

more-than-MHV amplitudes in QCD

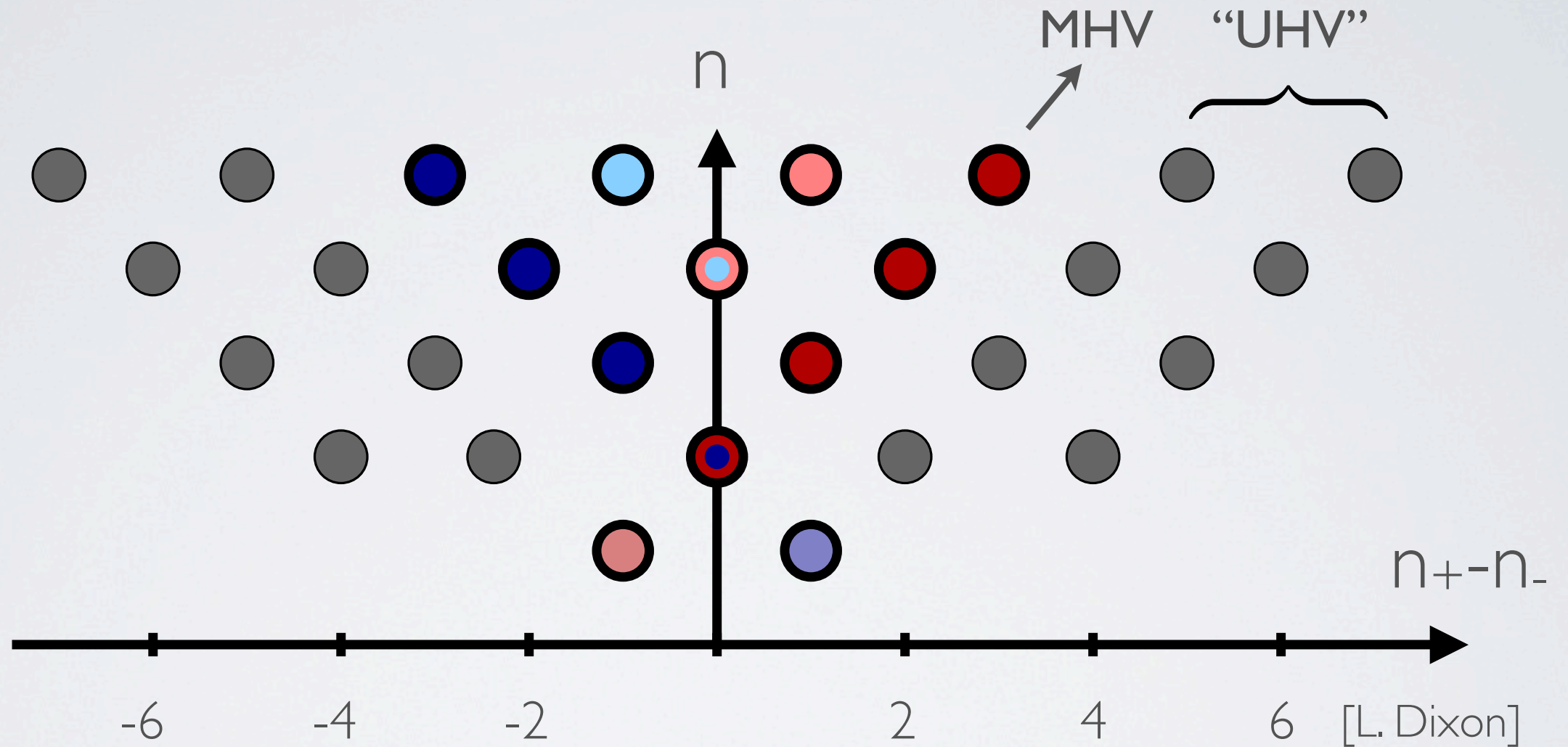
Simon Badger

18th March 2016

MHV@30, Fermilab, 16th-19th March 2016



helicity amplitudes



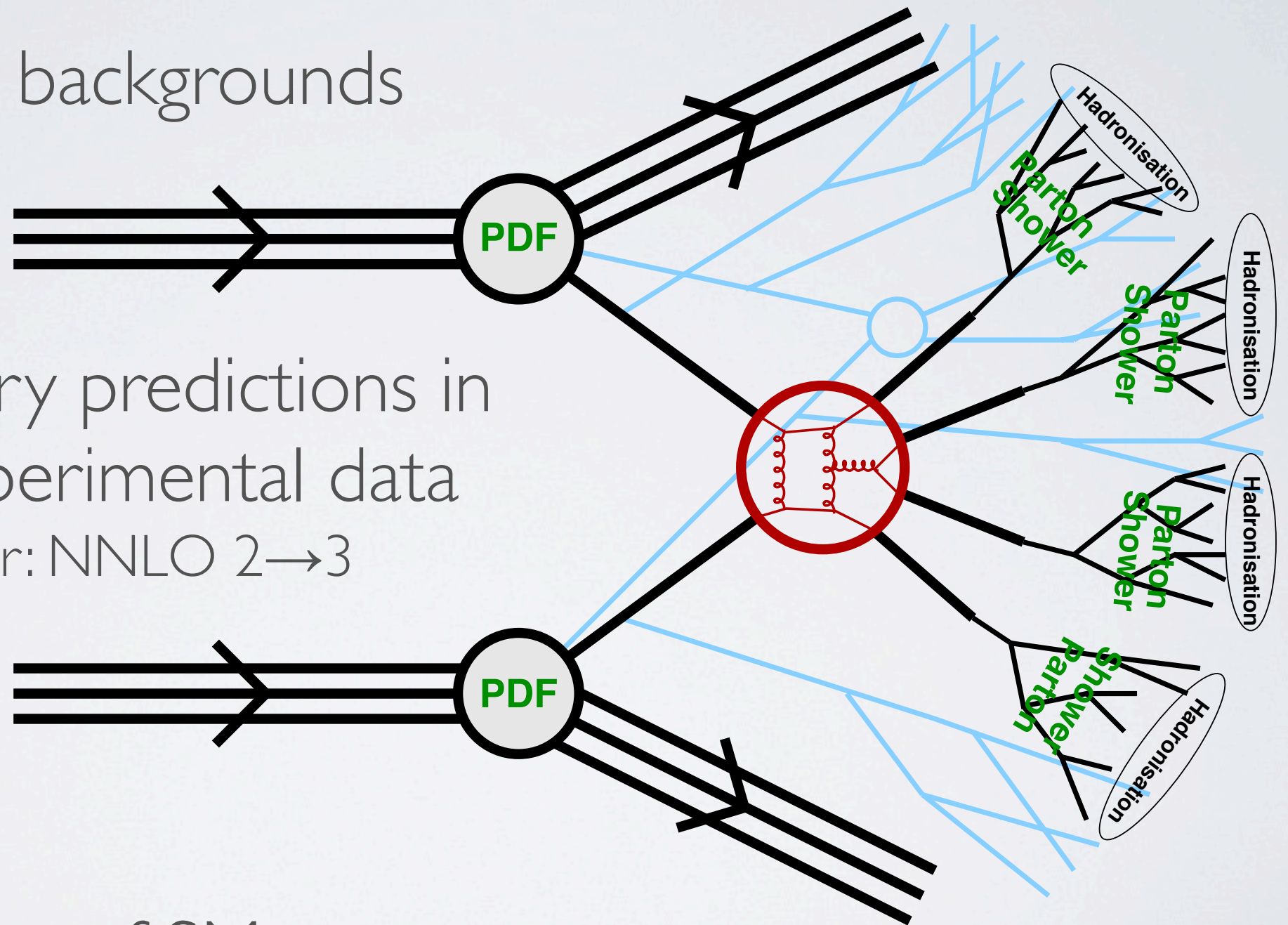
vanish at tree-level and to
all loops in super-symmetric theories

