Dijet Event Shapes at the LHC in SCET

Yiannis Makris
Duke University

In collaboration with
Andrew Hornig (LANL) and Thomas Mehen (Duke U.)

[arXiv: 1601.01319]

Jets and Heavy Flavor, Jan. 11-13 2016, Santa Fe, NM

Outline

Problem Setup - Boost Invariant Jet Shapes

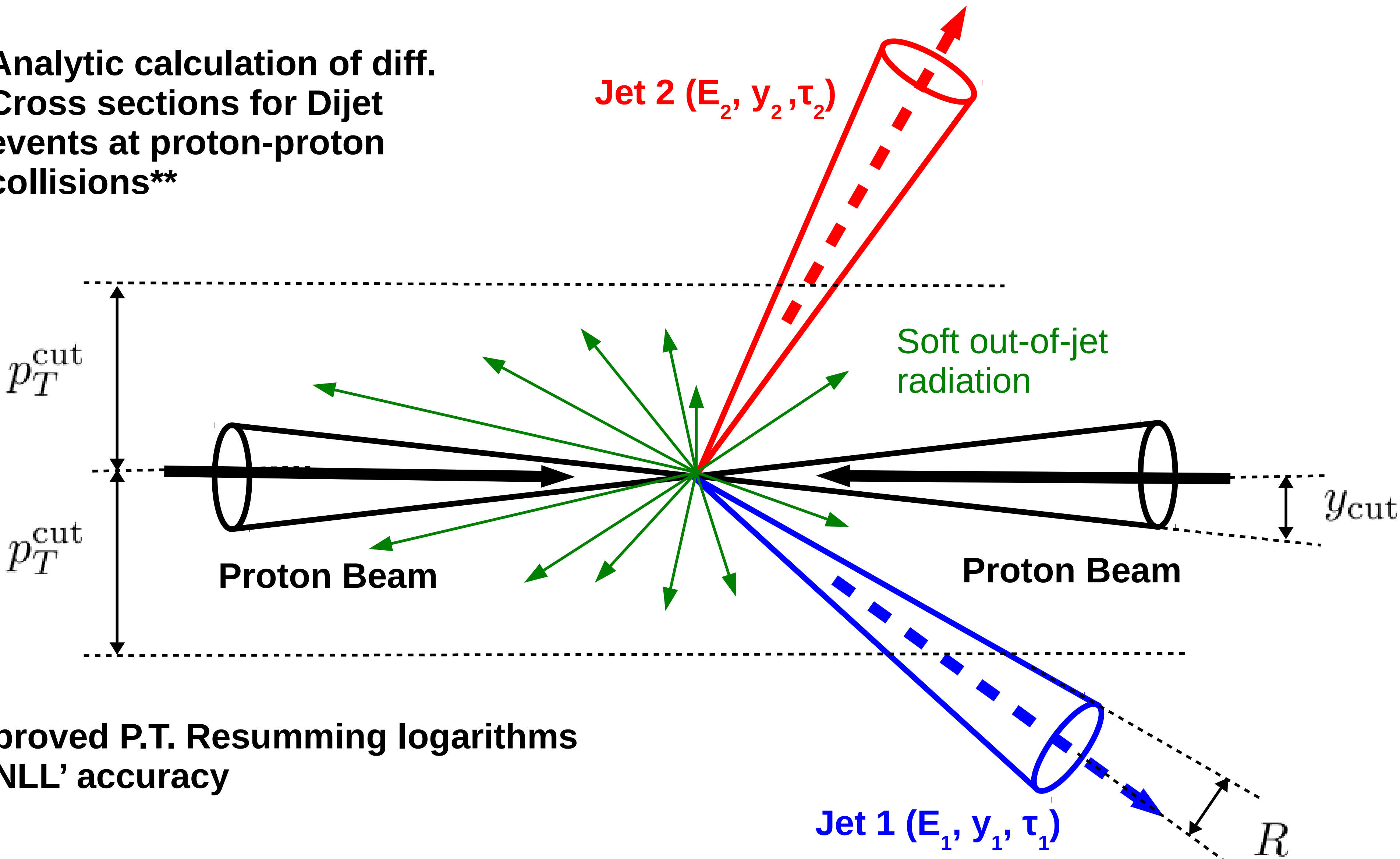
The Factorization Theorem in SCET (Jet, Hard, Beam and Soft Functions)

Scales and R.G. Evolution - Theoretical Uncertainties – Plots

Summary - Applications

Problem Setup

Analytic calculation of diff. Cross sections for Dijet events at proton-proton collisions**



Improved P.T. Resumming logarithms at NLL' accuracy

**Extension of the work on e+e- to N jets by Ellis, Vermilion, Walsh, Hornig and Lee, [arXiv: 1001.0014]

Angularities

Rotational invariant

Almeida et al. [arXiv: 0807.0234]
Berger, Kucs, and Sterman [hep-ph/ 0303051]

$$\tau_a^{e^+e^-} = \frac{1}{2E_J}\sum_i |p_T^{iJ}|e^{-(1-a)|y_{iJ}|}$$

$$= (2E_J)^{-(2-a)}(p_T)^{1-a}\sum_i |p_T^i|\left(\frac{\theta_{iJ}}{\sin\theta_J}\right)^{2-a}(1+\mathcal{O}(\theta_{iJ}^2))$$

$$\tau_a \equiv \tau_a^{pp} \equiv \frac{1}{p_T}\sum_i |p_T^i|(\Delta\mathcal{R}_{iJ})^{2-a}$$

Boost invariant

$$= \left(\frac{2E_J}{p_T}\right)^{2-a}\tau_a^{e^+e^-} + \mathcal{O}(\tau_a^2)$$

$$\text{where} \quad \Delta\mathcal{R}_{ij} \equiv \sqrt{(\Delta y_{ij})^2 + (\Delta\phi_{ij})^2}$$

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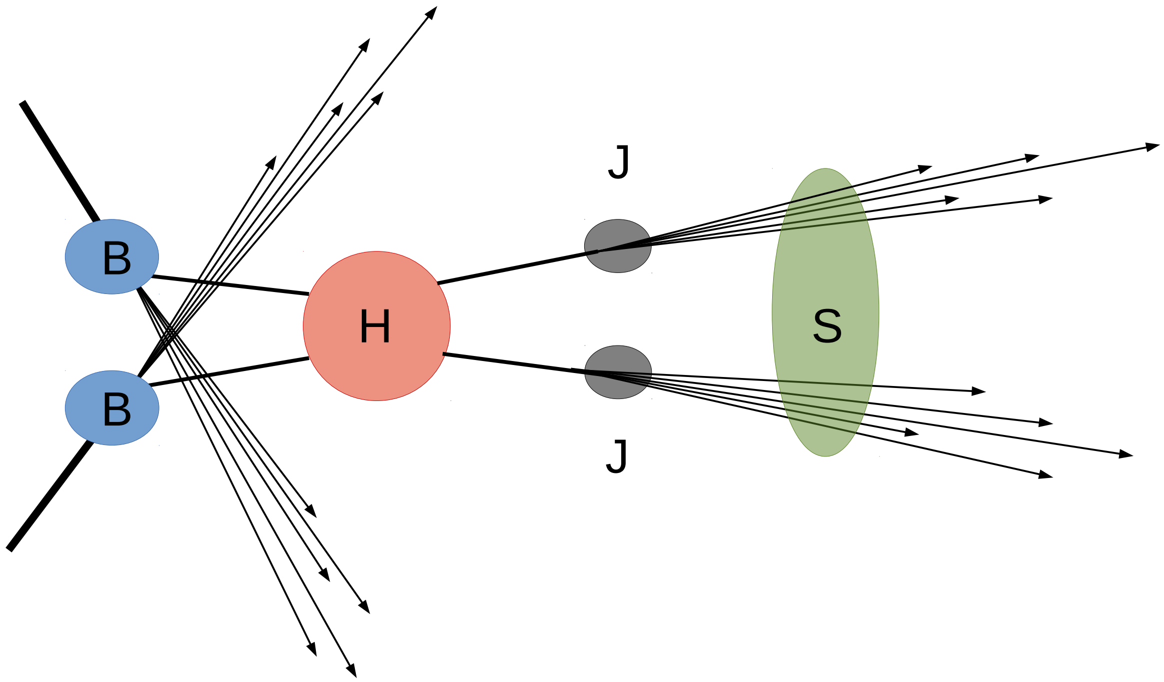
The Factorization Theorem in SCET

$$d\sigma(\tau_a^1, \tau_a^2) \equiv \frac{d\sigma}{dy_1 dy_2 dp_T d\tau_a^1 d\tau_a^2}$$

$$= \frac{p_T}{8\pi x_1 x_2 E_{\rm cm}^4} \frac{1}{N} B(x_1, \mu) \bar{B}(x_2, \mu)\, \mathrm{Tr}\{\mathbf{H}(\mu)\mathbf{S}(\tau_a^1, \tau_a^2, \mu)\} \otimes [J_1(\tau_a^1, \mu) J_2(\tau_a^2, \mu)]$$



