

Production of Jets in Hadronic Collisions beyond NLO

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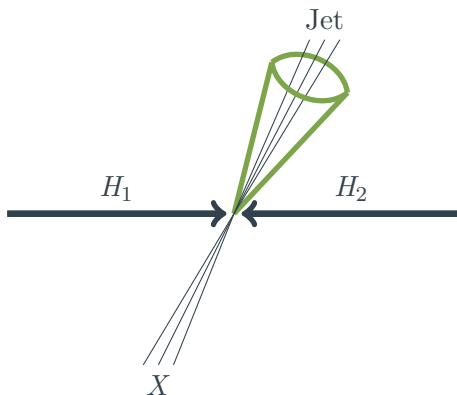
Single Inclusive Jet Production

$$H_1 + H_2 \rightarrow \text{Jet}(p_T, \eta) + X$$



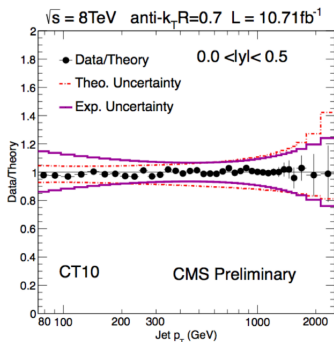
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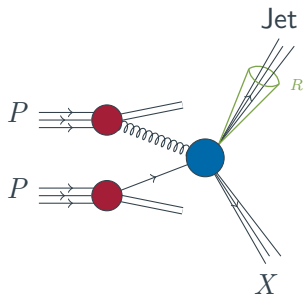


Single Inclusive Jet Production

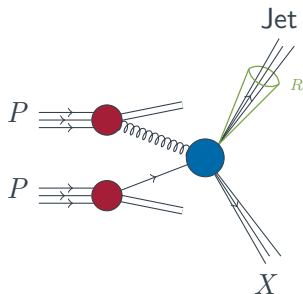
- Large theoretical uncertainties especially at high p_T
- PDFs are constrained by collider jet data
- Determination of α_s



Single Inclusive Jet Production - Cross Section



Single Inclusive Jet Production - Cross Section



anti-k_T algorithm $p = -1$

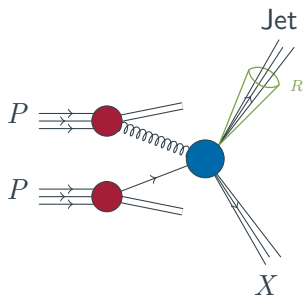
$$d_{jk} \equiv \min(k_{T_j}^{2p}, k_{T_k}^{2p}) \frac{R_{jk}^2}{R^2}, \quad d_{jB} \equiv k_{T_j}^{2p}$$

$$R_{jk}^2 = (\eta_j - \eta_k)^2 + (\Phi_j - \Phi_k)^2$$

(Cacciari, Salam, Soyez 2008)



Single Inclusive Jet Production - Cross Section



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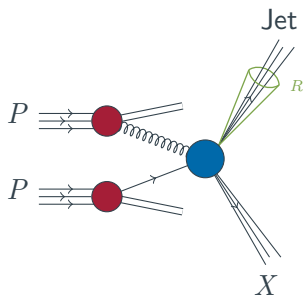
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$$v = \frac{u}{t+u}, \quad z = \frac{s_4}{s}$$

Single Inclusive Jet Production - Cross Section



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$$\frac{p_T^2 d^2\sigma}{dp_T^2 d\eta} = \sum_{ab} \int_0^{z_{\max}} dz \int_{v_{\min}}^{v_{\max}} dv x_a x_b f_a(x_a, \mu_F) f_b(x_b, \mu_F) \frac{d\hat{\sigma}_{ab}}{dv dz}(s, v, z, \mu_F, \mu_R; R)$$

$$z_{\max} = V(1 - W), \quad v_{\min} = \frac{VW}{1-z}, \quad v_{\max} = 1 - \frac{V-z}{1-z},$$

$$V = 1 - \frac{p_T}{\sqrt{s}} e^{-\eta}, \quad VW = \frac{p_T}{\sqrt{s}} e^{-\eta}$$



The Cross Section

$$\frac{p_T^2 d^2\sigma}{dp_T^2 d\eta} = \int_0^{z_{\max}} dz \int_{v_{\min}}^{v_{\max}} dv x_a x_b f_a(x_a, \mu_F) f_b(x_b, \mu_F) \frac{d\hat{\sigma}_{ab}}{dv dz}(s, v, z, \mu_F, \mu_R; R)$$

