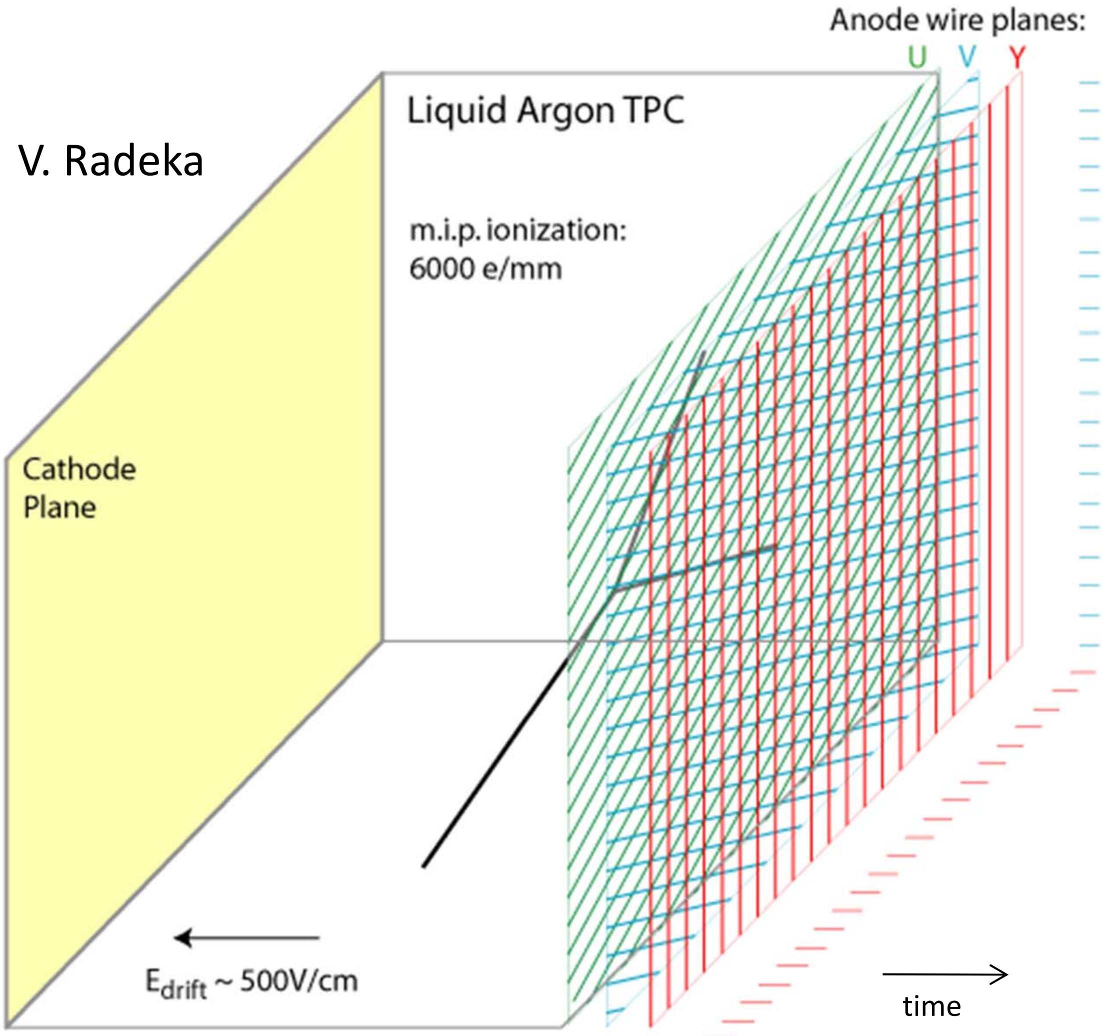
Overview of TPC Signal Processing

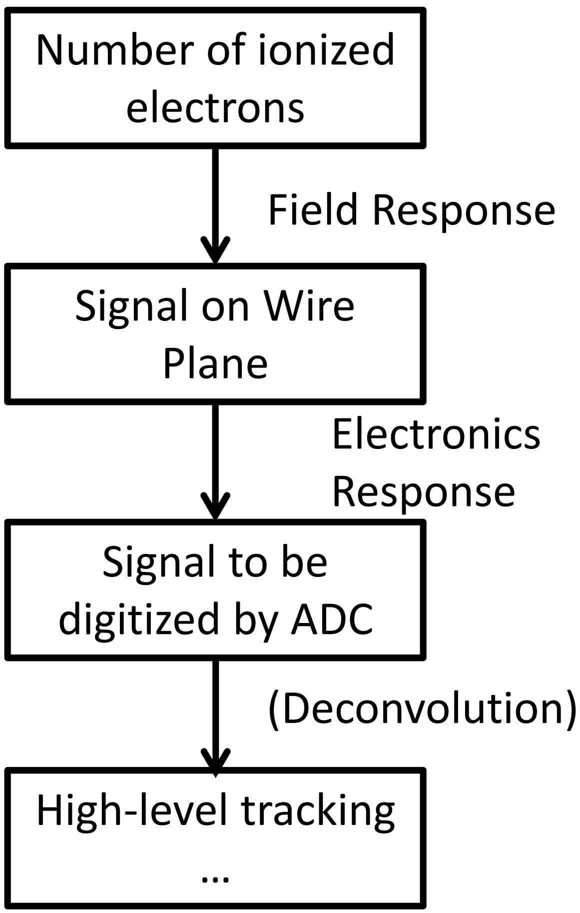
Xin Qian

BNL

Overview of Signal Processing

V. Radeka

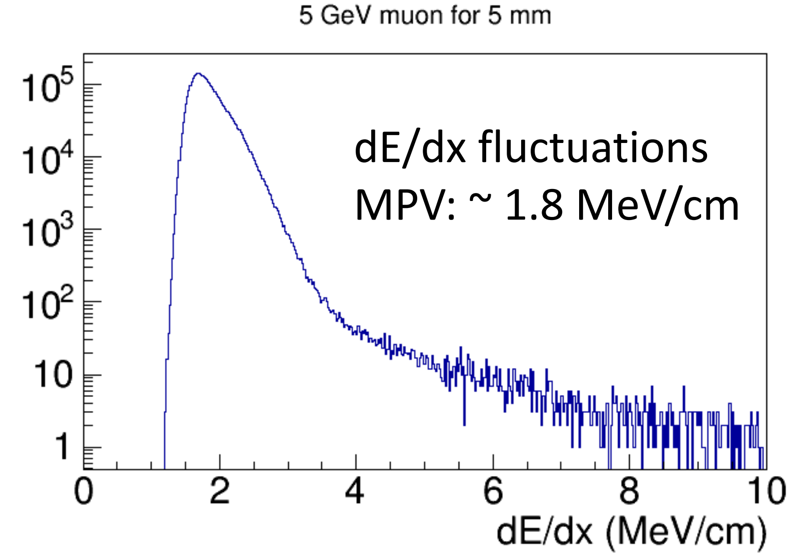


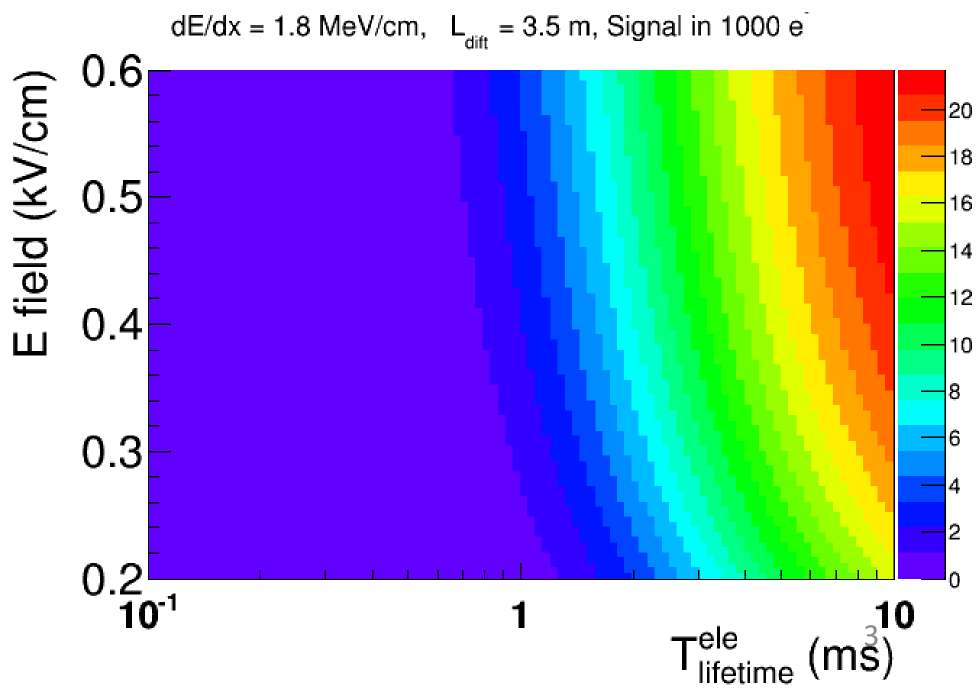


