COLLIER

A Complex One-Loop LIbrary with Extended Regularizations

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in collaboration with
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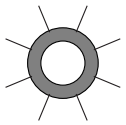
Loopfest, Buffalo, August 2015

One-loop amplitudes

general structure of one-loop amplitudes:



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with $D_i = (q + p_i)^2 - m_i^2$

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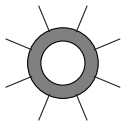

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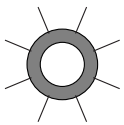
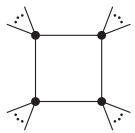
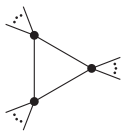


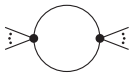
$$= \sum_l d_l$$



$$+ \sum_k c_k$$



$$+ \sum_j b_j$$



$$+ \sum_i a_i$$

$$+ R$$

$$= \sum_l d_l D_0(l) + \sum_k c_k C_0(k) + \sum_j b_j B_0(j) + \sum_i a_i A_0(i) + R$$

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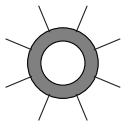

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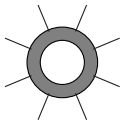

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different approaches for calculation:

- conventional method (Feynman diagrams) $\rightarrow$ TI's needed
- generalised unitarity $\rightarrow$ SI's needed

  [Ossola, Papadopoulos, Pittau '07, Bern, Dixon, Kosower, Britto, Cachazo, Feng,Ellis, Giele, Melnikov, . . .]
- recursive methods using tensor integrals $\rightarrow$ TI's needed

  [van Hameren'09; Cascioli,Maierhöfer,Pozzorini'11; Actis,Denner,LH,Scharf,Uccirati'12]

Tools for NLO

- Many tools for NLO calculatios, e.g.
  `FeynArts`, `FeynCalc`, `FormCalc`, `Blackhat`, `NGluon`, `HELAC-NLO`, `GoSam`, `CutTools`, `HELAC-1LOOP`, `Samurai`, `MadGraph5_aMC@NLO`, `OpenLoops`, `Recola`

- Libraries for scalar and tensor integrals, e.g.
  `FF` [van Oldenborgh], `LoopTools` [Hahn,Perez-Victoria], `QCDLoop` [R.K.Ellis,Zanderighi], `OneLOop` [van Hameren], `Golem95C` [Cullen,Guillet,Heinrich,Kleinschmidt,Pilon,...], `PJFry` [Fleischer,Riemann]

- This talk:

  COLLIER **A Complex One-Loop LIbrary with Extended Regularizations**

  fortran-library for fast and stable numerical evaluation of scalar and tensor integrals [Denner,Dittmaier,LH]

  Released on April 25:
  `http://collier.hepforge.org`, arXiv:1604.06792

Collier: Applications

successfully used in many calculations of

- NLO QCD corrections, e.g.
  $pp \to t\bar{t}b\bar{b}$ [Bredenstein,Denner,Dittmaier,Pozzorini '09]
  $pp \to WWb\bar{b}$ [Denner,Dittmaier,Kallweit,Pozzorini '11]
  $pp \to WWb\bar{b}H$ [Denner,Feger '15] (incl. 7-point integrals)

- NLO EW corrections, e.g.
  $e^+e^- \to$ 4 fermions [Denner,Dittmaier,Roth,Wieders '05]
  $pp \to Hjj$ via VBF [Ciccolini,Denner,Dittmaier '07]
  $pp \to l^+l^-jj$ [Denner,LH,Scharf,Uccirati '14]
  $pp \to \mu^+\mu^-e^+e^-, \mu^+e^-\nu_\mu\bar{\nu}_e$ [Biedermann et al. '16]
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- NNLO QCD corrections:
  real-virtual corrections require calculation of one-loop integrals
  for configurations with unresolved external particles
  $\to$ kinematics involves collinear/soft external momenta
  $pp \to Z\gamma$ [Grazini,Kallweit,Ratlev,Torre '13]
  $pp \to ZZ$ [Cascioli et al. '14], $pp \to WW$ [Gehrmann et al. '14]

Features of Collier (1)

