

HIGH ENERGY RESUMMATION AND THE HIGGS p_T SPECTRUM

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LOOPFEST 2016

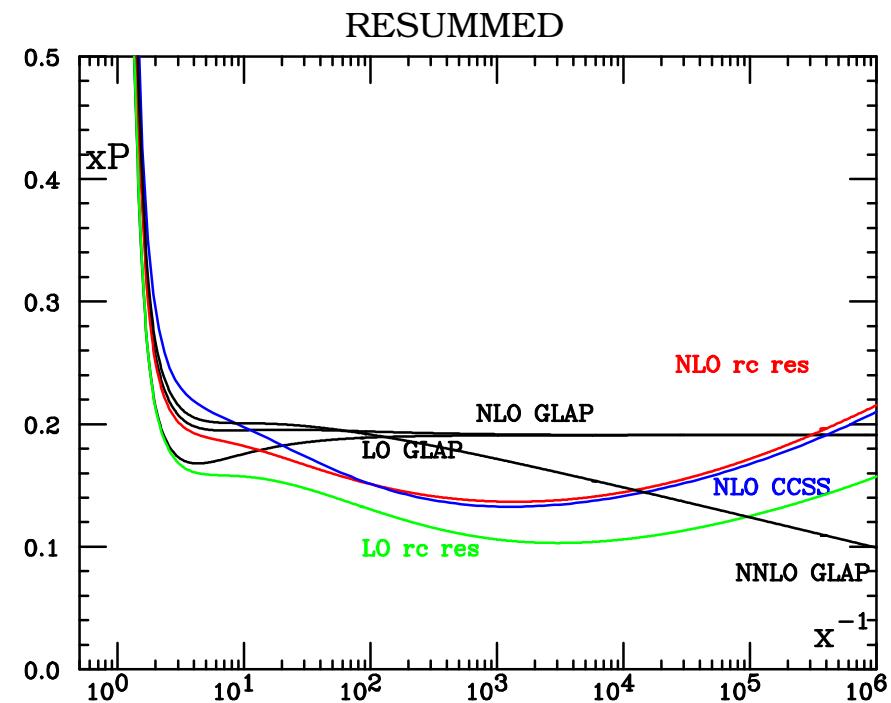
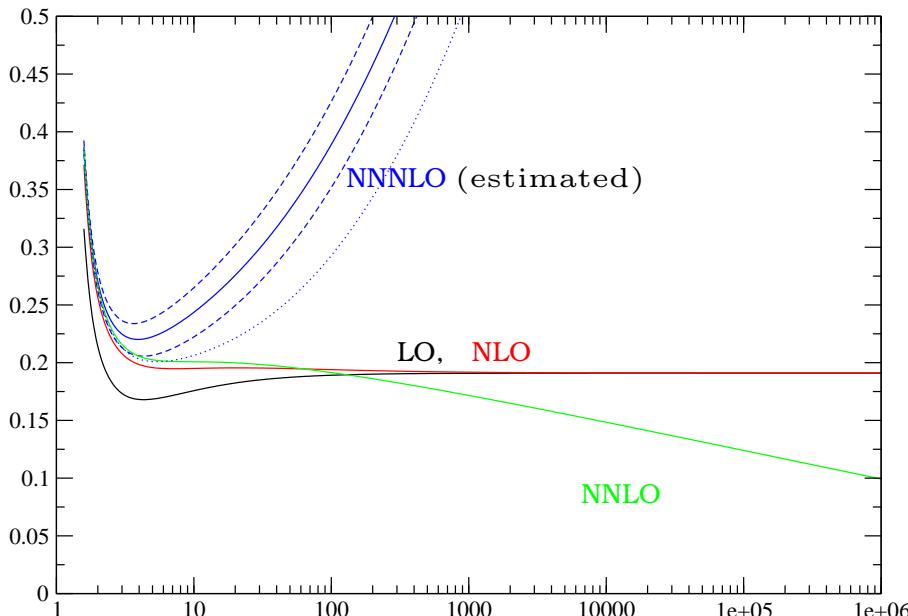
BUFFALO, AUGUST 15, 2016

WHY SMALL x RESUMMATION?

OLD ANSWER: PERTURBATIVE INSTABILITY OF THE SINGLET SPLITTING FUNCTION

$$xP(\alpha_s, x) \underset{x \rightarrow 0}{\sim} \alpha_s c_1^{(1)} + \alpha_s^2 c_2^{(1)} + \alpha_s^3 \left(c_3^{(2)} \ln x + c_3^{(1)} \right) + \alpha_s^4 \left(c_4^{(4)} \ln^3 x + c_4^{(3)} \ln^2 x + c_4^{(2)} \ln x + c_4^{(1)} \right) + \dots$$

FIXED ORDER



- DRAMATIC RISE AT SMALL x OF THE N^3LO SPLITTING FUNCTION
- TAMED BY RESUMMATION (& ONLY HAPPENS AT VERY SMALL x)

(Altarelli, Ball, sf & Ciafaloni, Colferai, Salam, Stasto, 1999-2008)

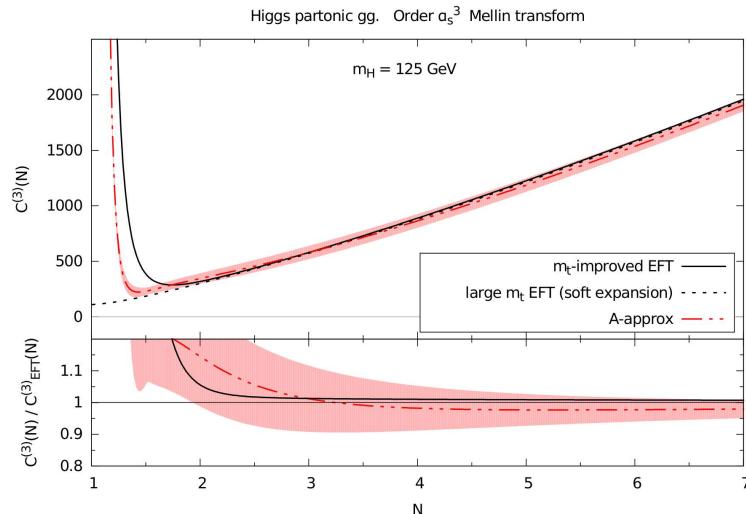
WHY SMALL x RESUMMATION?

NEWER ANSWER: INFORMATION ON HIGHER-ORDER CROSS-SECTIONS

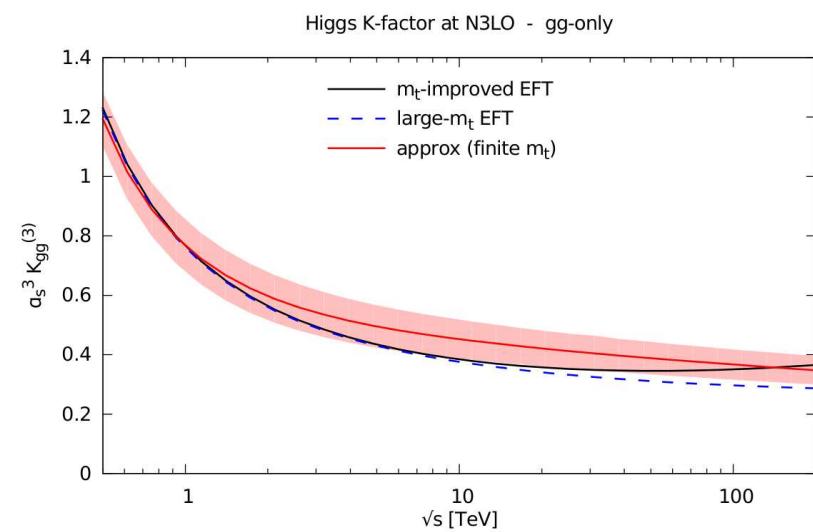
- SMALL x RESUMMATION PROVIDES INFORMATION ON LEADING RIGHTMOST LEADING SMALL- N SINGULARITY TO ALL ORDERS
- COMBINED WITH LARGE N SINGULARITY CAN BE USED TO CONSTRUCT APPROXIMATE HIGHER ORDER RESULTS
- FOR HIGGS IN GLUON FUSION, TOP MASS FULLY INCLUDED \Rightarrow CAN IMPROVE EFT RESULTS

(Ball, Bonvini, sf, Marzani, Ridolfi, 2013)

$N^3\text{LO}$ COEFFICIENT FUNCTION (MELLIN SPACE)



$N^3\text{LO}$ K-FACTOR



(Bonvini, Marzani, Muselli, Rottoli 2016)

SMALL x RESUMMATION: WHERE DO WE STAND?