Modelling hadron collisions

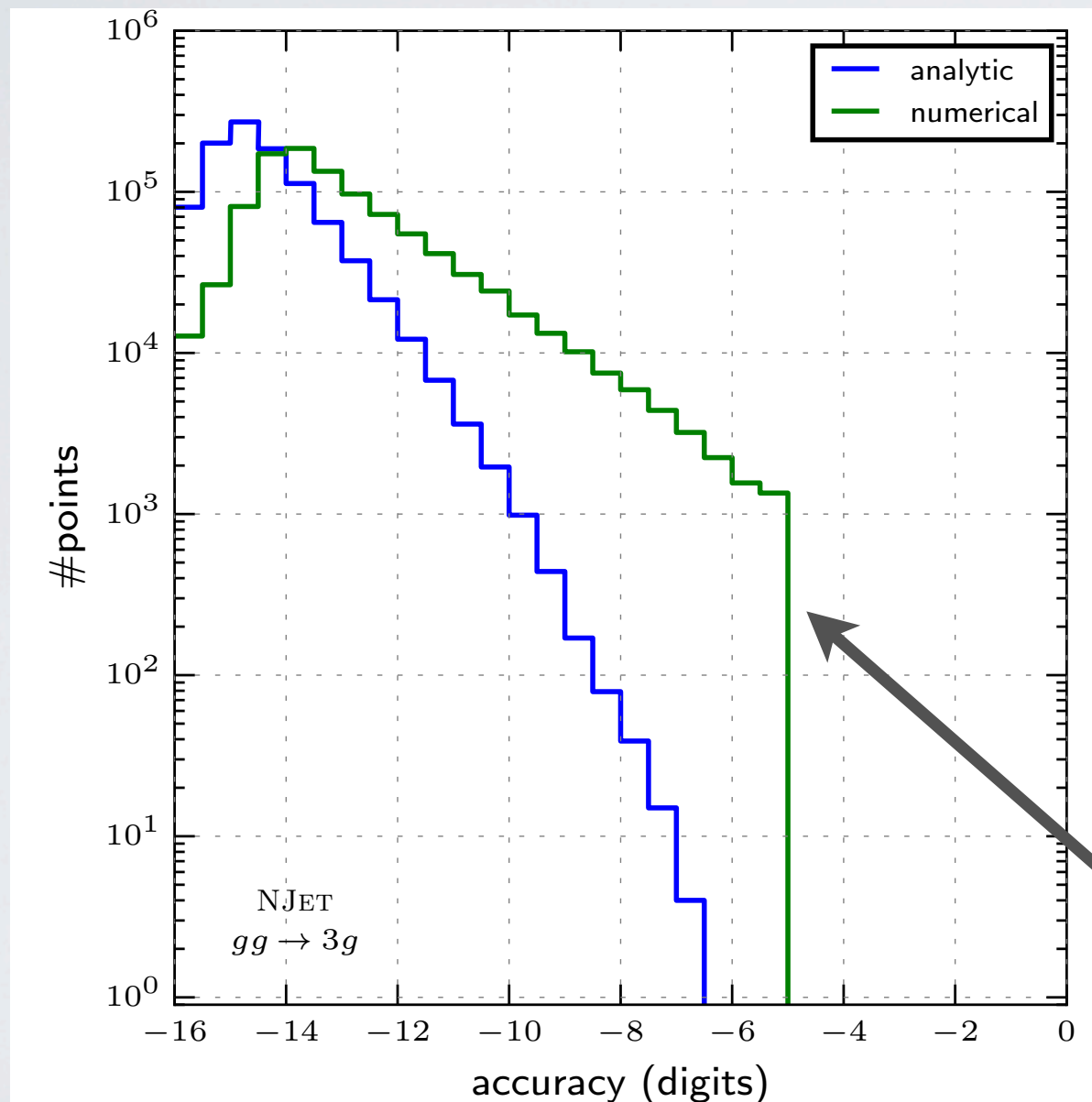
New physics backgrounds

Keeping theory predictions in line with experimental data
next frontier: NNLO $2 \rightarrow 3$

Determination of SM parameters



spurious singularities



numerical D-dimensional
generalised unitarity

vs

analytic computation with
finite integrals basis
(e.g. Bern, Dixon, Kosower
[hep-ph/9302280])

removing spurious poles also
simplifies coefficients \Rightarrow faster
($\sim 100x$ in this case)

need to switch to quadruple
precision evaluation

applications of loop amplitudes in NNLO
computations more intensive

one-loop finite amplitudes in QCD

all multiplicity expressions!

- Off-shell currents for one-loop amplitudes to $\mathcal{O}(\epsilon^0)$
[Mahlon (1993)]
- all-plus $\leftrightarrow \mathcal{N} = 4$ D-dimensional unitarity cuts
[Bern, Dixon, Dunbar, Kosower (1996)]
- BCFW recursion for $\mathcal{O}(\epsilon^0)$
[Bern, Dixon, Kosower (2005)]
- CSW(MHV) rules for D-dimensional integrands
[Elvang, Freedman, Kiermaier (2012)]

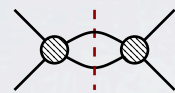
outline

- d-dimensional generalised unitarity
 - ⇒ multi-loops from trees
- two-loop all-plus amplitudes
- local integrands in d-dimensional amplitudes

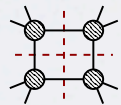
automated one-loop amplitudes

solving on-shell conditions requires **complex** momenta
 \Rightarrow factorise residues into **tree amplitudes**

