

Nonlinear proton dynamics in the IOTA ring

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Proton Accelerators for Science and Innovation

Fermi National Accelerator Laboratory

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Collaboration

- DOE HEP Phase II SBIR project
- Ongoing collaboration between RadiaSoft and Fermilab

D.L. Bruhwiler[&] N. Cook[&] S. Webb[&] A. Valishev[#]
R. Nagler[&] R. Kishek^{&,ψ} E. Prebys[#] S. Nagaitsev[#]



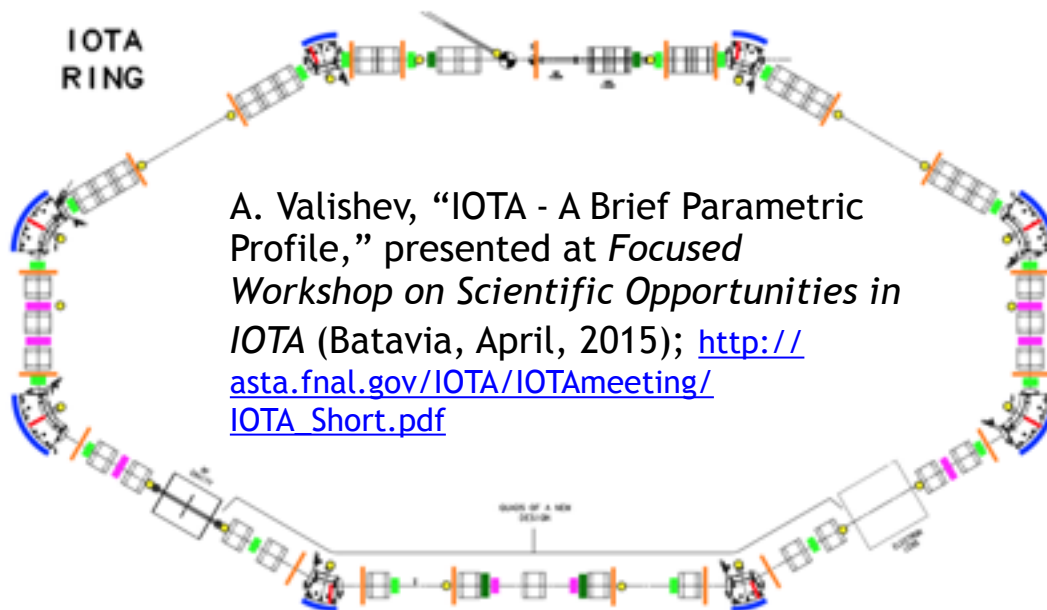
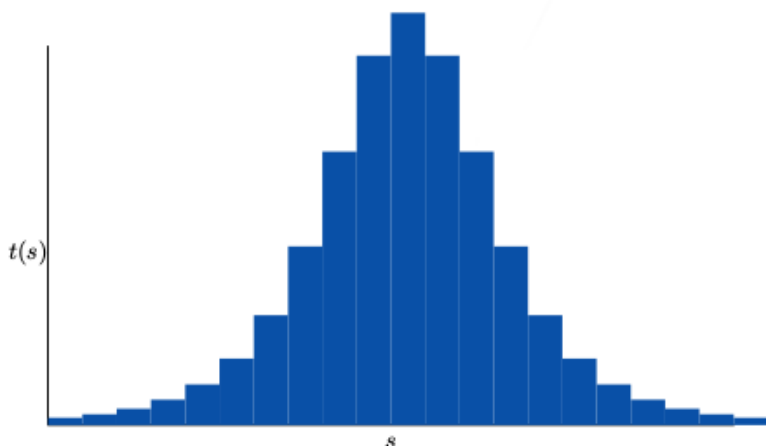
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Outline

- Integrable optics at IOTA
- Synergia 2.1 for beam dynamics simulations
- Nonlinear dynamics in the zero current limit
 - Higher order terms in the nonlinear Hamiltonian
 - Variation with H_0 and emittance
 - Correlations between 1st and 2nd invariant
 - Chromaticity correction schemes
- Initial studies of nonlinear dynamics with space charge
 - Tune depression
 - Diffusive dynamics and phase space mixing

