Decay of a bound muon

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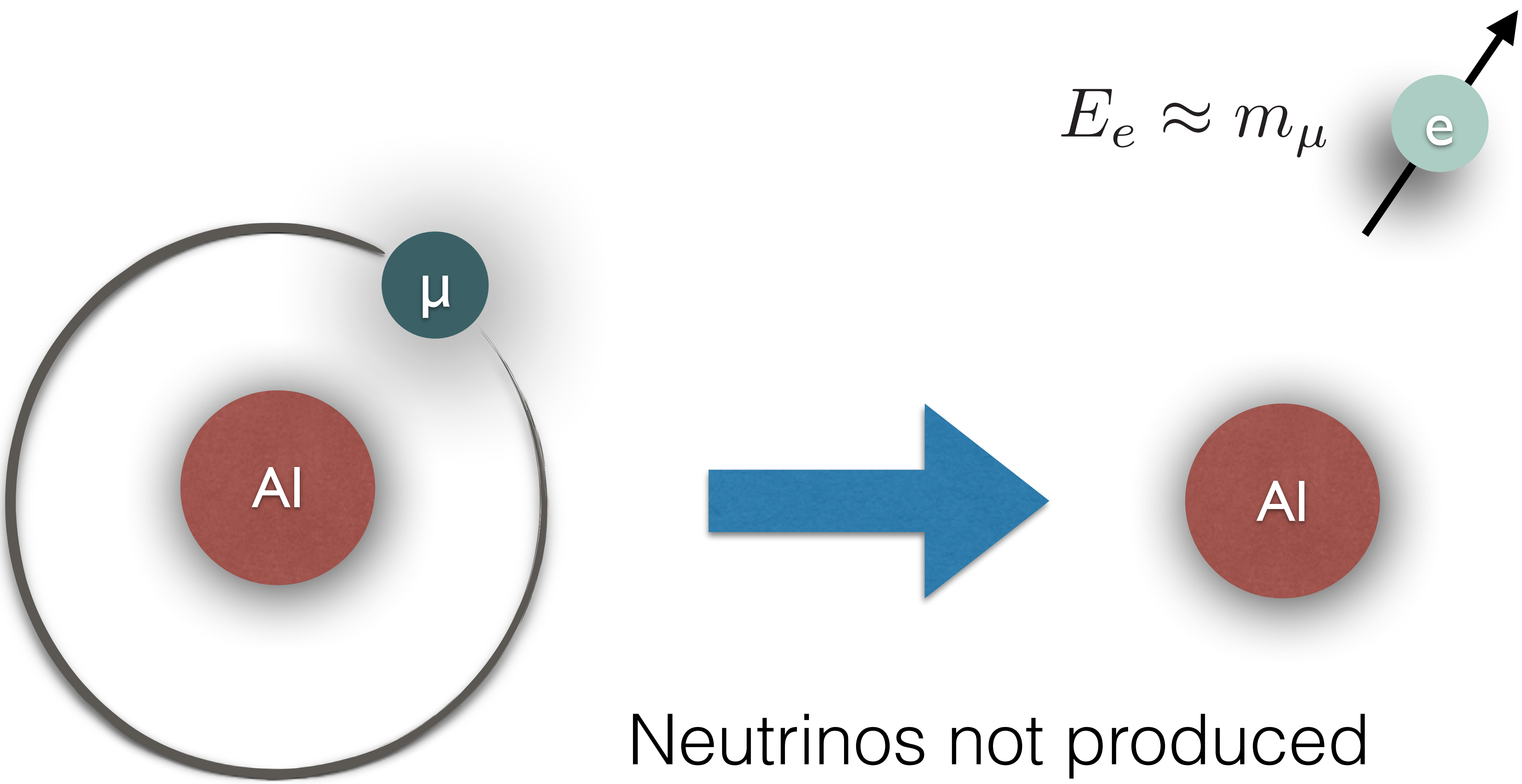
Outline

- Muonic atoms
- Muon electron coherent conversion
- Spectrum of the bound muons
  - Central region
  - Endpoint region
  - Radiative correction to the spectrum

General characteristic

* One of the electrons is replaced by a muon
* Muon orbit is much smaller than the electron orbit $\frac{r_\mu}{r_e} \sim \frac{m_e}{m_\mu}$
  * Much larger momentum
  * Muons are more sensitive to the structure of the nucleus $\frac{1}{m_\mu} < r_N$
* Muon can be captured by the nucleus or it can decay

