Analytical and Monte Carlo Studies of Jets with Heavy Mesons and Quarkonia

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Duke University

Jets and Heavy Flavor Workshop, Santa Fe NM
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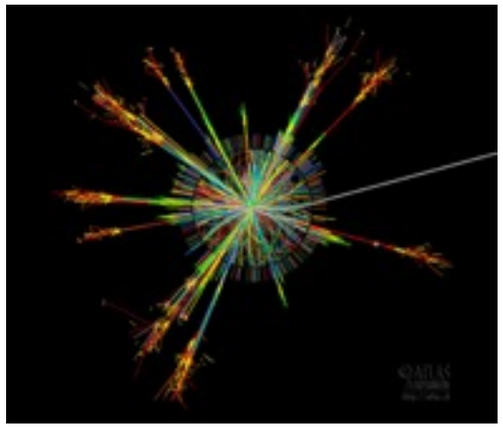
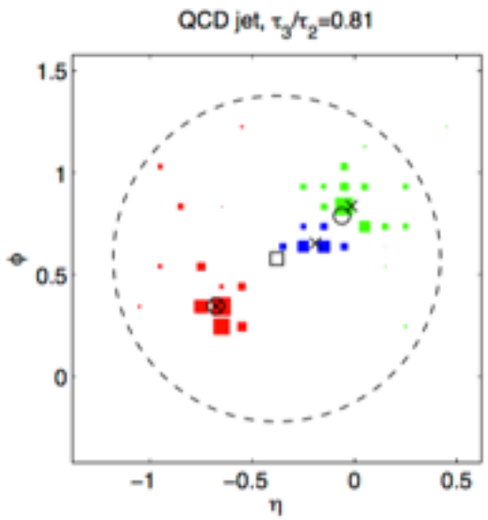
Motivations

Understand high energy jets at LHC



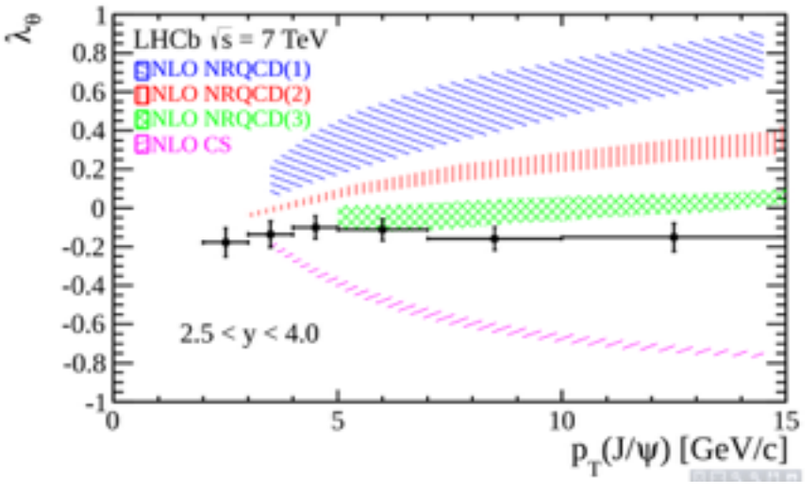
ATLAS Collaboration

Study wealth of jet substructure observables



Thaler, v. Tilberg, arXiv:1011.2268

Elucidate outstanding puzzles in quarkonia production



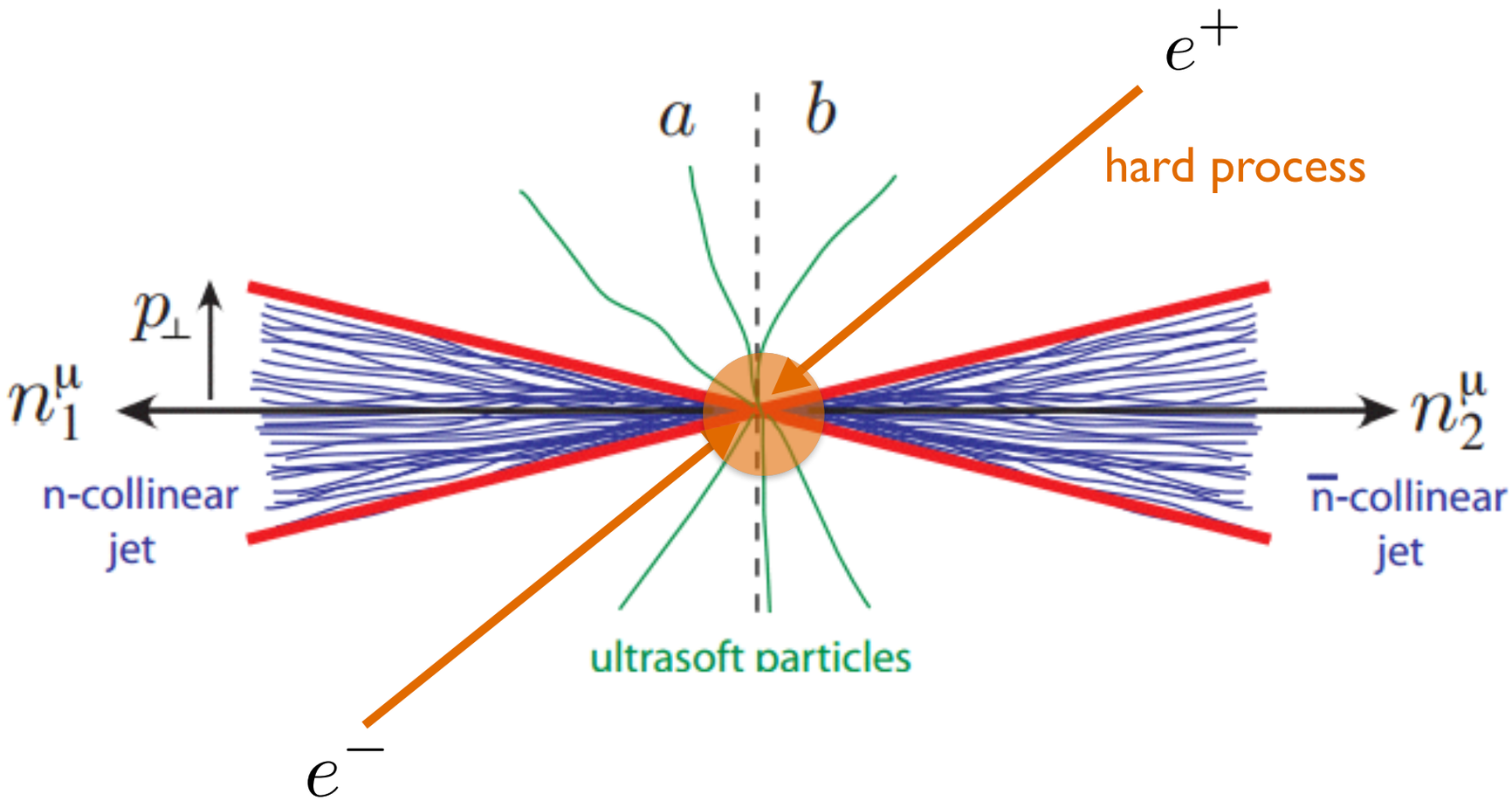
LHCb Collaboration

Outline

- Fragmenting jet functions & angularities
- Cross sections for e+e- $\longrightarrow$ B's
- Comparisons with Monte Carlo
- Applications to quarkonium production

Jet Cross-Sections in SCET

Factorization ⟷ Short Distance x Long Distance



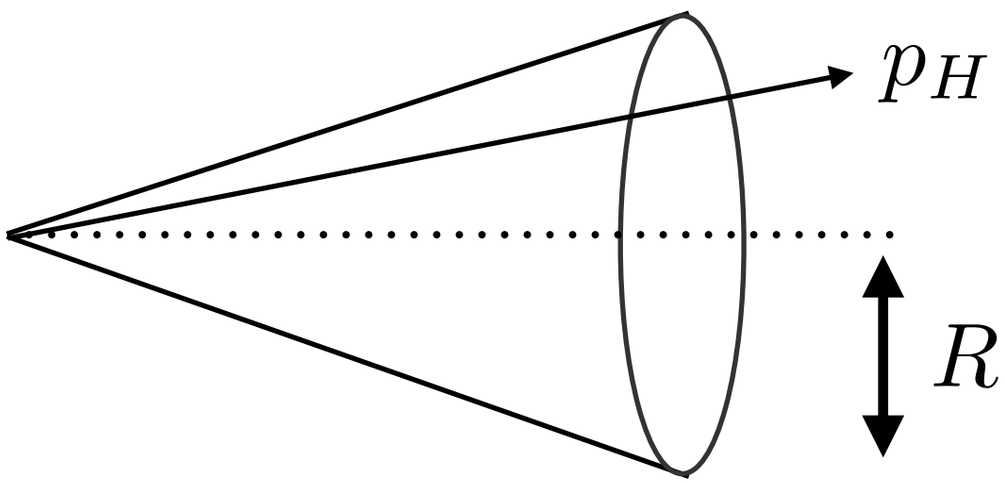
Factorization Theorem

$$d\sigma = H \times J^1 \otimes J^2 \otimes S$$

- Hard function $H(\mu)$
- Jet Functions $J^{(1)}(\mu)$ → Measured / Unmeasured
- Soft Function $S(\mu)$

Fragmenting Jet Functions (FJF's)

Jet with identified hadron H



Jet Energy	E_J
Plus Momentum	$p_H^+ = z p_J^+$
Cone Size	R

Additional Measured Observable

Measured energy $\mathcal{G}_i^H(E, R, \mu, z)$

Measured angularity $\mathcal{G}_i^H(\tau_a, R, \mu, z)$

Jain, Procura, Waalewijn, arXiv:1101.4953
Procura, Stewart, arXiv:0911.4980
Procura, Waalewijn, arXiv:1111.6605

Calculate Cross-Section with FJFs

Jet cross-section ⟶ Jet w/ Identified Hadron cross-section

$$J_i(s,\mu) \to \frac{1}{2(2\pi)^3}\mathcal{G}_i^H(s,z,\mu)dz$$

Convolution of Matching Coefficients & Fragmentation Functions (FF's)

$$\mathcal{G}_i^H(s,z,\mu) = \sum_j \left[\mathcal{J}_{ij}(s,\mu) \bullet D_j^H(\mu)\right](z) \quad \text{where} \quad [f \bullet g](z) \equiv \int_z^1 \frac{dx}{x} f(x) g(z/x)$$

Calculate $\mathcal{J}_{ij}(s,z,\mu)$ perturbatively for different observables

Procura, Stewart, arXiv:0911.4980

Our observable: Angularities τ_a

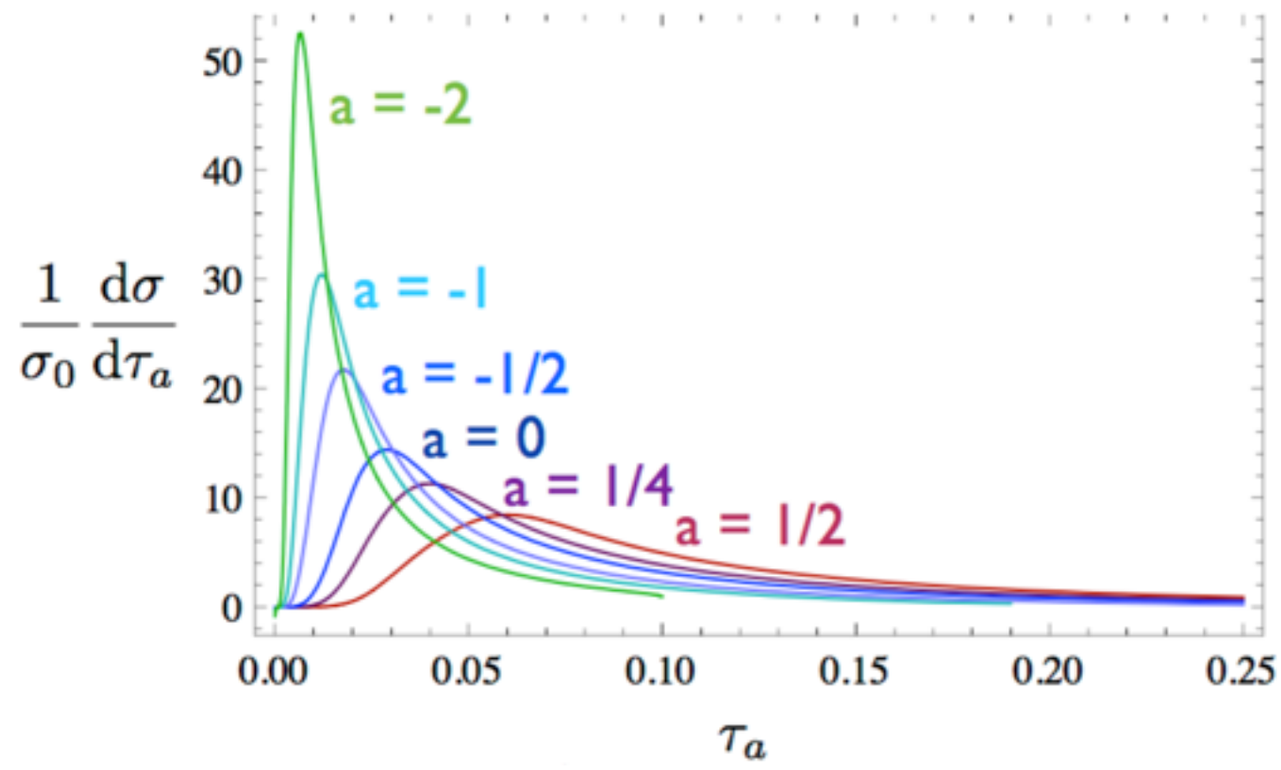
Generalization of jet thrust $\begin{cases} \text{a=0 thrust} \\ \text{a=1 broadening} \end{cases}$

$$\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2} \quad \begin{cases} \text{Sum over jet particles } i \\ \omega = \sum_i p_i^- \approx 2E_{jet} \end{cases}$$