- complete set of one-loop scalar integrals
- implementation of tensor integrals for (in principle) arbitrary number of external momenta N
  (tested in many physical processes up to $N = 8$)
- various expansion methods implemented for exceptional phase-space points
  (to arbitrary order in expansion parameter)
- mass- and dimensional regularisation supported for IR-singularities
- complex masses supported (unstable particles)
- output: coefficients $T^N_{0\cdots 0 i_1 \cdots i_k}$ or tensors $(T^N)^{\mu_1 \cdots \mu_P}$

Features of Collier (2)

- two independent implementations: COLI+DD
- cache-system to avoid recalculation of identical integrals
- uncertainty estimates are performed and returned with the results for the integrals
- problematic integrals can be reported to log-files
- demo programs illustrating the usage of the library

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integrated in automated NLO generators

- OpenLoops [Cascioli,Maierhöfer,Pozzorini]
- Recola [Actis,Denner,LH,Lang,Scharf,Uccirati]

  Released on May 3:
  `http://recola.hepforge.org`, arXiv:1605.01090

Tensor integrals in dim-reg

$$T^{N,\mu_1\dots\mu_P} = \frac{(2\pi\mu)^{2\epsilon}}{\mathrm{i}\pi^2}\int \mathrm{d}^D q\, \frac{q^{\mu_1}\cdots q^{\mu_P}}{N_0 N_1 \dots N_{N-1}}, \qquad N_i = (q+p_i)^2 - m_i^2$$

Structure UV- or IR-singular integrals in $D = 4 - 2\epsilon$ dimensions:

$$T^N = \tilde{T}^N_{\text{fin}} + \frac{a^{\text{UV}}}{\epsilon_{\text{UV}}}\mu_{\text{UV}}^{2\epsilon_{\text{UV}}} + \left(\frac{a^{\text{IR}}}{\epsilon_{\text{IR}}^2} + \frac{a^{\text{IR}}}{\epsilon_{\text{IR}}}\right)\mu_{\text{IR}}^{2\epsilon_{\text{IR}}}$$

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- change of normalization of tensor integral

  $$T^N \to c(\epsilon) T^N \qquad \text{with} \qquad c(\epsilon) = 1 + \mathcal{O}(\epsilon)$$

  is equivalent to a redefinition

  $$\frac{1}{\epsilon_{\text{UV}}} \to \Delta_{\text{UV}} = \frac{c(\epsilon)}{\epsilon_{\text{UV}}}, \qquad \frac{1}{\epsilon_{\text{IR}}} \to \Delta^{(1)}_{\text{IR}} = \frac{c(\epsilon)}{\epsilon_{\text{IR}}}, \qquad \frac{1}{\epsilon^2_{\text{IR}}} \to \Delta^{(2)}_{\text{IR}} = \frac{c(\epsilon)}{\epsilon^2_{\text{IR}}}$$

- convention in Collier: $c(\epsilon) = \Gamma(1+\epsilon)(4\pi)^\epsilon$

Output of Collier

$$T^N = T^N_{\text{fin}}(\mu^2_{\text{UV}}, \mu^2_{\text{IR}}) + a^{\text{UV}}\,\Delta_{\text{UV}} + a_2^{\text{IR}}\left(\Delta^{(2)}_{\text{IR}} + \Delta^{(1)}_{\text{IR}}\,\ln\mu^2_{\text{IR}}\right) + a_1^{\text{IR}}\Delta^{(1)}_{\text{IR}}$$

- scales $\mu^2_{\text{UV}}, \mu^2_{\text{IR}}$ and poles $\Delta_{\text{UV}}, \Delta^{(1)}_{\text{IR}}, \Delta^{(2)}_{\text{IR}}$ can be set to arbitrary real values

  $\Rightarrow$ output of Collier: numerical value for full T^N

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  $\Rightarrow$ output of Collier: numerical value for full T^N

- cancellation of poles can be checked numerically by varying $\Delta_{\text{UV}},\ \Delta^{(1)}_{\text{IR}},\ \Delta^{(2)}_{\text{IR}}$

- convention for prefactor $c(\epsilon)$ can be changed by shifting $\Delta_{\text{UV}},\ \Delta^{(1)}_{\text{IR}},\ \Delta^{(2)}_{\text{IR}}$ accordingly

- coefficient a^{UV} of $1/\epsilon_{\text{UV}}$ - pole returned also as separate output