Muon electron coherent conversion

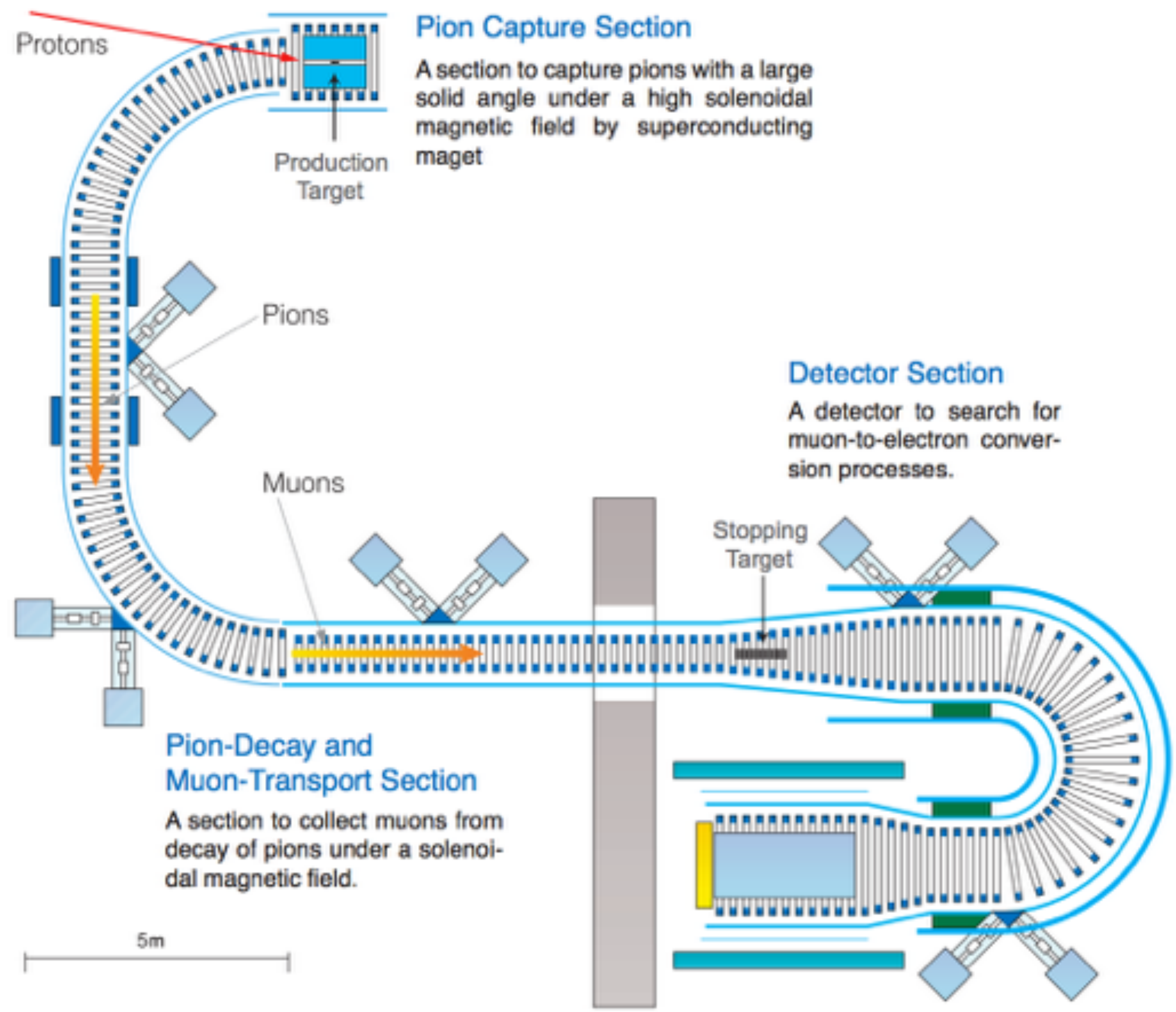


Neutrinos not produced

SM rate is negligible

Ongoing experimental efforts

- DeeMe in J-PARC MLF (2016/2017)
- COMET in J-PARC (Phase I 2018/2019)
- Mu2e in Fermilab (2021-)



