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Magnetic Field Analysis for Helical Undulators using OPERA

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June 26, 2007

The Second Special Workshop on Magnet Simulations for Particle Accelerators



U.S. Department
of Energy

UChicago ►
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Introduction: On-Axis Field B_0

(Helical) Solenoid W.R. Smythe (1939)

$$B_{tr} = \frac{\mu_0 I}{\lambda} \{kr_0 K_0(kr_0) + K_1(kr_0)\}$$

$$(k = 2\pi/\lambda)$$

K_n, I_n modified Bessel functions

(Current I in filamentary wire on radius r_0)

Helical Undulator B.M. Kincaid (1977)

$$B_0 = 2B_{tr}$$

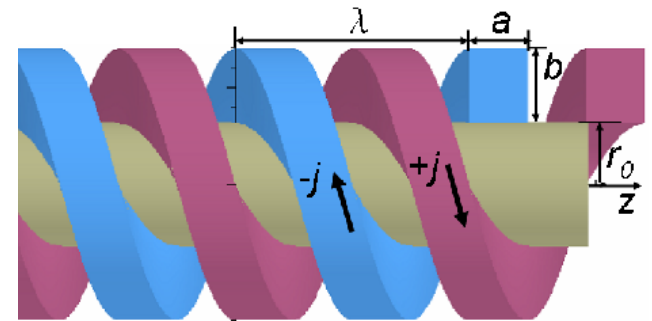
Helical Undulator with coil dimensions (a, b)

$$\mathbf{B} = B_0 \left\{ \hat{r} \left[I_0(kr) + I_2(kr) \right] \sin(kz - \phi) \right.$$

$$\left. + \hat{\phi} 2(-kr)^{-1} I_1(kr) \cos(kz - \phi) + \hat{z} 2I_1(kr) \cos(kz - \phi) \right\}$$

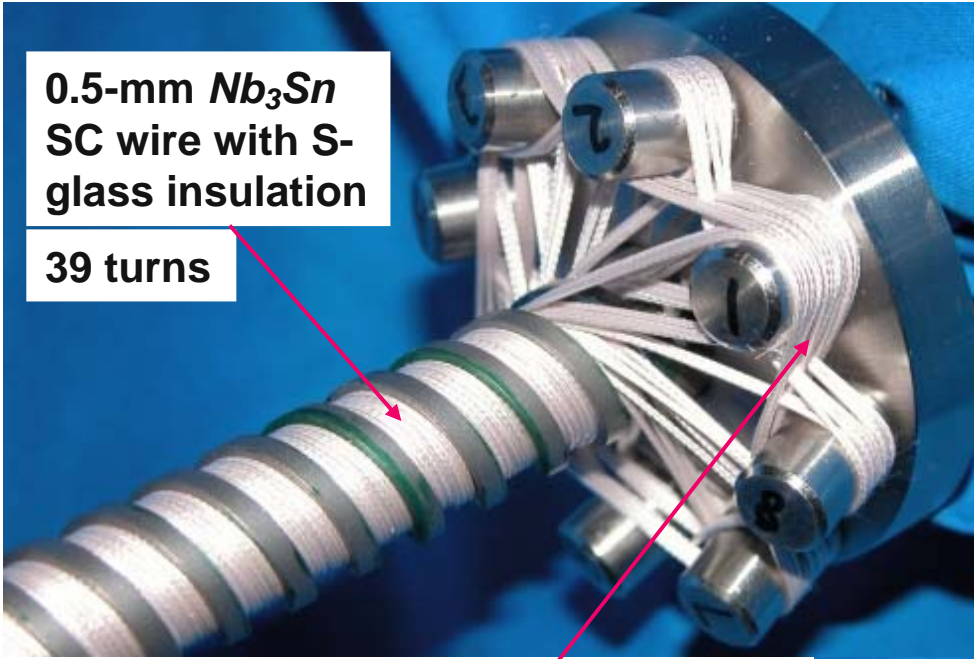
$$B_0 = \frac{2\mu_0 j \lambda}{\pi} \sin\left(k \frac{a}{2}\right) \int_{r_0}^{r_0+b} \{kr K_0(kr) + K_1(kr)\} \frac{dr}{\lambda}$$

$$\mathbf{B}(kz - \phi) = B_0 \left\{ \hat{r} \cos(kz - \phi) + \hat{\phi} \sin(kz - \phi) \right\}$$