Methods implemented in Collier

applied method depends on number N of propagators

- $N = 1, 2$: explicit analytical expressions
- $N = 3, 4$: exploit Lorentz-covariance

  standard PV reduction [Passarino,Veltman '79]

  $\rightarrow$ spurious $1/\det(Z)$ divergences in exceptional phase-space regions ($\det(Z)$: Gram determinant)

  $\Rightarrow$ stable expansions in $(\det(Z))^g$ [Denner,Dittmaier '05]
- $N \geq 5$: exploit 4-dimensionality of space-time

  [Melrose '65; Denner,Dittmaier '02,'05; Binoth et al. '05]

Basic scalar integrals from analytic expressions
['t Hooft,Veltman'79; Beenaker,Denner'90; Denner,Nierste,Scharf'91; Ellis,Zanderighi'08; Denner,Dittmaier'11]

$\Rightarrow$ fast and stable numerical reduction algorithm

Calculation of 3- and 4-point coefficients

1. Perform PV reduction:

   accuracy ΔT^N better than target precision η_{req}?

   yes → return result — done

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   ↓

   expansion to order $g = 0, 1, 2, \ldots, g_{\max}$

   $\Delta T^N_g < \eta_{\mathrm{req}}$?

   yes → return result — done

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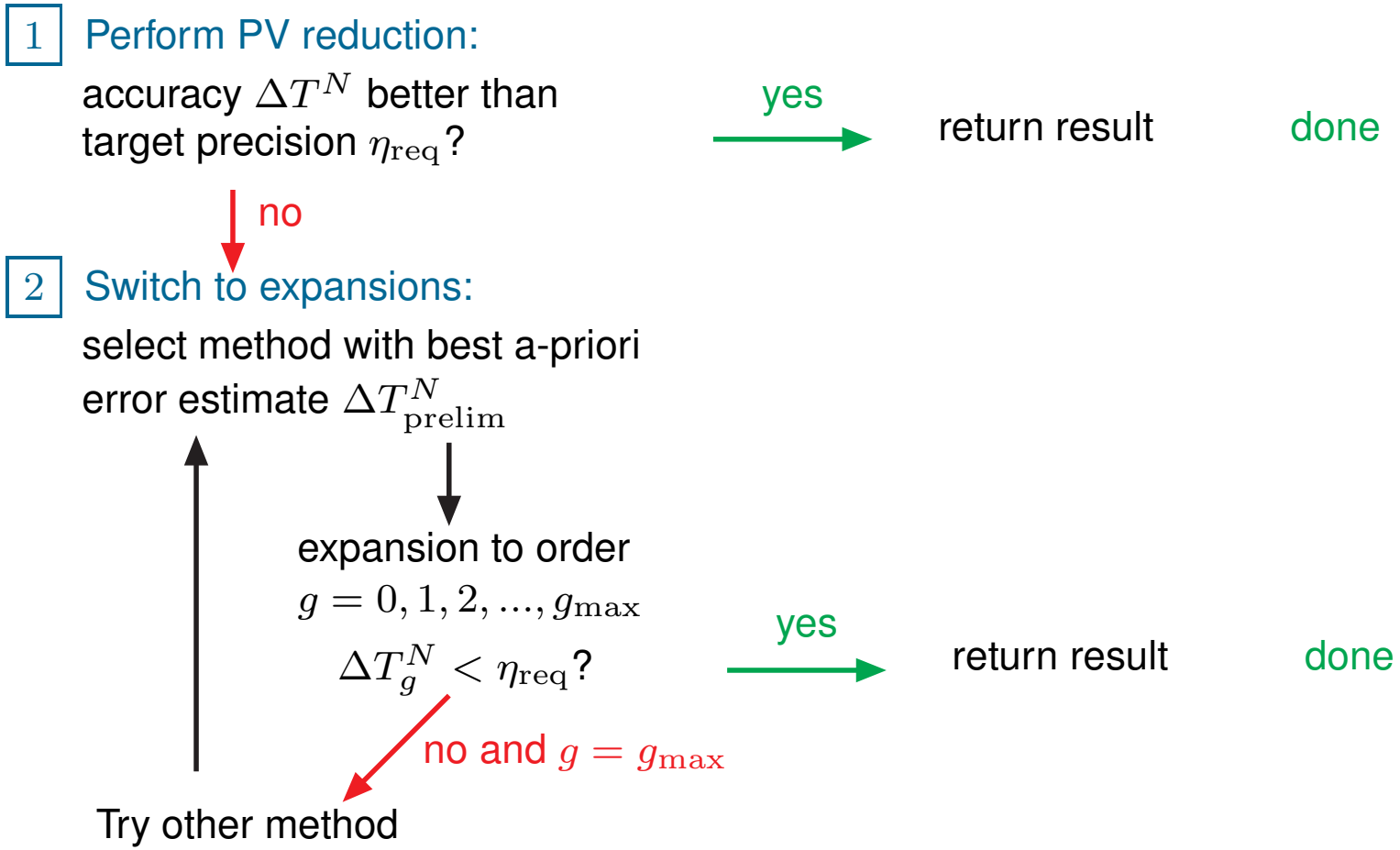

