In-medium $Q\bar{Q}$ potential and Heavy-Quark Diffusion in the QGP

Shuai Liu

Advisor: Ralf Rapp

Cyclotron Institute + Department of Physics and Astronomy
Texas A&M University

College Station

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Outline

1) Background and Motivation

2) New Way to Define Heavy-Quark Potential Based on Lattice Data

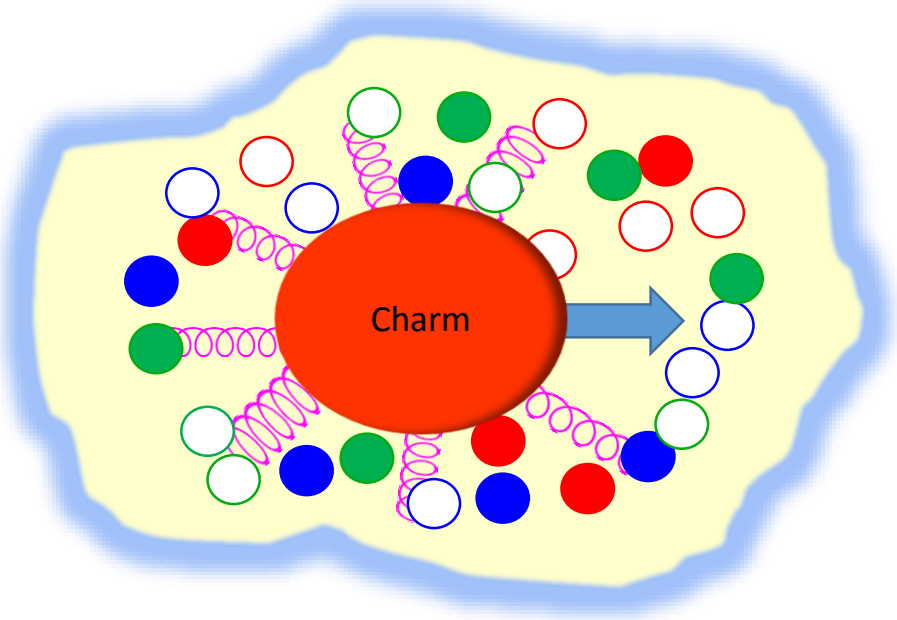
- Calculation of heavy-quark free energy from in-medium T matrix
- Application of the new method to extract a potential

3) Transport Model with the New Potential

- In-medium heavy-quark T-matrix and transport coefficient
- Langevin simulation and charm quark spectra

4) Conclusions

Heavy Quarks, Potential Model and Lattice QCD

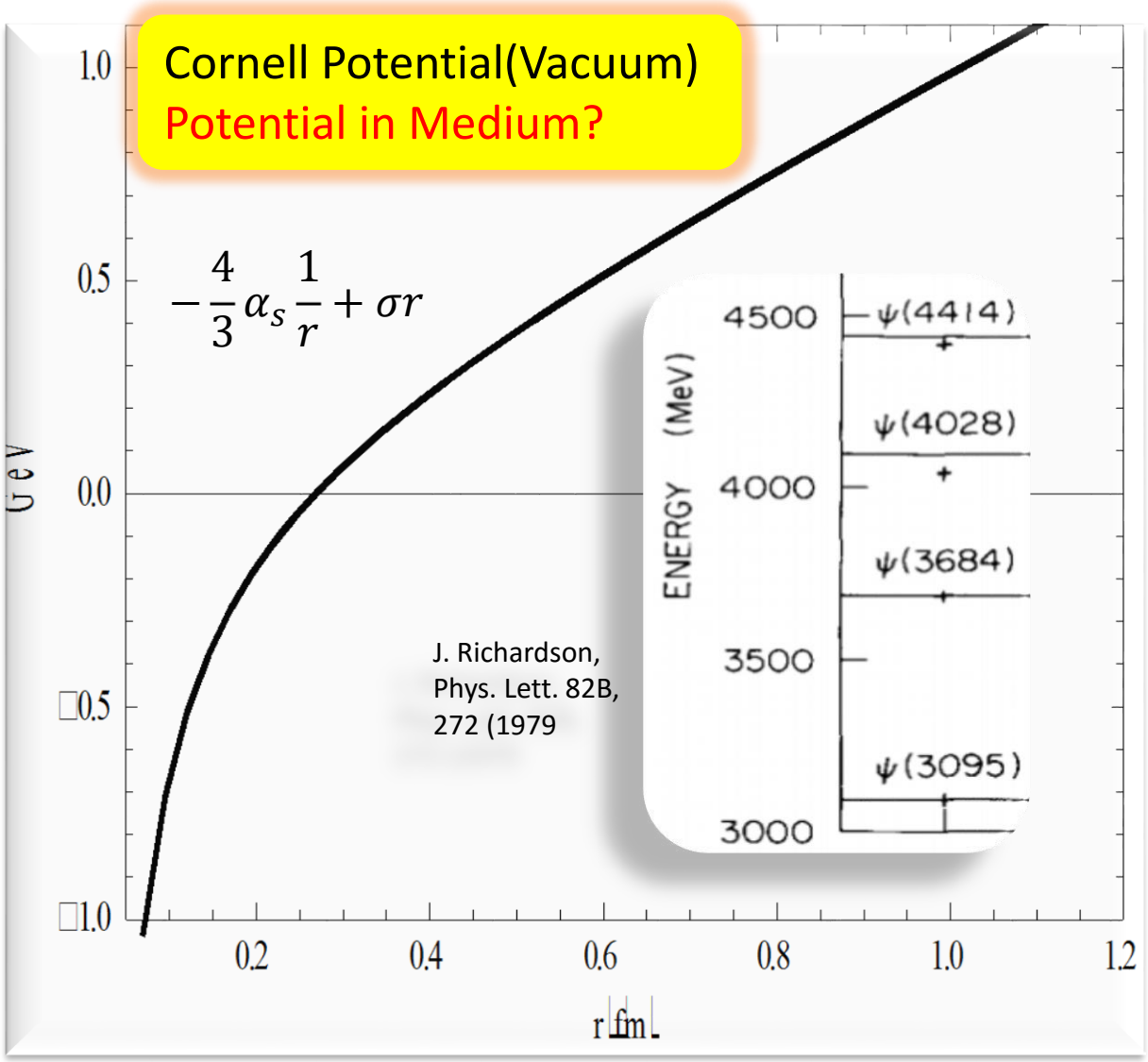


Lattice QCD

Potential Model

In-medium T-matrix Approach

Observables at Strong Coupling



Lattice Heavy-Quark Free Energy in Medium

In medium: $F_{Q\bar{Q}}(T,r)$=Change in Free Energy (Lattice)