Good analytic handle on τ_a

IR Safety $\quad -\infty < a < 2$

Factorizability $\quad -\infty < a < 1$



Hornig, Lee, Ovanesyan, arXiv:0905.0168

S.D.Ellis, et. al (2010) , arXiv:1001.0014

First steps: e^+e^- collisions

R. Bain, L. Dai, A. Hornig, A.Leibovich, Y. Makris, T. Mehen

$e^+e^- \to b\bar{b}$ vs. Monte Carlo

↳ B jet

$e^+e^- \to q\bar{q}g$ vs. Monte Carlo

↳ J/ψ jet

Goal: Working towards pp ⟶ B, J/ψ

Recent work on pp ⟶ light hadrons, D's (Chien et al. arXiv:1512.06851)

Matching Coefficients at NLO

We calculated all 4 NLO (1-loop) $\mathcal{J}_{ij}$ for measured angularities

$$
\begin{aligned}
\frac{\mathcal{J}_{qq}(\omega, z, \tau_a, \mu)}{2(2\pi)^3} = & \frac{C_F \alpha_s}{2\pi} \frac{1}{\omega^2} \Bigg\{ \delta(\tau_a)\delta(1-z) \frac{2-a}{1-a} \left(-\frac{\pi^2}{12} + \frac{1}{2} \ln^2 \left(\frac{\mu^2}{\omega^2} \right) \right) \\
& + \delta(\tau_a) \Bigg(1 - z - \left[\ln \left(\frac{\mu^2}{\omega^2} \right) + \frac{1}{1-a/2} \ln \left(1 + \frac{(1-z)^{1-a}}{z^{1-a}} \right) \right] \frac{1+z^2}{(1-z)_+} \\
& + \frac{1-a}{1-a/2} (1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ \Bigg) \\
& + \left[\frac{1}{\tau_a} \right]_+ \left(\frac{1}{1-a/2} \frac{1+z^2}{(1-z)_+} - \delta(1-z) \frac{2}{1-a} \ln \left(\frac{\mu^2}{\omega^2} \right) \right) \\
& + \frac{2\delta(1-z)}{(1-a)(1-a/2)} \left[\frac{\ln \tau_a}{\tau_a} \right]_+ \Bigg\}
\end{aligned}
$$

also… $\mathcal{J}_{qg}, \mathcal{J}_{gq}, \mathcal{J}_{gg}$

Consistency checks: ***1. a ⟶ 0 limit (arXiv:1101.4953)***

2. $J_i(s, \mu) = \frac{1}{2(2\pi)^3} \sum_j \int_0^1 dz z \mathcal{J}_{ij}(s, z, \mu)$

Cross Section for $b \longrightarrow B^+/B^0$

Re-summed to NLL' using renormalization group (RG)

FF for B's

$$\frac{1}{\sigma_0}\frac{d\sigma^{(b)}}{d\tau_a dz} = H(\mu_H) \times J^{(1)}_{\omega_1} \times (\mu_{J^1}) \times S^{unmeas}(\mu_s^\Lambda) \times \left\{ \sum_{j\neq b} \left[D_j^B(\mu_{J^2}) \bullet f_j^{bj}(\tau_a) \right](z) \right.$$

$$+ \left[D_b^B(\mu_{J^2}) \bullet \left(\delta(1-z)\,(1 + f_s(\tau_a, \mu_{S^2})) + f_J^{bb}(\tau_a, \mu_{J^2}) \right) \right](z)$$

coupled z & τ_a

$$\left. \times \frac{1}{\Gamma\left[-\omega_{J^2} - \omega_{S^2}\right]} \frac{1}{(\tau_a)^{1+\omega_{J^2}+\omega_{S^2}}} \right\}_+ \times \prod\left(\mu; \mu_H, \mu_{J^1}, \mu_{J^2}, \mu_{S^2}\right)$$

RG evolution kernel

Coupling of z and τ_a dependence appears first at NLO

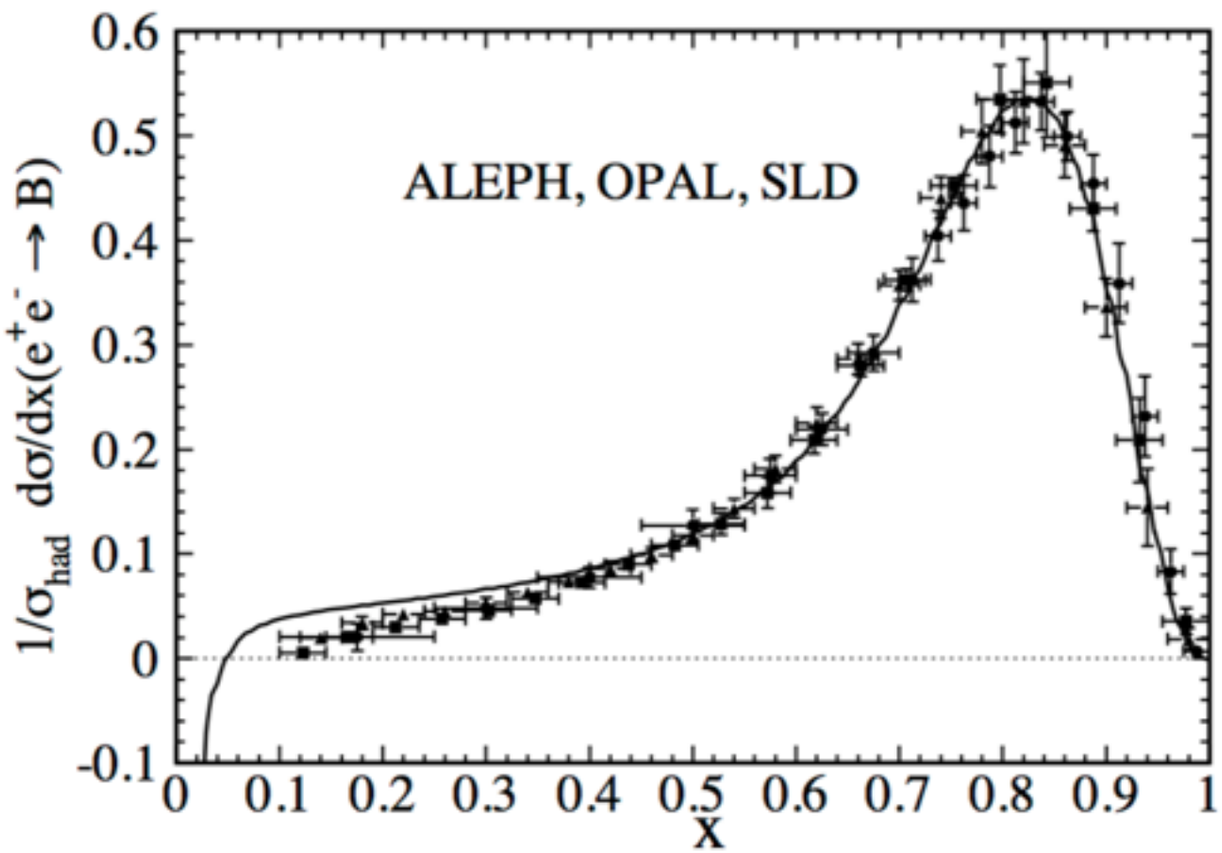
Can be easily extended to other identified hadrons

S.D.Ellis, et. al (2010) , arXiv:1001.0014

b quark Fragmentation Function

Fit power model to LEP data

Inclusive Cross-Section vs. z



$$D(x, \mu_0) = N x^{\alpha}(1-x)^{\beta}$$

$$N = 4684.1$$

$$\alpha = 16.87$$

$$\beta = 2.028$$

$$\mu_0 = m_b = 4.5 GeV$$

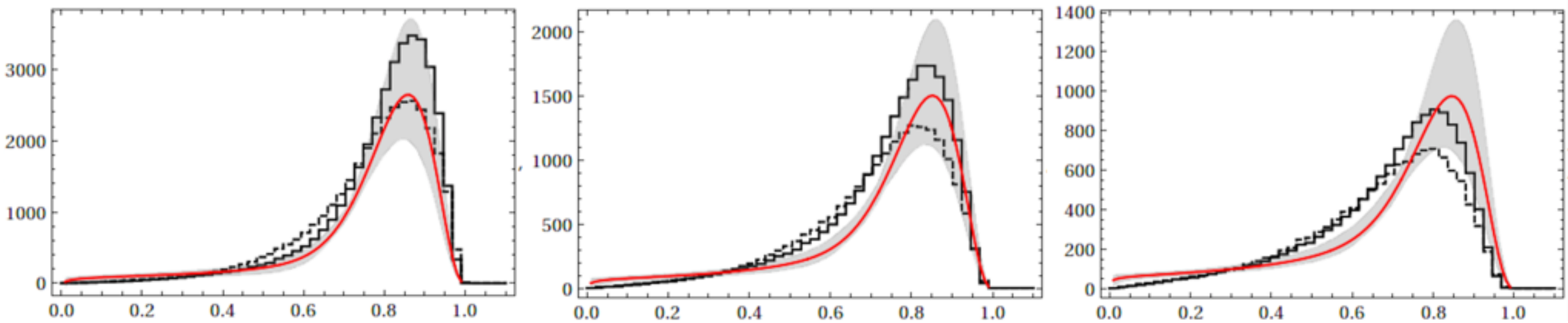
$$\chi^2_{dof} = 1.495$$

Kniehl, et al. arXiv:0705.4392

Analytic vs. Monte Carlo (B^+/B^0)

z distributions, E_{cm}= 600 GeV

τ_0 = *0.0004* τ_0 = *0.0006* τ_0 = *0.0008*



Pythia

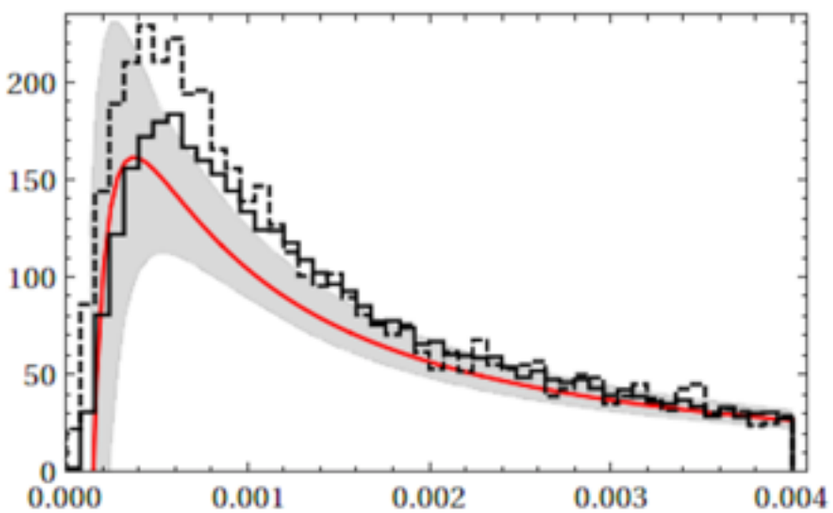
Analytic (NLL')

Herwig

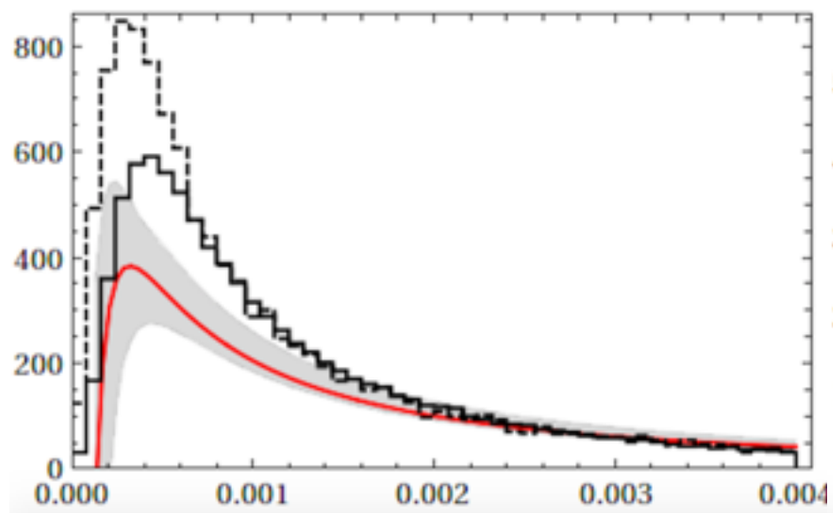
Analytic vs. Monte Carlo (B^+/B^0)