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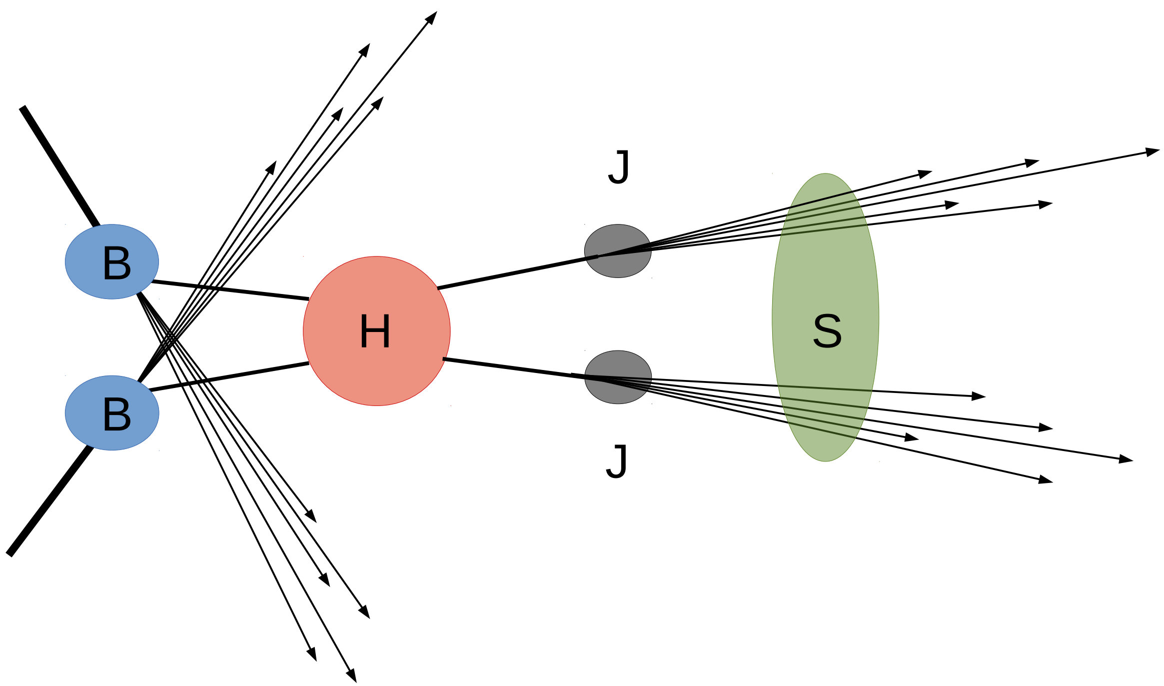
Hard
Function



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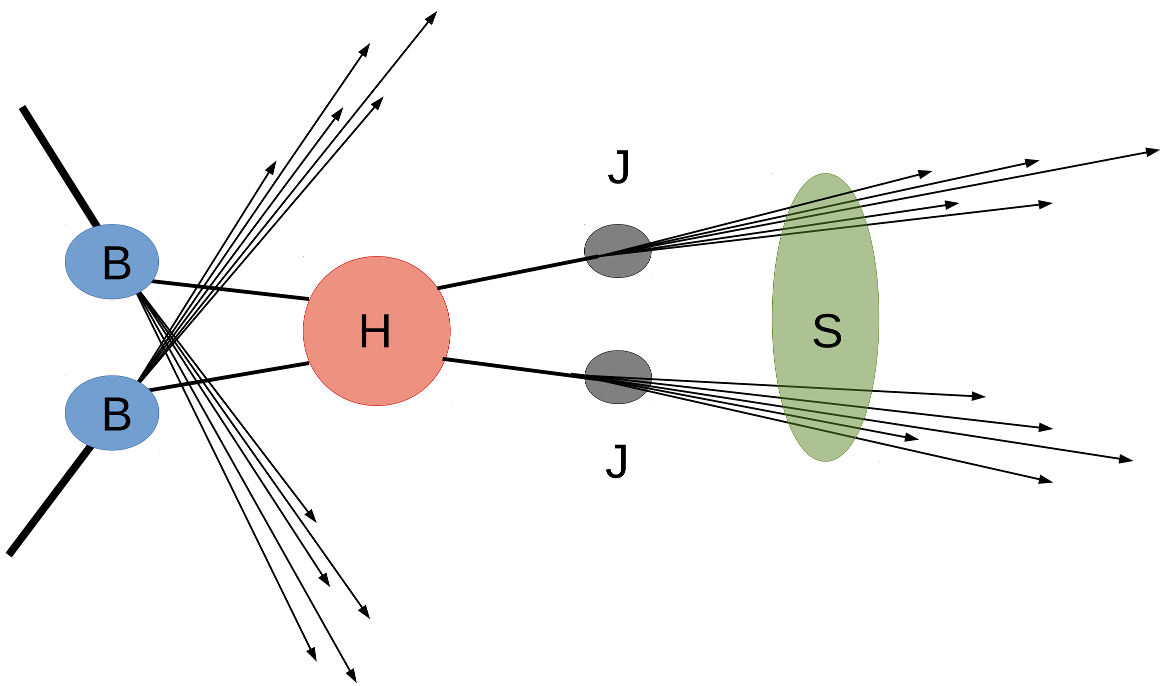
Hard Function

Soft Function



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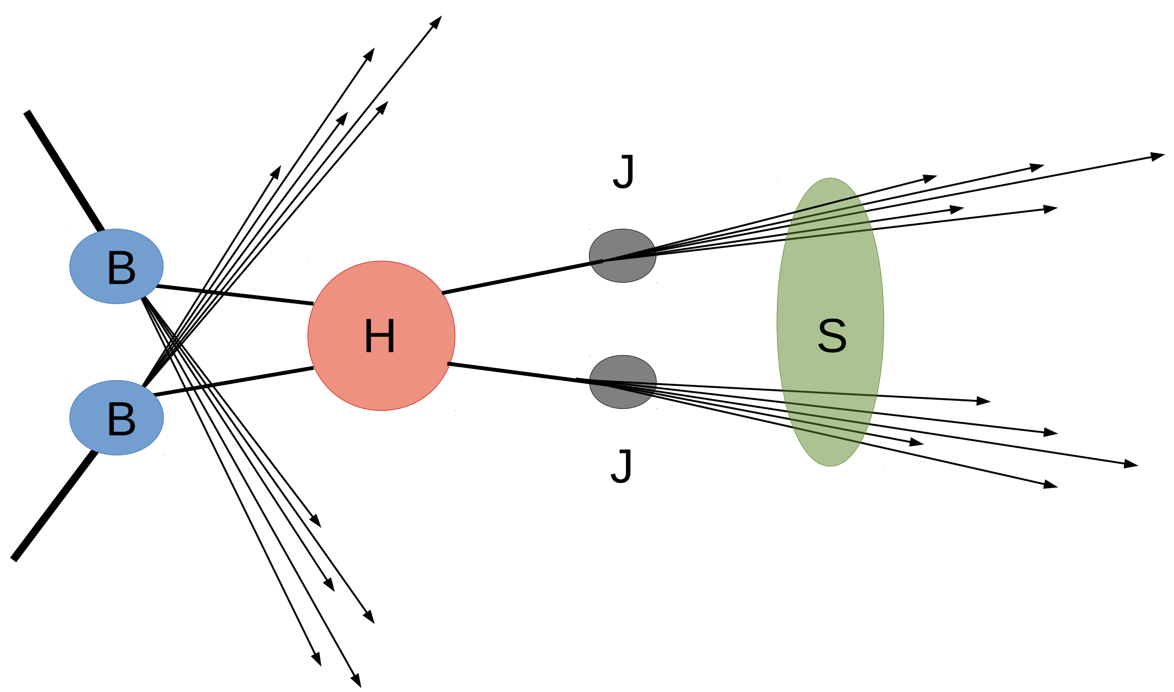
Hard Function | **Soft Function** | **Jet Functions**



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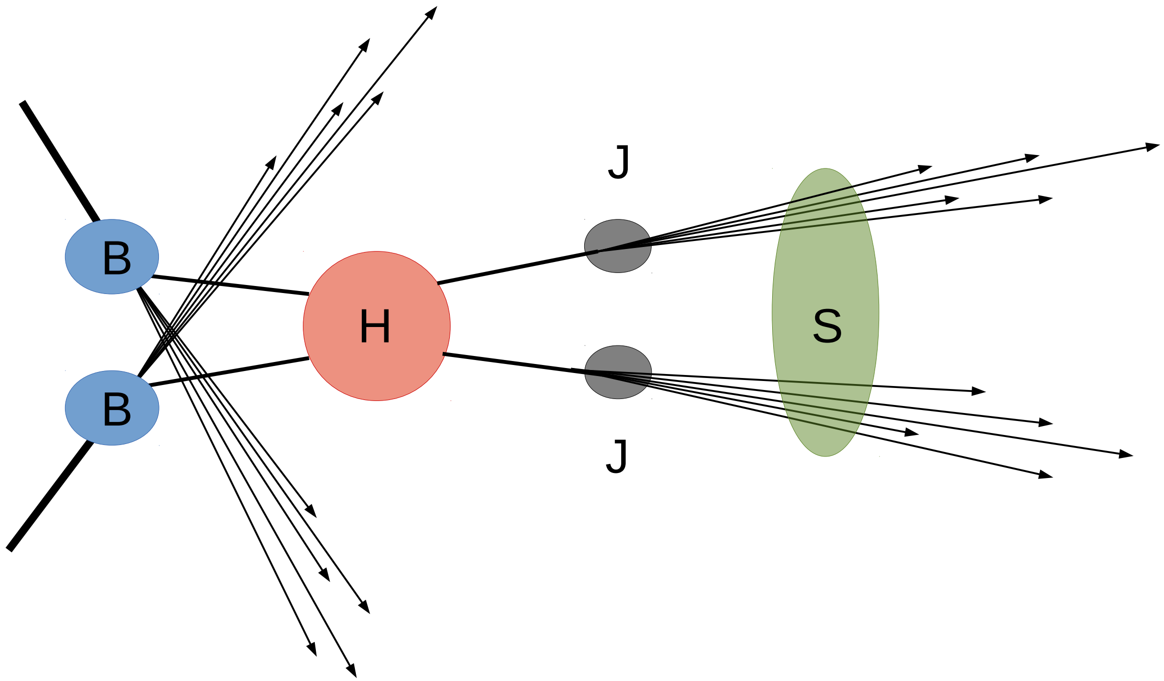
Beam Functions

Hard Function

Soft Function

Jet Functions



Jet Functions

$$\int \frac{dk}{(2\pi)^4} \exp(-ik\cdot x) J_{n,\omega}(\tau, k^-) \left(\frac{\not{n}}{2}\right)_{\alpha\beta} = \langle \Omega | \chi^{\alpha}_{n,\omega}(x) \delta(\tau - \hat{\tau}) \bar{\chi}^{\beta}_{n,\omega}(0) | \Omega \rangle$$