- **ANOMALOUS DIMENSIONS:** SMALL x TERMS IN DGLAP RESUMMED TO ALL ORDERS AT THE LEADING AND SUBLEADING LEVEL (BFKL 1975-1976, Fadin-Lipatov 1998)
- **DGLAP RESUMMATION:** TWO ALTERNATIVE APPROACHES
 - SMALL x RESUMMATION OF DGLAP (Altarelli, Ball, sf)
 - INCLUSION OF FIXED-ORDER DGLAP INTO BFKL(Ciafaloni, Colferai, Salam)
- **ELECTROPRODUCTION:** GENERAL RESUMMATION THEORY TO LL x (Catani, Ciafaloni, Hautmann, 1991) **SMALL x CORRECTIONS TO DEEP-INELASTIC COEFFICIENT FUNCTION KNOWN AT THE LEADING NONTRIVIAL LEVEL FOR INCLUSIVE DIS** (Catani, Hautmann, 1994) AND HQ PRODUCTION (Catani, Ciafaloni, Hautmann, 1991)
- **INCLUSIVE HADRONIC CROSS-SECTIONS:** GENERAL RESUMMATION THEORY TO LL x (Catani, Ciafaloni, Hautmann, 1991); **SMALL x CORRECTIONS TO INCLUSIVE HARD CROSS-SECTIONS KNOWN AT THE LEADING NONTRIVIAL LEVEL FOR HQ PRODUCTION** (Ball, K.Ellis, 2001); GG \rightarrow HIGGS EFT (Hautmann, 2002) & FULL HQ MASS DEPENDENCE (Marzani, Ball, Del Duca, sf, Vicini, 2008); DRELL-YAN (Marzani, Ball, 2009); ISOLATED PHOTON (Diana, 10)
- **RAPIDITY DISTRIBUTIONS:** GENERAL THEORY, & **HIGGS IN GLUON FUSION (FULL HQ MASS DEPENDENCE)** RESUMMED AT LL x (Caola, sf, Marzani, 2011);
- **TRANSVERSE MOMENTUM SPECTRUM DISTRIBUTIONS:** GENERAL THEORY,& **HIGGS IN GLUON FUSION (FULL HQ MASS DEPENDENCE)** RESUMMED AT LL x (sf, Muselli, 2015-2016);

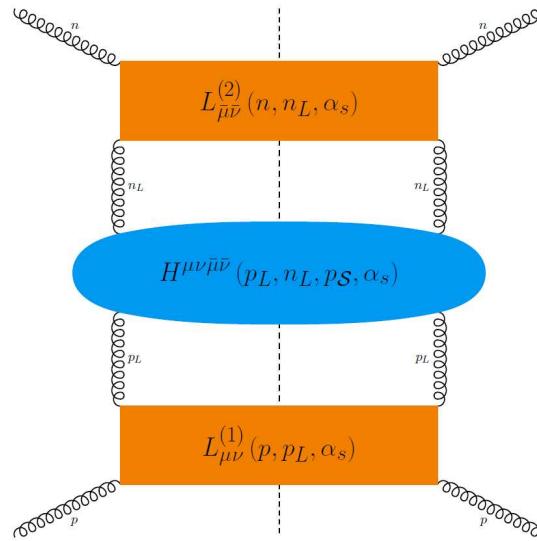
SUMMARY

- HIGH-ENERGY FACTORIZATION AND RESUMMATION OF p_t DISTRIBUTIONS
 - factorization and ladder expansion
 - off-shell cross-section & transverse momentum distribution
- THE HIGGS p_t DISTRIBUTION
 - qualitative behaviour: resolved vs. pointlike case
 - explicit results at parton level
- IMPLICATIONS OF THE RESUMMED RESULTS
 - the high-energy approximation up to NLO
 - bottom collinear logs

RESUMMATION

HIGH-ENERGY FACTORIZATION

(Catani, Ciafaloni, Hautmann, 91)

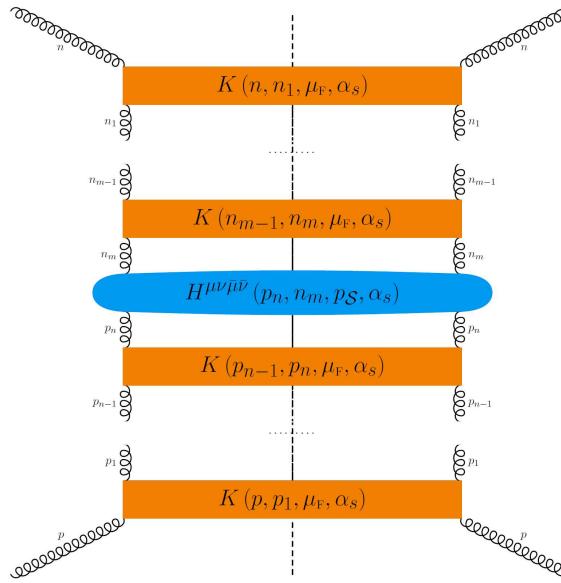


$$\sigma \left(\frac{Q^2}{s}, \frac{\mu_F^2}{Q^2}, \frac{\mu_R^2}{Q^2} \right) = \int \frac{Q^2}{2s} H^{\mu\nu\bar{\mu}\bar{\nu}} \left(n_L, p_L, \Omega_S, \mu_R^2, \mu_F^2, \alpha_s \right) L_{\mu\nu} \left(p_L, p, \mu_R^2, \mu_F^2, \alpha_s \right) L_{\bar{\mu}\bar{\nu}} \left(n_L, n, \mu_R^2, \mu_F^2, \alpha_s \right) [dp_L] [dn_L]$$

- SUDAKOV PARM. FOR p_L, n_l ; HIGH ENERGY LIMIT: $z \ll 1, \frac{k_t^2}{s} \ll 1$
- 2GI DIAGRAM CONTRIBUTE IN HIGH ENERGY LIMIT, UP TO POWER SUPPRESSION
- LORENTZ DECOMPOSITION OF LADDERS & HARD: ONLY LONGITUDINAL
CONTRIBUTES IN HIGH ENERGY LIMIT

THE LADDER EXPANSION

(Curci, Furmański, Petronzio, 1980; Caola, sf, Marzani, 2011)



$$\sigma^{n,m} \left(N, \frac{\mu_F^2}{Q^2}, \alpha_s; \epsilon \right) = \int_0^\infty \left[\gamma \left(N, \left(\frac{\mu_F^2}{Q^2 \xi_n} \right)^\epsilon, \alpha_s; \epsilon \right) \right] \frac{d\xi_n}{\xi_n^{1+\epsilon}} \times \int_0^\infty \left[\gamma \left(N, \left(\frac{\mu_F^2}{Q^2 \bar{\xi}_m} \right)^\epsilon, \alpha_s; \epsilon \right) \right] \frac{d\bar{\xi}_m}{\bar{\xi}_m^{1+\epsilon}} C(N, \xi, \bar{\xi}, \alpha_s; \epsilon) \\ \times \int_0^{\xi_n} \left[\gamma \left(N, \left(\frac{\mu_F^2}{Q^2 \xi_{n-1}} \right)^\epsilon, \alpha_s; \epsilon \right) \right] \frac{d\xi_{n-1}}{\xi_{n-1}^{1+\epsilon}} \times \cdots \times \int_0^{\xi_2} \left[\gamma \left(N, \left(\frac{\mu_F^2}{Q^2 \xi_1} \right)^\epsilon, \alpha_s; \epsilon \right) \right] \frac{d\xi_1}{\xi_1^{1+\epsilon}} \times \int_0^{\bar{\xi}_m} \left[\gamma \left(N, \left(\frac{\mu_F^2}{Q^2 \bar{\xi}_{m-1}} \right)^\epsilon, \alpha_s; \epsilon \right) \right] \frac{d\bar{\xi}_{m-1}}{\bar{\xi}_{m-1}^{1+\epsilon}} C(N, \bar{\xi}, \xi, \alpha_s; \epsilon)$$

- LADDER OBTAINED BY ITERATION OF A 2GI KERNEL
- THE INTEGRATED KERNEL IS A LLx ANOMALOUS DIMENSION, CAN BE OBTAINED FROM BFKL USING DUALITY: $K \left(N, \left(\frac{\mu_F^2}{Q^2 \bar{\xi}} \right)^\epsilon, \alpha_s; \epsilon \right) = \gamma \left(N, \left(\frac{\mu_F^2}{Q^2 \xi} \right)^\epsilon, \alpha_s; \epsilon \right)$ (Altarelli, Ball, sf , 2000)