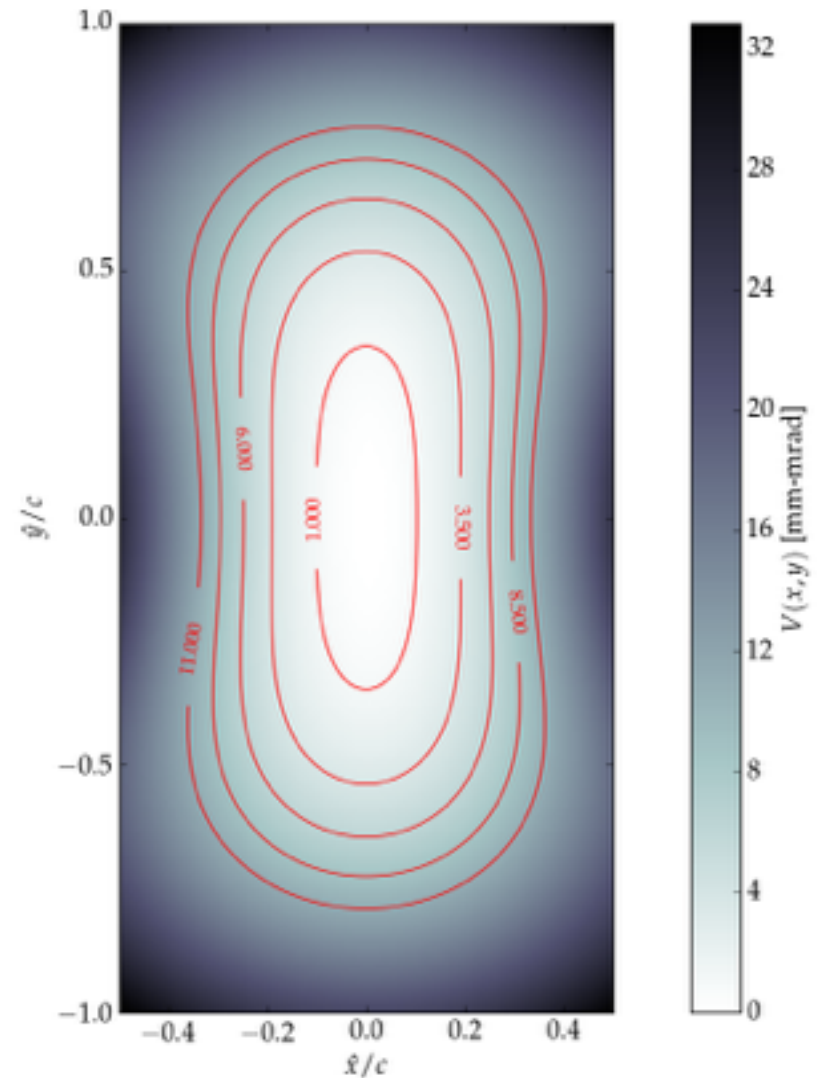
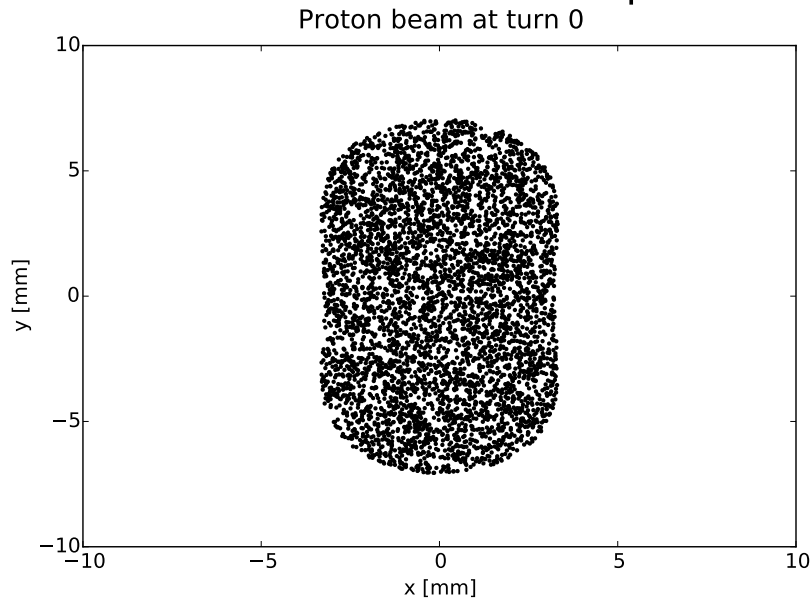
Integrable Optics and the IOTA lattice

- Experimental initiative to test nonlinear integrable optics
 - Danilov & Nagaitsev “Nonlinear accelerator lattices with one and two analytic invariants,” PRSTAB **13**, 084002 (2010)
- Use of special nonlinear magnet can result in a 2nd invariant of motion, completely integrable dynamics
 - Single particle trajectories are regular and bounded
 - Mitigate parametric resonances via nonlinear decoherence

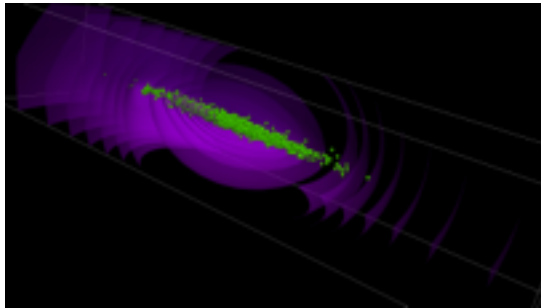


Bunch matching with the nonlinear lattice

- Danilov & Nagaitsev derived potential which produces Hamiltonian with two analytic invariants
 - Vary magnetic field strengths to obtain order unity tune spread
 - Symmetric motion through NL section preserves integrability
- A generalized K-V distribution with fixed H demonstrates shape



Beam dynamics with space charge via Synergia 2.1



Synergia: A comprehensive accelerator beam dynamics package

<http://web.fnal.gov/sites/synergia/SitePages/Synergia%20Home.aspx>

 **Fermilab**
Accelerator Simulation Group

James Amundson, Qiming Lu, Alexandru Macridin, Leo Michelotti, Chong Shik Park, (Panagiotis Spentzouris), Eric Stern and Timofey Zolkin

Computer time from INCITE



The ComPASS Project
High Performance Computing for Accelerator Design
and Optimization
<https://sharepoint.fnal.gov/sites/compass/SitePages/Home.aspx>



Funded by DOE SciDAC

CAMPA

Consortium for Advanced Modeling
of Particle Accelerators

Funded by DOE



Synergia 2.1 Overview

- Capabilities include:
 - symplectic tracking for single particle dynamics
 - PIC (validated with GSI space charge benchmark)
 - dynamic lattice element adjustments
 - Fit tunes or chromaticities to desired values
 - Construct arbitrary 6×6 matrix elements
- New capabilities added by RadiaSoft:
 - bunch matching with arbitrary distributions
 - interactive simulations with an IPython notebook

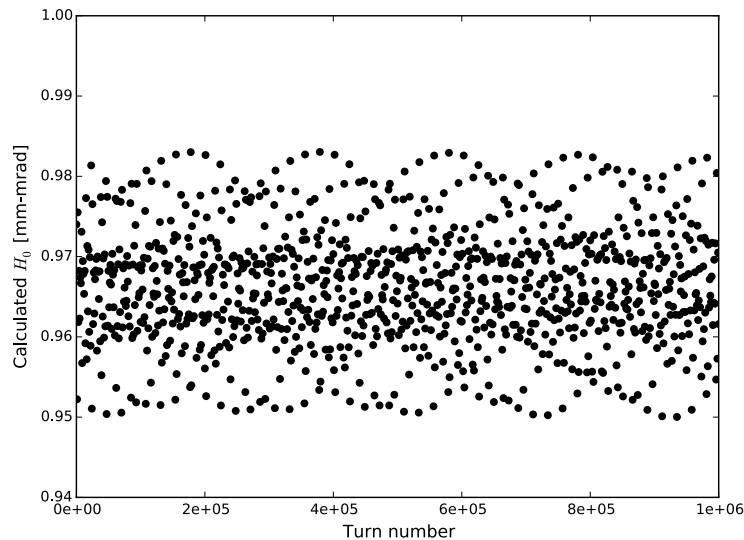
Thanks very much to the Synergia development team, from the Accelerator Simulation group within Fermilab's Scientific Computing Division!

