Large Angle Energy Flow in Medium Modified Jets

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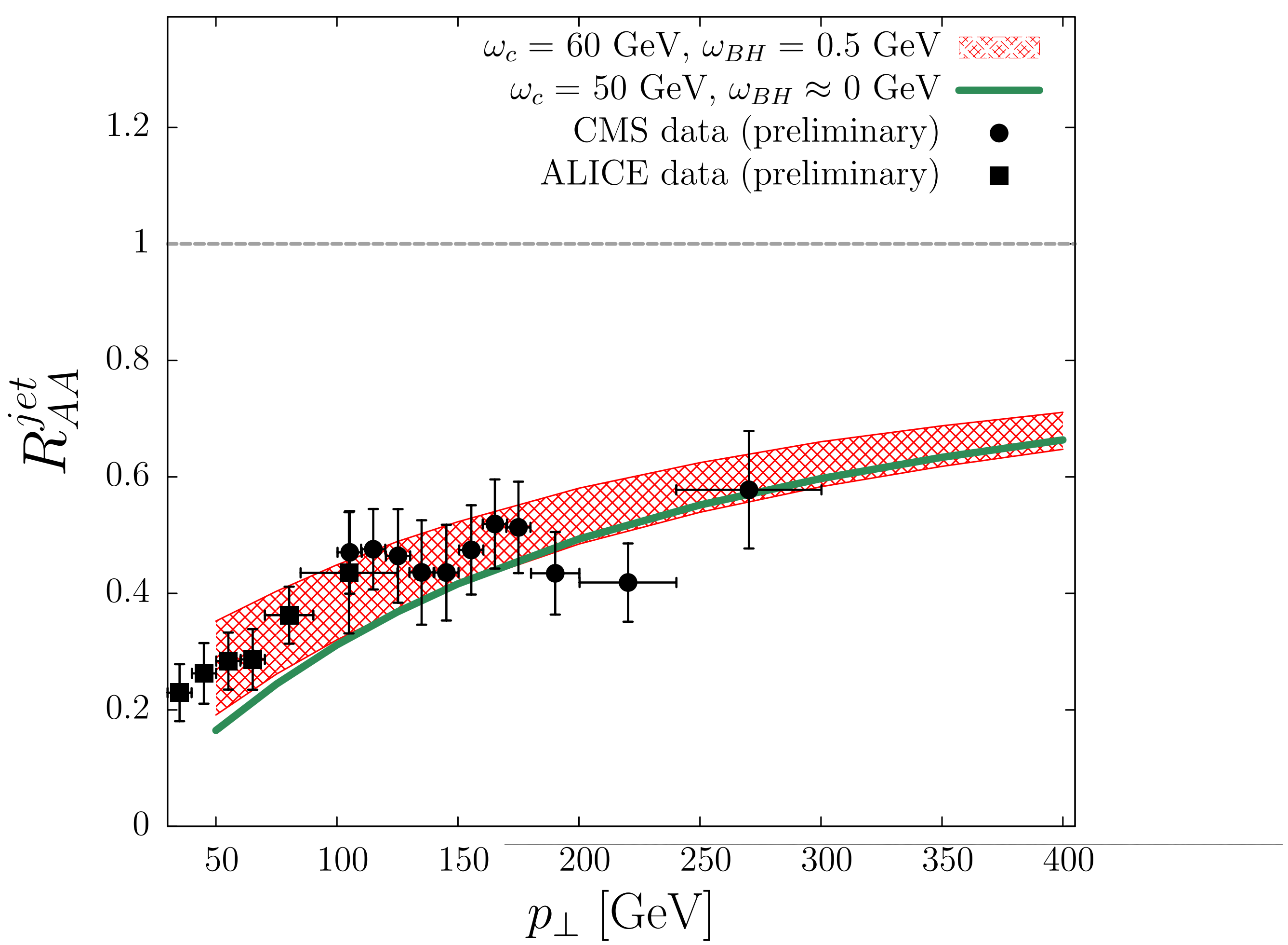
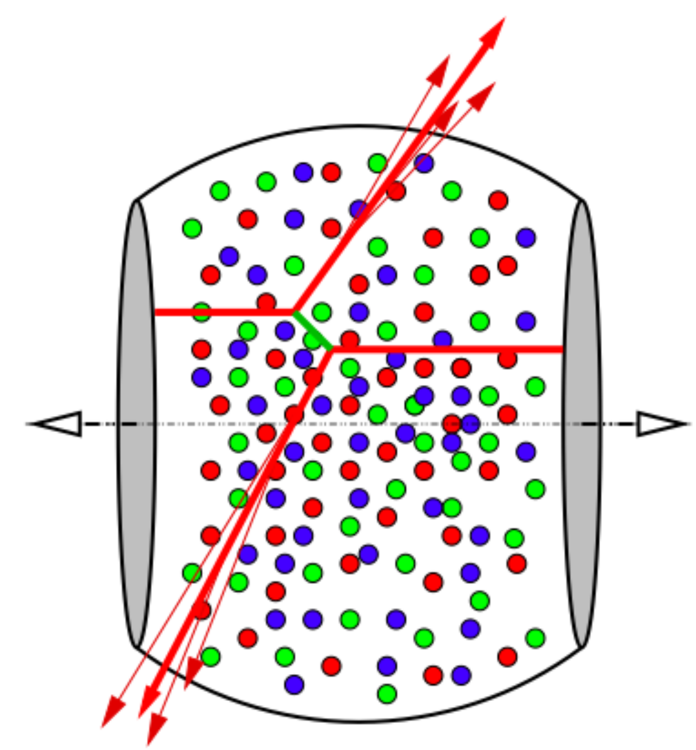
Phys.Rev.Lett. 114 (2015) 22 arXiv:1407.0326
Nucl.Phys. A940 (2015) 67 arXiv:1409.6202

Jet Nuclear Modification Factor

Jets or high pt partons lose energy mostly by radiating gluons at large angles: Jet in Pb-Pb collisions are strongly suppressed compared to proton-proton collisions

$$R_{AA} \equiv \frac{1}{N_{coll}} \frac{dN_{AA}}{dN_{pp}}$$

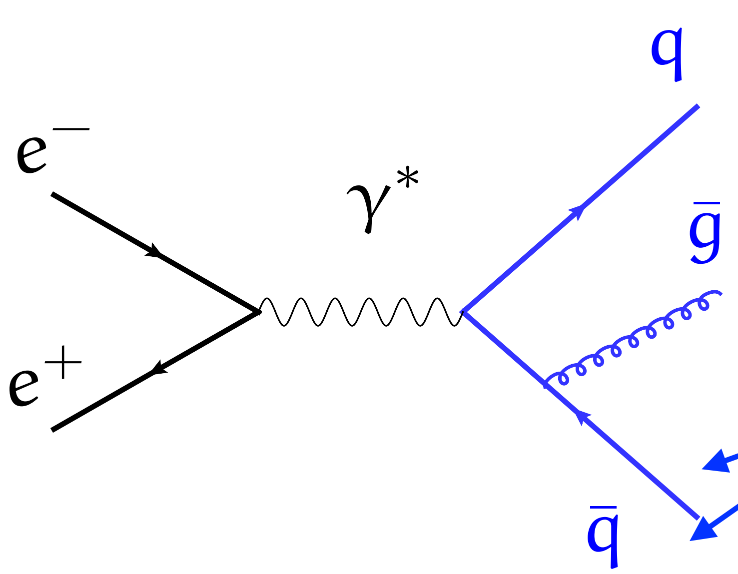


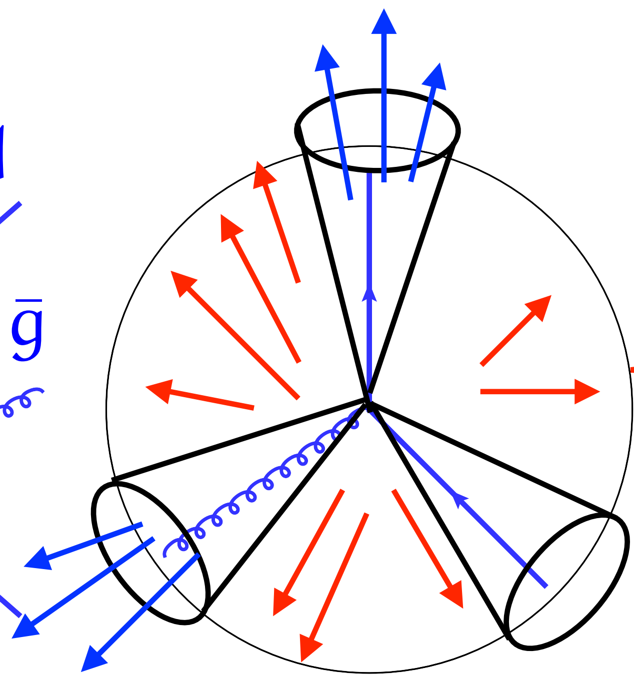


Jets in e^+e^-

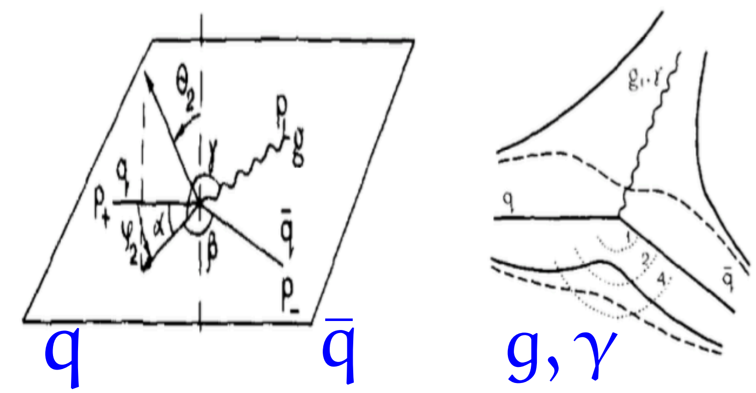
[Azimov, Dokshitzer, Khoze, Troyan (1985)]

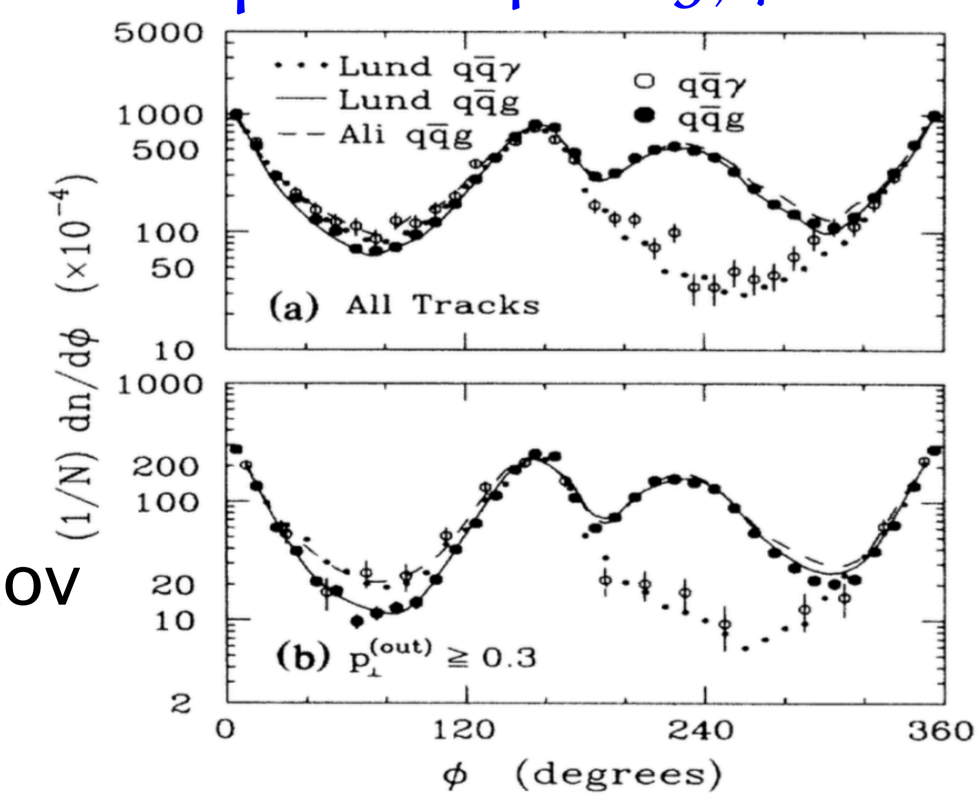
3-jet events





Interjet hadronic activity:
Dragg effect: “stringy” fragmentation from QCD





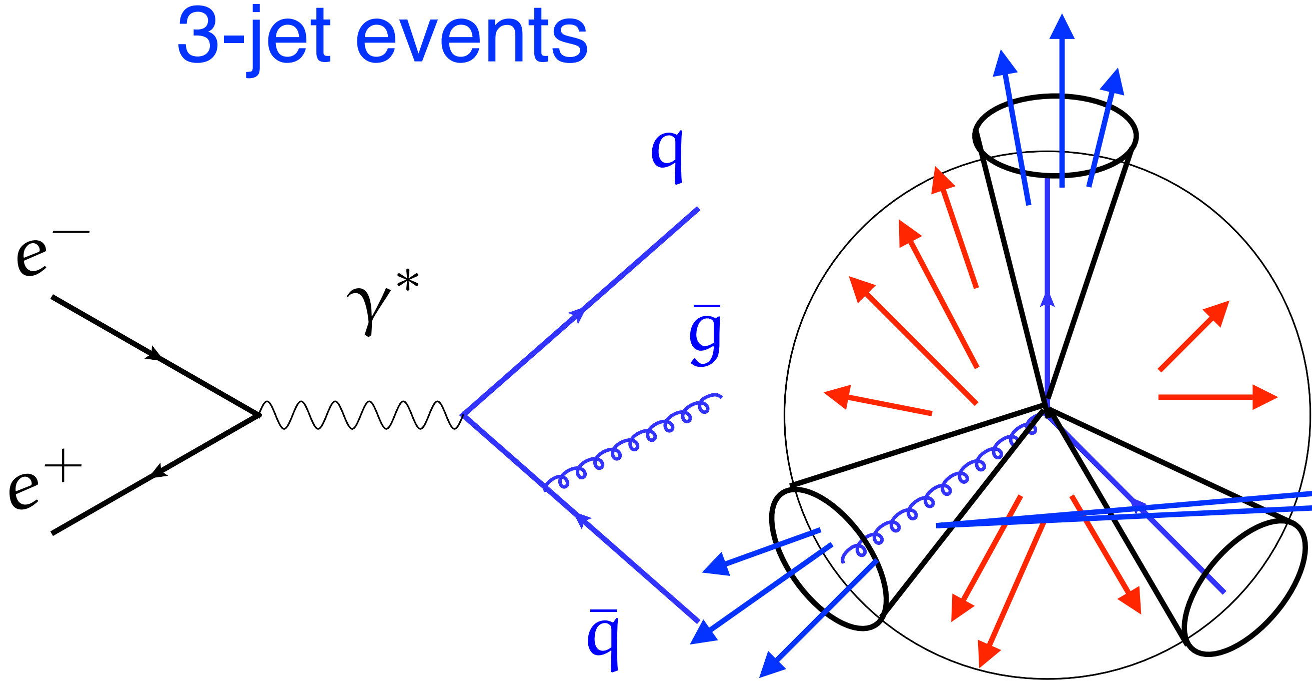
- Large angle soft gluon radiation: sensitive to total charge of the jet
- Coherence: destructive interferences at large angles
- Out-Of-Cone energy flow: Banfi-Marchesini- Smye Eq. (global logs, Sudakov suppression). Intrajet structure: Angular Ordering, MLLA Eq.

