Scanning the Earth by the Solar Neutrinos at DUNE, atmospheric neutrinos

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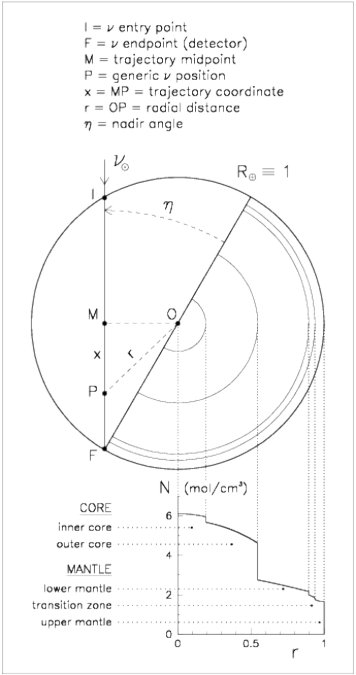

I = ν entry point
F = ν endpoint (detector)
M = trajectory midpoint
P = generic ν position
x = MP = trajectory coordinate
r = OP = radial distance
η = nadir angle
ν⊙
R⊕ ≡ 1
I
η
M
O
x
r
P
F
N (mol/cm³)
CORE
inner core
outer core
MANTLE
lower mantle
transition zone
upper mantle
6
4
2
0
0
0.5
1
r

$$\epsilon \equiv \frac{2V_e E}{\Delta m^2_{21}} \approx 0.03 \left(\frac{\rho}{3\frac{\mathrm{g}}{\mathrm{cm}^3}}\right)\left(\frac{7.5\cdot 10^{-5}\mathrm{eV}^2}{\Delta m^2_{21}}\right)\left(\frac{E}{10\ \mathrm{MeV}}\right)\left(\frac{Y_e}{0.5}\right)$$

$$l_\nu \approx 330 \left(\frac{7.5\times 10^{-5}\mathrm{eV}^2}{\Delta m^2_{21}}\right)\left(\frac{E}{10\ \mathrm{MeV}}\right)\mathrm{km}$$

$$\phi^m_{x_k\to x_n} \equiv \int_{x_k}^{x_n} dx\ \frac{\Delta m^2_{21}}{2E}\sqrt{(\cos 2\theta_{12} - \epsilon(x))^2 + \sin^2 2\theta_{12}}$$

$$\frac{P_N - P_D}{P_D} = -f(\Delta m^2_{21}, \theta_{12})\frac{1}{2}\int_0^L dx\ V(x) \sin\phi^m_{x\to L}$$

where

$$f(\Delta m^2_{21}, \theta_{12}) \ = \ \frac{2\ \cos 2\bar{\theta}^{\odot}_{12}\ \sin^2 2\theta_{12}}{1\ +\ \cos 2\bar{\theta}^{\odot}_{12} \cos 2\theta_{12}} \ = \frac{(2P_{ee} - 1)}{P_{ee}}\frac{\sin^2 2\theta_{12}}{\cos 2\theta_{12}}$$

$f(\Delta m^2_{21}, \theta_{12}) \simeq -2.3$ for $\tan^2\theta = 0.45$ ($\theta = 34^0$) and
$\Delta m^2 = 7.5\times 10^{-5}$ eV^2 ($P_{ee} \simeq 1/3$))
$V \to V\cdot\cos(\theta_{13})^2 \simeq V\cdot 0.98$

Averaging over neutrino energy

$$A_e = \int dE' g(E', E) \frac{P_N - P_D}{P_D} \ .$$

$$A_e = -f(\Delta m^2_{21}, \theta_{12}) \frac{1}{2} \int_0^L dx \ V(x) F(L-x) \sin \phi^m_{x \to L},$$

The decrease of F means that contributions from the large distances to the integral are suppressed.

