More precision, less work

(Semi)automated resummation for multijet processes

with MadGraph and SCET

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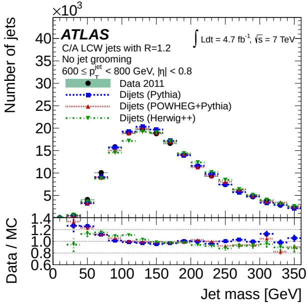
University of Oregon

Santa Fe — Jets and Heavy Flavor Workshop, January 13, 2016

David Farhi, Ilya Feige, MF, Matt Schwartz
(arXiv:1507.06315)

Jets and calculability at the LHC

- LHC measurements: full of jets

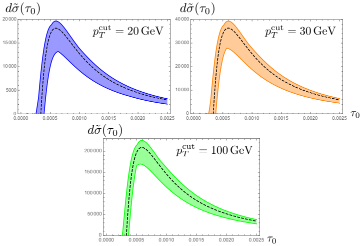


- QCD calculations of these observables are full of logs
- Want controlled calculations for precise predictions
- . . . so we should calculate distributions!

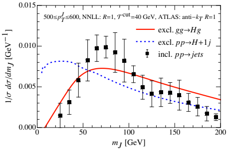
Multijet calculations

But…

- 2 QCD partons:
  e^+e^- thrust;
  Higgs + jet veto; …
- 3 QCD partons:
  $Z + j$ (N^3LL, 1207.1640),
  Higgs + j (1302.0846, NNLL),
  $\gamma + j$, (1208.0010, NNLL)
- 4+ QCD partons:
  dijets (1601.01319, NLL') just now!



arXiv:1601.01319



arXiv:1302.0846

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Would like much more

- $pp \to V + jj(j)$
- $pp \to VV + nh$
- $pp \to nj$
- Various cuts

While PS interation is hard, many distributions have the same IR structure…

Assembling a prediction

Factorization formula:

$$\frac{d\sigma}{d\mathcal{O}} = \sigma_0 \int d\Phi d^n s d^n k H(\Phi)\, S(\Phi, k_i)\, J(s_1) \cdots J(s_n) \delta(\mathcal{O} - \mathcal{O}(s_i, k_i))$$

- For NLL, need:
  - 2-loop cusp anomalous dimension (known)
  - 1-loop anomalous dimension of S and J (known)
  - Tree-level matrix element
  - Don't need anomalous dimensions of H
    – from scale cancellation
- Color mixing: Hard, soft functions are matrices
- Perform all phase space integrals

  Conceptually straightforward, practically can be prohibitive

Same SCET structure

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- Phase space integral depends on hard process, but not on observable
- Soft and jet functions are universal (if observables depend only on s, k)
- Integral over s and k depends on observable, but not hard process

Doing phase space integrals allows the same IR pieces to contribute distributions for many hard processes.

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Numerical integrals

$$\frac{d\sigma}{d\mathcal{O}} = \sigma_0 \int d\Phi\, \boldsymbol{H}_{IJ}(\Phi) \int d^4s d^4k$$
$$\boldsymbol{S}^{JI}(\Phi, \{k_i\})\, \boldsymbol{J}(s_1) \cdots \boldsymbol{J}(s_n) \delta^{(4)}(\mathcal{O} - \mathcal{O}(s, k))$$

- Compute IR-sensitive portion analytically at NLL

$$\frac{d\sigma}{d\mathcal{O}} = \sigma_0 \int d\Phi\, \boldsymbol{H}_{IJ}(\Phi) \boldsymbol{F}_{ji}(\Phi, \mathcal{O})$$

- F depends on $\mathcal{O}$ but not hard process; reusable
- Use existing MC for hard parton PS integration.
- Only use MC to do the integral; no events generated

Integration with Monte Carlo

$$\frac{d\sigma}{d\mathcal{O}} = \sigma_0 \int d\Phi\, \boldsymbol{H}_{IJ}(\Phi)\boldsymbol{F}_{ji}(\Phi,\mathcal{O}) = \int d\Phi\, |\mathcal{M}|^2 \boldsymbol{F}_{ji}(\Phi,\mathcal{O})$$

MC generators generate PS points Φ_i with weights w_i:

$$\int d\Phi\, |\mathcal{M}|^2 f(\Phi) = \sum_i w_i f(\Phi_i) \quad \Longrightarrow \quad \frac{d\sigma}{d\mathcal{O}} = \sum_i w_i F(\Phi_i,\mathcal{O})$$