THE OFF-SHELL CROSS-SECTION

$$\begin{array}{ccc}
 \text{Diagram 1:} & & \text{Diagram 2:} \\
 \begin{array}{c} n \\ | \\ \text{---} \\ | \\ \text{Diagram: } H(n, p_L, p_F, \alpha_s) \text{ (orange oval)} \\ | \\ \text{---} \\ | \\ p_L \end{array} & \equiv & \int d\Pi_{\mathcal{F}} \left[\frac{n}{2GI} \left(\frac{\partial}{\partial p_L} \right)^2 \delta_4(p + n - p_S - p_X) \right]^{\mathcal{F}}
 \end{array}$$

$$\sigma_{\text{res}}(N, \alpha_s) = \gamma \left(\frac{\alpha_s}{N} \right)^2 R \left(\frac{\alpha_s}{N} \right)^2 \int_0^\infty d\xi \xi^{\gamma(\frac{\alpha_s}{N})-1} \int_0^\infty d\bar{\xi} \bar{\xi}^{\gamma(\frac{\alpha_s}{N})-1} C(N, \xi, \bar{\xi}, \alpha_s)$$

- THE **ITERATED KERNEL** (ANOMALOUS DIMENSION) EXPONENTIATES
- THE **CONVOLUTIONS** LOOK LIKE k_t -SPACE **MELLIN-TRANSFORMS** ($\xi = \frac{k_t^2}{Q^2}$; k_t^2 GLUON OFF-SHELLNESS)
- .

THE OFF-SHELL CROSS-SECTION

The diagram shows the equivalence between a Feynman diagram and its off-shell cross-section representation. On the left, a Feynman diagram consists of a central orange oval labeled $H(n, p_L, p_{\mathcal{F}}, \alpha_s)$ with four external gluon lines. A vertical dashed line passes through the center of the oval. The top line is labeled n and the bottom line is labeled p_L . An equals sign follows this diagram. To the right is the off-shell cross-section expression:

$$\int d\Pi_{\mathcal{F}} \left[\frac{n}{p_L} \left(2GI \right) \mathcal{S} \right]_{\mathcal{F}}^2 \delta_4(p + n - p_S - p_X)$$

$$\sigma_{\text{res}}(N, \alpha_s) = h\left(N, \gamma\left(\frac{\alpha_s}{N}\right), \gamma\left(\frac{\alpha_s}{N}\right), \alpha_s\right)$$

$$h(N, M_1, M_2, \alpha_s) = M_1 M_2 R(M_1) R(M_2) \int_0^\infty d\xi \xi^{M_1-1} \int_0^\infty d\bar{\xi} \bar{\xi}^{M_2-1} C(N, \xi, \bar{\xi}, \alpha_s)$$

- THE **ITERATED KERNEL** (ANOMALOUS DIMENSION) EXPONENTIATES
- THE **CONVOLUTIONS** LOOK LIKE k_t -SPACE **MELLIN-TRANSFORMS** ($\xi = \frac{k_t^2}{Q^2}$; k_t^2 GLUON OFF-SHELLNESS)
- **RESUMMATION** \Leftrightarrow **OFF-SHELL CROSS-SECTION** WITH $M = \gamma$ (DUALITY) (Catani, Ciafaloni, Hautmann , 1991)

THE TRANSVERSE MOMENTUM DISTRIBUTION

(sf, Muselli, 2015)

$$p_1 = z_1 p - \mathbf{k}_1$$

$$q_1 = (1 - z_1) p + \mathbf{k}_1$$

$$p_2 = z_2 z_1 p - \mathbf{k}_2$$

$$q_2 = (1 - z_1 z_2) z_1 p + \mathbf{k}_2 - \mathbf{k}_1$$

.....

$$p_L = z p - \mathbf{k}$$

$$q_L = (1 - z) p + \mathbf{k} - \mathbf{k}_{\mathbf{n}-1}$$

$$n_1 = \bar{z}_1 p - \bar{\mathbf{k}}_1$$

$$r_1 = (1 - \bar{z}_1) p + \bar{\mathbf{k}}_1$$

$$n_2 = \bar{z}_2 \bar{z}_1 p - \bar{\mathbf{k}}_2$$

$$r_2 = (1 - \bar{z}_1 \bar{z}_2) \bar{z}_1 p + \bar{\mathbf{k}}_2 - \bar{\mathbf{k}}_1$$

.....

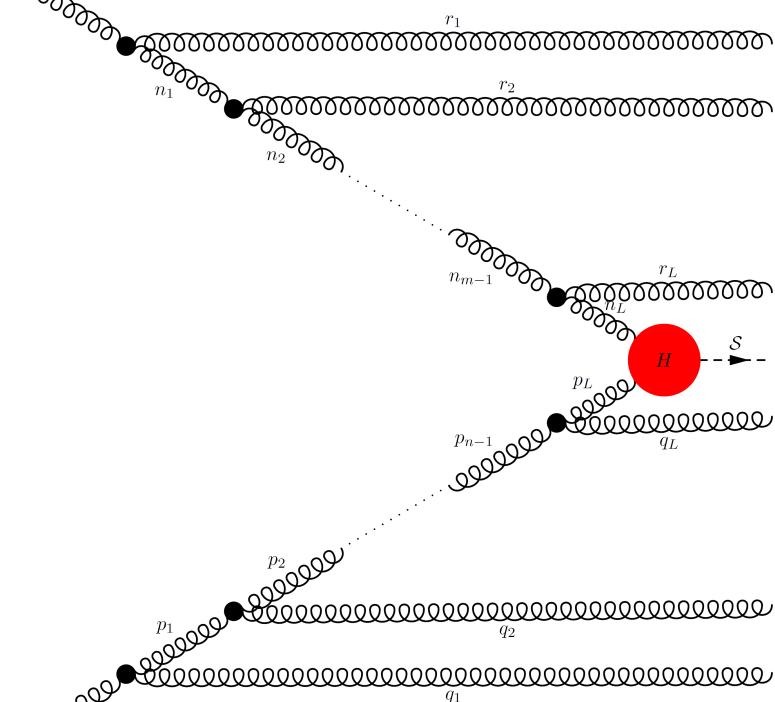
$$n_L = \bar{z} n - \bar{\mathbf{k}}$$

$$r_L = (1 - \bar{z}) n + \bar{\mathbf{k}} - \bar{\mathbf{k}}_{\mathbf{m}-1}.$$

(1)

$$n = \left(\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2} \right)$$

$$p = \left(\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2} \right)$$



- LADDER EXPANSION HOLDS ALSO FOR TRANSVERSE MOM. DISTN
- TRANSVERSE MOMENTUM INTEGRATIONS ARE INDEPENDENT OF EACH OTHER
- TRANSVERSE MOMENTUM DEPENDENCE LINKS INCOMING AND OUTGOING MOMENTA IN THE HARD PART

HIGGS: RESULTS

RESUMMED RESULTS: QUALITATIVE BEHAVIOUR

TOTAL CROSS-SECTION	TRANSVERSE MOMENTUM DISTRIBUTION
$\sigma \underset{x \rightarrow 0}{\sim} \sigma_{LO} \times \left\{ \begin{array}{l} \delta(1-x) + \sum_{k=1}^{\infty} c_k \alpha_s^k \ln^{2k-1} \frac{1}{x}, \text{ pointl.} \\ \delta(1-x) + \sum_{k=1}^{\infty} d_k \alpha_s^k \ln^{k-1} \frac{1}{x}, \text{ resolved} \end{array} \right. ; \quad \frac{d\sigma}{d\xi_p} \underset{x \rightarrow 0}{\sim} \frac{\sigma_{LO}}{\xi_p} \times \left\{ \begin{array}{l} \sum_{k=1}^{\infty} \alpha_s^k \ln^{k-1} \frac{1}{x} \sum_{n=0}^{k-1} c_{kn} \ln^n \xi_p, \\ \sum_{k=1}^{\infty} d_k (\xi_p) \alpha_s^k \ln^{k-1} \frac{1}{x} \end{array} \right. $	$\xi_p = \frac{p_T^2}{Q^2}$

- INCLUSIVE XSECT: DOUBLE ENERGY LOGS (POINTLIKE) VS SINGLE LOGS (RESOLVED)
- TRANSVERSE MOMENTUM DISTRIBUTION:
 - SINGLE ENERGY LOGS (ALWAYS)
 - POINTLIKE:
 - HARD PART IS p_t INDEP. \Rightarrow SERIES IN $\ln p_t$ WITH CONSTANT COEFFICIENTS
 - RESOLVED:
 - LARGE p_t : COEFFICIENTS VANISHING AS $\frac{1}{p_t}$; SMALL p_t : POINTLIKE LIMIT RECOVERED

