Resummation of Jet Rates

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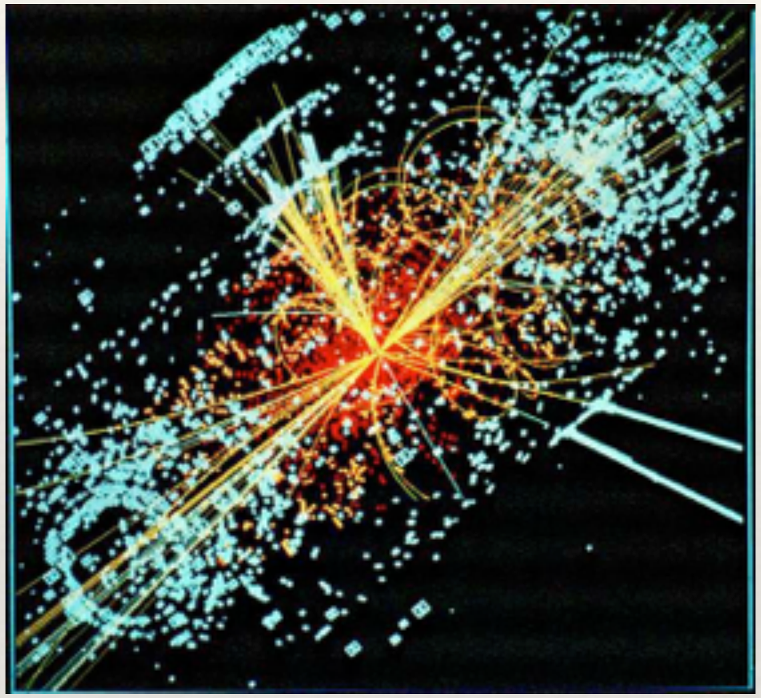
In collaboration with Y. Chien, C. Lee (arXiv:1509.04287)

What is a Jet?

- high-energy event:



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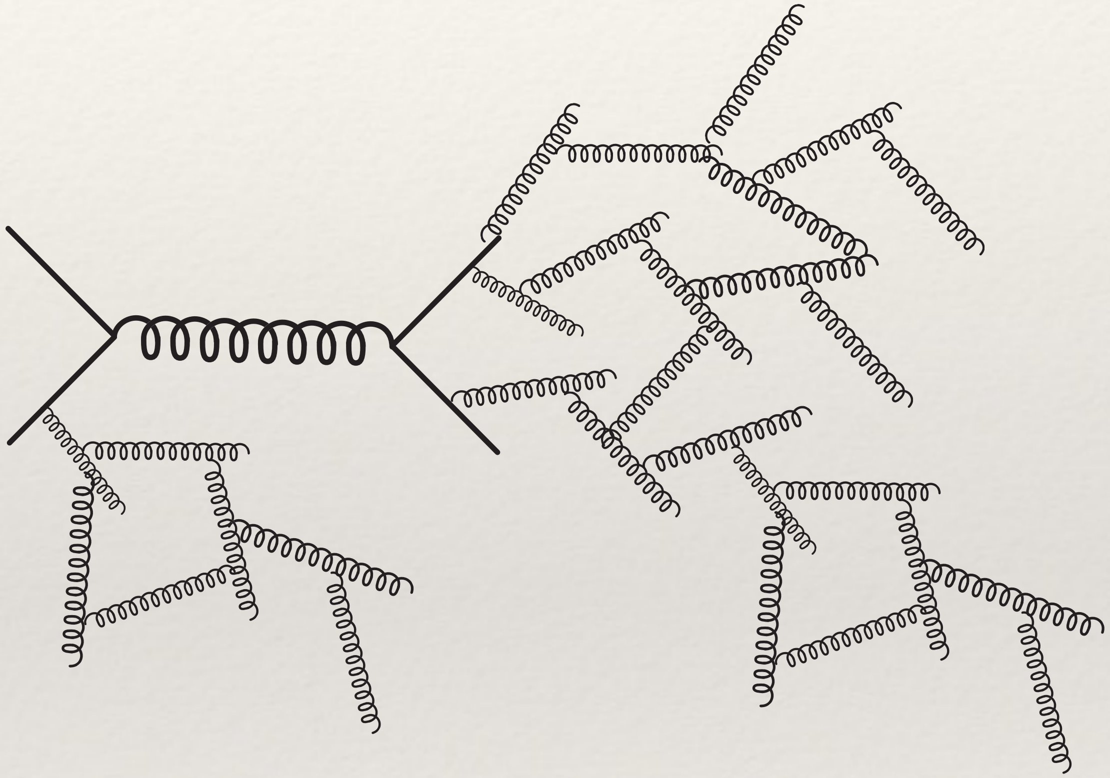
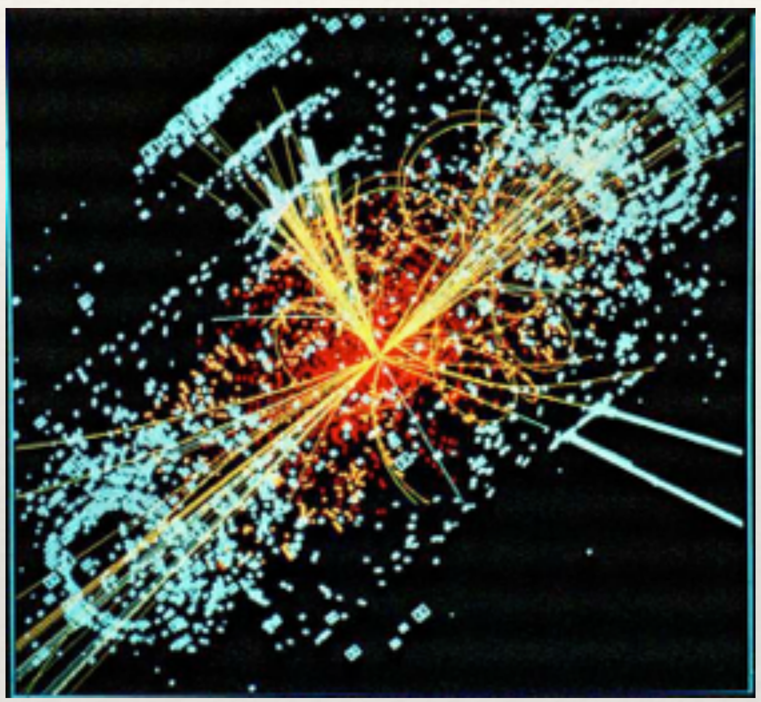


- organizing principle (beyond fixed-order calculation)?

What is a Jet?