COMET

Single event sensitivity

$\sim 10^{-14}$ DeeMe

$\sim 10^{-15}$ COMET, phase I

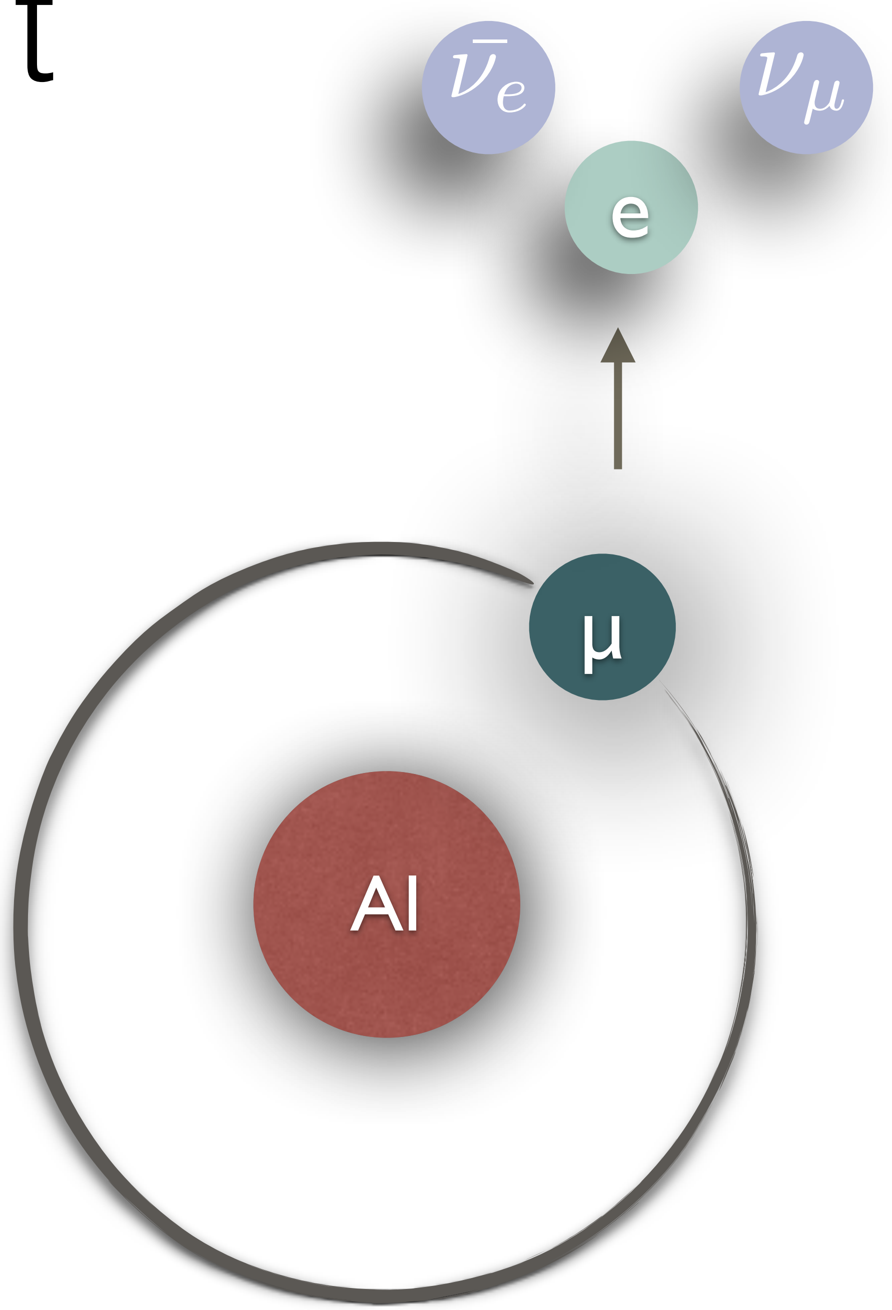
$\sim 10^{-17}$ COMET, phase II, Mu2e

Muon DIO

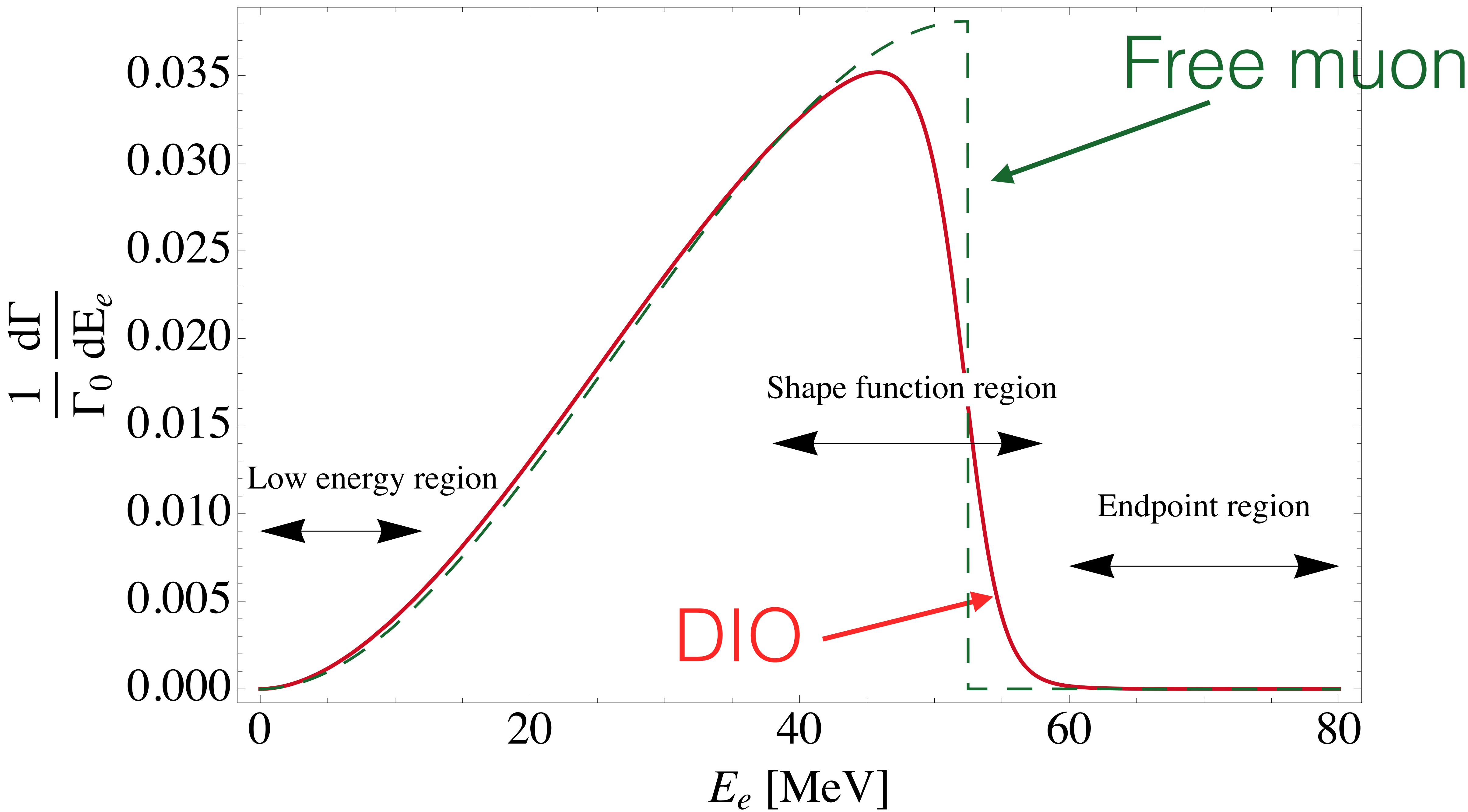
~39%

DIO — Decay In Orbit

- Muon DIO: standard muon decay into an electron and two neutrinos, with the muon and a nucleus forming a bound state
- For DIO momentum can be exchanged between the nucleus and both the muon and the electron