Especially: J. Admundson, E. Stern, C.S. Park, L. Michellotti, and P. Spentzouris

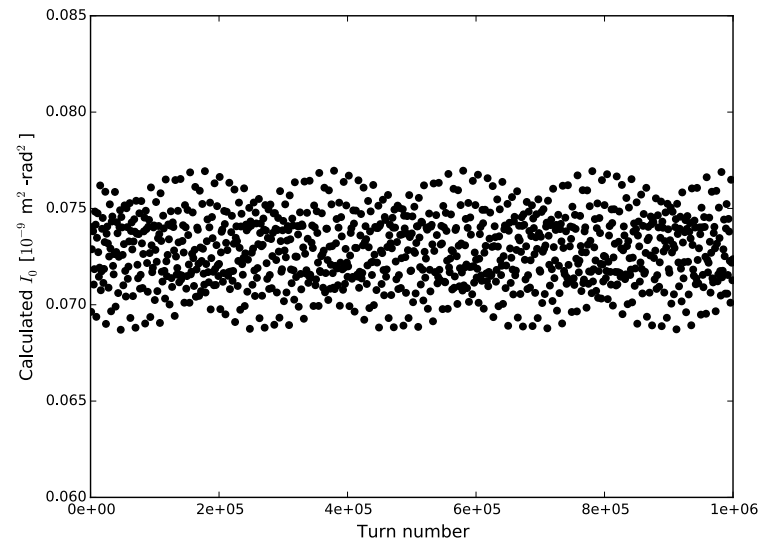
Initial simulations - 100k turns with NL element

- Tracking matched proton bunch in lattice
 - Generalized KV distribution with fixed H_0
 - $\varepsilon_x = 0.03$ mm-mrad normalized emittance
 - $\delta = 0\%$ — no variation in particle energy
- Resulting variation is consistent over 100k turns
 - $\sigma_H = 1.07\%$, $\sigma_I = 2.56\%$ for the particle shown below
 - Periodic variation which appears bounded

First Invariant with turn # for 100k turns



Second Invariant with turn # for 100k turns



Higher order terms in the Nonlinear Hamiltonian

- Original analysis computed the single turn map for the ideal integrable optics lattice
- Expansion of the Hamiltonian lowest order correction to H in ν_0 , the “tune-advance” through the nonlinear drift

$$\mathcal{H} = \nu_0 \underbrace{\left\{ \frac{1}{2} (\vec{p}^2 + \vec{x}^2) + tV(\hat{x}, \hat{y}) \right\}}_{\text{D.-N. Hamiltonian}} +$$

$$\frac{1}{6} \nu_0^3 \underbrace{\left\{ \frac{1}{4} [\vec{p}^2 + \vec{x}^2, [\vec{p}^2 + \vec{x}^2, tV(\hat{x}, \hat{y})]] - \frac{t^2}{2} [V(\hat{x}, \hat{y}), [V(\hat{x}, \hat{y}), \vec{p}^2 + \vec{x}^2]] \right\}}_{\text{Next-order correction}} + \mathcal{O}(\nu_0^5).$$

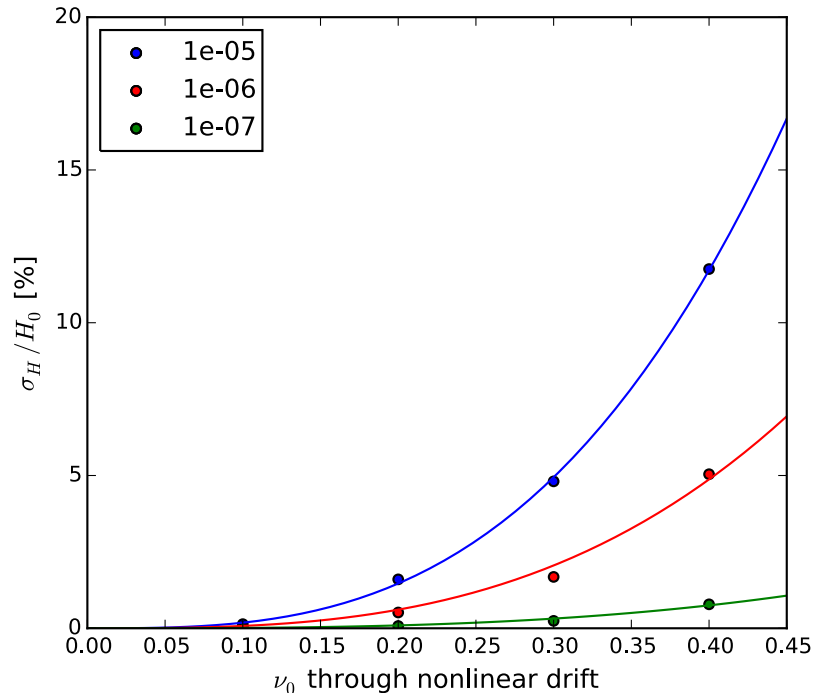
- *The leading order correction to the ideal Hamiltonian H_0 scales with ν_0^3 , thus implying a variation in H from the first order estimate H_0*

