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Magnetic Field Analysis for Helical Undulators using OPERA

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U.S. Department of Energy

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Office of Science U.S. DEPARTMENT OF ENERGY

Introduction: On-Axis Field B_0

(Helical) Solenoid W.R. Smythe (1939)

($k = 2\pi/\lambda$)

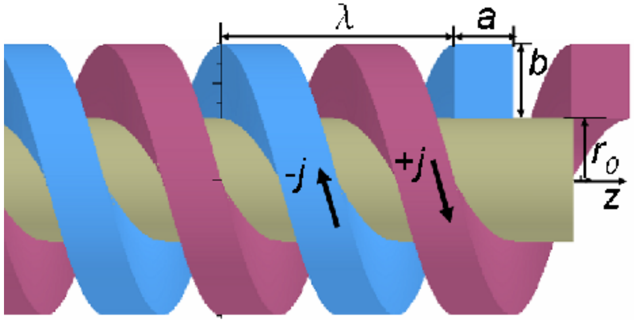
$$B_{tr} = \frac{\mu_0 I}{\lambda}\{kr_0 K_0(kr_0) + K_1(kr_0)\}$$

K_n, I_n modified Bessel functions

(Current I in filamentary wire on radius r_0)

Helical Undulator B.M. Kincaid (1977)

$$B_0 = 2B_{tr}$$



Helical Undulator with coil dimensions (a, b)

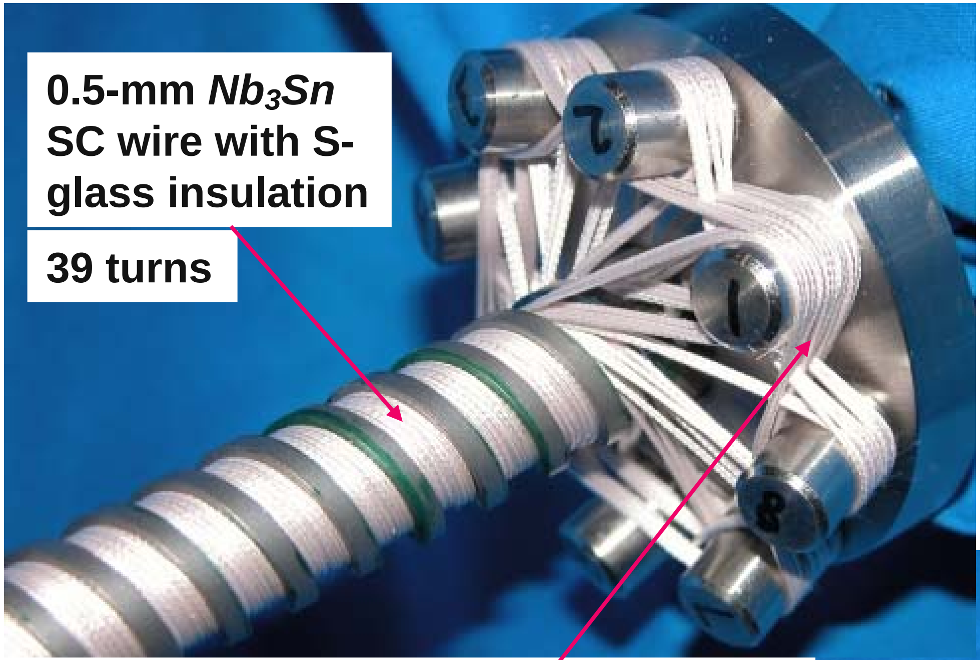
$$\mathbf{B} = B_0\left\{\hat{r}\left[I_0(kr) + I_2(kr)\right]\sin(kz-\phi) + \hat{\phi}2(-kr)^{-1}I_1(kr)\cos(kz-\phi) + \hat{z}2I_1(kr)\cos(kz-\phi)\right\}$$