RESUMMED RESULTS: POINTLIKE LIMIT

IMPACT FACTOR

$$h_{pT} = R(M_1) R(M_2) \sigma_{LO} \frac{\xi_p^{M_1 + M_2 - 1}}{(1 + \xi_p)^N} \left[\frac{\Gamma(1 + M_1)\Gamma(1 + M_2)\Gamma(2 - M_1 - M_2)}{\Gamma(2 - M_1)\Gamma(2 - M_2)\Gamma(M_1 + M_2)} \left(1 + \frac{2M_1 M_2}{1 - M_1 - M_2} \right) \right]$$

SUBSTITUTING $M_1 = \gamma_s \left(\frac{\alpha_s}{N} \right)$ $M_2 = \gamma_s \left(\frac{\alpha_s}{N} \right)$ & EXPANDING

p_t DISTRIBUTION

$$\frac{d\sigma}{d\xi_p} (N, \alpha_s) = \sigma_{LO} \sum_{k=1}^{\infty} C_k (\xi_p) \alpha_s^k \frac{\ln^{k-1} x}{(k-1)!}$$

$$C_1 (\xi_p) = \frac{2C_A}{\pi} \frac{1}{\xi_p}$$

$$C_2 (\xi_p) = \frac{4C_A^2}{\pi^2} \frac{\ln \xi_p}{\xi_p}$$

$$C_3 (\xi_p) = \frac{2C_A^3}{\pi^3} \frac{1 + 2 \ln^2 \xi_p}{\xi_p}$$

$$C_4 (\xi_p) = \frac{4C_A^4}{\pi^4} \frac{3 + 3 \ln \xi_p + 2 \ln^3 \xi_p + 17\zeta_3}{3\xi_p}$$

- AGREES WITH EXPECTED BEHAVIOUR
- LO AND NLO RESULTS CHECKED AGAINST EXACT EXPRESSIONS
- NNLO RESULT CHECKED AGAINST NNLL TRANSVERSE MOMENTUM RESUMMATION

RESUMMED RESULTS: RESOLVED CASE

THE IMPACT FACTOR

(Caola, sf, Marzani, Muselli, Vita, 2016)

$$h_{pT} = \sigma_0(y_i) R(M_1) R(M_2) \frac{\xi_p^{M_1 + M_2 - 1}}{(1 + \xi_p)^N} \left[c_0(\xi_p, y_i) (M_1 + M_2) + \sum_{j \geq k > 0} c_{j,k}(\xi_p, y_i) (M_1^k M_2^j + M_1^j M_2^k) \right]$$

$$\begin{aligned} c_0(\xi_p, \{y_i\}) &= \frac{2304\pi^4}{|\sum_i K(y_i)|^2} \left| \sum_i y_i A(0, \xi_p, \xi_p, y_i) \right|^2 \\ c_{j,k}(\xi_p, \{y_i\}) &= \frac{1}{(j-1)! (k-1)!} \frac{1}{1 + \delta_{jk}} \times \\ &\int_{-1}^1 \frac{2du}{\pi\sqrt{1-u^2}} \int_0^1 dz \frac{\ln^{j-1} a \ln^{k-1} b F(a, b, \xi_p, \{y_i\}) - \delta_{j,1} \ln^{k-1} z F(1, 0, \xi_p, \{y_i\})}{z} + (j \leftrightarrow k) \end{aligned}$$

$$F(\xi_1, \xi_2, \xi_p, \{y_i\}) = \frac{2304\pi^4}{|\sum_i K(y_i)|} |\sum_i y_i A(\xi_1 \xi_p, \xi_2 \xi_p, \xi_p, y_i)|^2$$

- COEFFICIENTS DEPEND ON $\xi_p = \frac{p_T^2}{Q^2}$, $y_t = \frac{m_t^2}{m_h^2}$, $y_b = \frac{m_b^2}{m_h^2}$
- $c_0 \Rightarrow$ CLOSED FORM; $c_{ij} \Rightarrow$ NUMERICAL INTEGRAL OVER HQ-MASS DEPENDENT FORM FACTOR A (Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld, 2001; Ellis, Zanderighi, 2008); MUST EXTRACT COLLINEAR SINGULARITY

THE FORM FACTORS

$$\begin{aligned}
A(\xi, \bar{\xi}, \xi_p, y) = & \frac{C_0(\xi, \bar{\xi}, y)}{\sqrt{\xi \bar{\xi}}} \left[\left(\frac{2y}{\Delta_3} + \frac{6\xi \bar{\xi}}{\Delta_3^2} \right) ((\xi_p - \xi - \bar{\xi})(1 + \xi + \bar{\xi}) + 4\xi \bar{\xi}) - \frac{\xi_p - \xi - \bar{\xi}}{2} + 2 \frac{\xi \bar{\xi}(1 - \xi_p)}{\Delta_3} \right] \\
& - \frac{1}{\sqrt{\xi \bar{\xi}}} [B_0(-\bar{\xi}, y) - B_0(1, y)] \left[-\frac{\bar{\xi}}{\Delta_3} (\xi_p - \bar{\xi} + \xi) + \frac{6\xi \bar{\xi}}{\Delta_3^2} (1 + \xi_p)(1 + \xi - \bar{\xi}) \right] \\
& - \frac{1}{\sqrt{\xi \bar{\xi}}} [B_0(-\xi, y) - B_0(1, y)] \left[-\frac{\xi}{\Delta_3} (\xi_p - \xi + \bar{\xi}) + \frac{6\xi \bar{\xi}}{\Delta_3^2} (1 + \xi_p)(1 + \bar{\xi} - \xi) \right] \\
& + \frac{1}{4\pi^2} \frac{1}{\Delta_3} \frac{1}{\sqrt{\xi \bar{\xi}}} ((\xi_p - \xi - \bar{\xi})(1 + \xi + \bar{\xi}) + \xi \bar{\xi})
\end{aligned}$$

$$\Delta_3 = (1 + \xi + \bar{\xi})^2 - 4\xi \bar{\xi}; \quad B_0(\rho, y) = -\frac{1}{16\pi^2} \sqrt{\frac{\rho - 4y}{\rho}} \ln \frac{\sqrt{\frac{\rho - 4y}{\rho}} + 1}{\sqrt{\frac{\rho - 4y}{\rho}} - 1}$$

$$\begin{aligned}
C_0(\xi, \bar{\xi}, y) = & \frac{1}{16\pi^2} \frac{1}{\sqrt{\Delta_3}} \left[\ln(1 - y_-) \ln \left(\frac{1 - y_- \delta_1^+}{1 - y_- \delta_1^-} \right) + \ln(1 - x_-) \ln \left(\frac{1 - x_- \delta_2^+}{1 - x_- \delta_2^-} \right) \right. \\
& + \ln(1 - z_-) \ln \left(\frac{1 - z_- \delta_3^+}{1 - z_- \delta_3^-} \right) + \text{Li}_2(y_+ \delta_1^+) + \text{Li}_2(y_- \delta_1^+) - \text{Li}_2(y_+ \delta_1^-) - \text{Li}_2(y_- \delta_1^-) + \text{Li}_2(x_+ \delta_2^+) \\
& \left. + \text{Li}_2(x_- \delta_2^+) - \text{Li}_2(x_+ \delta_2^-) - \text{Li}_2(x_- \delta_2^-) + \text{Li}_2(z_+ \delta_3^+) + \text{Li}_2(z_- \delta_3^+) - \text{Li}_2(z_+ \delta_3^-) - \text{Li}_2(z_- \delta_3^-) \right]
\end{aligned}$$

$$\begin{aligned}
A(0, \xi_p, \xi_p, y) = & \frac{1}{32\pi^2} \left(\frac{4y - 1 - \xi_p}{(1 + \xi_p)^2} \left[\ln 2 \frac{\sqrt{1 - 4y} - 1}{\sqrt{1 - 4y} + 1} - \ln 2 \frac{\sqrt{1 + \frac{4y}{\xi_p}} - 1}{\sqrt{1 + \frac{4y}{\xi_p}} + 1} \right] + \right. \\
& \left. \frac{4\xi_p}{(1 + \xi_p)^2} \left[\sqrt{1 - 4y} \ln \frac{\sqrt{1 - 4y} + 1}{\sqrt{1 - 4y} - 1} - \sqrt{1 + \frac{4y}{\xi_p}} \ln \frac{\sqrt{1 + \frac{4y}{\xi_p}} + 1}{\sqrt{1 + \frac{4y}{\xi_p}} - 1} \right] + \frac{4}{1 + \xi_p} \right)
\end{aligned}$$