$$\mathcal{H} = H_0 \nu_0 + H_2 \nu_0^3 + \dots$$

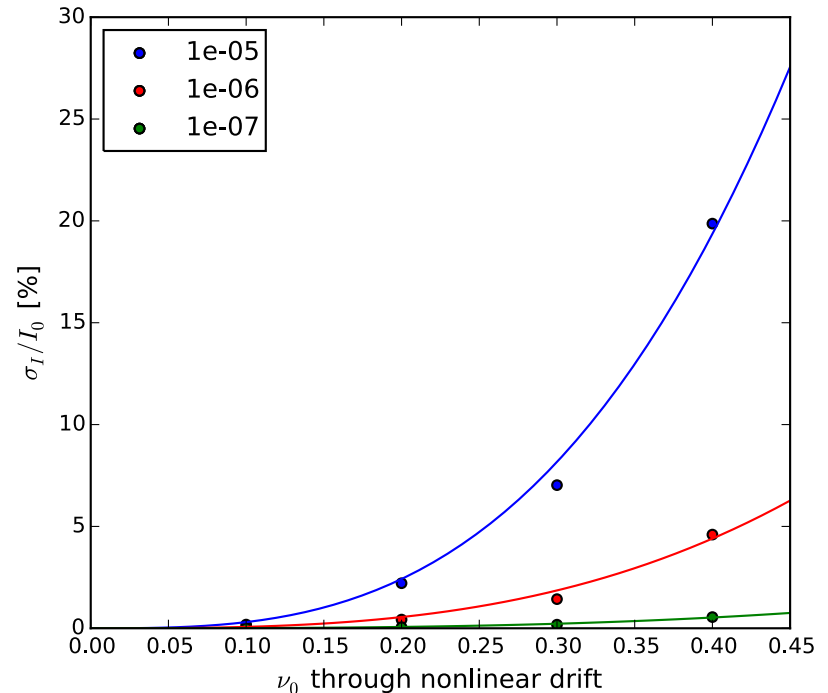
Variation in the first invariant - H_0

- Simulated a toy-model IOTA lattice, comprised of a nonlinear element followed by a corresponding 6x6 matrix representing a thin double-focusing lens.
 - Variations of the nonlinear element with different ν_0 are calculated using a MADX script

Standard Deviation in H versus phase advance ν_0

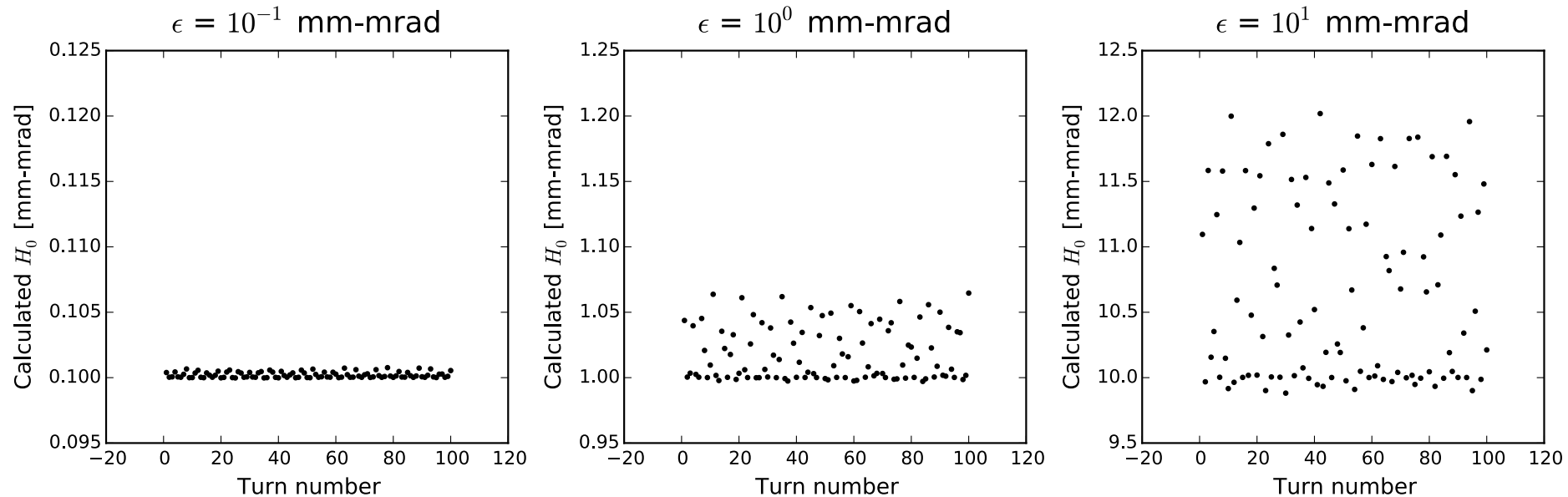


Standard Deviation in I versus phase advance ν_0



Variation in H_0 with increasing emittance

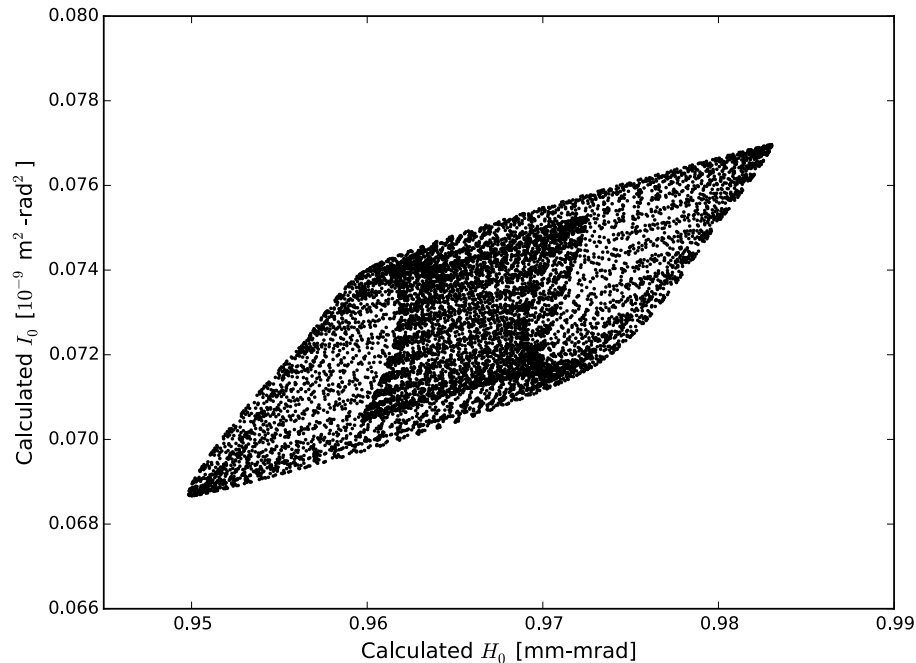
- Greater variation in H_0 with increasing emittance — coefficients in the expansion of the Hamiltonian vary with ϵ^3 .
- Ex. For a NL segment with $\nu_0 = 0.3$, a KV distribution with $H_0 = 10$ mm-mrad demonstrates an average r.m.s. variation of 5% in calculated value of H_0 .