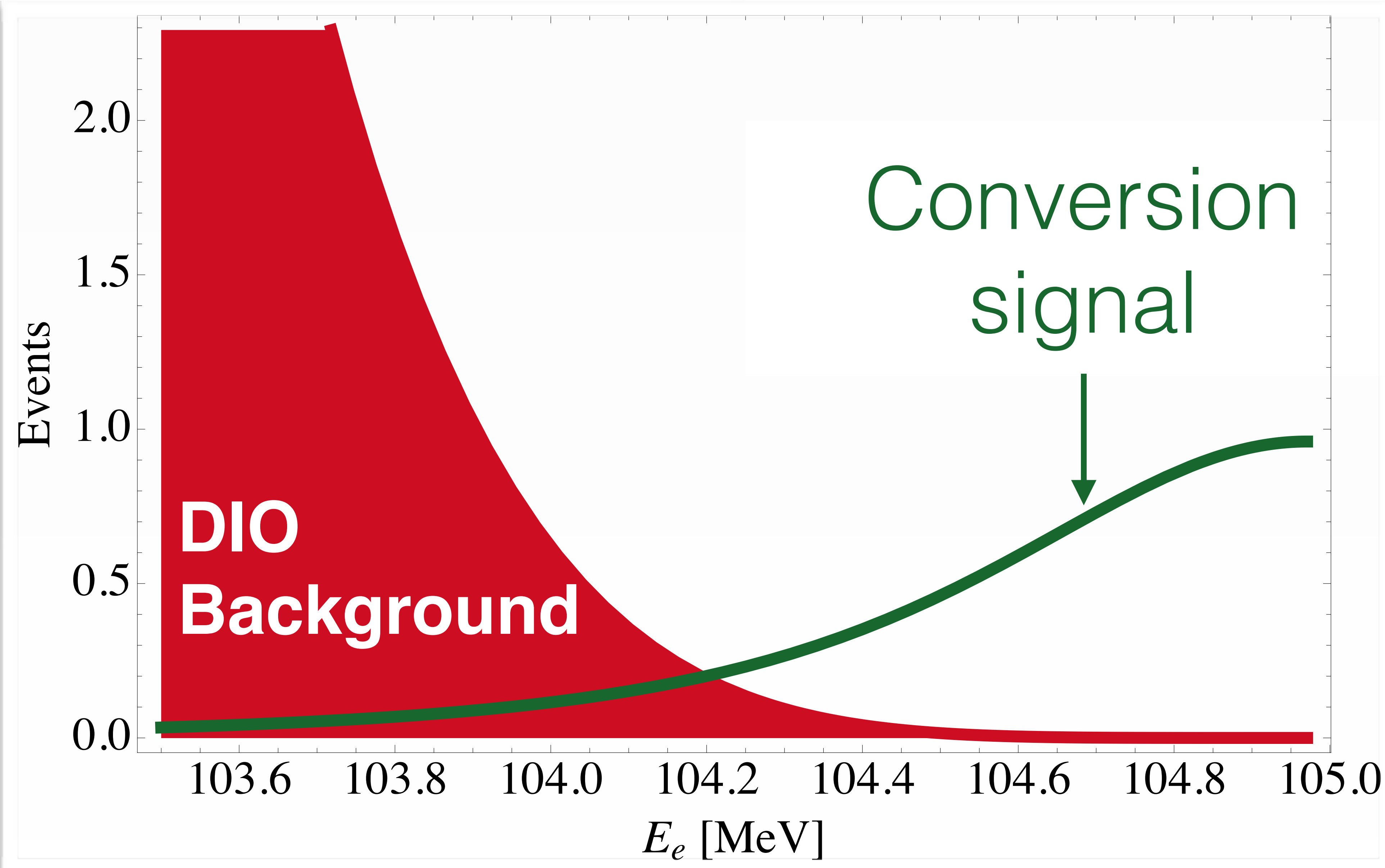


DIO Spectrum



DIO Spectrum

Endpoint region



Endpoint energy

$$E_{\max} = m_\mu + E_b + E_{\text{rec}}$$

$$E_b \approx -m_\mu \frac{(Z\alpha)^2}{2}$$

Binding energy

(+ higher orders)