Quark Jet Function

$$A^{-1}\delta\left(A^{-1}\tau - \hat{\tau}\right) = \delta\left(\tau - A\hat{\tau}\right)$$

Similarly for Gloun Jets

$$\longrightarrow \quad J_i(\tau_a) = \left(\frac{p_T}{2E_J}\right)^{2-a} J_i^{e^+e^-}\left(\left(\frac{p_T}{2E_J}\right)^{2-a} \tau_a\right)$$

Ellis, Vermilion, Walsh, Hornig and Lee, [arXiv: 1001.0014]

Hard Function

$$H_{IJ}(\mu) = C_I(\mu) C_J^*(\mu)$$

$C_I(\mu)$ **Wilson Coefficients**

Kelley and Schwartz [arXiv: 1008.2759]

$$\frac{d\,\mathbf{H}}{d\ln\mu} = \mathbf{\Gamma}_H\,\mathbf{H} + \mathbf{H}\,\mathbf{\Gamma}_H^\dagger$$

$$\mathbf{\Gamma}_H = \frac{1}{2}\Gamma_H \mathbf{1} + \Gamma_c \mathbf{M}(m_i)$$

$$\mathbf{H}(\mu, \mu_H) = \Pi_H(\mu, \mu_H)\,\mathbf{\Pi}_H(\mu, \mu_H)\mathbf{H}(\mu_H)\mathbf{\Pi}_H^\dagger(\mu, \mu_H)$$

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Beam Function ("Unmeasured")

Ritzmann and Waalewijn [arXiv:1407.3272]
Stewart, Tackmann and Waalewijn [arXiv:0910.0467]

$$B_i(x_i,\mu) \equiv B_i(\mathrm{E}_{\mathrm{cm}}, y_{\mathrm{cut}}, x_i, \mu)$$

$$= \sum_j \int \frac{dz}{z} \mathcal{J}_{ij}(x_i \mathrm{E}_{\mathrm{cm}} e^{-y_{\mathrm{cut}}}, z, \mu) f_j(x_i/z, \mu) + \mathcal{O}(\Lambda^2_{\mathrm{QCD}}/E^2)$$

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Short Distance Matching Coefficients.

Procura and Waalewijn, [arXiv: 1111.6605]

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Parton Distribution Functions (PDF)

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Parton Distribution Functions (PDF)

Procura and Waalewijn, [arXiv: 1111.6605]

$$\mu_J = 2E\tan(R/2) \longrightarrow \mu_{B_i} = x_i E_{\mathrm{CM}} \exp(-y^{\mathrm{cut}})$$

$$\Rightarrow \gamma_B(\mu_B, \mu) = \gamma_J(\mu_B, \mu)$$

Soft Function

$$S_{IJ}(\tau_1, \tau_2) = \mathcal{N} \sum_X \langle \Omega | \mathcal{W}_I^\dagger | X \rangle \langle X | \mathcal{W}_J | \Omega \rangle \; \delta(\tau_1 - \tau_1^X) \delta(\tau_2 - \tau_2^X)$$

$$\text{where} \quad \tau_i^X = \begin{cases} \tau^X & : \text{inside jet} \;\; \text{i} \\ 0 & : \text{outside jet} \;\; \text{i} \end{cases}$$

Soft Function

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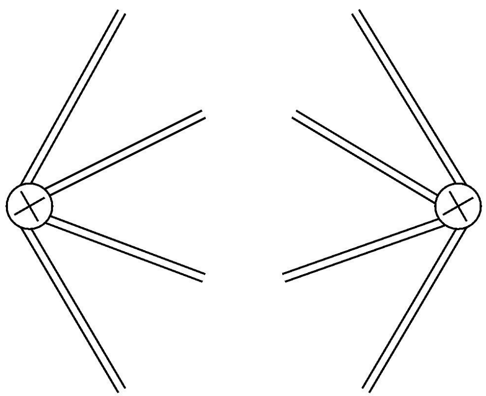
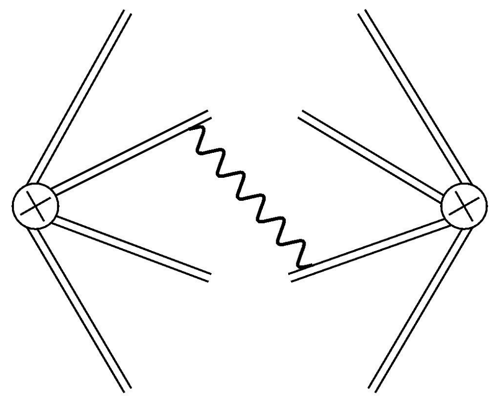
Time ordered product of Wilson lines.

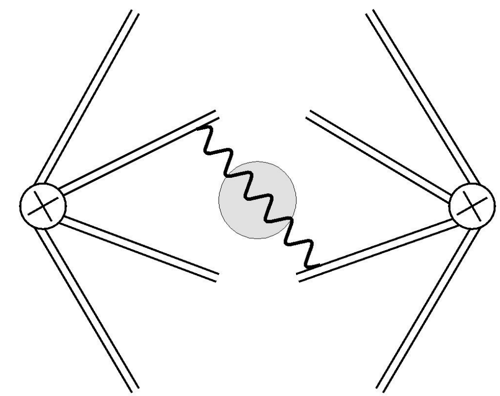
$$\text{where} \quad \tau_i^X = \begin{cases} \tau^X & : \text{inside jet} \;\; i \\ 0 & : \text{outside jet} \;\; i \end{cases}$$



Leading Order (LO) contribution



Next to Leading Order (NLO) contribution outside Jets



Next to Leading Order (NLO) contribution inside Jets

Next to Leading Order Form of the Soft Function

2-measured 0-unmeasured Jets

$$\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}}\delta(\tau_a^1)\delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1)\delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)$$

1-measured 1-unmeasured Jets

$$\mathbf{S}(\tau_a) = \mathbf{S}^{\text{unmeas}}\delta(\tau_a) + \mathbf{S}_0 S^{\text{meas}}(\tau_a) + \mathcal{O}(\alpha_s^2)$$

0-measured 2-unmeasured Jets

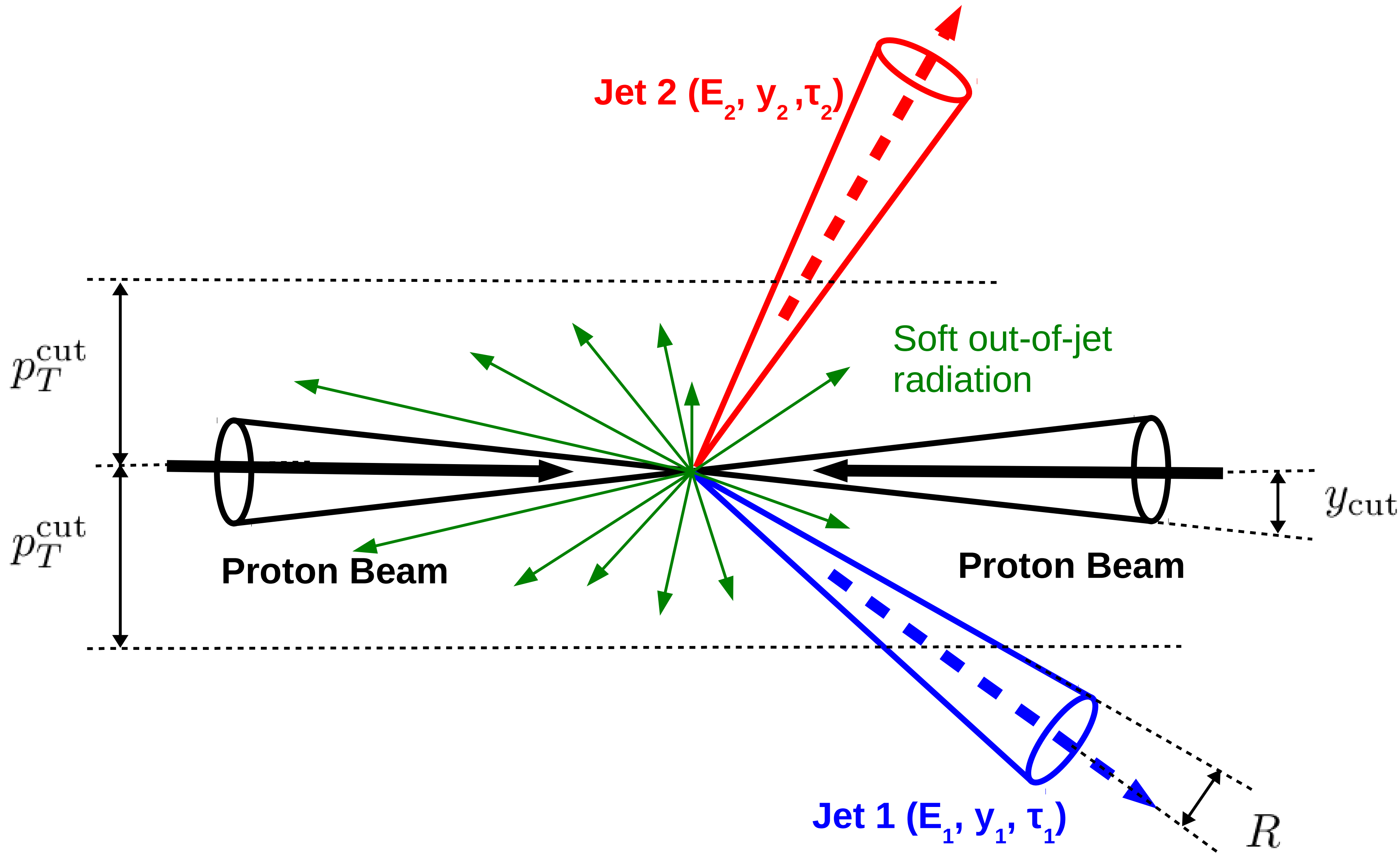
$$\mathbf{S} = \mathbf{S}^{\text{unmeas}} + \mathcal{O}(\alpha_s^2)$$