Perturbative expansion of the hard part:

$$s \frac{d\hat{\sigma}_{ab}}{dv dz} = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ \omega_{ab}^{(0)} + \left(\frac{\alpha_s}{\pi}\right) \omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}(\alpha_s^3) \right\}$$



Full analytical results at NLO in "Narrow Jet Approximation"

Jäger, Stratmann, Vogelsang 2004

Mukherjee, Vogelsang 2013

Kaufmann, Mukherjee, Vogelsang 2015

- $\omega_{ab}^{(1)} \sim A + B \log(R) + \mathcal{O}(R^2)$
- different algorithms
 - anti - k_T algorithm
 - cone algorithm
 - maximized jet function
- accuracy better than 2-3 % for $R \leq 0.7$



The Partonic Cross Section

Perturbative expansion of the hard part:

$$\frac{sd\hat{\sigma}_{ab}}{dv dz} = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ \omega_{ab}^{(0)} + \left(\frac{\alpha_s}{\pi}\right) \omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}(\alpha_s^3) \right\}$$



The Partonic Cross Section

Perturbative expansion of the hard part:

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- threshold logarithms $\alpha_s^k \omega_{ab}^{(k)} \sim \alpha_s^k \left(\frac{\log(z)^m}{z}\right)_+$, with $0 \leq m \leq 2k - 1$
- threshold logarithms dominate at partonic threshold: $z = \frac{s_4}{s} \rightarrow 0$



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- threshold logarithms dominate at partonic threshold: $z = \frac{s_4}{s} \rightarrow 0$

Threshold resummation determines all order structure of logarithms



mellin transformation

$$\tilde{\omega}_{ab}(v, N) = \int_0^1 dz (1-z)^{N-1} s \frac{d\sigma^2}{dv dz}$$

threshold logarithms

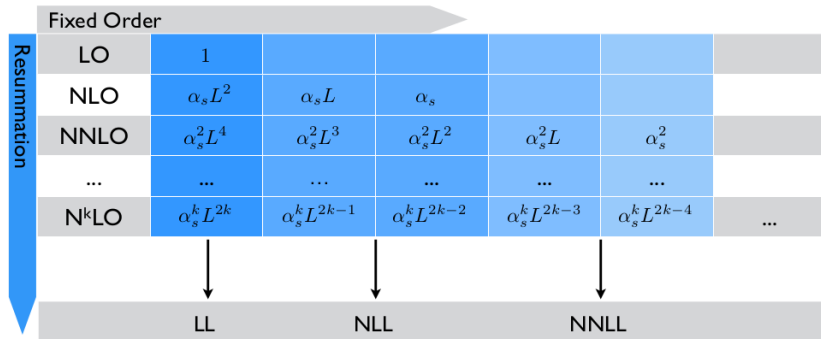
$$\left(\frac{\ln^{2k-1}(z)}{z} \right)_+ \rightarrow \ln^{2k} N + \dots$$

threshold limit

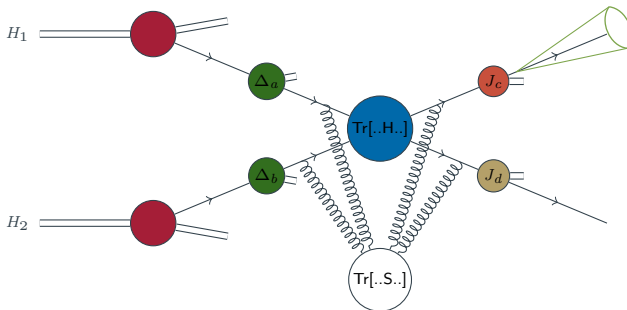
$$N \rightarrow \infty$$



Accuracy of Resummation



Resummed Cross Section



$$\Omega_{ab}^{\text{res}}(v, N) = \sum_{cd} \Delta_a(N_a) \Delta_b(N_b) J_c^{(\text{Jet})}(N) J_d^{(\text{recoil})}(N) \Delta_{ab \rightarrow cd}^{\text{int}}(N, v) \Delta_c^{(\text{ng})}(N)$$

Kidonakis, Oderda, Sterman 1998

$$\Delta_{ab \rightarrow cd}^{\text{int}}(N, v) = \text{Tr}(\dots S \dots H)$$

Resummed Cross Section

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Resummed Cross Section

$$\Omega_{ab}^{\text{res}}(v, N) = \sum_{cd} \left(\Delta_a(N_a) \Delta_b(N_b) \right) J_c^{(\text{Jet})}(N) J_d^{(\text{recoil})}(N) \Delta_{ab \rightarrow cd}^{\text{int}}(N, v) \Delta_c^{(\text{ng})}(N)$$