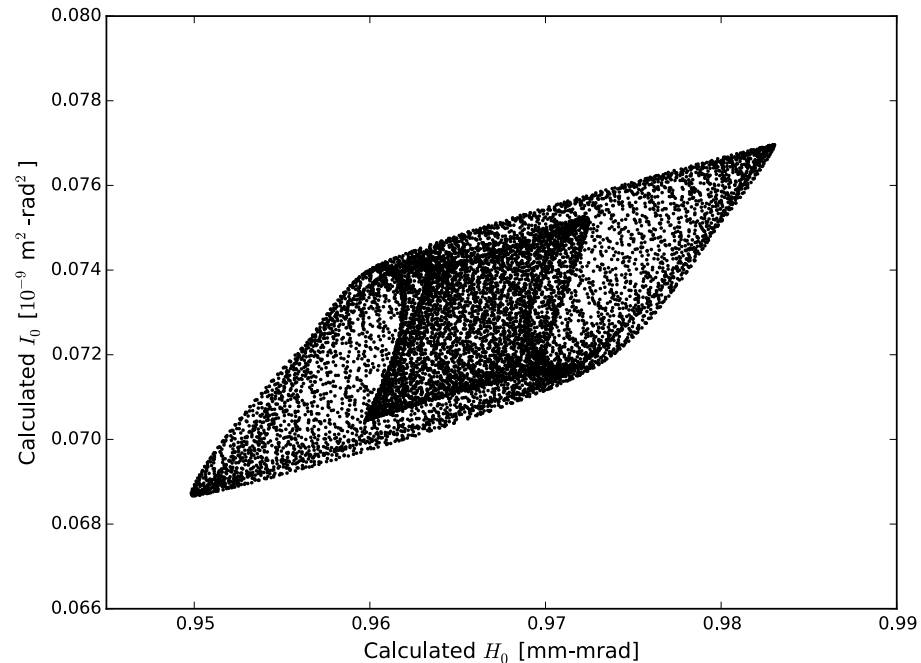
Higher order correlations between H and I

- Turn by turn variation observed over many turns
 - Variations appear bounded and well behaved
- Suggestive of some surface relating to a new invariant
 - Can we calculate this hyper-surface?

Correlation between 1st and 2nd invariants - 10K turns



Correlation between 1st and 2nd invariants - 100K turns



Chromatic Considerations in IOTA

- Tests performed for matched bunches with energy deviations $\delta \leq 0.5\%$, with and without chromaticity correction
- Simulations of chromaticity **corrected lattice**
 - Beam loss observed (4%) even for a bunch with $\delta = 0.1\%$ and a normalized emittance of $\epsilon_x = 0.03$ mm-mrad
 - Invariant clearly not preserved - $\sigma_H > 15\%$
 - As δ approaches 0.4%, the majority of the beam is lost
- **Uncorrected lattice** retains better single particle dynamics
 - For $\delta = 0.1\%$, no particle loss and $\sigma_H \approx 10\%$
- **Conclusion:** Off-energy particles in IOTA are more sensitive to sextupole fields than to natural chromaticity of the lattice.

Chromaticity Correction Schemes

- For realistic energy spreads, chromaticity significantly perturbs integrable motion

$$\overline{H} = \overline{H}_0 + \Delta_C (\overline{p}_x^2 + \overline{x}^2 - \overline{p}_y^2 - \overline{y}^2)$$

- Recent work by Webb et al. presents a chromaticity correction scheme which preserves integrability
 - Require equal horizontal and vertical chromaticities $C_x = C_y$
 - Sextupoles must be optically transparent ($\delta\phi = (2n + 1)\pi$)
- Recover normalized Hamiltonian with adjustment to nonlinear strength parameter, t
 - *Complete correction may not be needed!*
 - *Carefully placed sextuples at low strength may correct for energy spread*
- Current work aimed at adjusting IOTA-110 design to meet these requirements for basic testing