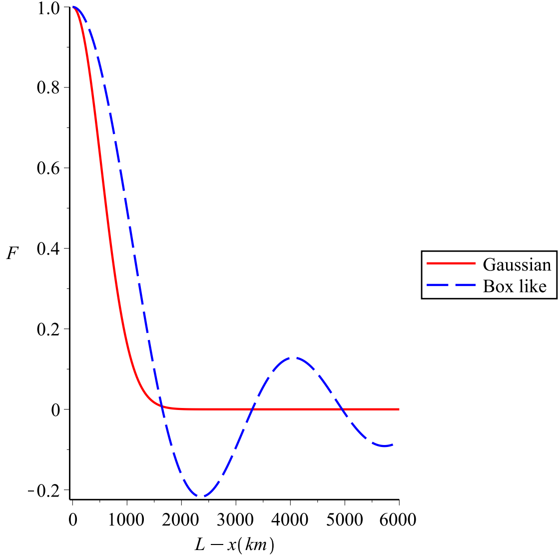
Gaussian energy resolution function

$$g(E, E') = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(E-E')^2}{2\sigma^2}}, \qquad F(L-x) \simeq e^{-2\left(\frac{\pi\sigma(L-x)}{E \ l_\nu}\right)^2}$$

Box like energy resolution function

$$A_e = \frac{1}{2\sigma} \int_{E-\sigma}^{E+\sigma} dE' \ \frac{P_N - P_D}{P_D}$$

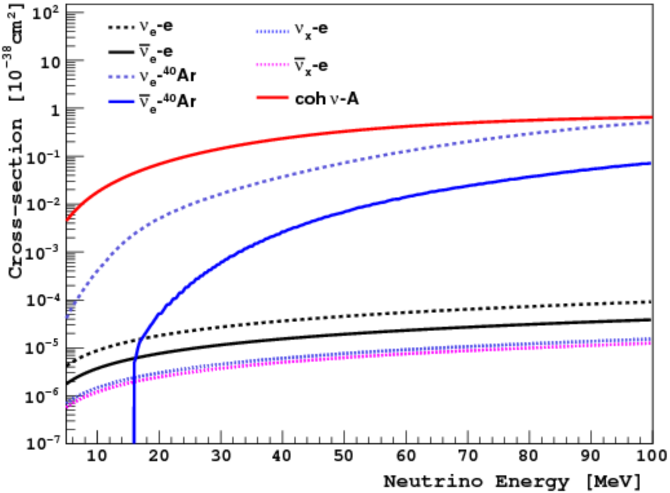
$$F(L-x) \simeq \frac{1}{Q(L-x)} \sin Q(L-x), \qquad Q(L-x) \equiv \frac{2\pi\sigma(L-x)}{E \ l_\nu} \ ,$$



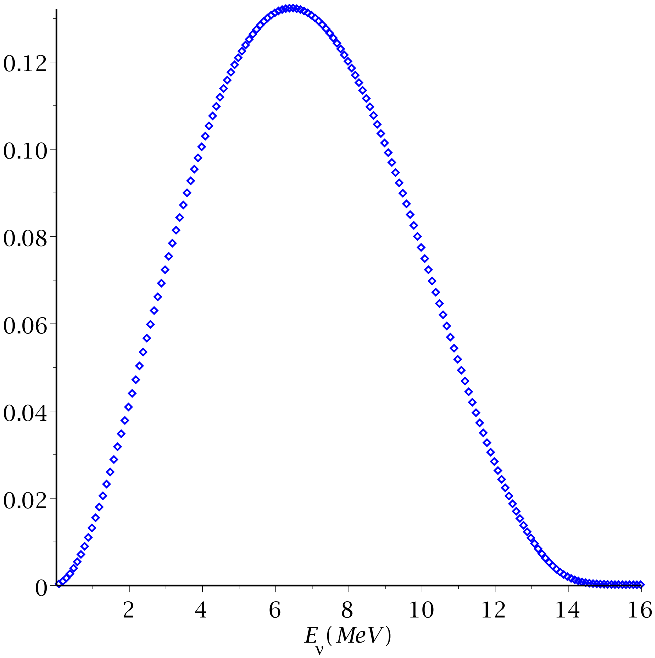
The attenuation factor F as function of $(L - x)$ (distance from detector). $E = 10$ MeV, $\sigma = 1$ MeV, and $\Delta m^2_{21} = 7.5 \cdot 10^{-5}$ eV^2.

$$A_e(E) = \frac{\int dE' \; g(E,E') \; \sigma_{\nu Ar}(E') \; f_{B8}(E')\Delta P_{ee}(E')}{\int dE' \; g(E,E') \; \sigma_{\nu Ar}(E') \; f_{B8}(E') P_{ee}(E')}$$