- (soft & collinear) singularities → organize through *factorization*



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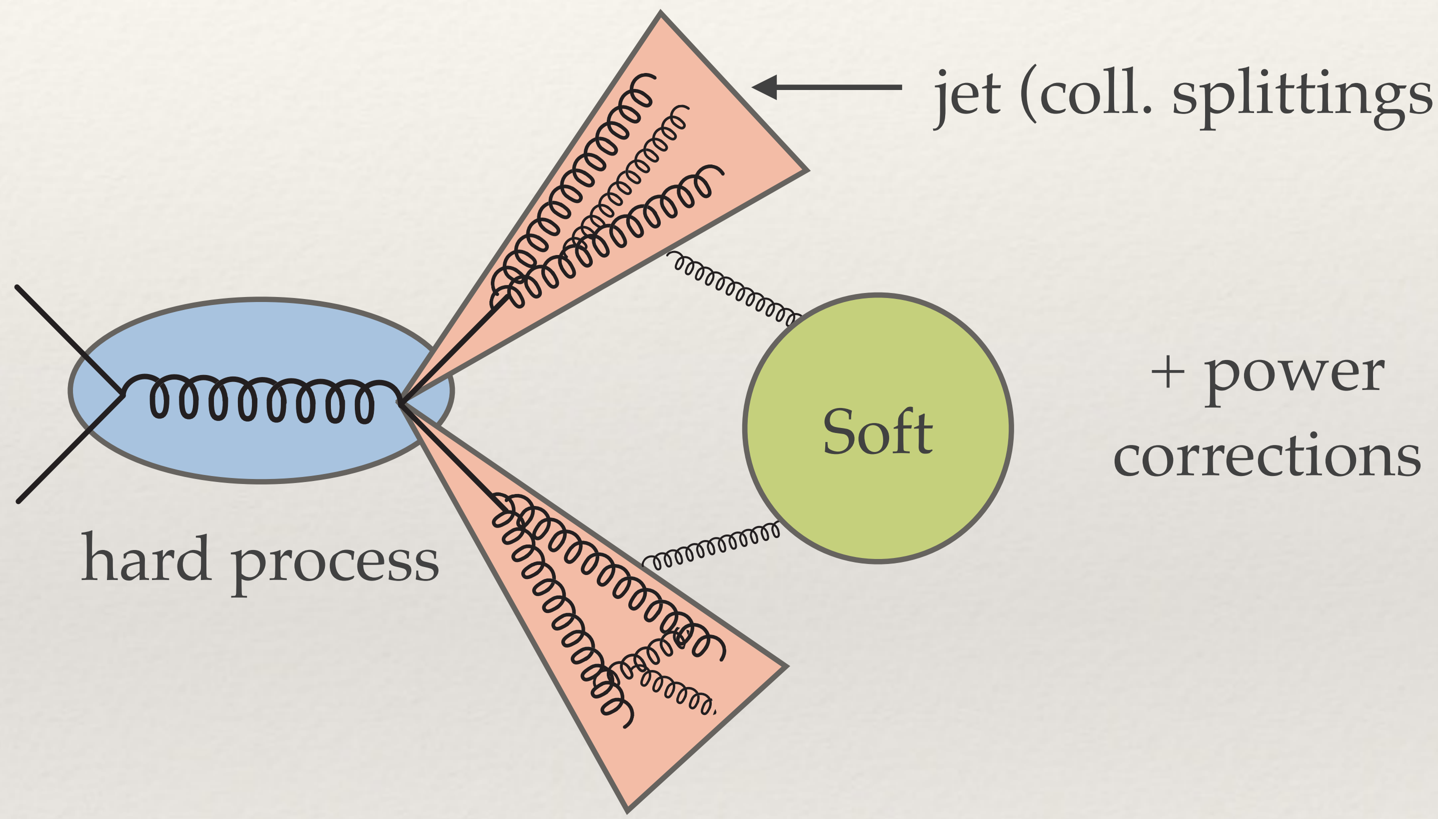


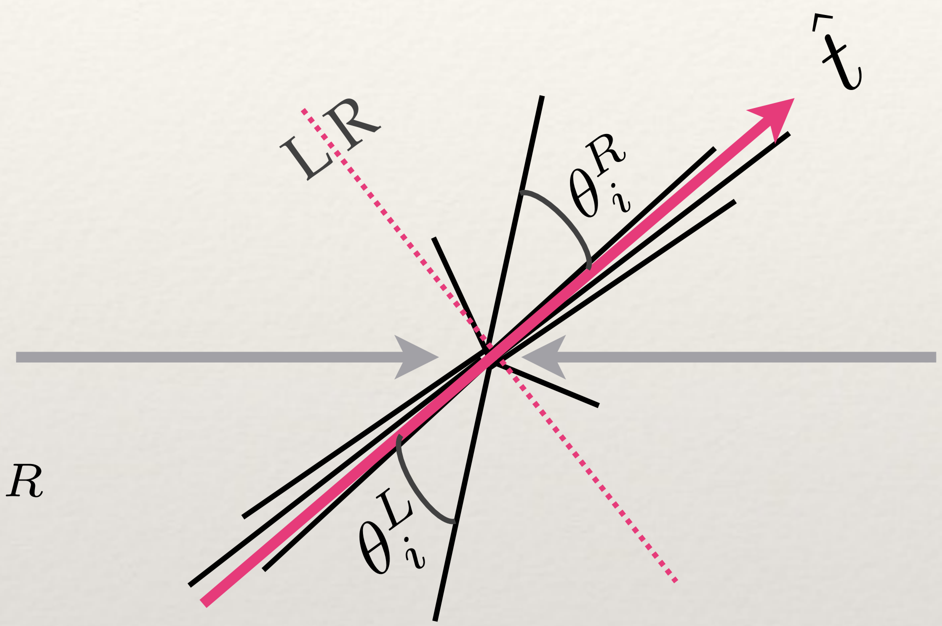
- can be achieved via Effective Field Theory (in particular, Soft-Collinear Effective Theory, or SCET)

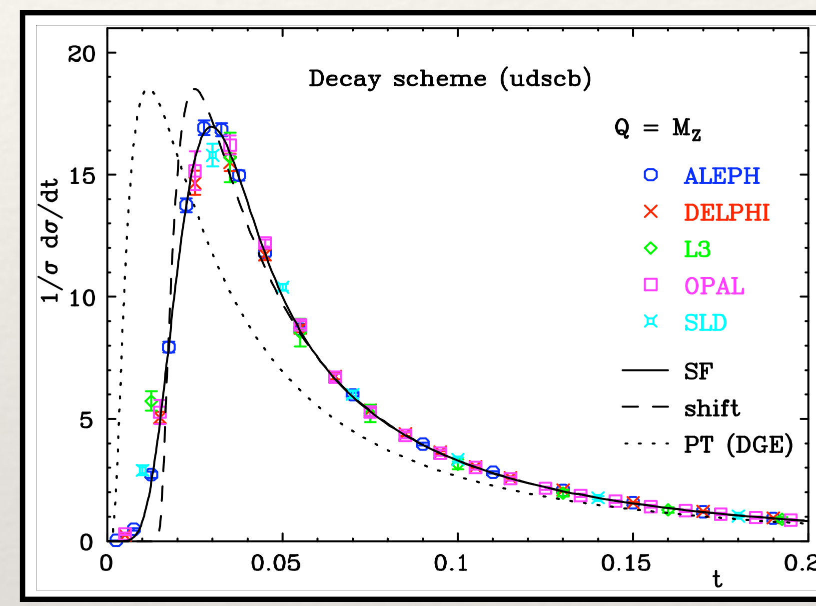
SCET & Factorization: Thrust

- thrust measures "jettiness" of e^+e^- events:

$$\tau = \tau_L + \tau_R$$

$$1 - \tau_{L,R} = \sum_{i \in L,R} E_i \cos\theta_i^{L,R}$$





- small thrust $\Longrightarrow$ all particles close to thrust axis (very jetty)
- fixed order calculation not possible in this region:

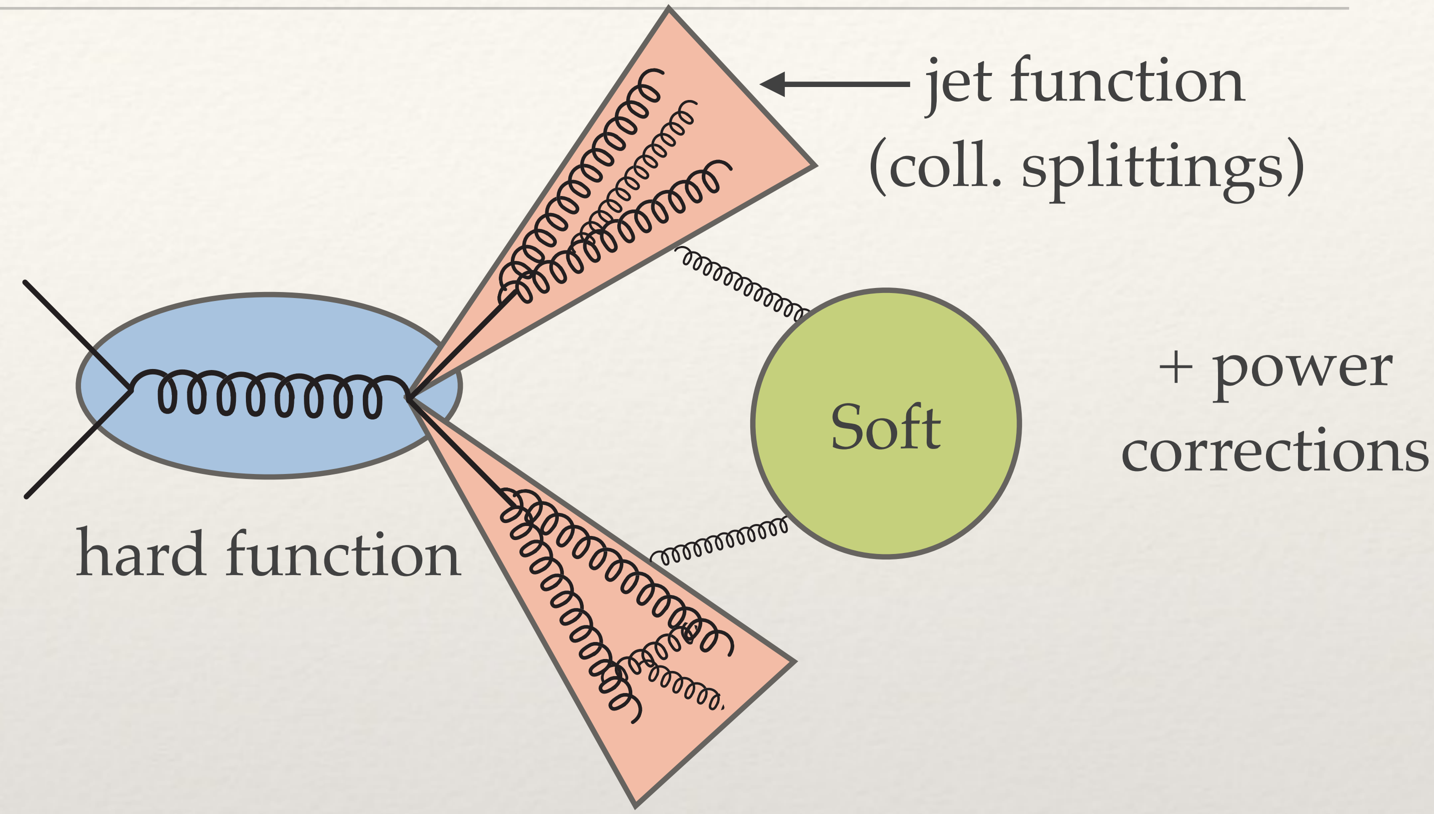
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = 1 + \alpha_s\Big(a_{12}\frac{\ln\tau}{\tau} + a_{11}\frac{1}{\tau} + a_{10}\Big) + \alpha_s^2\Big(a_{23}\frac{\ln^3\tau}{\tau} + a_{22}\frac{\ln^2\tau}{\tau} + a_{21}\frac{\ln\tau}{\tau} + a_{20}\Big) + \cdots$$

SCET & Factorization: Thrust

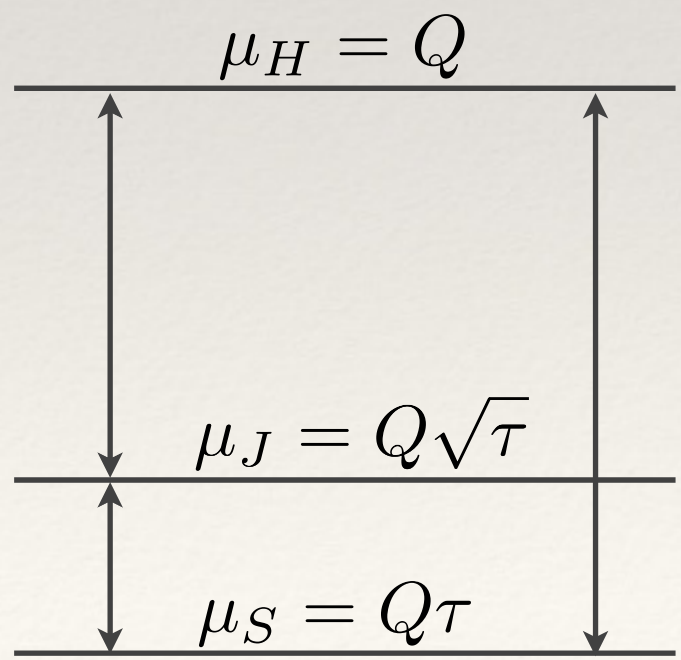
- factorization:

$$\frac{d\sigma}{d\tau} = H * J_n \otimes J_{\bar{n}} \otimes S_{n\bar{n}}$$

virtual   coll. real   soft real

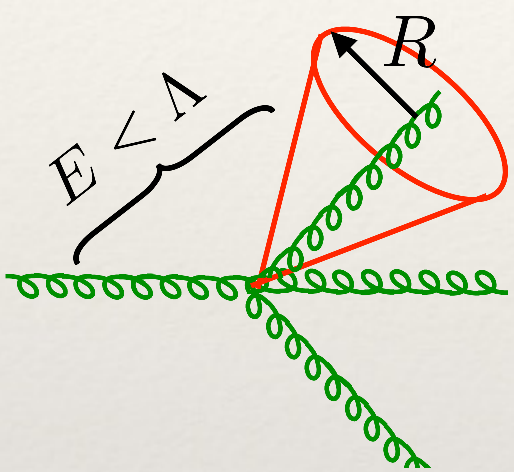


- resummation via RGE:



Factorization of Jet Rates

- "unmeasured jets" : tagged with algorithm but unprobed

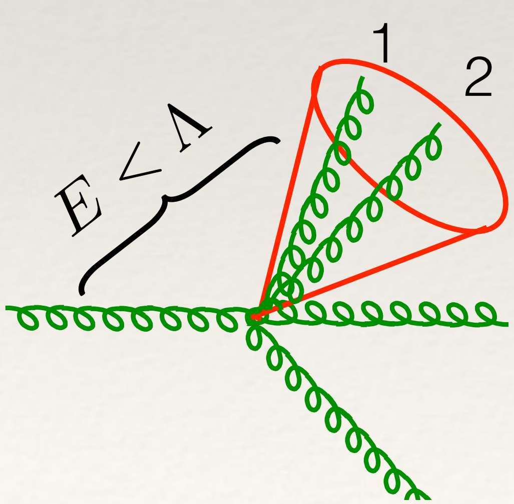


$\sigma(R, \Lambda)$ record rate (count events)

- "measured jets" : probed with mass, angularity, etc

"jet shapes" (not *the* jet shape $\Psi(r/R)$)

Ellis, Kunszt, Soper '91, '92



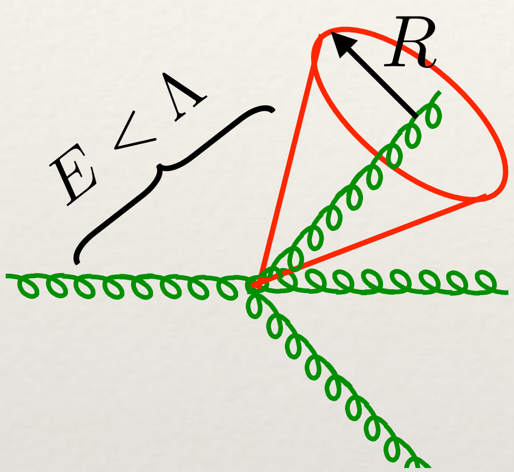
$$\frac{d\sigma(R)}{dm_J}$$ bin in (e.g.) mass

$$m_J^2 = (p_1 + p_2)^2$$

Factorization of Jet Rates

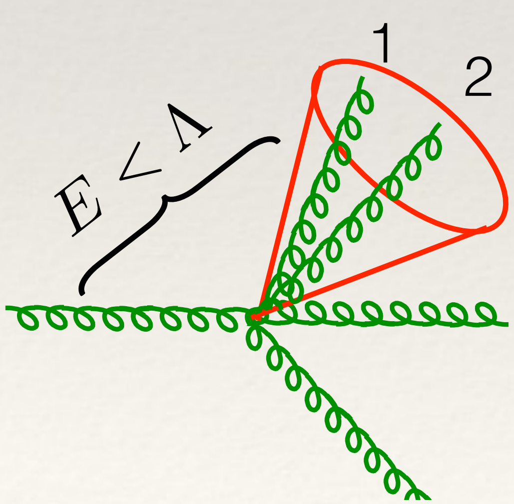
Ellis, AH, Lee, Vermilion, Walsh 1001.0014

- "unmeasured jets" : tagged with algorithm but unproved



$$\sigma(R, \Lambda) \overset{?}{=} H(Q) * J^{\text{unmeas}}(QR) * S^{\text{unmeas}}(R, \Lambda/Q)$$

- "measured jets" : probed with mass, angularity, etc



$$\frac{d\sigma(R)}{dm_J} \overset{?}{=} H(Q) * J^{\text{meas}}(m_J, R) * S^{\text{meas}}(R, \Lambda/Q, m_J)$$

valid for R << 1

Jet Rates from Integrating Shapes to α_s^1

- can get rates directly from integrating shapes:

$$\sigma(R) = \int_0^{\tau^{\max}(R)} d\tau \frac{d\sigma}{d\tau} = \begin{cases} \sigma_c^{\text{cone}}(\tau = R^2) = 1 + \frac{\alpha_s C_F}{2\pi}\Big(-8\ln R \ln\frac{2\Lambda}{Q} - 6\ln R + 6\ln 2 - 1\Big) \\ \sigma_c^{k_T}\Big(\tau = \frac{R^2}{4}\Big) = 1 + \frac{\alpha_s C_F}{2\pi}\Big(-8\ln R\ln\frac{2\Lambda}{Q} - 6\ln R + 5 - \frac{2\pi^2}{3}\Big) \end{cases}$$

$$\frac{d\sigma}{d\tau} \rightarrow = H * J^{\text{meas}}(\tau, R) * S^{\text{meas}}(R, \Lambda/Q, \tau)$$

$$\sigma(R) \rightarrow = H(Q) * J^{\text{unmeas}}(QR) * S^{\text{unmeas}}(R, \Lambda/Q)$$

note: $\int_0^{\tau^{\max}(R)} d\tau J^{\text{meas}}(\tau, R) \neq J^{\text{unmeas}}(QR)$

→ part of $S^{\text{meas}}(\tau)$ is needed (more later!)

Jet Shapes to α_s^1

- jet function with a jet algorithm (R dependence needed!):

$$J_n^{\mathrm{alg.}}(t_n, R, \mu) = J^{\mathrm{incl}}(t_n, \mu) + \Delta J^{\mathrm{alg.}}(t_n, R)$$

$$\Delta J^{\mathrm{cone}}(t, R) = \frac{\alpha_s C_F}{4\pi}\left[\theta(t)\theta(Q^2R^2 - t)\frac{6}{t + Q^2R^2} + \frac{\theta(t - Q^2R^2)}{t}\left(4\ln\frac{t}{Q^2R^2} + 3\right)\right]$$

→ power correction for $\tau << R$,
but needed in general!

- soft function:

$$S(k_n, k_{\bar{n}}, \Lambda, R, \mu) = \delta(k)\Big[1 + \underbrace{\frac{\alpha_s C_F}{4\pi}\Big(4\ln R \ln\frac{\mu^2}{4\Lambda^2 R} - \frac{\pi^2}{3}\Big)}\Big] - \sum_{i=n,\bar{n}} \frac{2\alpha_s C_F}{\pi}\frac{1}{\mu R}\left[\frac{\theta(k_i)\mu R}{k_i}\ln\frac{k_i}{\mu R}\right]_+$$

part associated with veto:
minimized for $\mu \sim 2\,\Lambda\,R^{1/2}$
CLUE??