RESUMMED RESULTS: RESOLVED CASE

THE IMPACT FACTOR

(Caola, sf, Marzani, Muselli, Vita, 2016)

$$h_{pT} = \sigma_0(y_i) R(M_1) R(M_2) \frac{\xi_p^{M_1+M_2-1}}{(1+\xi_p)^N} \left[c_0(\xi_p, y_i) (M_1 + M_2) + \sum_{j \geq k > 0} c_{j,k}(\xi_p, y_i) (M_1^k M_2^j + M_1^j M_2^k) \right]$$

$$\begin{aligned} c_0(\xi_p, \{y_i\}) &= \frac{2304\pi^4}{|\sum_i K(y_i)|^2} \left| \sum_i y_i A(0, \xi_p, \xi_p, y_i) \right|^2 \\ c_{j,k}(\xi_p, \{y_i\}) &= \frac{1}{(j-1)! (k-1)!} \frac{1}{1 + \delta_{jk}} \times \\ &\quad \int_{-1}^1 \frac{2du}{\pi\sqrt{1-u^2}} \int_0^1 dz \frac{\ln^{j-1} a \ln^{k-1} b F(a, b, \xi_p, \{y_i\}) - \delta_{j,1} \ln^{k-1} z F(1, 0, \xi_p, \{y_i\})}{z} + (j \leftrightarrow k) \end{aligned}$$

$$F(\xi_1, \xi_2, \xi_p, \{y_i\}) = \frac{2304\pi^4}{|\sum_i K(y_i)|} |\sum_i y_i A(\xi_1 \xi_p, \xi_2 \xi_p, \xi_p, y_i)|^2$$

- COEFFICIENTS DEPEND ON $\xi_p = \frac{p_T^2}{Q^2}$, $y_t = \frac{m_t^2}{m_h^2}$, $y_b = \frac{m_b^2}{m_h^2}$
- $c_0 \Rightarrow$ CLOSED FORM; $c_{ij} \Rightarrow$ NUMERICAL INTEGRAL OVER HQ-MASS DEPENDENT FORM FACTOR A ; MUST EXTRACT COLLINEAR SINGULARITY
- POINTLIKE LIMIT REPRODUCED WHEN $y_t \rightarrow 0$
- POINTLIKE LIMIT REPRODUCED UP TO WILSON COEFFN WHEN $p_T \rightarrow 0$: $c_{j,k}(\xi_p, y_t) \xrightarrow[\xi_p \rightarrow 0]{} c_{j,k}^{\text{PL}}$; EXPLICITLY CHECKED: NONTRIVIAL TEST

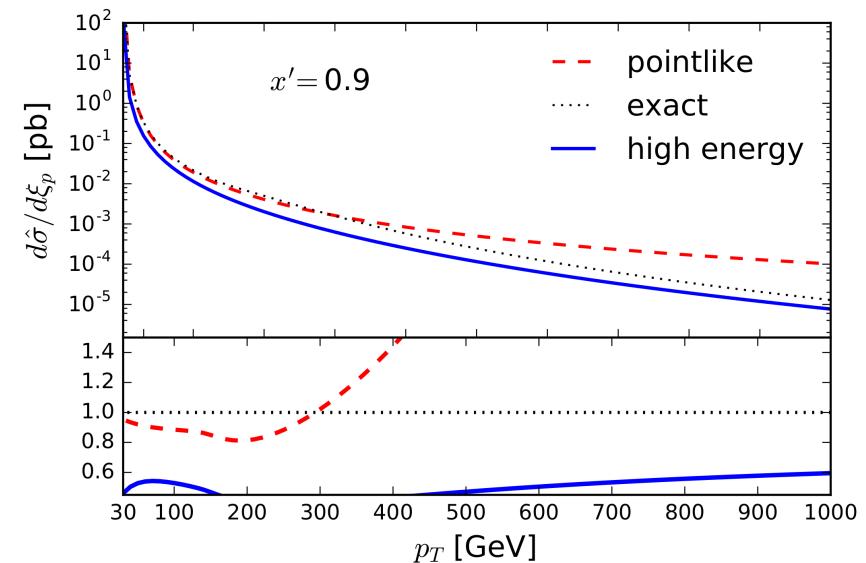
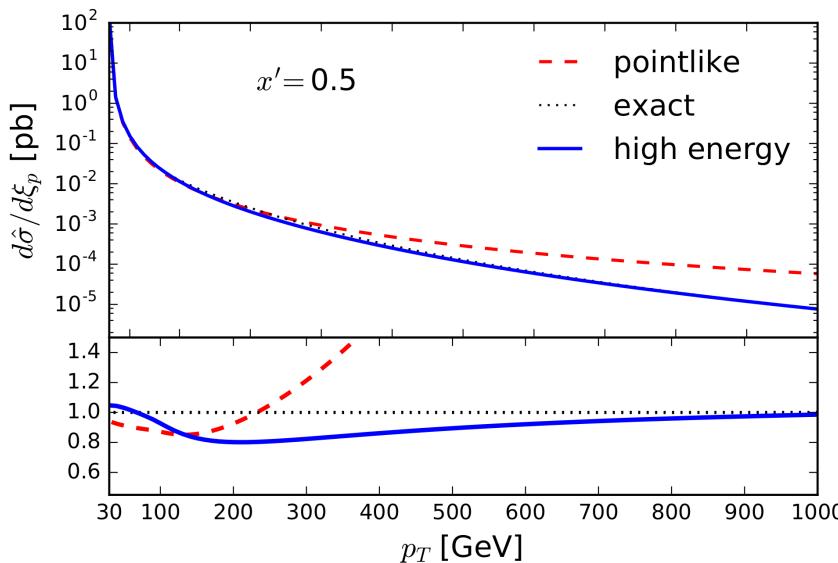
HIGGS: IMPLICATIONS

HIGH-ENERGY APPROXIMATION PARTON LEVEL AT LEADING ORDER

(Caola, sf, Marzani, Muselli, Vita, 2016)

- CAN COMPARE TO EXACT RESULT WITH MASS DEP. (Baur, Glover, 1990) & IN POINTLIKE LIMIT (Ellis, Hinchliffe, Soldate, van der Bij 1988)
- LO HE RESULT DOES NOT DEPEND ON x : $\frac{d\sigma^{\text{LLx-LO}}}{d\xi_p} = \sigma_0(y_b, y_t) c_0(\xi_p, y_t, y_b) \frac{2C_A \alpha_s}{\pi} \frac{1}{\xi_p}$

PARTON-LEVEL TRANSVERSE MOMENTUM DISTRIBUTION



- POINTLIKE LIMIT FAILS BADLY FOR $p_t \gtrsim m_t$: POINTLIKE $\underset{p_t \rightarrow \infty}{\sim} \frac{1}{p_t}$, MASSIVE $\underset{p_t \rightarrow \infty}{\sim} \frac{1}{p_t^2}$
- BOTTOM MASS CORRECTION (DUE TO INTERFERENCE) SMALL BUT VISIBLE FOR $m_b \lesssim p_t \lesssim m_t$
- HE APPROXIMATION EXCELLENT! FOR $x \lesssim 0.5$
- EVEN WHEN $x \gg 0.5$, HE APPROXIMATION DOES NOT DETERIORATE AT LARGE p_t