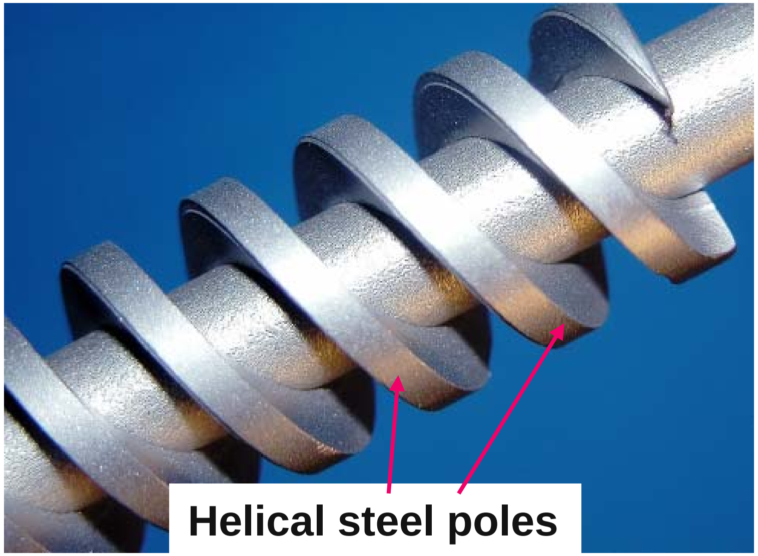
$$B_0 = \frac{2\mu_0 j\lambda}{\pi}\sin(k\frac{a}{2})\int_{r_0}^{r_0+b}\{krK_0(kr) + K_1(kr)\}\frac{dr}{\lambda}$$

$$\mathbf{B}(kz-\phi) = B_0\left\{\hat{r}\cos(kz-\phi) + \hat{\phi}\sin(kz-\phi)\right\}$$

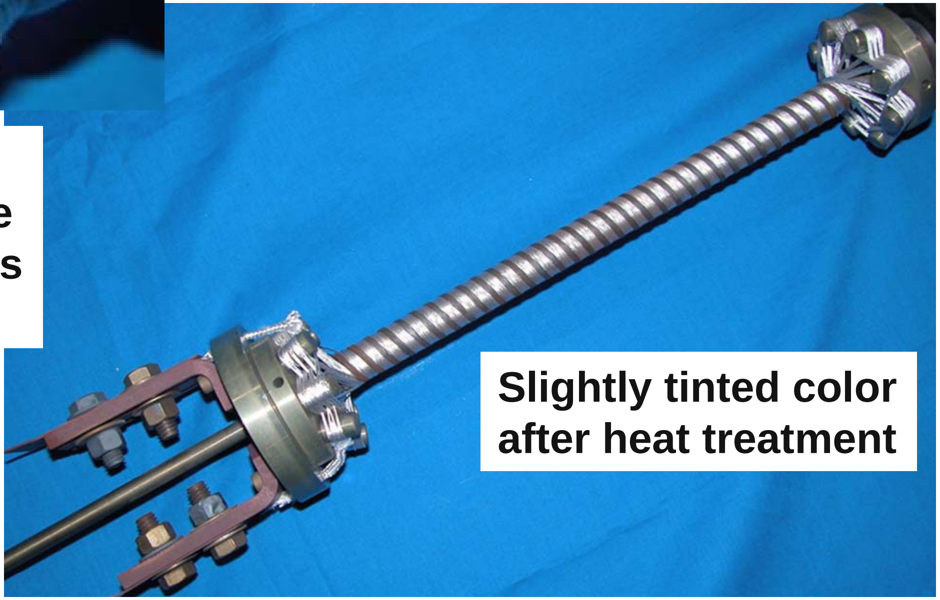
Compare with model undulator calculations