The $\alpha_s{}^2$ Result

Manteufell, Schabinger, Zhu 1309.3560

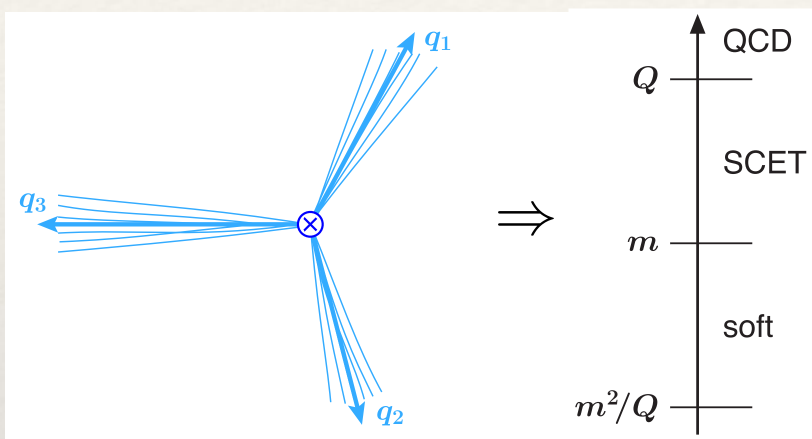
$$
\begin{aligned}
\bar{K}^{(2)}_{TC}(\tau_\omega,\omega,r\to 0,\mu) = C_A C_F \bigg[& -\frac{176}{9}\ln^3\left(\frac{\mu}{Q\tau_\omega}\right) + \left(-\frac{88\ln(r)}{3} + \frac{8\pi^2}{3} - \frac{536}{9}\right) \\
& \times \ln^2\left(\frac{\mu}{Q\tau_\omega}\right) + \left(-\frac{44}{3}\ln^2(r) + \frac{8}{3}\pi^2\ln(r) - \frac{536\ln(r)}{9} + 56\zeta_3 + \frac{44\pi^2}{9} - \frac{1616}{27}\right) \\
& \times \ln\left(\frac{\mu}{Q\tau_\omega}\right) + \left(-\frac{44}{3}\ln^2(r) - \frac{8}{3}\pi^2\ln(r) + \frac{536\ln(r)}{9} - \frac{44\pi^2}{9}\right)\ln\left(\frac{\mu}{2\omega}\right) + \frac{88}{3}\ln(r) \\
& \times \ln^2\left(\frac{\mu}{2\omega}\right) - \frac{8}{3}\pi^2\ln^2\left(\frac{Q\tau_\omega}{2r\omega}\right) + \left(-16\zeta_3 - \frac{8}{3} + \frac{88\pi^2}{9}\right)\ln\left(\frac{Q\tau_\omega}{2r\omega}\right) + \frac{4}{3}\pi^2\ln^2(r) \\
& -\frac{268\ln^2(r)}{9} - \frac{682\zeta_3}{9} + \frac{109\pi^4}{45} - \frac{1139\pi^2}{54} - \frac{1636}{81}\bigg] + C_F n_f T_F \bigg[\frac{64}{9}\ln^3\left(\frac{\mu}{Q\tau_\omega}\right) \\
& + \left(\frac{32\ln(r)}{3} + \frac{160}{9}\right)\ln^2\left(\frac{\mu}{Q\tau_\omega}\right) + \left(\frac{16\ln^2(r)}{3} + \frac{160\ln(r)}{9} - \frac{16\pi^2}{9} + \frac{448}{27}\right)\ln\left(\frac{\mu}{Q\tau_\omega}\right) \\
& -\frac{32}{3}\ln(r)\ln^2\left(\frac{\mu}{2\omega}\right) + \left(\frac{16\ln^2(r)}{3} - \frac{160\ln(r)}{9} + \frac{16\pi^2}{9}\right)\ln\left(\frac{\mu}{2\omega}\right) + \left(\frac{16}{3} - \frac{32\pi^2}{9}\right) \\
& \times \ln\left(\frac{Q\tau_\omega}{2r\omega}\right) + \frac{80\ln^2(r)}{9} + \frac{248\zeta_3}{9} + \frac{218\pi^2}{27} - \frac{928}{81}\bigg]\,. \qquad (5.14)
\end{aligned}
$$

- large logs at $\mu \sim 2\,\Lambda\,R^{1/2}$
- “refactorization??” (but not clear any *set* of scales will work)

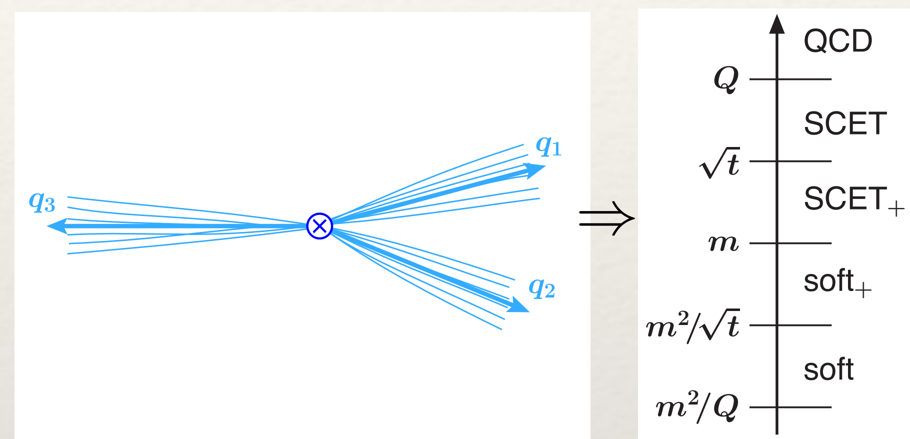
SCET$_+$

Bauer, Tackmann, Walsh, Zuberi 1106.6047

- originally used for when jets get close:



(a) All jets equally separated.



(b) Two jets close to each other.