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \mathcal{O}(\alpha_s)$$

$$[\mathbf{S}_0]_{IJ} = \mathcal{N} \sum_X \langle \Omega | \mathcal{W}_I^\dagger | X \rangle \langle X | \mathcal{W}_J | \Omega \rangle \Big|_{Y_i \to \mathbf{1}} = \mathcal{N} \, \mathrm{tr}[T_I T_J] \mathrm{tr}[T_I T_J]$$

Phase-Space of Integration

$$\int \frac{d^d k}{(2\pi)^{d-1}} \; \delta(k^2)\Theta(E>0)\Theta(k_T < p_T^{\mathrm{cut}})\Theta(\text{out of Jets and Beams})$$



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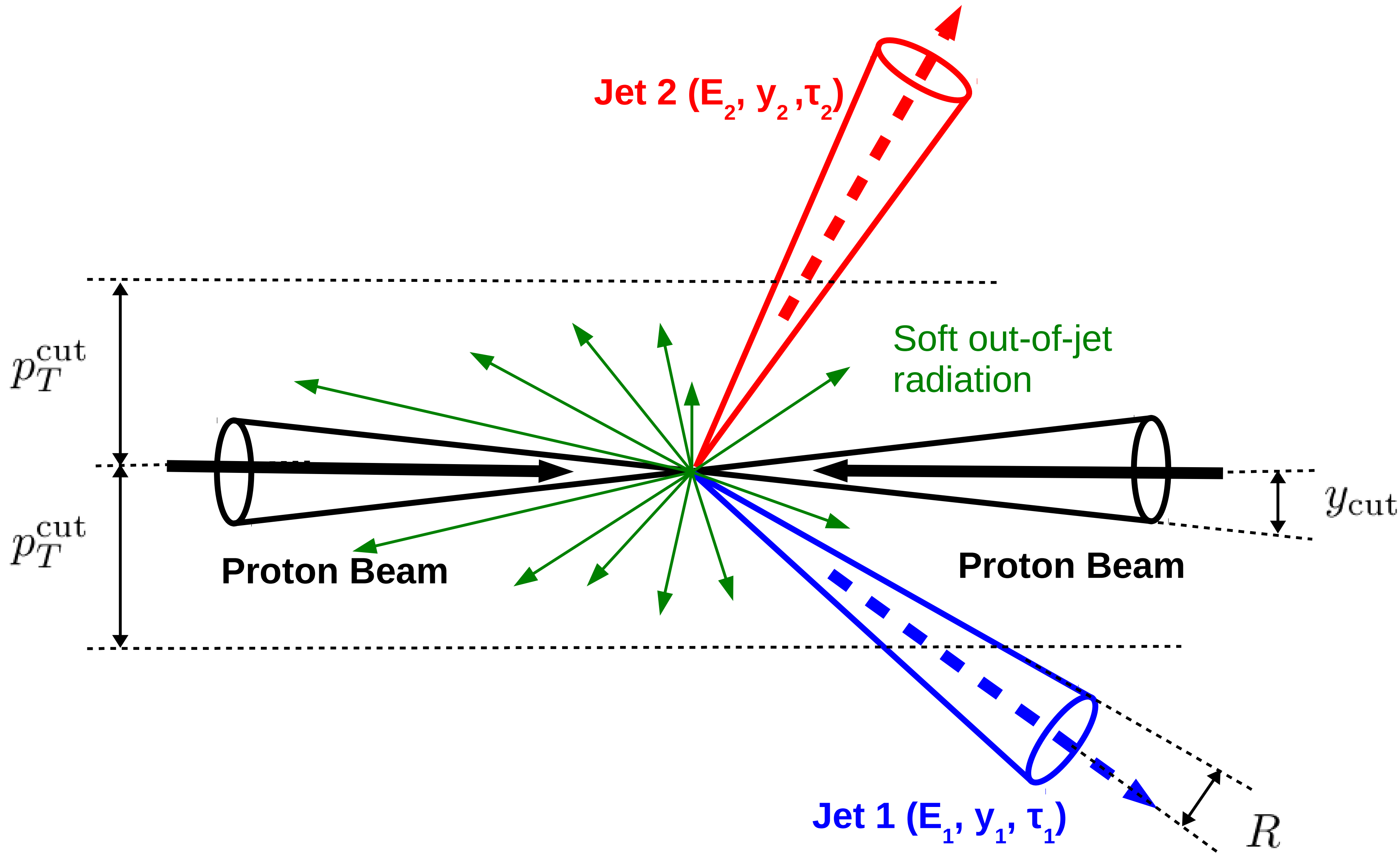

Unmeasured Evolution

$$\frac{d}{d\ln\mu}\mathbf{S}^{\mathrm{unmeas}} = \mathbf{S}^{\mathrm{unmeas}}\mathbf{\Gamma}_S^{\mathrm{unmeas}} + \mathrm{h.c.}$$

$$\mathbf{\Gamma}_S = \frac{1}{2}\Gamma_S\mathbf{1} - \Gamma_c\mathbf{M}(m_i)$$

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Unmeasured Evolution

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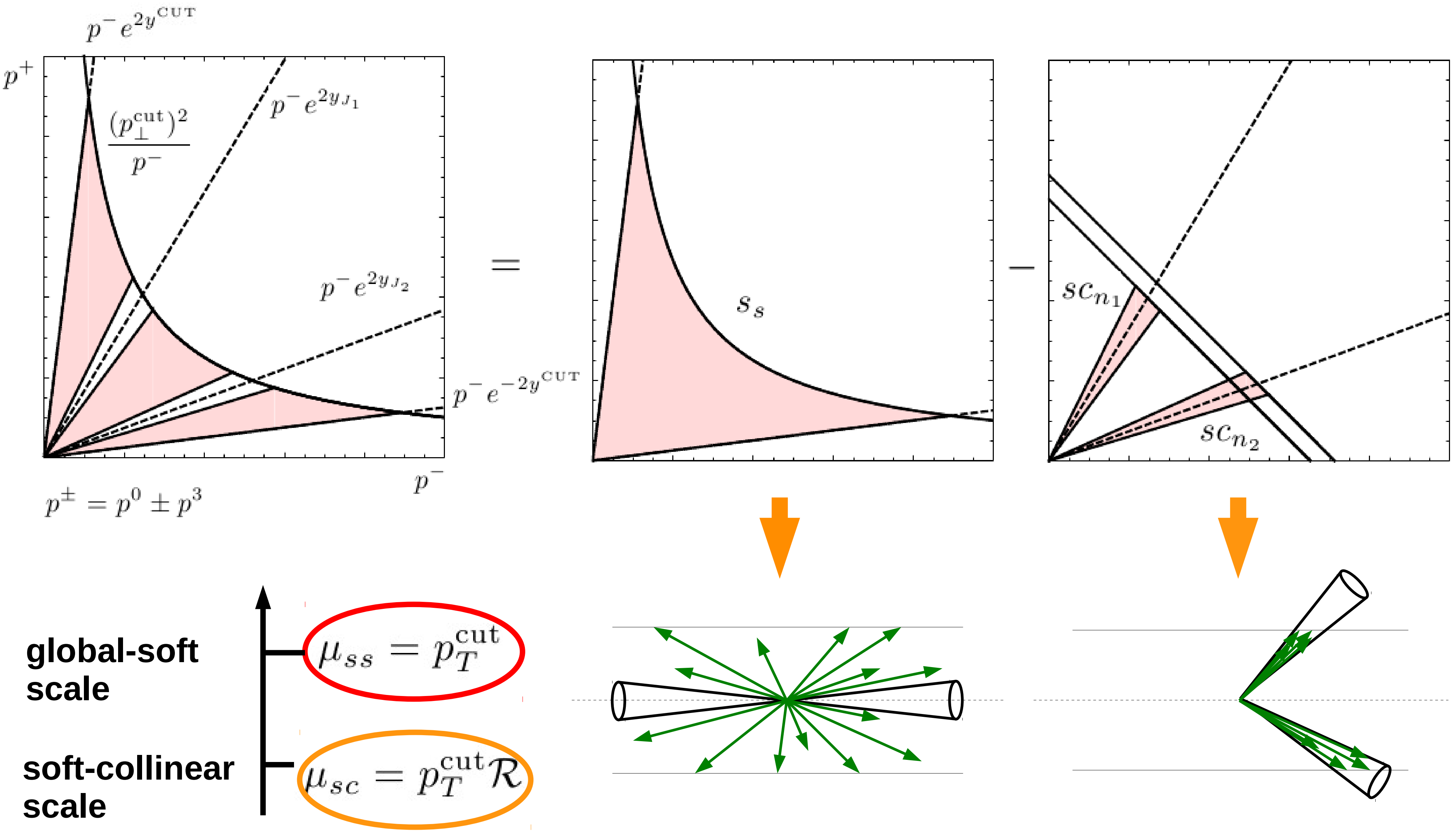
$$\boldsymbol{\Gamma}_S = \frac{1}{2}\Gamma_S\mathbf{1} - \Gamma_c\mathbf{M}(m_i)$$

$$\boldsymbol{\Gamma}_H = \frac{1}{2}\Gamma_H\mathbf{1} + \Gamma_c\mathbf{M}(m_i)$$