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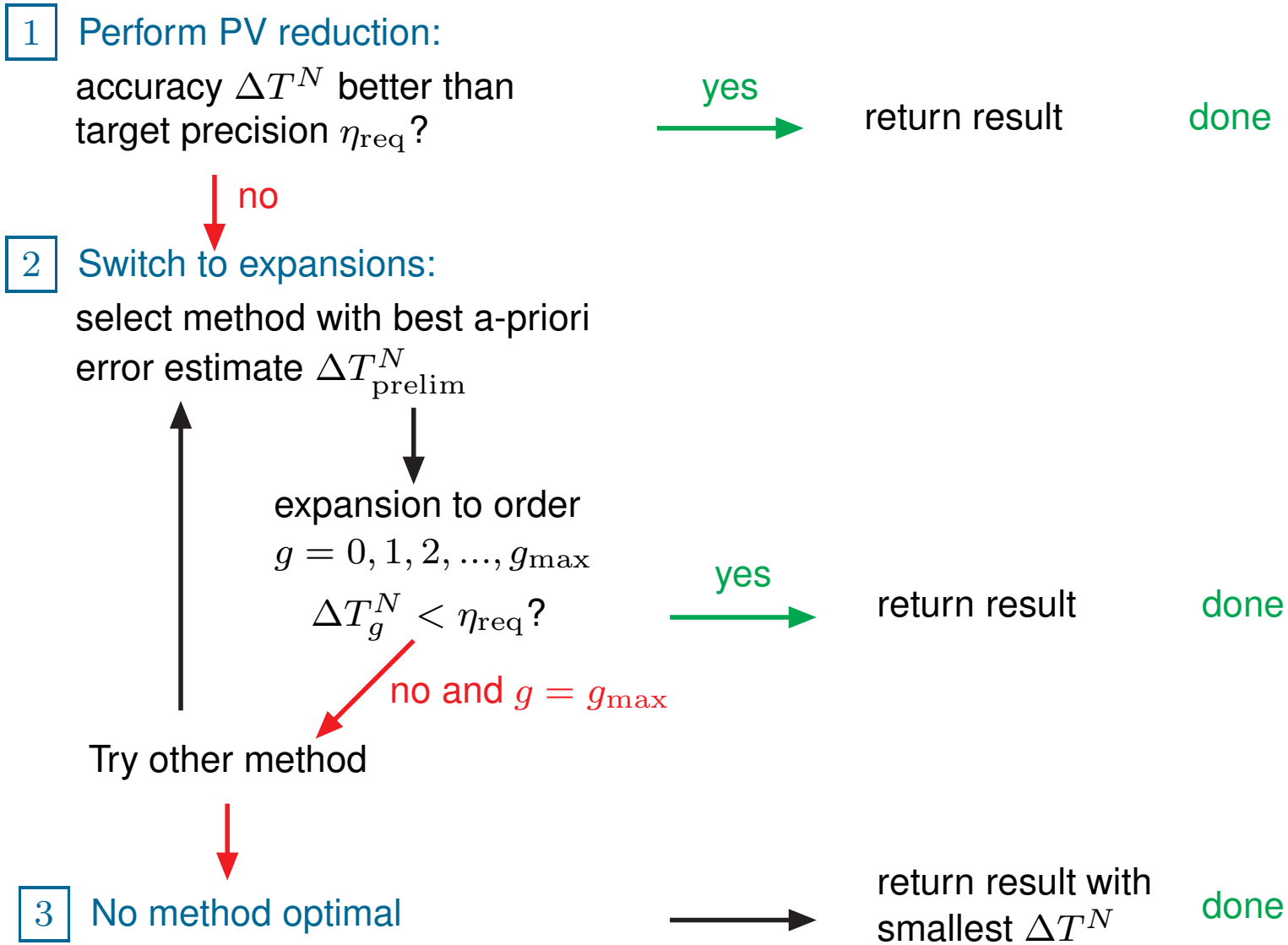