- initial state, soft-collinear gluon emission:

$$\Delta_i(N_i) = R_i(\alpha_s(\mu_r)) \exp \left\{ \int_0^1 dz z^{\frac{N_i-1}{1-z}} \left[\int_{\mu_F^2}^{(1-z)^2 s} \frac{d\mu^2}{\mu^2} A_i(\alpha_s(\mu)) + D_i(\alpha_s((1-z)s)) \right] \right\}$$



Resummed Cross Section

$$\Omega_{ab}^{\text{res}}(v, N) = \sum_{cd} \Delta_a(N_a) \Delta_b(N_b) J_c^{(\text{Jet})}(N) \boxed{J_d^{(\text{recoil})}(N)} \Delta_{ab \rightarrow cd}^{\text{int}}(N, v) \Delta_c^{(\text{ng})}(N)$$

- initial state, soft-collinear gluon emission:

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- unobserved final state, soft-collinear gluon emission:

$$J_i^{(\text{recoil})} = R_i^J(\alpha_s(\mu_r)) \exp \left\{ \int_0^1 dz \frac{z^{N-1}-1}{1-z} \left[\int_{(1-z)^2 s}^{(1-z)s} \frac{d\mu^2}{\mu^2} A_i(\alpha_s(\mu)) + \frac{1}{2} B_i(\alpha_s((1-z)s)) \right] \right\}$$



Resummed Cross Section

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- non global contribution $\Delta_c^{(NG)}$

3rd tower effect starting at NNLO, arising from boundary of the jet

Dasgupta, Salam 2001; Banfi, Dasgupta 2004



Resummed Cross Section

$$\Omega_{ab}^{\text{res}}(v, N) = \sum_{cd} \Delta_a(N_a) \Delta_b(N_b) J_c^{(\text{Jet})}(N) J_d^{(\text{recoil})}(N) \boxed{\Delta_{ab \rightarrow cd}^{\text{int}}(N, v)} \Delta_c^{(\text{ng})}(N)$$

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- trace part

$$\Delta_{ab \rightarrow cd}^{\text{int}}(N, v) = \text{Tr} \{ H(\alpha_s(\mu_r^2)) \mathcal{S}^\dagger S(\alpha_s(S/\bar{N})) \mathcal{S} \}$$

matrices in space of color exchange operators (2-,3-,8-dimensional)



Resummed Cross Section

$$\Delta_{ab \rightarrow cd}^{\text{int}}(N, v) = \text{Tr} \left\{ H(\alpha_s(\mu_r^2)) \mathcal{S}^\dagger \mathcal{S}(\alpha_s(S/\bar{N})) \mathcal{S} \right\}$$



$$\Delta_{ab \rightarrow cd}^{\text{int}}(N, v) = \text{Tr} \left\{ H(\alpha_s(\mu_r^2)) \mathcal{S}^\dagger \left(\mathcal{S}(\alpha_s(S/\bar{N})) \right) \mathcal{S} \right\}$$

- soft matrix
 - soft, large angle gluon emission

$$S = S^{(0)} + \frac{\alpha_s}{\pi} S^{(1)}$$



$$\Delta_{ab \rightarrow cd}^{\text{int}}(N, v) = \text{Tr} \left\{ H(\alpha_s(\mu_r^2)) \mathcal{S}^\dagger S(\alpha_s(S/\bar{N})) \mathcal{S} \right\}$$

$$\mathcal{S} = \mathcal{P} \exp \left[\frac{1}{2} \int_s^{s/\bar{N}^2} \frac{d\mu^2}{\mu^2} \Gamma_S^{(f)}(\alpha_s(\mu^2)) \right]$$

$$(\Gamma_S^{(f)})_{KL} = (\Gamma_{S'}^{(f)})_{KL} + \delta_{KL} \frac{\alpha_s}{\pi} \sum_{i=a,b,c,d} C_{fi} \frac{1}{2} [-\ln(2\nu_i) + 1 - i\pi]$$

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- contains far off shell part

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$$H = H^{(0)} + \frac{\alpha_s}{\pi} H^{(1)}$$

$$\left(\frac{\alpha_s}{\pi}\right)^k \frac{\sum_i A_i^{(1)}}{(k-1)!} \text{Tr} \left\{ \underbrace{H^{(1)} S^{(0)} + H^{(0)} S^{(1)}} \right\} \ln^{2k-2} \bar{N}$$

matching to NLO \rightarrow NLL accuracy



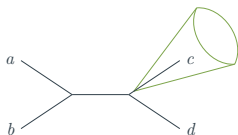
Resummed Cross Section

$$\Omega_{ab}^{\text{res}}(v, N) = \sum_{cd} \Delta_a(N_a) \Delta_b(N_b) \boxed{J_c^{(\text{Jet})}(N)} J_d^{(\text{recoil})}(N) \Delta_{ab \rightarrow cd}^{\text{int}}(N, v) \Delta_c^{(\text{ng})}(N)$$