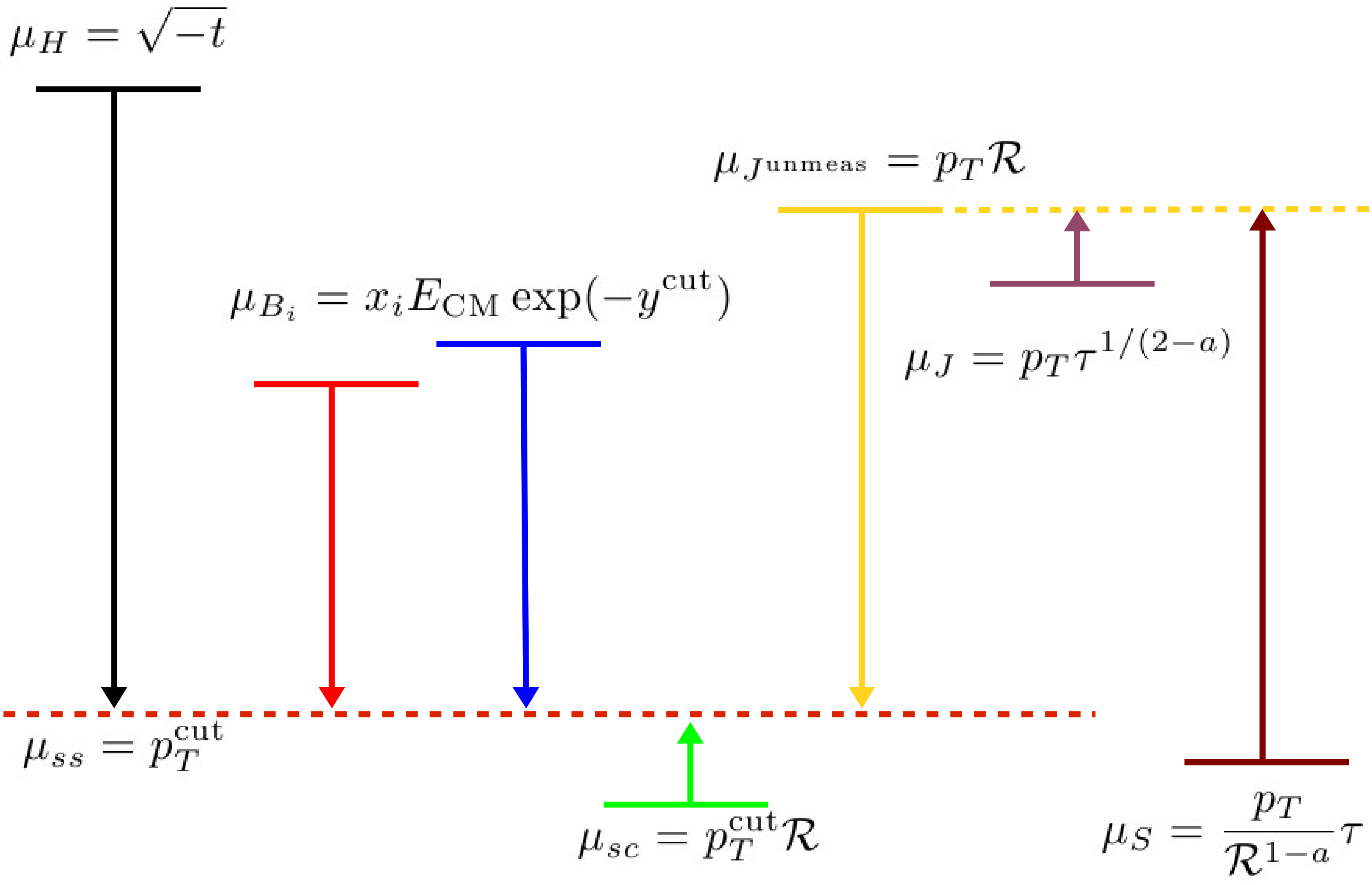
Necessary for the cancellation of μ dependence in the cross section

Soft-Collinear Refactorization

Chien, Hornig, and Lee, [arXiv:1509.04287]



Scales and R.G. Evolution



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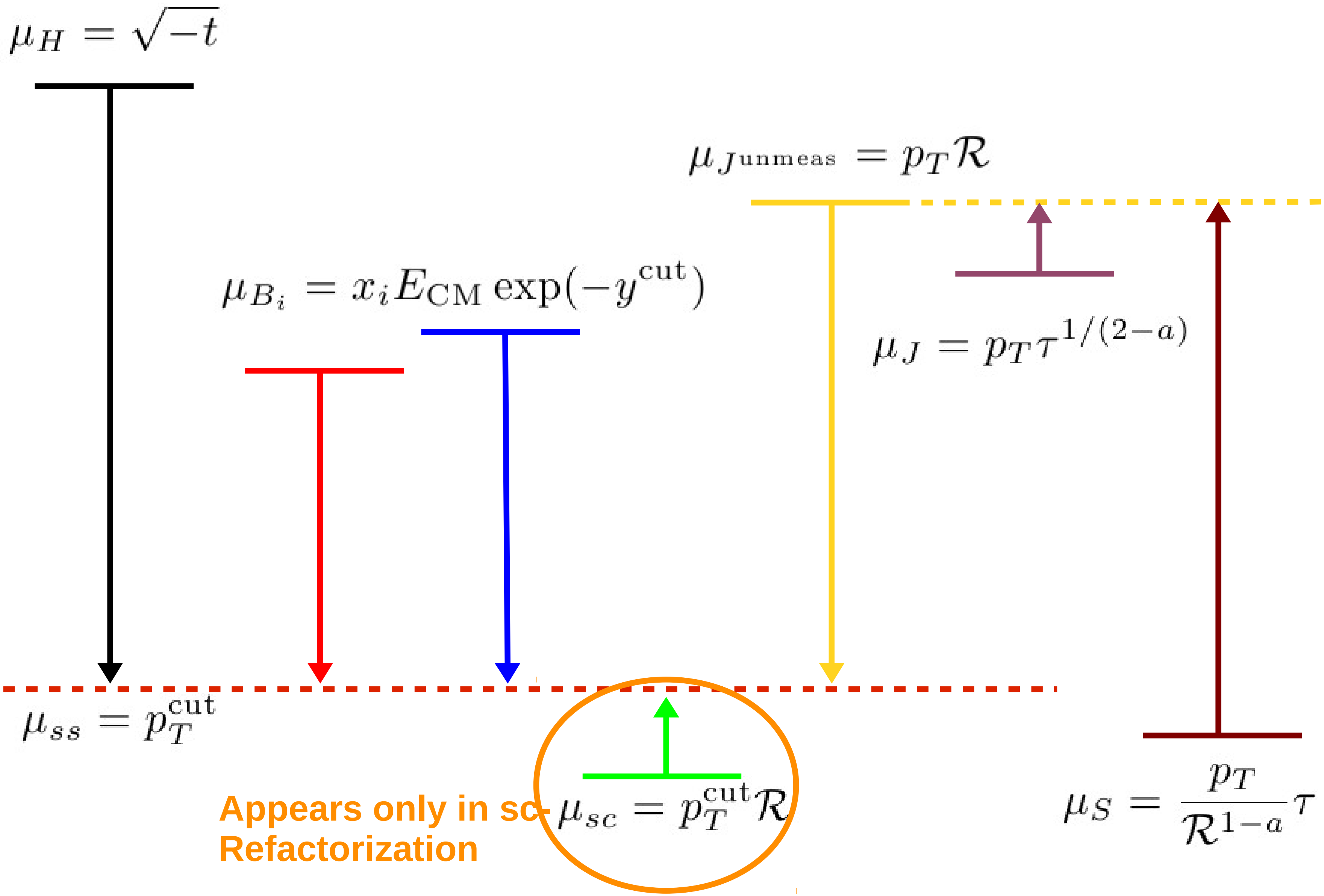


Theoretical Uncertainties

Variation of the characteristic scales

Hard

Soft (Unmeasured)

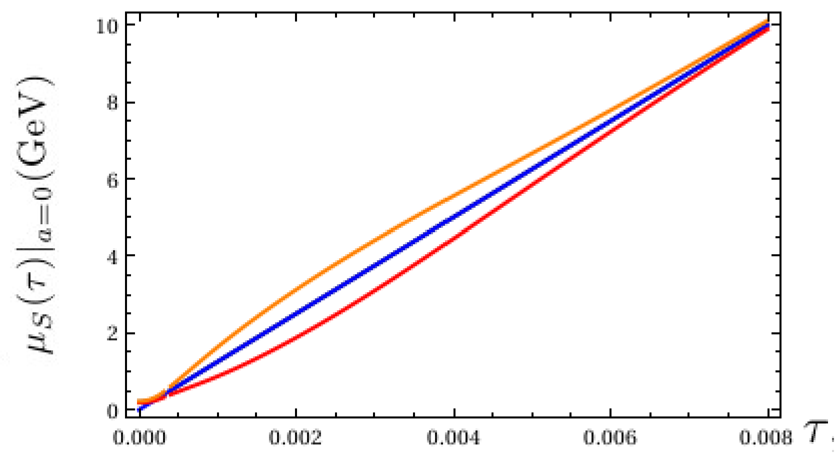
Beam

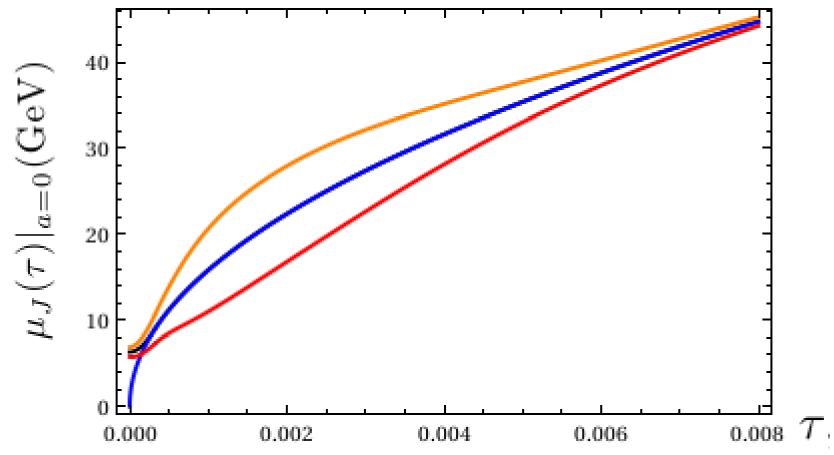
± 50 %

Jet (Measured)