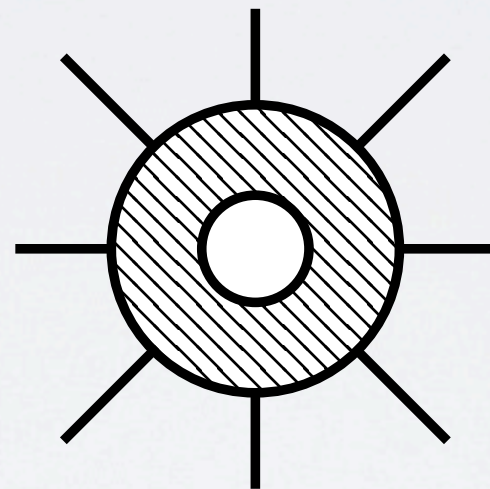
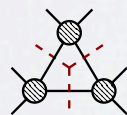
Unitarity: double cuts
 [BDDK '94]
 [triple cuts BDK '97]



Generalized unitarity:
 quadruple cuts [BCF '04]



triple cuts [e.g. Forde '07]



Integrand reduction [OPP '05]

$$\Delta_3 = \text{triangle diagram} - \text{square diagram}$$

D-dim. generalized unitarity [GKM '08]

$$A = \sum_i (\text{rational})_i (\text{integral})_i$$

find complex contour to isolate
 integral coefficient

multi-scale
 kinematic algebra
 performed
numerically

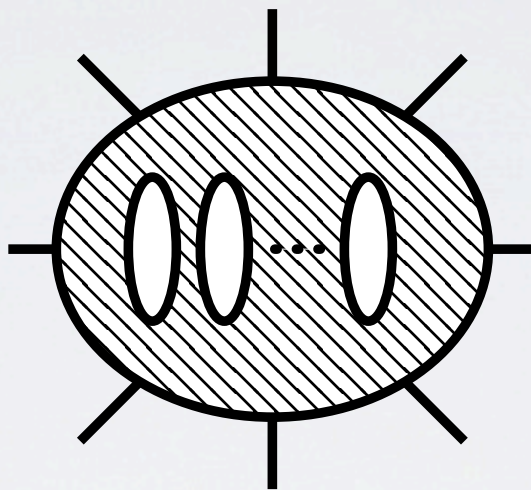
$$A = \int_k \sum_i \frac{\Delta_i(k, p)}{(\text{propagators})_i}$$

explicitly remove poles

multi-loop amplitudes from trees

Maximal unitarity

[Kosower, Larsen, Johansson,
Caron-Huot, Zhang, Søgaard]

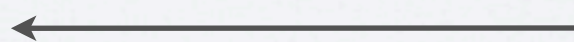


Integrand reduction via
polynomial division

[Mastrolia, Ossola, SB, Frellesvig,
Zhang, Mirabella, Peraro, Malamos,
Kleiss, Papadopolous, Verheyen,
Feng, Huang]

$$A = \sum_i (\text{rational})_i (\text{integral})_i$$

e.g. IBPs



$$A = \int_k \sum_i \frac{\Delta_i(k, p)}{(\text{propagators})_i}$$

IBPs must be free of
doubled propagator MI

[Gluza, Kosower, Kajda 1009.0472] [Schabinger 1111.4220]
[Ita 1510.05626] [Larsen, Zhang 1511.01071]

a toolbox for multi-loop integrands

momentum twistors

[Hodges (2009)]

six-dimensional
spinor-helicity

$$A = \int_k \sum_i \frac{\Delta_i(k, p)}{(\text{propagators})_i}$$

generalised unitarity
cuts

integrand reduction

$$A = \sum_i S_i \frac{C(\Delta_i) \Delta_i}{\prod D_\alpha}$$

colour/kinematics
BCJ relations

multi-loop integrand reduction

[Mastrolia, Ossola | 107.604 |] [SB, Frellesvig, Zhang | 202.2019 |]

[Zhang | 205.5707 |] [Mastrolia, Mirabella, Ossola, Peraro | 205.7087 |]

ISP monomials

$$\Delta_T(\{k\}) = \sum_{\{\alpha\}} c_{T;\alpha_1 \dots \alpha_n} (k_1 \cdot p_5)^{\alpha_1} (k_2 \cdot p_2)^{\alpha_2} \dots (k_1 \cdot \omega_2)^{\alpha_m} \dots \mu_{12}^{\alpha_n}$$

rational coefficients

spurious directions

extra-dimensional ISPs

$$\mu_{ij} = -k_i^{-2\epsilon} \cdot k_j^{-2\epsilon}$$

integrand reduction

$$\Delta_{c;T} \Big|_{\text{cut}} = \prod_i A_i^{(0)} - \sum_{T'} \frac{\Delta_{c;T'}}{\prod_{l \in T'/T} D_l} \Big|_{\text{cut}}$$

on-shell the numerators can be written as products of tree-level amplitudes

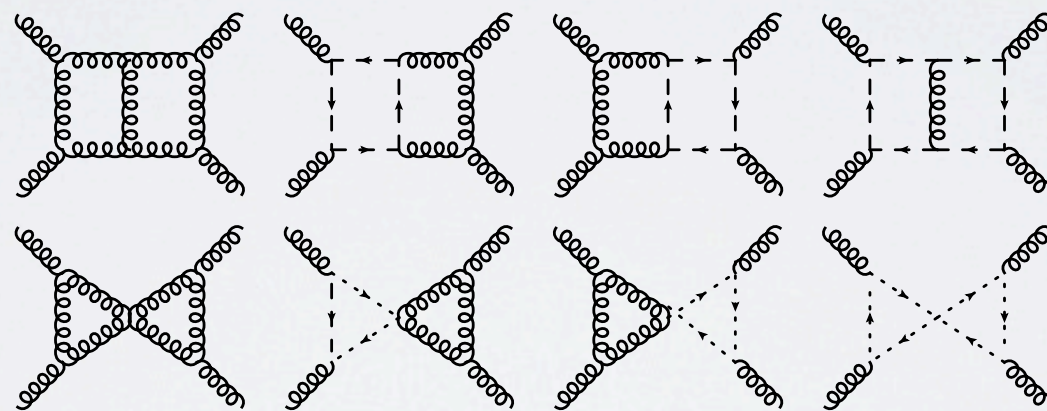
fix basis of **monomials** in **irreducible scalar products** via **polynomial division** (Gröbner basis)

integrand parameterisations not unique - freedom in the choices of ISP monomials

six-dimensional trees

six dimensional spinor helicity [Cheung, O'Connell 0902.0981]
 (one-loop applications [Bern et al. 1010.0494] [Davies 1108.0398])