Error estimates

1 PV-reduction

- error propagation during reduction:
  $\delta D_0 = 10 \times \text{machine precision}$
  $\delta D_r \sim a_{\max}\, \delta D_{r-1} \qquad (a_{\max} \sim 1/\det(Z))$
  (+error propagation from 3-point functions)
- symmetry of coefficients
  $$\delta D_r \sim |D_{i_1 i_2 \ldots i_r} - D_{i_2 i_1 \ldots i_r}|, \qquad (0 \neq i_1 \neq i_2 \neq 0)$$

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2 Expansions: $D_r = D_r^{(0)} + \ldots + D_r^{(g)}$

- a-prori estimate:
  neglected higher orders: $\delta D_r \sim (\det(Z))^{g+1}$
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- extrapolation after calculation: $\delta D_r = D_r^{(g)} \times \dfrac{D_r^{(g)}}{D_r^{(g-1)}}$

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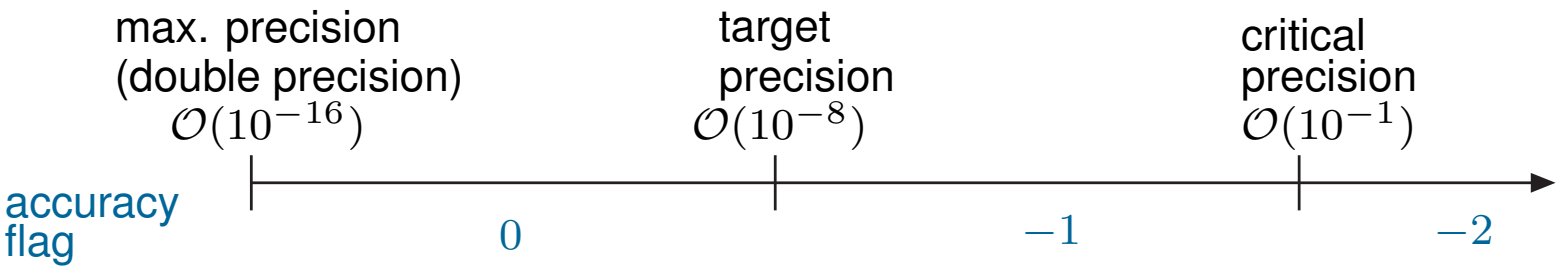
+ error estimate at almost zero cost in run-time
– only rough order-of-magnitude estimate, typically accurate within a factor of 10-100 (conservative in most cases)

Precision handling



- target precision:
  governs selection of expansion method and expansion depth
  $\rightarrow$ balancing between precision and run-time

- critical precision:
  arguments and results of function calls are reported to an output file if estimated accuracy is worse than critical precision

- accuracy flag:
  stores status of worst integral within all function calls of the same phase space point (reinitialized for new phase-space point)

Coefficients vs. tensors

$$(T^N)^{\mu_1\cdots\mu_P} = \sum_k \sum_{i_1,\ldots,i_k} T^{N,P}_{\underbrace{0\cdots0}_{P-k}\, i_1\cdots i_k} \{ \underbrace{g\cdots g}_{(P-k)/2}\, p_{i_1}\cdots p_{i_k}\}^{\mu_1\cdots\mu_P}$$

# of tensor coefficients (TC) vs. # of tensor elements (TE)

	$r=0$	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$
$N=3$	1	3	7	13	22	34	50
$N=4$	1	4	11	24	46	80	130
$N=5$	1	5	16	40	86	166	296
$N=6$	1	6	22	62	148	314	610
$N=7$	1	7	29	91	239	553	1163
tensor	1	5	15	35	70	126	210

#TC $<$ #TE

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NLO generators OpenLoops and Recola:
parametrisation of one-loop amplitude in terms of tensor integrals:

calculated by OpenLoops/Recola

$$\mathcal{M} = \sum_j c^{(j)}_{\mu_1\ldots\mu_{n_j}} T^{\mu_1\ldots\mu_{n_j}}_{(j)} \rightarrow \text{Tensor Integrals}$$