$$F_{Q\bar{Q}}(T,r) = -T\ln(\tilde{Z}_{Q\bar{Q}}) = (-T\ln(Z_{Q\bar{Q}})) - (-T\ln(Z))$$

$$\tilde{Z}_{Q\bar{Q}} = \frac{Z_{Q\bar{Q}}}{Z} = \frac{\sum_n \langle n|\chi(r_2)\psi(r_1)e^{-\beta H}\psi^+(r_1)\chi^+(r_2)|n\rangle}{Z}$$

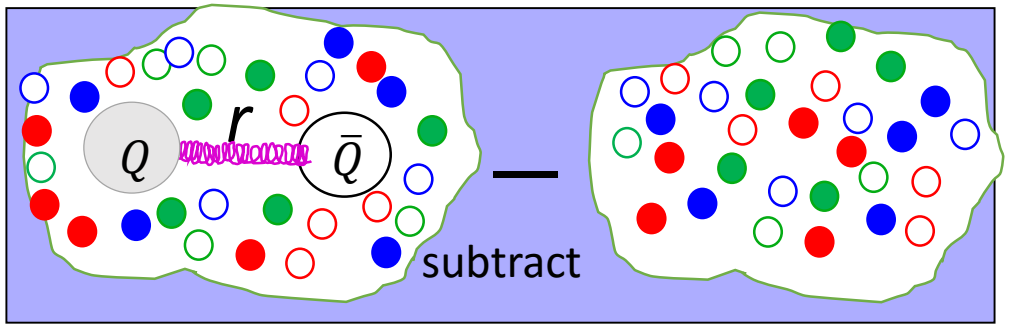
$$= G^>(-i\tau, r_1, r_2|r_1, r_2)|_{\tau=\beta} = \tilde{G}^>(-i\tau, r)|_{\tau=\beta}$$

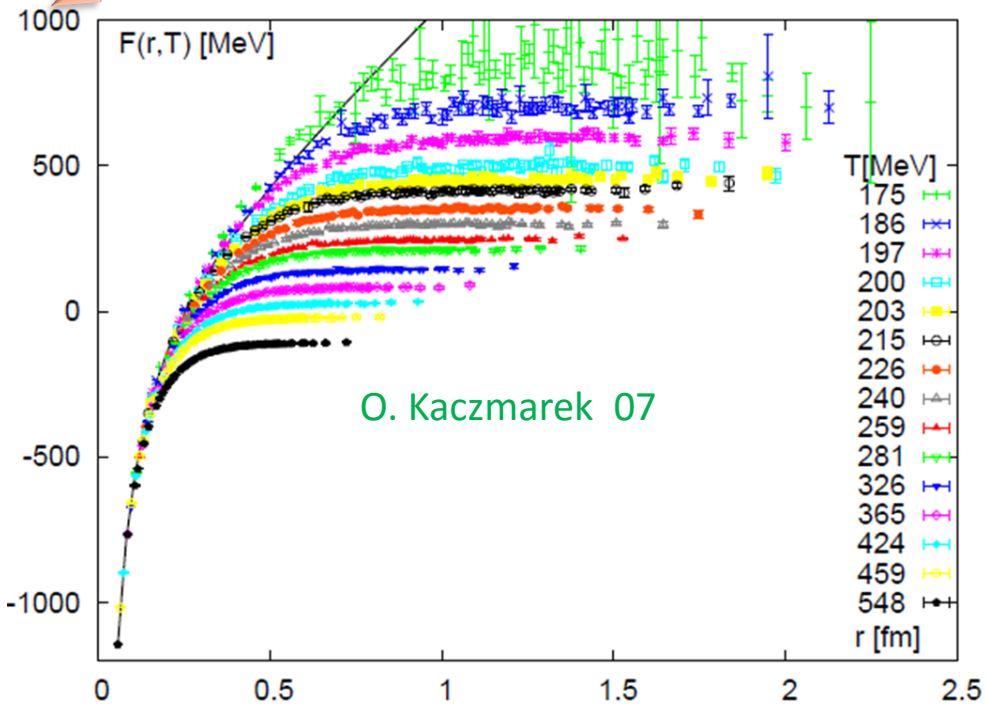
$$r = r_1 - r_2, \qquad \beta = 1/T$$

$$\boldsymbol{F_{Q\bar{Q}}(T,r) = -T\,ln\left(\tilde{G}^>(-i\tau,r)\right)|_{\tau=\beta}}$$

$\tilde{G}^>(-i\beta, r)$: 4 point $Q\bar{Q}$ Green function

Calculate $\tilde{G}^>$ from in-medium T-matrix





Heavy-Quark Free Energy From In-medium T-matrix

❖Lattice QCD can generate data for all r and τ.

