Analysis of exclusive kT algorithm in electron-positron annihilation

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Factorization formula for the cross section

- We have factorized dijet cross section as following.

$$\sigma_{\text{jet}} = \sigma_0 H(Q^2,\mu)\mathcal{J}_{n,\Theta}(\mu)\mathcal{J}_{\bar{n},\Theta}(\mu)\mathcal{S}_{\Theta}(\mu).$$

- The jet and soft function is defined as

$$\mathcal{J}_{n,\Theta}(\mu) = \int dp^2 J_{n,\Theta}(p^2,\mu),$$

$$\sum_{X_n} \langle 0|\chi_n^a|X_n\rangle \Theta_J \langle X_n|\overline{\chi}_n^b|0\rangle = \int \frac{d^4 p_{X_n}}{(2\pi)^3}\frac{\not{n}}{2}\bar{n}\cdot p_{X_n} J_{n,\Theta}(p_{X_n}^2,\mu)\delta^{ab}.$$

$$\mathcal{S}_{\Theta}(\mu) = \sum_{X_s}\frac{1}{N_c}\mathrm{Tr}\langle 0|\tilde{Y}_{\bar{n}}^{\dagger}\tilde{Y}_n|X_S\rangle\Theta_{\text{soft}}\langle X_S|\tilde{Y}_n^{\dagger}\tilde{Y}_{\bar{n}}|0\rangle.$$

Exclusive kt algorithm - The definition

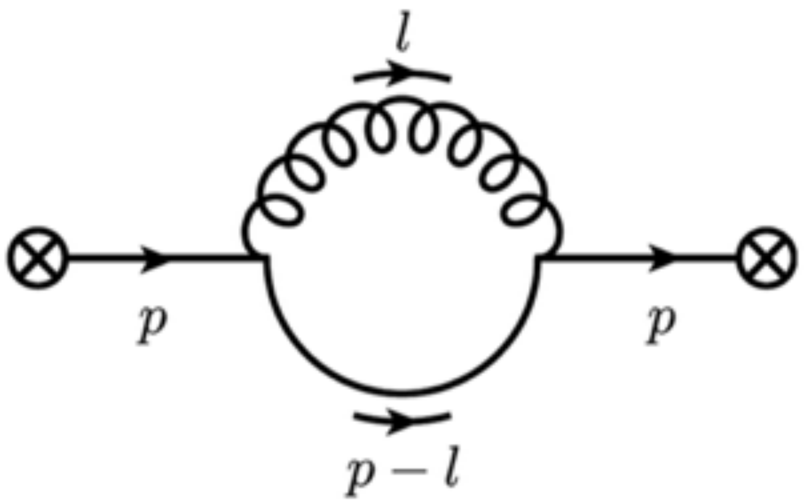
1. For each pair of partons i and j, work out the distance

$$y_{ij} = \frac{2}{Q^2}(1 - \cos\theta_{ij}) \min(E_i^2, E_j^2)$$

2. If y_{ij} is smaller than y_c, merge two partons into a single jet.
3. Repeat from step 1 until no particles are left.