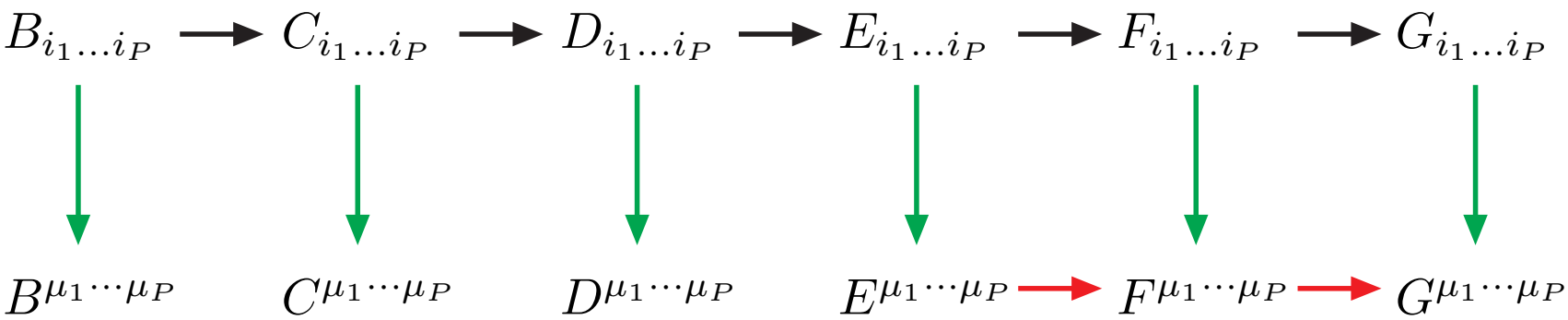
$\Rightarrow$ need full tensors!

From coefficients to tensors

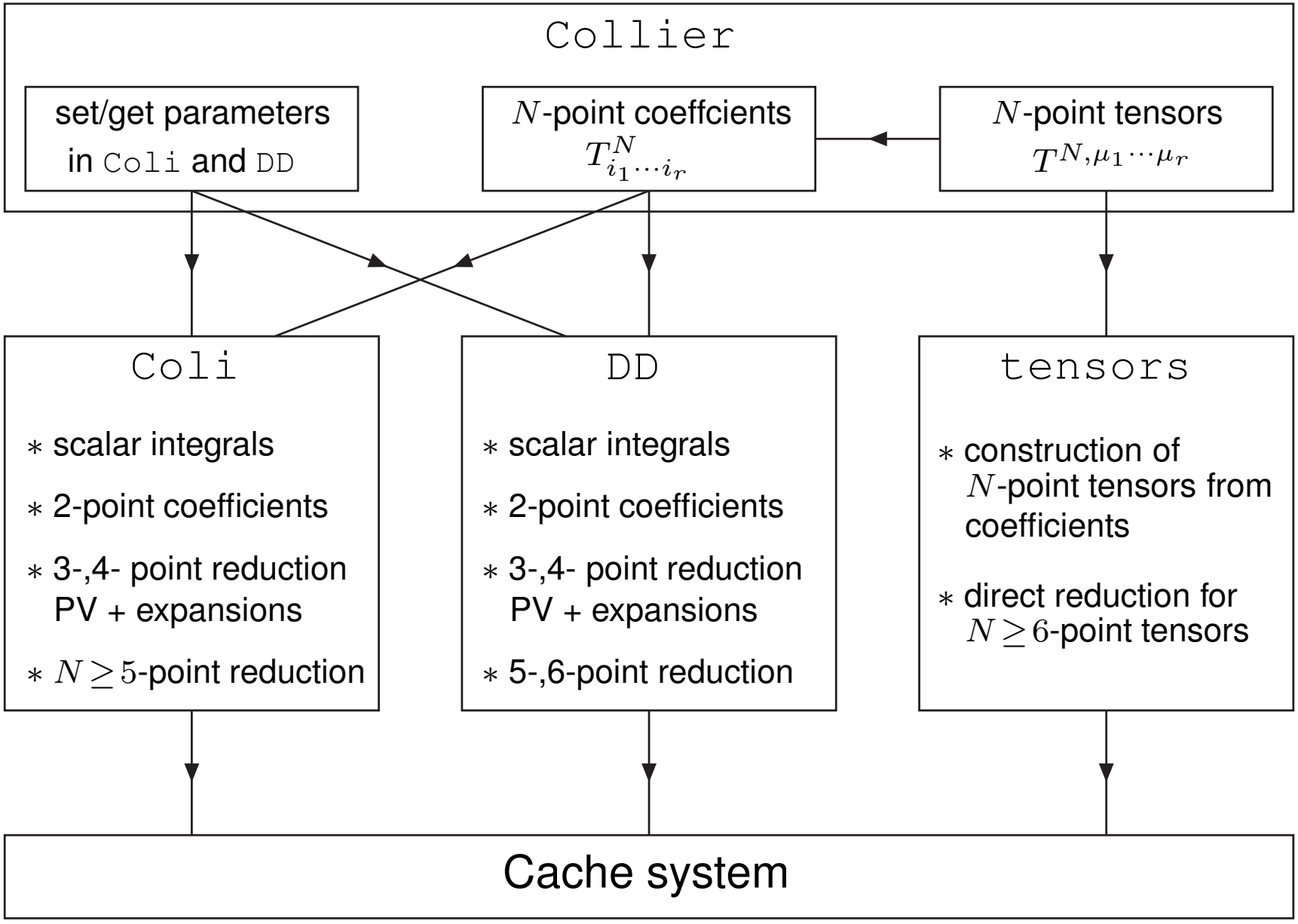
$$(T^N)^{\mu_1\cdots\mu_P} = \sum_k \sum_{i_1,\ldots,i_k} T^{N,P}_{\underbrace{0\cdots0}_{P-k} i_1\cdots i_k} \{ \underbrace{g\cdots g}_{(P-k)/2}\, p_{i_1}\cdots p_{i_k}\}^{\mu_1\cdots\mu_P}$$