[MARK II (1986)]

Jets in e^+e^-

3-jet events

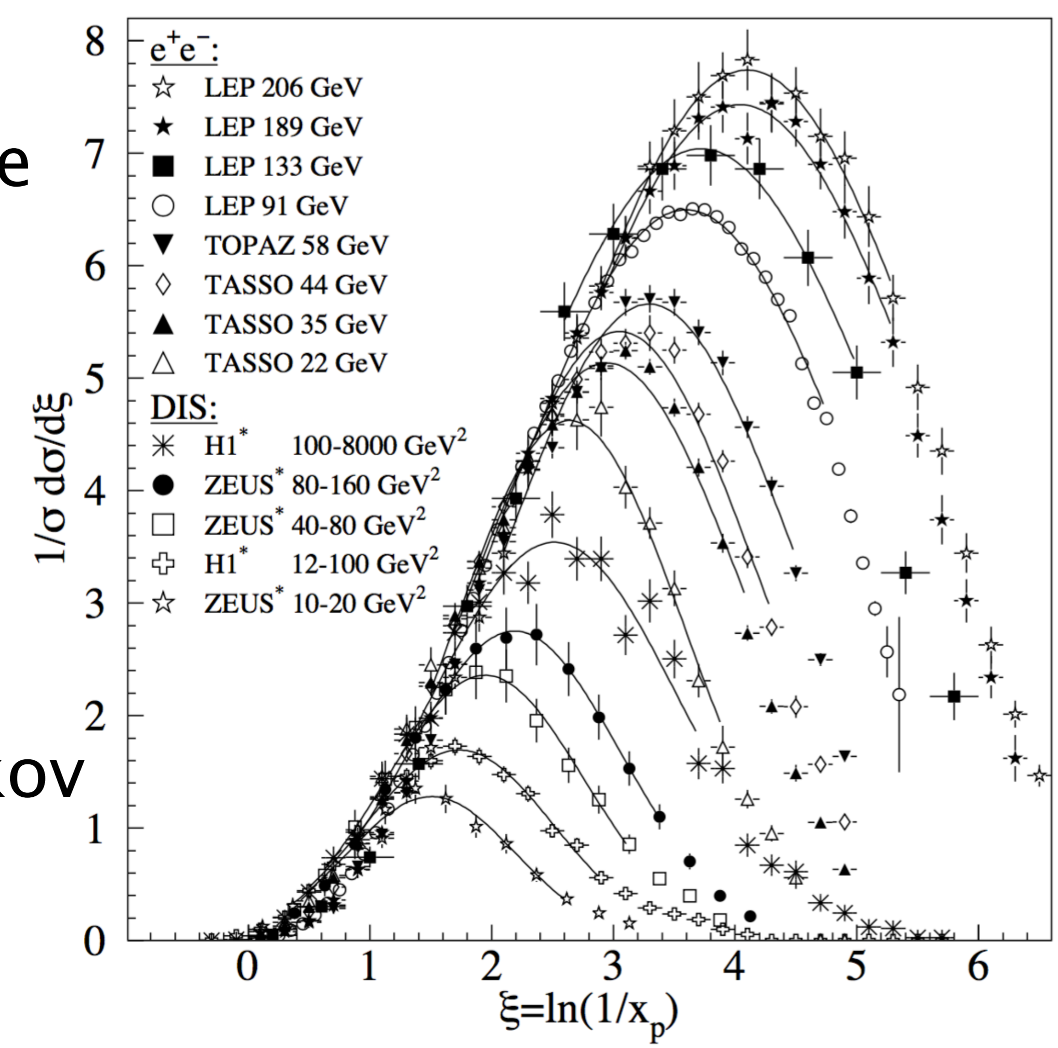
[Azimov, Dokshitzer, Khoze, Troyan (1985)]



Fragmentation function:

Intrajet distribution

- Large angle soft gluon radiation: sensitive to total charge of the jet
- Coherence: destructive interferences at large angles
- Out-Of-Cone energy flow: Banfi-Marchesini- Smye Eq. (global logs, Sudakov suppression). Intrajet structure: Angular Ordering, MLLA Eq.



Jets in the QGP

- Color Decoherence: Coherence suppressed by in-medium color randomization

[MT, Salgado, Tywoniuk (2010-2011)
Casalderrey-Solana, Iancu (2011)]

- Additional component: large angle medium-induced gluon cascade (This talk)

- No logarithmic enhancements for the medium-induced part but length enhancement

New (obvious) time scale: the medium length L

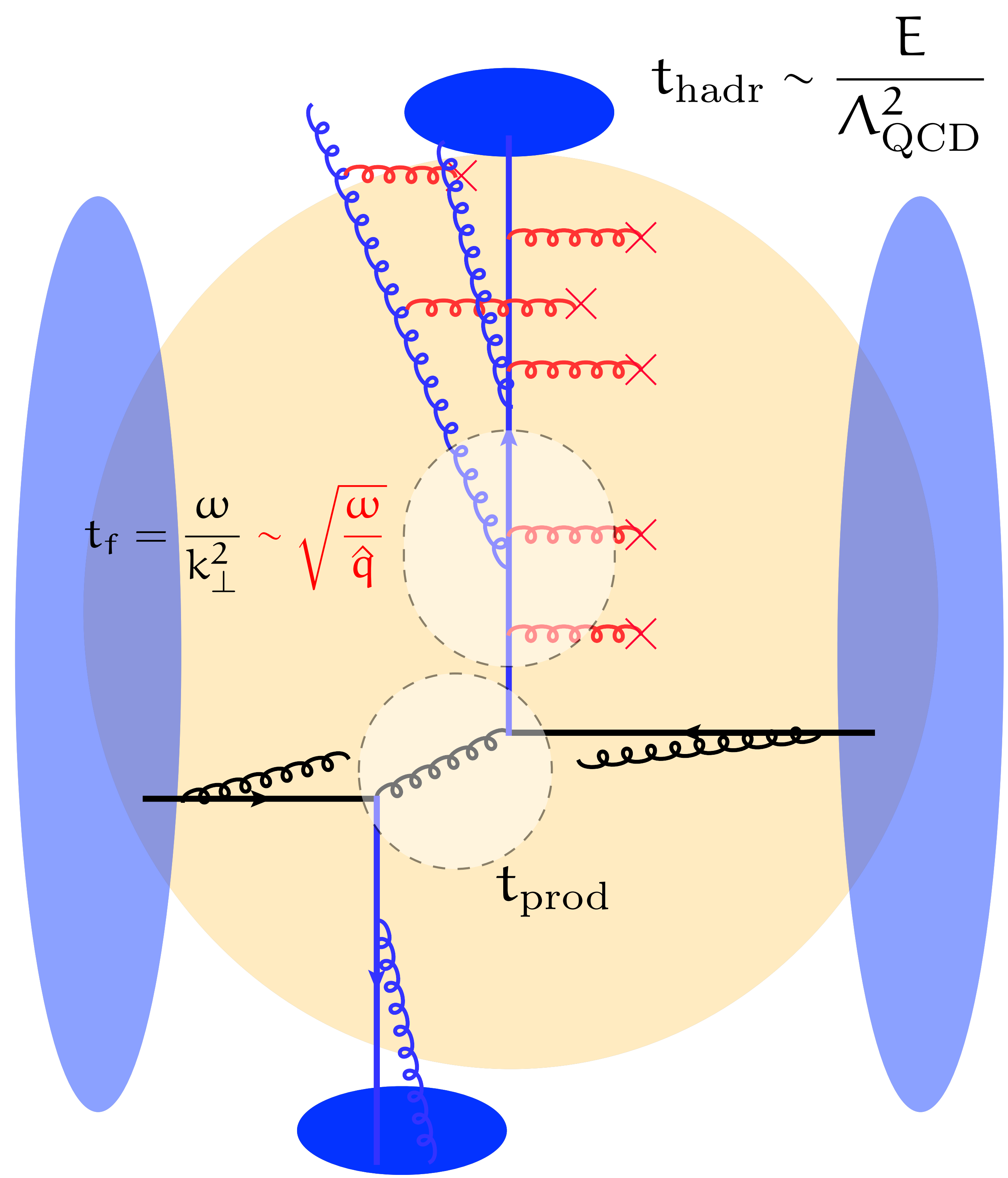
- In the presence of QCD medium:

  - Final state rescattering

$$\langle p_\perp^2 \rangle \equiv \hat{q}\, L$$

  - Coherent medium-induced soft gluon radiation: no logarithm enhancement but length enhancement

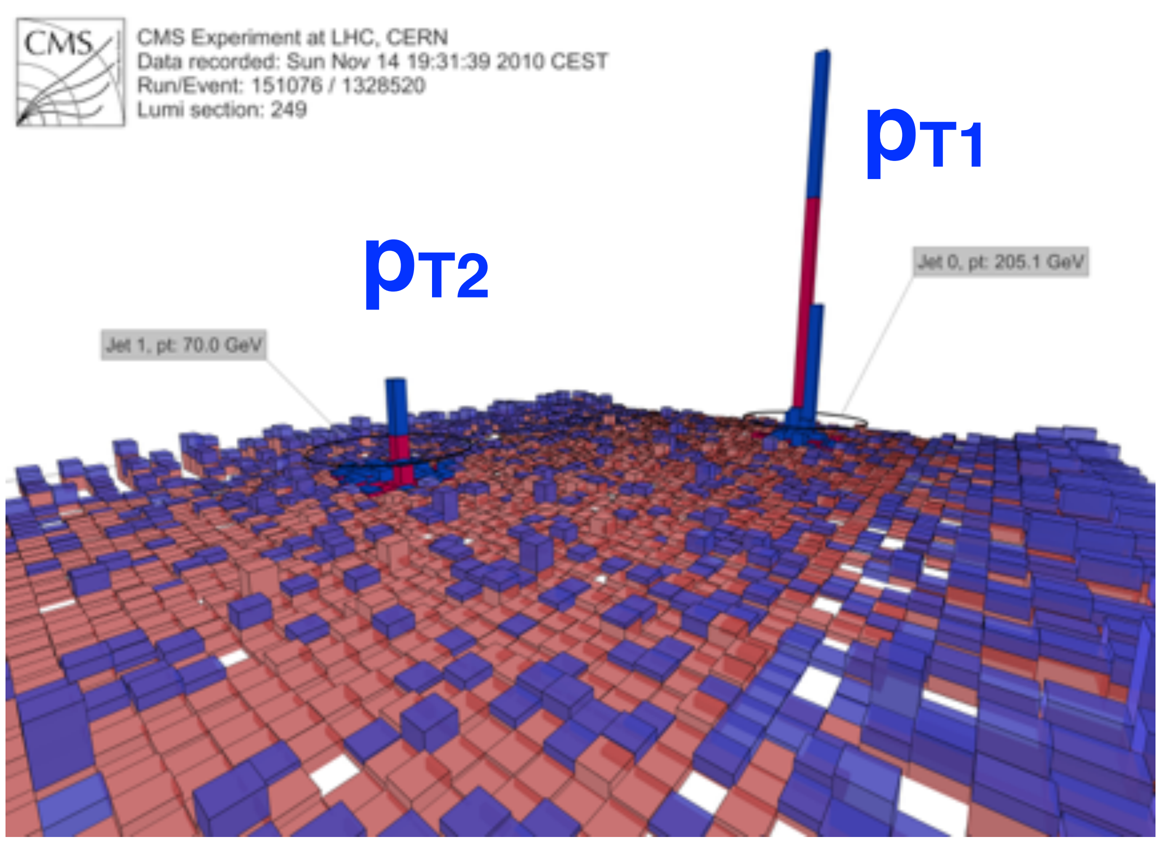
$$\omega \frac{dN}{d\omega} = \alpha_s \frac{L}{t_f} \equiv \alpha_s N_{eff}$$