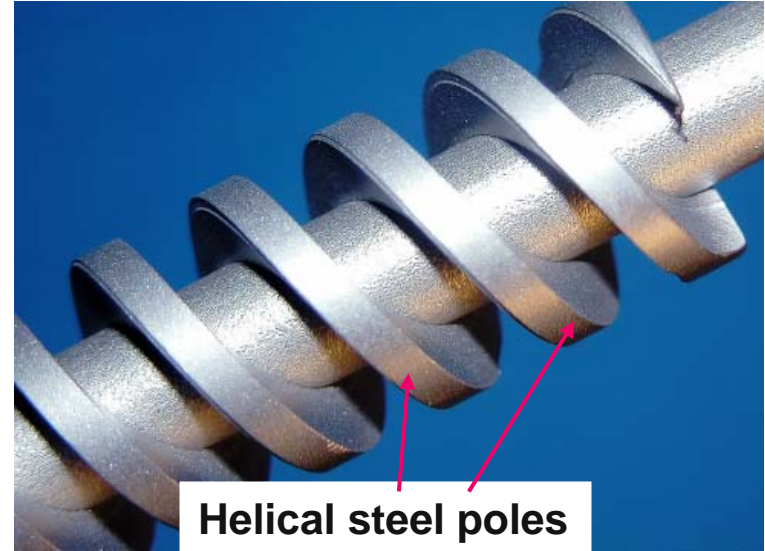
Compare with model undulator calculations