- TPC signal consists of **time** and **charge** information from induction and collection planes
  - **Same amount charge** was seen by all the wire planes
- The goal of signal processing is to extract both time and charge information reliably

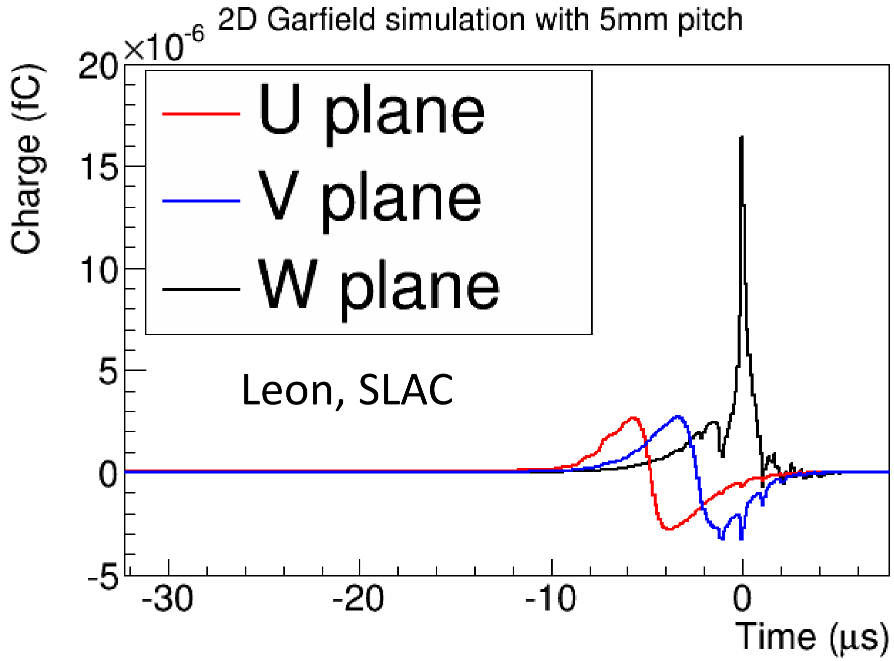
What will affect TPC signal?