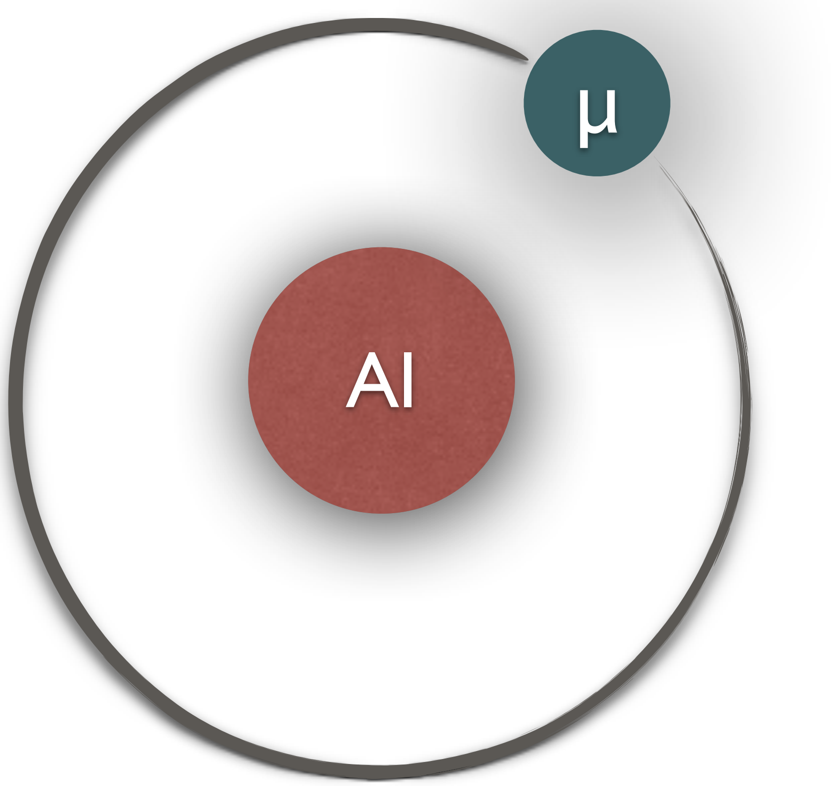
$$E_{rec} \approx -\frac{m_\mu^2}{2m_N}$$

Recoil energy

(kinetic energy of the nucleus)

Both corrections decrease the endpoint energy

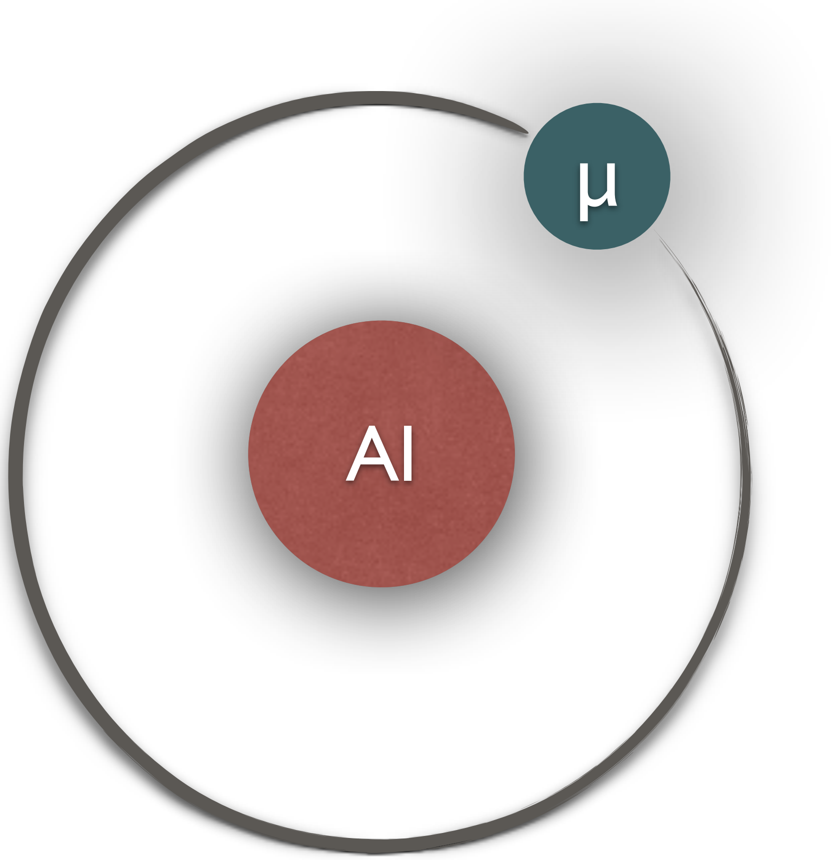
Characteristic scales of muonic atom



nucleus mass	M_{Al}
muon mass	m_μ
muon momentum	$Z\alpha m_\mu$
muon binding energy	$(Z\alpha)^2 m_\mu$
electron cloud	$\sim m_e$