Helical Nb₃Sn SCU Fabrication, $\lambda = 14$ mm

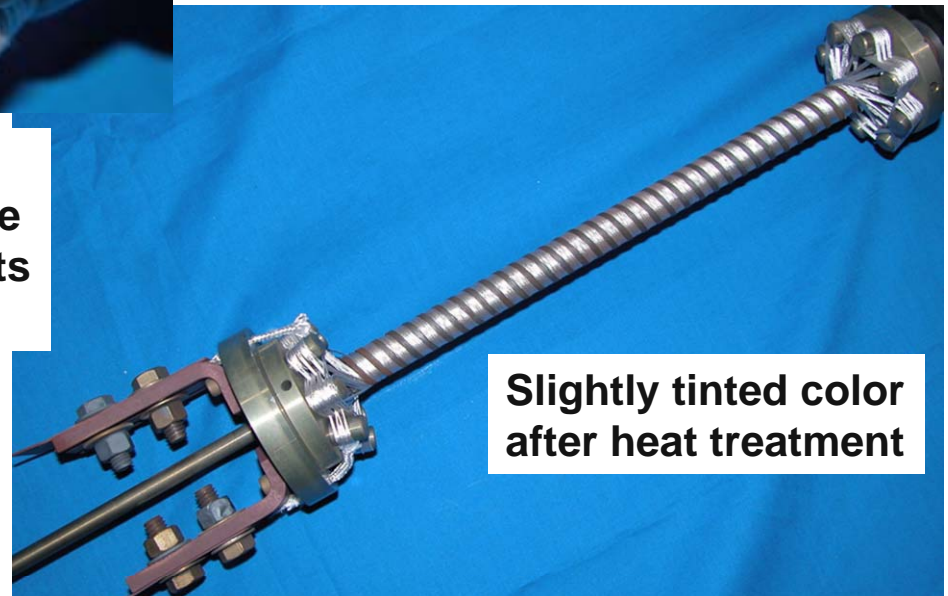


0.5-mm Nb₃Sn
SC wire with S-
glass insulation

39 turns

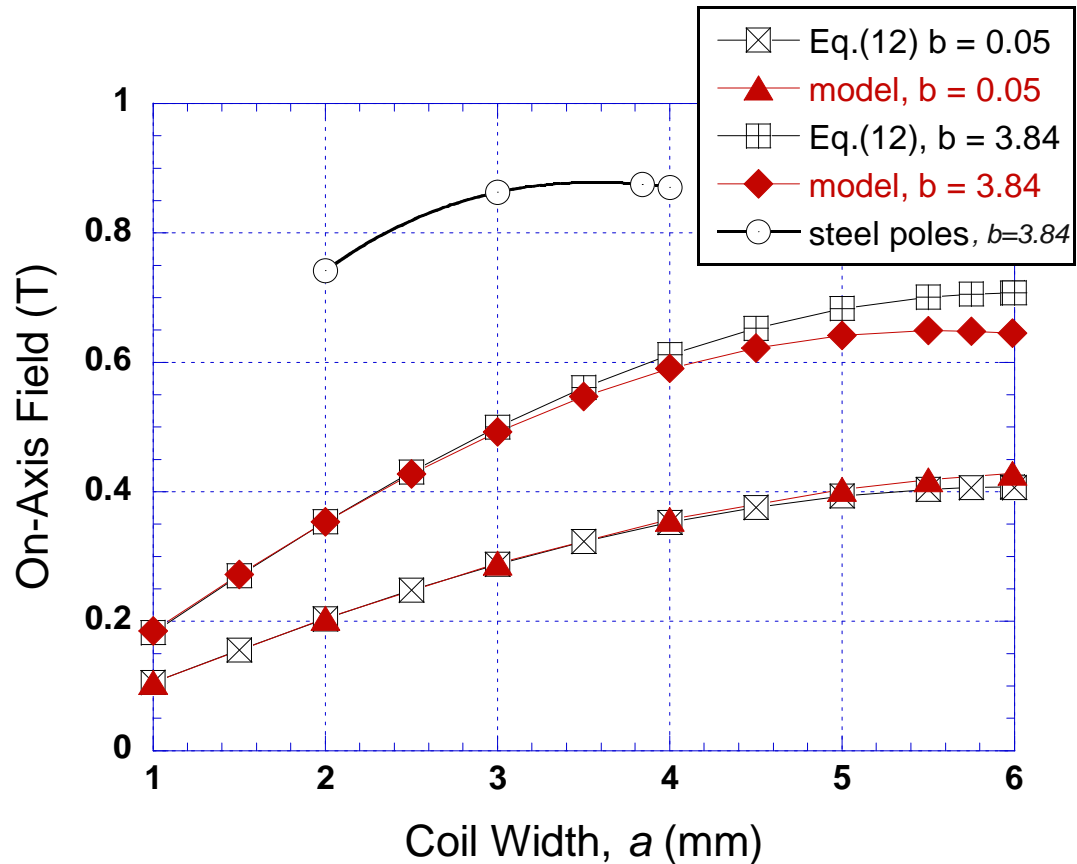


Undulator ends: designed for
continuous winding of the double
helix without any conductor joints
and to minimize the stray field