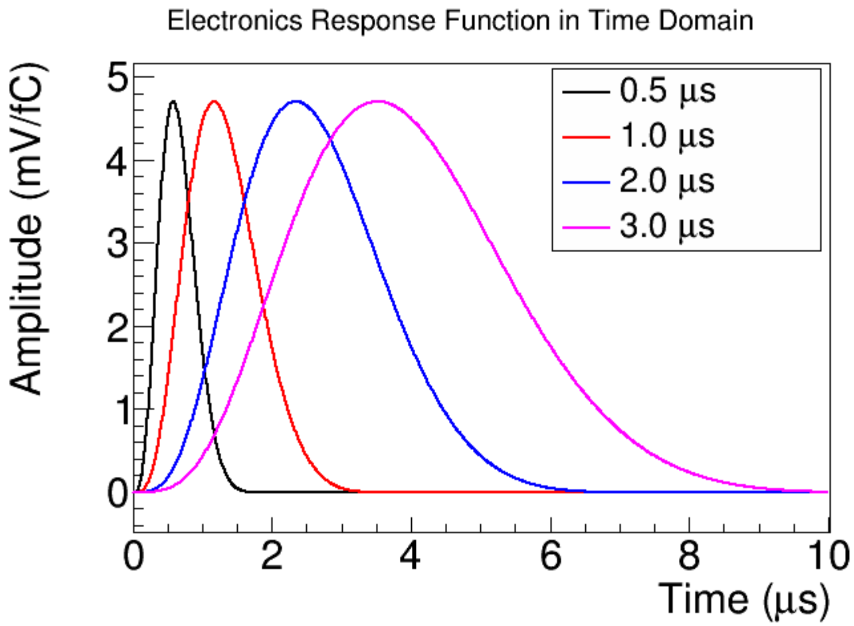
- E-field:
  - Electron drift velocity
  - Recombination factor
- Electron lifetime
  - Drift distance → drift time → signal size
- Track angle
  - Time structure of signal
- dE/dx
  - Recombination etc.
- Diffusion





Field and Electronics Response





- Charge vs. Time averaged for a single electron
- Difference between simulation and data (bigger signal in LongBo arXiv:1504:00398)

Cold electronics:

- 4 shaping time (0.5, 1.0, 2.0, 3.0 us)
- 4 gain (4.7, 7.8, 14, 25 mV/fC)

$$[\text{Digitized (ADC)}] = [\#\ \text{of}\ e^-] \times [\text{Field res. (fC/}e^-)] \times [\text{Ele. res. (mV/fC)}] \times 2.5(\text{ADC/ mV})$$

Deconvolution

Time domain

$$M(t_0) = \int_t R(t - t_0) \cdot S(t) \cdot dt$$

Fourier transformation

$$M(\omega) = R(\omega) \cdot S(\omega)$$

Frequency domain

$$S(\omega) = \frac{M(\omega)}{R(\omega)} \cdot F(\omega)$$

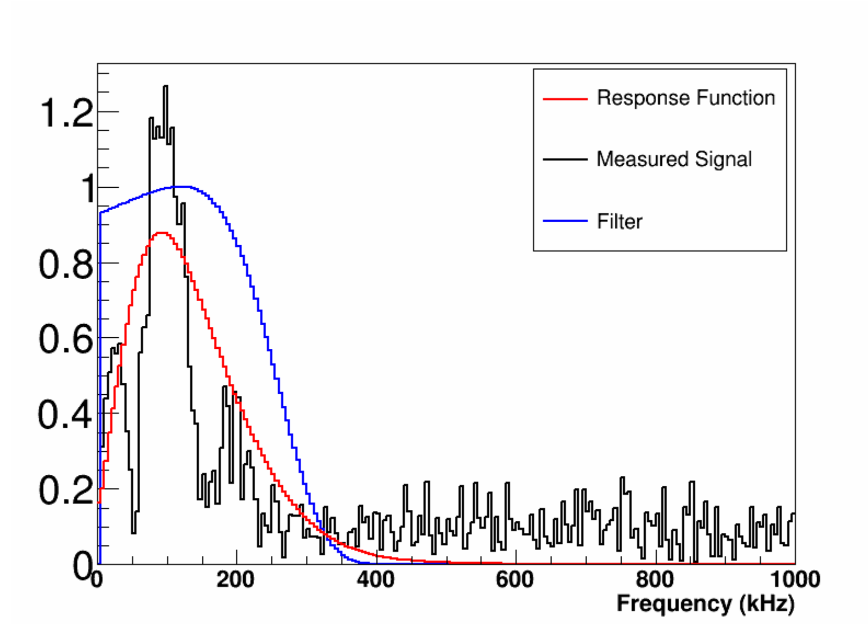
Back to time domain

$$S(t)$$

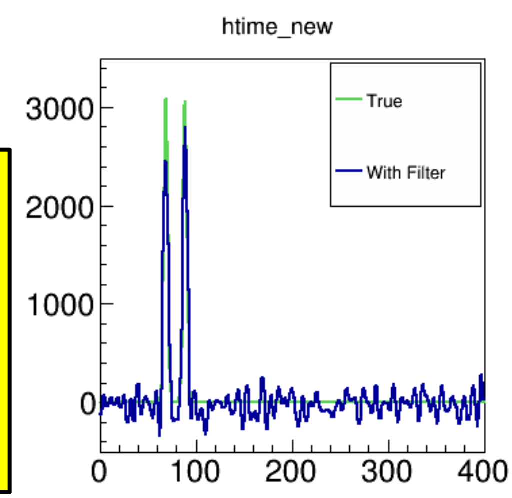
Anti-Fourier transformation

The goal of deconvolution is to help extract charge and time information from TPC signals