τ_0 distributions, E_{cm}= 600 GeV

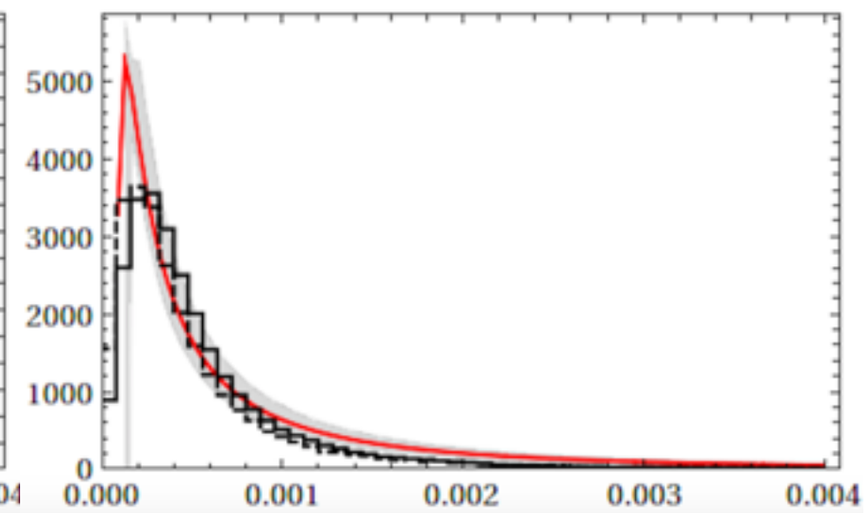
z = 0.4



z = 0.6



z = 0.8



Pythia

Analytic (NLL')

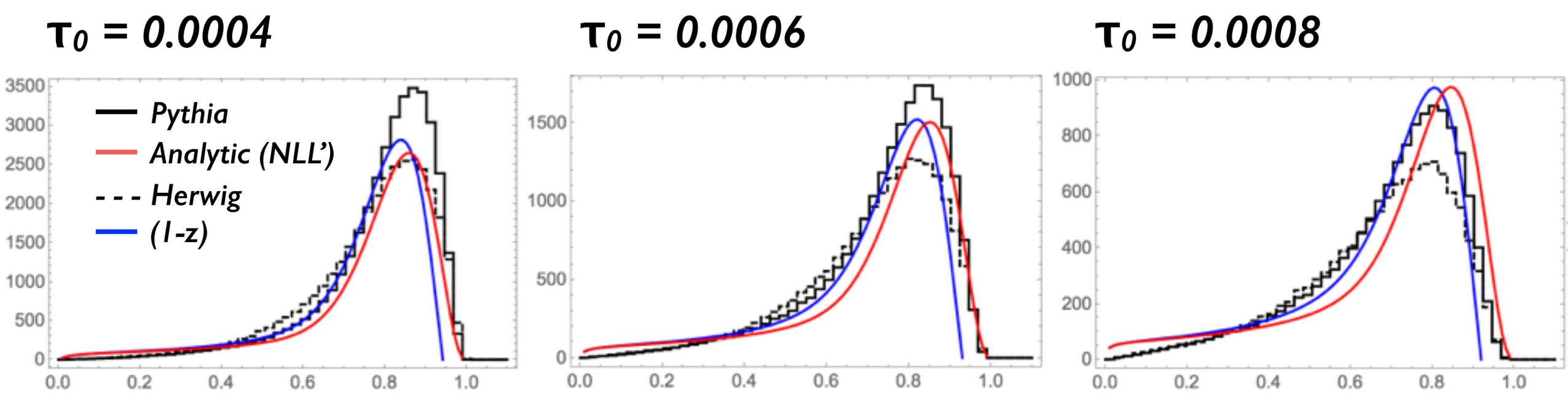
Herwig

Eliminating Logs of 1-z

Minimize Log(1-z) in FJF with z-dependent measured jet scale

$$[f_{\mathcal{J}}^{ij} \bullet D](z) \sim \log\left(\frac{\mu}{\mu_J(z)}\right) \quad \text{with} \quad \mu_J(z) = \omega\tau^{1/(2-a)}(1-z)^{(1-a)/(2-a)}$$

z-distributions w/ minimized Log(1-z)



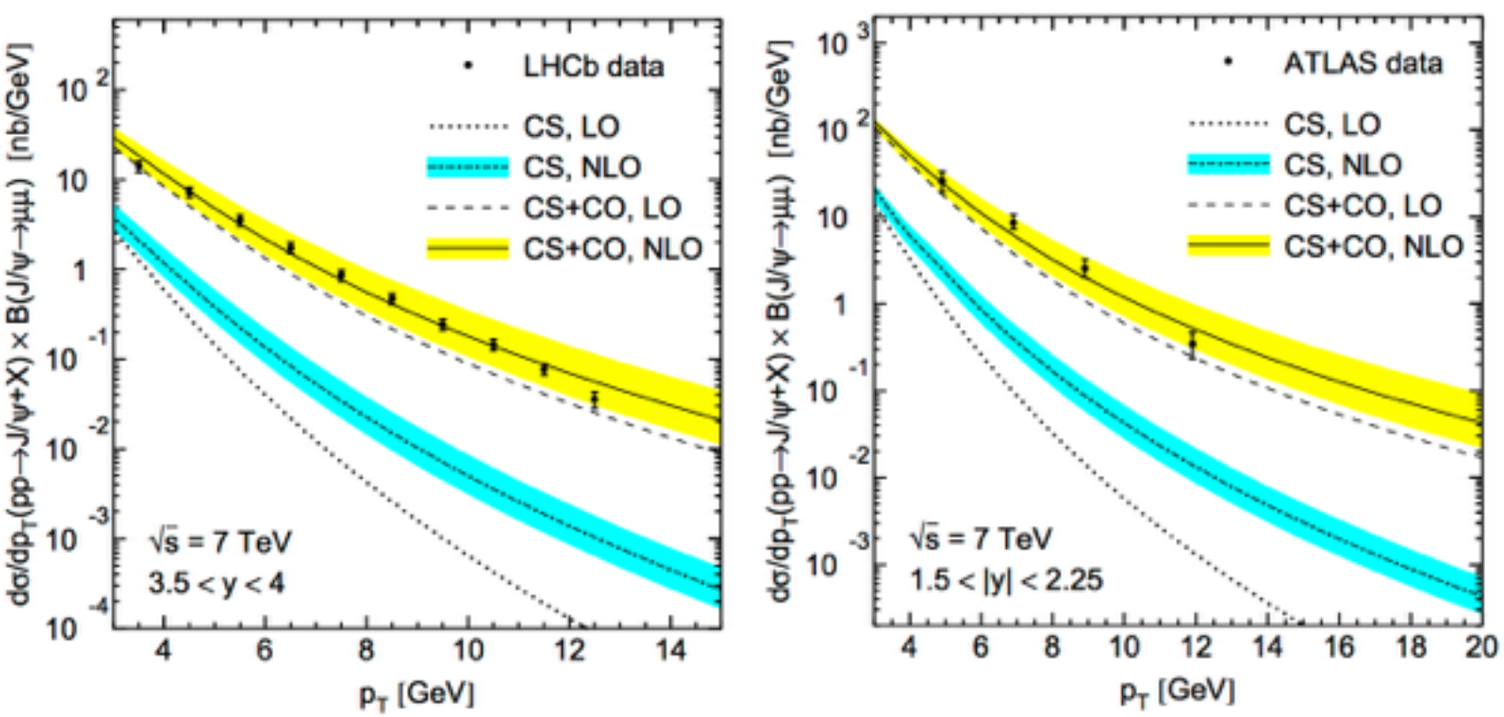
Procura, Waalewijn, arXiv:1111.6605

Apply to Heavy Quarkonium?

Non-relativistic QCD Factorization Formalism

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

Double expansion in α_s, v with $n - {}^{2S+1}L_J^{(1,8)}$



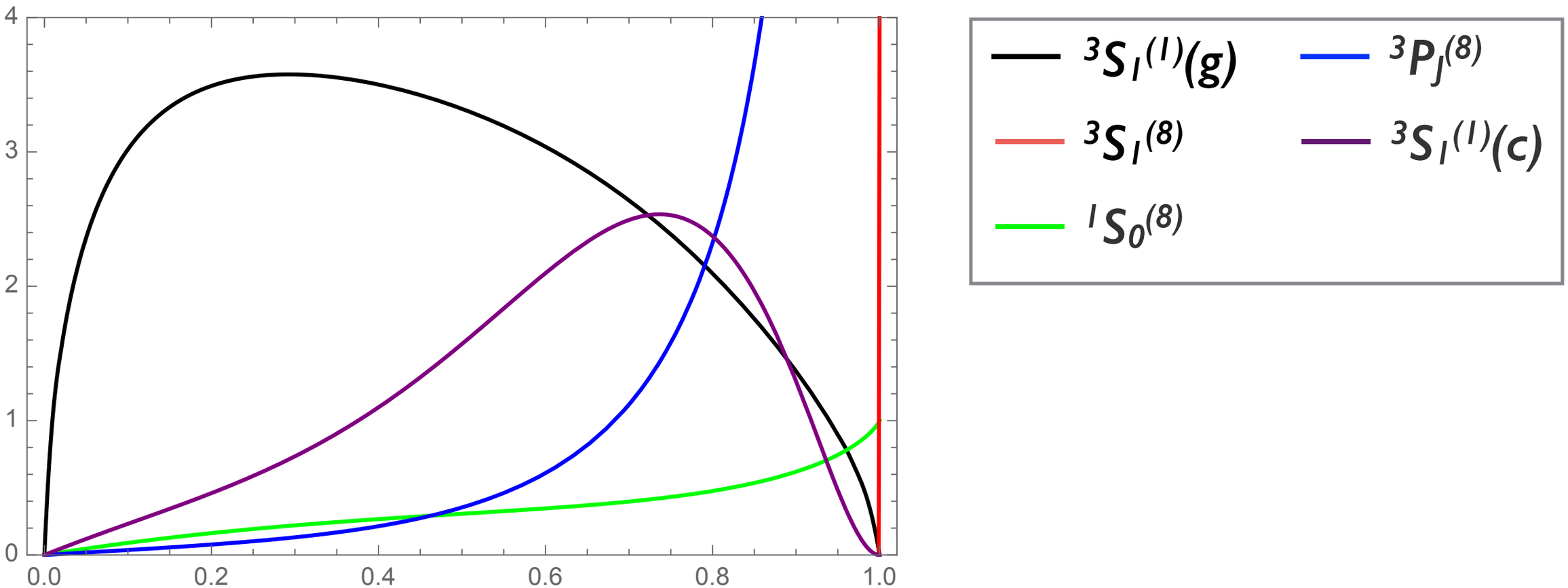
Fits to world data

Mechanism	Fitted Value
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{(1)})\rangle$	$1.32\,GeV^3$
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{(8)})\rangle$	$(2.24 \pm 0.59) \times 10^{-3}\,GeV^3$
$\langle \mathcal{O}^{J/\psi}({}^1S_0^{(8)})\rangle$	$(4.97 \pm 0.44) \times 10^{-2}\,GeV^3$
$\langle \mathcal{O}^{J/\psi}({}^3P_0^{(8)})\rangle$	$(-1.61 \pm 0.20) \times 10^{-2}\,GeV^5$