On-axis field B_0 : Analytical and Model Calculations

$\lambda = 12$ mm
 $r_0 = 3.15$ mm



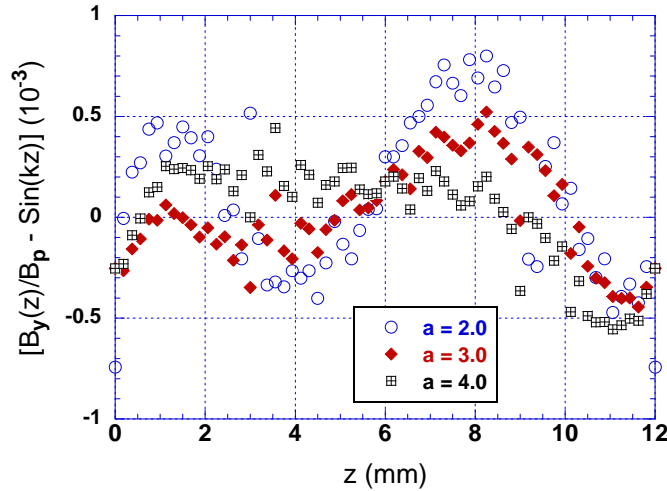
$$B_0 = \frac{2\mu_0 j\lambda}{\pi} \sin\left(k \frac{a}{2}\right) \int_{r_0}^{r_0+b} \{krK_0(kr) + K_1(kr)\} \frac{dr}{\lambda}$$

Higher Harmonics: two models and a test undulator

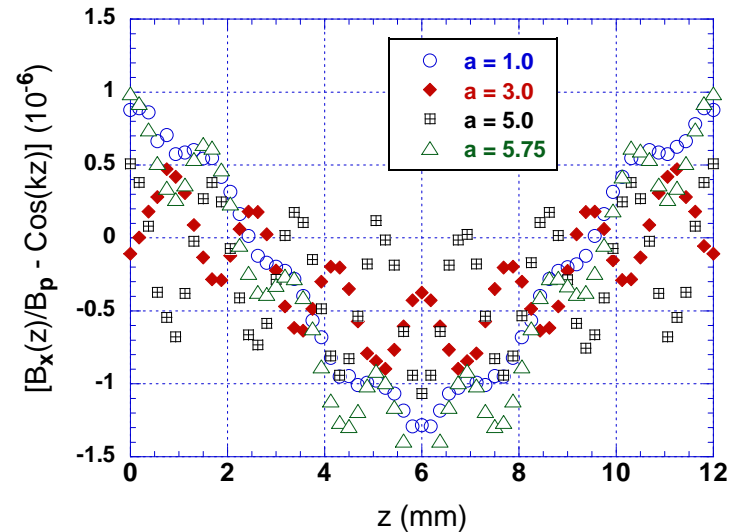
Model calculations for steel poles: $< 2 \times 10^{-3}$