Frequency Content



Deconvoluted Signals



Time Domain

Deconvolution and Matrix Inversion

Time domain

$$M(t_0) = \int_t R(t - t_0) \cdot S(t) \cdot dt$$

Fourier transformation

$$M(\omega) = R(\omega) \cdot S(\omega)$$

Frequency domain

$$S(\omega) = \frac{M(\omega)}{R(\omega)} \cdot F(\omega)$$

Anti-Fourier transformation

$$S(t)$$

Back to time domain

- We can also write the formula in a discrete manner

$$M_j = \sum_i R_{ij} \cdot S_i$$

- If there is no filter function in the deconvolution process , the FFT deconvolution is basically equivalent to the matrix inversion problem

$$S = R^{-1} \cdot M$$

Matrix Inversion and Chi-square

- Matrix inversion can be derived through Chi-square

$$\chi^2 = \sum_i \left(M_i - \sum_j R_{ij} \cdot S_j \right)^2$$

$$\frac{\partial \chi^2}{\partial S_k} = 0 \rightarrow \sum_i 2 \cdot \left(M_i - \sum_j R_{ij} \cdot S_j \right) \cdot R_{ik} = 0$$

$$\text{Solution} \rightarrow M_i - \sum_j R_{ij} \cdot S_j = 0 \text{ or}$$

$$\sum_i M_i R_{ik} = \sum_j \left(\sum_i R_{ij} R_{ik} \right) \cdot S_j \rightarrow R \cdot M = R^2 \cdot S$$

Now, we recovered the solution in the previous slice
The last equation is in matrix format

Adding a Penalty in Chi-square

$$\chi^2 = \sum_i \left(M_i - \sum_j R_{ij} \cdot S_j \right)^2 + \chi^2_{penalty}$$

- Let's use the second derivative to penalty

$$\chi^2_{penalty} = c^2 \sum_i \left(S_i '' \right)^2$$

- The second derivative can be written as
- $S_i '' \sim S_{i+1} - 2S_i + S_{i-1}$, in this case

$$\chi^2 = \sum_i \left(M_i - \sum_j R_{ij} \cdot S_j \right)^2 + \sum_i \left(\sum_j F_{ij} \cdot S_j \right)^2$$

$$\frac{\partial \chi^2}{\partial S_k} = 0 \rightarrow \sum_i -2 \cdot \left(M_i - \sum_j R_{ij} \cdot S_j \right) \cdot R_{ik} + 2 \cdot \left(\sum_j F_{ij} \cdot S_j \right) \cdot F_{ik} = 0$$

Continue

$$\sum_i \left(\left(M_i - \sum_j R_{ij} \cdot S_j \right) \cdot R_{ik} - \left(\sum_j F_{ij} \cdot S_j \right) \cdot F_{ik} \right) = 0$$

$$\sum_i \left(M_i - \sum_j \left(R_{ij} + \frac{F_{ij} \cdot F_{ik}}{R_{ik}} \right) \cdot S_j \right) \cdot R_{ik} = 0$$

- The solution is thus

$$\sum_i \left(M_i \cdot R_{ik} - \sum_j R_{ij} \cdot R_{ik} \cdot \left(1 + \frac{F_{ij} \cdot F_{ik}}{R_{ij} \cdot R_{ik}} \right) \cdot S_j \right) = 0$$

In Matrix Format, we have

$$\mathrm{R} \cdot \mathrm{M} = \left(R^2 + F^2 \right) \cdot S$$

$$\mathrm{S} = (1 + \frac{F^2}{R^2})^{-1} R^{-1} M$$

- So, we have an additional term of $\mathrm{A} = (1 + \frac{F^2}{R^2})^{-1}$ applied to the original solution

A in Matrix vs. Filter in Deconvolution

- A in matrix format is multiplied to the original solution → effective a smearing function
- Filter in deconvolution is effective a smearing function

- Therefore, we can conclude that they are equivalent!
  - Filter is equivalent to a penalty in the chi-square
  - Filter is equivalent to a smearing function

TPC Optimization

- The measured signal inside TPC contains both signal and (electronic) noise
  - Assuming deconvolution is correctly performed, we recover the signal completely
  - Therefore, deconvolution of noise would become uncertainty of charge → smaller the better!
  - The relevant factors are noise level and response functions

$$S(\omega) = \frac{N(\omega)}{R(\omega)} \cdot F(\omega)$$

  - Noise level → electronics, wire length …
  - Response function → Gap between planes, E-field between planes
  - **This study does not require high-level reconstruction**

A new Challenge: Induced Charge in Adjacent Wires

$$i_1 = -q_m \cdot \mathbf{E}_w \mathbf{v}.$$

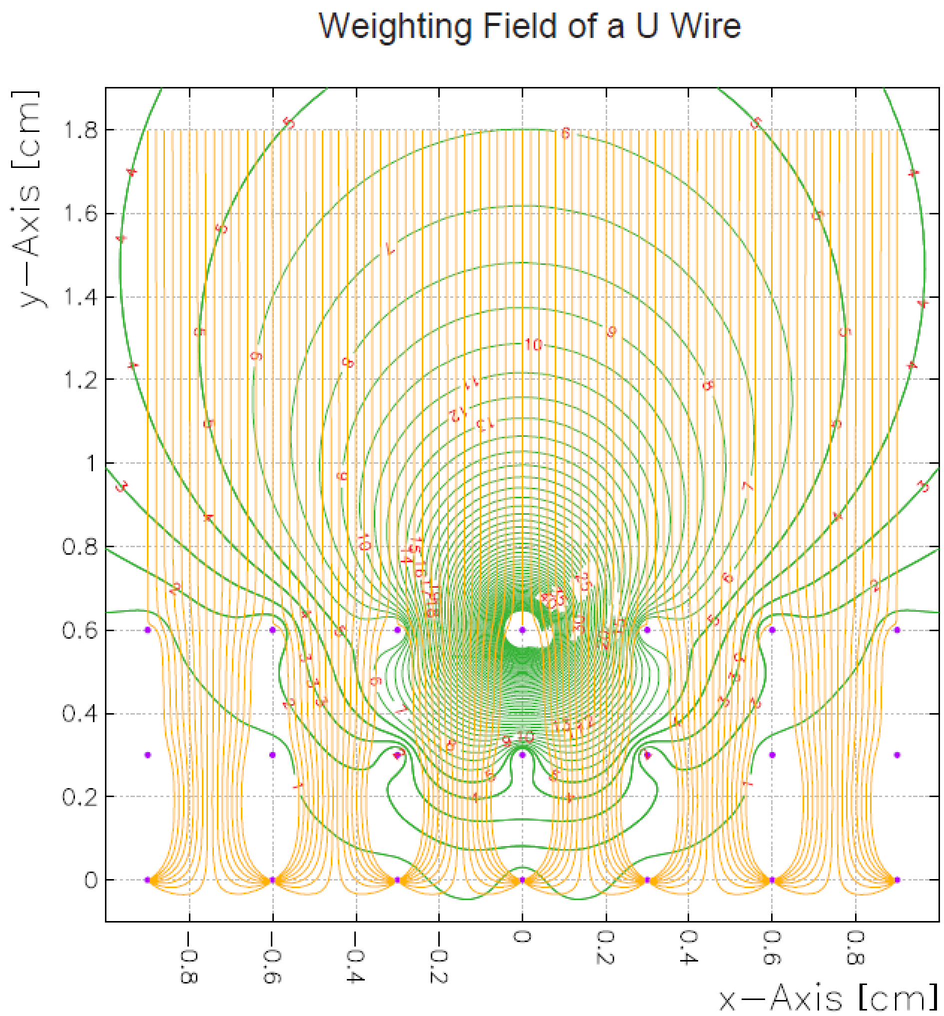
V. Radeka Ann. Rev. Nucl. Part. Sci. 38, 217, 1988