Buttenschon, Kniehl (2011), arXiv:1105.0820
Bodwin, Braaten, Lepage

NRQCD Fragmentation Functions

Fragmentation Function vs. z of J/ψ



Perturbative coefficients x Long Distance Matrix Element

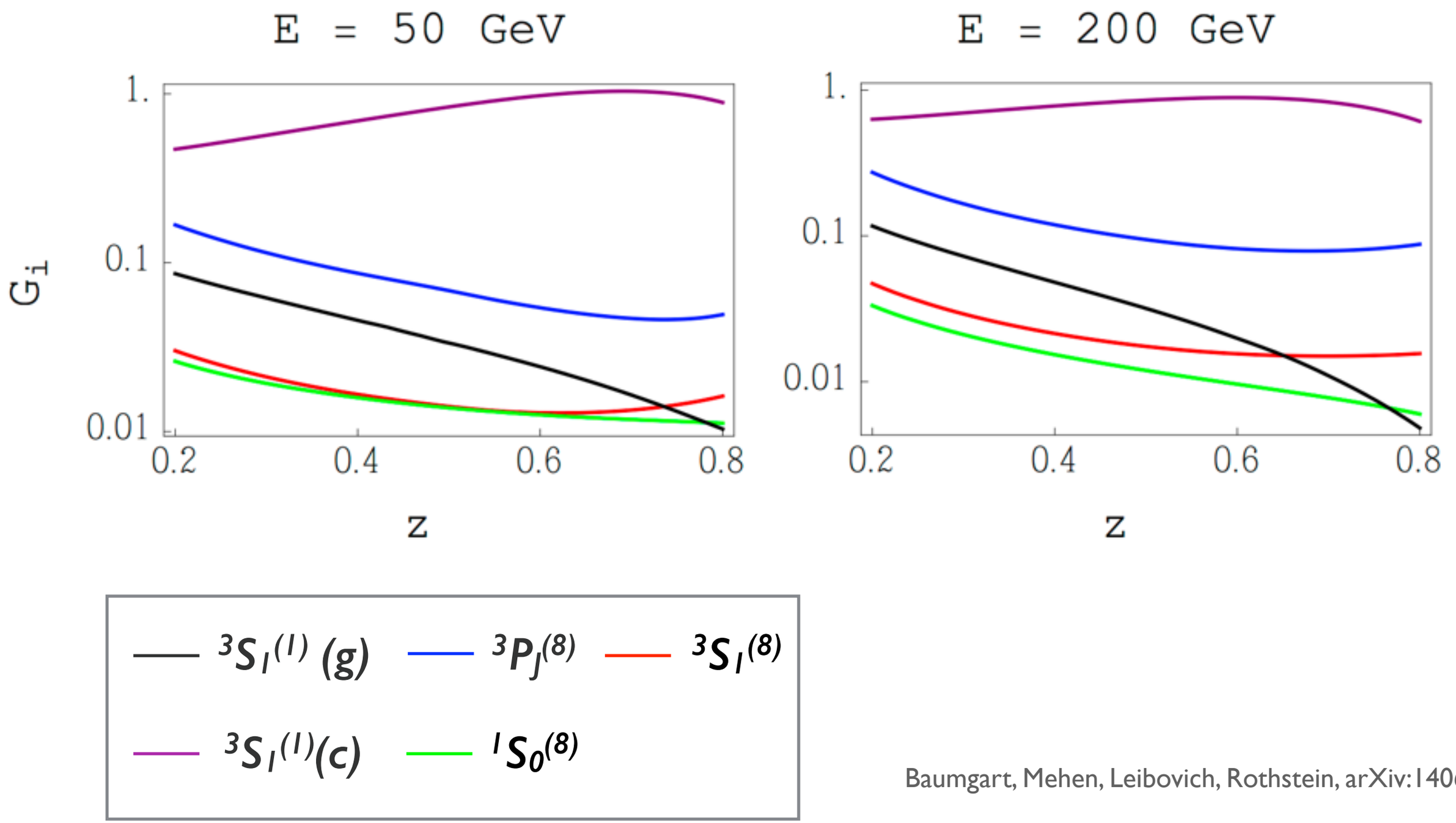
$$D_{g\to J/\psi}^{^3S_1^{(8)}}(z,2m_c) = \frac{\pi\alpha_s(2m_c)}{24m_c^3}\langle\mathcal{O}^{J/\psi}(^3S_1^{(8)})\rangle\delta(1-z)$$

$$D_{g\to J/\psi}^{^1S_0^{(8)}}(z,2m_c) = \frac{5\alpha_s(2m_c)}{96m_c^3}\langle\mathcal{O}^{J/\psi}(^1S_0^{(8)})\rangle\left(3z-2z^2+2(1-z)\log(1-z)\right)$$

Braaten, Chen, hep-ph/9610401
Braaten, Chen, hep-ph/9604237
Braaten, Yuan, hep-ph/9302307

FJF's and Quarkonia Production

Discriminating power between NRQCD production mechanisms



Baumgart, Mehen, Leibovich, Rothstein, arXiv:1406.2295

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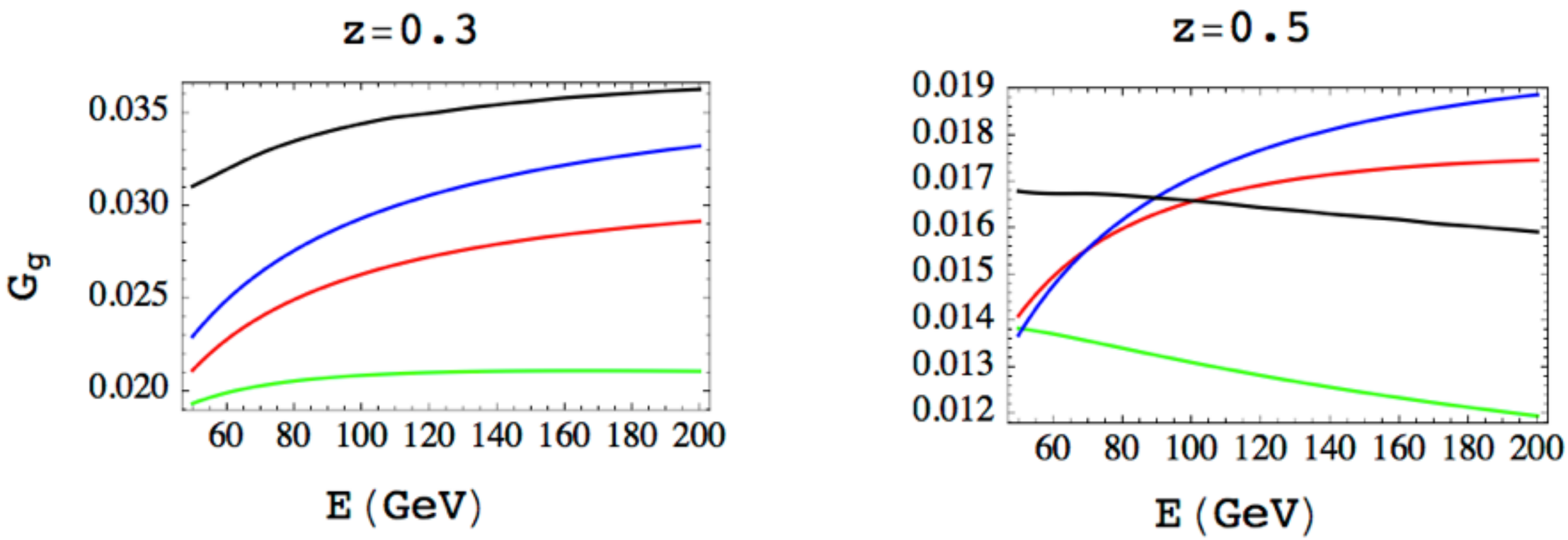
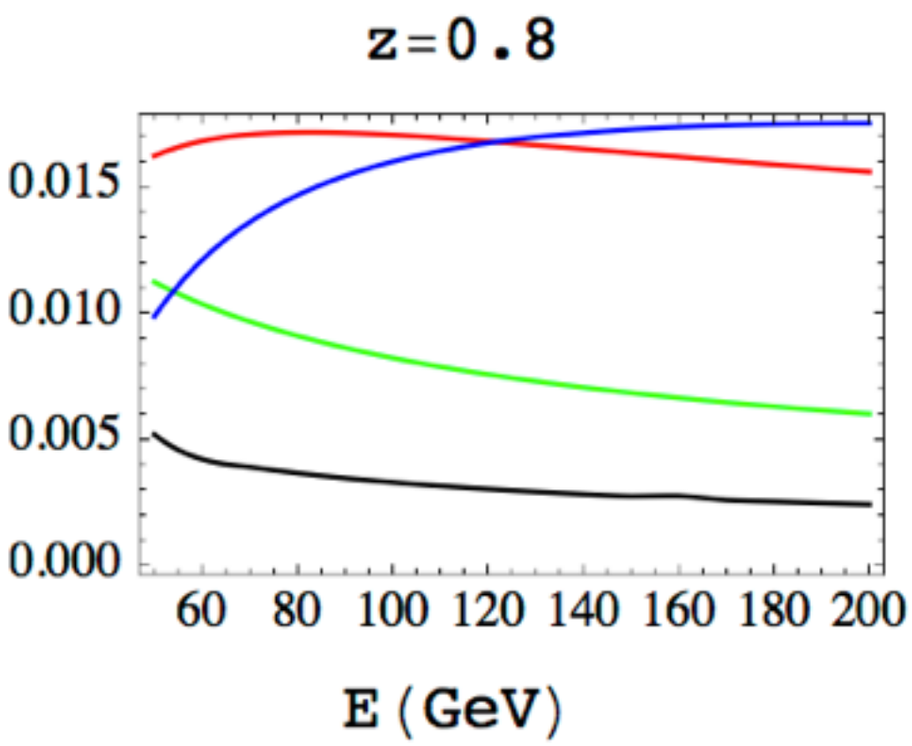
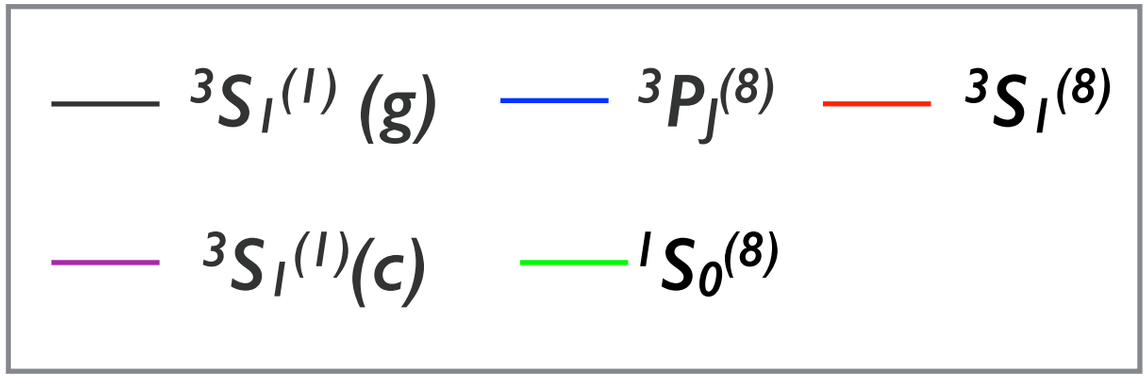