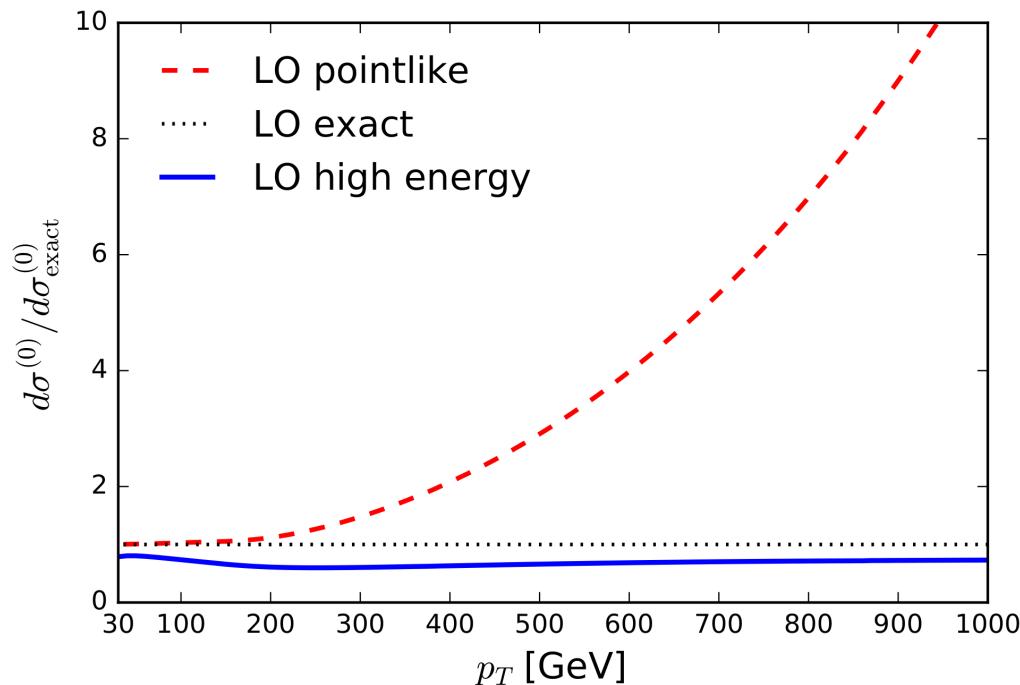
THE HIGH ENERGY APPROXIMATION HADRON LEVEL AT LO

(Caola, sf, Marzani, Muselli, Vita, 2016)

THE FACTORIZED RESULT: $\frac{d\sigma_{ij}}{d\xi_p}(\tau, \xi_p, y_t, \alpha_s) = \tau' \int_{\tau'}^1 \frac{dx'}{x'} \mathcal{L}_{ij}\left(\frac{\tau'}{x'}\right) \left[\frac{1}{x'} \frac{d\hat{\sigma}}{d\xi_p}(x', \xi_p, y_t, \alpha_s) \right]$

SCALE: $Q^2 = \left(\sqrt{m_H^2 + p_T^2} + \sqrt{p_T^2} \right)^2$; $0 \leq \tau', x' \leq 1$; LOW p_t : $Q^2 \approx m_h^2$; HIGH p_t : $Q^2 \approx 4p_t^2$

p_t SPECTRUM AT LHC 13: RATIO TO EXACT RESULT (Baur, Glover, 1990)



HIGH ENERGY IS MORE ACCURATE THAN POINTLIKE FOR $p_t \gtrsim m_t$

THE HIGH ENERGY APPROXIMATION HADRON LEVEL AT NLO

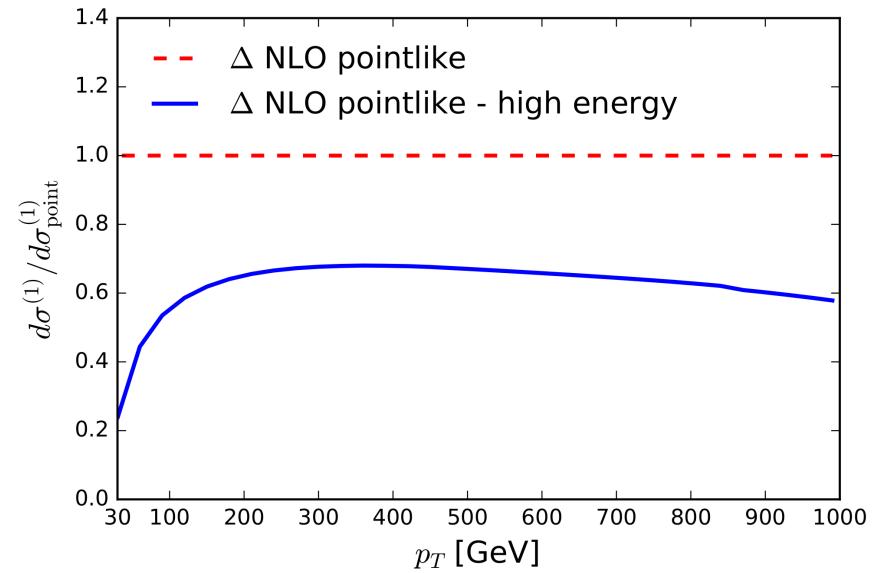
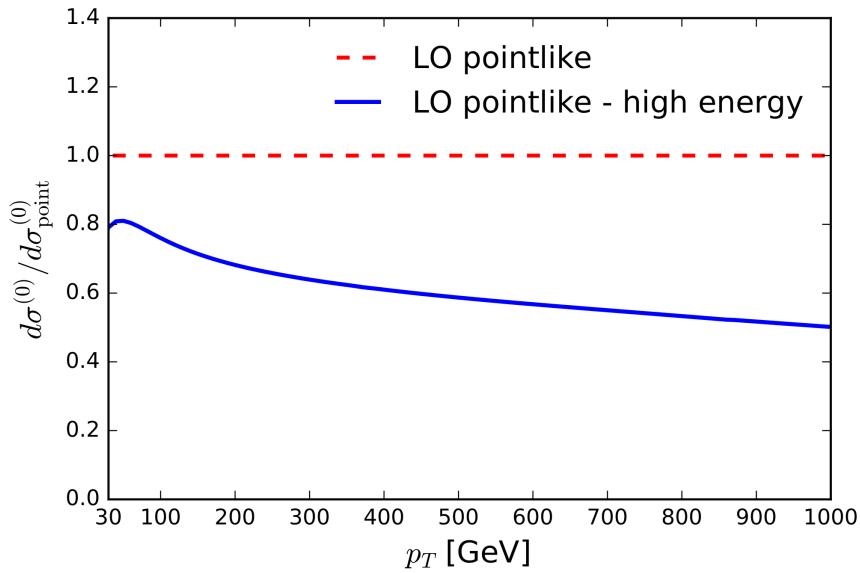
(Caola, sf, Marzani, Muselli, Vita, 2016)

BEYOND LO, ONLY EFT RESULT KNOWN (NLO: Glosser, Schmidt, 2002; NNLO: Boughezal, Caola, Melnikov, Petriello, Schulze, 2015) \Rightarrow **HIGH-ENERGY VS. EXACT ONLY IN POINTLIKE LIMIT**

HIGH ENERGY VS. EXACT IN POINTLIKE CASE: LHC 13

$$\frac{d\sigma}{d\xi_p}(\tau', \xi_p, y_t, \alpha_s) = \alpha_s \frac{d\sigma^{(0)}}{d\xi_p} + \alpha_s^2 \frac{d\sigma^{(1)}}{d\xi_p} + \mathcal{O}(\alpha_s^3)$$

$$\frac{d\sigma^{(0)}}{d\sigma_{\text{point}}^{(0)}} \qquad \qquad \qquad \frac{d\sigma^{(1)}}{d\sigma_{\text{point}}^{(1)}}$$



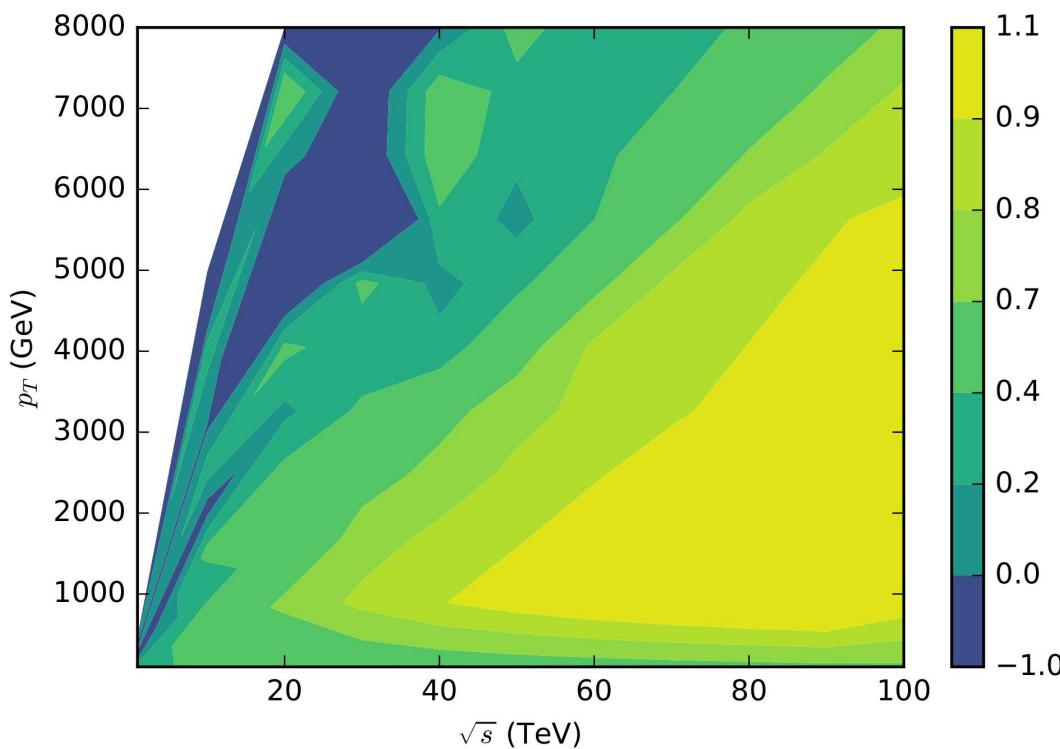
QUALITY OF HIGH ENERGY APPROX SIMILAR AT LO AND NLO