$\lambda = 12 \text{ mm}$

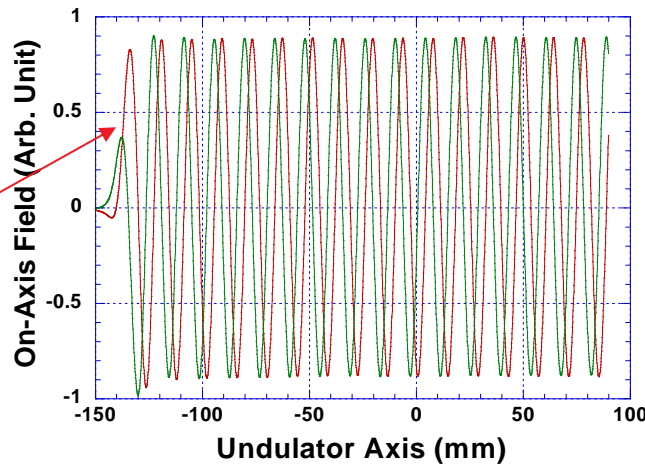
$r_0 = 3.15 \text{ mm}$



Model calculations for air poles: $< 2.5 \times 10^{-6}$



data at ϕ_0
and $\phi_0 + \pi/2$



$\lambda = 14 \text{ mm}$

$r_0 = 3.97 \text{ mm}$

Measurements of test undulator with steel poles: $< 5 \times 10^{-3}$

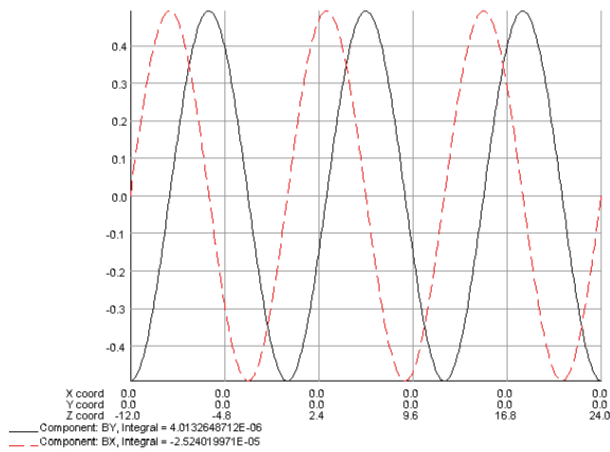
41-Period Air Poles

$$\lambda = 12 \text{ mm}$$

$$a = 3.0 \text{ mm}$$

$$r_0 = 3.15 \text{ mm}$$

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UNITS	
Length	mm
Magn Flux	T
Density	
Magn Field	A m ⁻¹
Magn Scalar Pot	A
Magn Vector	Wb
Pot	m ⁻¹
Eleo Flux	C m ⁻²
Density	
Eleo Field	V m ⁻¹
Conductivity	S m ⁻¹
Current Density	A
	mm ⁻²
Power	W
Force	N
Energy	J

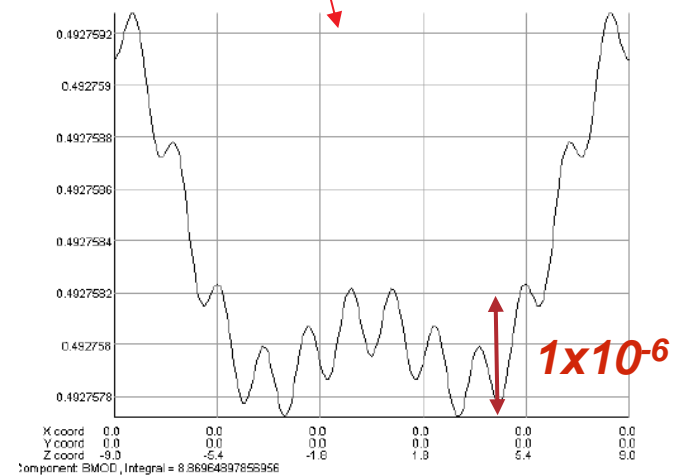
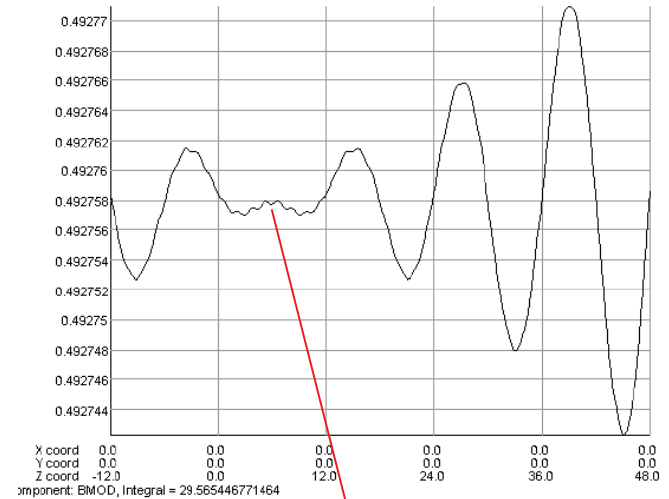
PROBLEM DATA	
656 conductors	