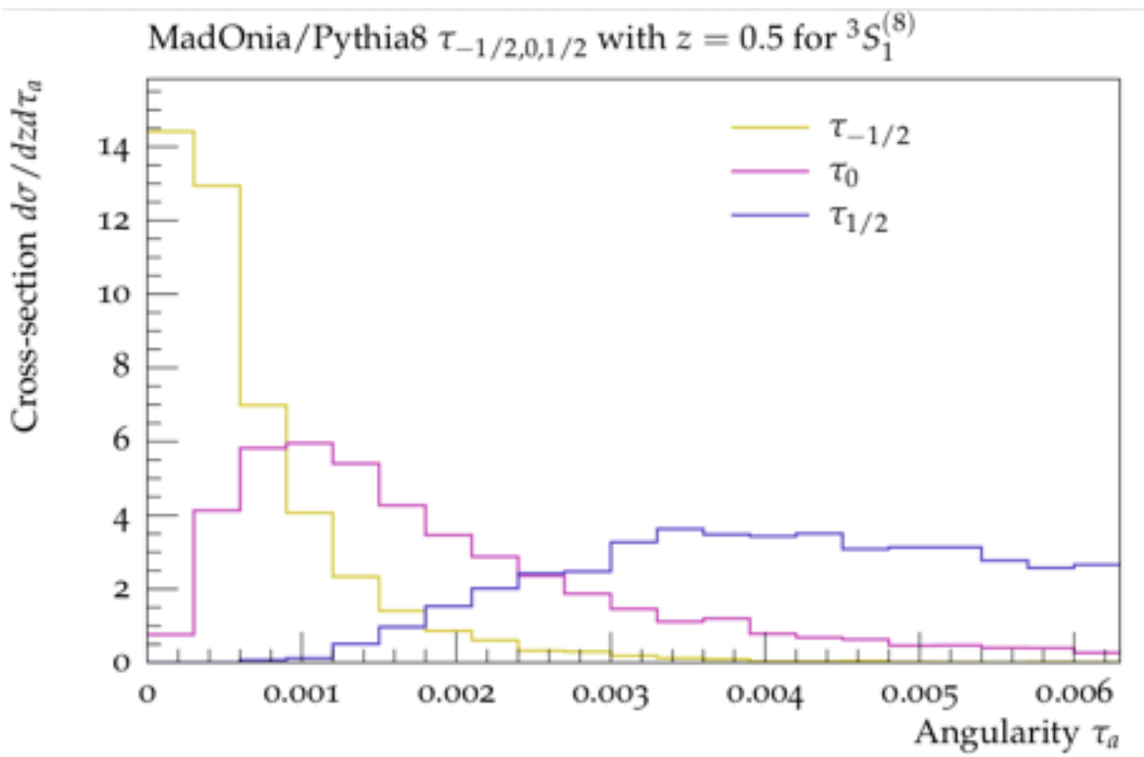


Baumgart, Mehen, Leibovich, Rothstein, arXiv:1406.2295

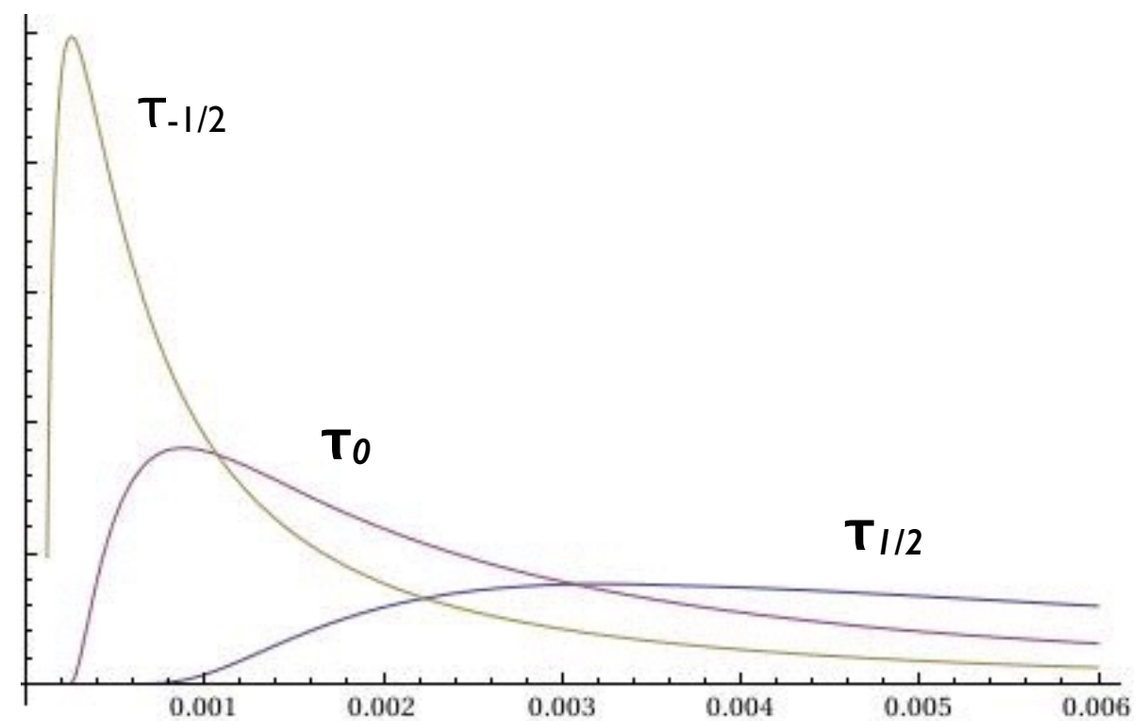
Issues with J/ψ in Monte Carlo

Angularity distributions for fixed z=0.5

PYTHIA



Analytic Angularity Distribution

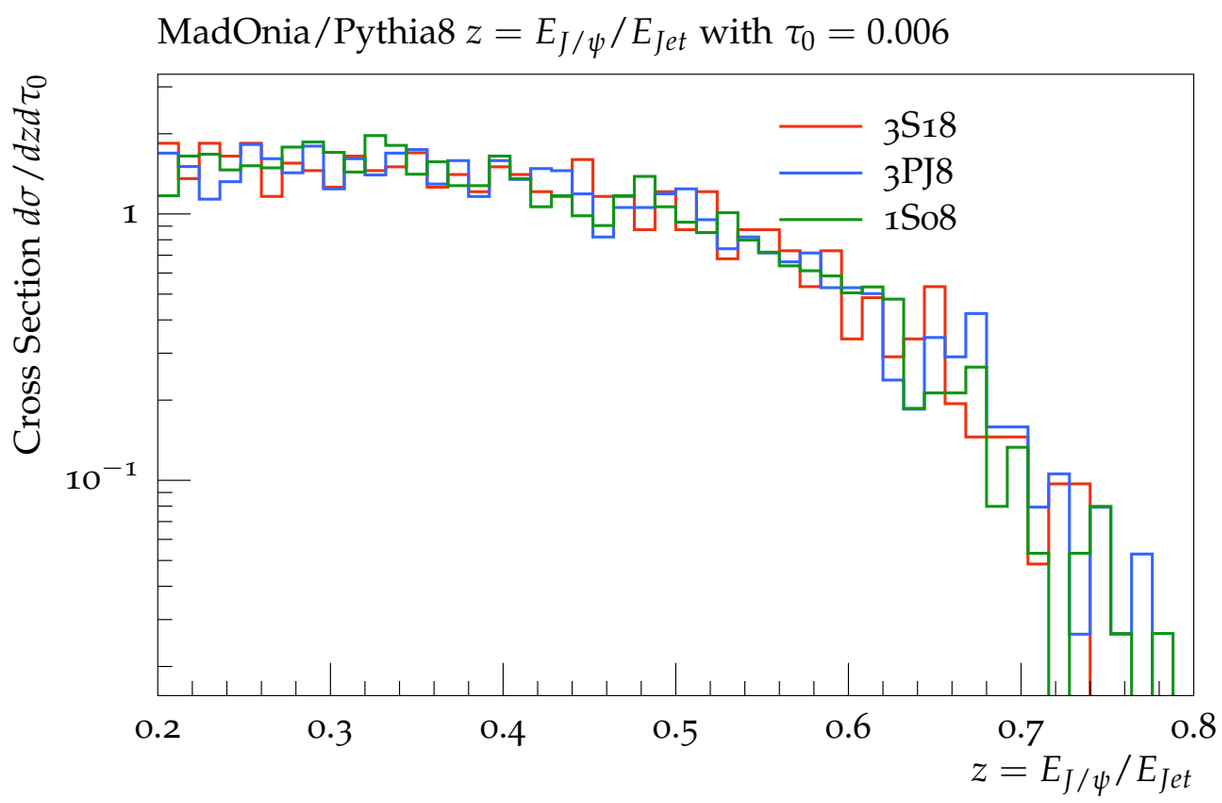


PYTHIA seems to match our model for substructure

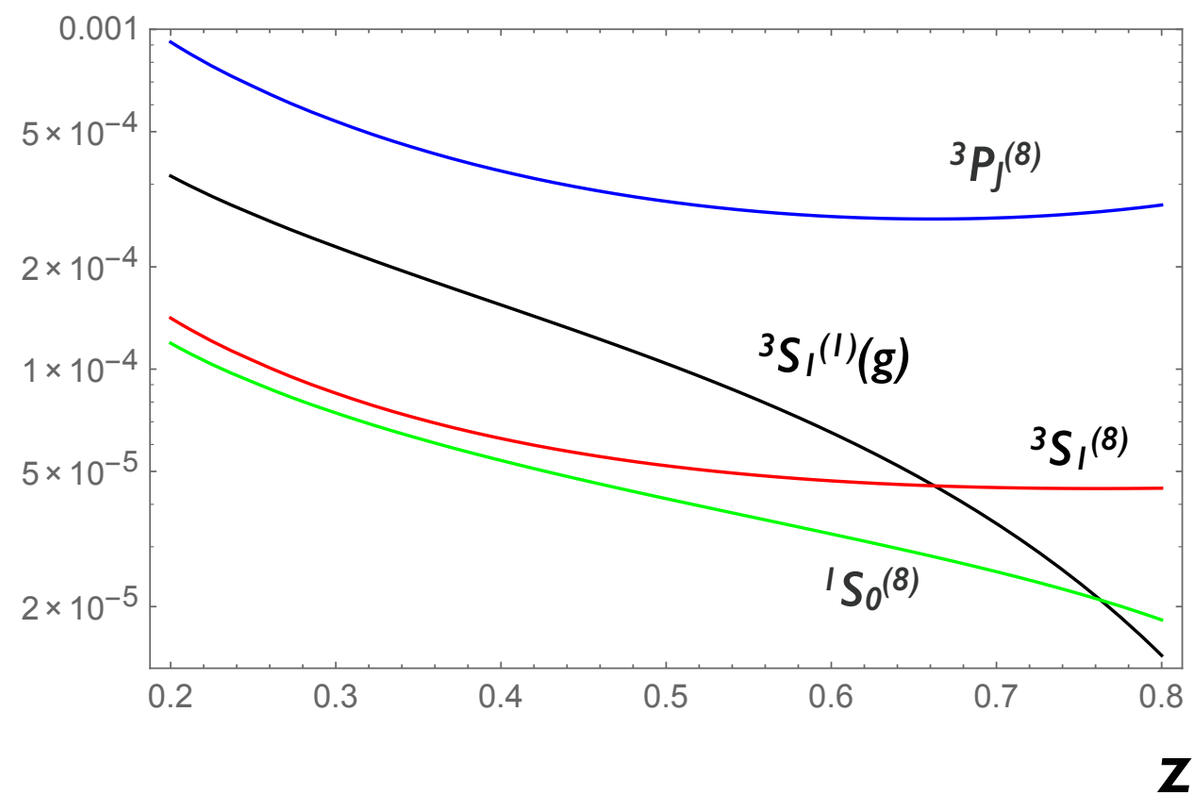
Issues with J/ψ in Monte Carlo

z-distributions for fixed angularity

PYTHIA



Analytic z distribution



PYTHIA treats octet mechanisms identically; frag. model incomplete

(Work in progress)

Summary & Conclusions

- New calculation: FJF for measured angularities
- FJF's help us calculate more exclusive jet cross-sections
- Our calculation fits B production in Monte Carlo ($d\sigma/d\tau dz$)
- Method could yield insights into quarkonium production questions
- Monte Carlo fragmentation model for J/ψ does not match ours

Future Work

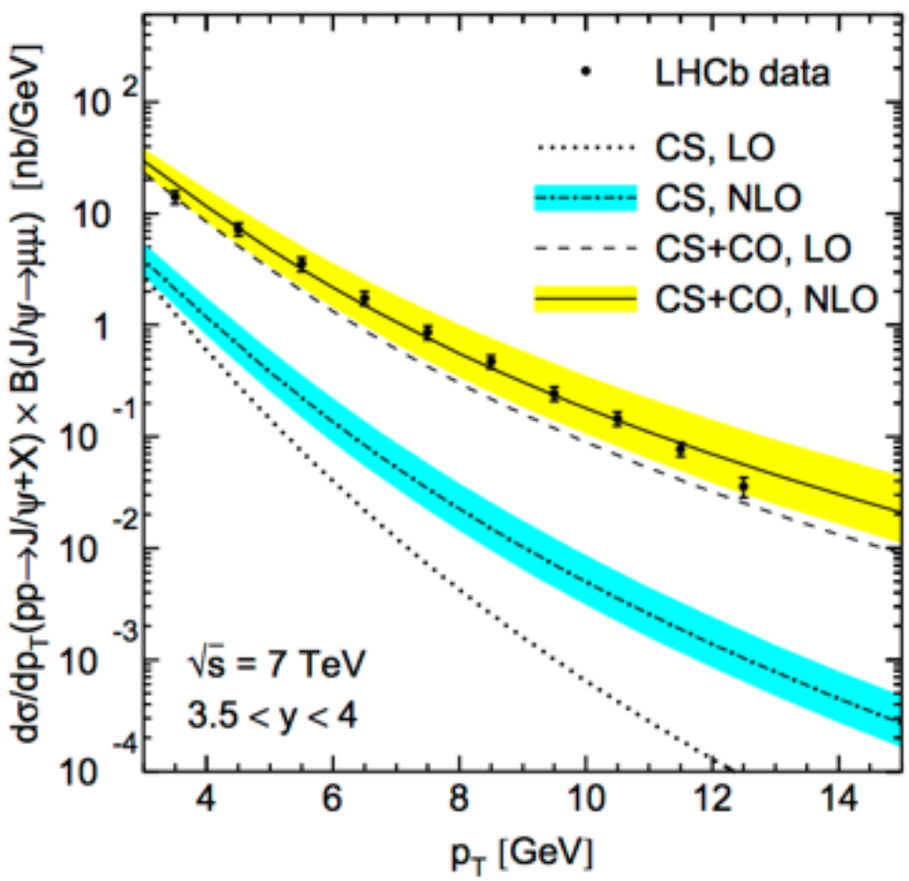
- Further study of g fragmentation in analytic and Monte Carlo calc.
- Calculate cross-section for pp w/ measured angularity
- Apply soft-collinear re-factorization (see Andrew Hornig's talk)
- Extend to Next-to-next-to-leading log (NNLL) accuracy