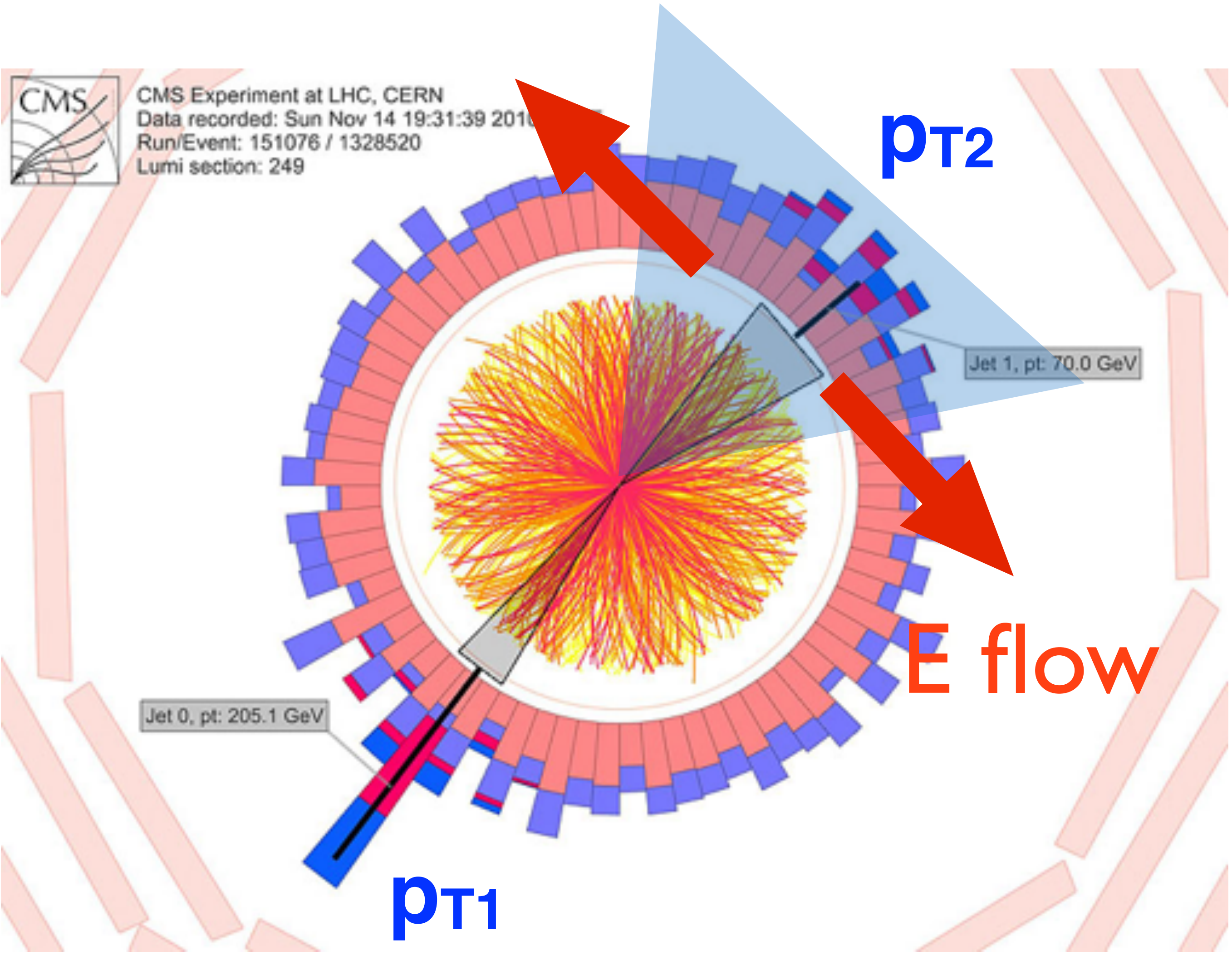


[Guylassy, Wang, Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov, Vitev, Levai, Wiedemann, Arnold ,Moore, Yaffe (1992-2000)]

Missing energy in asymmetric dijets

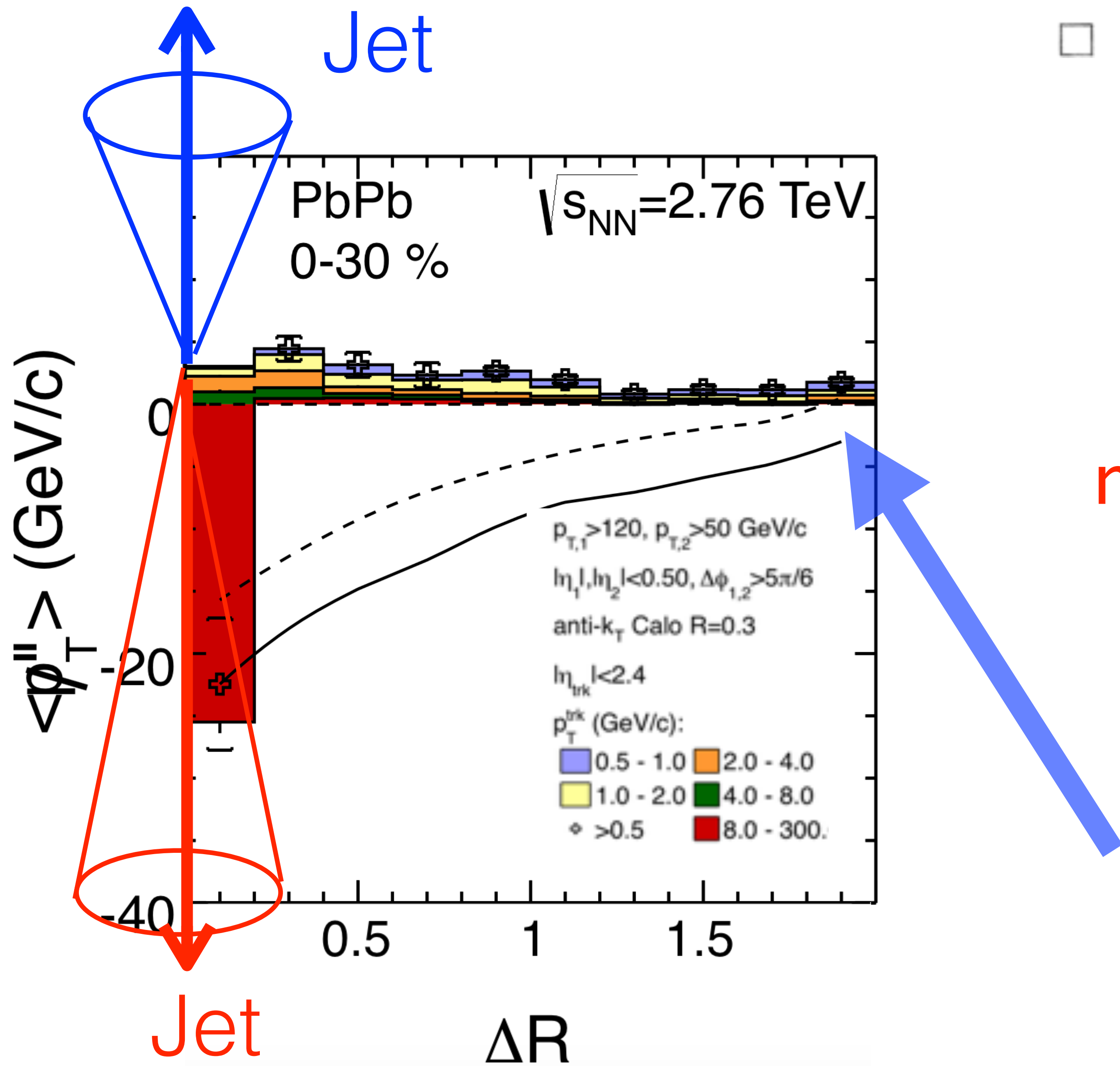
- Selection of dijet events with large momentum Imbalance $p_{T1}>120$ GeV and $p_{T2}>50$ GeV





CMS: energy is lost in soft particles at large angles

Out-of-cone energy distribution



- Recovering the missing energy (angular distribution of particles away from the jet axis)

momentum imbalance:

Projection of particle p_T's along the jet axis.

$$\langle p^{||} \rangle = (k_1 - k_2)^{||}$$

Vanishes due to mom. conservation when ΔR=π

CMS (2014–2015): energy is lost in soft particles at large angles

Coupling to the medium

- The jet couples to the medium via (local) transport coefficient

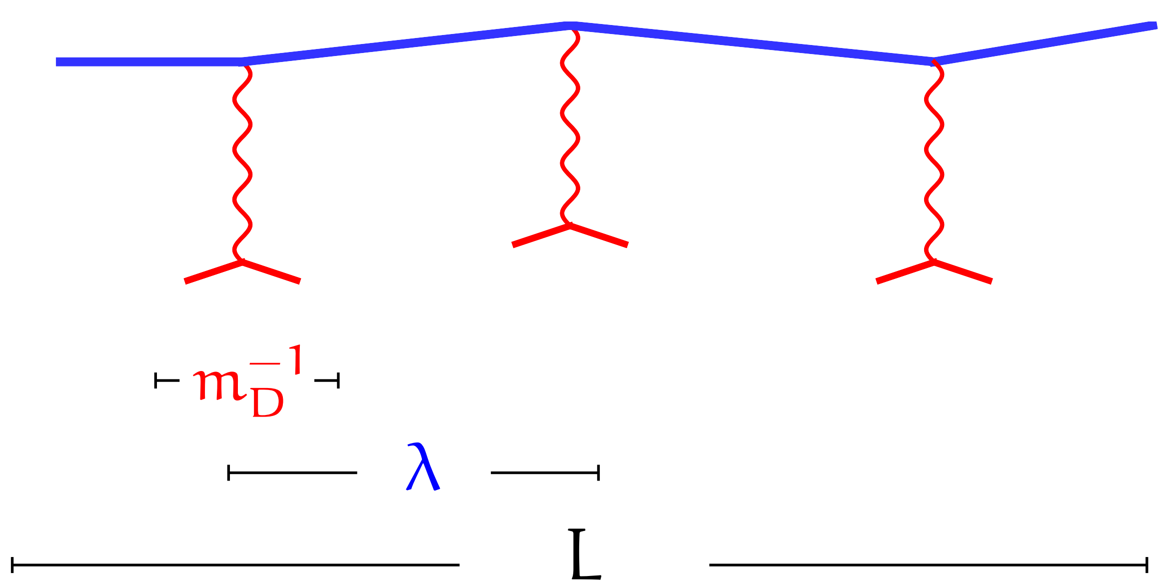
$$\hat{q} \equiv \frac{m_D^2}{\lambda} \sim \frac{(\text{Debye mass})^2}{\text{mean free path}}$$

pt-broadening $\langle k_\perp^2 \rangle \sim \hat{q}\, L$

- Weak coupling:
  Independent multiple scattering approximation

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000)]

correlation length « mean-free-path « L

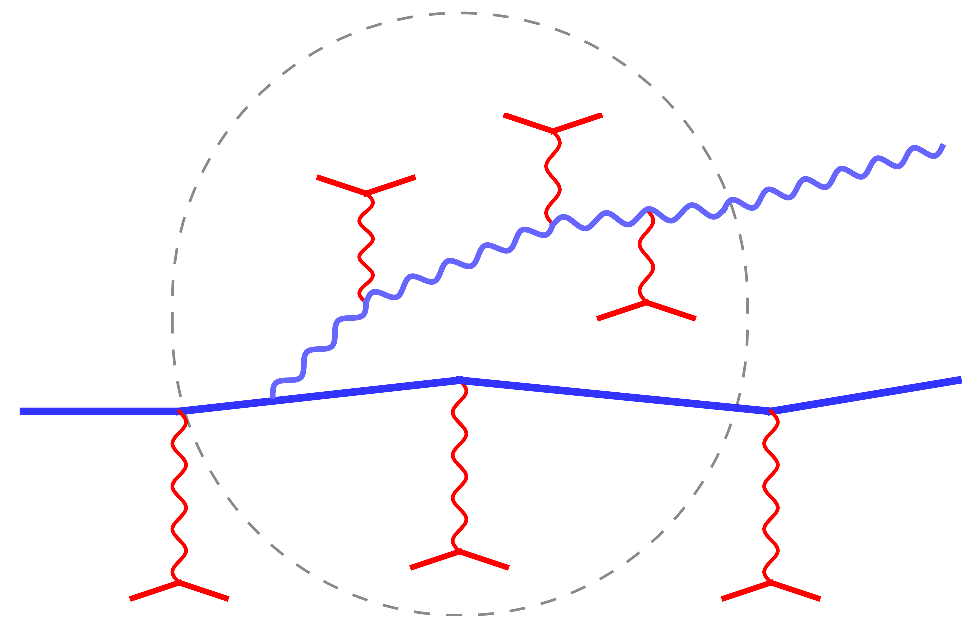


- Formally: Wilson lines

$$U(\boldsymbol{x}) \equiv \mathcal{P} \exp\left(ig \int_0^L dx^+ A^-(\boldsymbol{x}, x^+) \right)$$

In-medium radiation mechanism

- Radiation triggered by multiple scatterings



- Landau-Pomeranchuk-Migdal suppression (coherent radiation)

$$\omega \frac{dN}{d\omega} = \alpha_s \frac{L}{t_f} \equiv \alpha_s N_{eff}$$

$$t_f = \frac{\omega}{k_\perp^2} \sim \sqrt{\frac{\omega}{\hat{q}}}$$

formation time

- Maximum suppression when $t_f \gtrsim L \quad \Rightarrow \quad \omega > \omega_c = \hat{q} L^2$

- Minimum radiation angle $\theta > \theta_c \equiv 1/\sqrt{\hat{q} L^3}$

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]
[Gyulassy, Levai, Litev (2001) Wiedemann (2001) Arnold, Moore,Yaffe (2002)]