Helical Nb_3Sn SCU Fabrication, λ = 14 mm

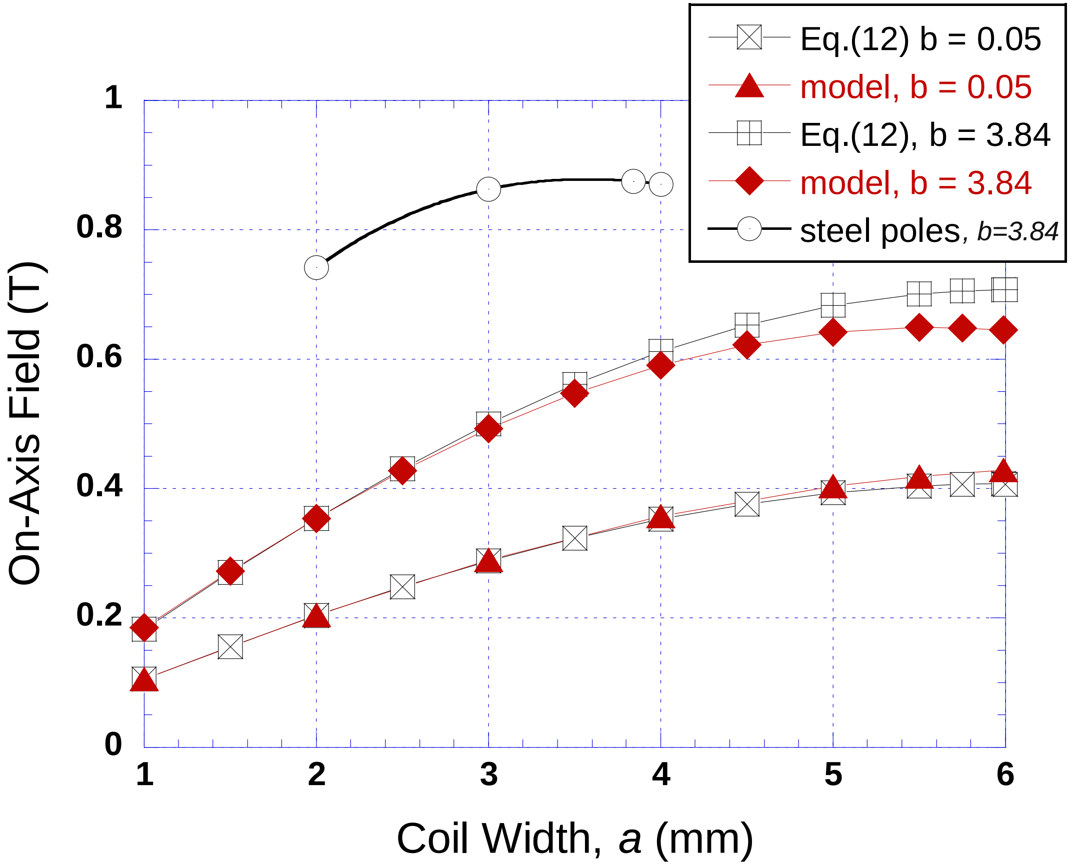




Undulator ends: designed for continuous winding of the double helix without any conductor joints and to minimize the stray field



On-axis field B_0:
Analytical and Model Calculations



λ = 12 mm

r_0 = 3.15 mm

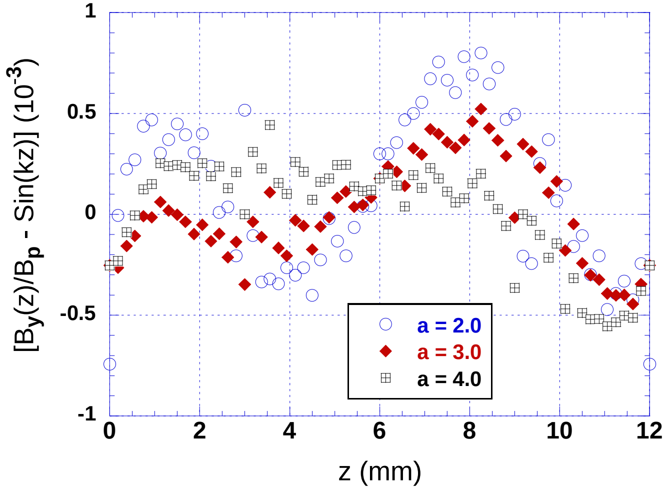
$$B_0 = \frac{2\mu_0 j\lambda}{\pi}\sin(k\frac{a}{2})\int_{r_0}^{r_0+b}\{krK_0(kr)+K_1(kr)\}\frac{dr}{\lambda}$$

Higher Harmonics: two models and a test undulator

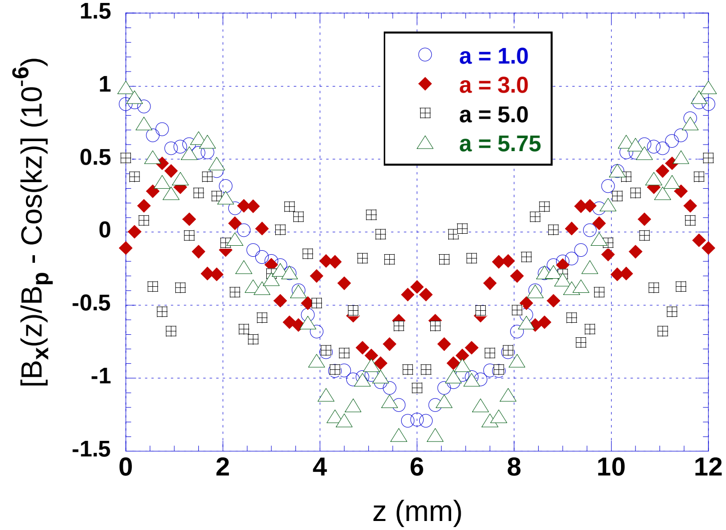
Model calculations for steel poles: < 2 x 10^{-3}