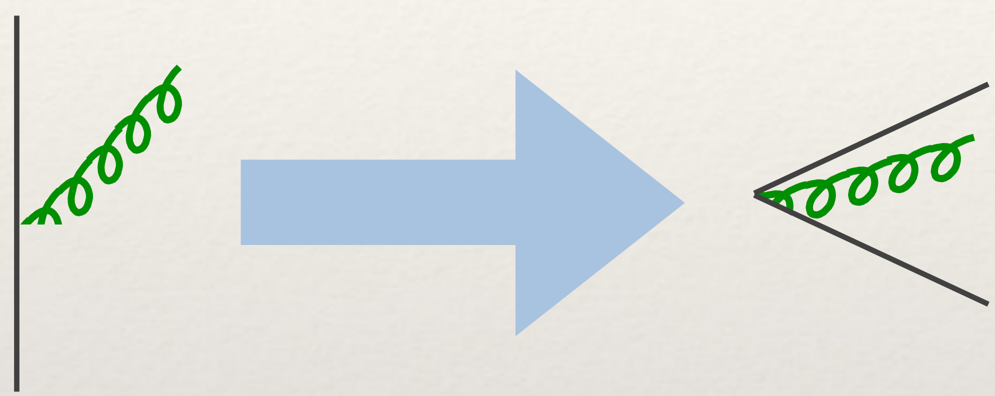
- requires a new "csoft" mode

$$p_{cs} \sim Q(\lambda^2, \eta^2, \eta\lambda)$$

$$\lambda = \frac{m}{Q} \qquad \eta = \frac{\lambda}{\lambda_t} = \frac{m}{\sqrt{t}} .$$

SCET$_+$ for Jet Rates

- we also fix small component and decrease $\perp$

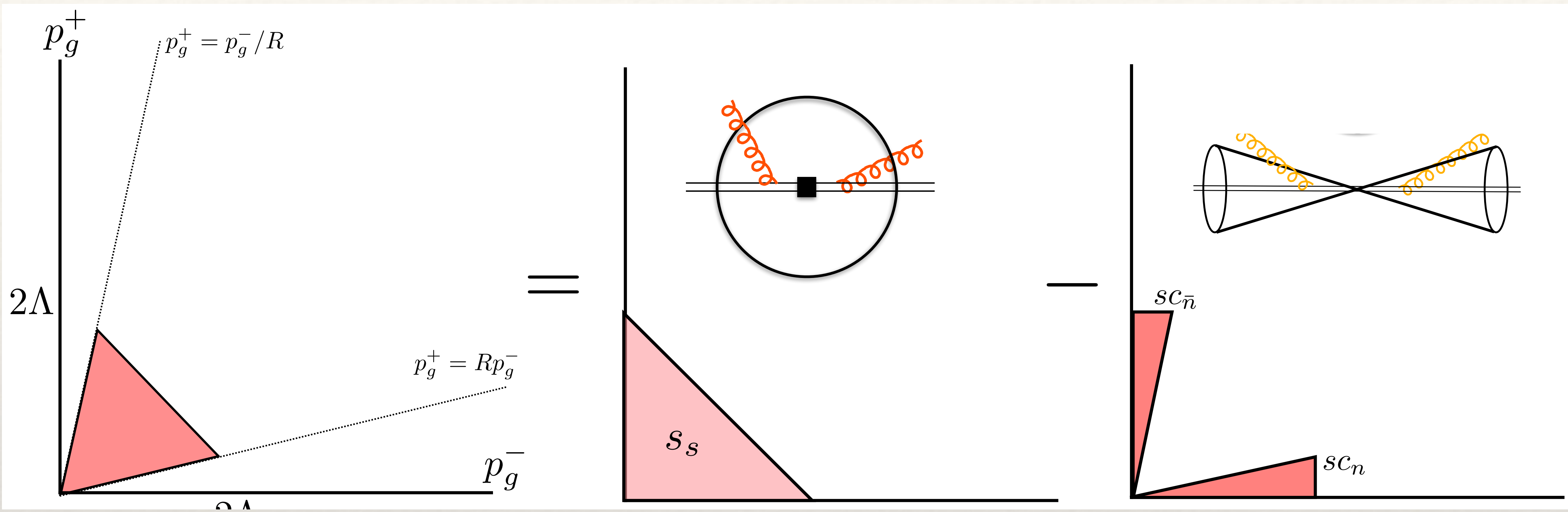


small R

$$\left.\begin{array}{ll} p^+ = Q\tau & \text{fixed by } \tau \text{ meas} \\[1.5em] p^\perp \propto p^- / R & \text{inside R (the jet)} \end{array}\right\} \quad p = Q\tau(1, 1/R^2, 1/R)$$

virtuality increased due to R!

The Soft-Collinear Mode (new!)



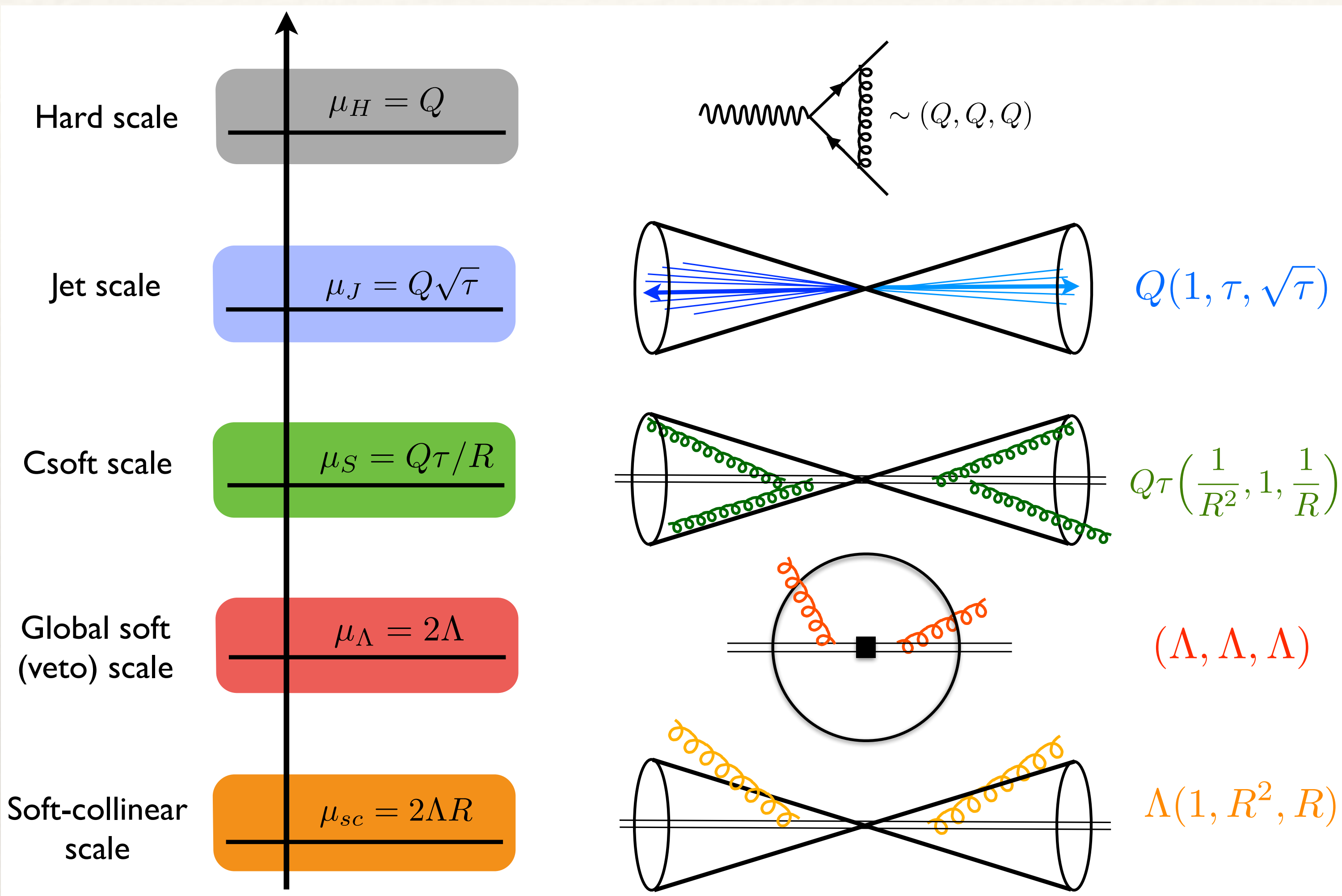
soft anywhere

$(\Lambda, \Lambda, \Lambda)$

soft but confined to jet
"soft-collinear"

