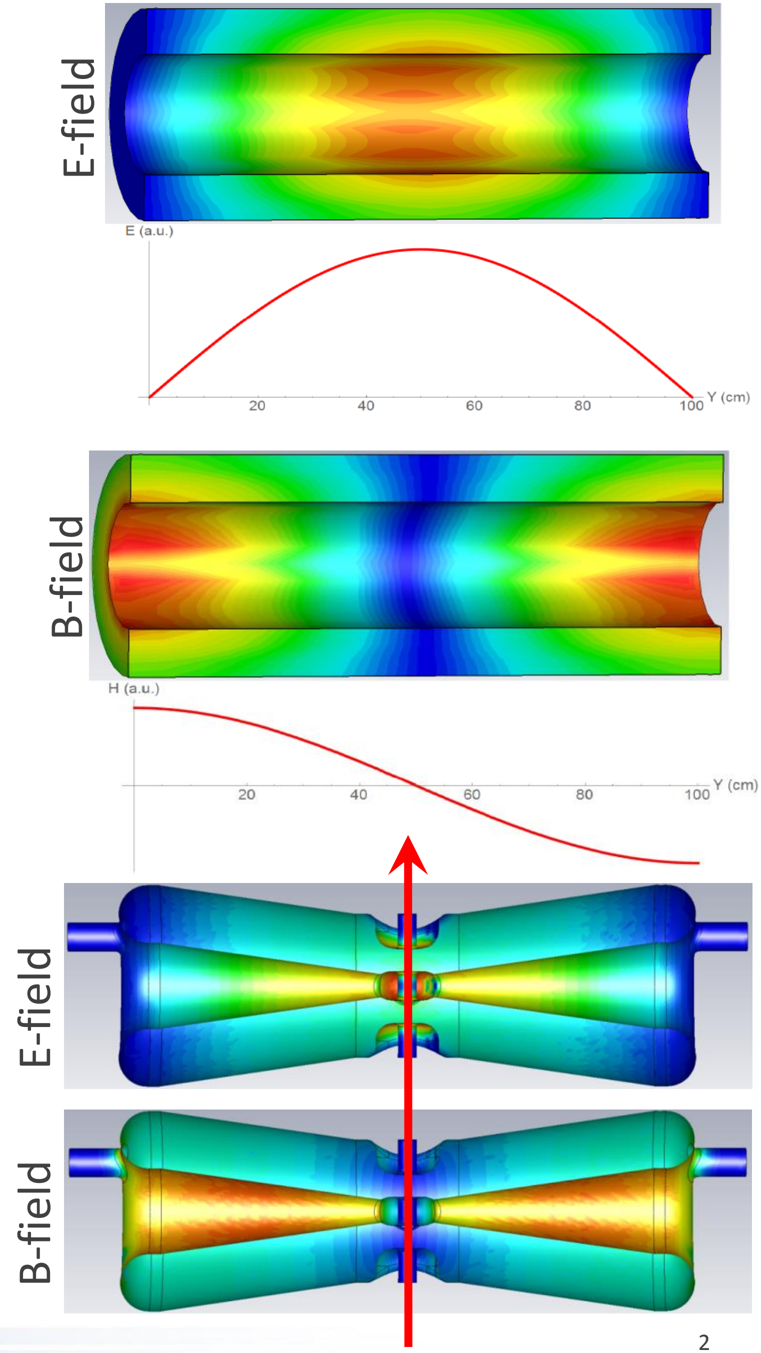
Half-wave resonators

- Accelerating cavities use resonant electromagnetic fields to accelerate and focus particle bunches.
- We use a half-wave resonator (HWR)
  - Coaxial line
  - Resonates transverse electromagnetic modes
- Need to monitor EM field Amplitude and Phase.
  - Amplitude determines the magnitude of acceleration of the bunch.
  - The phase relationship between the beam and the EM fields determines both the magnitude of acceleration and the longitudinal focusing of the bunch.



Above: The electric field on a cross-section of the HWR. The arrows show the direction of the field, and their size represents the magnitude of field.