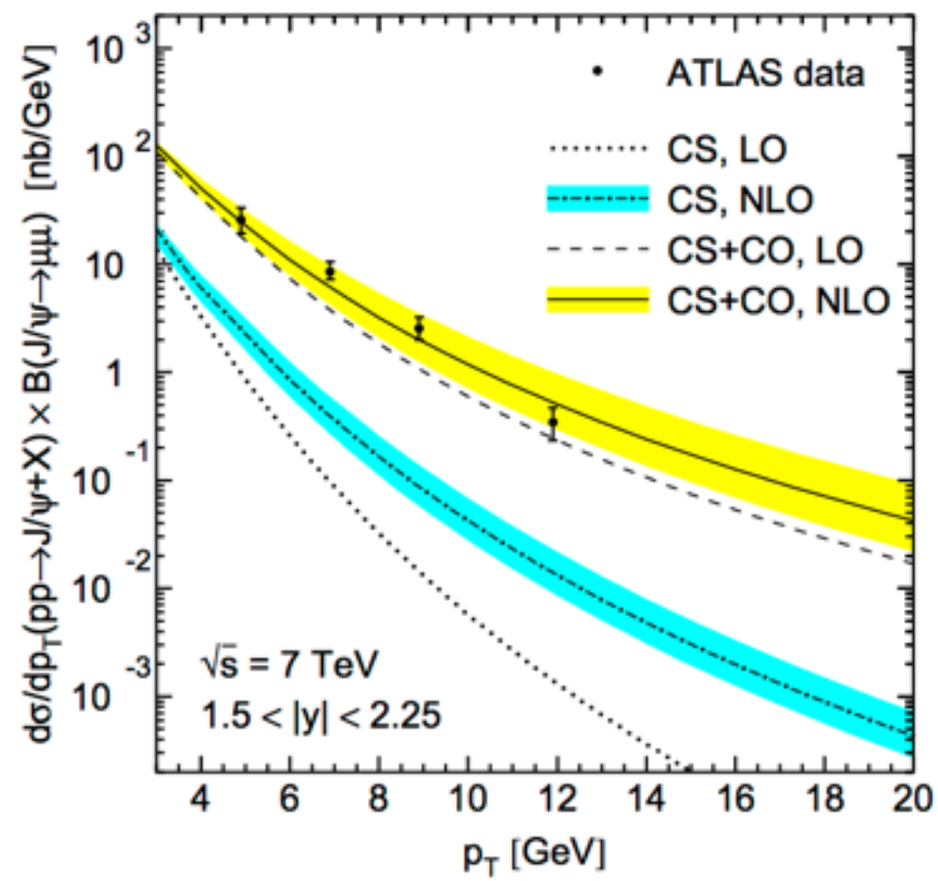
Thank you!

Backup Slides

Extract LDME's from World's Data

Need CS+CO at NLO to fit data from various experiments





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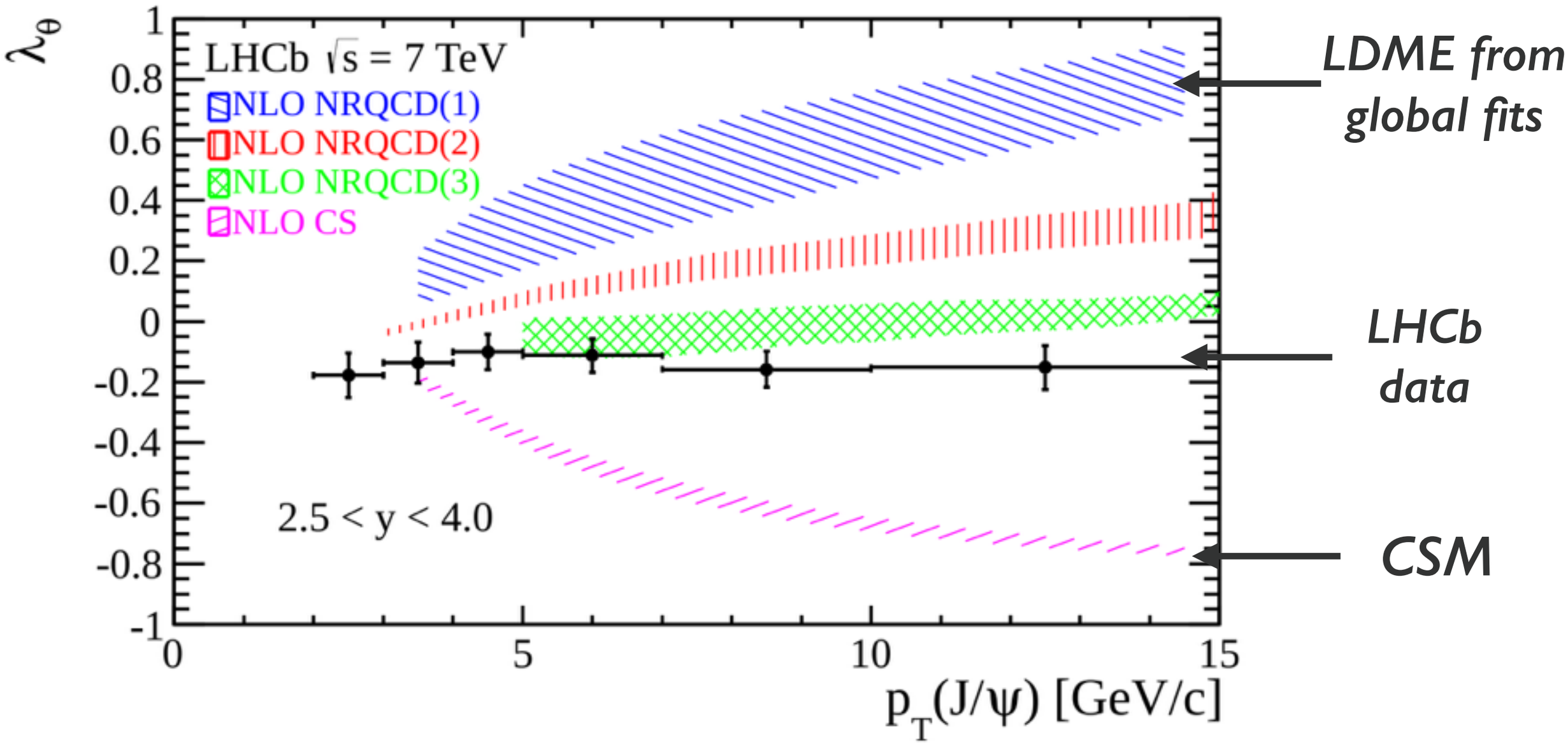
2 of 26 data sets shown

$$e^+e^-,\ \gamma\gamma,\ \gamma p,\ p\bar{p}, pp \to J/\psi + X$$

$$\chi^2 = 857/194 = 4.42$$

Buttenschon, Kniehl (2011), arXiv:1105.0820

Polarization Problem



λ_θ = +1 (trans.), 0 (unpol.), -1 (long.)

θ = J/ψ and μ+ momentum polar angle

Blue = No feed down, pT > 3 GeV; Buttenschon et. al (2012)

Red = Chi_cJ and Psi(2S) feed down, pT > 7 GeV; Gong et al. (2013)

Green = No feed down, pT > 7 GeV; Chao et. al (2012)

Magenta = Color singlet at NLO; Buttenschon et al (2012)

Apply to Heavy Quarkonium?

Non-relativistic QCD Factorization Formalism

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

Double expansion in α_s, v with $n - {}^{2S+1}L_J^{(1,8)}$

Color-singlet model $\langle \mathcal{O}^{J/\psi}({}^3S_1^{(1)})\rangle \sim v^3$

combine w/ fits or polarization problem

Color-octet mechanisms

$\langle \mathcal{O}^{J/\psi}({}^3S_1^{(8)})\rangle$, $\langle \mathcal{O}^{J/\psi}({}^1S_0^{(8)})\rangle$, $\langle \mathcal{O}^{J/\psi}({}^3P_J^{(8)})\rangle \sim v^7$

Bodwin, Braaten, Lepage

Resumming Logarithms

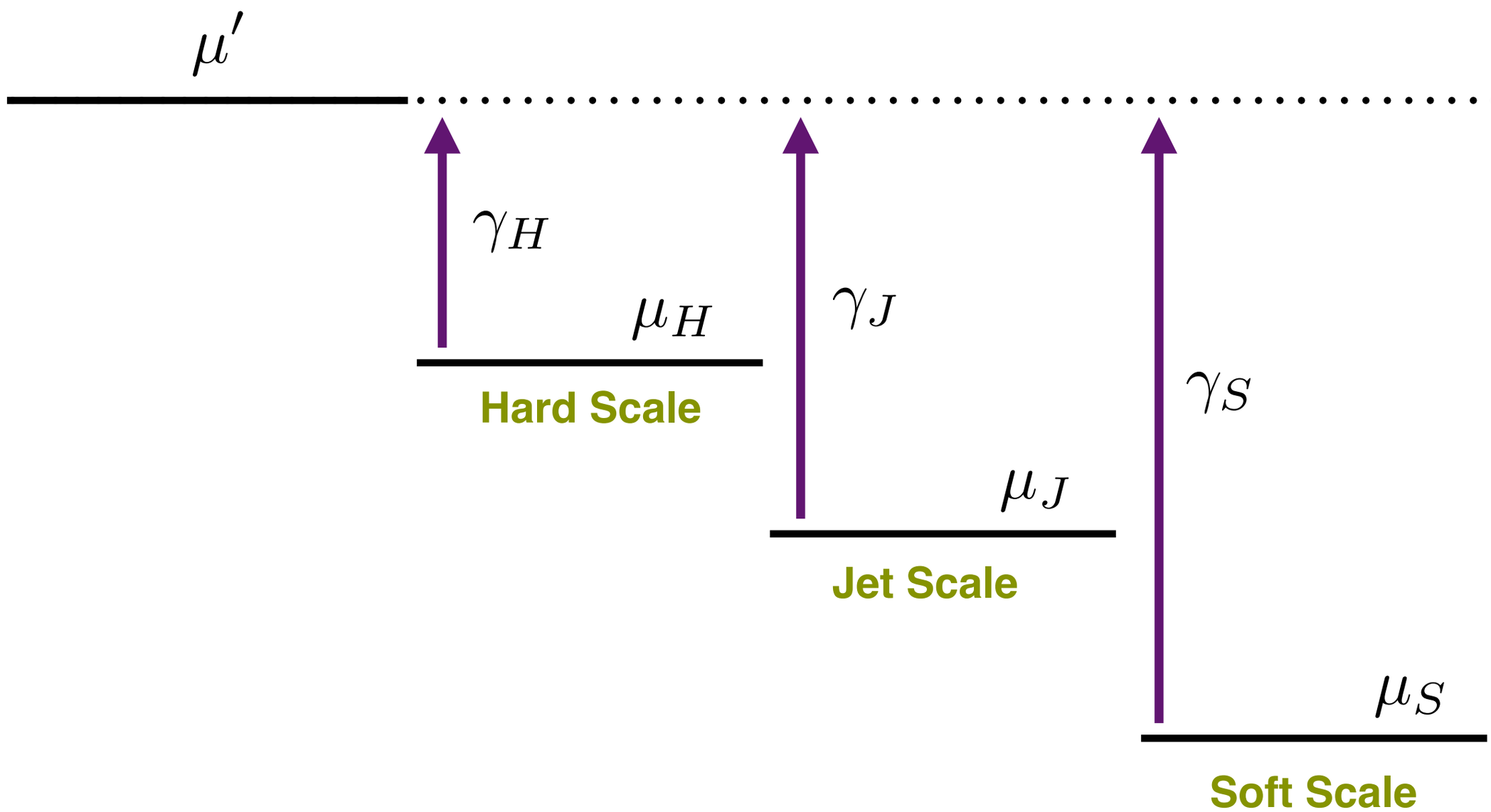
$$
\begin{aligned}
A = a &+ \alpha \left(b_1 + b_2 \log\left(\frac{\mu}{\mu_0}\right)\right) \\
&+ \alpha^2 \left(c_1 + c_2 \log\left(\frac{\mu}{\mu_0}\right) + c_3 \log^2\left(\frac{\mu}{\mu_0}\right)\right) \\
&+ \alpha^3 \left(d_1 + d_2 \log\left(\frac{\mu}{\mu_0}\right) + d_3 \log^2\left(\frac{\mu}{\mu_0}\right) + d_4 \log^3\left(\frac{\mu}{\mu_0}\right)\right) \\
&+ \alpha^4 \left(e_1 + e_2 \log\left(\frac{\mu}{\mu_0}\right) + e_3 \log^2\left(\frac{\mu}{\mu_0}\right) + e_4 \log^3\left(\frac{\mu}{\mu_0}\right) + e_5 \log^4\left(\frac{\mu}{\mu_0}\right)\right) \\
&+ \ldots
\end{aligned}
$$