THE HIGH ENERGY APPROXIMATION HADRON LEVEL AT NLO

(Caola, sf, Marzani, Muselli, Vita, 2016)

BEYOND LO, ONLY EFT RESULT KNOWN \Rightarrow HIGH-ENERGY VS. EXACT IN POINTLIKE LIMIT

$$d\sigma^{(1)}/d\sigma_{\text{point}}^{(1)}$$

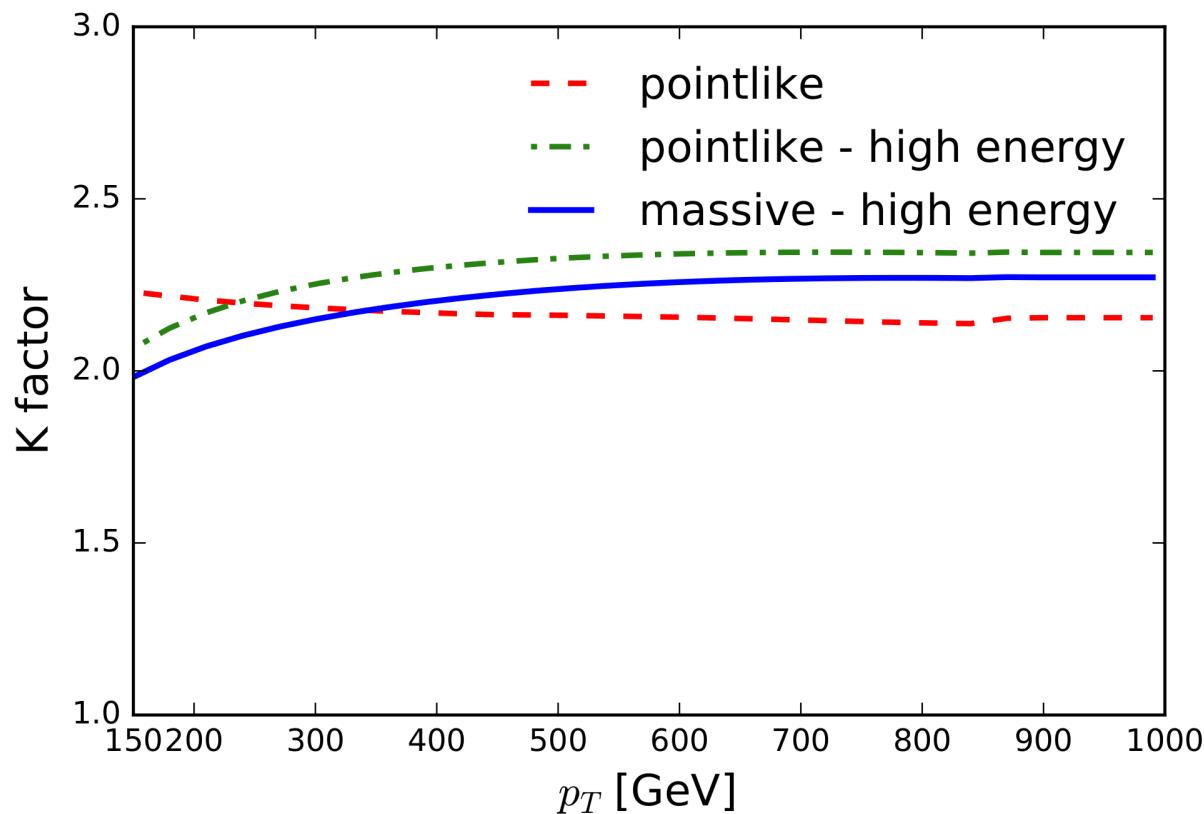


HIGH ENERGY APPROX. GOOD FOR $2m_t \lesssim p_t \lesssim p_t^{\max}/5$

THE NLO K -FACTOR (LHC 13)

(Caola, sf, Marzani, Muselli, Vita, 2016)

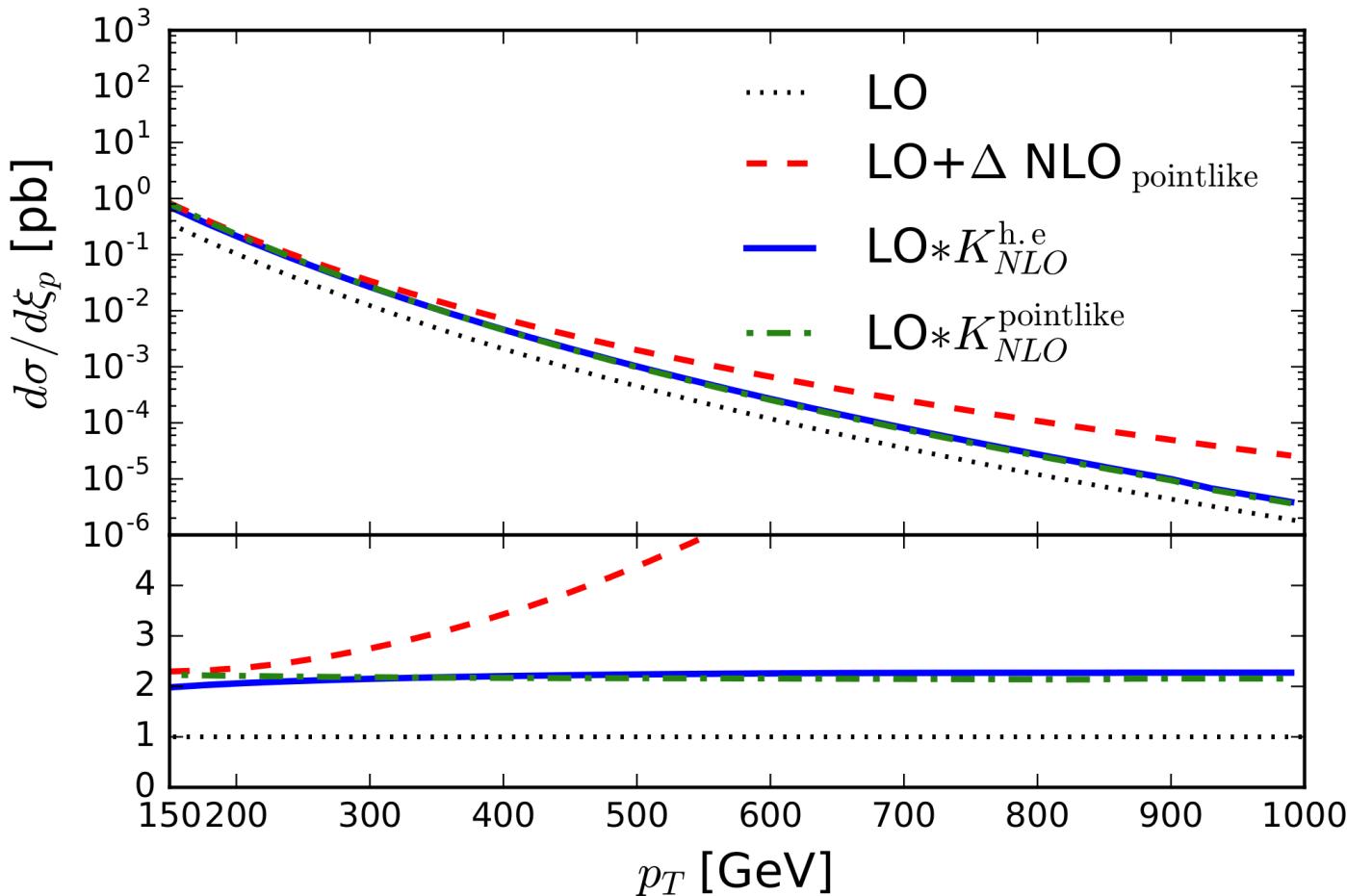
$$K^{NLO} = 1 + \frac{d\sigma^{(1)}/d\xi}{d\sigma^{(0)}/d\xi}$$



- HIGH-ENERGY APPROX. IS 20% ACCURATE FOR ALL $p_t \gtrsim 2m_t$
- POINTLIKE AND MASSIVE K -FACTORS SIMILAR, APPROX. p_t -INDEP.