Reintroduce color matrix after resummation by RG evoltion:

$$\frac{d\sigma}{d\mathcal{O}} = \sigma_0 \int d\Phi\, \mathrm{Tr}\, \boldsymbol{H}(\Phi)\boldsymbol{F}(\Phi,\mathcal{O}) = \sum_i w_i \frac{\mathrm{Tr}\, \boldsymbol{H}(\Phi)\boldsymbol{F}(\Phi,\mathcal{O})}{\mathrm{Tr}\, \boldsymbol{H}(\Phi) S_{\mathrm{tree}}}$$

Processes

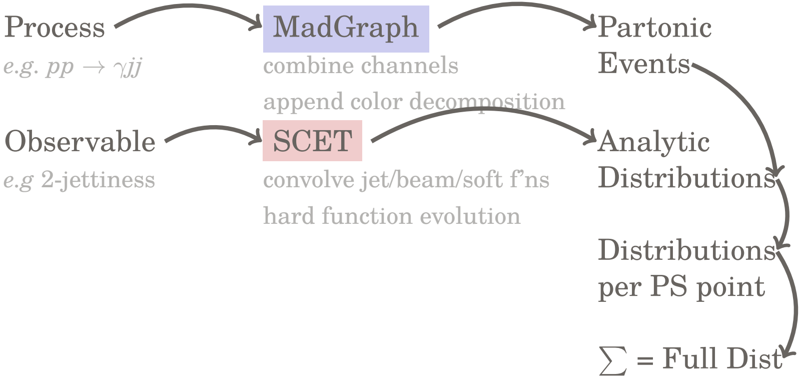
Look at processes with 4 colored partons at tree level

- $e^+e^- \to 4j$
- $pp \to \gamma + 2j$

Measure variations of N-jettiness observable

1. Sum all channels and crossing — MadGraph ✓
2. Compute 5/6-particle matrix elemtents — MadGraph ✓
3. Compute 4-parton jet/soft functions — Universal input
4. Integrate 3/4-particle PS with cuts — MadGraph ✓