dimensional reduction to $4 - 2\epsilon$ using additional scalar amplitudes



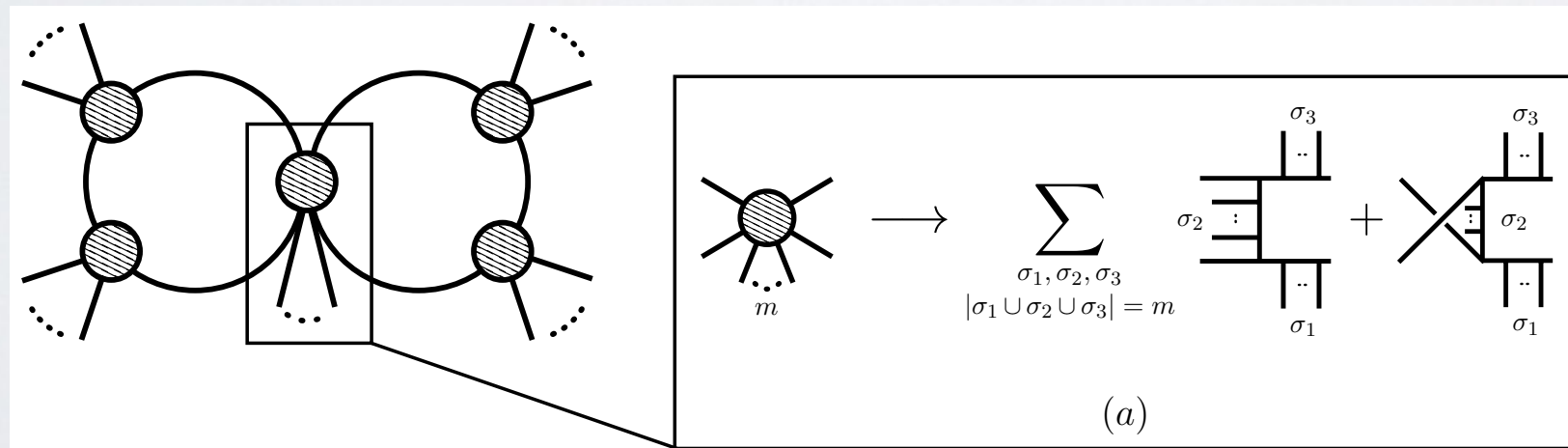
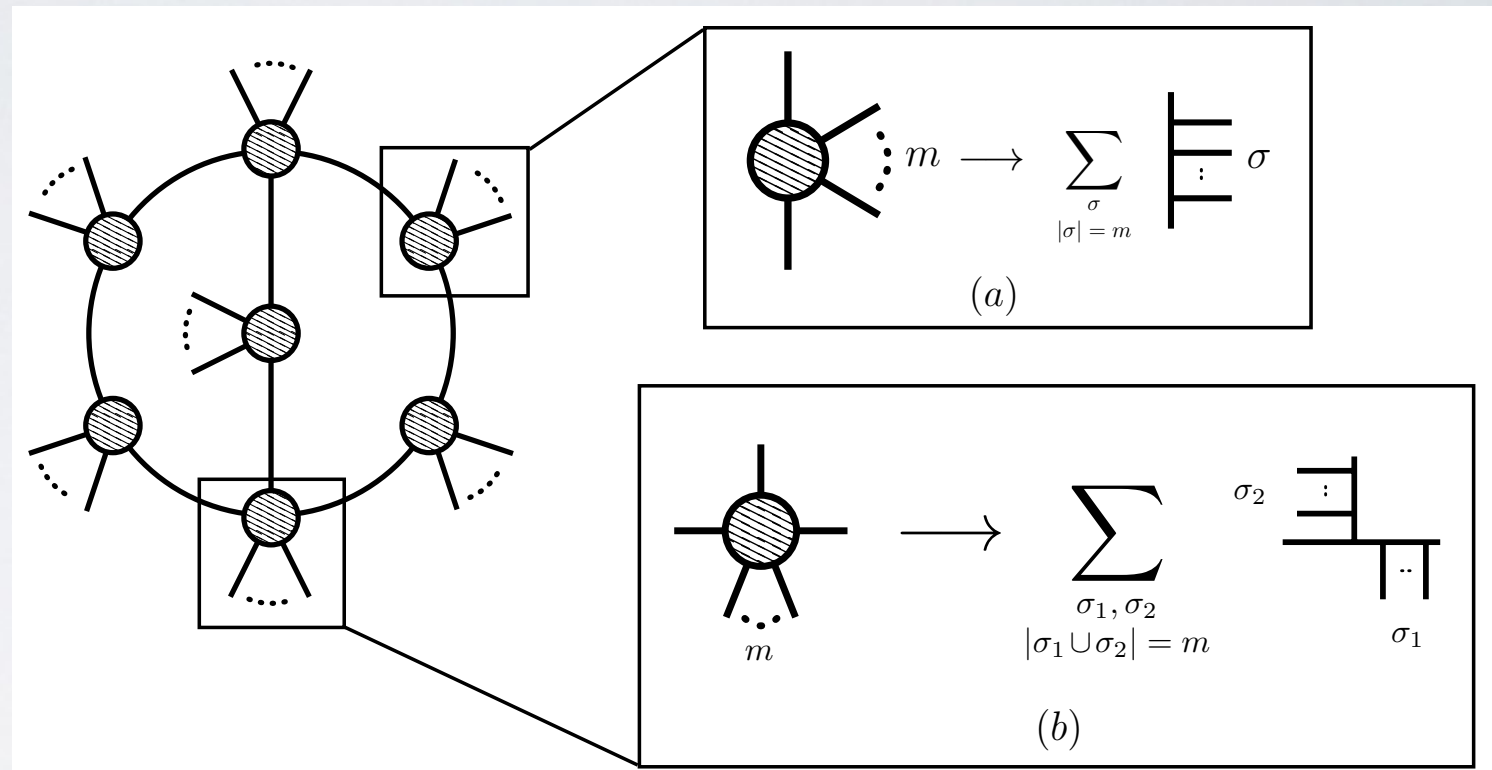
6D is a convenient way to manage all helicities simultaneously

$$A(1_{a\dot{a}}, 2_{b\dot{b}}, 3_{d\dot{d}}, 4_{d\dot{d}}) = \frac{i}{st} \langle 1_a 2_b 3_c 4_d \rangle [1_{\dot{a}} 2_{\dot{b}} 3_{\dot{c}} 4_{\dot{d}}]$$

General colour decompositions

[Dixon, Del Duca, Maltoni (1999)]