Field Point Local	
Coordinates	Local = Global

Vector Fields

$$\mathbf{B}(kz - \phi) = B_0 \left\{ \hat{r} \cos(kz - \phi) + \hat{\phi} \sin(kz - \phi) \right\}$$

$$\mathbf{B}(x, y) = B_0 \left\{ \hat{x} \cos(kz) + \hat{y} \sin(kz) \right\}$$



81-Period Air Poles

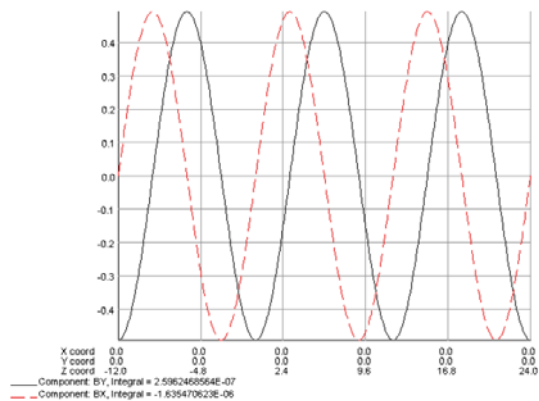
$$\lambda = 12 \text{ mm}$$

$$r_0 = 3.15 \text{ mm}$$

$$a = 3.0 \text{ mm}$$

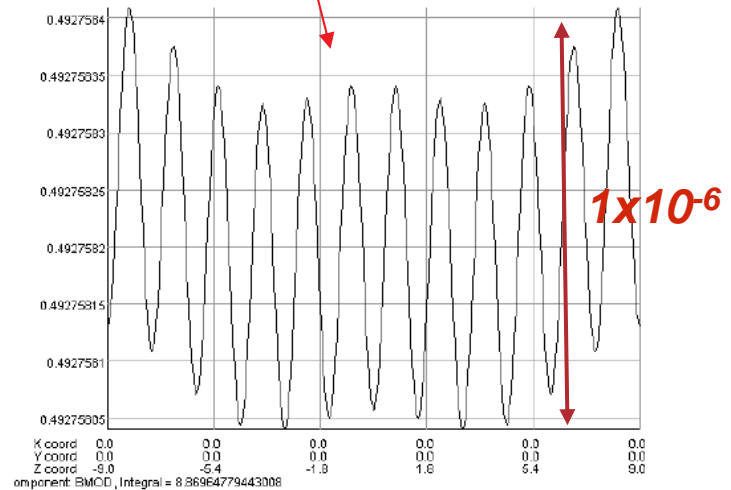
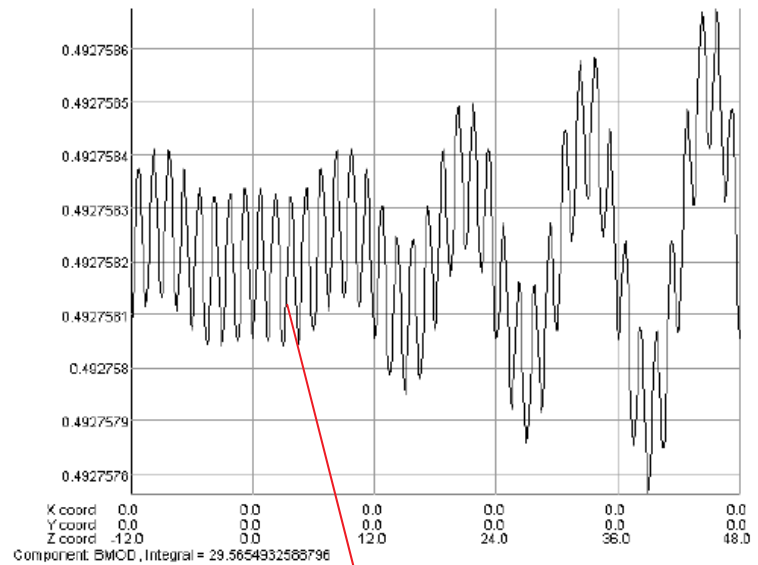
B_x and B_y

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UNITS	
Length	mm
Magn Flux	T
Density	
Magn Field	A m ⁻¹
Magn Scalar Pot	A
Magn Vector Pot	Wb m ⁻¹
Eleo Flux Density	C m ⁻²
Eleo Field	V m ⁻¹
Conductivity	S m ⁻¹
Current Density	A mm ⁻²
Power	W
Force	N
Energy	J
PROBLEM DATA	
1296 conductors	
Field Point Local Coordinates	
Local = Global	