v: velocity

E_w: weighting field

q_m: charge

Bo Yu for MicroBooNE configuration



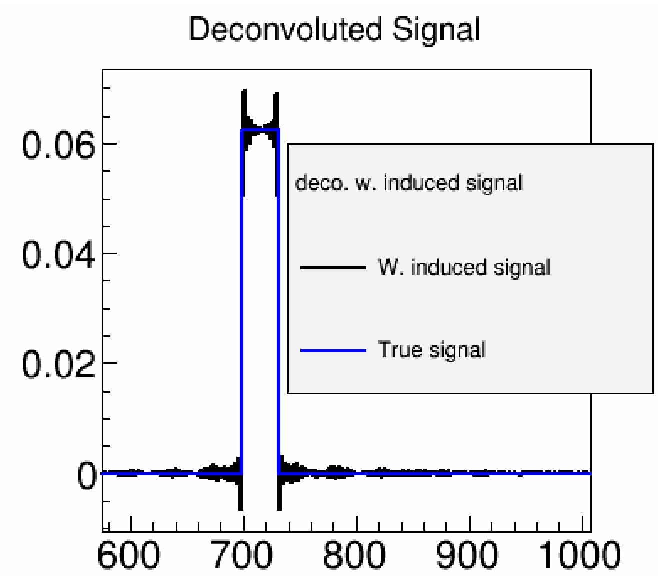
Double Deconvolution

$$M_i(t_0) = \int_t \left(R_0(t-t_0)\cdot S_i(t) + R_1(t-t_0)\cdot S_{i+1}(t) + \ldots \right) \cdot dt$$

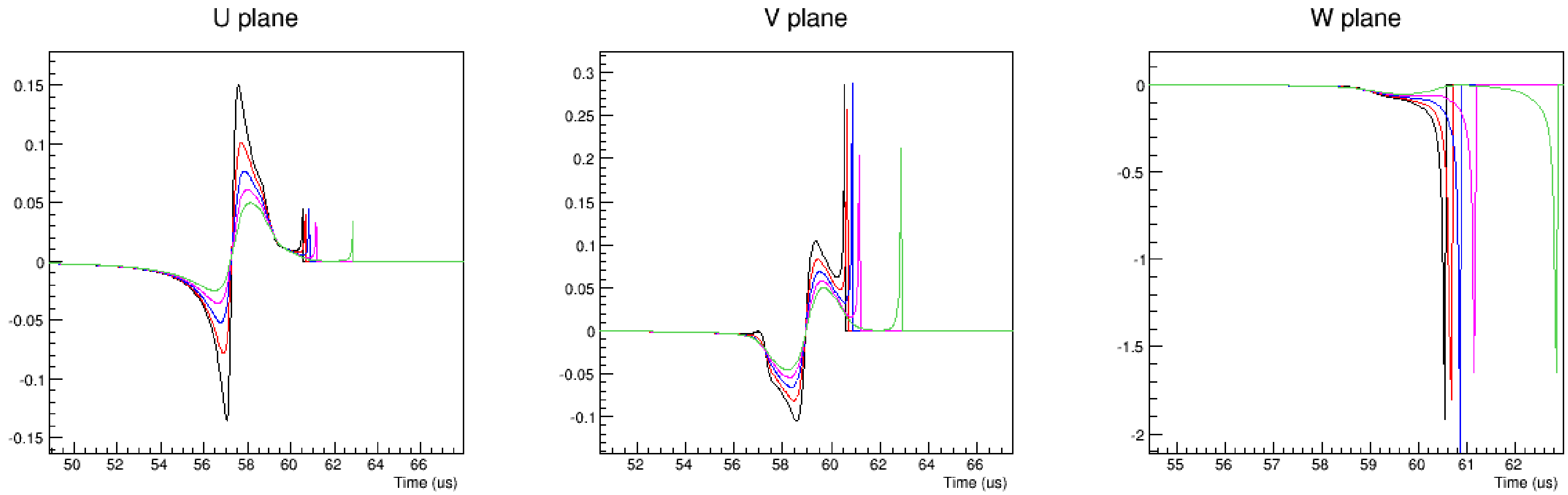
$$M_i(\omega) = R_0(\omega)\cdot S_i(\omega) + R_1(\omega)\cdot S_{i+1}(\omega) + \ldots$$