The external quality factor

- We need to couple power out of the cavity.
  - Faraday's law: use changing magnetic field

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\iint \boldsymbol{B}\cdot d\boldsymbol{a}$$

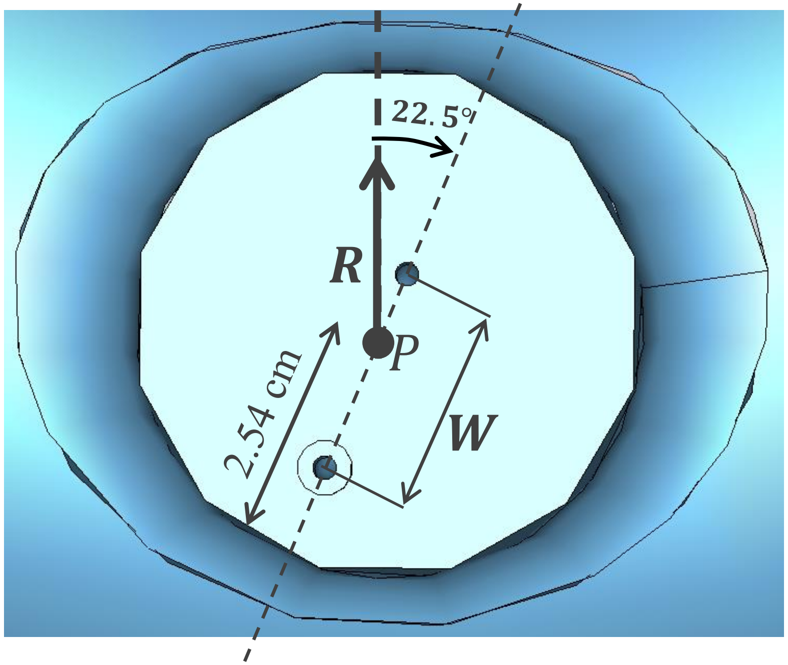
  - calculate power from induced voltage

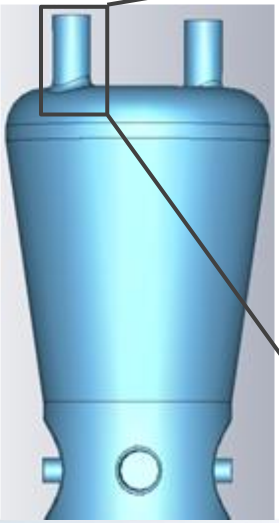
$$P_{out} = \mathcal{E}_{rms} I = \frac{\mathcal{E}_{rms}^2}{Z}$$

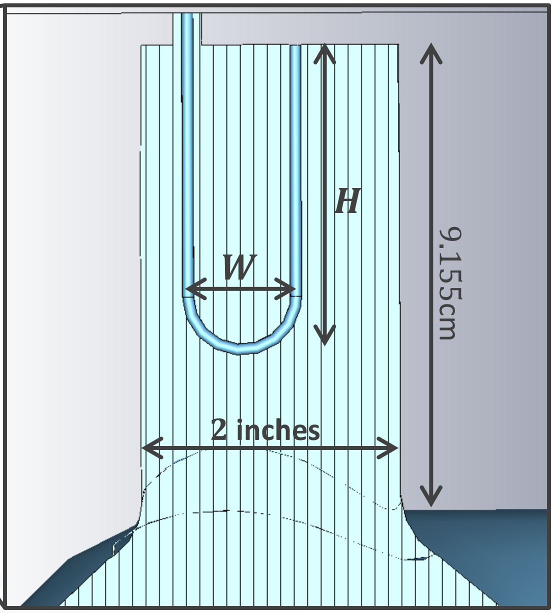
- Convenient parameter is the external quality factor