$\Lambda(1, R^2, R)$

Re-Factorization



SCET+

Bauer, Tackmann, Walsh, Zuberi 1106.6047

SCET$_{++}$

Chien, AH, Lee 1509.04287

Soft-collinear mode (new!)

depends on choice of shape

$$\frac{d\sigma}{d\tau} = H(Q) * J^{\text{meas}}(\tau, R) * S^{\text{meas}}(R, \Lambda/Q, \tau)$$

$$S^{\text{meas}}(R, \Lambda/Q, \tau) \to S_{\text{in}}\left(\frac{Q\tau}{R}\right) S_s(\Lambda)\, S_c(\Lambda R)$$

radiation everywhere w/ E ~ Λ

radiation in jet w/ E ~ Λ

Predicting the $\alpha_s{}^2$ Result

$$S^{c}(k,\Lambda,R,\mu) = S_{C_F}(k,\Lambda,R,\mu) + \left(\frac{\alpha_s}{4\pi}\right)^2 S^{(2)}_{nA}(k,\Lambda,R,\mu)$$

$$\begin{aligned}S_{C_F}(k,\Lambda,R,\mu) = 1 &+ \frac{\alpha_s}{4\pi}\left[2\Gamma_0\left(-\ln^2\frac{\mu R}{k} + \ln R\ln\frac{\mu^2}{4\Lambda^2 R}\right) - \frac{\pi^2}{3}C_F\right] + \left(\frac{\alpha_s}{4\pi}\right)^2\Bigg\{2(\Gamma_0)^2\left(-\ln^2\frac{\mu R}{k} + \ln R\ln\frac{\mu^2}{4\Lambda^2 R}\right)^2 \\ &+ 2\Gamma_0\frac{\pi^2}{3}C_F\left(\ln^2\frac{\mu R}{k} - \ln R\ln\frac{\mu^2}{4\Lambda^2 R}\right) - \frac{4\pi^2}{3}(\Gamma_0)^2\left(\ln^2\frac{\mu R}{k} + \ln^2 R\right) - 16\zeta_3\Gamma_0^2\ln\frac{\mu R}{k} + c^{(2)}_{C_F}\Bigg\}, \quad (65)\end{aligned}$$

$$\begin{aligned}S^{(2)}_{nA}(k,\Lambda,R,\mu) = &\frac{4}{3}\Gamma_0\beta_0\left(-\ln^3\frac{\mu R}{k} + \ln^3\frac{\mu}{2\Lambda} - \ln^3\frac{\mu}{2\Lambda R}\right) + 2\Gamma_1\left(-\ln^2\frac{\mu R}{k} + \ln R\ln\frac{\mu^2}{4\Lambda^2 R}\right) + S^{c(2)}_{ng}(k,\Lambda,R,\mu) \\ &+ 2(\gamma^1_{\rm in} + 2\beta_0 c^1_{\rm in})\ln\frac{\mu R}{k} + (\gamma^1_{ss} + 2\beta_0 c^1_{ss})\ln\frac{\mu}{2\Lambda} + 2(\gamma^1_{sc} + 2\beta_0 c^1_{sc})\ln\frac{\mu}{2\Lambda R} + c^{(2)}_{nA}\,.\end{aligned}$$

- comparison to α^2 result $\Rightarrow$ all logs of 2Λ, $2\Lambda R$, and $Q\tau/R$!
- this also gives the anom. dimensions to α^2 for free!!

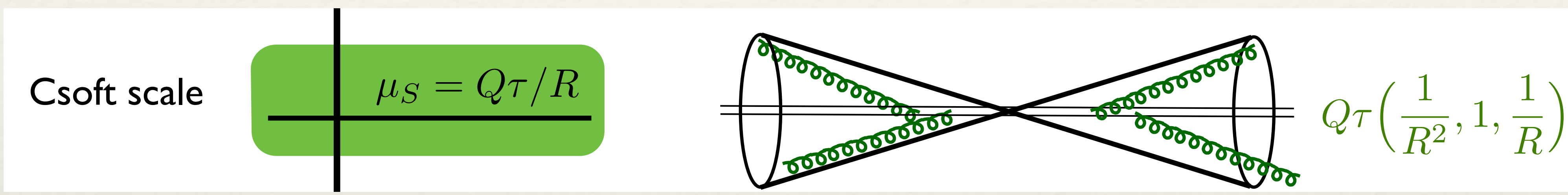
$$\boxed{\begin{aligned}\gamma^1_{ss} &= -2\gamma^1_{\rm in} = -2\gamma^1_{sc} \\ &= C_F\left[\left(\frac{1616}{27} - 56\zeta_3\right)C_A - \frac{448}{27}T_F n_f - \frac{2\pi^2}{3}\beta_0\right].\end{aligned}}$$

- can argue to all orders (ingredients known to α^3)!!!

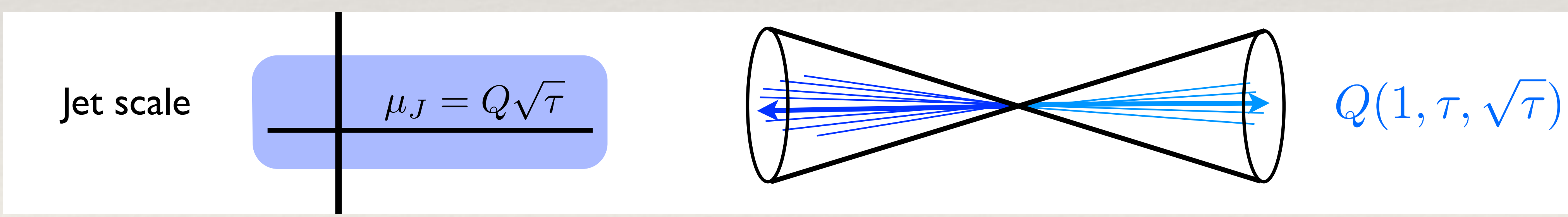
$$\boxed{\gamma_{\rm hemi} = \gamma_{\rm in} = \gamma_{sc} = -\frac{\gamma_{ss}}{2}}$$