Inserting the DDM decomposition into colour dressed cuts leads to a compact loop decomposition



general tree-level DDM colour bases including fermions [Johansson, Ochirov arXiv:1507.00332]

non-planar from planar

e.g. [Bern, Carrasco, Dixon, Johansson, Roiban 1201.5366]

$$A_4(1, 2, 3, 4) = \frac{s_{13}}{s_{12}} A_4(1, 3, 2, 4)$$

see talks by Carrasco and Johansson

factorization

$$\Rightarrow A_3(1, 2, -P_{12}) A_3(P_{12}, 3, 4) = \text{Res}_{s_{12}=0} (A_4(1, 2, 3, 4)) = s_{13} A_4(1, 3, 2, 4) \Big|_{s_{12}=0}$$

$$\Rightarrow \left(\text{Diagram 1} \right) = (k_1 - P_{123})^2 \left(\text{Diagram 2} \right) \Big|_{(k_1 + k_2 + p_3)^2}$$

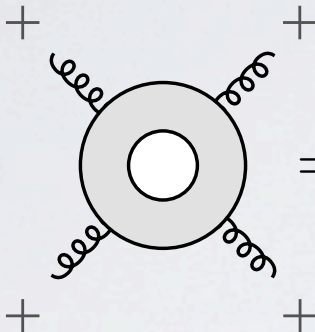
The diagram shows a factorization of a non-planar box diagram into a planar box diagram squared, with a denominator. The non-planar diagram has a vertical internal line. The planar diagram is a standard box with a vertical internal line. The denominator is the square of the sum of the momenta of the top and bottom edges and the internal vertical line.

$$\Rightarrow \Delta \left(\text{Diagram 1} \right) \Big|_{\text{cut}} = \left((k_1 - P_{123})^2 \Delta \left(\text{Diagram 2} \right) + \Delta \left(\text{Diagram 3} \right) - \Delta \left(\text{Diagram 4} \right) \right) \Big|_{\text{cut}}$$

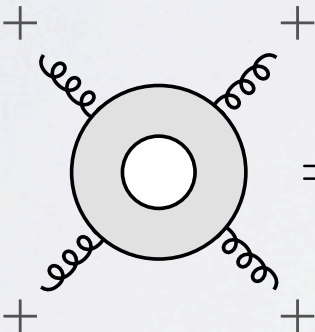
The diagram shows a cut equation for the box diagram. The left side is the cut of the non-planar box diagram. The right side is the cut of the sum of three terms: the planar box diagram squared, a non-planar diagram with a different internal line, and another non-planar diagram with a different internal line.

applications:
all-plus amplitudes in QCD

one-loop 4pt all-plus



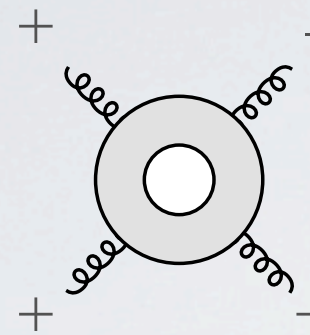
$$= \frac{D_s - 2}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \{-st\} \cdot \left\{ I \left(\text{square diagram} \right) [\mu_{11}^2] \right\}$$



$$= \frac{D_s - 2}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \frac{-st}{6} + \mathcal{O}(\epsilon)$$

[Bern, Kosower (1991)]

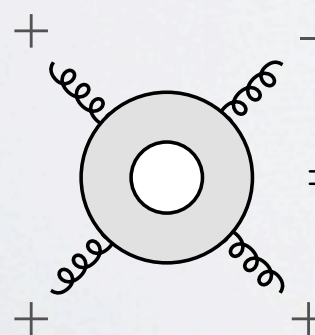
one-loop 4pt single minus



$$= \vec{c} \cdot \left\{ I \left(\text{square} \right) [\mu_{11}^2], I \left(\text{square} \right) [\mu_{11}], \right. \\ \left. I \left(\text{triangle} \right) [\mu_{11}], I \left(\text{triangle} \right) [\mu_{11}], \right. \\ \left. I \left(\text{circle} \right) [\mu_{11}], I \left(\text{circle} \right) [\mu_{11}] \right\}$$

$$\vec{c} = \frac{(D_s - 2)[24]^2}{[12]\langle 23\rangle\langle 34\rangle[41]} \left\{ \frac{st}{u}, \frac{s^2 t^2}{u^2}, \frac{t^2(s-u)}{u^2}, \frac{s^2(t-u)}{u^2}, -\frac{t(t-u)}{tu}, -\frac{s(s-u)}{su} \right\}$$

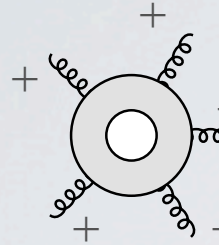
[Bern, Morgan (1995)]



$$= \frac{(D_s - 2)[24]^2}{[12]\langle 23\rangle\langle 34\rangle[41]} \frac{u}{6} + \mathcal{O}(\epsilon)$$

[Bern, Kosower (1991)]

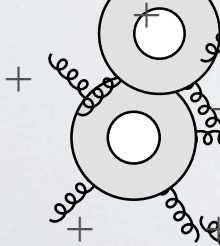
one-loop 5pt all-plus



$$= \vec{c} \cdot \left\{ I \left(\text{pentagon} \right) [\mu_{11}^2], \right. \\ \left. I \left(\text{square} \right) [\mu_{11}^2], I \left(\text{square} \right) [\mu_{11}^2], I \left(\text{square} \right) [\mu_{11}^2], I \left(\text{square} \right) [\mu_{11}^2], I \left(\text{square} \right) [\mu_{11}^2] \right\}$$

$$\vec{c} = \frac{(D_s - 2)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle \text{tr}_5(1234)} \left\{ -s_{12}s_{23}s_{34}s_{45}s_{51}, \right. \\ \left. s_{12}s_{23}\text{tr}_-(1345), s_{23}s_{34}\text{tr}_-(2451), s_{34}s_{45}\text{tr}_-(3512), s_{45}s_{51}\text{tr}_-(4123), s_{51}s_{12}\text{tr}_-(5234) \right\}$$

[Bern, Morgan (1995)]



$$= \frac{(D_s - 2)}{6} \frac{s_{12}s_{23} + s_{23}s_{34} + s_{34}s_{45} + s_{45}s_{51} + s_{51}s_{12} + \text{tr}_5(1234)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} + \mathcal{O}(\epsilon)$$

[Bern, Dixon, Kosower (1993)]

one-loop 5pt all-plus

$$\Delta^{(1)}(1^+, 2^+, 3^+, \dots, n^+) = \frac{D_s - 2}{\langle 12 \rangle^4} \mu_{11}^2 \Delta^{(1), [\mathcal{N}=4]}(1^-, 2^-, 3^+, \dots, n^+)$$

[Bern, Dixon, Dunbar, Kosower (1996)]

$$\begin{aligned}
 & \text{Diagram: a circle with five external legs, each with a '+' sign, representing the one-loop 5pt all-plus amplitude.} \\
 & = \vec{c} \cdot \left\{ I \left(\text{Diagram: a pentagon with a diagonal line} \right) [\mu_{11}^3], \right. \\
 & \quad \left. I \left(\text{Diagram: a square with two internal lines} \right) [\mu_{11}^2], I \left(\text{Diagram: a square with two internal lines} \right) [\mu_{11}^2], I \left(\text{Diagram: a square with two internal lines} \right) [\mu_{11}^2], I \left(\text{Diagram: a square with two internal lines} \right) [\mu_{11}^2], I \left(\text{Diagram: a square with two internal lines} \right) [\mu_{11}^2] \right\}
 \end{aligned}$$

$$\vec{c} = \frac{(D_s - 2)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} \{ 2\text{tr}_5(1234), s_{12}s_{23}, s_{23}s_{34}, s_{34}s_{45}, s_{45}s_{51}, s_{51}s_{12} \}$$

two-loop 4pt all-plus

(drop Parke-Taylor
pre-factors from now on...)