Initial simulations with space charge

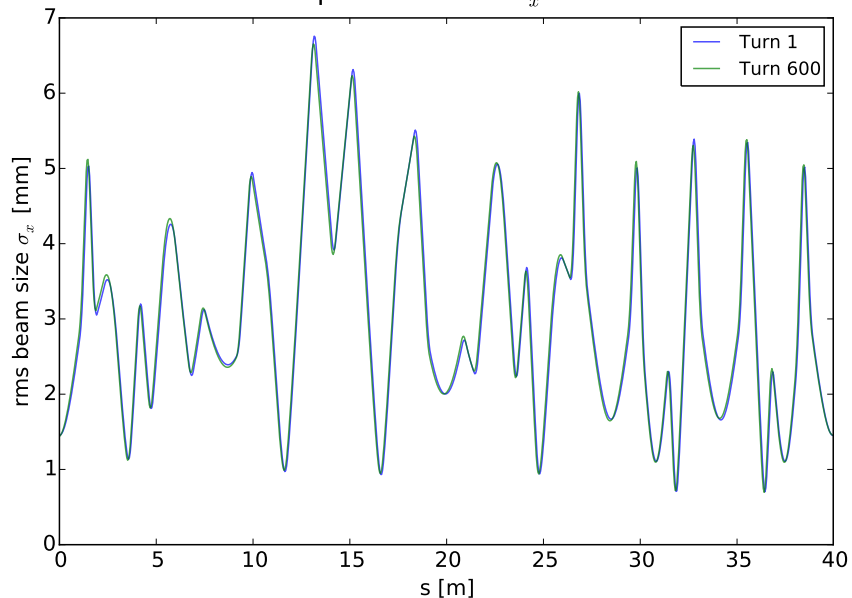
- Lattice Parameters:
 - Iota v6.6 lattice with a single NL magnet
 - zero sextupole strength, zero RF
 - CHEF propagation of all lattice elements, 4 slices per element
 - **Nonlinear strength parameter $t = 0.4$**
 - Nonlinear aperture parameter $c = 0.01 \text{ m}^{1/2}$
 - diagnostics at center of NL magnet
- Space Charge parameters:
 - 2D open solver with 64x64 mesh, periodic in z
 - 100 particles per cell
- Proton beam parameters:
 - 2.5 MeV protons, matched at center of NL element
 - $dp/p = 0$, to minimize chromatic effects
 - generalized KV distribution with first invariant $H_0 = 9.74 \text{ mm-mrad}$
 - corresponds to $\epsilon_{x,n} = 0.3 \text{ mm-mrad}$, and $\epsilon_{y,n} = 0.6 \text{ mm-mrad}$

Space charge tune shift in IOTA

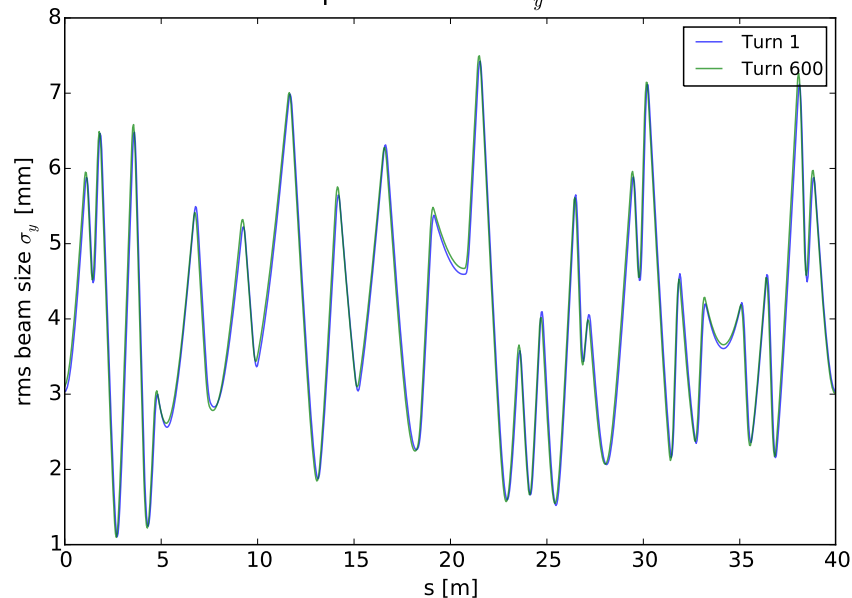
- Linear lattice with zero current
 - tunes are equal and vary weakly with amplitude
 - $Q_x = Q_y = 0.3$
- Nonlinear lattice with zero current
 - tunes are asymmetric and vary strongly with amplitude
 - $Q_x = 0.403$ $Q_y = 0.232$ (very near the axis)
- With space charge, tune varies orbit to orbit
 - Compute incoherent tune shift $\Delta Q_x = -\frac{r_0 N}{2\pi \epsilon_x \beta^2 \gamma^3}$
 - Compare against single particle orbits
 - Observe $dQ_x \sim dQ_y$ (near axis)
 - evidence of strong coupling in x and y

Beam evolution at $dQ = -0.01$ IOTA lattice

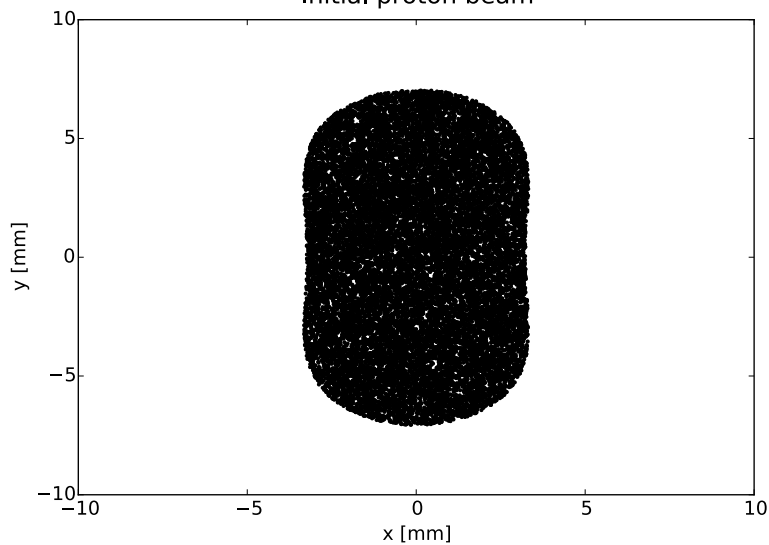
Beam envelope evolution - σ_x over 600 turns



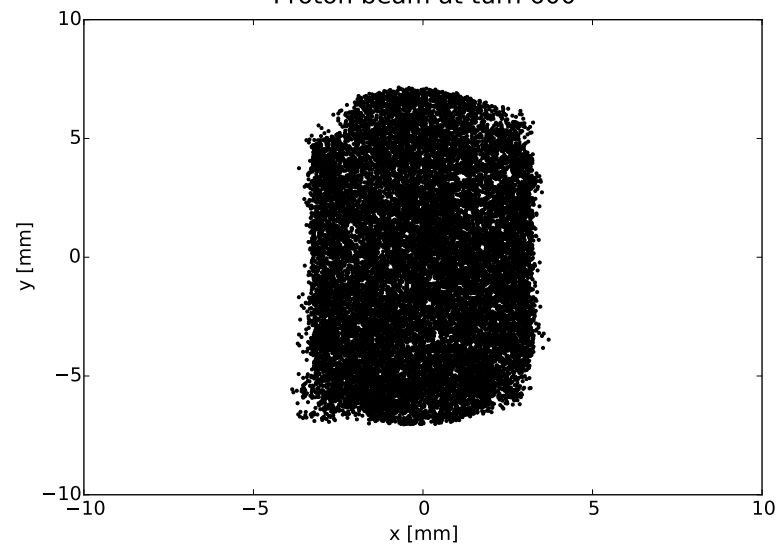
Beam envelope evolution - σ_y over 600 turns