Jet Mass at Threshold

LO:

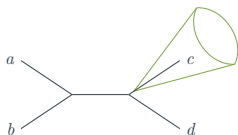


\Rightarrow jet massless



Jet Mass at Threshold

LO:



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- Scheme 1: keep jet massless at threshold

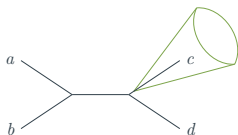
$$\exp \left[\frac{\alpha_s}{\pi} \ln^2 \bar{N} \left(C_a + C_b - \frac{1}{2} C_c - \frac{1}{2} C_d \right) \right]$$

Kidonakis, Owens 2000
Moch, Kumar 2013



Jet Mass at Threshold

LO:



\Rightarrow jet massless

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no dependence on R

Kidonakis, Owens 2000
Moch, Kumar 2013



Threshold NLO Results

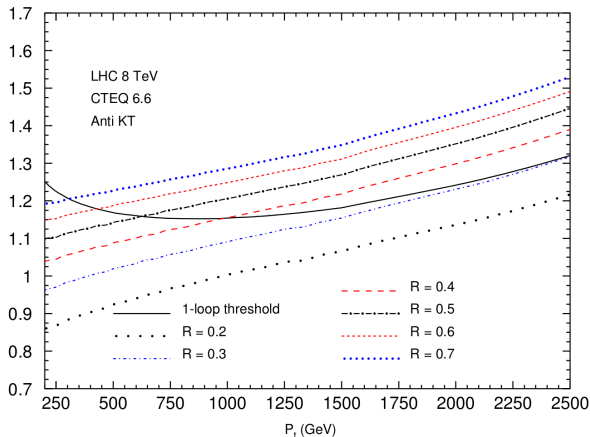
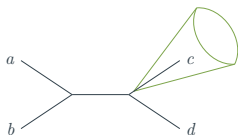


Figure : Moch/Kumar 13'



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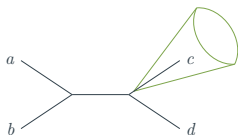
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no dependence on R

Kidonakis, Owens 2000
Moch, Kumar 2013

- Scheme 2: jet allowed to be massive at threshold

$$\exp \left[\frac{\alpha_s}{\pi} \ln^2 \bar{N} \left(C_a + C_b - \frac{1}{2} C_d \right) + \frac{\alpha_s}{\pi} C_c \ln(R) \ln(\bar{N}) \right]$$



Jet Mass At Threshold



Jet Mass At Threshold



Jet massless at threshold:

- Two partons in the jet, restrictions:
 - one must be arbitrarily soft or
 - they are exactly collinear

→ no R -dependence in $J_c^{(\text{Jet})}$

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Jet massive at threshold:

- more possible final states

→ generates R -dependence in $J_c^{(\text{Jet})}$



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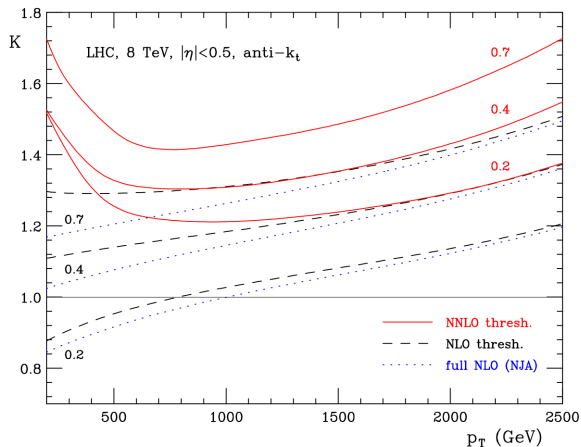
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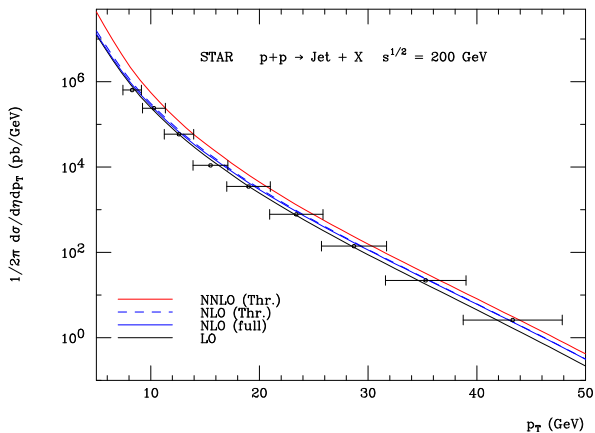
→ generates R -dependence in $J_c^{(\text{Jet})}$

$$J_c^{(\text{Jet})}(\bar{N}, R) = \exp \left\{ \int_s^{s/\bar{N}^2} \frac{dq^2}{q^2} \alpha_s(q^2) \left[-\frac{C_c}{2\pi} \log \left(\frac{p_T^2 R^2}{s} \right) \right] \right\}$$