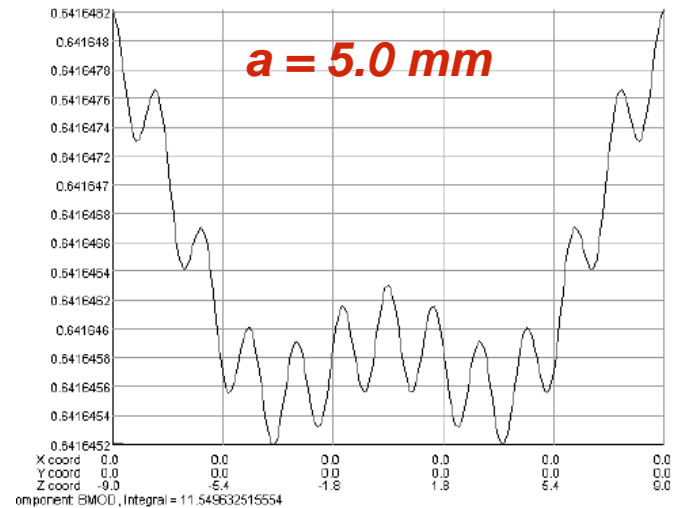
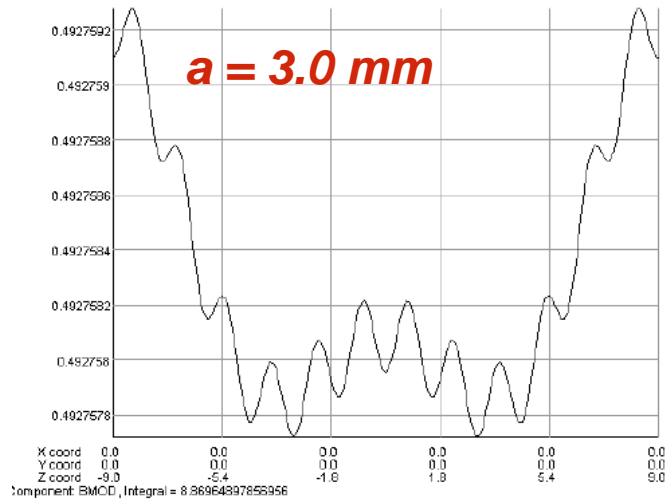
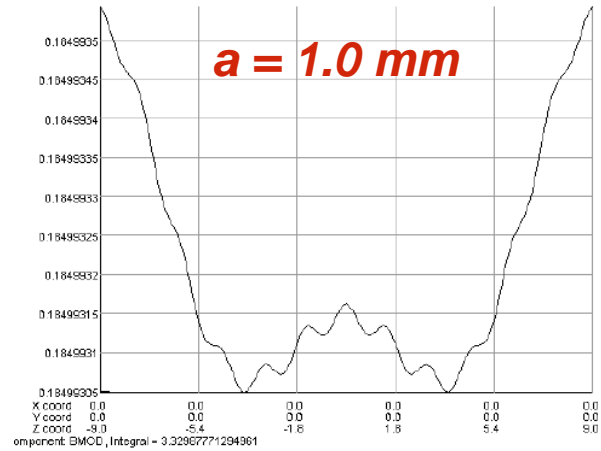
Vector Fields
software for electromagnetic design