Exclusive kt algorithm - Momentum assignments

- We assign the momentum flow as



- Then the energies of two partons and the invariant-mass squared is given by

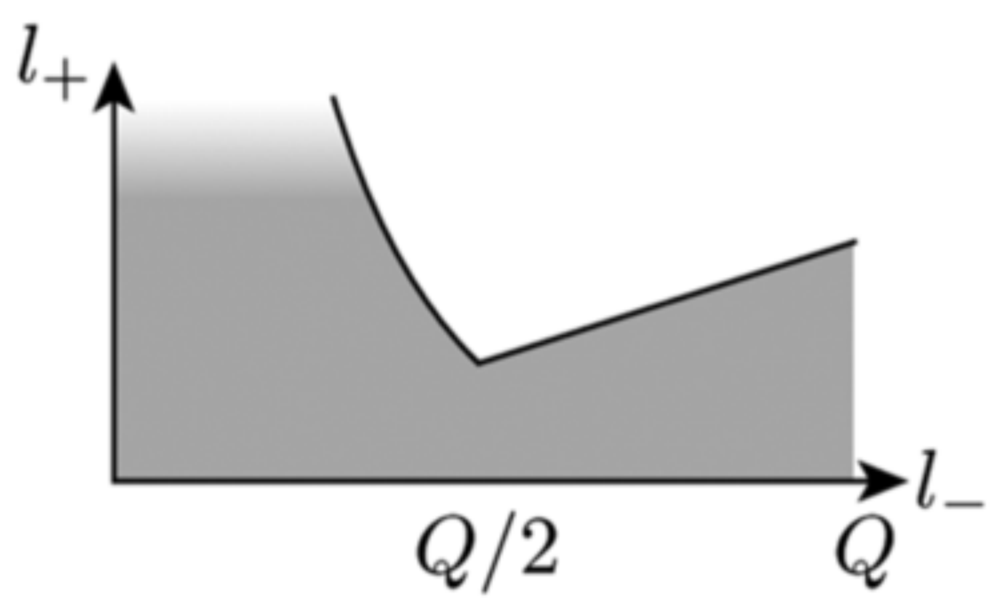
$$E_q = \frac{1}{2}\left(Q - l_- + \frac{p^2}{Q} - l_+\right), \quad E_g = \frac{1}{2}(l_- + l_+),$$

$$p^2 = (p_q + p_g)^2 = \frac{Ql_+}{1 - l_-/Q}.$$

Exclusive kt algorithm - Constraint functions

- With the previous variables, we get the jet constraint function.

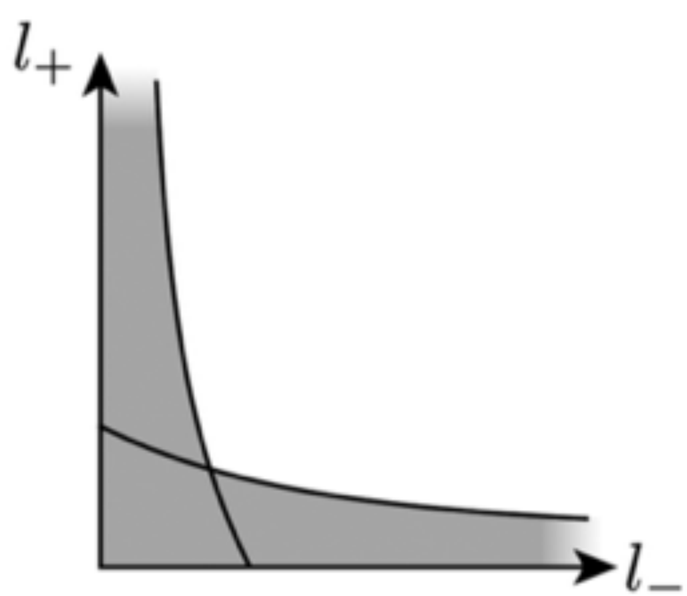
$$y_c > y_{qg} = \frac{p^2}{Q^2} \min\left(\frac{E_q}{E_g}, \frac{E_g}{E_q}\right) = \begin{cases} \dfrac{l_+ l_-}{(Q-l_-)^2}, & 0 < l_- < \dfrac{Q}{2}, \\[2ex] \dfrac{l_+}{l_-}, & \dfrac{Q}{2} < l_- < Q. \end{cases}$$