THE NLO SPECTRUM (LHC 13)

(Caola, sf, Marzani, Muselli, Vita, 2016)



- BEST APPROX AT LARGE p_t : EXACT LO \times HE K-FACTOR
- UNCERTAINTY DRIVEN BY HE APPROX (ABOUT 30% AT LHC 13)
- POINTLIKE NLO FAILS FOR $p_t \gtrsim m_t$

THE ROLE OF BOTTOM COLLINEAR LOGS

(Caola, sf, Marzani, Muselli, Vita, 2016)

- WHEN $m_b \lesssim p_t \lesssim m_h$ (i.e. $p_t \rightarrow 0$; $\frac{m_b}{p_t} \rightarrow 0$) \Rightarrow COLLINEAR & IR LOGS (Banfi, Monni, Zanderighi, 2014)
- DO THEY EXPONENTIATE?

THE COLLINEAR BOTTOM LOGS IN HE LIMIT

HE LO CROSS-SECTION: $\frac{d\sigma^{LO}}{d\xi_p} (x, xp, \xi_p, y_b) \underset{y_b \rightarrow 0}{\sim} \frac{G_F \alpha_s^2}{256\pi^2} \frac{2C_A \alpha_s}{\pi \xi_p} y_b^2 \left| \ln^2 \frac{\xi_p}{y_b} - \ln^2 (-y_b) + 4 \right|^2$

IMPACT FACTOR:

$${}^h p_T \underset{y_b \rightarrow 0}{\sim} \sigma_0^{\text{PL}} R(M_1) R(M_2) \frac{\xi_p^{M_1 + M_2 - 1}}{(1 + \xi_p)^N} \left[c_0^{\text{PL}} (M_1 + M_2) + \sum_{j > k > 0} c_{j,k}^{\text{PL}} (M_1^j M_2^k + M_1^k M_2^j) \right] \ln^4 \frac{\xi_p}{y_b}$$

- ONLY COLLINEAR LOGS PRESENT AT HE (EXTERNAL LEGS)
- $\frac{\xi_p}{y_b} = \frac{p_t^2}{m_b^2} \Rightarrow$ LO COLLINEAR DOUBLE LOG REPRODUCED
- AT LL x APPEARS TO ALL ORDERS, BUT AS PREFACCTOR (NO EXPONENTIATION)
- HE FIXED POWER OF LOG \Rightarrow CANNOT EXCLUDE SUBLEADING EXPONENTIATION;

NOTE INFRARED DOUBLE LOGS FROM ABELIAN LOOPS DO EXPONENTIATE (Melnikov, Penin, 2016), BUT ABSENT AT LL x (BOXES AND HIGHER ORDERS)

OUTLOOK

- **FULLY DIFFERENTIAL HIGH-ENERGY RESUMMATION?**
 - INCLUDING p_t & RAPIDITY DEPENDENCE
- **DOUBLE AND TRIPLE RESUMMATION?**
 - INCLUDING HIGH-ENERGY, p_t & THRESHOLD RESUMMATIONS
- **MATCHED APPROXIMATIONS?**
 - MATCH $1/m_t$ TO HIGH-ENERGY (AKIN TO Harlander, Mantler, Marzani, Ozeren 2010
FOR INCLUSIVE CROSS-SECTION)
 - MATCH HIGH-ENERGY TO THRESHOLD (AKIN TO Ball, Bonvini, sf, Marzani, Ridolfi 2013
FOR INCLUSIVE CROSS-SECTION)
- **OTHER PROCESSES?**
 - DRELL-YAN
 - JETS???

EXTRAS

HIGH ENERGY vs. TRANSVERSE MOMENTUM RESUMMATION

(sf, Muselli, 2015; Marzani, 2015)

THE STRUCTURE OF TRANSVERSE MOMENTUM RESUMMATION:

$$\frac{d\sigma_{a_1 a_2}^{\text{res}}}{d\mathbf{p}_T} (N, \mathbf{p}_T, Q^2) = \sum_{ij} \sigma_{ij}^{(0)} \int d^2 \mathbf{b} e^{i \mathbf{b} \cdot \mathbf{p}_T} S_{ij}(b^2, Q^2) \times \\ \sum_{lm} H_g(\alpha_s) [C_{gl}(N, \mathbf{b}) C_{gm}(N, \mathbf{b}) + G_{gl}(N, \mathbf{b}) G_{gm}(N, \mathbf{b})] \times \Gamma_{la_1}[\alpha_s, b^2, Q^2] \Gamma_{ma_2}[\alpha_s, b^2, Q^2],$$

SUMS RUN OVER PARTON INDICES, $\Gamma \Rightarrow$ EVOLUTION; C, G UNIVERSAL PARTONIC FUNCTIONS

(Catani, Grazzini, 2012)

THE b -SPACE IMPACT FACTOR

$$h_{p_T}(N, M_1, M_2, b, \alpha_s) = \int_0^\infty d\xi_p J_0(\sqrt{\xi_p} b m_h) h_{p_T}(N, M_1, M_2, \xi_p, \alpha_s)$$

- AT HIGH-ENERGY (POINTLIKE), ALL p_t DEPENDENCE IN (EVOLUTION) PREFACCTOR
- IN RESOLVED CASE, THIS WILL HAPPEN FOR SMALL p_t (p_t DEP. KICKS IN AS p_t GROWS)
- SUDAKOV AND HARD DO NOT DEPEND ON $N \Rightarrow$ AT LLx ONLY $O(\alpha_s)$ PART CONTRIBUTE
- THE N -DEPENDENCE OF THE b -SPACE IMPACT FACTOR HAS THE UNIVERSAL STRUCTURE REQUIRED BY SMALL- p_T RESUMMATION

$$h_{p_T}(0, M_1, M_2, b, \alpha_s) = \\ \sigma_0 e^{-(M_1 + M_2) \ln \frac{b^2 m_h^2}{4}} R(M_1) R(M_2) \times \left[\frac{\Gamma(1+M_1)}{\Gamma(1-M_1)} \frac{\Gamma(1+M_2)}{\Gamma(1-M_2)} + M_1 \frac{\Gamma(1+M_1)}{\Gamma(2-M_1)} M_2 \frac{\Gamma(1+M_2)}{\Gamma(2-M_2)} \right]$$

STRUCTURE OF THE RESUMMED RESULT: RESOLVED CASE IMPACT FACTOR

$$h_{pT} = \sigma_0(y_i) R(M_1) R(M_2) \frac{\xi_p^{M_1 + M_2 - 1}}{(1 + \xi_p)^N} [c_0(\xi_p, y_i)(M_1 + M_2) + \sum_{j \geq k > 0} c_{j,k}(\xi_p, y_i)(M_1^k M_2^j + M_1^j M_2^k)]$$

SUBSTITUTING $M_1 = \gamma_s(\frac{\alpha_s}{N})$ $M_2 = \gamma_s(\frac{\alpha_s}{N})$ & EXPANDING

p_t DISTRIBUTION

$$\frac{d\sigma}{d\xi_p}(x, \xi_p, y_t, y_b) = \sigma_0(y_t, y_b) \sum_{k=1}^{\infty} C_k(\xi_p, y_t, y_b) \alpha_s^k (-1)^{k+1} \frac{\ln^{k-1} x}{(k-1)!}$$

$$C_1(\xi_p, y_t, y_b) = \frac{2C_A}{\pi} \frac{c_0(\xi_p, y_t, y_b)}{\xi_p}$$

$$C_2(\xi_p, y_t, y_b) = \frac{2C_A^2}{\pi^2} \frac{2c_0(\xi_p, y_t, y_b) \ln \xi_p + c_{1,1}(\xi_p, y_t, y_b)}{\xi_p}$$

$$C_3(\xi_p, y_t, y_b) = \frac{2C_A^3}{\pi^3} \frac{2c_0(\xi_p, y_t, y_b) \ln^2 \xi_p + 2c_{1,1}(\xi_p, y_t, y_b) \ln \xi_p + c_{2,1}(\xi_p, y_t, y_b)}{\xi_p}$$

\Rightarrow LEADING POWER OF $\ln \xi_p$ PROPORTIONAL TO LO c_0