Initial proton beam

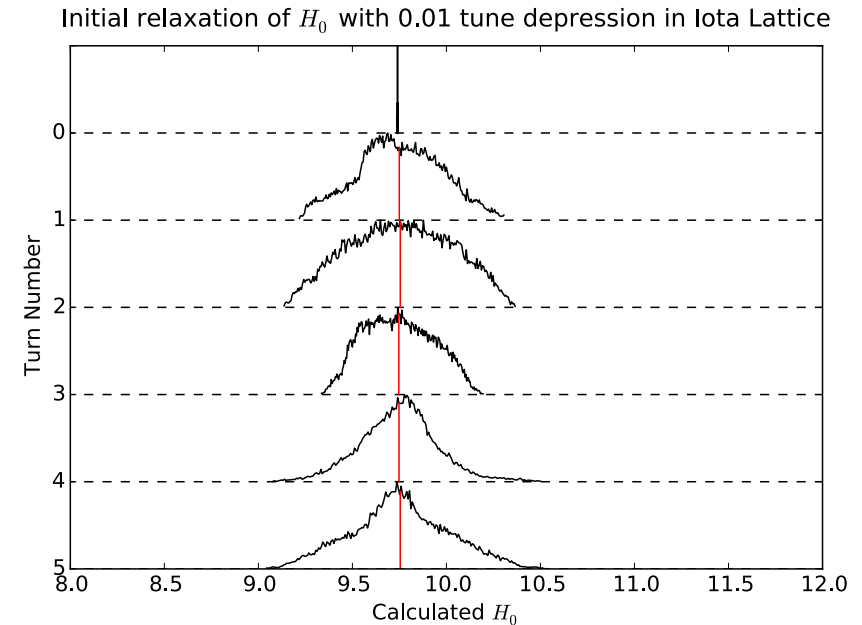
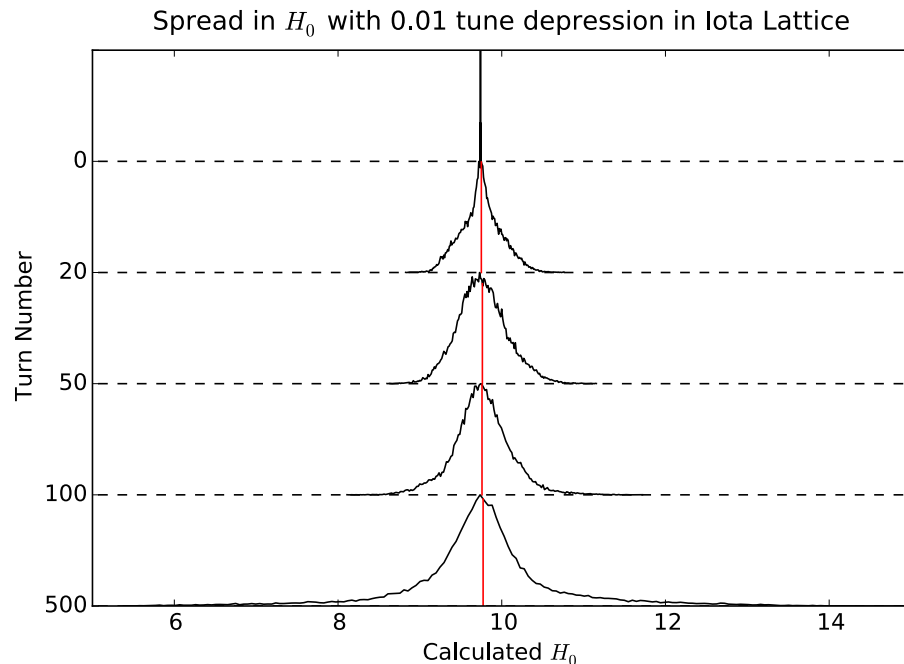


Proton beam at turn 600



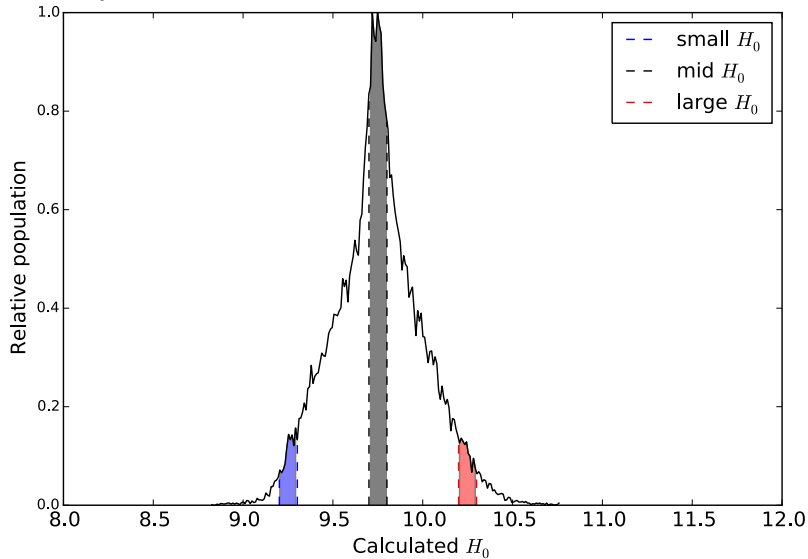
Particle dynamics - Equilibration and diffusion