$$F_{Q\bar{Q}}(T,r) = -T\ln\left(\tilde{G}^{>}(-i\tau,r)\right)|_{\tau=\beta}$$

❖Infinite mass limit: T-matrix can calculate $\tilde{G}^{>}(-i\tau,r)$ as:

$$\left[G_0^{(2)}\right]^{-1} = z - \Sigma(z)$$

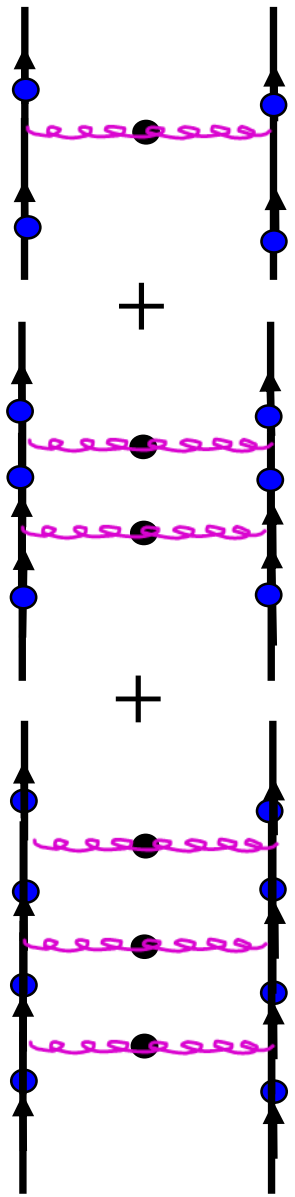
Energy space

$$G(z,r) = G_0^{(2)}(z) + G_0^{(2)}(z)V(z,r)G(z,r) \qquad G(z,r) = \frac{1}{[G_0^{(2)}(z)]^{-1} - V(z,r)}$$

Time space

$$\tilde{G}^{>}(-i\tau,r) = \int_{-\infty}^{\infty} dE\, \frac{-1}{\pi}\mathrm{Im}\left[\frac{1}{\left[G_0^{(2)}(E+i\epsilon)\right]^{-1} - V(E+i\epsilon,r)}\right] e^{-\tau E}$$

$$F_{Q\bar{Q}}(T,r) = -T\ln\left(\int_{-\infty}^{\infty} dE\, \frac{-1}{\pi}\mathrm{Im}\left[\frac{1}{\left[G_0^{(2)}(E+i\epsilon)\right]^{-1} - V(E+i\epsilon,r)}\right] e^{-\beta E}\right)$$



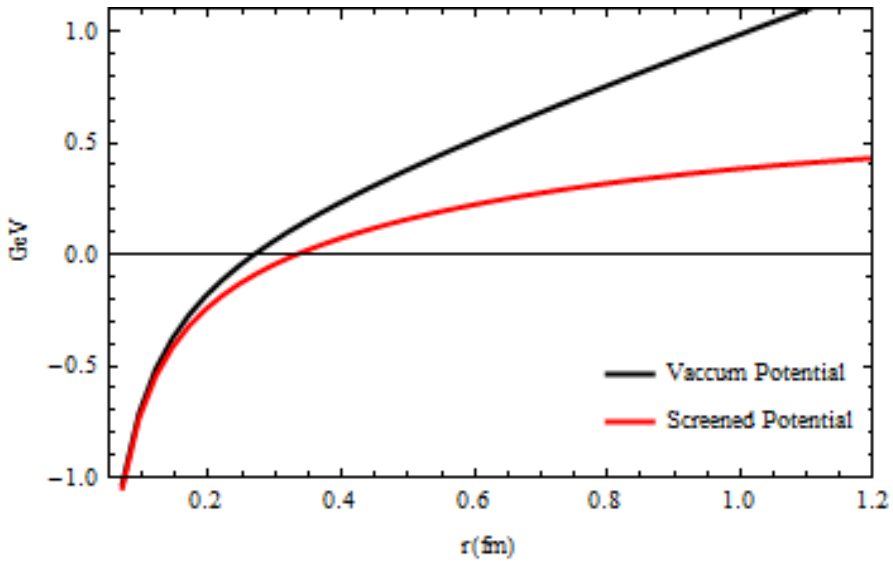
S.YF Liu + Rapp, NPA 941

Compare T-matrix $F_{Q\bar{Q}}(T,r)$ with Lattice $F_{Q\bar{Q}}(T,r)$ to extract in-medium $V(r)$ and $G_0^{(2)}$.

Concrete Ansatz to Extract a Potential

$$F_{Q\bar{Q}}(T,r) = -T\ln\left(\tilde{G}^{>}(-i\beta,r)\right)$$

$$= -T\ln\left(\int_{-\infty}^{\infty} dE\,\frac{1}{\pi}\frac{(V+\Sigma)_I(E)}{(E-(V+\Sigma)_R)^2+(V+\Sigma)_I^2(E)}e^{-\beta E}\right)$$



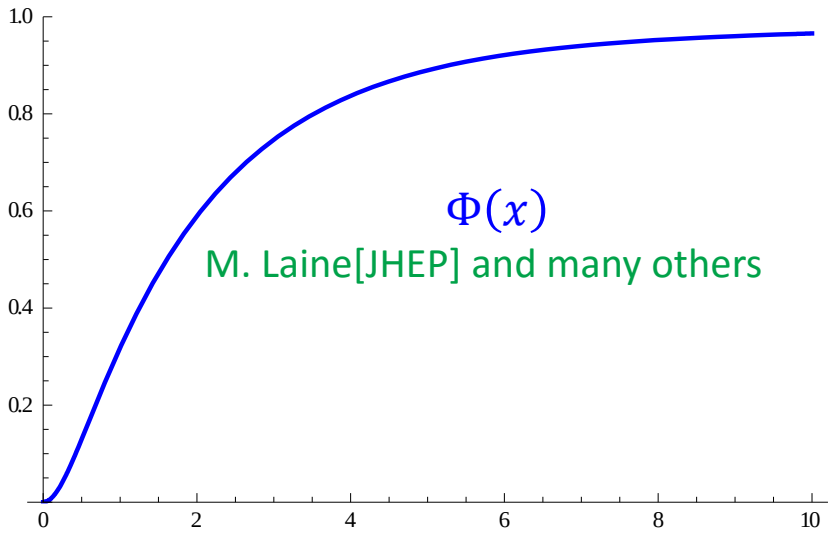
❖Screened Cornell $V(r)$ with imaginary part

- Real part of the potential + self energy:

$$(V+\Sigma)_R = -\frac{4}{3}\alpha_s\frac{e^{-m_D r}}{r} - \frac{4}{3}\alpha_s m_D - \sigma\left(\frac{e^{-m_s r}}{m_s} - \frac{1}{m_s}\right)$$