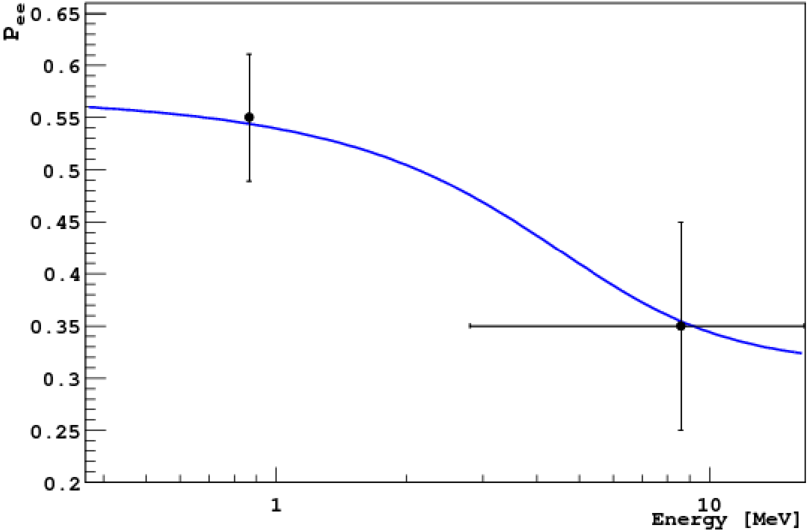
$$\Delta P_{ee} = P_N - P_D = (P_{ee} - \frac{1}{2})\frac{\sin^2 2\theta_{12}}{\cos 2\theta_{12}} \int_0^L dx \; V(x) \sin \phi^m_{x \to L}$$



Cross-section [10^-38 cm^2]
Neutrino Energy [MeV]
ν_e-e
$\bar{\nu}_e$-e
ν_e-^{40}Ar
$\bar{\nu}_e$-^{40}Ar
ν_x-e
$\bar{\nu}_x$-e
coh ν-A



0.12
0.10
0.08
0.06
0.04
0.02
0
2
4
6
8
10
12
14
16
$E_\nu (MeV)$

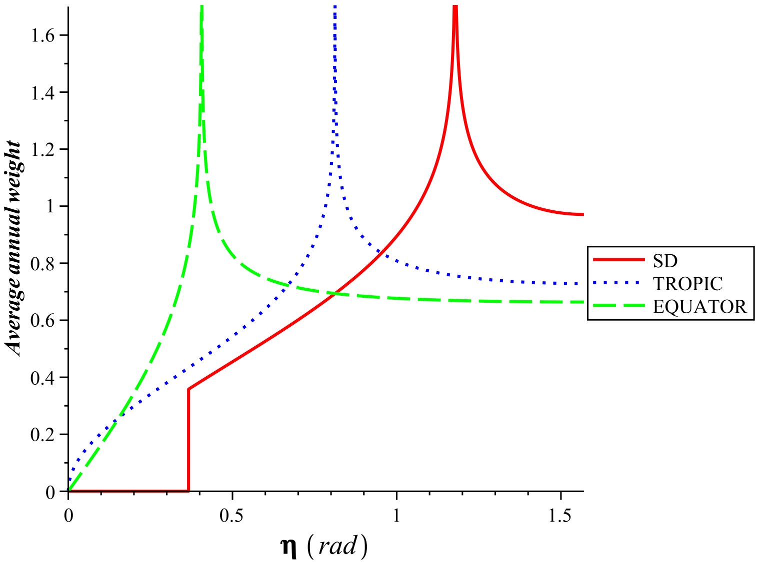


$$P_D \equiv P_{ee} = \frac{1}{2}(1 + \cos 2\bar{\theta}_{12}^{\odot} \cos 2\theta) + s_{13}^4$$