41-Period Air Poles

$\lambda = 12 \text{ mm}$

$r_0 = 3.15 \text{ mm}$

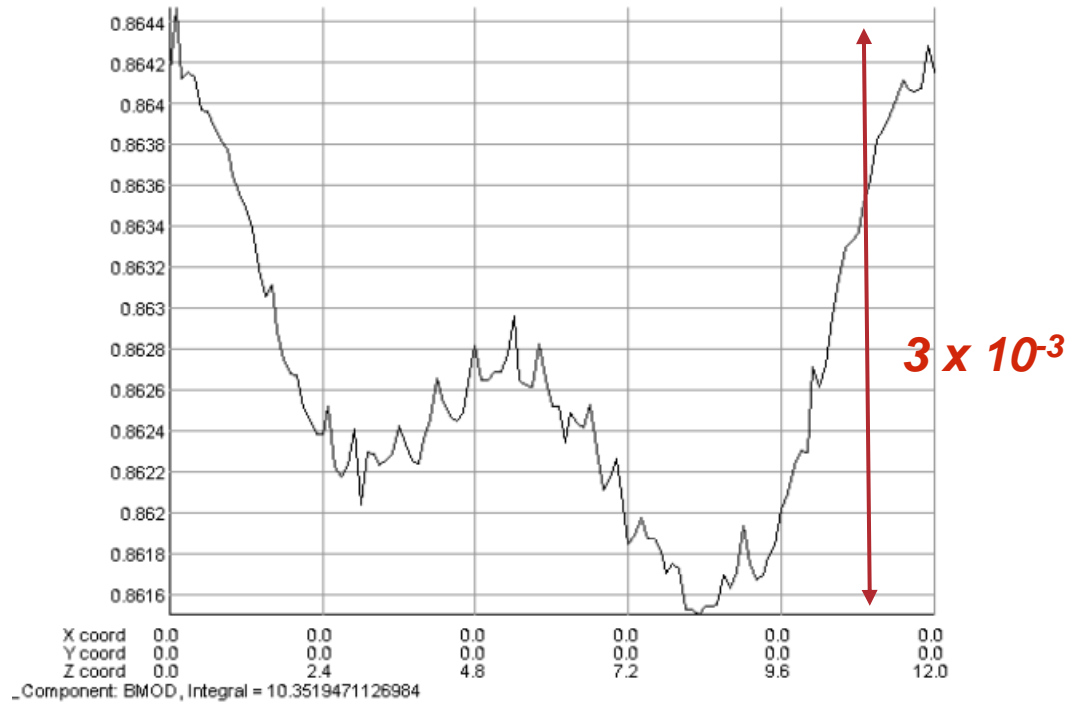
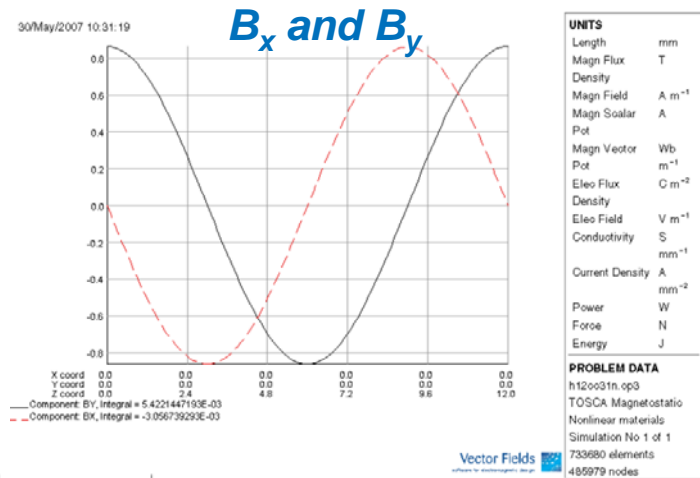


31-Period Steel Poles

$$\lambda = 12 \text{ mm}$$

$$r_0 = 3.15 \text{ mm}$$

$$a = 3.0 \text{ mm}$$



Conclusion

- **On-axis field B_0 :**
 - **For coil/pole ratio < 2 , discrepancy $< 4\%$**
- **Higher harmonics:**
 - **It appears that the calculated higher harmonics for both linear and nonlinear poles are due to calculation errors and end fields for linear poles.**
 - **It also appears that the calculated higher harmonics do not depend on the coil/pole ratios indicating that the analytical result may be correct.**