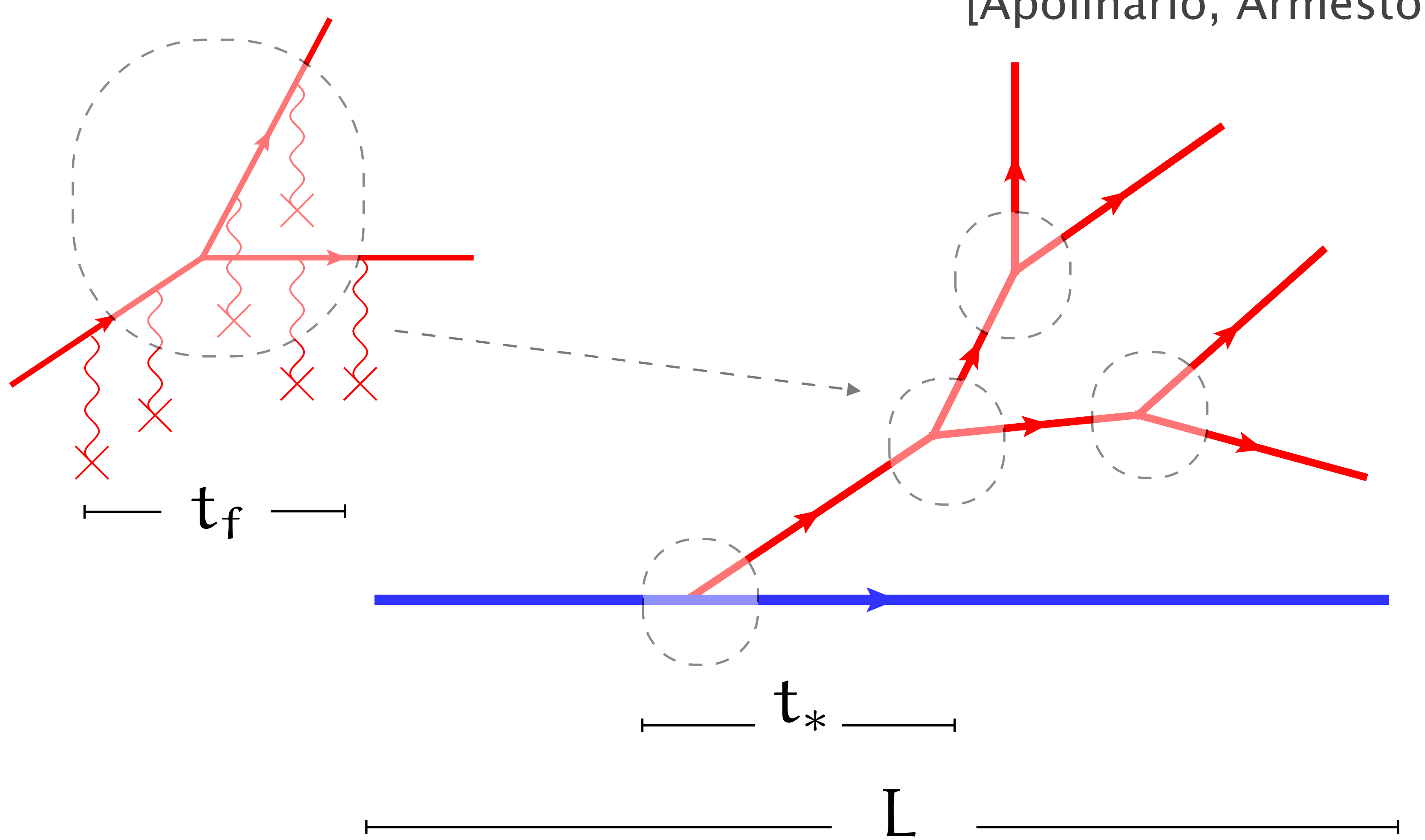
Probabilistic picture

- Multiple (independent) branchings regime:

$$t_f \ll t_* \ll L$$

- Incoherent branchings: randomization of color due to rescatterings

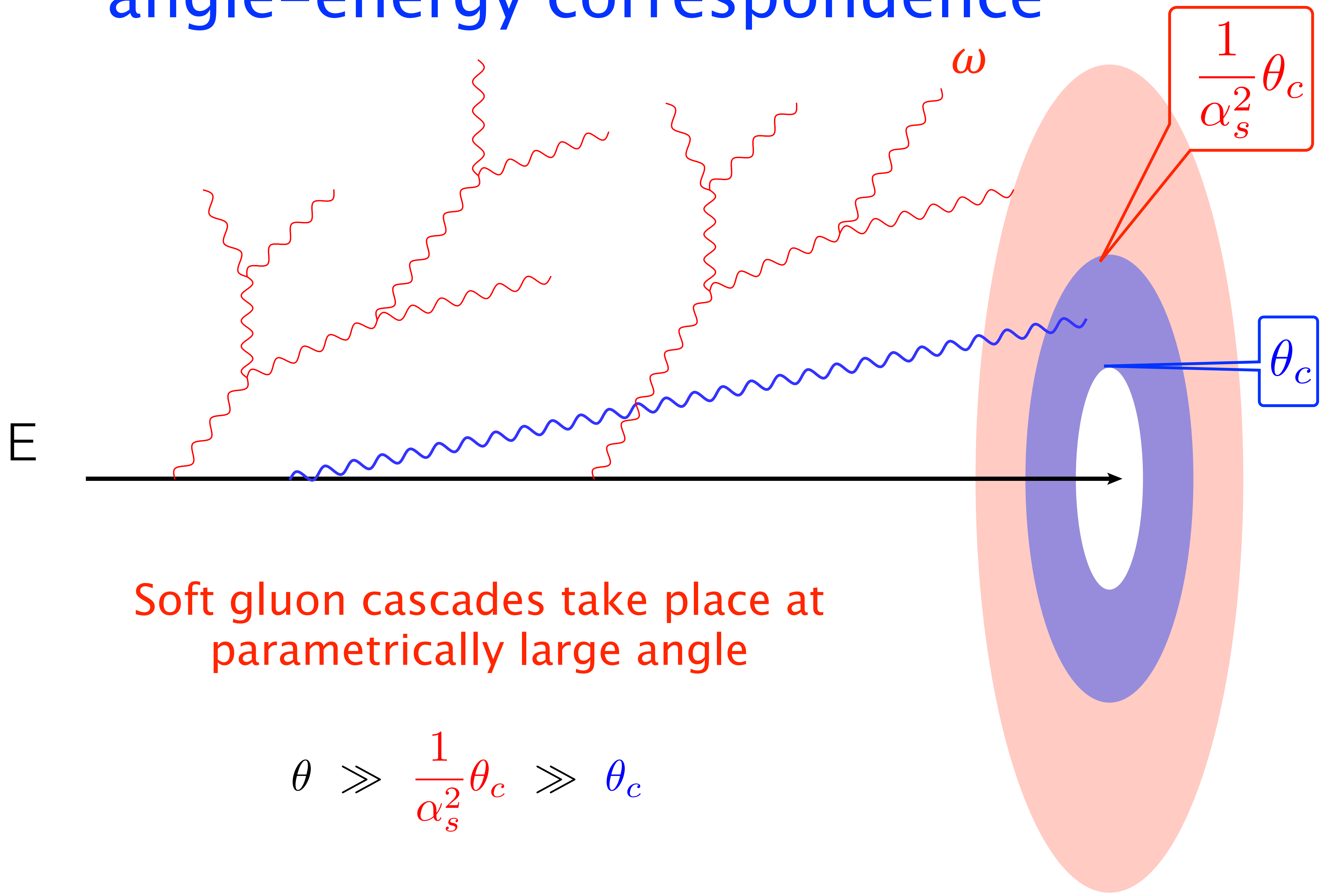
[Blaizot, Dominguez, Iancu, MT (2013-2014)]
[Apolinário, Armesto, Milhano, Salgado (2014)]



effective inelastic mean free path

$$t_*(\omega) \sim \frac{1}{\alpha_s} t_f(\omega)$$

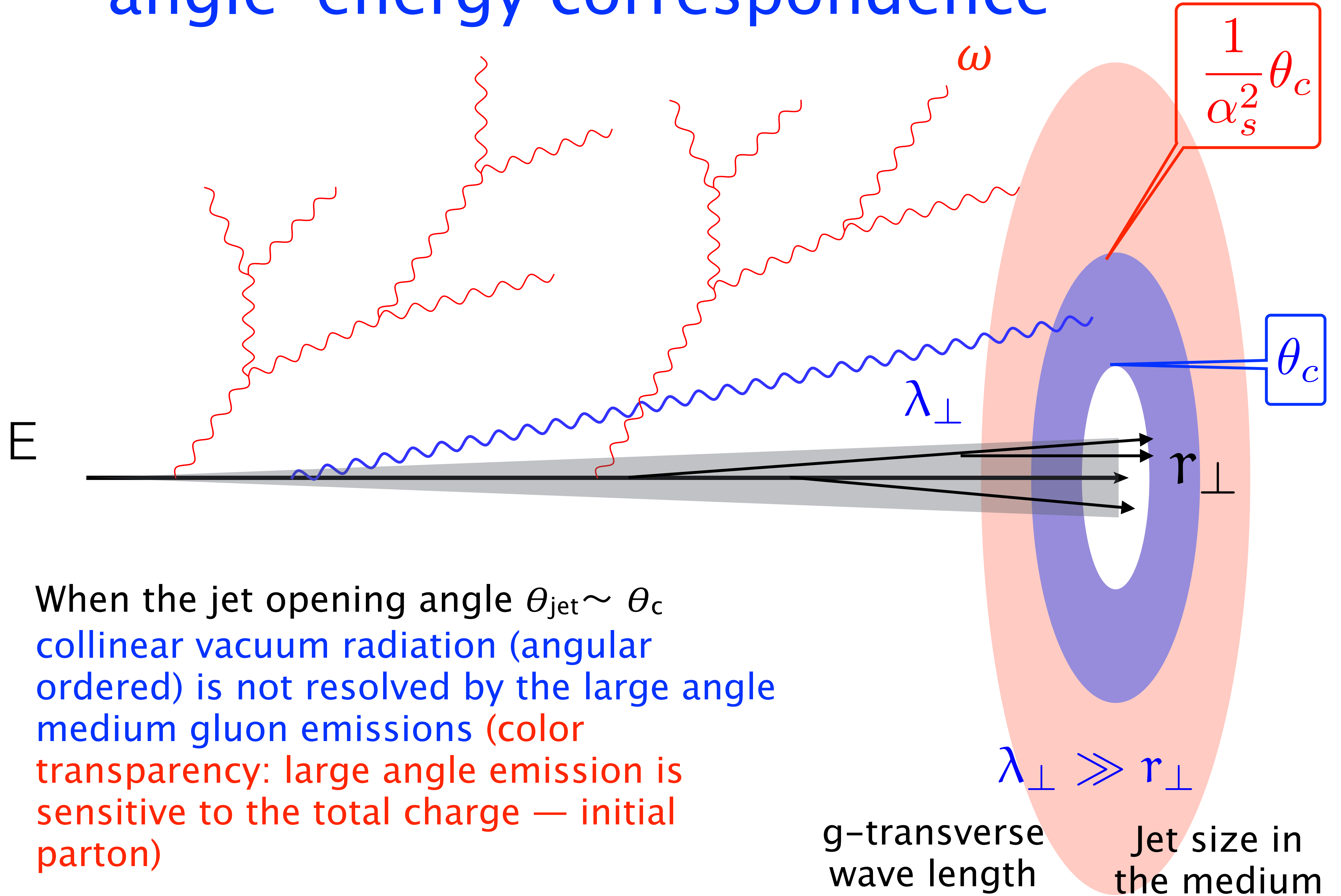
Structure of branchings: angle-energy correspondence



Soft gluon cascades take place at parametrically large angle

$$\theta \gg \frac{1}{\alpha_s^2}\theta_c \gg \theta_c$$

Structure of branchings: angle-energy correspondence



When the jet opening angle $\theta_{\text{jet}} \sim \theta_c$

collinear vacuum radiation (angular ordered) is not resolved by the large angle medium gluon emissions (color transparency: large angle emission is sensitive to the total charge — initial parton)

Structure of branchings: angle–energy correspondence

Numerical estimate: L= 5 fm, $\hat{q}$= 2 GeV^2/fm, α_s=0.3

$$\frac{1}{\alpha_s^2}\theta_c \sim 1 \;\gg\; \theta_{\rm jet} = 0.3$$

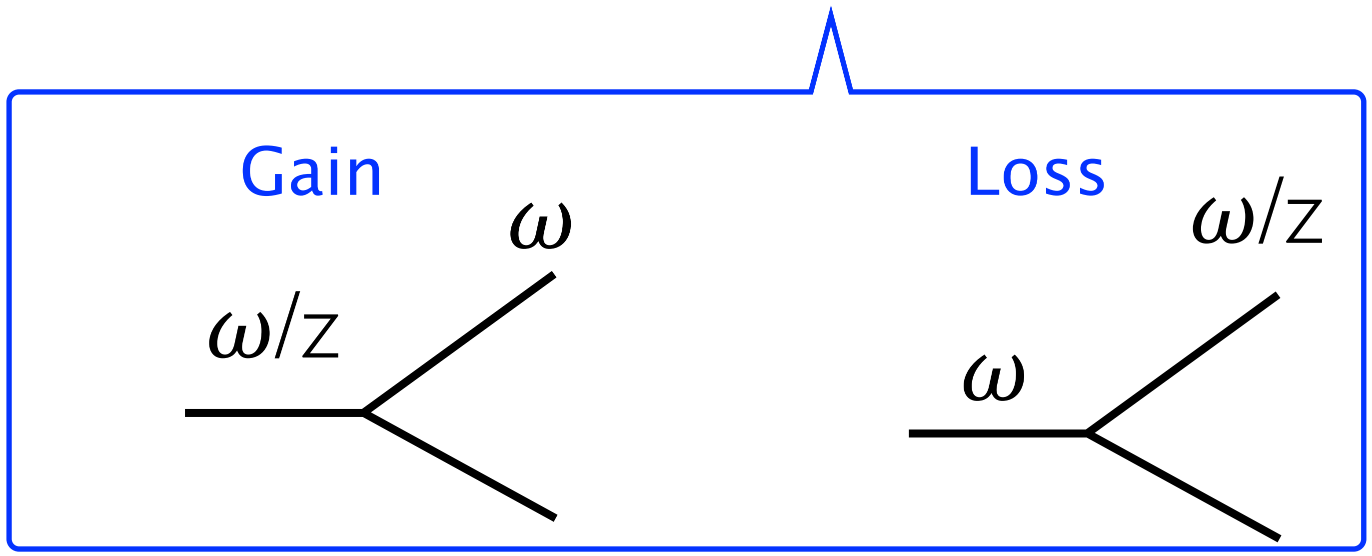
⇒ Geometrical separation between medium–induced multiple gluon branchings and collimated vacuum shower

Rate equation for the gluon distribution

Evolution of the gluon distribution
up to t= L in collinear branching approx.

$$D(\omega,\theta) \equiv \omega \frac{dN}{d\omega d\theta^2}$$

$$\frac{\partial}{\partial t} D(\omega,\theta) = \int_0^1 dz \mathcal{K}(z) \left[\frac{D(\omega/z,\theta)}{t_*(\omega/z)} - \frac{D(\omega,\theta)}{t_*(\omega)} \right] - \frac{\hat{q}}{\omega^2} \nabla_\theta^2 D(\omega,\theta)$$



Diffusion btw
2 branchings

Energy loss: Baier, Mueller, Schiff, Son [2001], Moore, Jeon [2003]
Energy recovery: Blaizot, Dominguez, Iancu, MT [2013]