Approximate NNLO Results



Approximate NNLO Results



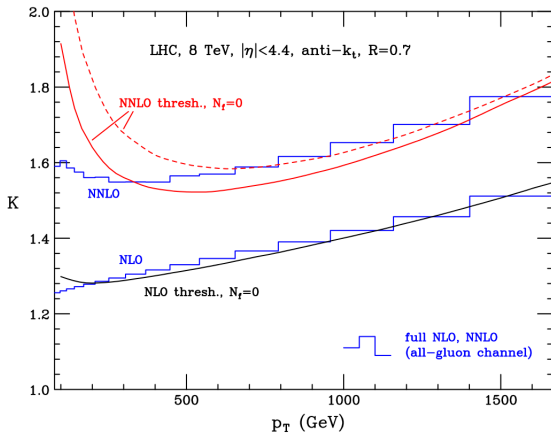
K-factors in the gluon only channel

$$v = \frac{u}{t+u}$$

$$v' = \frac{t+s}{s}$$

scale choice

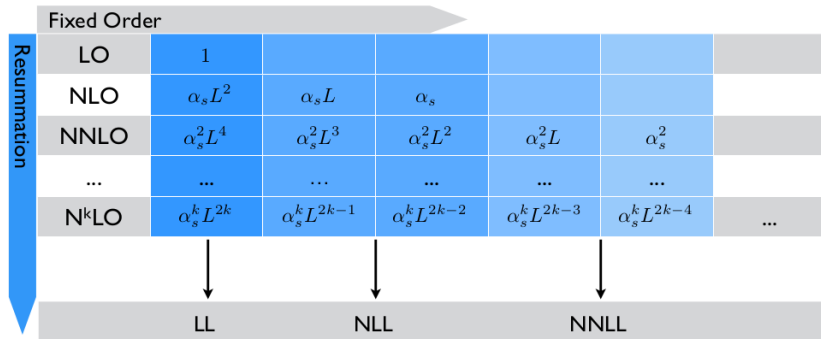
$$p_{T,1} \neq p_T$$



Currie, Gehrmann-De-Ritter, Glover, Pires 2013



Towards NNLL accuracy



Towards NNLL accuracy

- resummed exponents $\Delta(N_a)$, $J_C^{\text{jet}}(N, R)$, $J_d^{\text{recoil}}(N)$
- Gamma Matrix $\Gamma^{(2)}$ Aybat, Dixon, Sterman 2006
- Matrices $H^{(1)}$, $S^{(1)}$ needed for 4th tower

$$H = H^{(0)} + \frac{\alpha_s}{\pi} H^{(1)}$$

Hinderer, Ringer, Sterman, Vogelsang 2014

Kelley, Schwartz 2010

$$S = S^{(0)} + \frac{\alpha_s}{\pi} S^{(1)}$$

- for Di-Hadron Production

Hinderer, Ringer, Sterman, Vogelsang 2014

- for Single Inclusive Hadron and Jets

Hinderer, Ringer, Sterman, Vogelsang (work in progress)



Approximate NNLO results for A_{LL}

- measured at STAR

$$\Omega_{ab}^{\text{res}}(v, N) = \sum_{cd} \Delta_a(N_a) \Delta_b(N_b) J_c^{\text{Jet}}(N) J_d^{\text{recoil}}(N) \text{Tr}(\dots S \dots H) \Delta_c^{(\text{ng})}(N)$$

Soft gluon emission is spin independent

- $\Delta_i(N_i)$, $J_c^{\text{Jet}}(N)$, J_d^{recoil} , $\Delta^{\text{NG}}(N)$, S remain unchanged

see eg. de Florian, Vogelsang, Wagner 2008

- $H_{UU} \neq H_{LL}$, they are related to hard radiation



- Single inclusive jet production
- Full result is available up to NLO (NJA)
- NLL-NNLO
- Observed jet should be massive at threshold
- Extension to NNLL-NNLO is crucial
- A_{LL}