λ = *12* mm

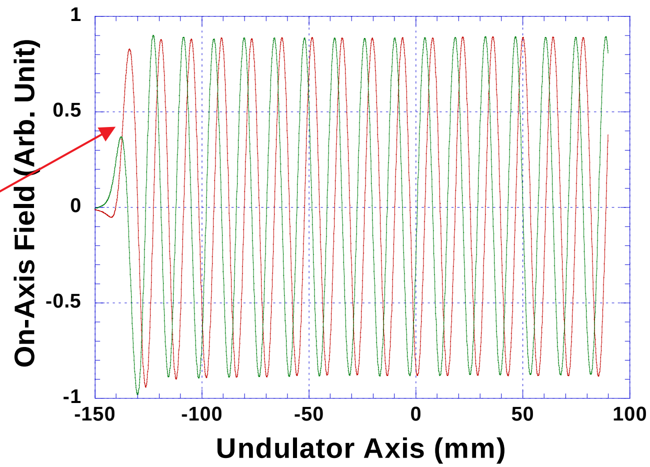
r_0 = 3.15 mm



Model calculations for air poles: < 2.5 x 10^{-6}



data at ϕ_0 and $\phi_0+\pi/2$



λ = *14* mm

r_0 = 3.97 mm

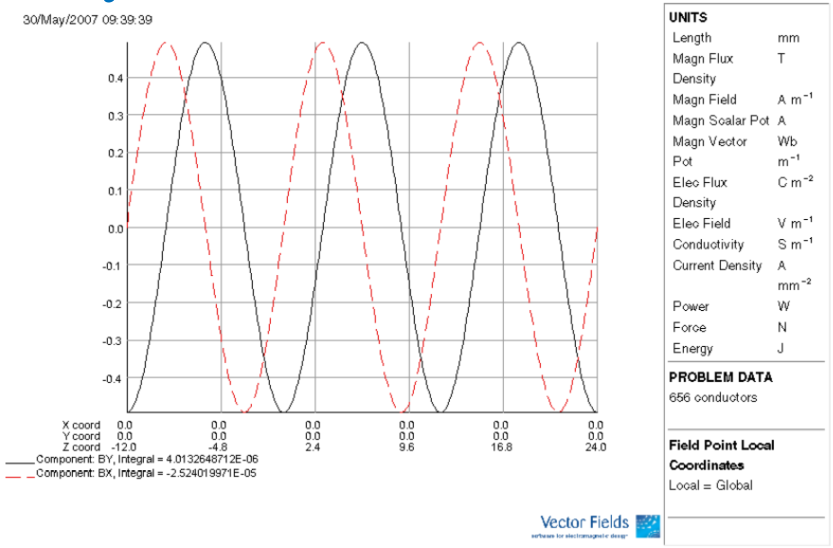
Measurements of test undulator with steel poles: < 5 x 10^{-3}