Splitting kernel

$$\mathcal{K}(z) \sim \frac{1}{z^{3/2}(1-z)^{3/2}}$$

Inelastic mean-free-path

$$t_*(\omega) = \frac{1}{\alpha_s} \sqrt{\frac{\omega}{\hat{q}}}$$

Rate equation for the gluon distribution

$$\frac{\partial}{\partial t}D(\omega,\theta) = \int_0^1 \mathrm{d}z\mathcal{K}(z,\hat{q})\left[\frac{D(\omega/z,\theta)}{t_*(\omega/z)} - \frac{D(\omega,\theta)}{t_*(\omega)}\right] - \frac{\hat{q}}{\omega^2}\nabla_\theta^2\, D(\omega,\theta)$$

Broadening due to branchings logarithmically enhanced

Mueller, Liou, Wu [2013]

This large correction can be fully absorbed in a renormalization of the quenching parameter

$$\hat{q} \equiv \hat{q}_0\left(1+\frac{2\alpha_s N_c}{\pi}\ln^2\frac{k_\perp^2}{m_D^2}\right)$$

Blaizot, MT [2014]

Solution of the rate equation

Blaizot, Dominguez, Iancu, MT, PRL 111 (2013)

Integrating over angles $$D(\omega) = \int \mathrm{d}^2\theta D(\omega,\theta)$$

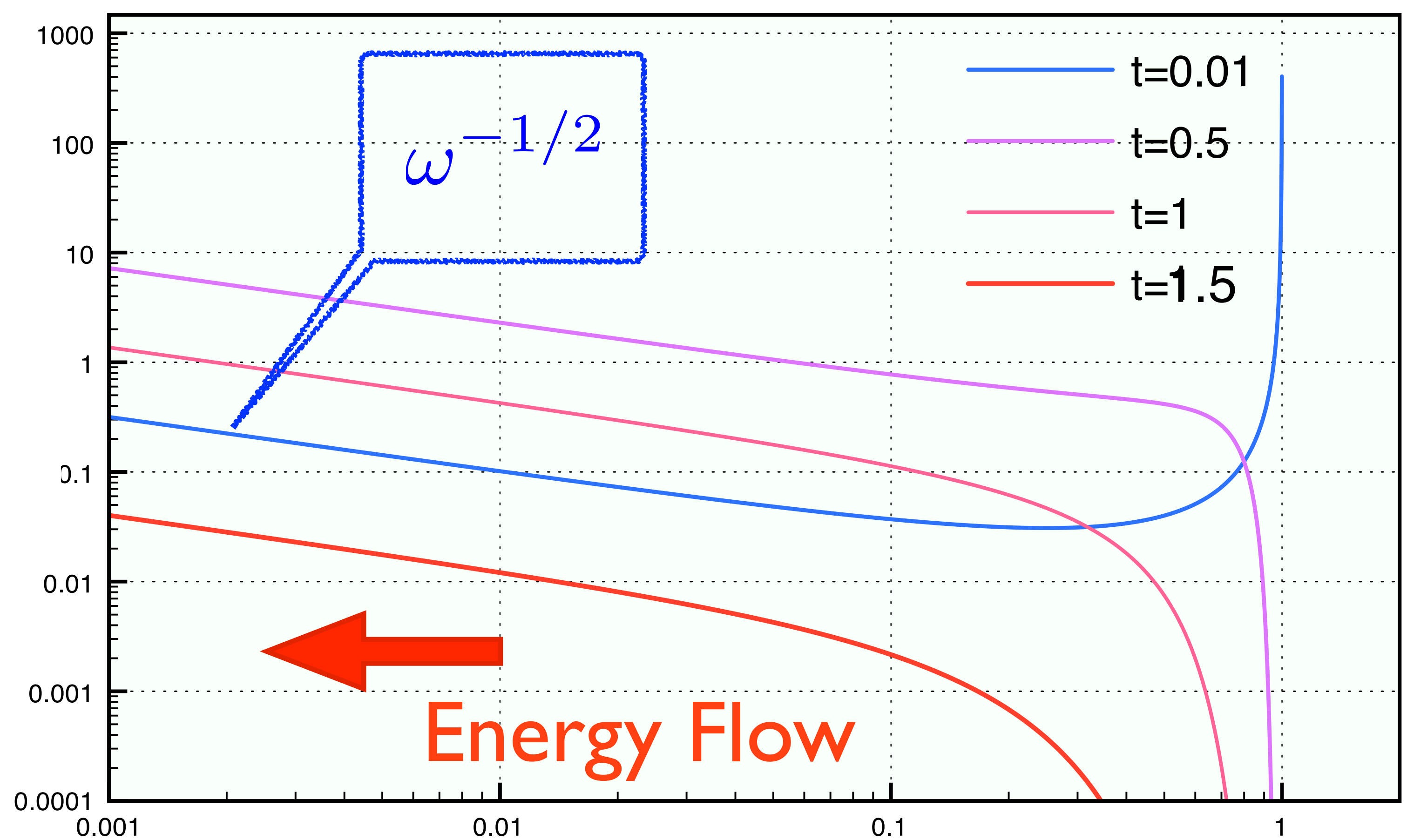
Initial condition $$D_0(\omega) = \delta(\omega - E)$$

Scaling solution (for $\omega \ll$ E) $\Rightarrow$ Fixed-point of the collision term

$$D(\omega) \sim \frac{\tau}{\sqrt{\omega}}\,\mathrm{e}^{-\pi\tau^2}$$

where

$$\tau = \frac{L}{t_*(E)}$$



Energy flow at low frequencies

Although the rate equation conserves energy at each branching, the integrated energy is not conserved

$$\int_0^E d\omega D(\omega) = E\,\mathrm{e}^{-\pi\tau} < E$$

Where does the missing energy go?

Energy flow at low frequencies

The flow of energy is positive and constant in the soft sector

$$\mathcal{F}(\omega) = \frac{\partial}{\partial t} \int_{\omega}^{E} \mathrm{d}\omega' D(\omega')$$

$$\tau = \frac{L}{t_*(E)}$$

At low frequencies $\omega \ll \omega_s$

$$\mathcal{F}(\omega) = 2\pi\tau \mathrm{e}^{-\pi\tau^2}$$

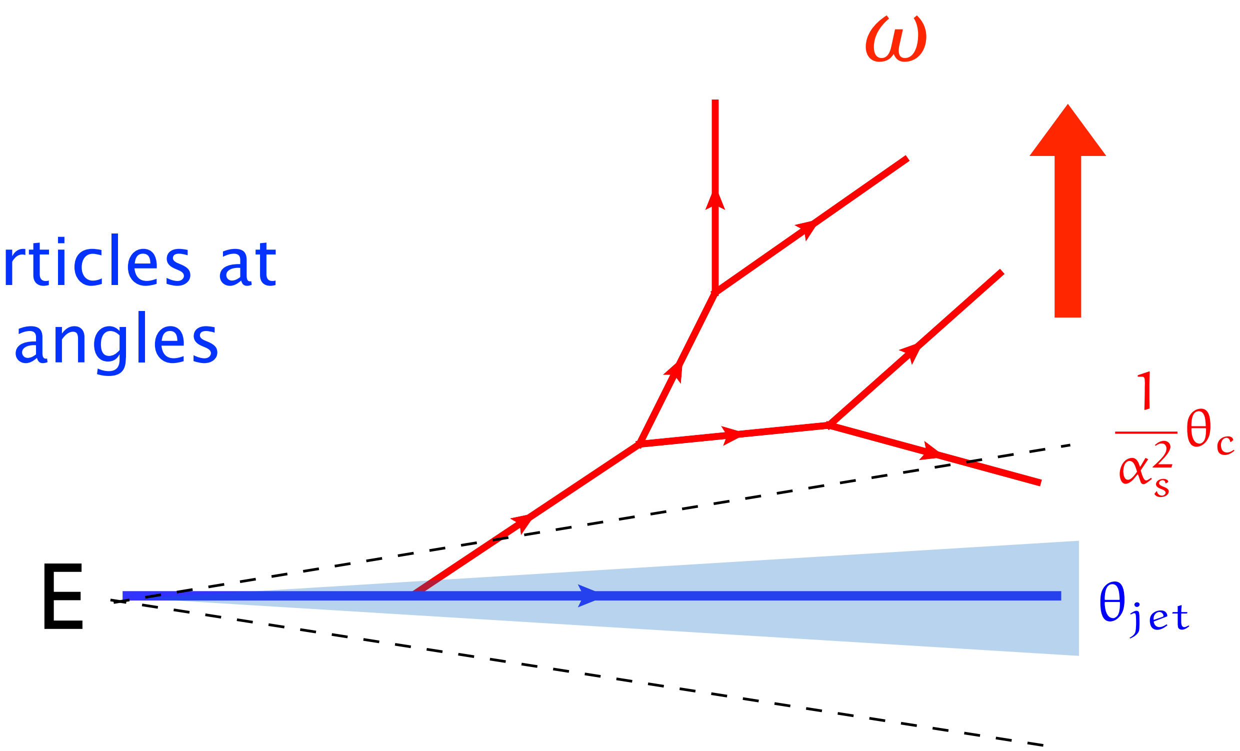
Condensation: Energy accumulates at $\omega = 0$ (in real life energy dissipates at ω = temperature of the QGP)

For a jet that escapes the medium $t_*(E) \gg L$ (or $\tau \ll 1$) the energy that is injected in the medium is

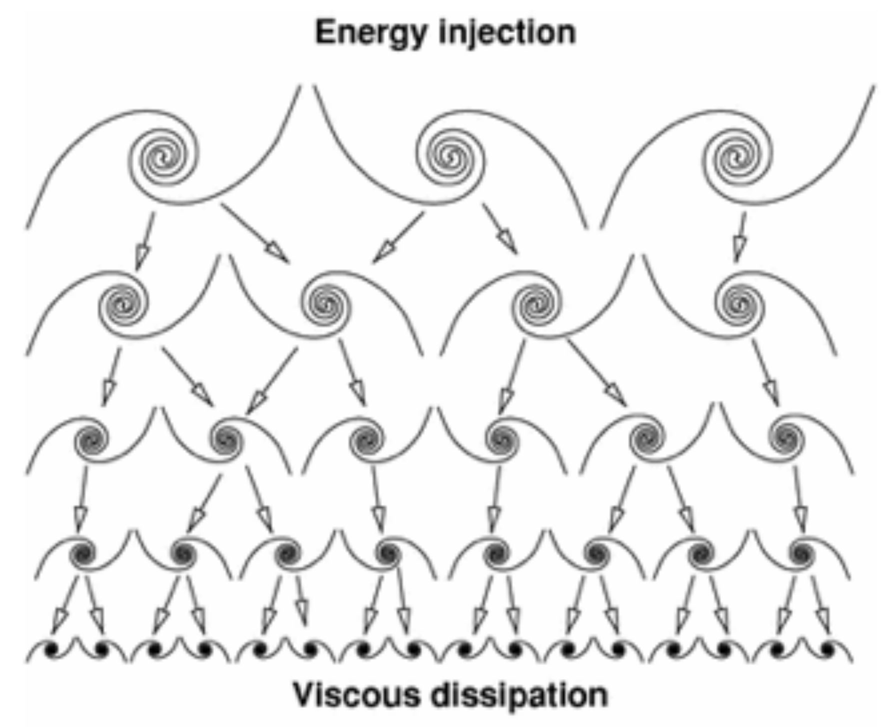
$$E_0 = \pi\,\omega_s \sim \alpha_s^2 \hat{q} L^2$$

Wave Turbulence in Jets

Soft particles at large angles



Richardson Cascade 1921



- Gain = Loss $\Longrightarrow$ Constant Energy Flow
- Inertial range: Energy flows from high to low frequencies without accumulating (inverse energy cascade)

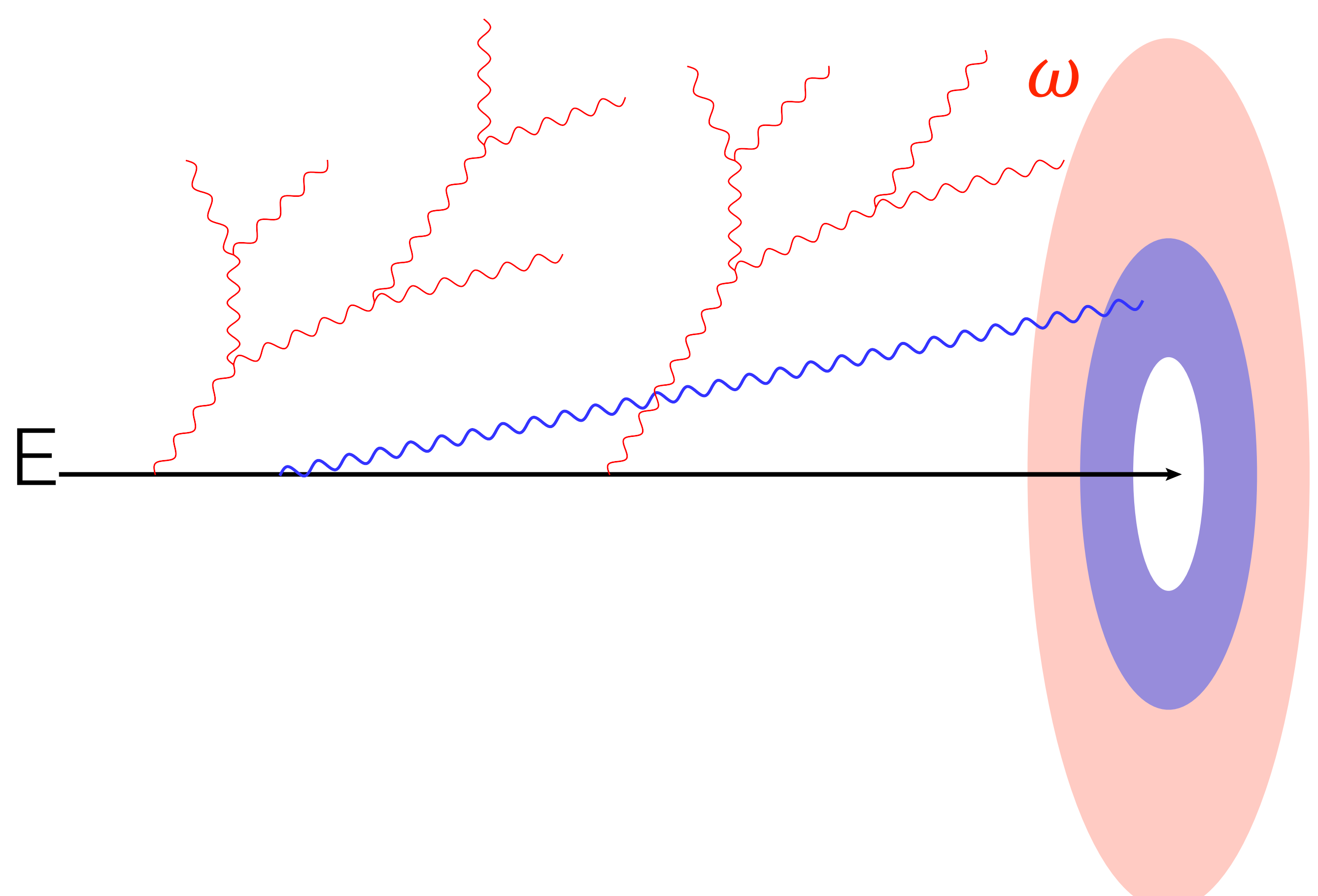
Efficient mechanism for energy transport to large angles