- Imaginary part of the potential + self energy:

$$(V+\Sigma)_I(E) = D\exp\left(-\frac{E^2}{2(CT)^2}\right)\frac{4}{3}\alpha_s T\Phi(m_D r)$$



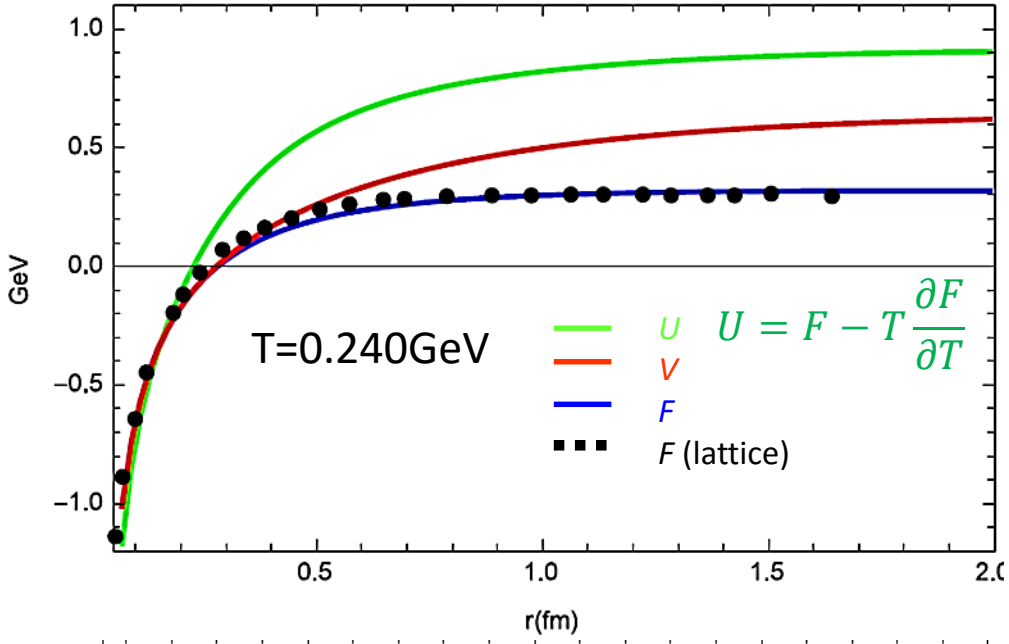
Large imaginary part make $(V+\Sigma)_R > F$

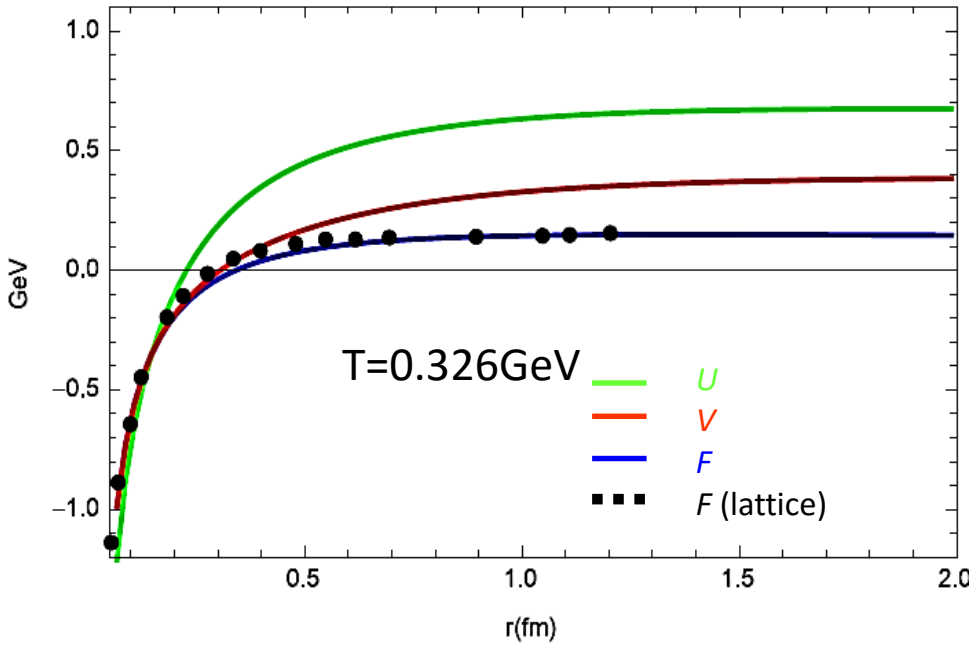
F. Riek, R. Rapp, New J. Phys 13 (2011)

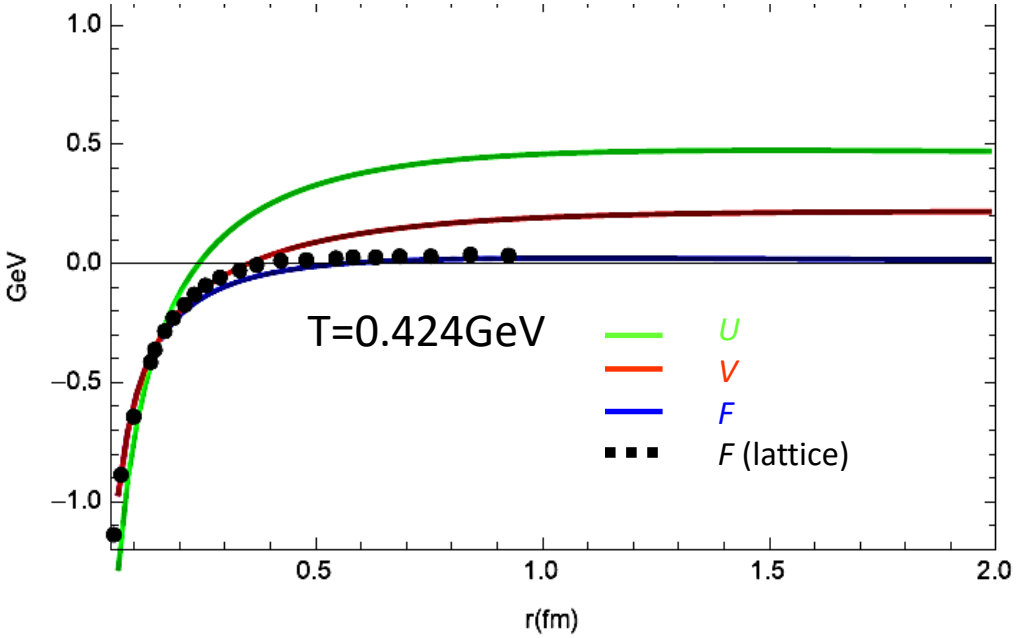
Fit to Lattice Data:

❖ Features of *V*

- More binding than *F*
- Long force range

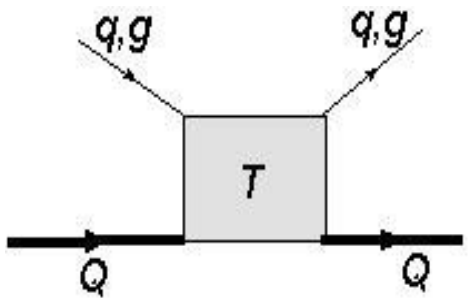


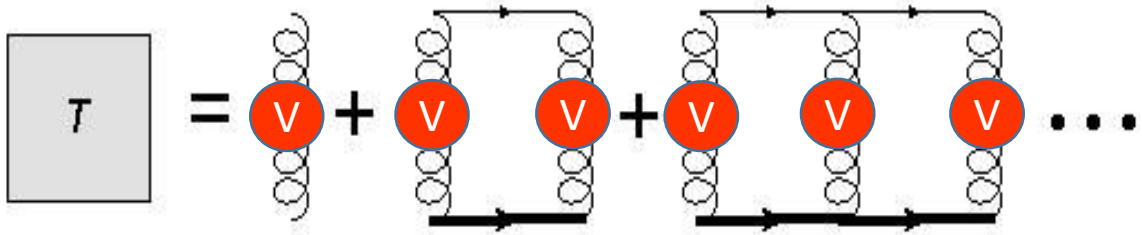




Heavy-Light T-matrix and Heavy-Quark Transport in QGP

$$T(E|\mathbf{p},\mathbf{p}') = R(\mathbf{p},\mathbf{p}')V(\mathbf{p}-\mathbf{p}') + \int \mathrm{d}^3\tilde{\boldsymbol{k}}\, R(\mathbf{p},\mathbf{k})V(\mathbf{p}-\mathbf{k})G_0(E|\mathbf{k})T(E|\mathbf{k},\mathbf{p}')$$





$R(\mathbf{p},\mathbf{p}')$ • Relativistic correction for heavy-heavy potential for heavy-light scattering

F Riek+ Rapp, PRC 82

V $V(\mathbf{p}-\mathbf{p}')$ • Potential includes Non-perturbative string interaction

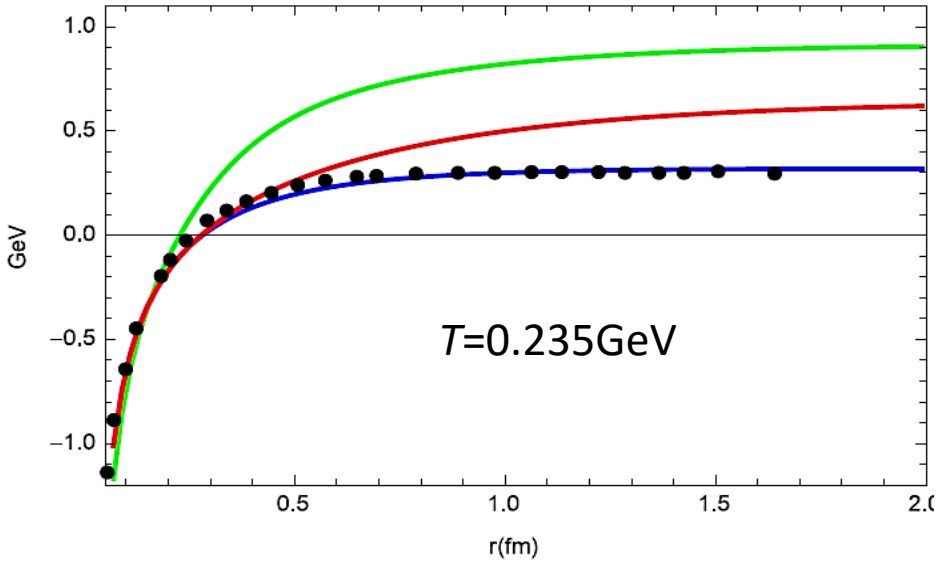
Friction Coefficient

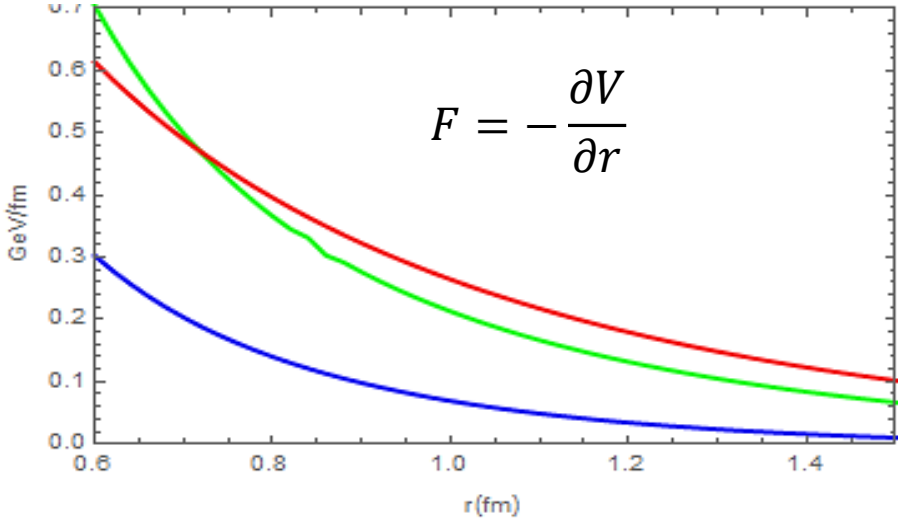
$$A(p) = \frac{1}{2\omega_Q(p)}\sum\int d^3\tilde{q}\, d^3\tilde{q}'\, d^3\tilde{p}'\, n_i(\omega_q)\cdot\frac{(2\pi)^4}{d_c}C_f|T(E_{cm}|\mathbf{p}_{cm},\mathbf{p}'_{cm})|^2\delta^4(p+q-p'-q')\left(1-\frac{p\,p'}{p^2}\right)$$