Soft (Measured)

Profile Functions





Ligeti, Stewart and Tackmann
[arXiv: 0807.1926]

Plots

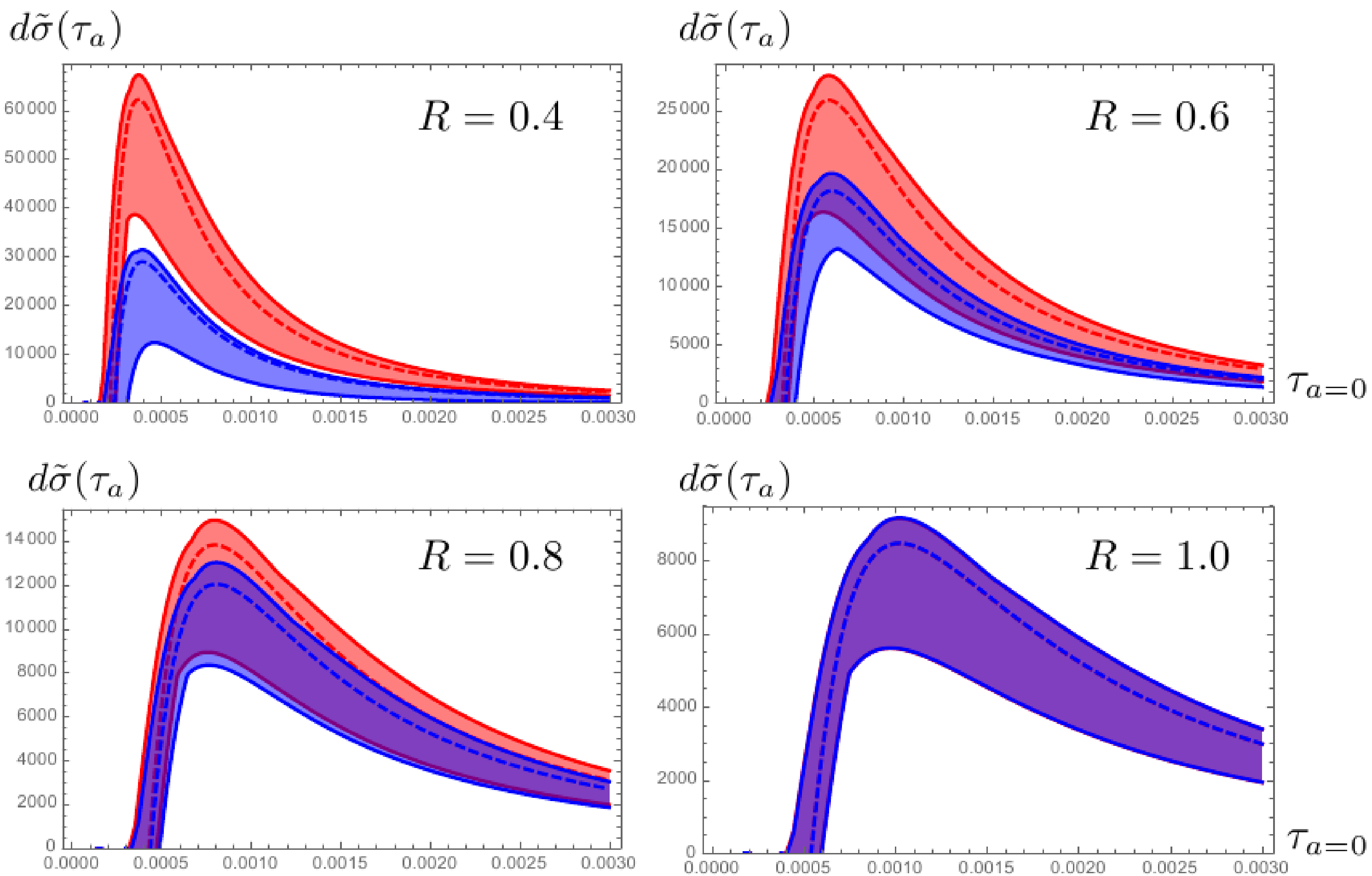
$$\boxed{d\tilde{\sigma}(\tau_a) \equiv \frac{B(x_1, \mu = \mu_H)\bar{B}(x_2, \mu = \mu_H)}{B(x_1, \mu = \mu_B^1)\bar{B}(x_2, \mu = \mu_B^2)} \frac{d\sigma(\tau_a^1, \tau_a^2)}{\sigma^{\mathrm{LO}}(\mu = \mu_H)}\bigg|_{\tau_a^1 = \tau_a^2 = \tau_a}}$$

$\boldsymbol{\tau_a^1 = \tau_a^2 = \tau_a}$

Partonic Channel: qq' $\longrightarrow$ qq'

$\mathrm{E_{cm}} = 10\ \mathrm{TeV}$	$y_1 = 1.0$	$p_T = 500\ \mathrm{GeV}$	$R = 0.6$
$a = 0$	$y_2 = 1.4$	$p_T^{\mathrm{cut}} = 20\ \mathrm{GeV}$	$y_{\mathrm{cut}} = 5.0$

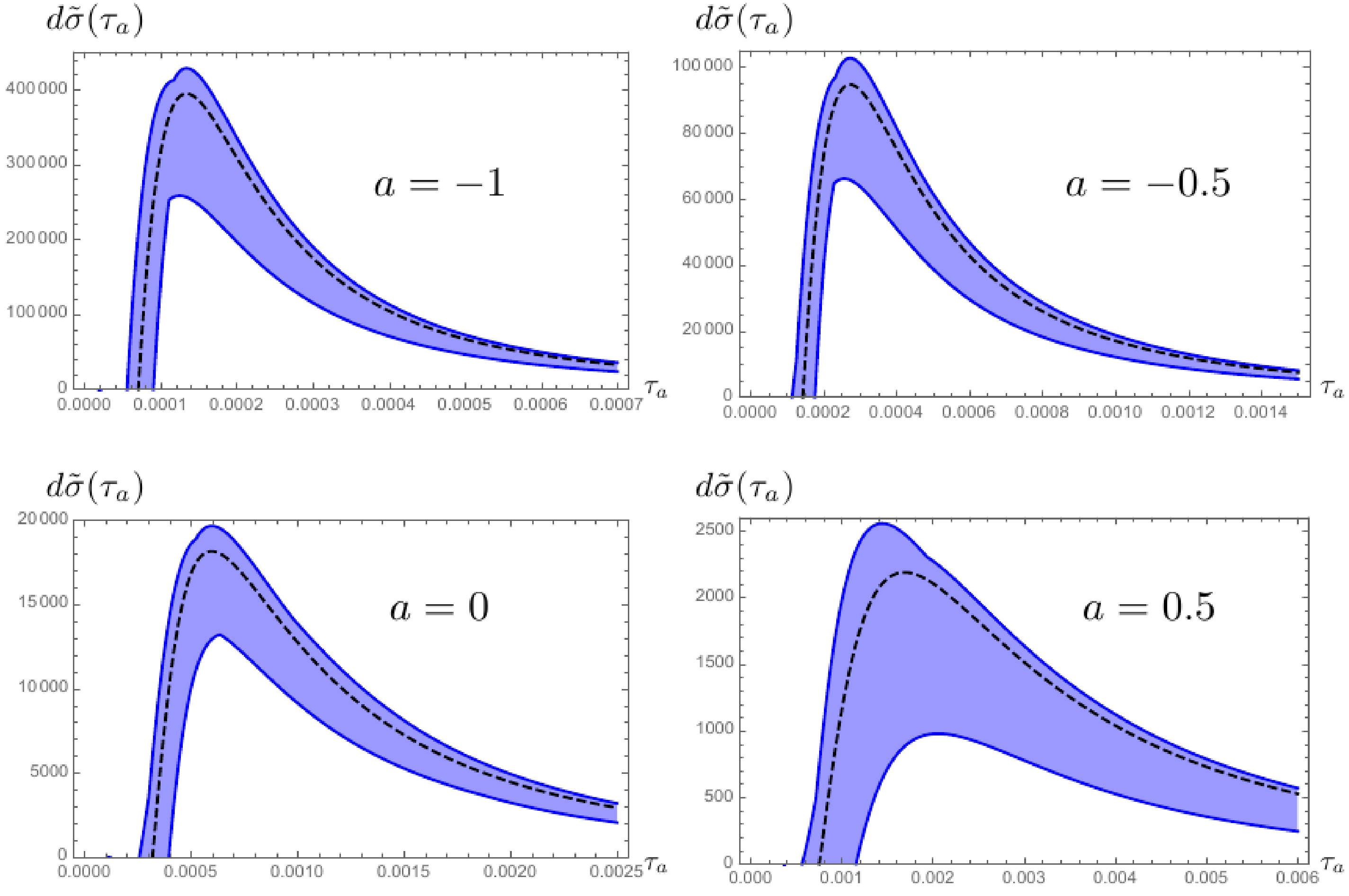
Plots - Variation of cone size R



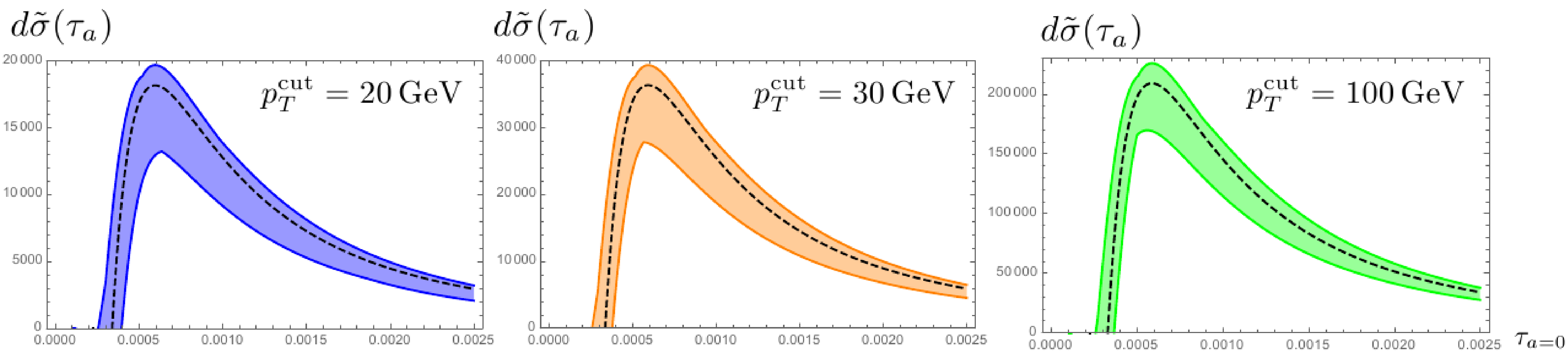
Without S-C Refactorization

With S-C Refactorization

Plots - Variation of a



Plots - Variation of p_T^{cut}



Increase of p_T^{cut} corresponds to increase of normalization

Peak location and shape independent of p_T^{cut}

Non-Global-Logarithms : $\alpha_s^n \ln^n(p_T^{\rm cut}\, \mathcal{R}^2 / p_T^J\, \tau_a)$ not included