Exclusive kt algorithm - Constraint functions

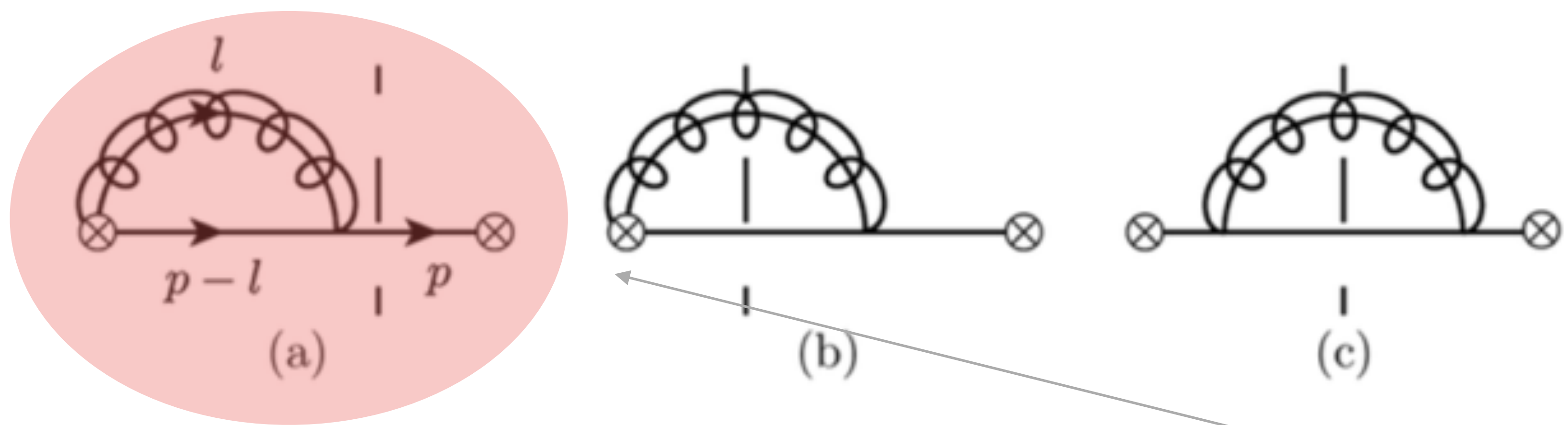
- Similarly, one can derive the soft constraint function as follows.

$$y_c > y_{qg} = \begin{cases} \dfrac{l_+(l_+ + l_-)}{Q^2}, & n - \text{collinear}, \\ \\ \dfrac{l_-(l_+ + l_-)}{Q^2}, & \bar{n} - \text{collinear}. \end{cases}$$

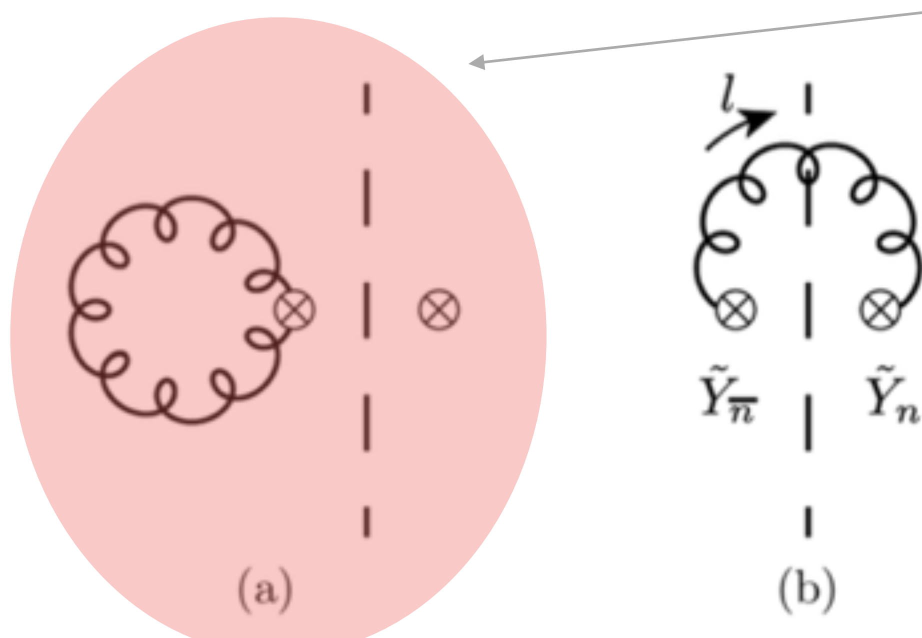


Exclusive kt algorithm - Virtual contributions 1

- The virtual contribution to the one loop correction of the scattering cross section is independent of the algorithm.