How the Modes Integrate

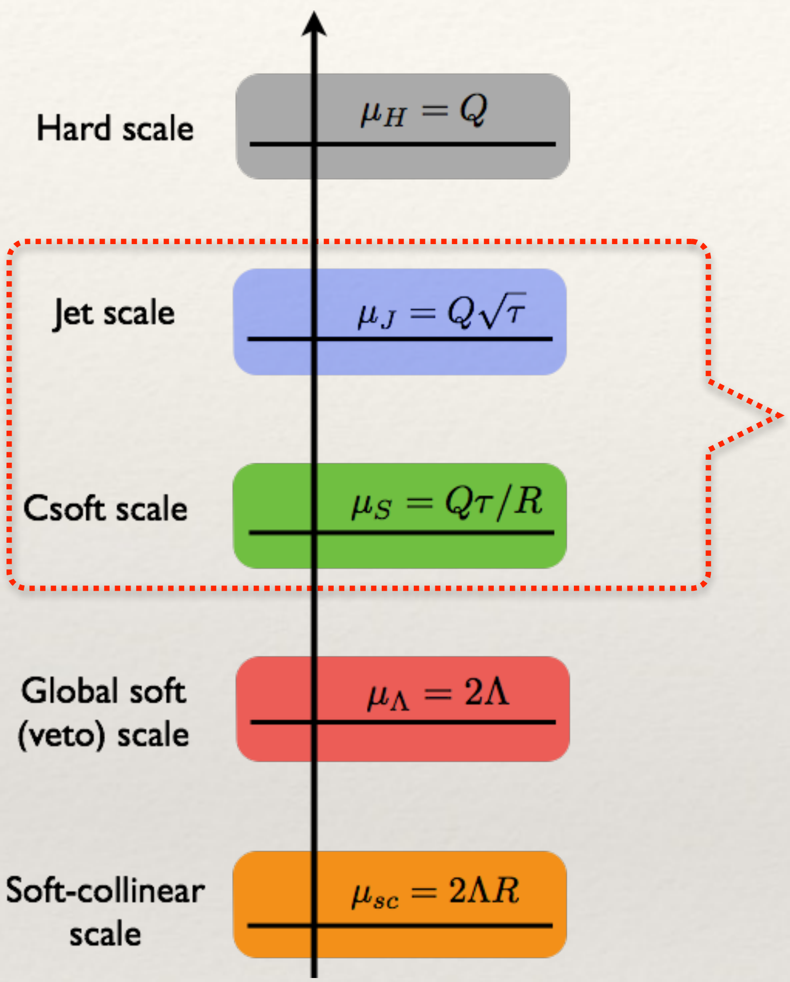
- complete EFT over all physical values of τ



$$\tau \to \tau^{\max} \sim R^2$$



Jet Rate Factorization (Proof)



these modes coincide @ $\tau^{\max} \sim R^2$

$$\frac{d\sigma}{d\tau} = H(Q) * J^{\rm meas}(\tau, R) * S^{\rm meas}(R, \Lambda/Q, \tau)$$

$$\downarrow$$

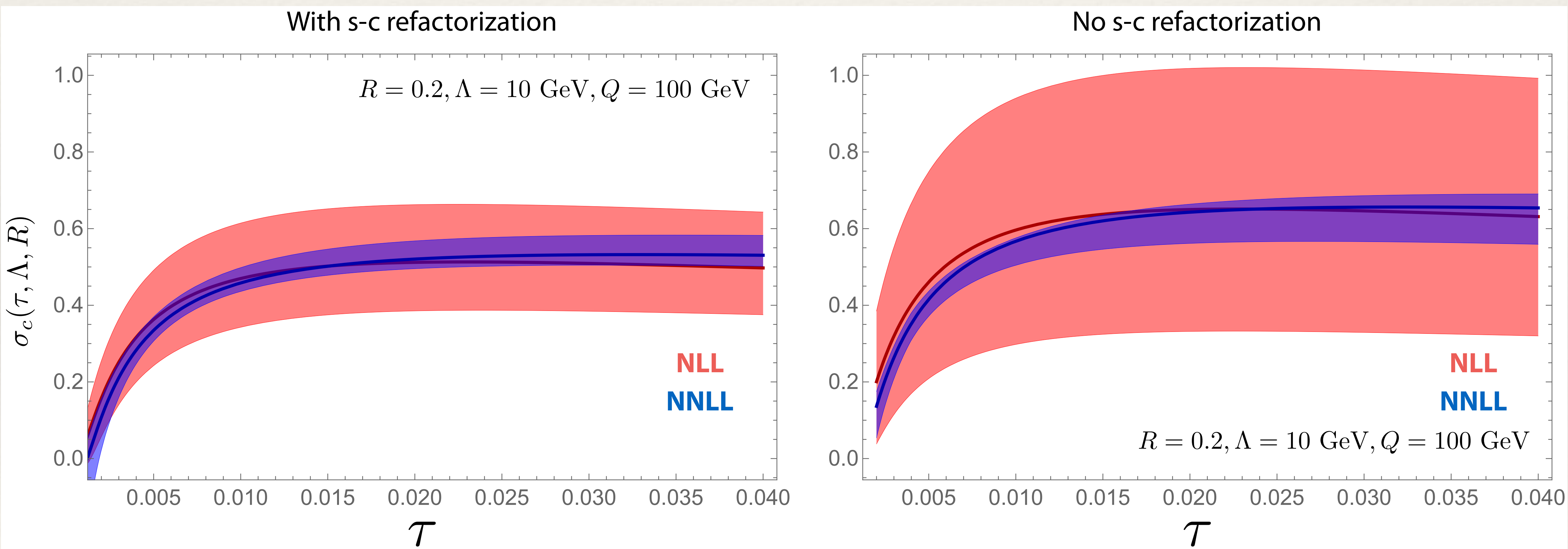
$$S_{\rm in}\Big(\frac{Q\tau}{R}\Big)\, S_s(\Lambda)\, S_c(\Lambda R)$$

- now we have: $\sigma(R,\Lambda) \to H(Q) J^{\rm unmeas}(QR) S_s(\Lambda) S_c(\Lambda R)$

$$J^{\rm unmeas}(QR) = \int_0^{\tau^{\max}(R)} d\tau\; J^{\rm meas}(\tau, R)\, S_{\rm in}\Big(\frac{Q\tau}{R}\Big)$$

Plots

- reduced normalization and scale uncertainty:



Conclusions

- can resum logs or R with 2 additional modes:
    1. "csoft" mode of SCET_+
    2. soft-collinear mode (new)

  $\Big\}$ SCET_{++}

- all anomalous dimensions known to α^3
- can integrate jet shapes to get jet rates
    1. jet rate fact. thms now proven (with J^{unmeas})
    2. understand relation of unmeas. and meas. funcs