Summary

Establish framework for calculation of dijet events in proton-proton collisions with a veto on out-of-jet transverse momentum radiation and rapidity constrains

Calculate differential cross section at NLL’ accuracy

Apply s-c refactorization for improved accuracy

Use profile functions for measured scale variation

Future Work

Apply to different partonic channels and compute physically observable cross section

NNLL calculation

Study other jet substructure observables

Exclusive cross sections for heavy meson and quarkonium production (In collaboration with Bain, Dai, Hornig, Leibovich, Mehen)

Compare to Monte Carlo simulations and experimental data

Thank you!

Scales and R.G. Evolution (2/3)

$$\frac{d}{d\ln\mu}F(\mu) = \left(\Gamma_F[\alpha]\ln\frac{\mu^2}{m_F^2} + \gamma_F[\alpha]\right)F(\mu)$$

$$F(\mu) = \exp[K_F(\mu,\mu_0)]\left(\frac{\mu_0}{m_F}\right)^{\omega_F(\mu,\mu_0)}F(\mu_0)$$

Unmeasured

$$\frac{d}{d\ln\mu}F(\tau,\mu) = \left[\Gamma_F[\alpha]\left(\ln\frac{\mu^2}{m_F^2}\delta(\tau) - \frac{2}{j_F}\left[\frac{\Theta(\tau)}{\tau}\right]_+\right) + \gamma_F[\alpha]\delta(\tau)\right]\otimes F(\tau,\mu)$$

$$F(\tau,\mu) = \frac{\exp[K_F(\mu,\mu_0) + \gamma_E\omega(\mu,\mu_0)]}{\Gamma(-\omega(\mu,\mu_0))}\left(\frac{\mu_0}{m_F}\right)^{j_F\omega_F(\mu,\mu_0)}\left[\frac{\Theta(\tau)}{(\tau)^{1+\omega(\mu,\mu_0)}}\right]_+\otimes F(\tau,\mu_0)$$

Measured

$$\omega_F(\mu,\mu_0) \equiv \frac{2}{j_F}\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta[\alpha]}\Gamma_F[\alpha]\,,$$

$$K_F(\mu,\mu_0) \equiv \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta[\alpha]}\gamma_F[\alpha] + 2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta[\alpha]}\Gamma_F[\alpha]\int_{\alpha_s(\mu_0)}^{\alpha}\frac{d\alpha'}{\beta[\alpha']}$$

Scales and R.G. Evolution (3/3)

	$\Gamma_F[\alpha_s]$	$\gamma_F[\alpha_s]$	j_F	m_F	μ_F
γ_H	$-\Gamma \sum_i C_i$	$-\sum_i \frac{\alpha_s}{\pi}\gamma_i$	1	$\prod_i m_i^{C_i/\sum_j C_j}$	m_i
$\gamma_{J_i}(\tau_a^i)$	$\Gamma C_i \frac{2-a}{1-a}$	$\frac{\alpha_s}{\pi}\gamma_i$	$2-a$	p_T	$p_T(\tau_a^i)^{1/(2-a)}$
$\gamma_S^{\rm meas}(\tau_a^i)$	$-\Gamma C_i \frac{1}{1-a}$	0	1	$p_T/\mathcal{R}^{1-a}$	$p_T\, \tau_a^i/\mathcal{R}^{1-a}$
γ_{J_i}	ΓC_i	$\frac{\alpha_s}{\pi}\gamma_i$	1	$p_T\mathcal{R}$	$p_T\mathcal{R}$
γ_{B_i}	ΓC_i	$\frac{\alpha_s}{\pi}\gamma_i$	1	$x_i \mathrm{E}_{\rm cm} e^{-y_{\rm cut}}$	$x_i \mathrm{E}_{\rm cm} e^{-y_{\rm cut}}$
$\gamma_S^{\rm unmeas}$	0	$\frac{2\alpha_s}{\pi}\Delta\gamma_{ss}(m_i)$ $+\frac{2\alpha_s}{\pi}(C_1+C_2)\ln\mathcal{R}$	1	—	$p_T^{\rm cut}$
γ_{ss}	$\Gamma(C_1+C_2)$	$\frac{2\alpha_s}{\pi}\Delta\gamma_{ss}(m_i)$	1	$p_T^{\rm cut}$	$p_T^{\rm cut}$
γ_{sc}^i	$-\Gamma C_i$	0	1	$p_T^{\rm cut}\mathcal{R}$	$p_T^{\rm cut}\mathcal{R}$

Profile Functions

$$\mu_S^i(\tau_a^i) = (1 + e_S g(\tau))\mu(\tau_a^i) \qquad \mu_J^i(\tau_a^i) = (1 + e_J g(\tau))\big(p_T \mathcal{R}\big)^{\frac{1-a}{2-a}}\big(\mu(\tau_a^i)\big)^{\frac{1}{2-a}}$$

$$\mu(\tau) = \begin{cases} \mu_0 + \alpha \tau^\beta \sqrt{-t}, & \tau < \tau^{\min} \\ \dfrac{p_T\, \tau}{\mathcal{R}^{1-a}}, & \tau > \tau^{\min}, \end{cases}$$

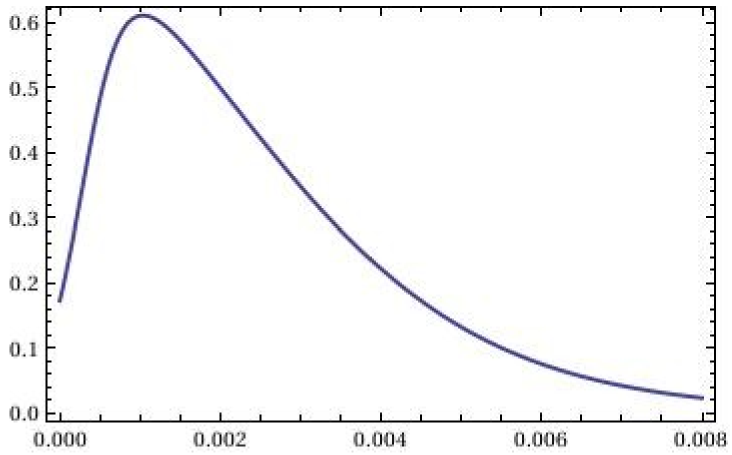
$$\alpha = \frac{p_T}{\beta (\tau^{\min})^{\beta-1} \mathcal{R}^{1-a} \sqrt{-t}}$$

$$\beta = \left(1 - \frac{\mu_0 R^{1-a}}{p_T \tau^{\min}}\right)^{-1},$$

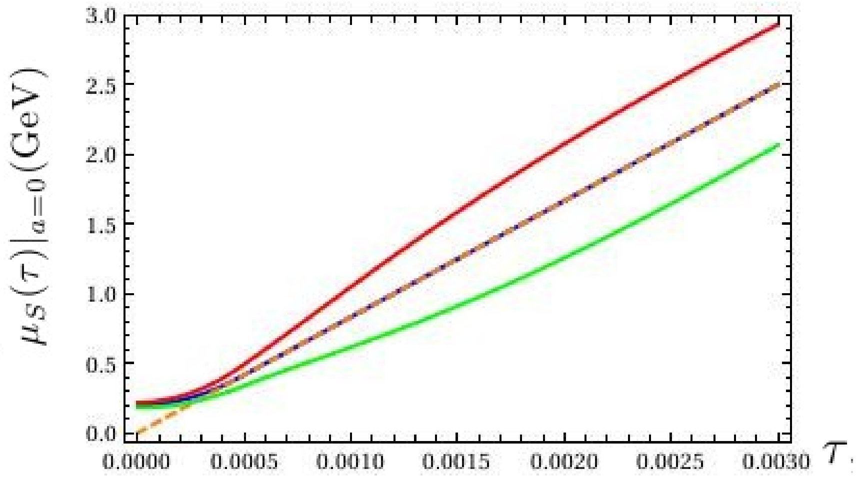
$$g(\tau) = \theta_{\epsilon_1}(\tau - \tau^{\min})\, \theta_{\epsilon_2}(\tau^{\max} - \tau)$$