N³LL **NNLL** **NLL** **LL**

$$N^{n-m}LL \sim \sum \alpha_s^n \log^m\left(\frac{\mu}{\mu_0}\right)$$

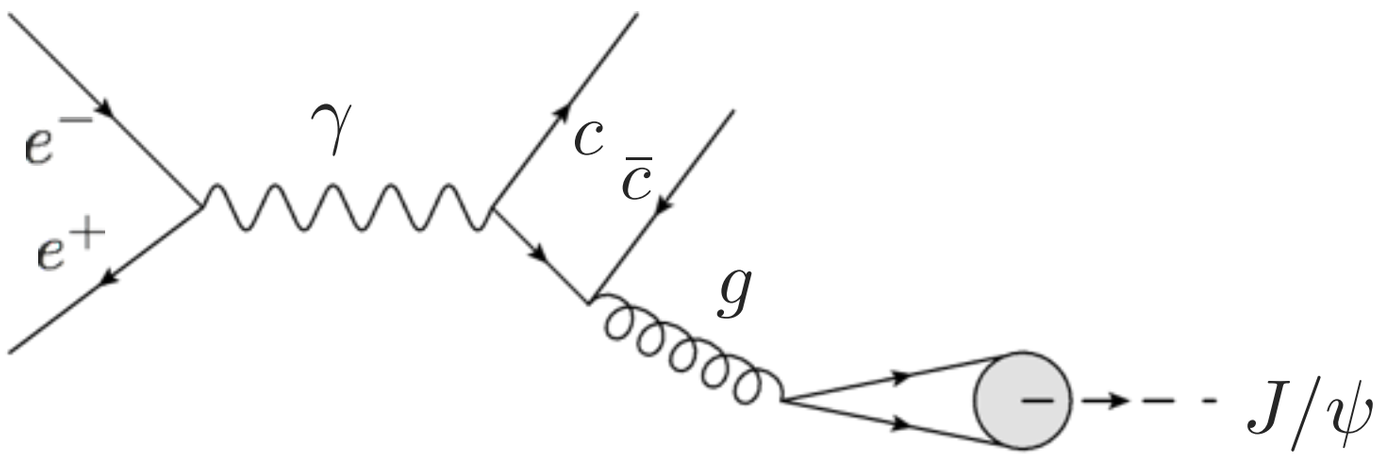
Resummation of Logarithms

Evolve each function to common scale using RG

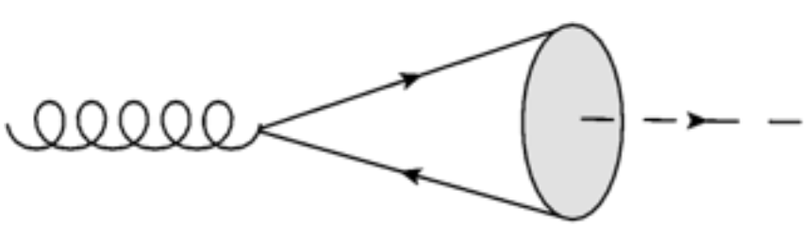


J/ψ Production Mechanisms

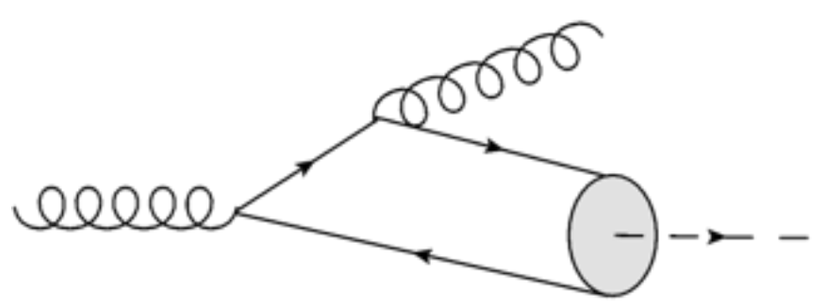
Diagrams for each singlet/octet channels



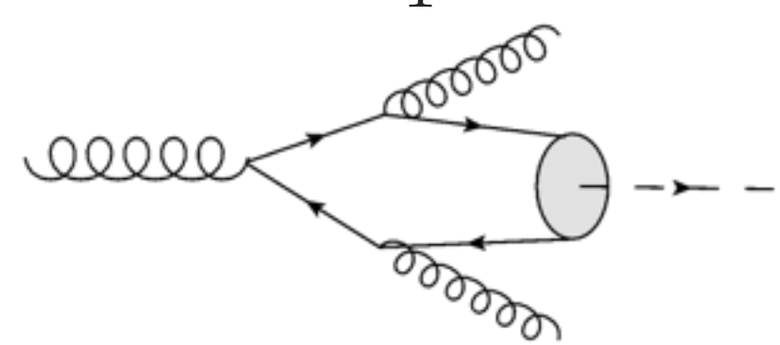
$^3S_1^{(8)}$



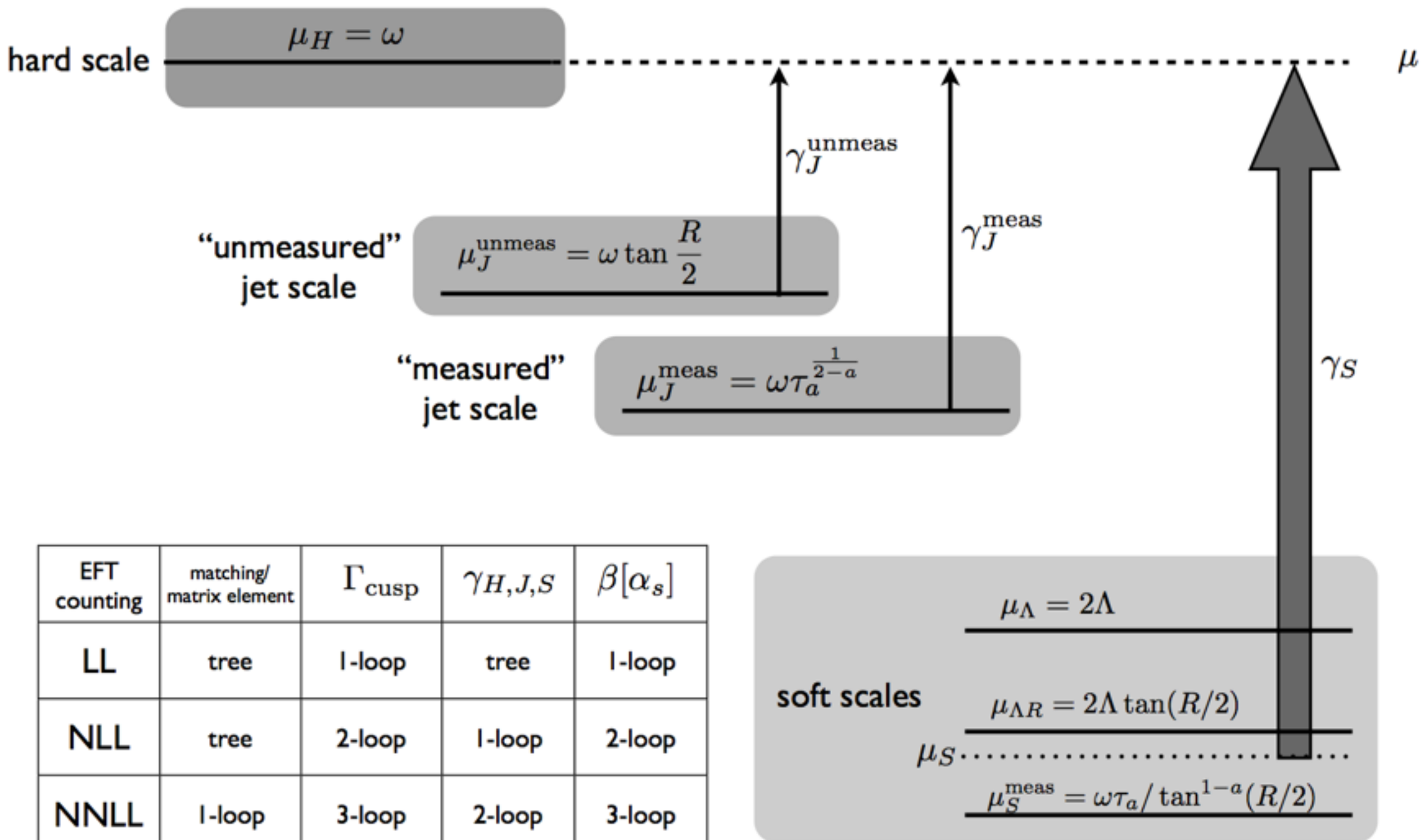
$^1S_0^{(8)}$ $^3P_J^{(8)}$



$^3S_1^{(1)}$



Canonical Scales in Cross-Section

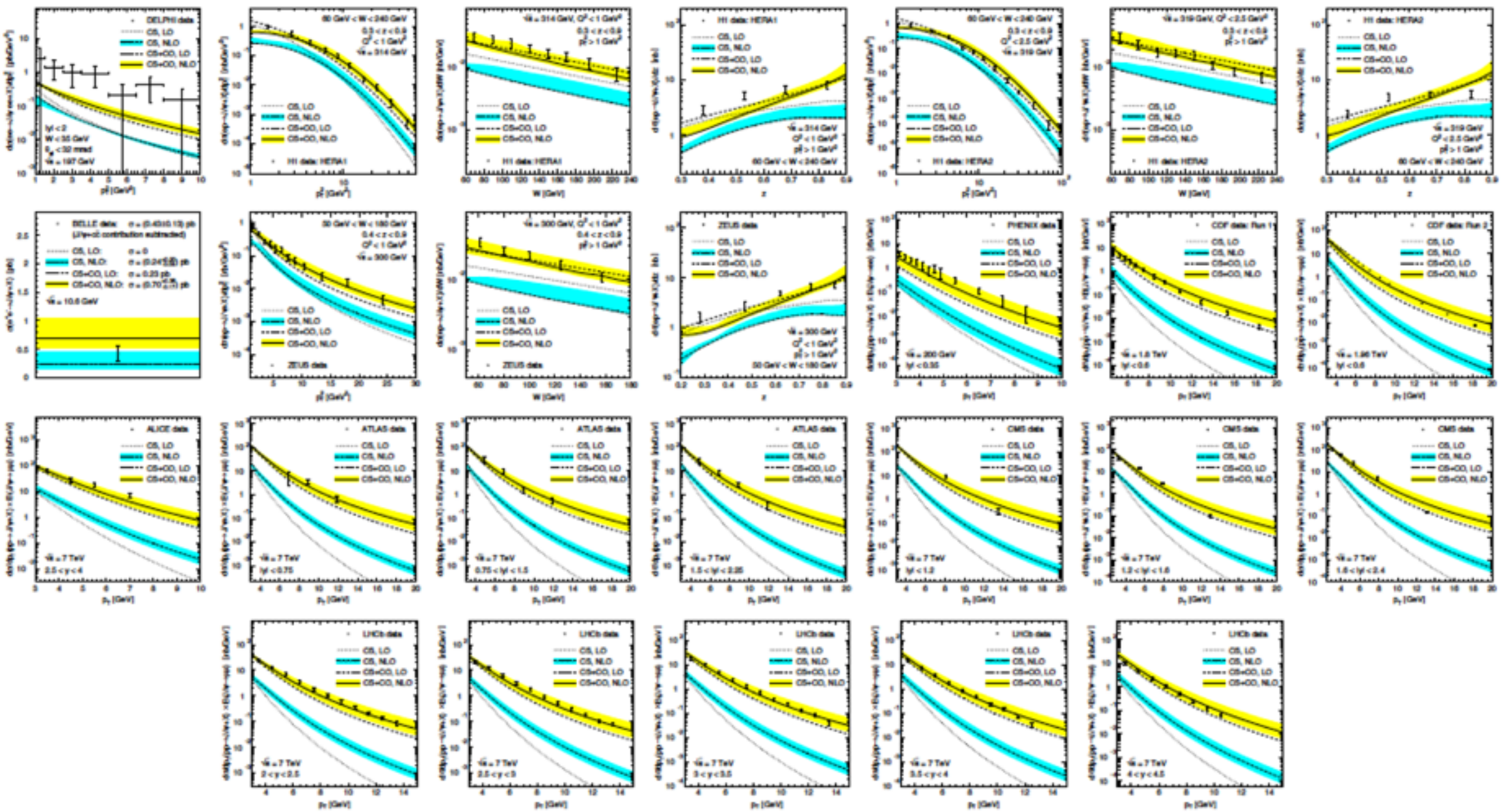


EFT counting	matching/ matrix element	$\Gamma_{\rm cusp}$	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop

Global Fits to World's Data

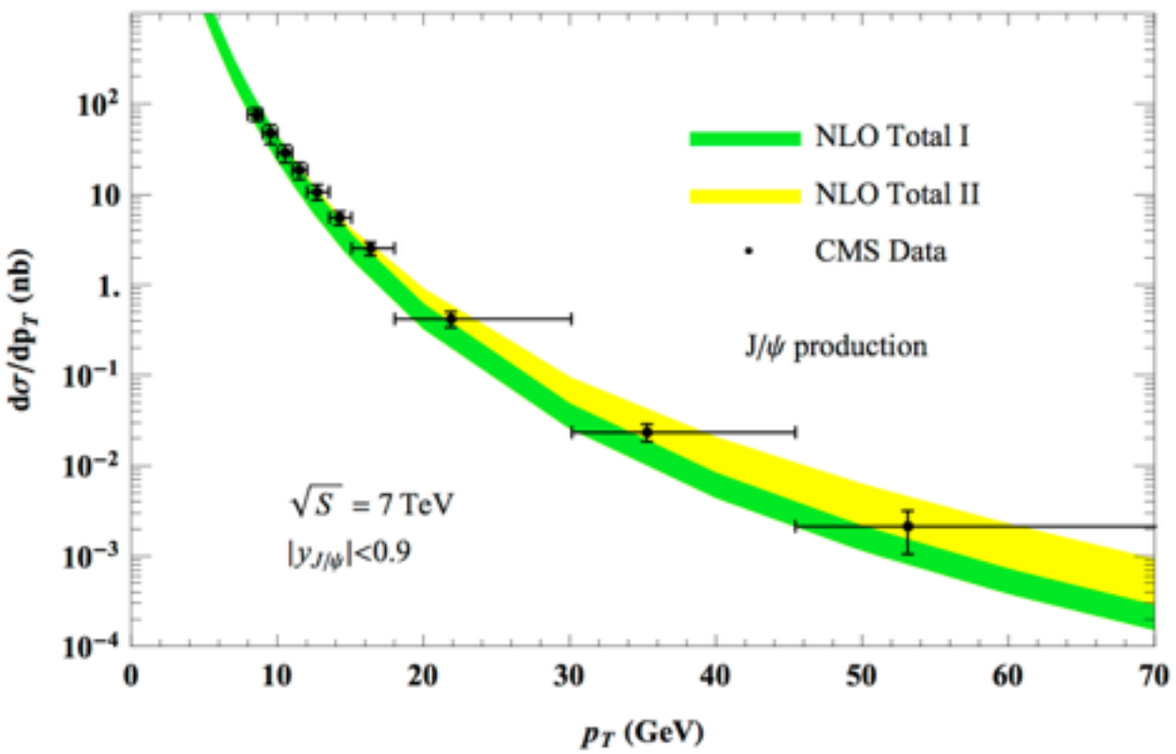
Buttenschon, Kniehl (2011), arXiv:1105.0820

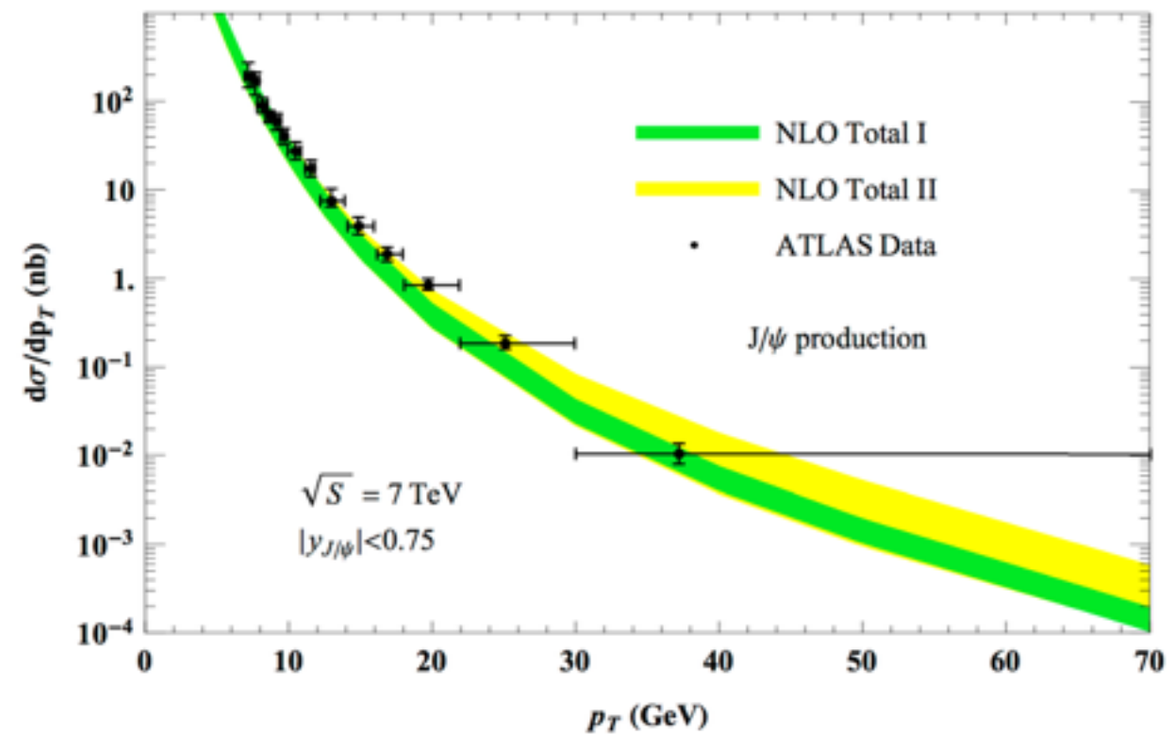
Fit done on 194 data points, 26 data sets

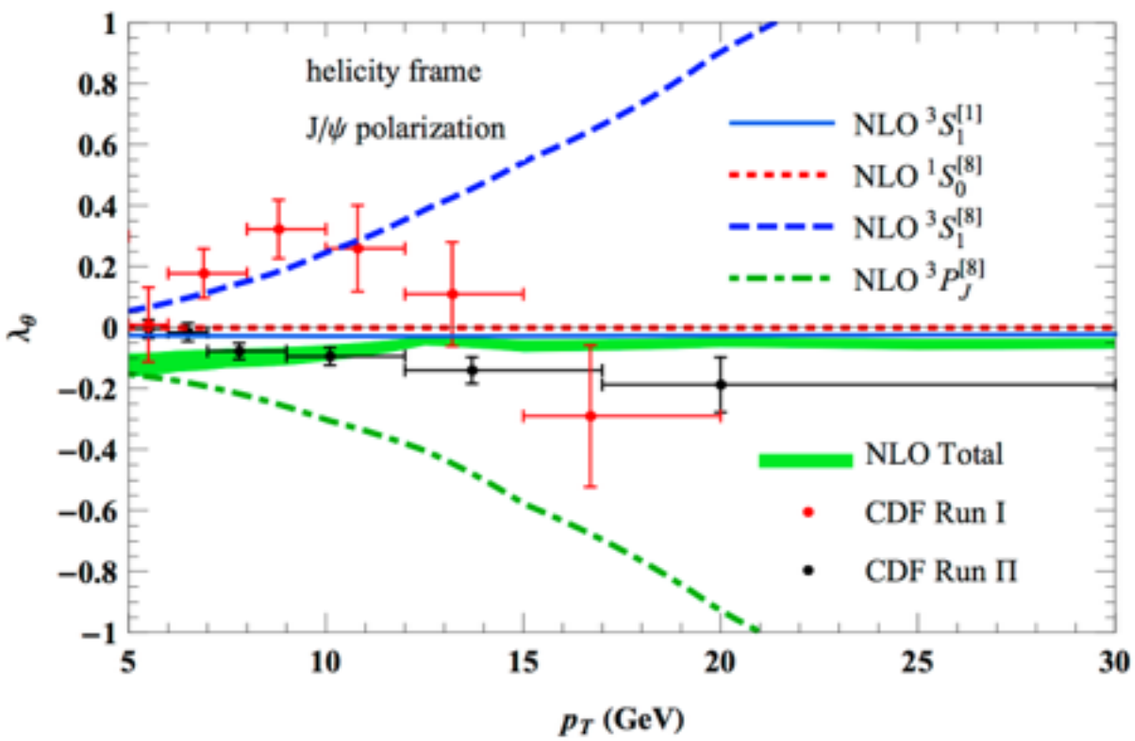


Attempts to Fix Polarization Problem

Simultaneous NLO fit to CMS, ATLAS high p_t production, polarization







Chao, et al. (2012),arXiv:1201.2675

$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2}GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2}GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle/m_c^2$ 10^{-2}GeV^3
1.16	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
1.16	0	1.4	2.4
1.16	11	0	0

Inconsistent with global fits!

Jet Cross-Sections with Angularities

Jet substructure technology ⟶ New observables

Factorization Theorem (SCET)

$$d\sigma \sim H \times J^{(1)} \otimes J^{(2)} \otimes J^{(3)} \otimes S$$

- Hard function $H(\mu)$
- Jet Functions $J^{(1)}(\mu)$
- Soft Function $S(\mu)$

Angularities

$$\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2}$$

- Sum over jet particles i
- Good analytical handle on them
- $\omega = \sum_i p_i^- \approx 2E_{jet}$

IR Safety $-\infty < a < 2$

Factorizability $-\infty < a < 1$

S.D.Ellis, et. al (2010) , arXiv:1001.0014

Deriving the Cross Section

Measure Hadron z and Jet τ

$$\frac{1}{\sigma_0}\frac{d\sigma^{(i)}}{d\tau_a dz} = H(\mu)S^{unmeas}(\mu)J^{(1)}_{\omega_1}(\mu)\sum_j\left[\left(S^{meas}(\mu)\otimes\frac{\mathcal{J}_{ij}(\mu)}{2(2\pi)^3}\right)(\tau_a)\bullet D^H_j(\mu)\right](z)$$

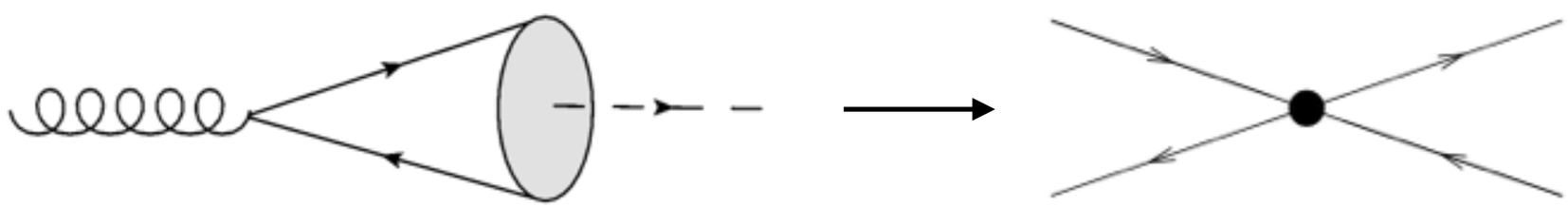
Convolutions of the form

$$[f\otimes g](\tau_a)\equiv\int d\tau' f(\tau-\tau')g(\tau') \qquad [f\bullet g](z)\equiv\int_z^1\frac{dx}{x}f(x)g(z/x)$$

$$\bar{\omega}_H = \prod_{i=1}^N \omega_i^{\mathbf{T}_i^2/\mathbf{T}^2} \qquad \mathbf{T}^2 = \sum_{i=1}^N \mathbf{T}_i^2$$

NRQCD Fragmentation Functions

Matching QCD and NRQCD



Perturbatively Calculable Frag. Functions

$$D_{g\to J/\psi}^{{}^3S_1^{(8)}}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{24m_c^3}\langle\mathcal{O}^{J/\psi}({}^3S_1^{(8)})\rangle\delta(1-z)$$

Braaten, Chen, hep-ph/9610401

Braaten, Chen, hep-ph/9604237

Braaten, Yuan, hep-ph/9302307

Definitions of Operators

QCD Fragmentation Function

$$D_q^h(z) = z \int \frac{dx^+}{4\pi} e^{ik^- x^+/2} \frac{1}{4N_c} \mathrm{Tr} \sum_X \langle 0| \not{\bar{n}} \psi(x^+, 0, 0_\perp) |Xh\rangle \langle Xh| \bar{\psi}(0) |0\rangle \big|_{p_h^\perp = 0}$$

SCET Fragmentation Function

$$D_q^h(\frac{p_h^-}{\omega}, \mu) = \pi\omega \int dp_h^+ \frac{1}{4N_c} \mathrm{Tr} \sum_X \not{\bar{n}} \langle 0| \delta_{\omega,\bar{\mathcal{P}}} \delta_{0,\mathcal{P}_\perp} \chi_n(0) |Xh\rangle \langle Xh| \bar{\chi_n}(0) |0\rangle$$

SCET Jet Function

$$J(p^\mu) = \frac{1}{8\pi N(\bar{n}\cdot p)} \sum_X \int d^4x e^{ipx} \mathrm{Tr} \left[\langle \Omega| \bar{\chi}_n(x) |X_n\rangle \langle X_n| \not{\bar{n}} \chi_n(0) |\Omega\rangle \right]$$

SCET Fragmenting Jet Function

$$\mathcal{G}^h_{q,\mathrm{bare}}(s,z) = \int \mathrm{d}^4y\, e^{\mathrm{i}k^+y^-/2} \int \mathrm{d}p_h^+ \sum_X \frac{1}{4N_c} \mathrm{tr} \left[\frac{\not{\bar{n}}}{2} \langle 0| [\delta_{\omega,\overline{\mathcal{P}}} \delta_{0,\mathcal{P}_\perp} \chi_n(y)] |Xh\rangle \langle Xh| \bar{\chi}_n(0) |0\rangle \right]$$