$$M_{Al} \gg m_\mu \gg m_\mu Z\alpha \gg m_\mu (Z\alpha)^2$$

Characteristic scales of muonic atom



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muon mass m_μ

muon momentum $Z\alpha m_\mu$

muon binding energy $(Z\alpha)^2 m_\mu$

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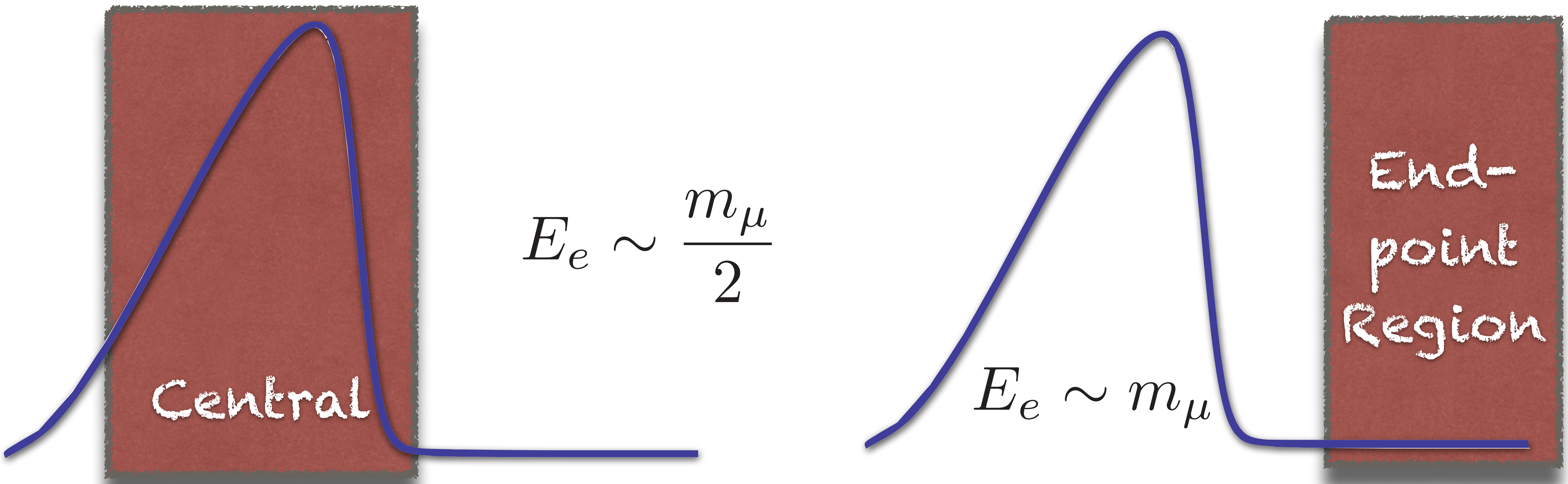
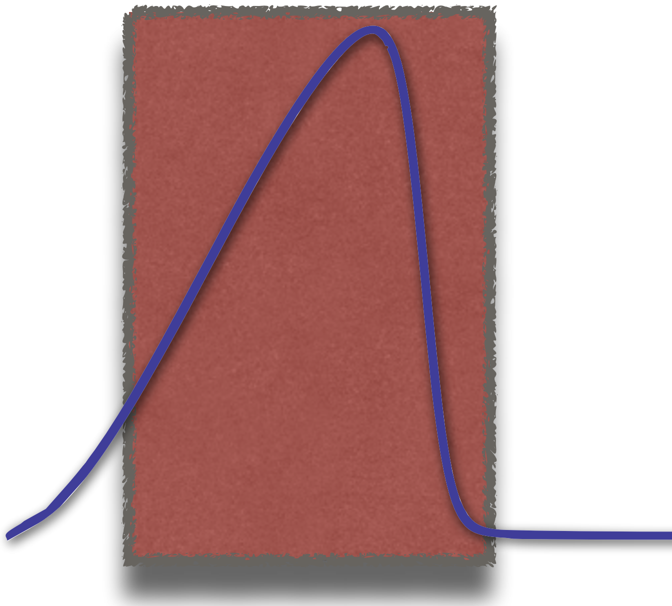
DIO spectrum regions



- Measured by the TWIST experiment in 2009
- Muon motion dominates

- Background for the conversion experiments
- Will be measured in conversion experiments

Central region



- Free muon decay is the Leading Order effect
- Binding effects are only a correction
- Typical momentum transfer between nucleus and muon is of the order of $m_\mu Z\alpha$
- Binding effects need to be re-summed; wave-function cannot be expanded

$$\psi(q) \sim \frac{1}{\left[q^2 + m_\mu^2 (Z\alpha)^2\right]^2}$$

Factorization

(shape function)

QCD case:
Neubert 1993; Mannel, Neubert 1994; Bigi, Shifman, Uraltsev, Vainshtein, 1994

Following QCD approach a factorization theorem can be derived

$$\frac{d\Gamma_{\mathrm{DIO}}}{dE_e} = \frac{d\Gamma_{\mathrm{free}}}{dE_e} \otimes S$$