Virtual contributions are independent of algorithms



Exclusive kt algorithm - Virtual contributions 2

- The results for the virtual contributions are as follows.

$$M^{\mathrm{v}}_{\mathrm{jet}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)\left(\frac{1}{\epsilon_{\mathrm{UV}}} + 1 + \ln\frac{\mu}{Q}\right),$$

$$M^{\mathrm{v}}_{\mathrm{soft}} = -\frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)^2 .$$

- We used dimensional regularization to regulate both UV and IR divergences.

$$\int_0^\infty du u^{-1-\epsilon} = \frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}$$

Exclusive kt algorithm - Real gluon emissions

- In the calculation with the exclusive kt algorithm, we have problematic integral.

$$
\begin{aligned}
M_{\text{jet}}^{(b)} &= \frac{\alpha_s C_F}{2\pi} \frac{(e^{\gamma_{\text{E}}}\mu^2)^\epsilon}{\Gamma(1-\epsilon)} \int dl_+ dl_- \frac{Q-l_-}{Q} (l_- l_+)^{-1-\epsilon} \Theta_{\text{jet}} \\
&= \frac{\alpha_s C_F}{2\pi} \frac{(e^{\gamma_{\text{E}}}\mu^2)^\epsilon}{\Gamma(1-\epsilon)} \int_0^{Q/2} dl_- \frac{Q-l_-}{Q} l_-^{-1-\epsilon} \int_0^{y_c (Q-l_-)^2/l_-} dl_+ l_+^{-1-\epsilon} + \cdots \\
&= \frac{\alpha_s C_F}{2\pi} \frac{(e^{\gamma_{\text{E}}}\mu^2)^\epsilon}{\Gamma(1-\epsilon)} \frac{-1}{\epsilon_{\text{IR}}} \int_0^{Q/2} dl_- \frac{Q-l_-}{Q} l_-^{-1-\epsilon} \frac{y_c^{-\epsilon}(Q-l_-)^{-2\epsilon}}{l_-^{-\epsilon}} + \cdots \\
&= ???
\end{aligned}
$$

Generalized kt algorithm - The definition

1. For each pair of partons i and j, work out the distance

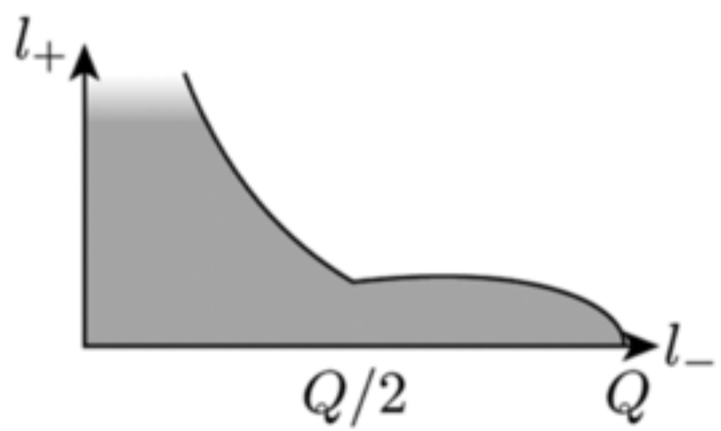
$$y_{ij} = \frac{2}{Q^\alpha}(1 - \cos\theta_{ij}) \min(E_i^\alpha, E_j^\alpha)$$

2. If y_{ij} is smaller than y_c, merge two partons into a single jet.
3. Repeat from step 1 until no particles are left.