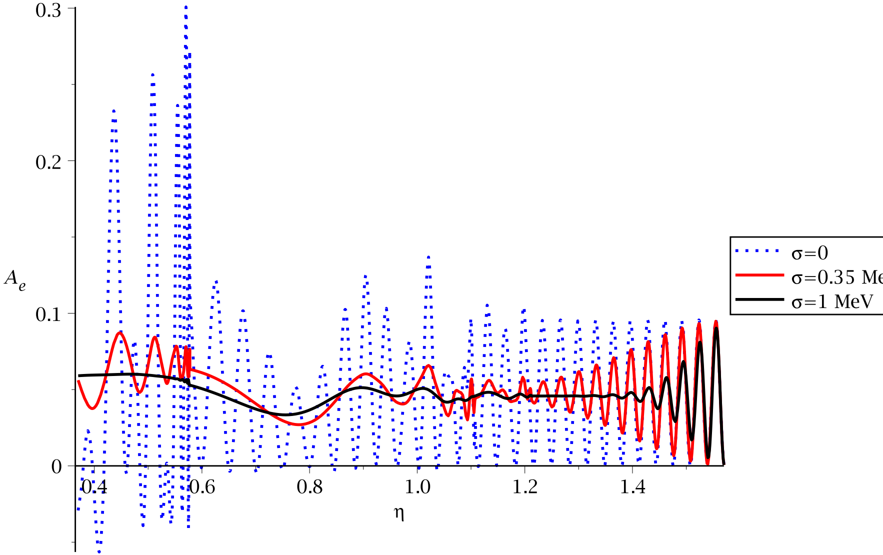
Averaging over solar neutrino production size

From the Earth ^8B neutrino production region in the Sun is seen as a disk where more neutrinos come from the center of the disk. It turns out that that distribution can be appoximated as a normal one with a variance $\delta_\eta \simeq 1.9 \times 10^{-4}$:

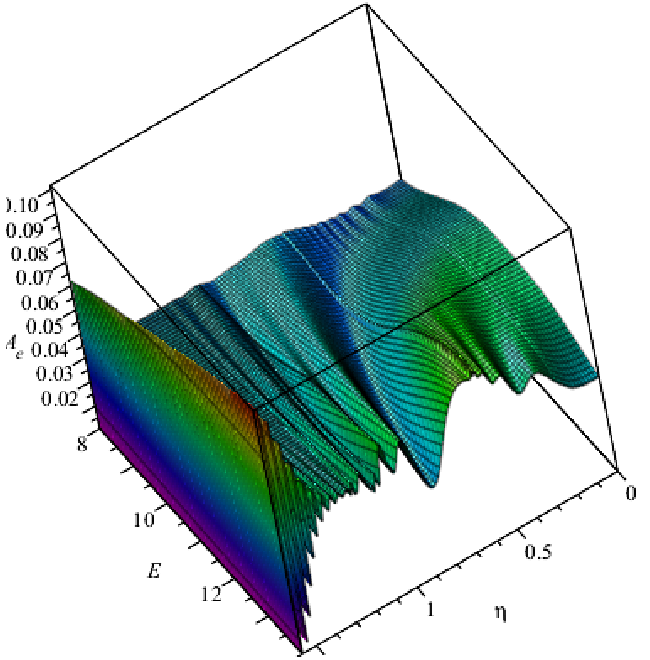
$$f(\eta', \eta) = \frac{1}{\delta_\eta \sqrt{2\pi}} \; e^{-\frac{(\eta-\eta')^2}{2\delta_\eta^2}} = \frac{1}{\delta_\eta \sqrt{2\pi}} \; e^{-\frac{(L-2R_\oplus \cos\eta)^2}{8\delta_\eta^2 R_\oplus^2 \sin^2\eta}}$$



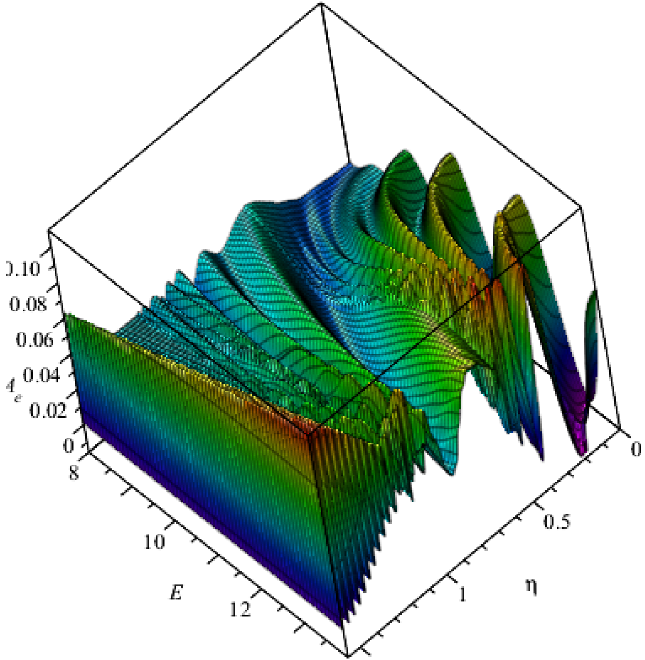
Dependance of average annual weight function on the nadir angle of neutrino trajectory(η is in radians). Outer core is "visable" at SD site about 9% of time