Angular distribution

I — Leading particle $\omega \sim E$

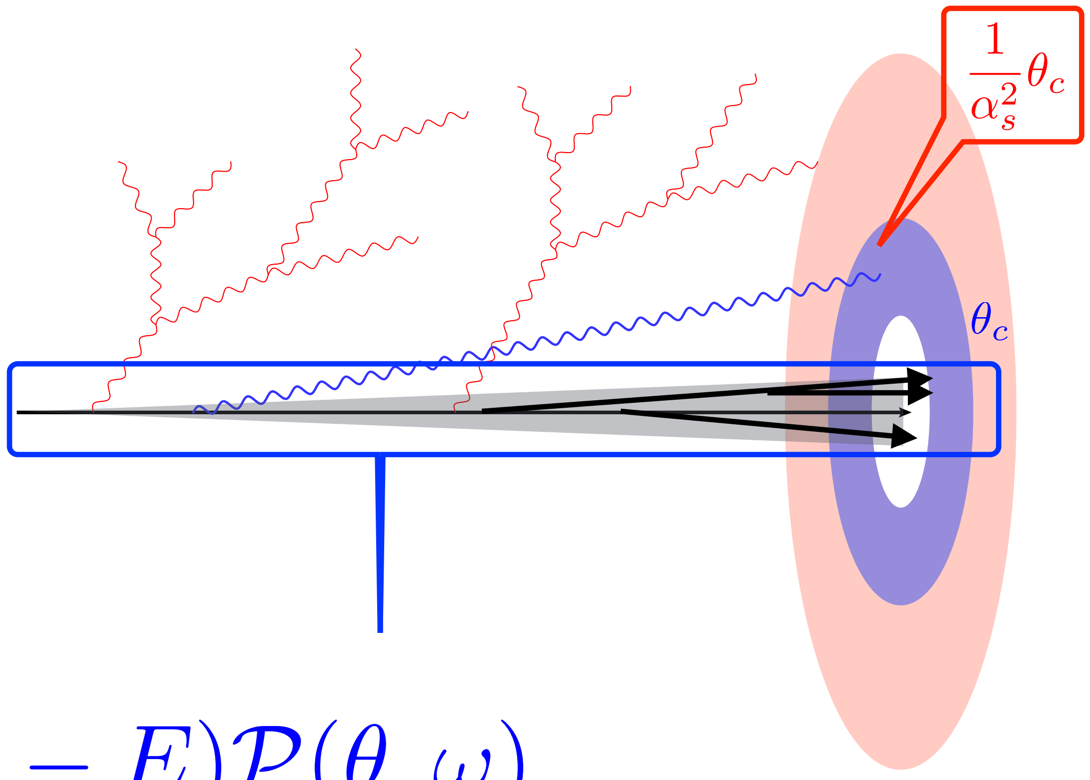
II — Rare BDMPS radiation: $\omega_s \ll \omega \ll \omega_c < E$

III — Multiple branching regime $\omega \ll \omega_s \ll E$



I — Leading particle $\omega \sim E$

Not sensitive to gluon radiation (or only via the renormalized quenching parameter)



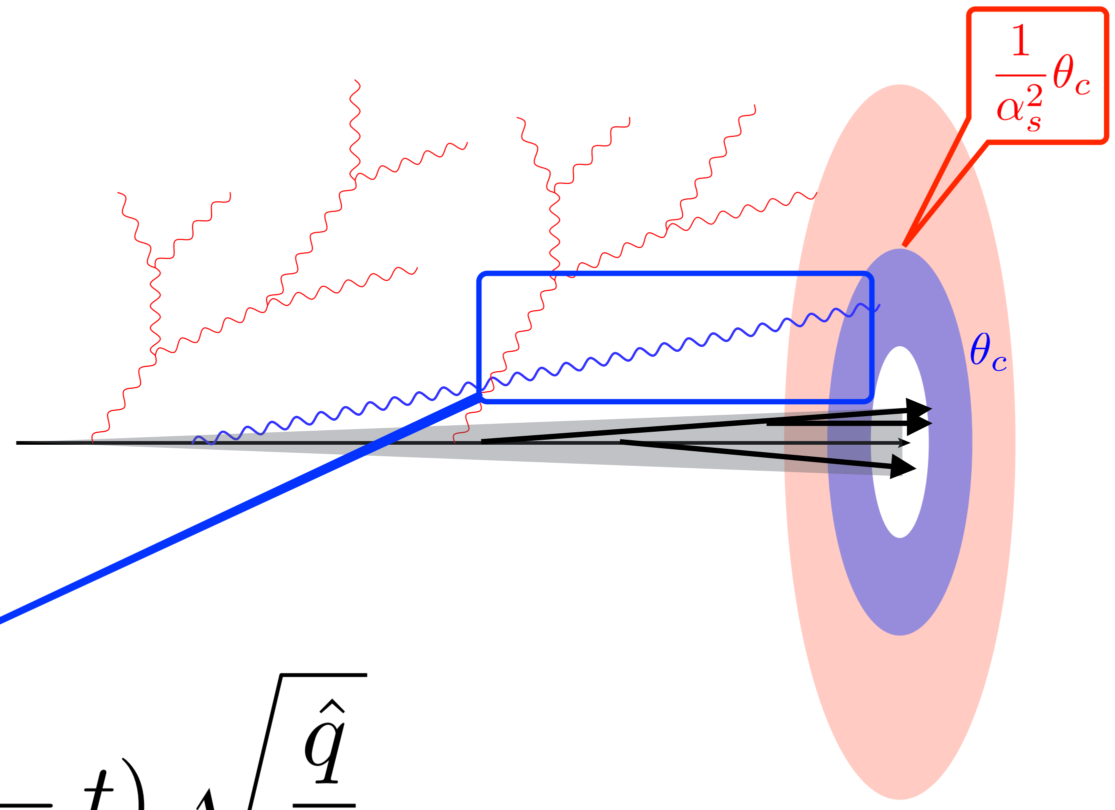
$$D(\omega,\theta) \sim D(\omega)\mathcal{P}(\theta,\omega) \sim \omega\delta(\omega - E)\mathcal{P}(\theta,\omega)$$

- Broadening Prob. $$\mathcal{P}(\theta,\omega) \equiv \frac{4\pi}{\langle\theta\rangle^2}\mathrm{e}^{-\theta^2/\langle\theta\rangle^2}$$

$$\langle\theta\rangle^2 \equiv \frac{\hat{q}L}{E^2} \quad (\sim 0.001)$$

The deflection of the jet is negligible!

II — Rare BDMPS radiation: $\omega_s \ll \omega \ll \omega_c < E$



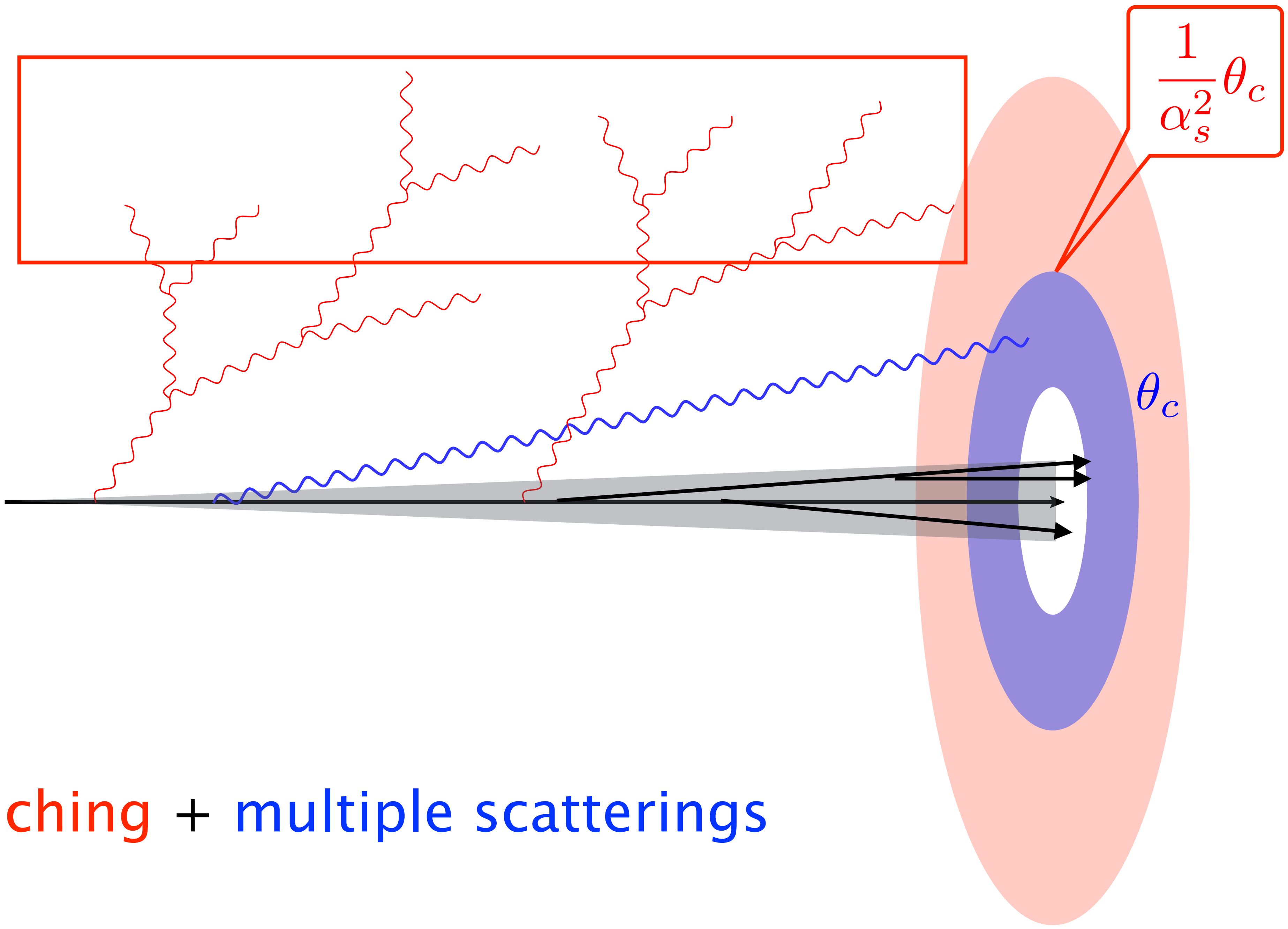
Single gluon radiation O(α_s).
The broadening is determined by the diffusion of the radiated gluon

$$D(\omega,\theta) \simeq \alpha_s \int_0^L \mathrm{d}t\ \mathcal{P}(\theta,\omega,L-t)\ \sqrt{\frac{\hat{q}}{\omega}}$$

The typical angular broadening reads (the factor 1/2 comes from the time integral)

$$\langle \theta^2 \rangle \equiv \frac{\langle k_\perp^2 \rangle}{\omega^2} = \frac{\hat{q}L}{2\omega^2} \qquad > \ \theta_c^2$$

III — Multiple branching regime $\omega \ll \omega_s \ll E$



Multiple branching + multiple scatterings

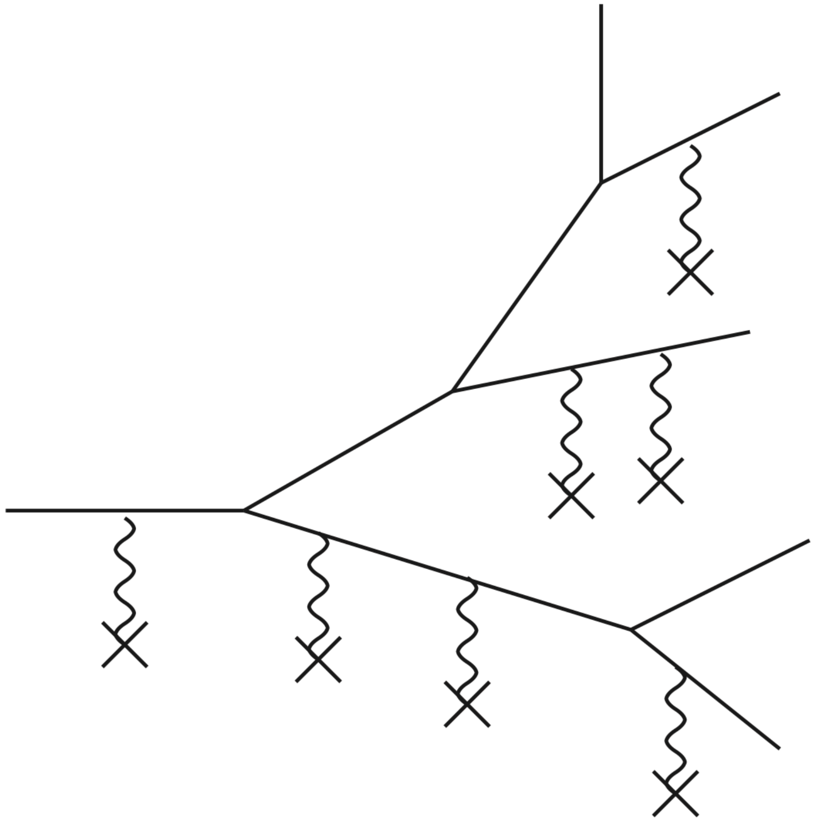
III — Multiple branching regime $\omega \ll \omega_s \ll E$