- Mixing on the scale of 1 turn
 - $t = 0.4$ is rather large, so tune spread leads to fast mixing
- Diffusion varies with H_0
 - consistent at median values
 - strong tails at large H_0

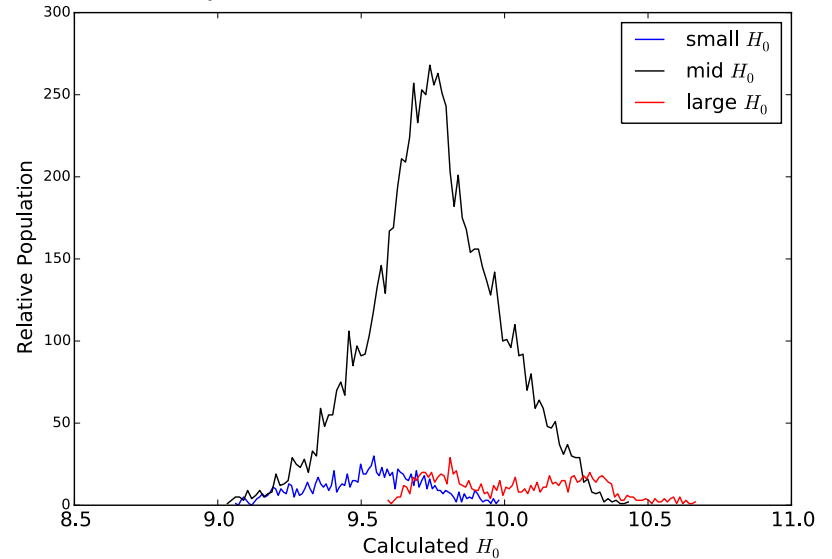


Dependence of diffusion on H_0

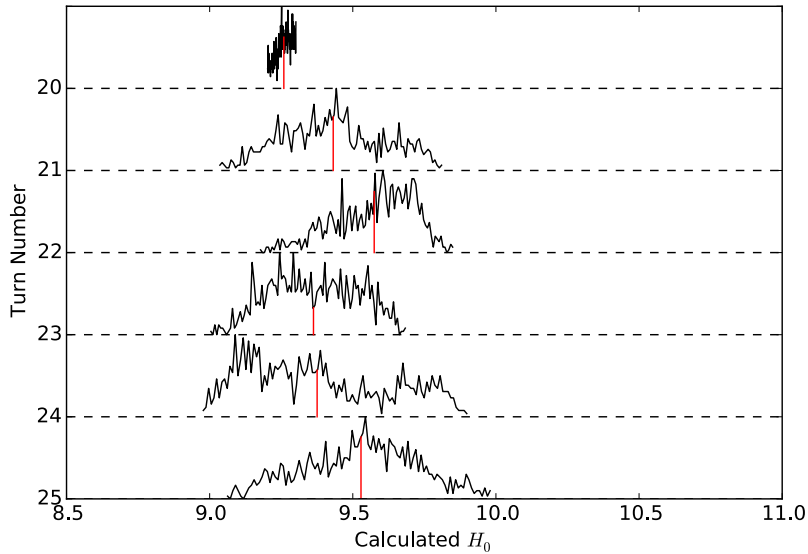
H_0 distribution after 20 turns - 14 mA current in Iota Lattice



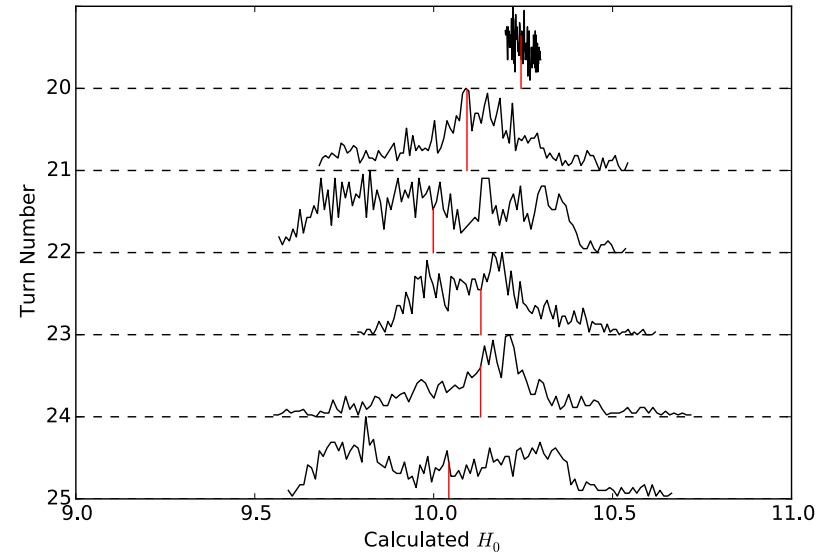
H_0 distribution for full population - Turn 25



Spread in H for low H_0 population Turns 20-25



Spread in H for high H_0 population Turns 20-25



Conclusions

- Simulated IOTA lattice with segmented NL element
 - Invariants appear well behaved over 1 million turns
- Higher order contributions of NL magnet at zero current
 - Variation in the invariants scales with v_0^3 and ϵ^3
- Large energy spread breaks integrability
 - Dynamics are especially sensitive to sextupole fields
- Initial simulations with space charge
 - Tune depression appears to break integrability
 - But there may be other nearby invariants
 - Look for bounded behavior to retain nonlinear decoherence
 - Operation at modest currents will likely require adjustments to the magnets surrounding the integrable optics insert

Future Work

- Understand dynamics with space charge
 - Investigate single particle dynamics with low nonlinearity
 - Adjust lattice incrementally to recover integrability
 - Calculate diffusion coefficient in perturbative regime
- Implement chromaticity correction scheme
 - Optically transparent sextuples to equate chromaticities
 - Optimize dynamic aperture
- Simulate wakefields in IOTA
 - Quadrupolar term along vertical axis may limit peak current
- Simulate injection of matched beam
- Consider machine perturbations
 - misalignments and mismatches