E=12 MeV



0.10
0.09
0.08
0.07
0.06
0.05
A_e
0.04
0.03
0.02
8
10
E
12
1
η
0.5
0



0.08
0.06
0.04
0.02
0
8
10
E
12
0
0.5
η
1

the change of the solar neutrino flux due to the eccentricity of the Earth orbit ($\pm 3\%$) must be taken into account.
Each bin mush cover nadir angle diapason larger then the visible sun size ($(\eta_{i+1} - \eta_i) > 6 * \delta_\eta \simeq 4 * 10^{-4}$)
if one choose the bins in such a way that in each bin (without regeneration effect) it is expected to have a equal detected neutrinos

$$(\eta_{i+1} - \eta_i) = y_0 / W(\eta_i)$$

where $W(\eta)$ is average annual weight function.
Then the χ^2 method gives

$$\chi^2 = \frac{1}{N} \sum_i^n (y_(\eta_i) - y_0 - y_0 A(\eta_i))^2$$

Here y_i is number of registered neutrinos at the i-rd bin and $A(\eta_i)$ is expected regeneration effect at nadir angle η_i. N is number of all detected neutrinos during the night time.

Let rewrite it

$$\chi^2 = 1/N \sum_i^n (y_(\eta_i) - y_0)^2 + y_0^2 A(\eta_i)^2 + y_0^2 A(\eta_i) - 2y_(\eta_i) A_(\eta_i))$$

During the run of variables (Δm_{21}^2 and θ_{12}) for minimization of χ^2 the second and thierd terms under the sum are averaged to $\simeq y_0^2 \bar{A}$ and $\simeq y_0^2 \bar{A}^2 3/2$. ($\bar{A}$ is an average value of relative change of the electron neutrino flux). And they are weakly depend on neutrino oscillation parametres.
Only the last term variate strongly and
we may write the following periodogram

$$\sum_i^n y_(\eta_i) A_(\eta_i)$$

and look for its *maximum* for determination of solar Δ_{21}^2 with high precision.

$$i\frac{d}{dx}\nu = \mathcal{H}\nu \qquad V \ll \frac{\Delta m_a^2}{2E}, \quad \epsilon_a \equiv \frac{2EV(x)}{\Delta m_a^2} \ll 1 \quad (\epsilon_a < 1/4)$$

$$\mathcal{H} = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_\odot^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_a^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$U = U_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} U_{13} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} U_{12}$$

$$U \to U \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix}$$

$$U = U_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} U_{13}\ U_{12}$$

$$S_{\alpha\beta} = T\ e^{-i\int_{x_0}^{x_f} \mathcal{H}(x)dx} =$$

$$= T\ \left[U_m(x) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi_{21}} & 0 \\ 0 & 0 & e^{-i\phi_{31}} \end{pmatrix} U_m^\dagger(x) \right]$$

$$U_m(x) = U \cdot U'(x), \quad U'(x) = U'_{23}(x) \cdot U'_{13}(x) \cdot U'_{12}(x)$$

$$\theta'_{23}(x) = s_{12}\frac{s_{13}c_{12}2EV(x)}{\Delta m_a^2}, \quad \theta'_{13}(x) = c_{12}\frac{s_{13}c_{13}2EV(x)}{\Delta m_a^2}$$

$$\theta'_{23}(x) = s_{12}\frac{\sin 2\theta_{13}}{2}\epsilon_a, \quad \theta'_{13}(x) = c_{12}\frac{\sin 2\theta_{13}}{2}\epsilon_a, \qquad \epsilon_a = \frac{2EV(x)}{\Delta m_a^2}$$

$$\theta'_{12}(x) = \theta_{12}^m(x) - \theta_{12} \quad (V \to c_{13}^2 V(1 - s_{13}^2\epsilon_a))$$