In Collier:

- output: coefficients $T^N_{0\cdots0 i_1\cdots i_k}$ or tensors $(T^N)^{\mu_1\cdots\mu_P}$
- efficient algorithm to construct tensors from invariant coefficients for arbitrary N, P via recursive calculation of tensor structures
- for $N \geq 6$: Direct reduction at tensor level

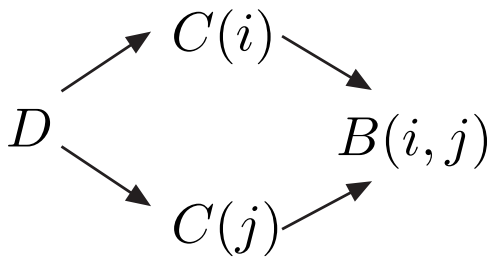


Structure of Collier



Cache system

Evaluation of one-loop amplitude leads to multiple calls for the same tensor integral (TI):



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  same TI appears several times
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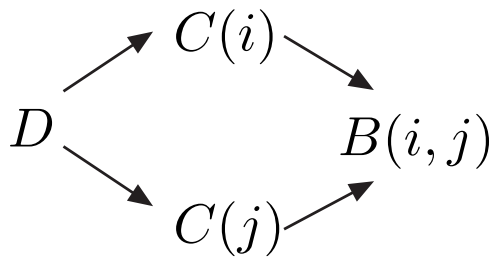

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Cache system in Collier:

- Identify each TI-call via index pair (N, i):
  N= number of external master call
  i = binary index for internal calls (propagated in reduction)
- pointers for each pair (N, i) point to same address in cache if arguments of TI's are identical
  first call: write cache further calls: read cache
- internal calls: always cached
  external calls: only cached if global cache is switched on by user
  $\rightarrow$ requires same sequence of integral calls for each event!