Free muon spectrum
It is associated with the hard scale m_μ

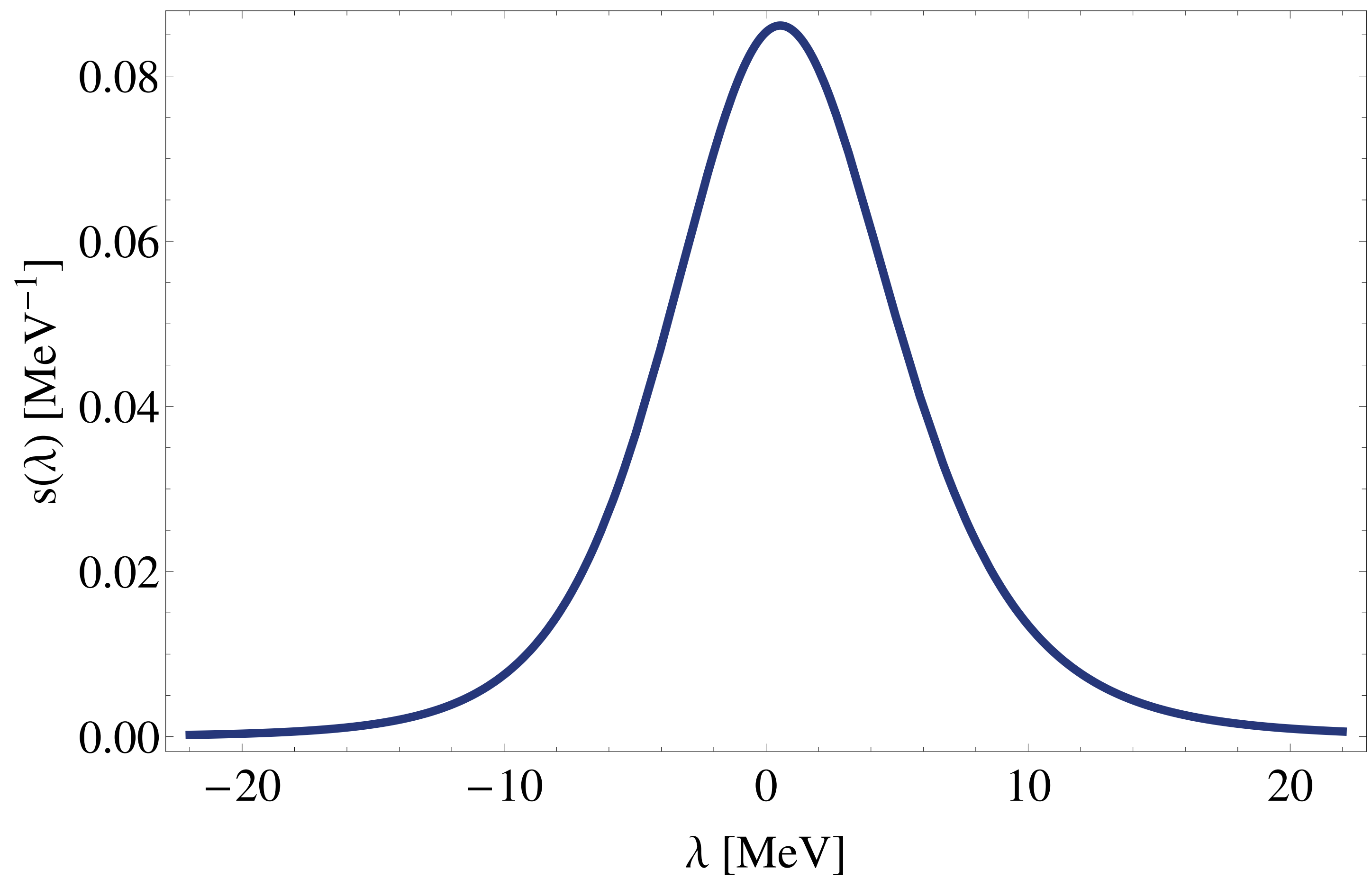
QED Shape function
It is associated with the soft scale $m_\mu Z\alpha$

Separation of scales $\quad m_\mu Z\alpha \ll m_\mu$

Shape function

For a point-like nucleus, the LO shape function can be calculated analytically

$$S(\lambda) = \frac{8m_\mu^5 Z^5 \alpha^5}{3\pi \left[\lambda^2 + m_\mu^2 Z^2 \alpha^2\right]^3}.$$



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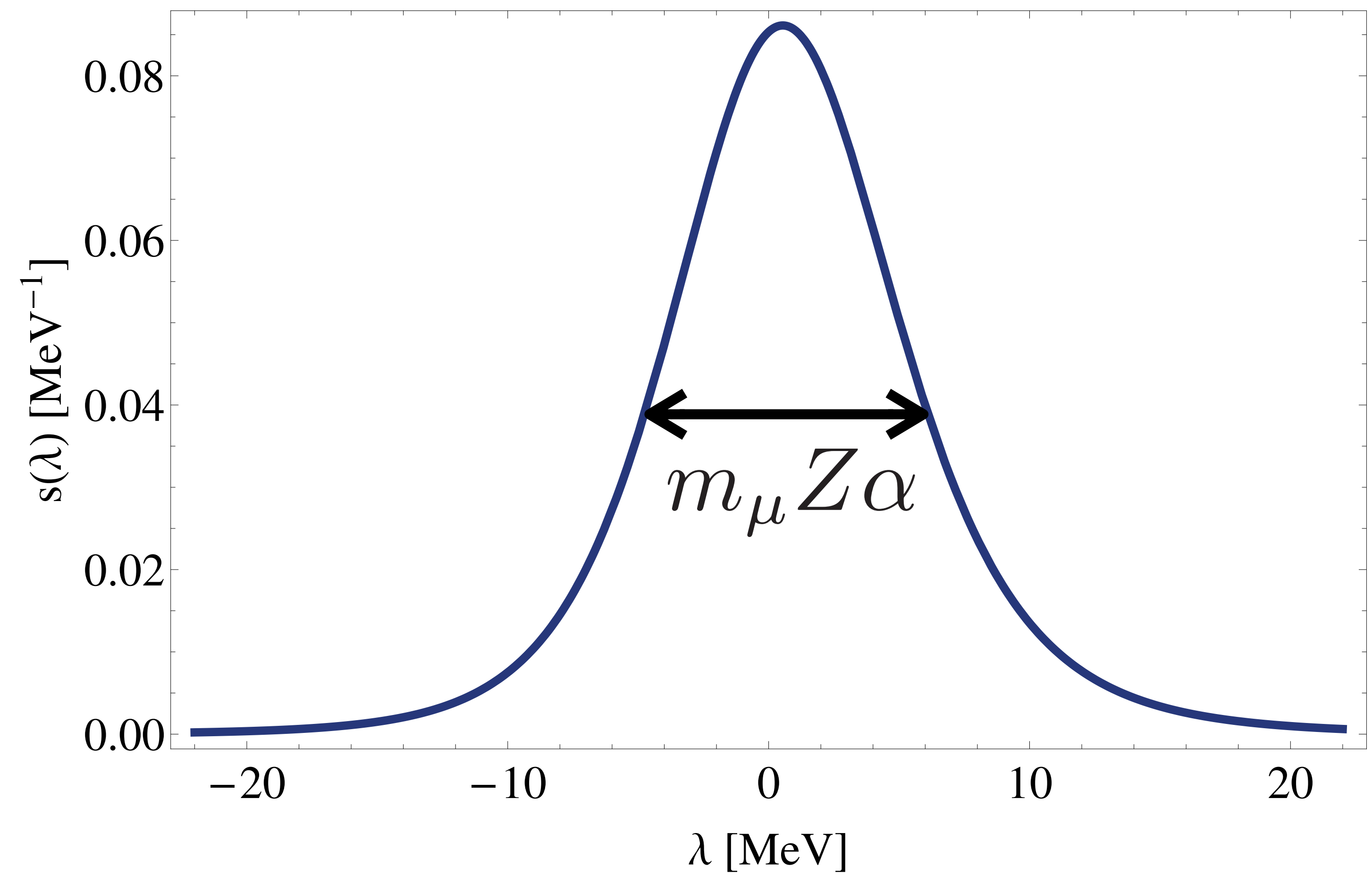