$$U_{12} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} \cos\theta_{12}^m & \sin\theta_{12}^m & \eta \\ -\sin\theta_{12}^m & \cos\theta_{12}^m & 0 \\ -\eta\cos\theta_{12}^m & -\eta\sin\theta_{12}^m & 1 \end{pmatrix}$$

$$\eta = \epsilon_a \frac{\sin 2\theta_{13}}{2} \qquad (s_{13} \approx 1/7, if\ \epsilon_a \simeq 1/4, \eta = 3\%)$$

$$U_m = U_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} U_{13} \begin{pmatrix} \cos\theta_{12}^m & \sin\theta_{12}^m & \eta \\ -\sin\theta_{12}^m & \cos\theta_{12}^m & 0 \\ -\eta\cos\theta_{12}^m & -\eta\sin\theta_{12}^m & 1 \end{pmatrix}$$

$$\sin 2\theta_{12}^m = \frac{\sin 2\theta_{12}}{\sqrt{(\cos 2\theta_{12} - \epsilon_\odot)^2 + \sin^2 2\theta_{12}}}$$

$$\Delta m_{21}^2 = \Delta m_\odot^2 \sqrt{(\cos 2\theta_{12} - \epsilon_\odot)^2 + \sin^2 2\theta_{12}}, \qquad \epsilon_\odot = \frac{c_{13}^2 2\ E\ V}{\Delta m_\odot^2}(1 -$$

$$\Delta m_{31}^2 = \Delta m_a^2 + \frac{1}{2}(\Delta m_{21}^2 - \Delta m_\odot^2 - c_{13}^2 2EV) + s_{13}^2 2EV + \frac{3}{2} c_{13}^2 s_{13}^2 \frac{(2}{\Delta}$$

$$\frac{\Delta m_{31}^2}{2E} = \frac{\Delta m_a^2}{2E} + \frac{1}{2}(\frac{\Delta m_{21}^2}{2E} - \frac{\Delta m_\odot^2}{2E} - c_{13}^2 V) + s_{13}^2 V + \frac{3}{2} c_{13}^2 s_{13}^2 \frac{2EV}{\Delta m_a^2}$$

$$\phi_{21} = \frac{\Delta m_{21}^2}{2E} \cdot x, \quad \phi_{31} = \frac{\Delta m_{31}^2}{2E} \cdot x$$

$$P_{\nu_\mu\to\nu_e} = P^{0m}_{\nu_\mu\to\nu_e} + P'_{\nu_\mu\to\nu_e}, \qquad P^{0m}_{\nu_\mu\to\nu_e} \quad \theta_{12}\to\theta^m_{12}$$

$$P'_{\nu_\mu\to\nu_e} \simeq \frac{1}{4}\frac{2EV}{\Delta m^2_a}\sin 2\theta_{13}\sin 2\theta_{23}\sin 2\theta^m_{12}[\sin\delta(\sin\phi_{32}-\sin\phi_{31}+\sin$$

$$+\cos\delta(\cos\phi_{32}-\cos\phi_{31}+\cos 2\theta^m_{12}(1-\cos\phi_{21})]$$

$$+8\frac{2EV}{\Delta m^2_a}\sin^2\theta_{13}[s^2_{23}s^{m2}_{12}\sin^2\frac{\phi_{32}}{2}+s^2_{23}c^{m2}_{12}\sin^2\frac{\phi_{31}}{2}-s^{m2}_{12}c^{m2}_{12}\sin^2\frac{\phi_{12}}{2}]$$

$$+O(s^3_{13})$$

$$P'_{\nu_\mu\to\nu_e} \simeq \sin 4\theta_{13}\ \sin 2\theta_{13}\ \frac{2EV}{\Delta m^2_a}\ \sin^2\theta_{23}\ \sin^2\frac{\phi_{31}}{2} \qquad \phi_{21}\ll 1$$

Conclusions

WC detectors are shortsighted. They see only nearby to detector matter.

Due to excellent energy resolution LiAr DUNE far detector may see deep layers of the Earth

It can determine solar $\Delta m^2_\odot$ with 10 % accuracy (or better).

independent measurement of electron number density $\rightarrow$ chemical composition of the Earth matter.

some lights on chemical composition of the Earth's core.

THANK YOU