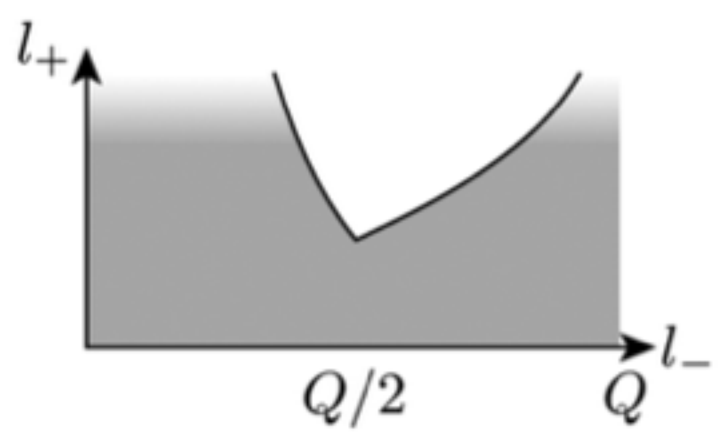
Generalized kt algorithm - Constraint functions

- With the previous variables, we get the jet constraint function.

$$y_c > y_{qg} = \begin{cases} \dfrac{2^{2-\alpha}}{Q^\alpha} \dfrac{l_+ l_-^{\alpha-1}}{(1 - l_-/Q)^2}, & 0 < l_- < \dfrac{Q}{2}, \\[2ex] 2^{2-\alpha} \left(1 - \dfrac{l_-}{Q}\right)^{\alpha-2} \dfrac{l_+}{l_-}, & \dfrac{Q}{2} < l_- < Q. \end{cases}$$



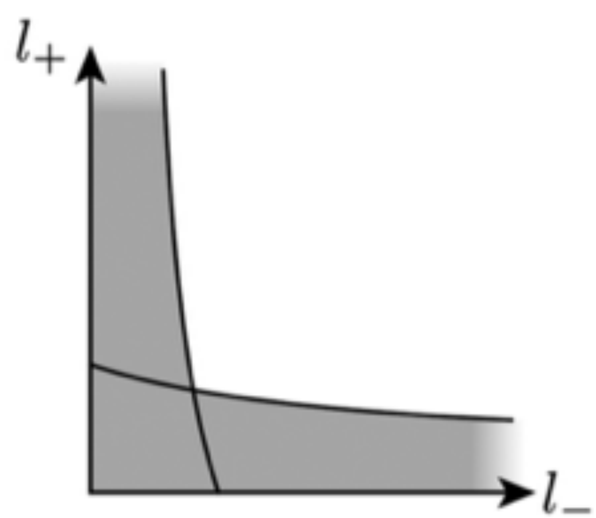
$1 < \alpha < 2$



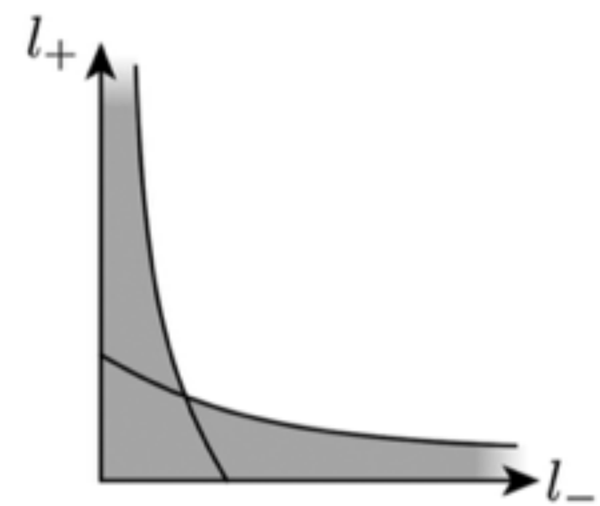
$\alpha > 2$

- Similarly, one can derive the soft constraint function as follows.

$$y_c > y_{qg} = \begin{cases} 2^{2-\alpha}\dfrac{l_+(l_+ + l_-)^{\alpha-1}}{Q^\alpha}, & n-\text{collinear}, \\ 2^{2-\alpha}\dfrac{l_-(l_+ + l_-)^{\alpha-1}}{Q^\alpha}, & \bar{n}-\text{collinear}. \end{cases}$$