- With induced signals, the signal is still linear sum of direct signal and induced signal
  - R_1 represents the induced signal from i+1th wire signal to ith wire
  - S_i and S_{i+1} are not directly related

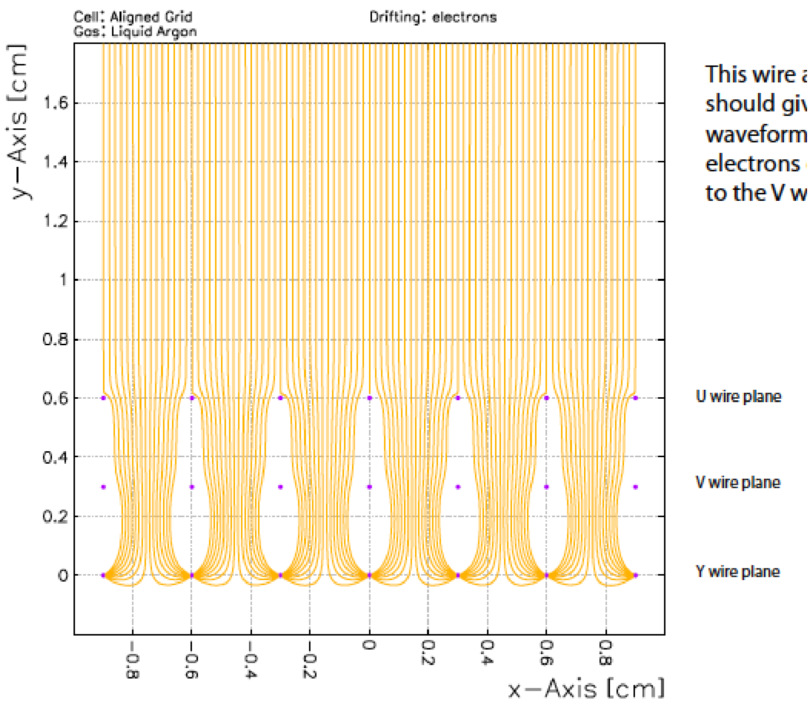
$$\begin{pmatrix} M_1(\omega) \\ M_2(\omega) \\ \ldots \\ M_{n-1}(\omega) \\ M_n(\omega) \end{pmatrix} = \begin{pmatrix} R_0(\omega) & R_1(\omega) & \ldots & R_{n-1}(\omega) & R_n(\omega) \\ R_1(\omega) & R_0(\omega) & \ldots & R_{n-2}(\omega) & R_{n-1}(\omega) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ R_{n-1}(\omega) & R_{n-2}(\omega) & \ldots & R_0(\omega) & R_1(\omega) \\ R_n(\omega) & R_{n-1}(\omega) & \ldots & R_1(\omega) & R_0(\omega) \end{pmatrix} \cdot \begin{pmatrix} S_1(\omega) \\ S_2(\omega) \\ \ldots \\ S_{n-1}(\omega) \\ S_n(\omega) \end{pmatrix}$$



The inversion of matrix R can again be done with deconvolution through 2-D FFT



Simulated by Yichen Li (BNL)



Different curves represent different positions for initial electron
Black → closest to the wire
Green → furthest from the wire

For induction plane, the signal size depends on the position → uncertainty in the charge as we used “average” response function

For collection plane, the signal arrival time depends on the position → uncertainty in the charge

About Wire Pitch

- Bigger wire pitch → bigger signal
  - With same electronics noise, the noise/signal will be smaller → good
- Bigger wire pitch → bigger effect due to position-dependent field response → bigger spread of reconstructed signal → bad
- Naively, we expect the signal has a form of

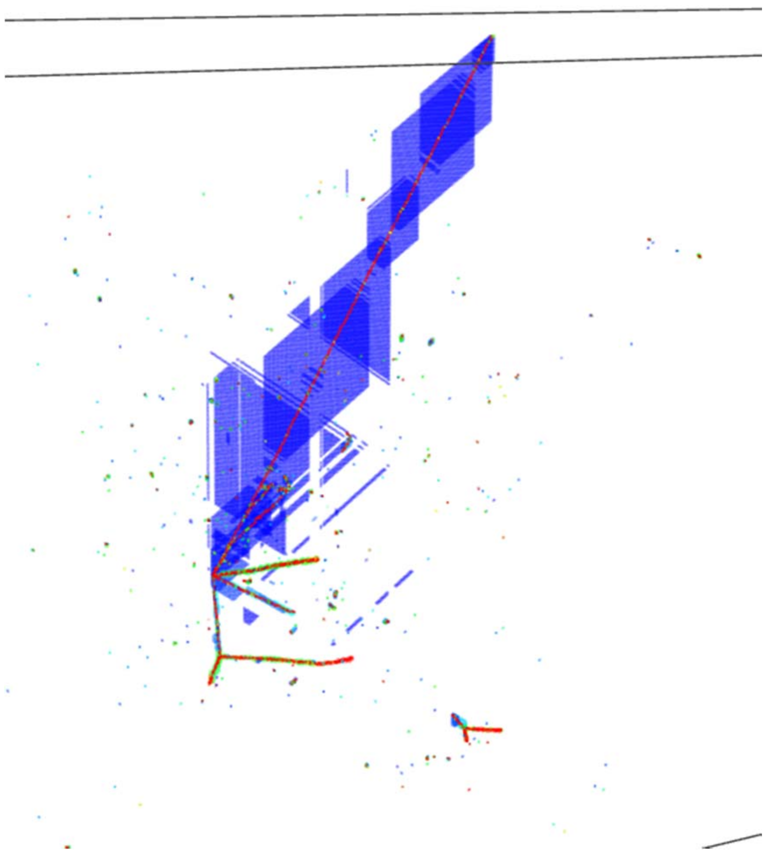
$$\frac{\delta S}{S} = a \oplus \frac{b}{S}$$

a: position-dependent field response,
b: electronics noise

Figure out a&b with detector parameters

Angle of TPC

- In a neutrino event, because of incident neutrino energy, the entire event is more boosted toward forward region
- Due to nucleon response, the lepton tends to going more forward in the center-of-mass frame
- TPC with wire readout has more trouble to reconstruct "tracks" which are parallel to the wire plane → charges arrive at various wire-plane at same time → ambiguity to figure out the proper correlation
- http://www.phy.bnl.gov/wire-cell/bee/set/4/event/2/