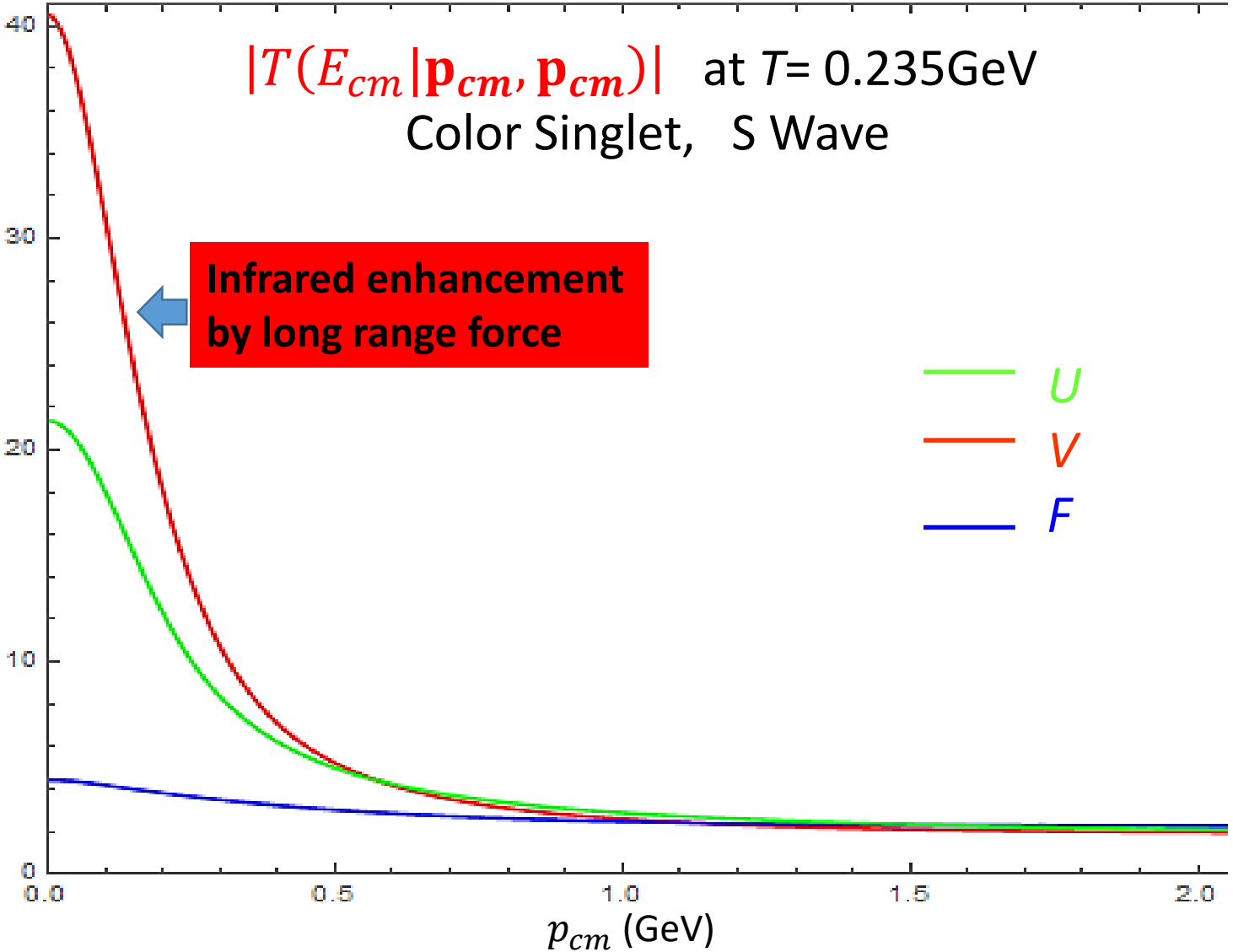
B Svetitsky, PRD 37 (1988)

T-matrix Amplitude for *U*, *V*, *F*

$$T(E|\mathbf{p},\mathbf{p}') = R(\mathbf{p},\mathbf{p}')V(\mathbf{p}-\mathbf{p}') + \int \mathrm{d}^3\tilde{\mathbf{k}}\, R(\mathbf{p},\mathbf{k})V(\mathbf{p}-\mathbf{k})G_0(E|\mathbf{k})T(E|\mathbf{k},\mathbf{p}')$$