Download & installation

- download `collier-X.Y.Z.tar.gz` from
  `http://collier.hepforge.org`
  and unpack the archive:
  `tar -zxvf collier-X.Y.Z.tar.gz`

- easy installation via the CMAKE build system:
  ```
  cd COLLIER-X.Y.Z/build
  cmake [options] ..
  make
  ```

- options:

  `-Dstatic=ON`

  → create static instead of dynamic library

  `-DCMAKE_Fortran_COMPILER=comp`

  → use comp as fortran compiler (comp can be gfortran, ifort, pgf95, ... or the full path to a compiler)

Initialization

```
! include module COLLIER
use COLLIER
```

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use COLLIER

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! example for up to 6-point integrals of rank up to 4
! with file output directed to folder "output_cll"
! (empty string suppresses file output)
call Init_cll(Nmax=6,rmax=4,"output_cll")
! --> all parameters are set to default values
!     (can be modified later)
```

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! set UV renormalization scale to 100GeV (default=1GeV)
call SetMuUV2_cll(1d4)

! choose mode (default=1)
! mode=1: use COLI
! mode=2: use DD
! mode=3: use COLI+DD and report differences
call setMode_cll(3)
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! initialize global cache system:
! use 1 cache, integrals up to 6-point functions cached
call InitCacheSystem_cll(1,Nmax=6)
```

Collier parameters

parameter	type	set with	default
mode	integer $\in \{1,2,3\}$	SetMode_cll	1
η_{req}	double precision	SetReqAcc_cll	1d-8
η_{crit}	double precision	SetCritAcc_cll	1d-1
η_{check}	double precision	SetCheckAcc_cll	1d-4
μ^2_{UV}	double precision	SetMuUV2_cll	1d0
μ^2_{IR}	double precision	SetMuIR2_cll	1d0
Δ_{UV}	double precision	SetDeltaUV_cll	0d0
$\Delta^{(1)}_{\text{IR}}, \Delta^{(2)}_{\text{IR}}$	double precision	SetDeltaIR_cll	0d0,0d0
$\{\overline{m}^2_1, \ldots, \overline{m}^2_{n_{\text{reg}}}\}$	double complex (n_{reg})	SetMinf2_cll	{}
σ_{stop}	integer < 0	SetErrStop_cll	-8
N_{tenred}	integer ≥ 6	SetTenRed_cll	6
n_{cache}	integer ≥ 0	InitCacheSystem_cll	0
$N^{\text{max}}_{\text{cache}}$	integer (n_{cache}) ≥ 1	SetCacheLevel_cll	–
n_{err}	integer ≥ 0	SetMaxErrOut_cll	100
$n^{N,\text{max}}_{\text{check}}$	integer (N_{max}) ≥ 0	SetMaxCheck_cll	$\{50, \ldots, 50\}$
$n^{N,\text{max}}_{\text{crit}}$	integer (N_{max}) ≥ 0	SetMaxCrit_cll	$\{50, \ldots, 50\}$
$\widehat{P}^{\text{max}}$	integer ≥ 6	SetRitmax_cll	14

Usage of Collier in a MC generator

```
! loop over MC events
do nevent=1,nevent_tot

  ! loop over channels
  do nchan=1,nchan_tot

    ! initialize new event in Collier
    call InitEvent(ncache(nchan))
    ! --> reinitializes corresponding cache
    ! --> resets error and accuracy flags

    ! reset parameters if needed
    call SetMuUV2_cll(Q2run)

    ! sequence of integral calls
    ! SAME ORDER FOR EACH EVENT IF CACHE USED!!!
    call TN_cll(TN1,TN1uv,MomVec1,MomInv1,masses1,N1,r1,TN1err)
    ...

    ! read out error and accuracy flags
    call GetErrFlag(eflag)
    call GetAccFlag(aflag)

  end do
end do
```

Features of RECOLA + COLLIER

- tree-level and one-loop amplitudes in the full SM
- recursive calculation employing Dyson-Schwinger-like recursion relations for computation of tensor coefficients
- complete set of SM counterterms and rational terms included
- complex mass scheme supported $\Rightarrow$ application to processes involving unstable particles possible
- efficient treatment of colour using colour-structures
- colour- and spin-correlated matrix elements for dipole subtraction

$\Rightarrow$ download combined Recola + Collier package from

`http://recola.hepforge.org`

Conclusions

- Collier= fortran library for numerical calculation of scalar and tensor integrals
- numerical stable results thanks to expansion methods for 3-,4-point integrals
- dimensional and mass regularization supported, as well as complex masses for unstable particles
- two independent implementations: Collier = Coli + DD
- the combination OpenLoops + Recola allows a fast and numerically stable evaluation of QCD and EW one-loop amplitudes to arbitrary SM processes
- Collier and Recola have been published recently and are available from hepforge