Calculation outline



Details of MadGraph output

Wrestle with MadGraph to get color decomposition:

```
<event>
 6   0  0.2609800E-07  0.50,00000E+03  0.7546771E-02  0.9399810E-01
   -11  -1   0   0    0    0  0.0000E+00  0.0000E+00  0.2500E+03  0.2500E+03  0.0E+00 0. -1.
    11  -1   0   0    0    0 -0.0000E+00 -0.0000E+00 -0.2500E+03  0.2500E+03  0.0E+00 0.  1.
     1   1   1   2  501    0 -0.1007E+03 -0.2251E+02 -0.1204E+02  0.1039E+03  0.0E+00 0.  1.
     2   1   1   2  502    0 -0.5886E+02  0.1209E+03 -0.1251E+02  0.1351E+03  0.0E+00 0.  1.
    -1   1   1   2    0  502  0.1321E+03  0.1112E+02 -0.2925E+02  0.1358E+03  0.0E+00 0. -1.
    -2   1   1   2    0  501  0.2749E+02 -0.1095E+03  0.5380E+02  0.1251E+03  0.0E+00 0. -1.
<mgrwt>
  ...MadGraph5 scale information for reweighting here...
</mgrwt>
<colordecomposition>
  0.11342428E+15 -.34027285E+15 -.34027285E+15 0.10208186E+16
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H_{IJ} - hard function

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```

H_{IJ} - hard function $\qquad$ S^{IJ}_{tree} - soft function/color basis

Details of SCET component

Focus on generalizations of N-jettiness observables:

$$\mathcal{T}^i = \sum_{k \in \text{jet}} n_i \cdot p_k \quad \Longrightarrow \quad \begin{aligned} \mathcal{T}_4 &= \mathcal{T}^1 + \mathcal{T}^2 + \mathcal{T}^3 + \mathcal{T}^4 \\ \mathcal{T}_2^{\text{cut}} &= (\mathcal{T}^1 + \mathcal{T}^2)\theta(\mathcal{T}_{\text{cut}} - \mathcal{T}^a - \mathcal{T}^b) \end{aligned}$$

$$\begin{aligned} \frac{d\sigma}{d^n\mathcal{T}} &= \sigma_0 \int d\Phi \, H_{IJ}(\Phi) \int d^4 s_i d^4 k_i \delta^{(4)}(\mathcal{T}_i - s_i/Q_i - k_i) \\ &\qquad S^{JI}(\Phi, \{k_i\}) \, B(s_a, x_a) B(s_b, x_b) J(s_1) \cdots J(s_n) \\ &= \sigma_0 \int d\Phi, H_{IJ}(\Phi) F^{JI}(\Phi, \{\mathcal{T}_i\}) \end{aligned}$$

with $F^{JI}(\Phi, \{\mathcal{T}_i\}) = U^H_{JK} \int d^4 s_i d^4 k_i \delta^{(4)}(\mathcal{T}_i - s_i/Q_i - k_i) S^{KI}(\cdots$

Details of SCET component

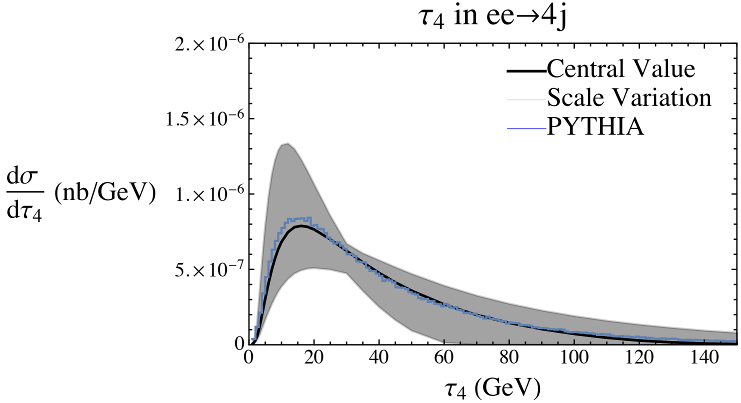
$$\frac{d\sigma}{d^n\mathcal{T}} = \sigma_0 \int d\Phi, H_{IJ}(\Phi) F^{JI}(\Phi, \{\mathcal{T}_i\})$$

$$F^{JI}(\Phi, \{\mathcal{T}_i\}) = U^H_{JK} \int d^4 s_i d^4 k_i \delta^{(4)}(\mathcal{T}_i - s_i/Q_i - k_i) S^{KI}(\cdots$$

- U^H_{JK} and S^{KI} are matrices in color space. Evolution by matrix exponentiation
- Different F for each channel ($q\bar{q}q\bar{q}$, $q\bar{q}gg$, $\ldots$)
- MadGraph convolves over PDFs in partonic calculation. Modify $B(s_i, x_i)$ by scaling out PDF dependence

Results

$e^+e^- \to 4\, jets$



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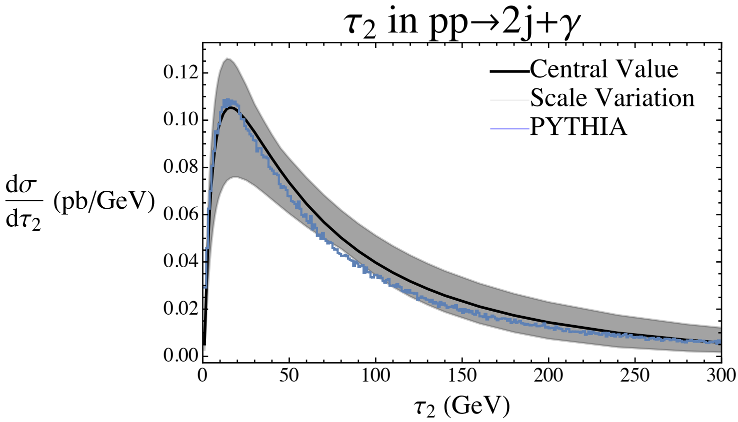
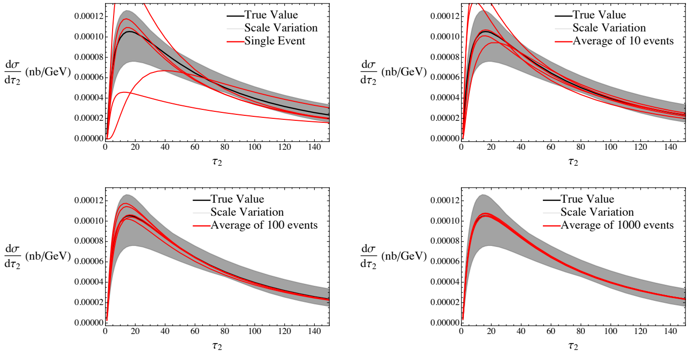


Convergence



Excellent convergence after O(few 100) phase space points

Extensibilty & Conclusions

	Observable	New Code	New Calculation
easier	Same observable, new process	—	—
⋮	Different function of $\mathcal{T}^i$'s	New Integral of F	—
⋮	More colored particles	New soft/hard anom. dims.	—
⋮	New observable not a function of cT^i	—	New beam/jet/ soft functions
harder	NNLL	Interface to NLO generators	All components at NLO

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Code included with arXiv submission. Try it out for yourself!

Thank you!