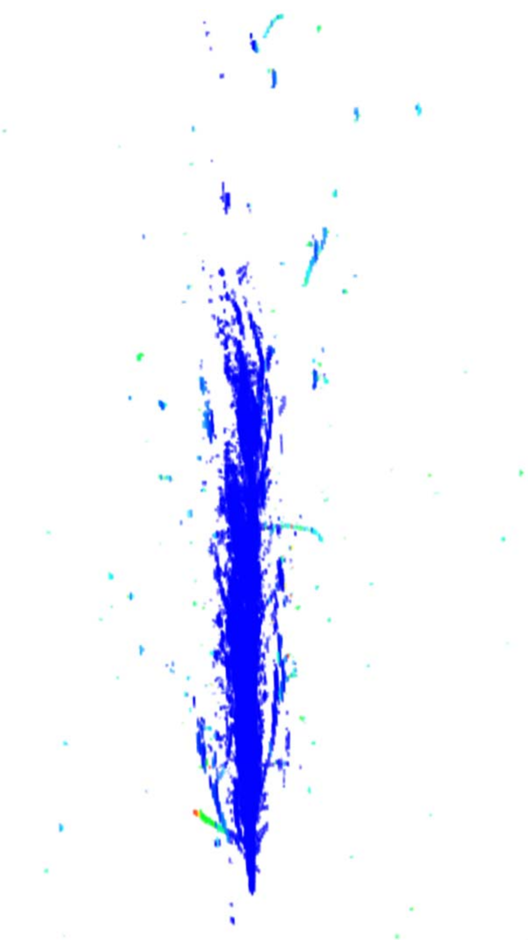


Detector Angle

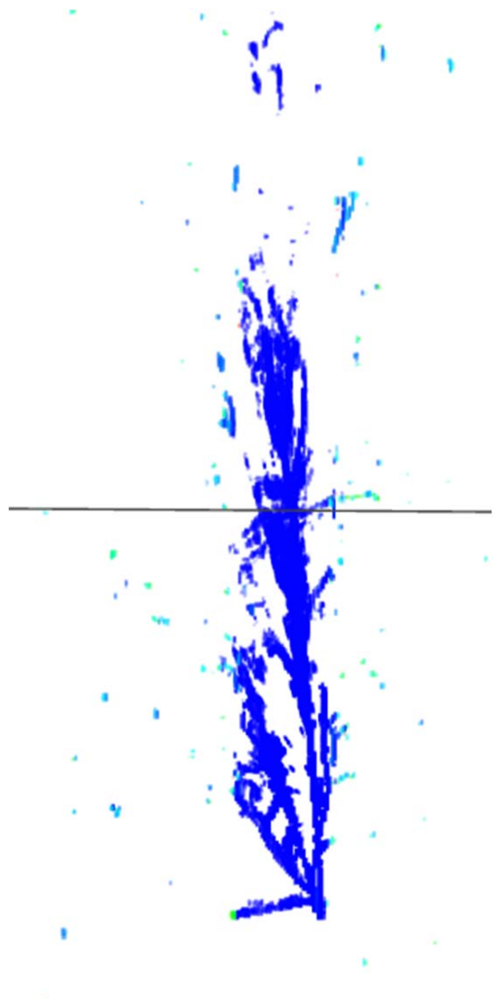
- http://www.phy.bnl.gov/wire-cell/bee/set/7515fe16-d163-4df8-ad87-09f7c809dc7e/event/0/
- Same events (rotate 0-19 degrees)
  - Event 0 → 0 degrees
  - Event 10 → 10 degrees
  - Event 19 → 19 degrees

One can click the web-site to look at events in 3D and rotate to get a better understanding.