$1 < \alpha < 2$

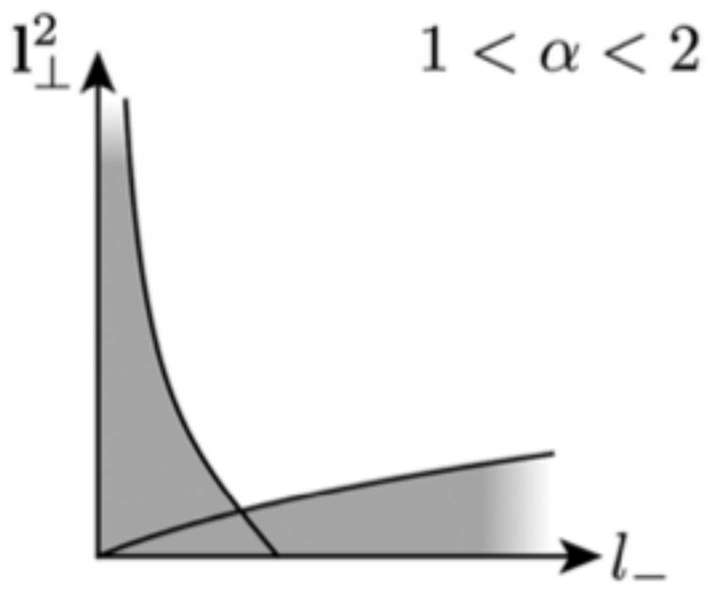


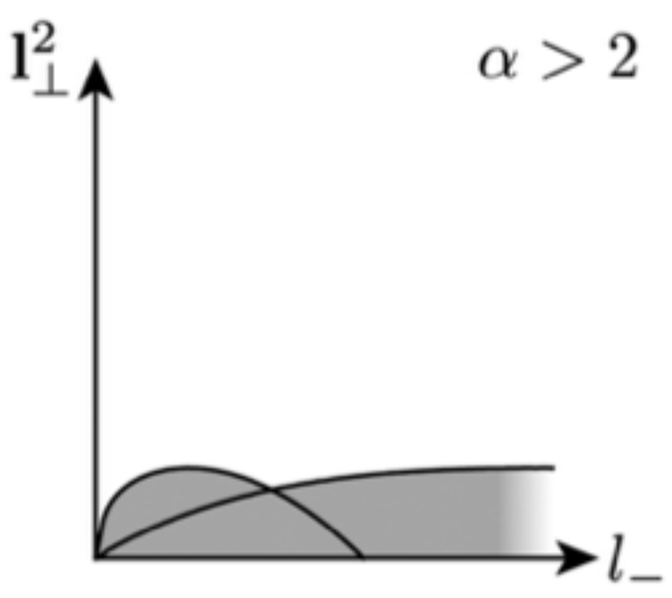
$\alpha > 2$

Generalized kt algorithm - Constraint functions

- Similarly, one can derive the soft constraint function as follows.

$$y_c > y_{qg} = \begin{cases} 2^{2-\alpha} \dfrac{l_+ (l_+ + l_-)^{\alpha-1}}{Q^\alpha}, & n - \text{collinear}, \\ 2^{2-\alpha} \dfrac{l_- (l_+ + l_-)^{\alpha-1}}{Q^\alpha}, & \bar{n} - \text{collinear}. \end{cases}$$





- With the generalized jet algorithm, we can calculate the previous integral.