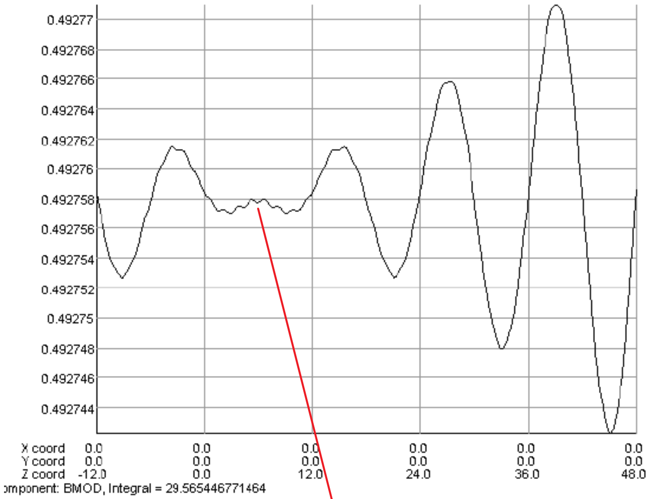
41-Period Air Poles

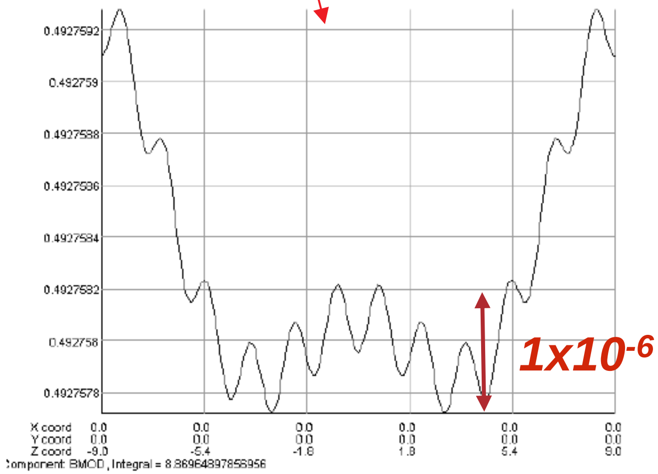
λ = 12 mm

r_0 = 3.15 mm

a = 3.0 mm







$$\mathbf{B}(kz-\phi) = B_0\left\{\hat{r}\cos(kz-\phi)+\hat{\phi}\sin(kz-\phi)\right\}$$

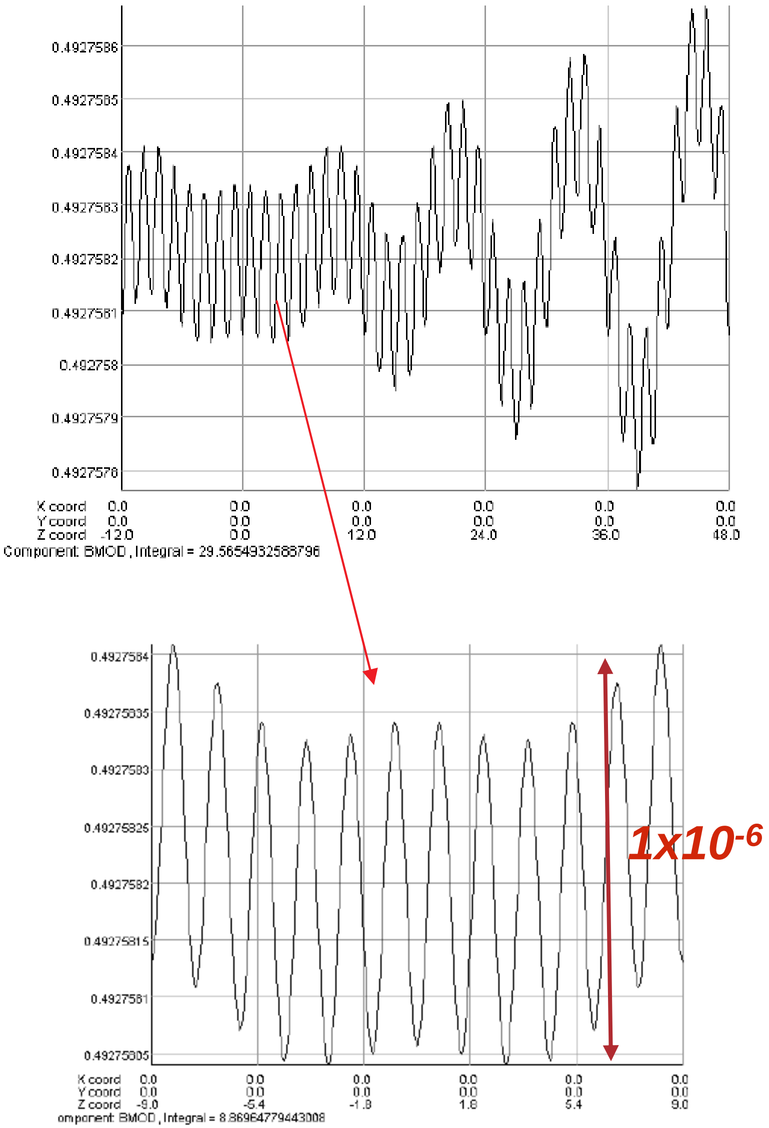
$$\mathbf{B}(x,y) = B_0\left\{\hat{x}\cos(kz)+\hat{y}\sin(kz)\right\}$$