The evolution equation may be solved in Fourier space ($r_T \sim u_T/\omega$ ~ transverse dipole size)

$$D(\omega, \boldsymbol{u}) \equiv \int \mathrm{d}^2\boldsymbol{\theta}\, D(\omega, \boldsymbol{\theta})\, \mathrm{e}^{-i\boldsymbol{u}\cdot\boldsymbol{\theta}}$$

Opacity Expansion: (order by order in elastic scatterings but all order in branchings)

$$D(\omega, \boldsymbol{u}) = \sum_{n=0}^{\infty} D_n(\omega, \boldsymbol{u})\,,$$



III — Multiple branching regime $\omega \ll \omega_s \ll E$

The general term reads

$$D_n(\omega, \boldsymbol{u}) = c_n \left[\sigma(\omega, \boldsymbol{u})\, t_*(\omega)\right]^n D(\omega)$$

dipole cross-section

where the coefficients c_n are solved recursively

$$c_n = \prod_{m=1}^{n} \frac{2}{\sqrt{\pi}} \frac{\Gamma(\frac{3m}{2})}{\Gamma(\frac{3m+1}{2})}$$

Why $t_*(\omega)$ and not L? a gluon ω can not survive in the medium longer than $t_*(\omega)$ therefore, to be measured it must be produced close to the surface within the shell $L - t_*(\omega)$

III — Multiple branching regime $\omega \ll \omega_s \ll E$

The solution can be written in the factorized form

$$\boxed{D(\omega,\theta) = D(\omega)\,\eta(\theta^2/\theta_*^2(\omega))}$$

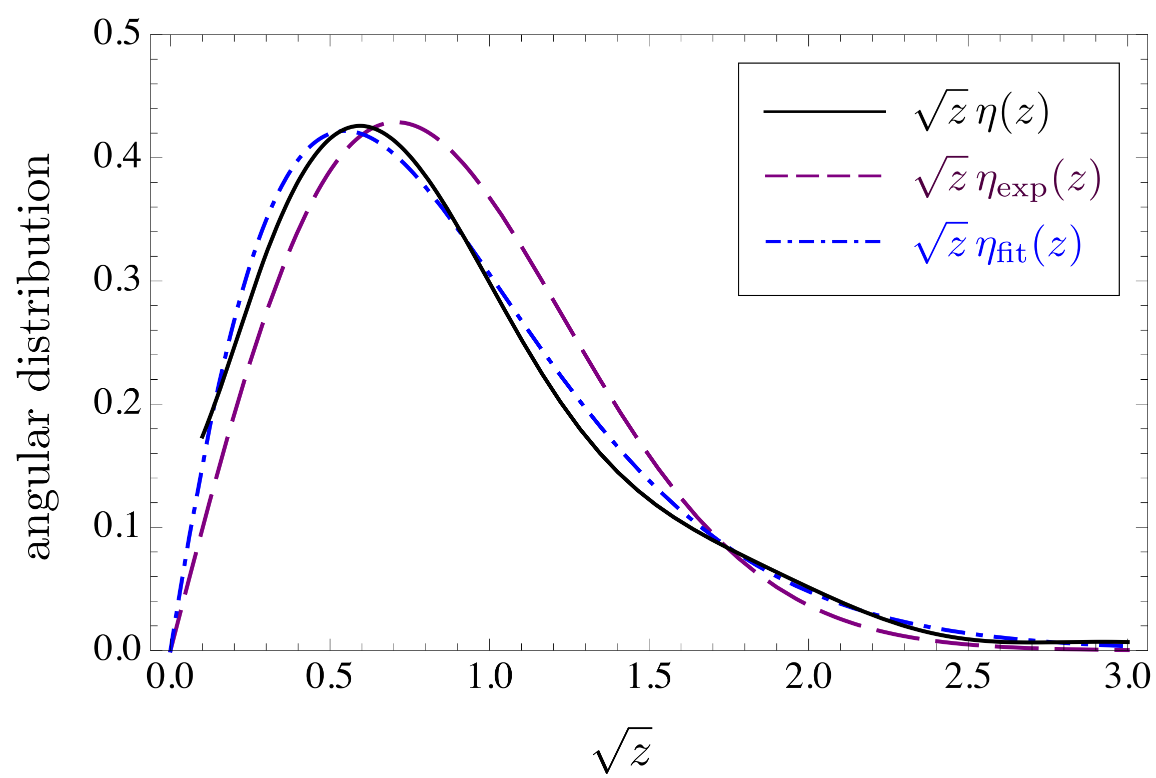
where $\theta_*^2(\omega) = \dfrac{1}{\alpha_s}\left(\dfrac{\hat{q}}{\omega}\right)^{1/2}$

Normalized angular distribution (to all orders in opacity expansion)

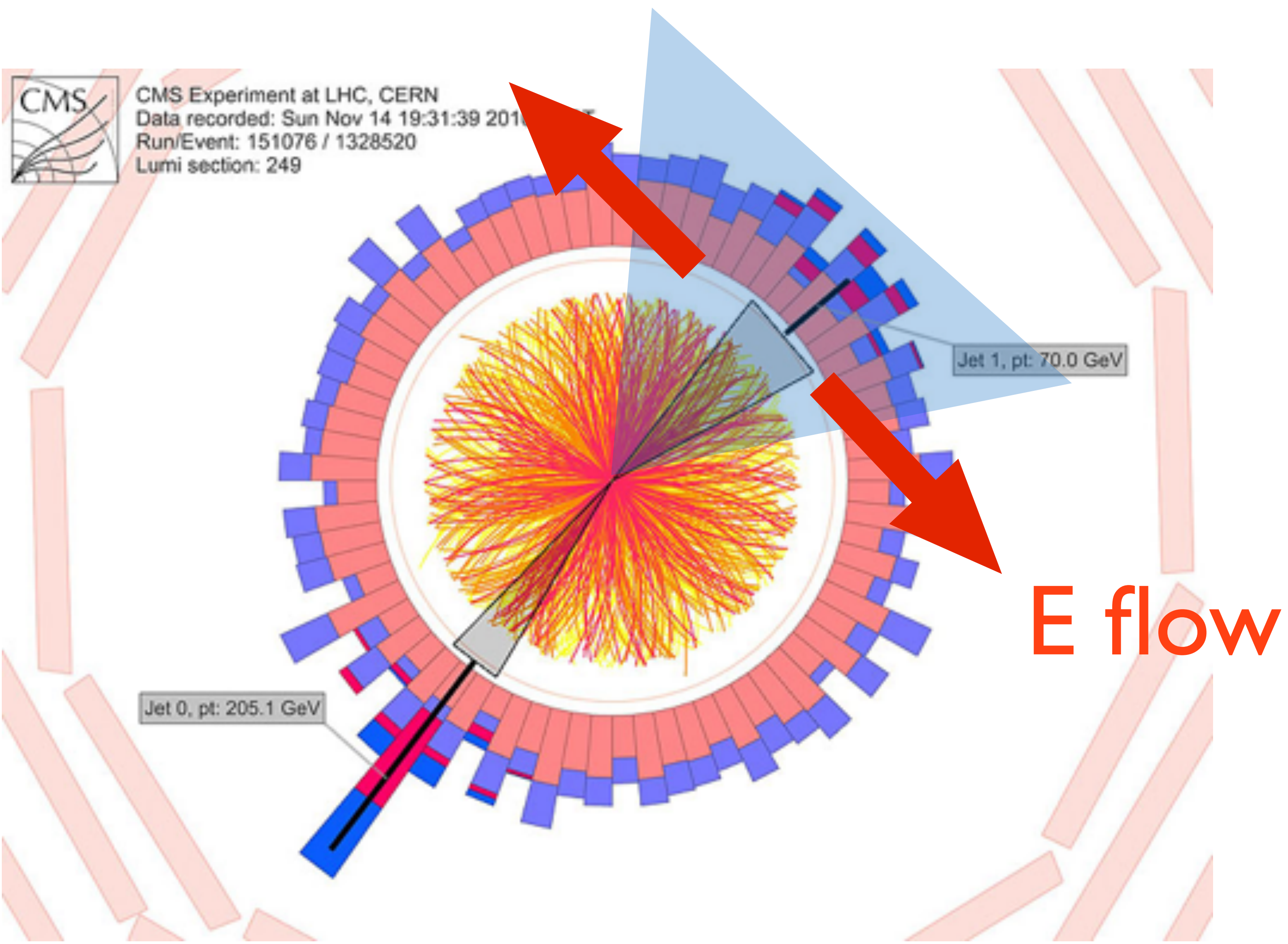
$$\eta(z) = \int_0^\infty \mathrm{d}\beta\, J_0(2\sqrt{z\beta})\,\sum_{n=0}^{\infty}(-1)^n\, c_n\,\beta^{2n}$$

$$\eta_{\rm fit}(z) = \frac{4a^{3/2}}{3\sqrt{\pi}}\mathrm{e}^{-az^{2/3}}$$

$$z = \theta^2/\theta_*^2 \qquad a \simeq 1.68$$



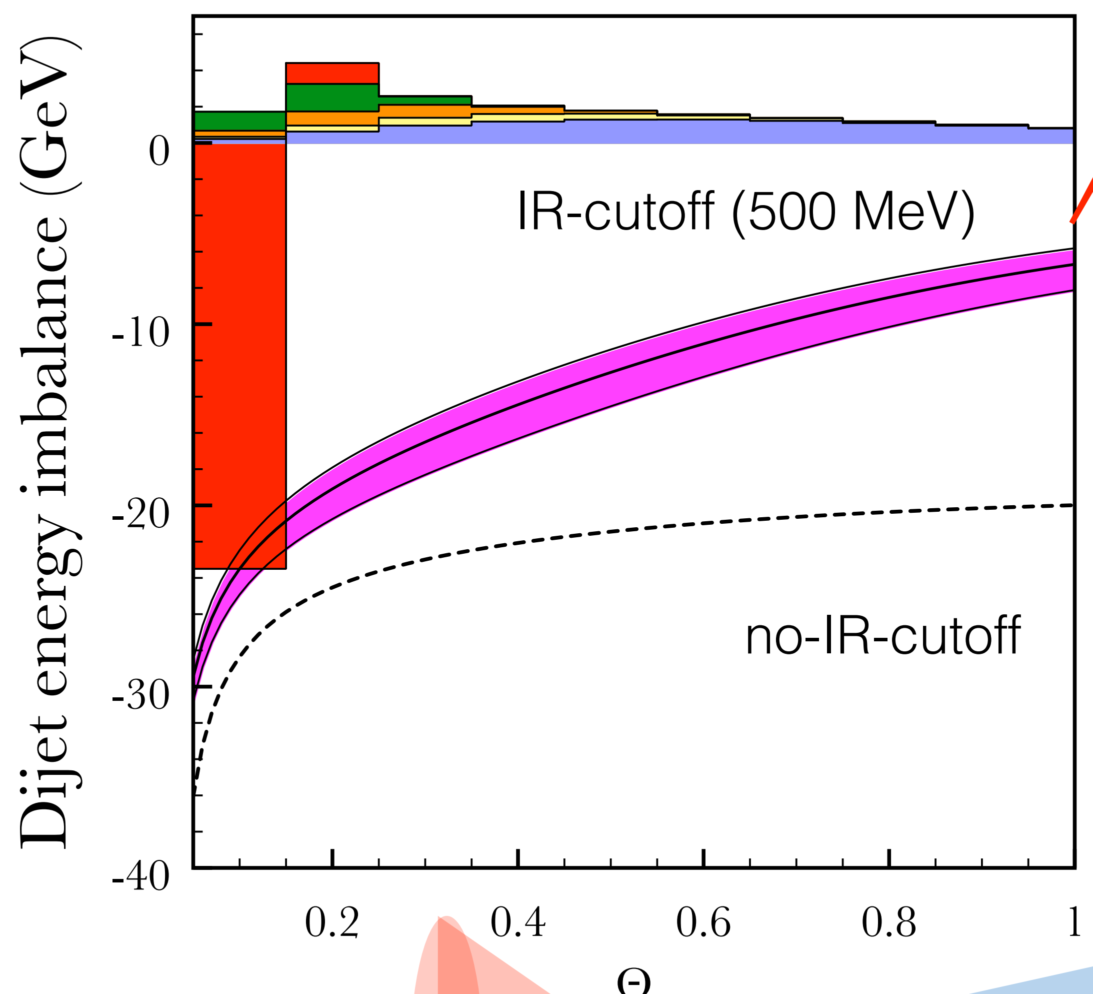
Understanding Dijet asymmetry?