$$Q_{ext} = \frac{\omega_{rf} U}{P_{out}}$$

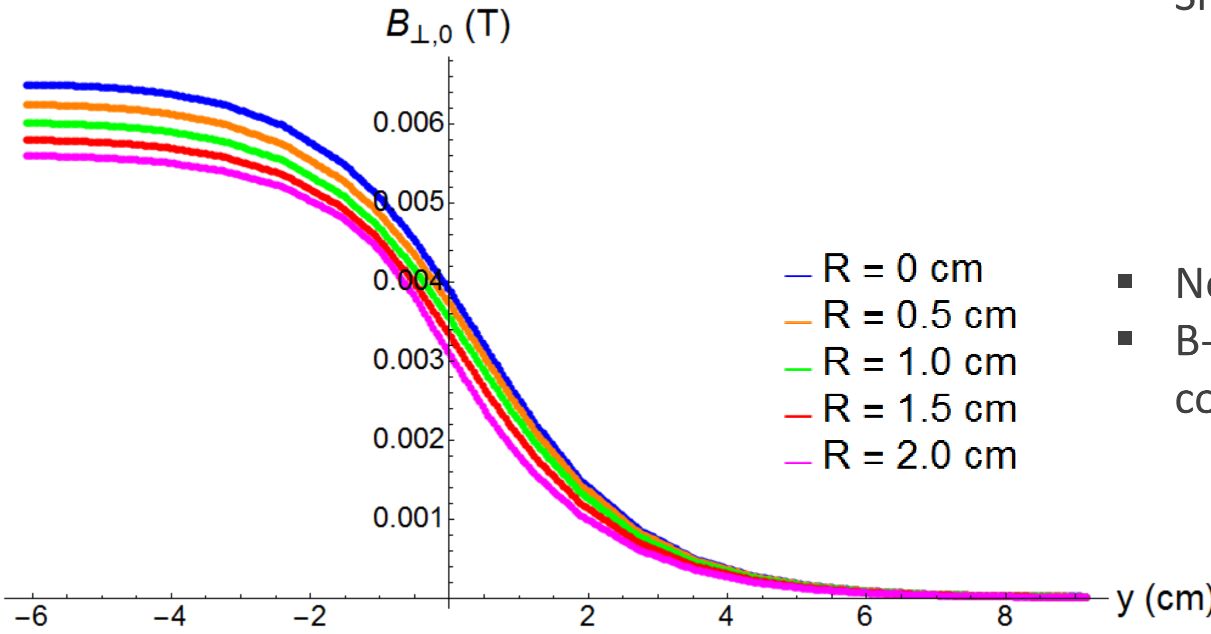
  - External quality Q_{ext}: losses out of coupler
  - Loaded quality Q_L: total losses
- I calculate Q_{ext} using Microwave Studio (MWS) and by hand