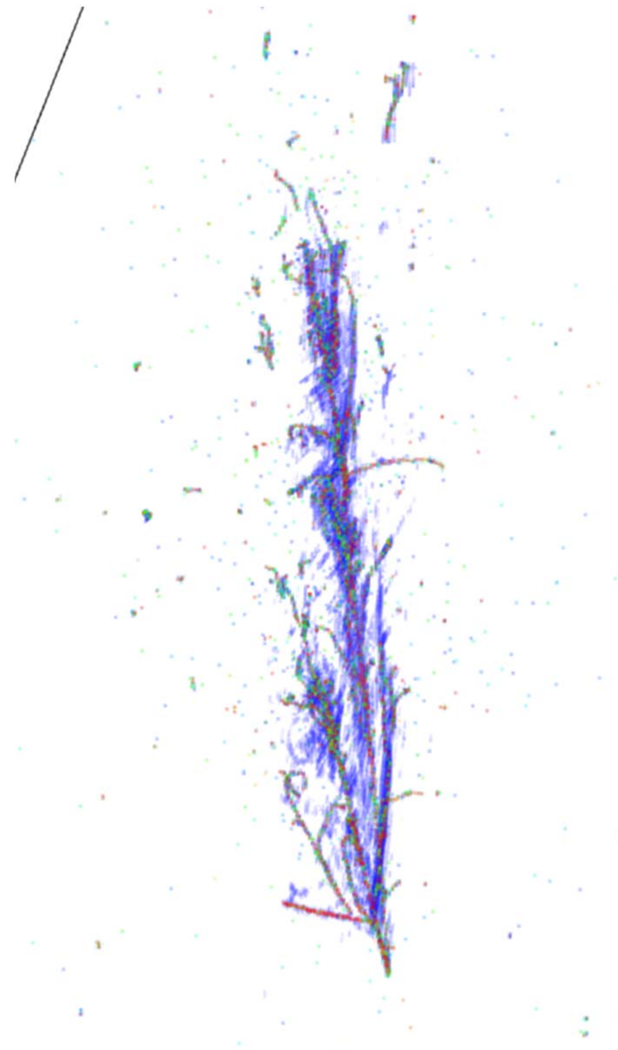
0 degrees



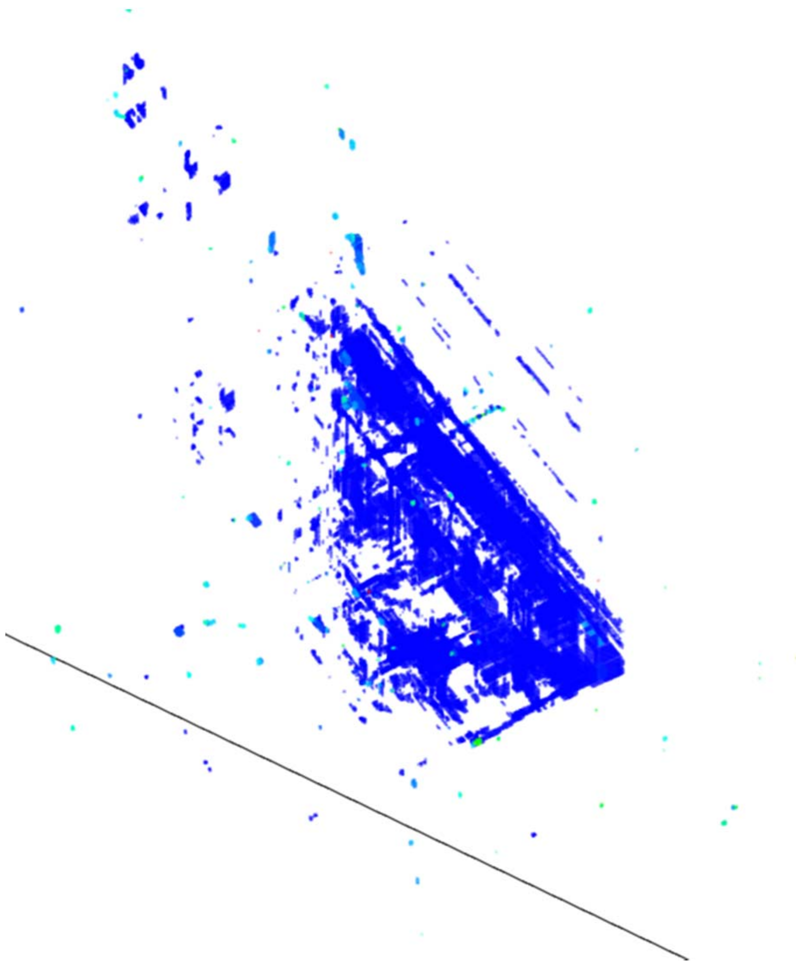
10 degrees



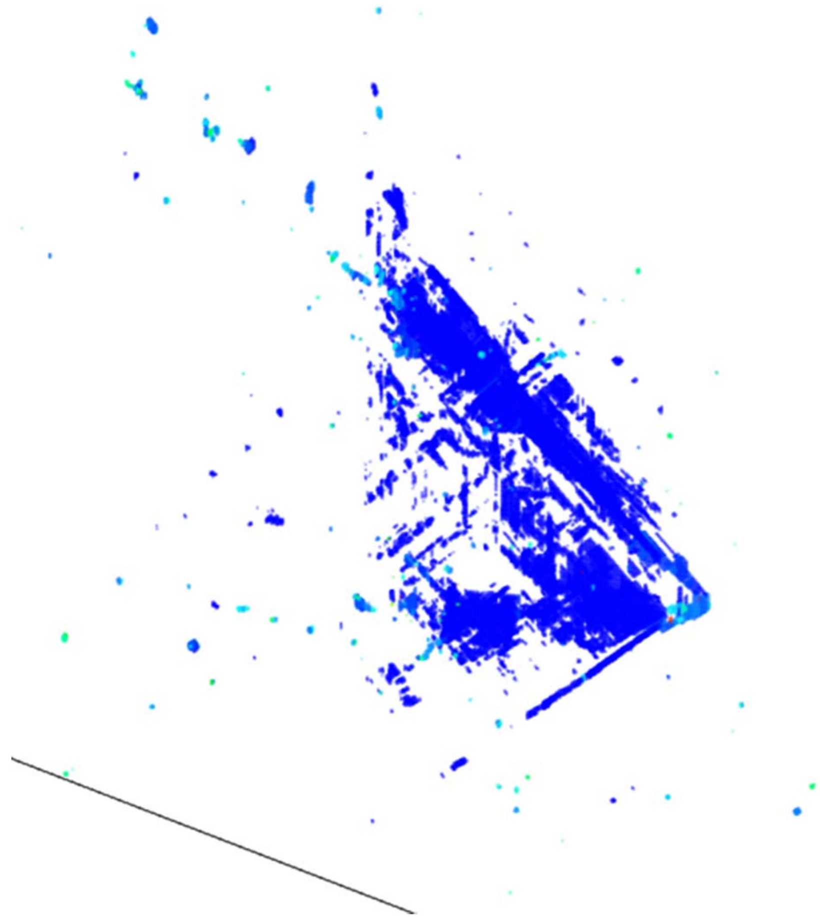
10 degrees (truth)



10 degrees



19 degrees



http://www.phy.bnl.gov/wire-cell/bee/set/6/event/8/

MC truth: one electron, one neutral pion, and one positve pion