[Bern, Dixon, Kosower (2000)]

$$\begin{aligned}
 & \text{Diagram} = \{s^2 t, t^2 s, st\} \cdot \left\{ I \left(\text{Diagram 1} \right) [F_1], I \left(\text{Diagram 2} \right) [F_1], I \left(\text{Diagram 3} \right) \left[F_2 + F_3 \frac{s + (l_1 + l_2)^2}{s} \right] \right\}
 \end{aligned}$$

dimension shifting
numerators

$$F_1 = (D_s - 2)(\mu_{11}\mu_{22} + (\mu_{11} + \mu_{22})^2 + 2(\mu_{11} + \mu_{22})\mu_{12}) + 16(\mu_{12}^2 - \mu_{11}\mu_{22}),$$

$$F_2 = 4(D_s - 2)(\mu_{11} + \mu_{22})\mu_{12},$$

$$F_3 = (D_s - 2)^2 \mu_{11}\mu_{22}.$$

two-loop 5pt all-plus

[SB, Frellesvig, Zhang (2013)]

$$\begin{aligned}
 \text{cyclic} \left(\text{circle with two internal lines and five external lines} \right) &= \sum_{\text{cyclic}} \left\{ \Delta \left(\text{diagram 1} \right) + \Delta \left(\text{diagram 2} \right) + \Delta \left(\text{diagram 3} \right) + \Delta \left(\text{diagram 4} \right) \right. \\
 &\quad \left. + \Delta \left(\text{diagram 5} \right) + \Delta \left(\text{diagram 6} \right) + \Delta \left(\text{diagram 7} \right) + \Delta \left(\text{diagram 8} \right) \right\}
 \end{aligned}$$

$$\Delta \left(\text{diagram 1} \right) = \frac{s_{12}s_{23}s_{45}}{\text{tr}_5} \{s_{34}s_{45}s_{15}, \text{tr}_+(1345)\} \cdot \{I \left(\text{diagram 1} \right) [F_1], I \left(\text{diagram 1} \right) [F_1]\}$$

$$\Delta \left(\text{diagram 2} \right) = \left\{ -\frac{s_{34}s_{45}^2 \text{tr}_+(1235)}{\text{tr}_5} \right\} \cdot \{I \left(\text{diagram 2} \right) [F_1]\}$$

$$\Delta \left(\text{diagram 3} \right) = \left\{ \frac{s_{12}s_{23}s_{34}s_{45}s_{15}}{\text{tr}_5} \right\} \cdot \{I \left(\text{diagram 3} \right) [F_1]\}$$

$$\begin{aligned}
 \Delta \left(\text{diagram 5} \right) &= -\frac{s_{12} \text{tr}_+(1345)}{2s_{13}} \{s_{23}, 1\} \cdot \{ \\
 &\quad I \left(\text{diagram 2} \right) [F_2 + F_3 \frac{s_{45} + (l_1 + l_2)^2}{s_{45}}], I \left(\text{diagram 2} \right) [(2k_1 \cdot \omega)(F_2 + F_3 \frac{s_{45} + (l_1 + l_2)^2}{s_{45}})] \}
 \end{aligned}$$

$$\Delta \left(\text{diagram 6} \right) = \left\{ -\frac{(s_{45} - s_{12}) \text{tr}_+(1345)}{2s_{13}} \right\} \cdot \{I \left(\text{diagram 6} \right) [F_2 + F_3 \frac{s_{45} + (l_1 + l_2)^2}{s_{45}}]\}$$

$$\begin{aligned}
 \Delta \left(\text{diagram 7} \right) &= \vec{c} \cdot \{I \left(\text{diagram 7} \right) [F_2], I \left(\text{diagram 7} \right) [F_3], I \left(\text{diagram 7} \right) [F_3(l_1 + l_2)^2], \\
 &\quad I \left(\text{diagram 7} \right) [F_3(k_1 \cdot 3)(k_2 \cdot 3)], I \left(\text{diagram 7} \right) [F_3(k_1 \cdot 3)], I \left(\text{diagram 7} \right) [F_3(k_2 \cdot 3)], \dots \}
 \end{aligned}$$

two-loop 5pt all-plus

$$\Delta^{(2)}(1^+, 2^+, 3^+, \dots, n^+) = \frac{D_s - 2}{\langle 12 \rangle^4} F_1 \Delta^{(2), [\mathcal{N}=4]}(1^-, 2^-, 3^+, \dots, n^+) + (1\text{-loop})^2$$

all genuine two-loop topologies related to $\mathcal{N}=4$ MHV

non-planar cuts via BCJ

[SB, Mogull, Ochirov, O'Connell (2015)]

complete BCJ numerator

representation

[Mogull, O'Connell (2015)]

$$\begin{aligned} \mathcal{A}_5^{(2)}(1^+, 2^+, 3^+, 4^+, 5^+) = & \sum_{\sigma \in S_5} I \left[C \left(\text{diagram} \right) \left(\frac{1}{2} \Delta \left(\text{diagram} \right) + \Delta \left(\text{diagram} \right) + \frac{1}{2} \Delta \left(\text{diagram} \right) \right. \right. \\ & \left. \left. + \frac{1}{2} \Delta \left(\text{diagram} \right) + \Delta \left(\text{diagram} \right) + \frac{1}{2} \Delta \left(\text{diagram} \right) \right) \right. \\ & \left. + C \left(\text{diagram} \right) \left(\frac{1}{4} \Delta \left(\text{diagram} \right) + \frac{1}{2} \Delta \left(\text{diagram} \right) + \frac{1}{2} \Delta \left(\text{diagram} \right) \right. \right. \\ & \left. \left. - \Delta \left(\text{diagram} \right) + \frac{1}{4} \Delta \left(\text{diagram} \right) \right) \right. \\ & \left. + C \left(\text{diagram} \right) \left(\frac{1}{4} \Delta \left(\text{diagram} \right) + \frac{1}{2} \Delta \left(\text{diagram} \right) + \frac{1}{2} \Delta \left(\text{diagram} \right) \right) \right] \end{aligned}$$