CMS: energy is lost in soft particles at large angles

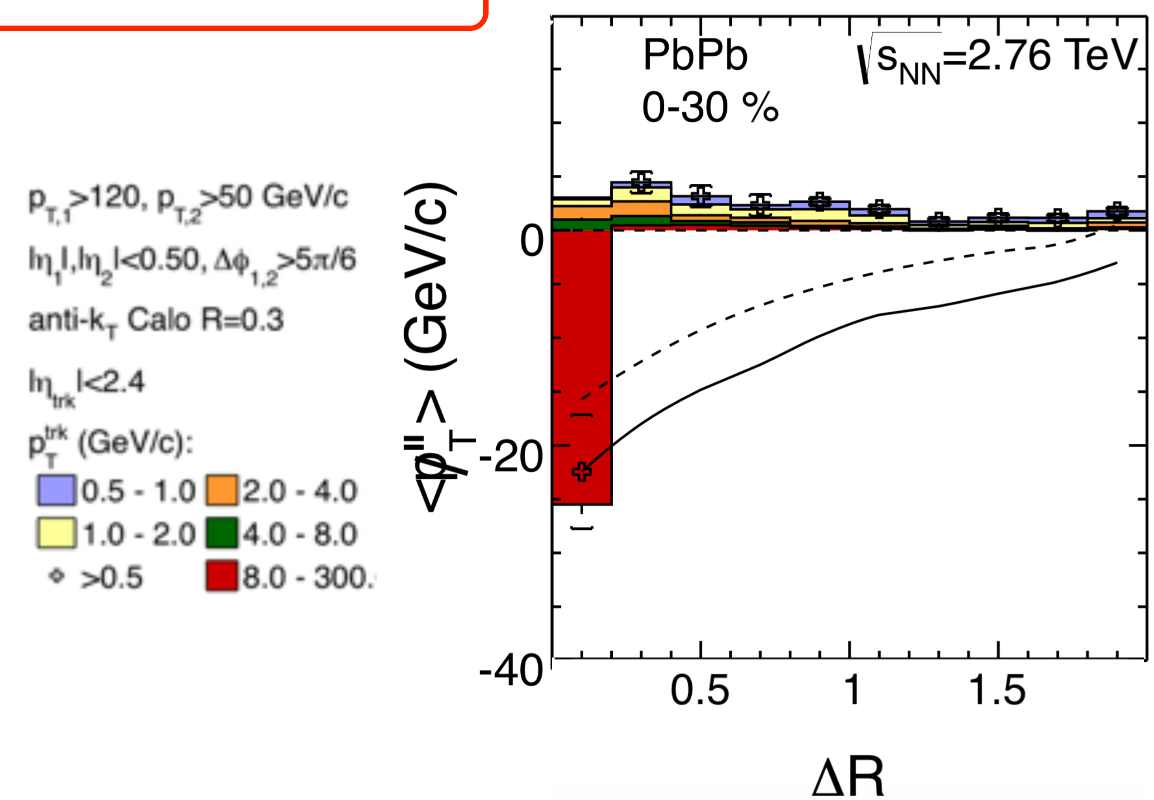
Dijet asymmetry (model vs. data)

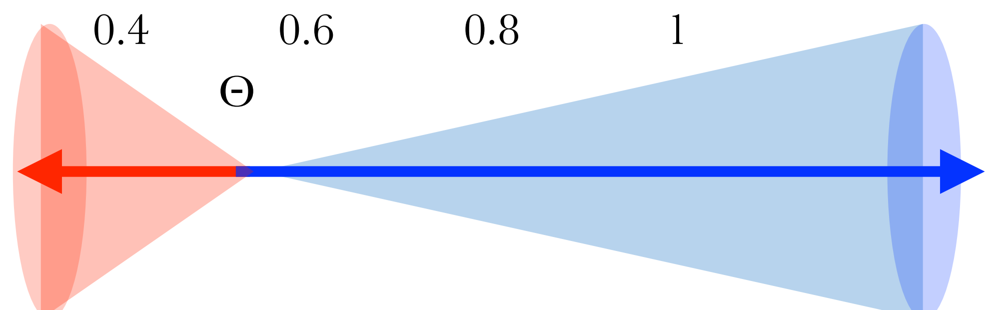
Ideal cascade (model)



Cumulative Energy

CMS DATA





with E = 120 GeV, $\hat{q}$ = 2 GeV^2/fm, α_s=0.3

Leading jet: L_1 = 1 fm, Subleading jet: L_2 = 5 fm,

Summary and outlook

- Jets in HIC are composed of a coherent inner core and large angle decoherent gluon cascades that are characterized by a constant energy flow from large to low momenta down to the QCD scale where energy is dissipated.
- Geometrical separation : The cascade develop at parametrically large angles away from the jet axis. Genuine QCD phenomenon. Seen in CMS data on missing energy in imbalanced dijet events?
- Comparison with CMS data: need a Monte Carlo event generator (to deal with experimental biases) and a realistic treatment of the geometry of the collision, etc.

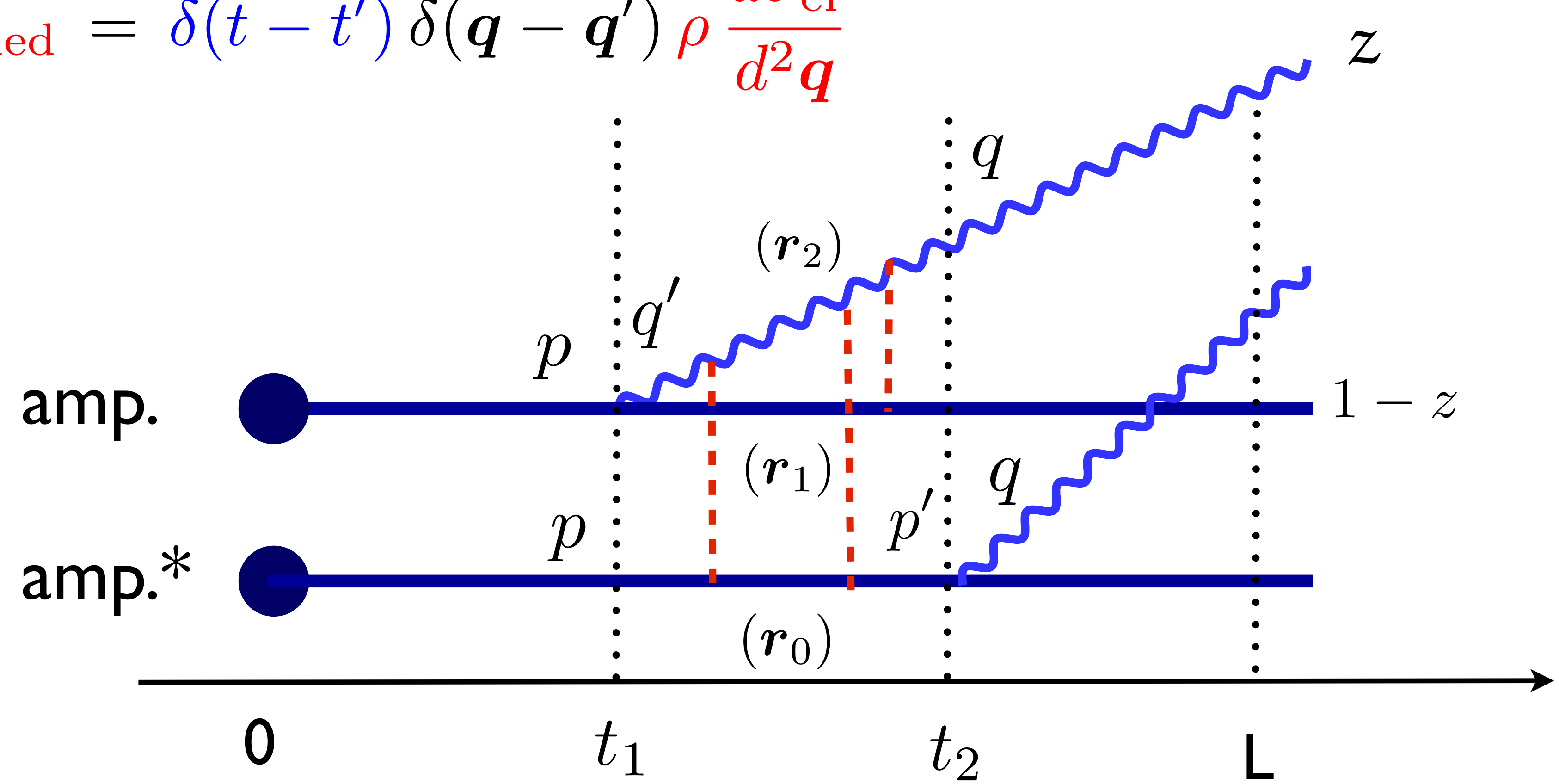
Backup

Inelastic rate $\mathcal{K}(z)$

Medium average

$$g^2 \langle A^-(\boldsymbol{q}, t)\, A^-(\boldsymbol{q}', t') \rangle_{\rm med} = \delta(t-t')\, \delta(\boldsymbol{q}-\boldsymbol{q}')\, \rho\, \frac{d\sigma_{\rm el}}{d^2\boldsymbol{q}}$$

- 3-point function correlator accounts for multiple instantaneous scatterings of a 3 dipole system



$$S^{(3)}(t_2, t_1) = S_0^{(3)}(t_2, t_1) + \int_{t_1}^{t_2} dt'\, S_0^{(3)}(t_2, t')\, \sigma_3(t')\, S^{(3)}(t', t_1)$$

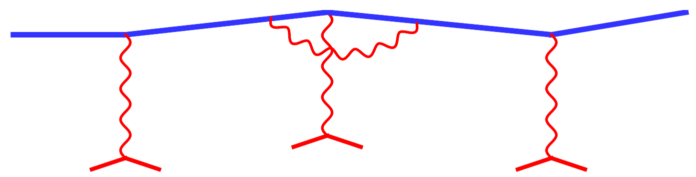
- It is related to the expectation value of 3 wilson lines at time-dependent transverse coordinates (Brownian motion in T-space)

$$\mathcal{K}(z) \sim \; S^{(3)} \sim \langle \operatorname{tr} T^a\, U_F(\boldsymbol{r}_1)\, T^b\, U_F^\dagger(\boldsymbol{r}_0) U_{ab}(\boldsymbol{r}_2) \rangle_{\rm med}$$

(Universal) radiative corrections

- Radiative corrections to pt-broadening to **Double Log** accuracy

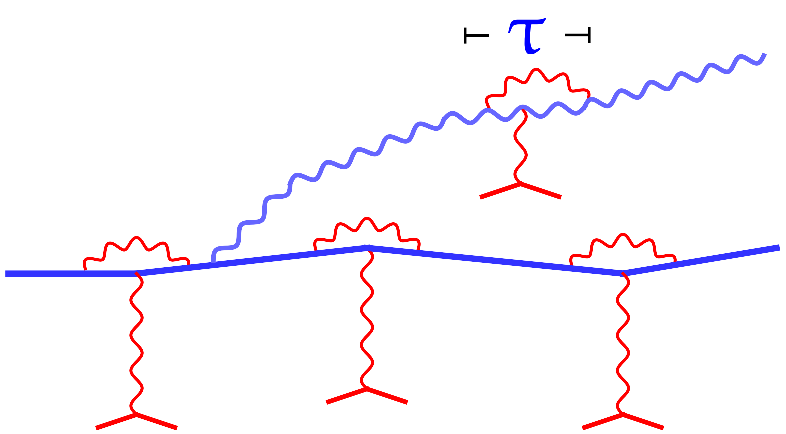
$$\langle k_\perp^2 \rangle = \hat{q} L \left(1 + \frac{\alpha_s N_c}{2\pi} \ln^2 \frac{L}{\tau_0} \right)$$



[Wu (2011) Liou, Mueller, Wu (2014) Blaizot, Iancu, Dominguez, MT (2014)]

- Radiative corrections to energy loss

$$\Delta E \sim \alpha_s C_R \hat{q} L^2 \left(1 + \frac{\alpha_s N_c}{2\pi} \ln^2 \frac{L}{\tau_0} \right)$$



[Blaizot, MT (2014) Wu (2014)]

- **Universality** and renormalization of $\hat{q}$

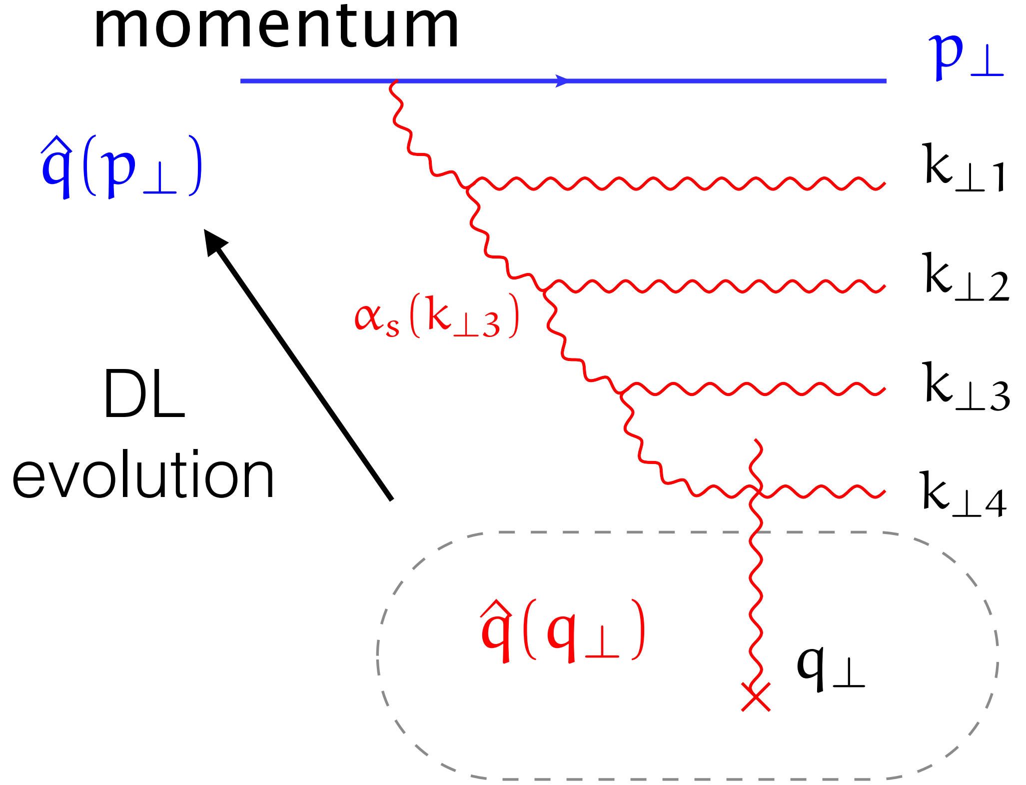
$$\frac{\partial}{\partial \tau} \hat{q}(\mathbf{k}, \tau) = \frac{\alpha_s N_c}{\pi} \int_{\hat{q}\tau}^{k^2} \frac{d\mathbf{k}'^2}{\mathbf{k}'^2} \hat{q}(\mathbf{k}', \tau)$$

[Blaizot, MT (2014) Iancu (2014)]

How is the jet coupled to the medium?

- QCD evolution of jet quenching parameter smoothly interpolates between hard medium scale and "non-perturbative" scale
- Strong ordering in transverse momentum

$$q_\perp \sim m_D \ll k_{\perp 1} \ll \ldots \ll p_\perp \sim Q_s = \hat{q} L$$



"Non-perturbative'' initial condition for the evolution of the quenching parameter:

- ☐ From HTL (LO+NLO)

  [Aurenche, Gelis, Zaraket (2000) Caron-Huot (2008)]
  Ghirglieri, Hong, Kurkela, Moore, Teaney (2013-2015)]

- ☐ Lattice

  [Majumder (2012)]
  [Panero, Rummukainen, Schäfer (2013)]

- ☐ AdS/CFT

☐ **Eloss anomalous scaling**: $\Delta E \sim L^{2+\gamma}$ with $\gamma \equiv \sqrt{\frac{4\alpha_s N_c}{\pi}}$

☐ **Between BDMPS** L^2 **and AdS/CFT results** L^3 [Blaizot, MT (2014)]