$$M_{\text{jet}}^{(b),1<\alpha<2} = \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\alpha-2}\left(-\frac{1}{\epsilon_{\text{UV}}^2} - \frac{1}{\epsilon_{\text{UV}}}\ln\frac{\mu^2}{2^{\alpha-2}y_c Q^2} - \frac{1}{2}\ln^2\frac{\mu^2}{2^{\alpha-2}y_c Q^2} + \frac{\pi^2}{12}\right)\right.$$
$$+\frac{1}{\epsilon_{\text{UV}}} + \ln\frac{\mu^2}{2^{\alpha-2}y_c Q^2} - \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right)\left(\frac{1}{\epsilon_{\text{UV}}} + 1 + \ln\frac{\mu}{Q}\right)$$
$$\left. + (4-\alpha)\left(1-\frac{\pi^2}{12}\right) - \alpha\ln 2\right].$$

$$M_{\text{jet}}^{\text{v}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right)\left(\frac{1}{\epsilon_{\text{UV}}} + 1 + \ln\frac{\mu}{Q}\right)$$

Generalized kt algorithm - Real gluon emissions 2

- $\alpha > 2$ case is IR divergent.

$$M_{\text{jet}}^{(b),\alpha>2} = \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\alpha-2}\left(-\frac{1}{\epsilon_{\text{IR}}^2} - \frac{1}{\epsilon_{\text{IR}}}\ln\frac{\mu^2}{2^{\alpha-2}y_c Q^2} - \frac{1}{2}\ln^2\frac{\mu^2}{2^{\alpha-2}y_c Q^2} + \frac{\pi^2}{12}\right)\right.$$
$$+ \frac{1}{\epsilon_{\text{IR}}} + \ln\frac{\mu^2}{2^{\alpha-2}y_c Q^2}$$
$$\left. + (4-\alpha)\left(1-\frac{\pi^2}{12}\right) - \alpha\ln 2\right].$$

$$M_{\text{jet}}^{\text{v}} = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right)\left(\frac{1}{\epsilon_{\text{UV}}} + 1 + \ln\frac{\mu}{Q}\right)$$

Generalized kt algorithm - Real gluon emissions 3

- The soft contribution has similar features.

$$M_{\rm soft}^{1<\alpha<2} = \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\alpha-2}\left(\frac{2}{\epsilon_{\rm UV}^2} - \frac{2}{\epsilon_{\rm UV}}\ln 2^{\alpha-2} y_c \left(\frac{Q}{\mu}\right)^\alpha + \ln^2 2^{\alpha-2} y_c \left(\frac{Q}{\mu}\right)^\alpha - \frac{\pi^2}{6}\right)\right.$$
$$\left. + \frac{1}{\epsilon_{\rm UV}^2} + \left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right)^2 - \frac{1}{\alpha}\ln^2 2^{\alpha-2} y_c \left(\frac{Q}{\mu}\right)^\alpha + \frac{\pi^2}{2}\left(\frac{1}{3\alpha} - \frac{1}{2}\right)\right],$$
$$M_{\rm soft}^{\alpha>2} = \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\alpha-2}\left(\frac{2}{\epsilon_{\rm IR}^2} - \frac{2}{\epsilon_{\rm IR}}\ln 2^{\alpha-2} y_c \left(\frac{Q}{\mu}\right)^\alpha + \ln^2 2^{\alpha-2} y_c \left(\frac{Q}{\mu}\right)^\alpha - \frac{\pi^2}{6}\right)\right.$$
$$\left. + \frac{1}{\epsilon_{\rm IR}^2} - \frac{1}{\alpha}\ln^2 2^{\alpha-2} y_c \left(\frac{Q}{\mu}\right)^\alpha + \frac{\pi^2}{2}\left(\frac{1}{3\alpha} - \frac{1}{2}\right)\right].$$

$$M_{\rm soft}^{\rm v} = -\frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right)^2$$

Conclusions

- With the exclusive jet algorithm, there is an integral which cannot be regularized by dimensional regularization.
- We can investigate the divergence structure of the kt algorithm as a limiting behavior of the generalized kt algorithm.
- We have IR finite jet and soft functions for α is smaller than 2, but, for $\alpha > 2$, each factorized parts has IR divergence.
- Since the IR divergence remains for $\alpha > 2$, the factorization breaks down.

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