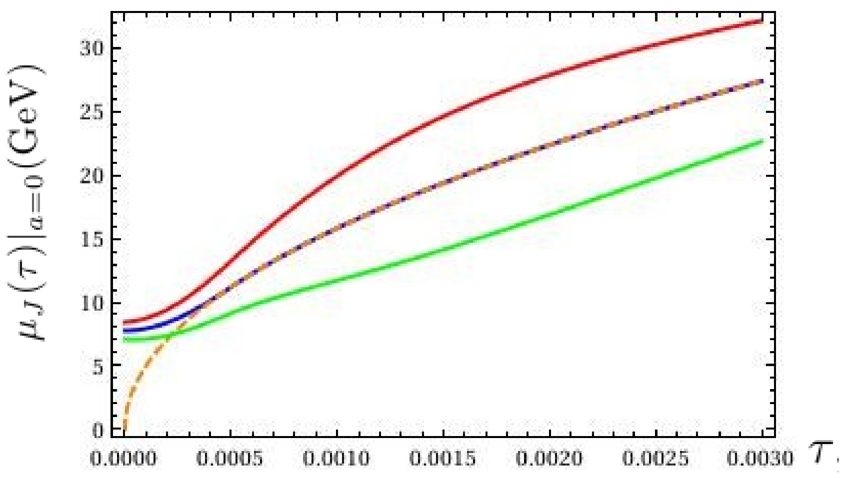
$$\theta_\epsilon(x) \equiv \frac{1}{1 + \exp\left(-x/\epsilon\right)}$$

$$g(\tau) = \theta_{\epsilon_1}(\tau - \tau^{\min})\, \theta_{\epsilon_2}(\tau^{\max} - \tau)$$



Profile Functions (2/2)





$$\tau^{\min} = 2(1-a)\mu_0 \mathcal{R}^{1-a}/p_T = .00032(1-a)$$
$$\tau^{\max} = .002$$

and

$$\frac{\epsilon_1}{\tau^{\min}} = \frac{\epsilon_2}{\tau^{\max}} = 10^{-0.1}$$
$$\mu_0 = 200\,\mathrm{MeV}$$

Soft Function (6/6)

Without Refactorization

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \left[\mathbf{S}_0 \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left(S_{ij}^{\text{incl}} + \sum_{k=1}^{N} S_{ij}^{k} \right) + \text{h.c.} \right]$$

With Refactorization

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \frac{\alpha_s}{4\pi} \frac{1}{2} \left[\mathbf{S}_0 \left(\mathbf{S}_s^{(1)}(p_T^{\text{cut}}) + \sum_{k=1,2} S_{sc}^{k(1)}(p_T^{\text{cut}} \mathcal{R}) \right) + \text{h.c.} \right] + \mathcal{O}(\alpha_s^2)$$

$$\mathbf{S}_s^{(1)}(p_T^{\text{cut}}) = \frac{4}{\epsilon} \left(\frac{\mu}{p_T^{\text{cut}}} \right)^{2\epsilon} \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left[\mathcal{I}_{ij}^{\text{incl}} + (\delta_{iB} + \delta_{i\bar{B}})(\delta_{jJ_1} + \delta_{jJ_2}) \mathcal{I}_{ij}^{i} + \delta_{iB}\delta_{i\bar{B}} (\mathcal{I}_{ij}^{i} + \mathcal{I}_{ij}^{j}) \right]$$

$$S_{sc}^{k(1)}(p_T^{\text{cut}} \mathcal{R}) = \frac{4}{\epsilon} \left(\frac{\mu}{p_T^{\text{cut}}} \right)^{2\epsilon} \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left[\delta_{ik} \mathcal{I}_{ij}^{i} \right]$$

Results (1/2)

Soft function after RG Evolution

$$\mathbf{S}(\tau_a^1, \tau_a^2, \mu, \mu_S^1, \mu_S^2, \bar{\mu}_S) = U_S^1(\tau_a^1, \mu, \mu_S^1) U_S^1(\tau_a^2, \mu, \mu_S^2) \big[1 + (f_S^1(\tau_a^1; \omega_S^1, \mu_S^1) + f_S^2(\tau_a^2; \omega_S^2, \mu_S^2))\big]$$

$$\times\, \Pi_S^{\text{unmeas}}(\mu, \bar{\mu}_S) \big[\mathbf{\Pi}_S^\dagger(\mu, \bar{\mu}_S) \mathbf{S}^{\text{unmeas}}(\bar{\mu}_S) \mathbf{\Pi}_S(\mu, \bar{\mu}_S)\big]$$

$$f_S^i(\tau; \Omega, \mu) = \frac{\alpha_s C_i}{\pi(1-a)} \left[\psi^{(1)}(-\Omega) - \left(H(-1-\Omega) + \ln \frac{\mu \mathcal{R}^{1-a}}{p_T\, \tau}\right)^2 - \frac{\pi^2}{8}\right]$$

Without s-c Refactorization

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \frac{\alpha_s}{\pi} \left\{ \mathbf{S}_0 \left[\left(\frac{1}{2\epsilon} + \ln \frac{\mu}{p_T^{\text{cut}}} \right) \left(\mathbf{S}^{\text{div}} + \sum_{i=1,2} C_i \ln \mathcal{R} \right) - \frac{1}{2} \sum_{i=1,2} C_i \ln^2 \mathcal{R} \right.\right.$$

$$\left.\left. - \mathbf{T}_1 \cdot \mathbf{T}_2 \ln\left(1 + e^{\Delta y}\right) \ln\left(1 + e^{-\Delta y}\right) \right] + \text{h.c.} \right\} + \mathcal{O}(\alpha_s^2)$$

$$\mathbf{S}^{\text{div}} = \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{n_i \cdot n_j}{2} - y_{\text{cut}} (C_B + C_{\bar{B}}) - \sum_{i=1,2} C_i \ln(2 \cosh y_i)$$

Results (2/2)

With s-c Refactorization

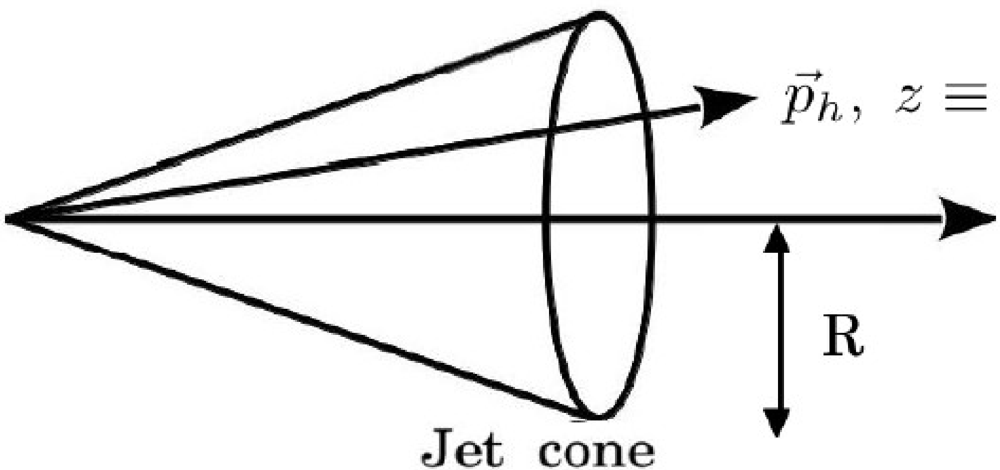
$$\begin{aligned}\mathbf{S}^{\text{unmeas}}(\Omega,\mu_{sc},\mu_{ss}) \equiv \mathbf{S}_0 + \Bigg\{ \mathbf{S}_0 \Bigg[& \frac{\alpha_s(\mu_{ss})}{4\pi}\bigg(\frac{1}{2}\mathbf{f}_s^2 + \mathbf{f}_s^1\Big(\ln\frac{\mu_{ss}}{p_T^{\text{cut}}} + H(-\Omega)\Big) \\ & + \mathbf{f}_s^0\Big(\frac{\pi^2}{6} - \psi^{(1)}(1-\Omega) + \Big(\ln\frac{\mu_{ss}}{p_T^{\text{cut}}} + H(-\Omega)\Big)^2\Big)\bigg) \\ & + \frac{\alpha_s(\mu_{sc})}{4\pi}\bigg(\frac{1}{2}f_c^2 + f_c^1\Big(\ln\frac{\mu_{sc}}{p_T^{\text{cut}}\mathcal{R}} + H(-\Omega)\Big) \\ & + f_c^0\Big(\frac{\pi^2}{6} - \psi^{(1)}(1-\Omega) + \Big(\ln\frac{\mu_{sc}}{p_T^{\text{cut}}\mathcal{R}} + H(-\Omega)\Big)^2\Big)\bigg)\Bigg] + \text{h.c.}\Bigg\}\end{aligned}$$

$$f_c^0 = -2(C_1 + C_2) \qquad \mathbf{f}_s^0 = -f_c^0$$

$$f_c^1 = 0 \qquad \mathbf{f}_s^1 = 4\mathbf{S}^{\text{div}}$$

$$f_c^2 = \frac{\pi^2}{6}(C_1 + C_2) \qquad \mathbf{f}_s^2 = -8\mathbf{T}_1\cdot\mathbf{T}_2\ln(1+e^{\Delta y})\ln(1+e^{-\Delta y}) - f_c^2$$

Applications in heavy meson and quarkonium production



$$\vec{p}_h, \ z \equiv \frac{p_h^-}{k^-}$$

$$\text{Jet} - \text{axis}: \ \vec{k} = \frac{\omega}{2}\hat{\mathbf{n}}$$

$$\text{where} \ \ \omega = k^- = s/k^+$$

$$\text{Identified Jets:} \quad J_i(p^2, \tau, \mu) \longrightarrow \mathcal{G}_i^h(z, \tau, \mu)$$

$$\text{OPE}: \quad \mathcal{G}(z, \tau, \mu) = \sum_j \Big[\mathcal{J}_i^j(\tau, \mu) \bullet D_{j\to h}(\mu)\Big](z)$$

$$[g \bullet f](z) = [f \bullet g](z) \equiv \int_z^1 \frac{dx}{x} g\left(\frac{z}{x}\right) f(x)$$

[arXiv:0911.4980]
M. Procura and I. W. Stewart
[arXiv:1111.6605]
M. Procura and W. J. Waalewijn
[arXiv:1101.4953]
A. Jain, M. Procura, and W. J. Waalewijn