two-loop 5pt all-plus amplitude

[Gehrmann, Henn, Lo Presti 1511.05409]

planar master integrals using canonical
differential equation approach

$$A_5^{(2)} = A_5^{(1)} \left[- \sum_{i=1}^5 \frac{1}{\epsilon^2} \left(\frac{\mu^2}{-v_i} \right)^\epsilon \right] + R F_5^{(2)} + \mathcal{O}(\epsilon)$$

$$F_5^{(2)} = \frac{5\pi^2}{12} F_5^{(1)} + \sum_{i=0}^4 \sigma^i \left\{ \frac{v_5 \operatorname{tr} \left[(1 - \gamma_5) \not{p}_4 \not{p}_5 \not{p}_1 \not{p}_2 \right]}{(v_2 + v_3 - v_5)} I_{23,5} \right. \\ \left. + \frac{1}{6} \frac{\operatorname{tr} \left[(1 - \gamma_5) \not{p}_4 \not{p}_5 \not{p}_1 \not{p}_2 \right]^2}{v_1 v_4} + \frac{10}{3} v_1 v_2 + \frac{2}{3} v_1 v_3 \right\}. \quad (8)$$

simple
function of Li_2

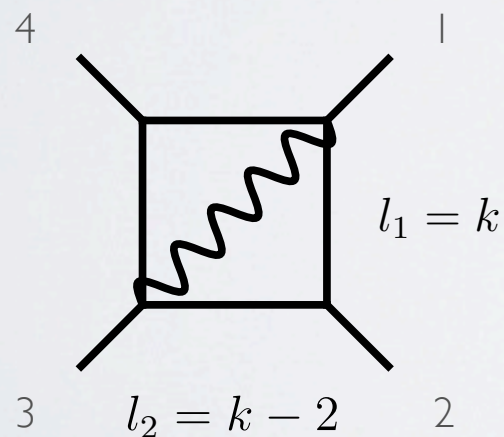
local integrands

[Arkani-Hamed, Bourjaily, Cachazo, Trnka 1012.6032]

[Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka 1008.2958]

manage infra-red
divergences at the
integrand level

simple integrals with
unit leading
singularities



$$= I_4^{4-2\epsilon} [\text{tr} (1l_1l_23)] = I_4^{6-2\epsilon} [1]$$

one-loop integrand bases

$$\begin{aligned}
 \text{Diagram 1} &= \sum c_4 I(\text{Diagram 2}) + \sum c_3 I(\text{Diagram 3}) + \sum c_2 I(\text{Diagram 4}) + \text{rational} \\
 &= \text{Diagram 5} \sum -P^2 I(\text{Diagram 3}) \\
 &\quad + \sum c_4 I(\text{Diagram 6}) + \sum c_3^{3m} I(\text{Diagram 3}) + \sum c_2 I(\text{Diagram 4}) + \text{rational}
 \end{aligned}$$

still complicated functions at high multiplicity

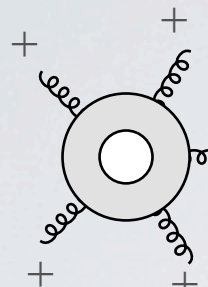
one-loop 4pt single minus

$$\begin{array}{c} + \\ \diagup \\ \text{---} \\ \diagdown \\ + \end{array} \begin{array}{c} - \\ \diagdown \\ \text{---} \\ \diagup \\ + \end{array} = \vec{c} \cdot \left\{ I \left(\begin{array}{c} \text{---} \\ \diagup \\ \text{---} \\ \diagdown \\ \text{---} \end{array} \right) [\mu_{11}], I \left(\begin{array}{c} \text{---} \\ \diagdown \\ \text{---} \\ \diagup \\ \text{---} \end{array} \right) [\mu_{11}], I \left(\begin{array}{c} \text{---} \\ \diagdown \\ \text{---} \\ \diagup \\ \text{---} \end{array} \right) [\mu_{11}], I \left(\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right) [\mu_{11}], I \left(\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right) [\mu_{11}] \right\}$$

$$\vec{c} = \frac{(D_s - 2)[24]^2}{[12]\langle 23 \rangle \langle 34 \rangle [41]} \left\{ \frac{1}{u^2}, s, t, \frac{2t}{s}, \frac{2s}{t} \right\}$$

$$I \left(\begin{array}{c} \text{---} \\ \diagup \\ \text{---} \\ \diagdown \\ \text{---} \end{array} \right) [\mu_{11}] = \mu_{11} \text{tr}_+ (1l_1 l_2 3)^2 - \text{bubbles}$$

one-loop 5pt all-plus



$$= \vec{c} \cdot \left\{ I \left(\text{pentagon} \right) [1], \right. \\ \left. I \left(\text{bubble} \right) [\mu_{11}^2], I \left(\text{bubble} \right) [\mu_{11}^2], I \left(\text{bubble} \right) [\mu_{11}^2], I \left(\text{bubble} \right) [\mu_{11}^2], I \left(\text{bubble} \right) [\mu_{11}^2], \right\}$$

$$\vec{c} = \frac{(D_s - 2)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} \left\{ \frac{\langle 13 \rangle \langle 24 \rangle \langle 35 \rangle \langle 41 \rangle \langle 52 \rangle}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}, s_{12} s_{23}, s_{23} s_{34}, s_{34} s_{45}, s_{45} s_{51}, s_{51} s_{12} \right\}$$

$$I \left(\text{pentagon} \right) [1] = \prod_{i=1}^5 \text{tr}_+ (k l_k l_{k+1} (k + 2)) - \text{bubbles}$$

two-loop 5pt all-plus

[SB, Mogull (in progress)]

$$\begin{array}{c} + \\ + \\ + \\ + \\ + \end{array} \text{ (cyclic) } = \sum_{\text{cyclic}} \Delta \left(\text{Diagram 1} \right) + \Delta \left(\text{Diagram 2} \right) + \Delta \left(\text{Diagram 3} \right) + \Delta \left(\text{Diagram 4} \right)$$

$$\Delta \left(\text{Diagram 1} \right) = \{s_{45}\} \cdot \{I \left(\text{Diagram 1} \right) [F_1]\}$$