81-Period Air Poles

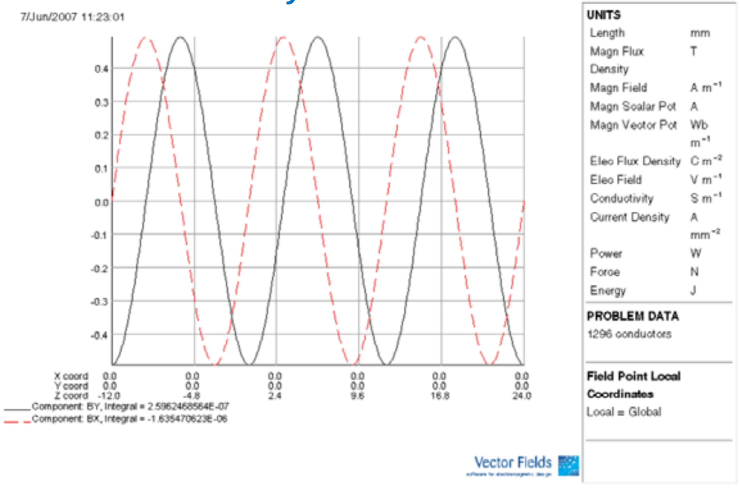
λ = 12 mm

r_0 = 3.15 mm

a = 3.0 mm



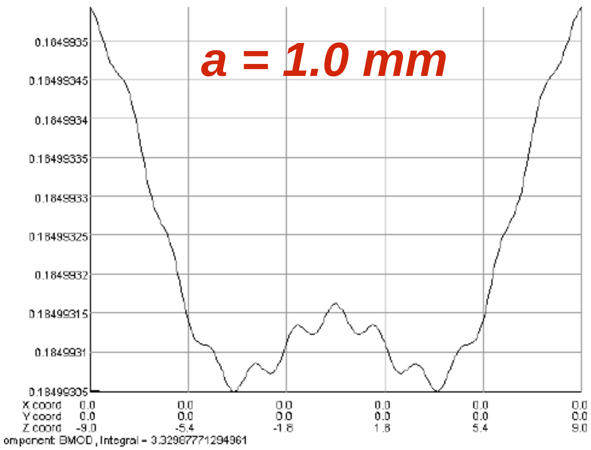
B_x and B_y

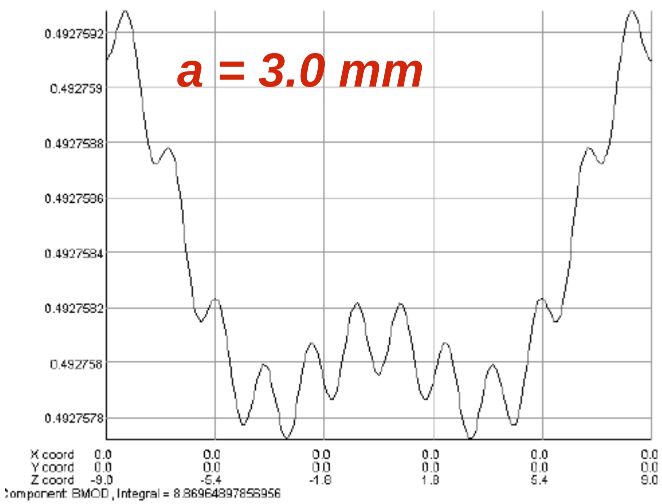


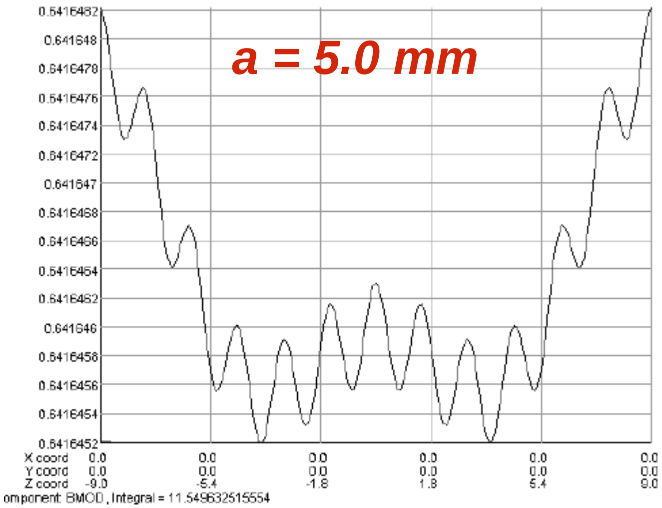
41-Period Air Poles

λ = ***12* mm**

r_0 = 3.15 mm





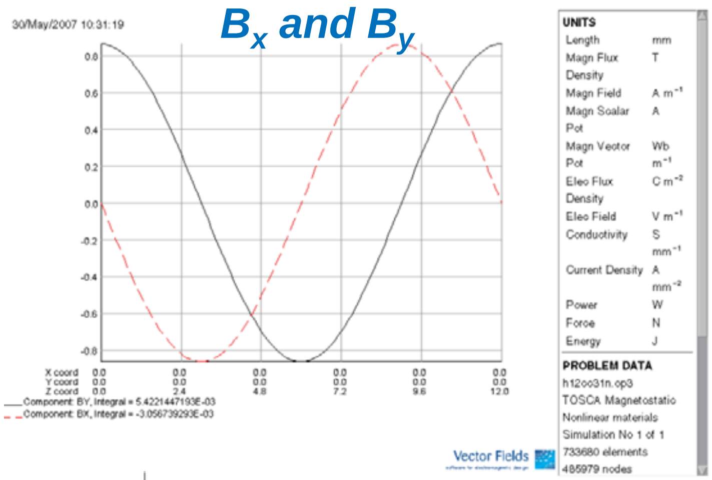


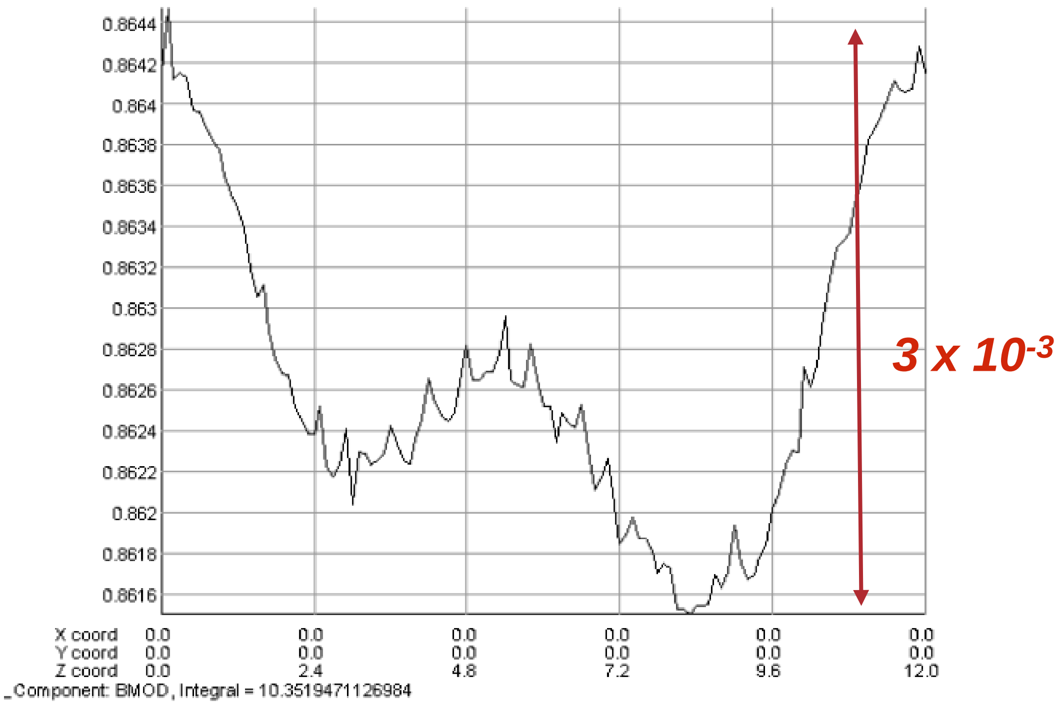
31-Period Steel Poles

λ = *12* **mm**

r_0 = *3.15 mm*

a = 3.0 mm





Conclusion

- **On-axis field B_0:**
  - **For coil/pole ratio < 2, discrepancy < 4%**

- **Higher harmonics:**
  - **It appears that the calculated higher harmonics for both linear and nonlinear poles are due to calculation errors and end fields for linear poles.**
  - **It also appears that the calculated higher harmonics do not depend on the coil/pole ratios indicating that the analytical result may be correct.**