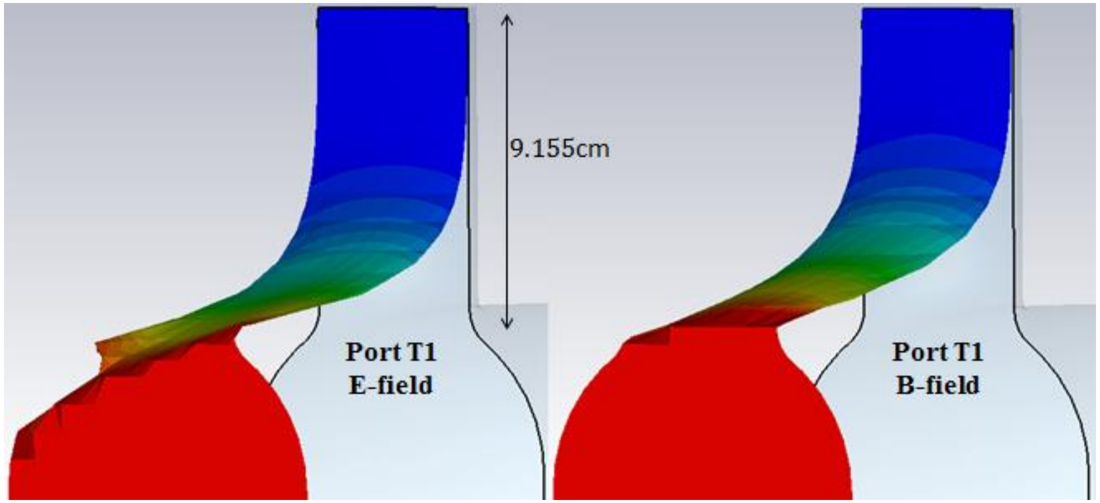




EM field in port



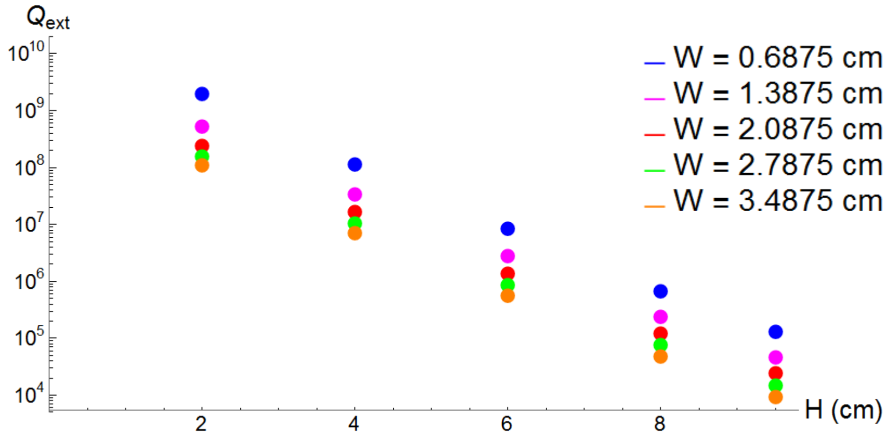
- Shape is an S-curve
  - asymptotically goes to maximum outside of port
  - asymptotically goes to zero inside port
- Not uniform along x
- B-field closer to the cavity contributes much more to flux

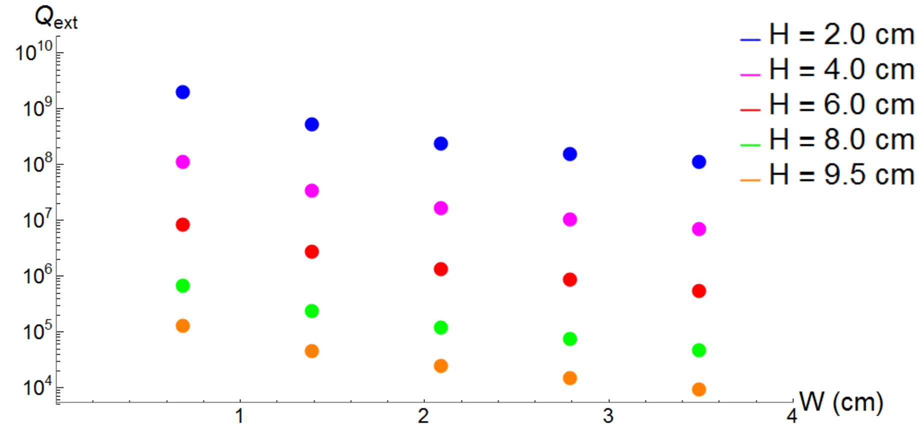


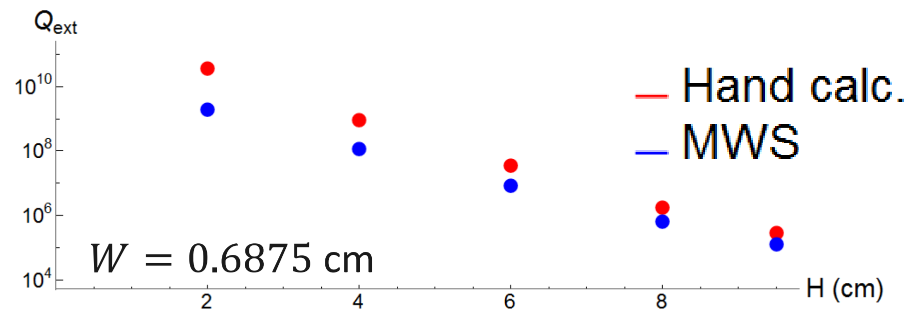
Top: A plot of the magnetic field normal to the plane of the conducting loop. Different curves correspond to different distances away from the center of the port.