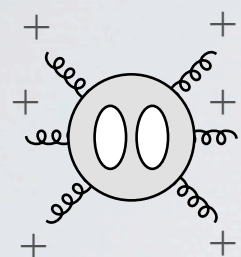
$$\Delta \left(\text{Diagram 2} \right) = \{s_{12}s_{45}s_{15}\} \cdot \{I \left(\text{Diagram 2} \right) [F_1]\}$$

$$\Delta \left(\text{Diagram 3} \right) = \{1\} \cdot \{I \left(\text{Diagram 3} \right) [F_2 + F_3 \frac{s_{45} + (l_1 + l_2)^2}{s_{45}}]\}$$

$$\begin{aligned}
 \Delta \left(\text{Diagram 4} \right) = & \{ \text{tr}_+(1245), s_{15}, -\text{tr}_+(1345), -\text{tr}_+(1235) \} \cdot \{ \\
 & \left(\text{Diagram 4a} \right) [F_2 + F_3 \frac{4(l_1 \cdot 3)(l_1 \cdot 3) + s_{12}s_{45} + (s_{12} + s_{45})(l_1 + l_2)^2}{s_{12}s_{45}}], \left(\text{Diagram 4b} \right) [F_3(l_1 + l_2)^2], \\
 & \left(\text{Diagram 4c} \right) [F_3 2(l_2 \cdot 3)(s_{12} + \frac{\text{tr}_+(1l_2l_33)}{s_{13}})], \left(\text{Diagram 4d} \right) [F_3 2(l_1 \cdot 3)(s_{12} + \frac{\text{tr}_+(5l_5l_43)}{s_{53}})] \}
 \end{aligned}$$

towards two-loop 6pt all-plus

[SB, Mogull (in progress)]



$$= \sum_{\text{cyclic}} \Delta \left(\text{diagram 1} \right) + \Delta \left(\text{diagram 2} \right) + \Delta \left(\text{diagram 3} \right) + \Delta \left(\text{diagram 4} \right) + \Delta \left(\text{diagram 5} \right) + \Delta \left(\text{diagram 6} \right)$$

$$+ \Delta \left(\text{diagram 7} \right) + \Delta \left(\text{diagram 8} \right) + \Delta \left(\text{diagram 9} \right) + \Delta \left(\text{diagram 10} \right) + \Delta \left(\text{diagram 11} \right) + \Delta \left(\text{diagram 12} \right)$$

in progress

follows the expected $\mathcal{N}=4 \times F_1$ structure

[Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich (2008)]

[Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka (2010)]

outlook

- Automated loop amplitude methods
 - purely algebraic algorithms with rational momenta (momentum twistors)
 - automated D-dimensional cuts with 6D spinor-helicity
- First complete five-point amplitude in Yang-Mills
- Aim: apply to high multiplicity amplitudes at one and two loops
 - compact analytic expressions using local integrands: all multiplicity all-plus
 - general helicity configurations for $2 \rightarrow 3$: LHC physics at NNLO

Backup

momentum twistors

[Hodges (2009)]

an all multiplicity parameterisation (not unique)

$$Z_{iA} = \left(\frac{\Sigma_i}{s_{12}}, 1 - \delta_{1i}, \frac{\langle 123i \rangle \langle 34 \rangle [23]}{\langle 1234 \rangle \langle 1i \rangle [12]}, \frac{-\langle 13 \rangle \langle 124i \rangle + \langle 14 \rangle \langle 123i \rangle}{\langle 1234 \rangle \langle 1i \rangle} \right)$$

$$\Sigma_i = \begin{cases} s_{12} & i = 1 \\ \frac{\langle 13 \rangle \langle 2i \rangle}{\langle 23 \rangle \langle 1i \rangle} & i \neq 1 \end{cases}$$

build spinors products
etc. from rational
phase-space points

In[21]:=

```

NN = 6;
GetMTrepALT[NN];
mtZ // MatrixForm // SymbolForm
    
```

Out[23]//DisplayForm=

$$\begin{pmatrix} 1 & 0 & \frac{1}{s_{12}} & \frac{\sigma_4}{s_{12}} & \frac{\sigma_5}{s_{12}} & \frac{\sigma_6}{s_{12}} \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & x_{1234} & x_{1235} & 1 \\ 0 & 0 & 1 & 1 & \frac{x_{1235} - x_{1245}}{x_{1234}} & \frac{1 - x_{1246}}{x_{1234}} \end{pmatrix}$$

momentum twistors

example: BCFW using rational kinematics

```
In[143]:= NN = 7;
GetMTrepALT[NN];
mtZ // MatrixForm // SymbolForm
mtZ = mtZ /. lpS[1,2]→1; mtW = mtW /. lpS[1,2]→1;
{p1,p2,p3,p4,p5,p6,p7} = Map[ToSPN4[MT[#]]&,Range@NN];
Map[Simplify,BCFW4[AMP[{p1,1,gluon},{p2,1,gluon},{p3,1,gluon},{p4,2,gluon},{p5,2,gluon},{p6,2,gluon},{p7,2,gluon}],1,2]] // SymbolForm
```

Out[145]//DisplayForm=

$$\begin{pmatrix} 1 & 0 & \frac{1}{s_{12}} & \frac{\sigma_4}{s_{12}} & \frac{\sigma_5}{s_{12}} & \frac{\sigma_6}{s_{12}} & \frac{\sigma_7}{s_{12}} \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & x_{1234} & x_{1235} & x_{1236} & 1 \\ 0 & 0 & 1 & 1 & \frac{x_{1235}-x_{1245}}{x_{1234}} & \frac{x_{1236}-x_{1246}}{x_{1234}} & \frac{1-x_{1247}}{x_{1234}} \end{pmatrix}$$

Out[148]//DisplayForm=

$$\frac{i (x_{1234} - x_{1235} + x_{1245})^3}{(-1 + \sigma_4) (\sigma_4 - \sigma_5) (\sigma_5 - \sigma_6) (\sigma_6 - \sigma_7) x_{1234} x_{1235} x_{1245} (\sigma_5 x_{1234} - \sigma_4 x_{1235} + x_{1245})} - \frac{i (x_{1234} (x_{1235} - x_{1236}) - x_{1236} x_{1245} + x_{1235} x_{1246})^3}{(-1 + \sigma_4) (\sigma_4 - \sigma_5) (\sigma_5 - \sigma_6) (\sigma_6 - \sigma_7) x_{1234} x_{1235} x_{1236} (-x_{1236} x_{1245} + x_{1235} x_{1246}) (-x_{1236} (\sigma_5 x_{1234} + x_{1245}) + x_{1235} (\sigma_6 x_{1234} + x_{1246}))} + \frac{i (x_{1234} (-1 + x_{1236}) - x_{1246} + x_{1236} x_{1247})^3}{(-1 + \sigma_4) (\sigma_4 - \sigma_5) (\sigma_5 - \sigma_6) (\sigma_6 - \sigma_7) x_{1234} x_{1236} (-x_{1246} + x_{1236} x_{1247}) (\sigma_6 x_{1234} + x_{1246} - x_{1236} (\sigma_7 x_{1234} + x_{1247}))}$$

[c.f. bcfw mathematica package Bourjaily (2010)]