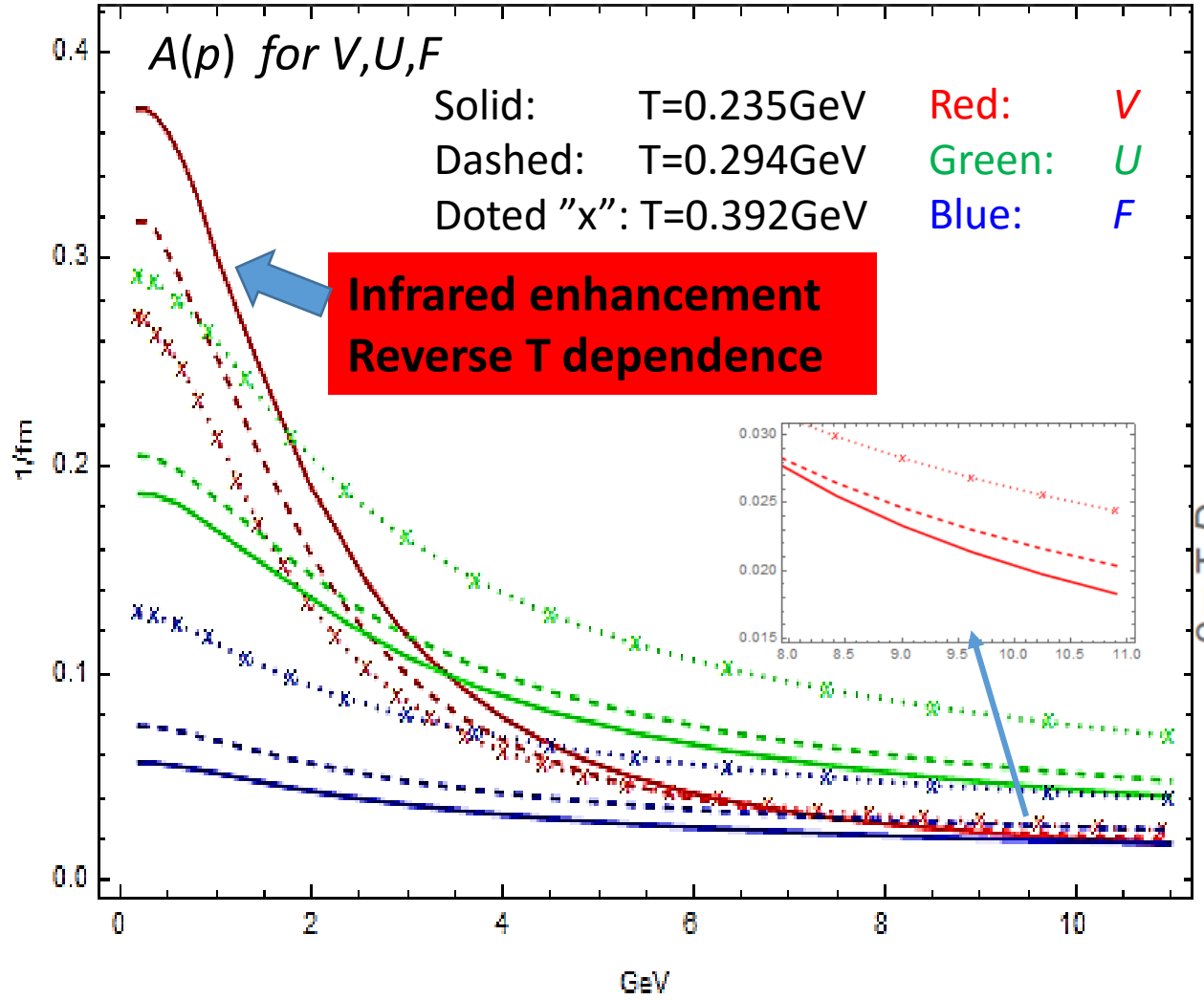






Heavy-Quark Transport Coefficient

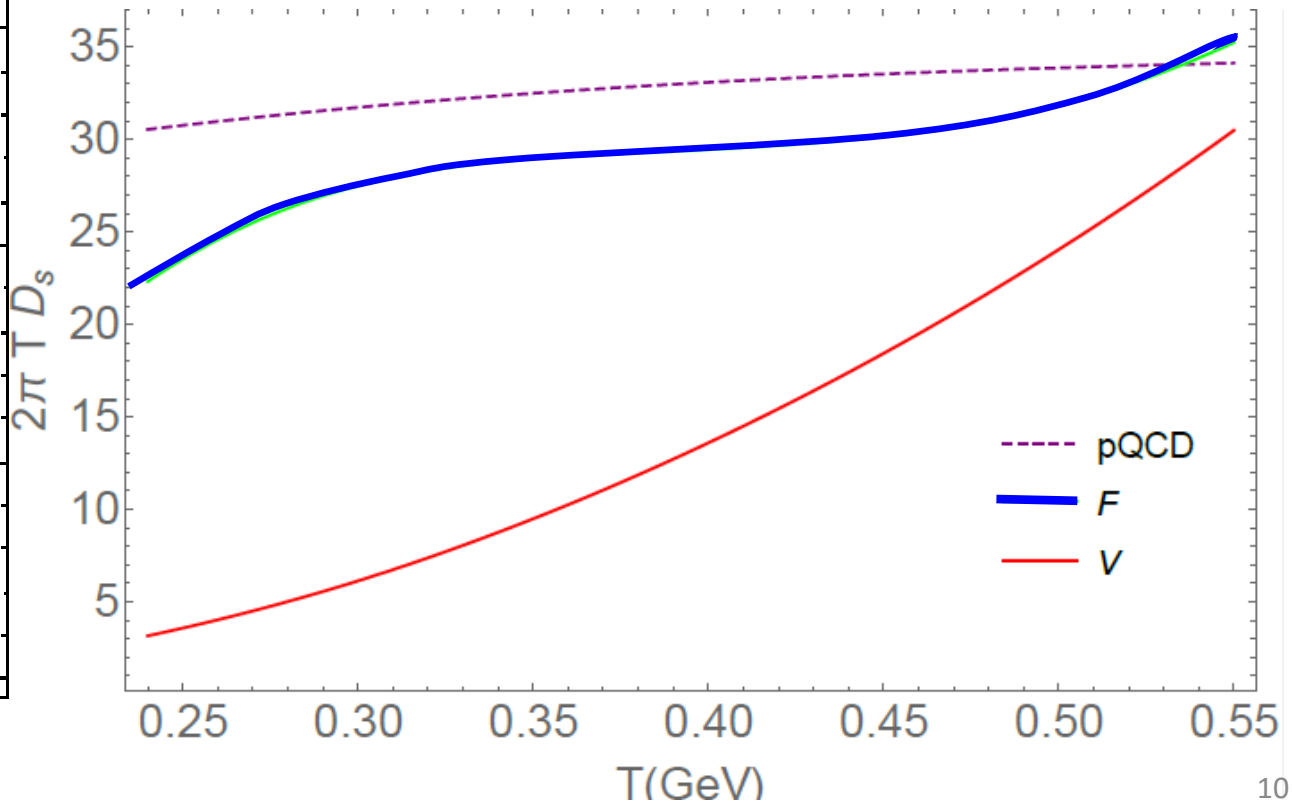
$$A(p) = \frac{1}{2\omega_Q(p)} \sum \int d^3\tilde{q} d^3\tilde{q}' d^3\tilde{p}' n_i(\omega_q) \cdot \frac{(2\pi)^4}{d_c} C_f |T(E_{cm}|\mathbf{p}_{cm}, \mathbf{p}'_{cm})|^2 \delta^4(p+q-p'-q') \left(1 - \frac{p\,p'}{p^2}\right)$$