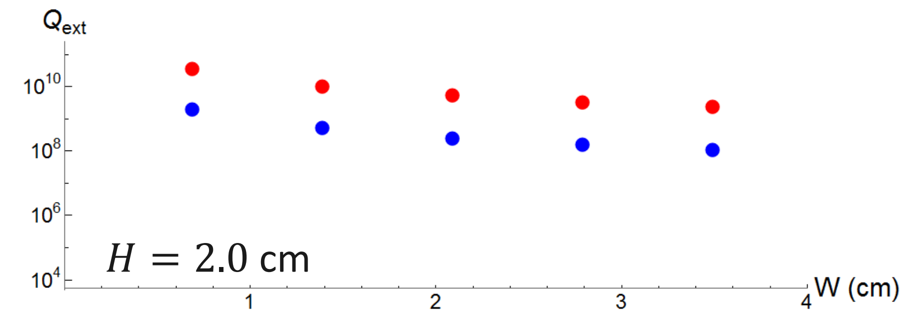
Bottom: Carpet plots of the electric (left) and magnetic (right) field magnitudes inside the pick-up probe port. Blue corresponds to low magnitudes, red corresponds to high magnitudes.

MWS vs. hand calculations





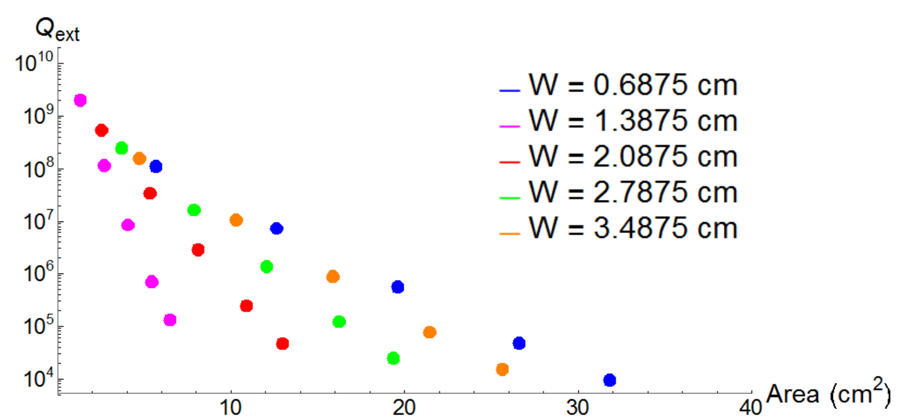




The external quality factor against loop height.

The external quality factor against loop width.

$$Q_{ext} = \frac{\omega_{rf} U}{P_{out}}$$



The external quality factor against loop area.

Summary

- The phase and amplitude of the EM fields of an accelerating cavity need to be monitored.
  - I designed and simulated a power coupler for this monitoring.
  - We used a conducting loop to couple to the magnetic fields for HWRs.

- The external quality factor Q_{ext} is useful for comparing the coupling strength for different loop geometries
  - lower Q_{ext} have higher emitted power

- Q_{ext} was calculated two ways: MWS and hand calculations
  - hand calc. overestimated Q_{ext}, but had the same dependence on height and width

- Smallest loop → greatest Q_{ext}: $H = 2.0$ cm, $W = 0.6875$ cm
  - lowest P_{out}
- Largest loop → lowest Q_{ext}: $H = 9.5$ cm, $W = 3.4875$ cm
  - greatest P_{out}
- Actual loop could be as large as $H = 9.036$ cm, $W = 4.008$ cm
  - MWS: $Q_{ext} \approx 1.098 \cdot 10^4$
  - Hand calc: $Q_{ext} = 2.1250 \cdot 10^4$

Hand Calculations

- Need to calculate magnetic flux through the coupling loop.

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\iint \boldsymbol{B}\cdot d\boldsymbol{a} = -\iint \frac{d}{dt}[B_\perp]\cdot da$$

- We assume that the B-field is
  - sinusoidal with time: $B_\perp(x,y,t) = B_{\perp,0}(x,y)\sin(\omega_{RF}t)$
  - uniform across the width of the coupling loop: $B_{\perp,0}(x,y) = B_{\perp,0}(y)$
  - $B_{\perp,0}(y')$ is a fit to B-field in port (at $R = 0$)

$$B_{\perp,0}(y) = \frac{a}{b + ce^{d\cdot y}}$$

- Calculate power using

$$P_{out} = \mathcal{E}_{rms}I = \frac{\mathcal{E}_{rms}^2}{Z}$$