Spatial Diffusion Coefficient

$$\gamma_c = A(p)|_{p=0}$$

$$D_s = \frac{T}{M_Q \gamma_c}$$



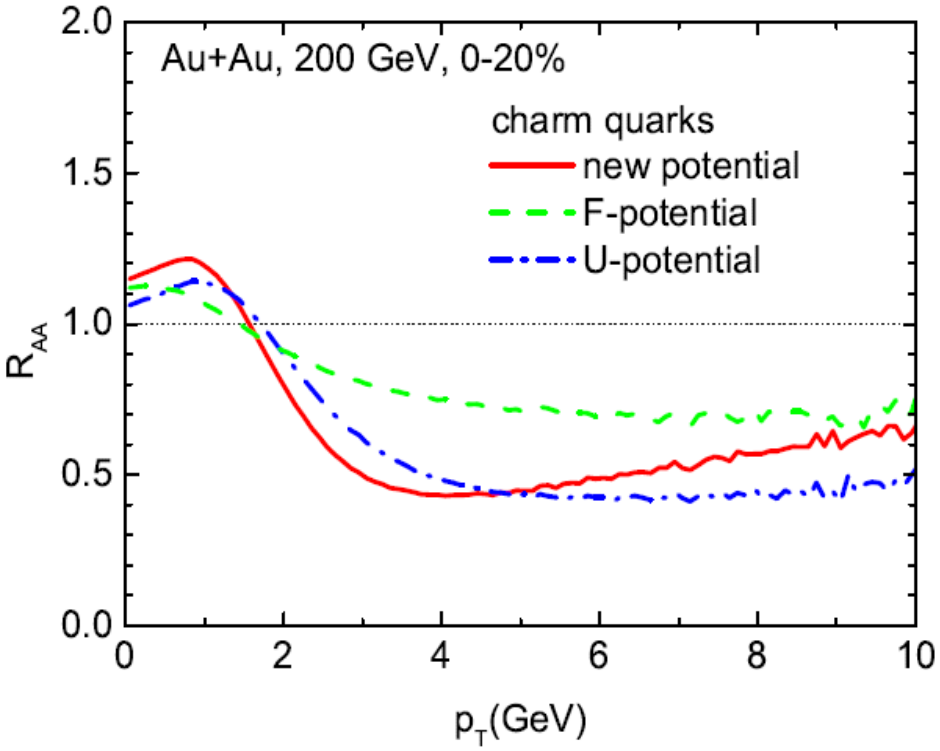
Langevin Simulation and Charm-Quark Spectra

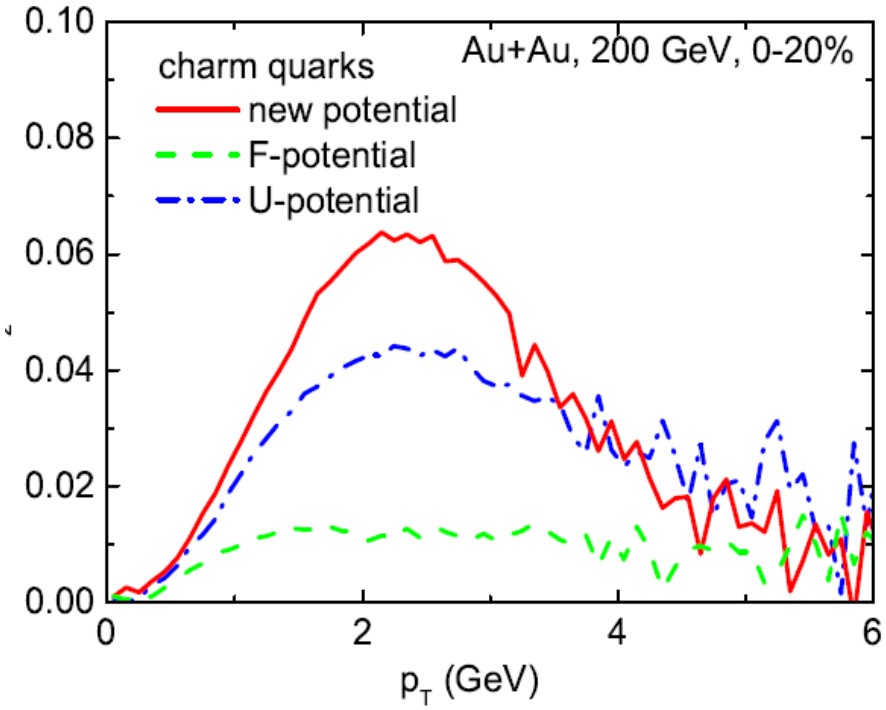
Transport Coefficient for Heavy Quark

$$\Gamma = A(p) + O\left(\frac{T}{M_Q}\right); \quad D(p) = \Gamma(p)E(p)T$$

Langevin Equation

$$d\mathbf{x} = \mathbf{p}dt$$
$$d\mathbf{p} = -\Gamma\mathbf{p}\,dt + \sqrt{D(p+dp)dt\rho}$$





M He+S.YF Liu+Ralf Rapp inprep

Conclusions and Perspectives

❖ Present Findings

- Developed approach to define in-medium V
- Extracted potential from lattice $F_{Q\bar{Q}}(T,r)$
- Potential generates large transport coefficient (strongly coupled)
- Langevin simulations indicate sensitivity of heavy-quark v_2 to underlying potential

❖ Future work:

- Systematic self-consistent formalism to eliminate free parameters in fit to $F_{Q\bar{Q}}$
- Include off-shell (quantum) effects for heavy-quark transport coefficient

Thanks!