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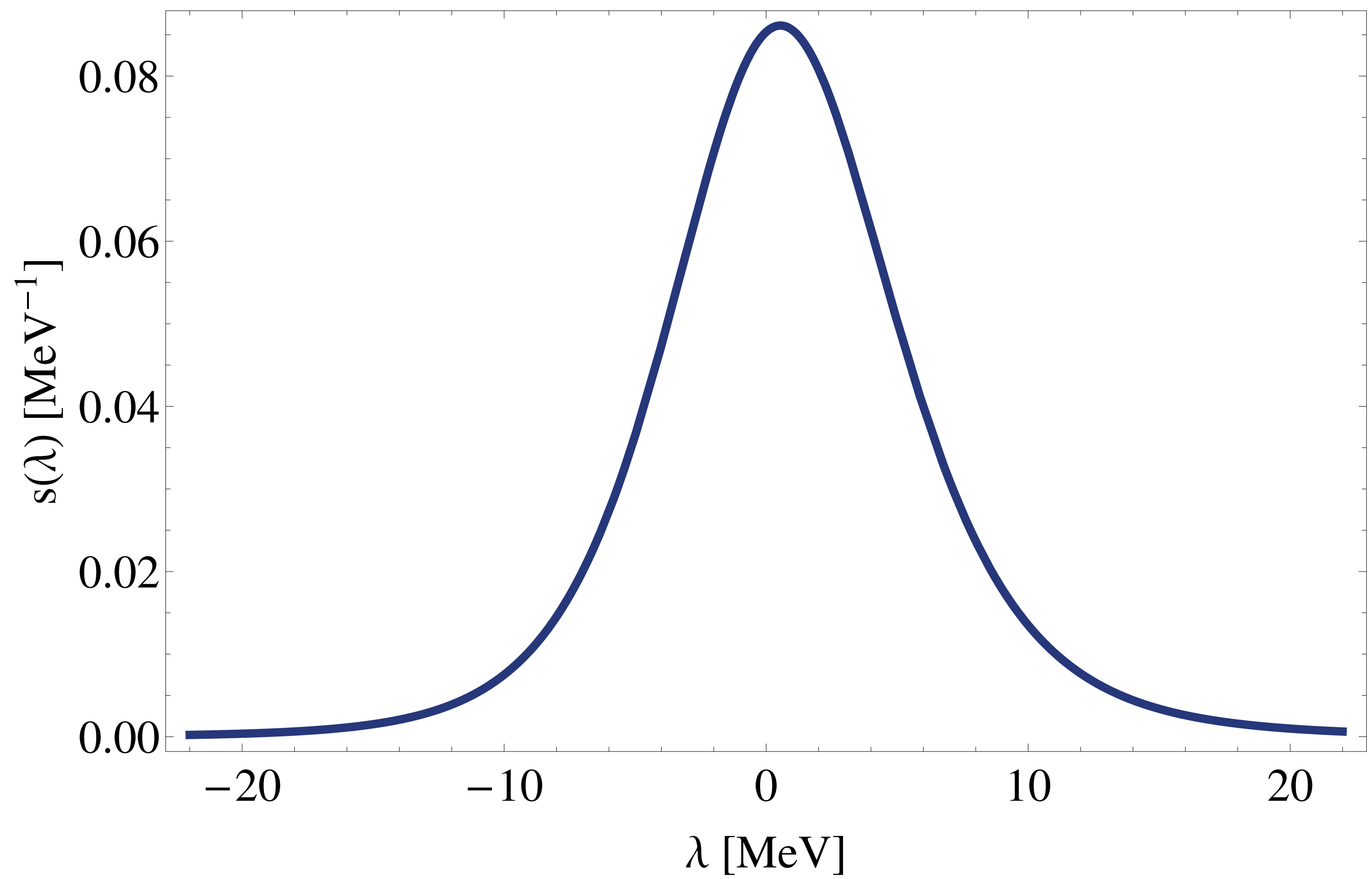

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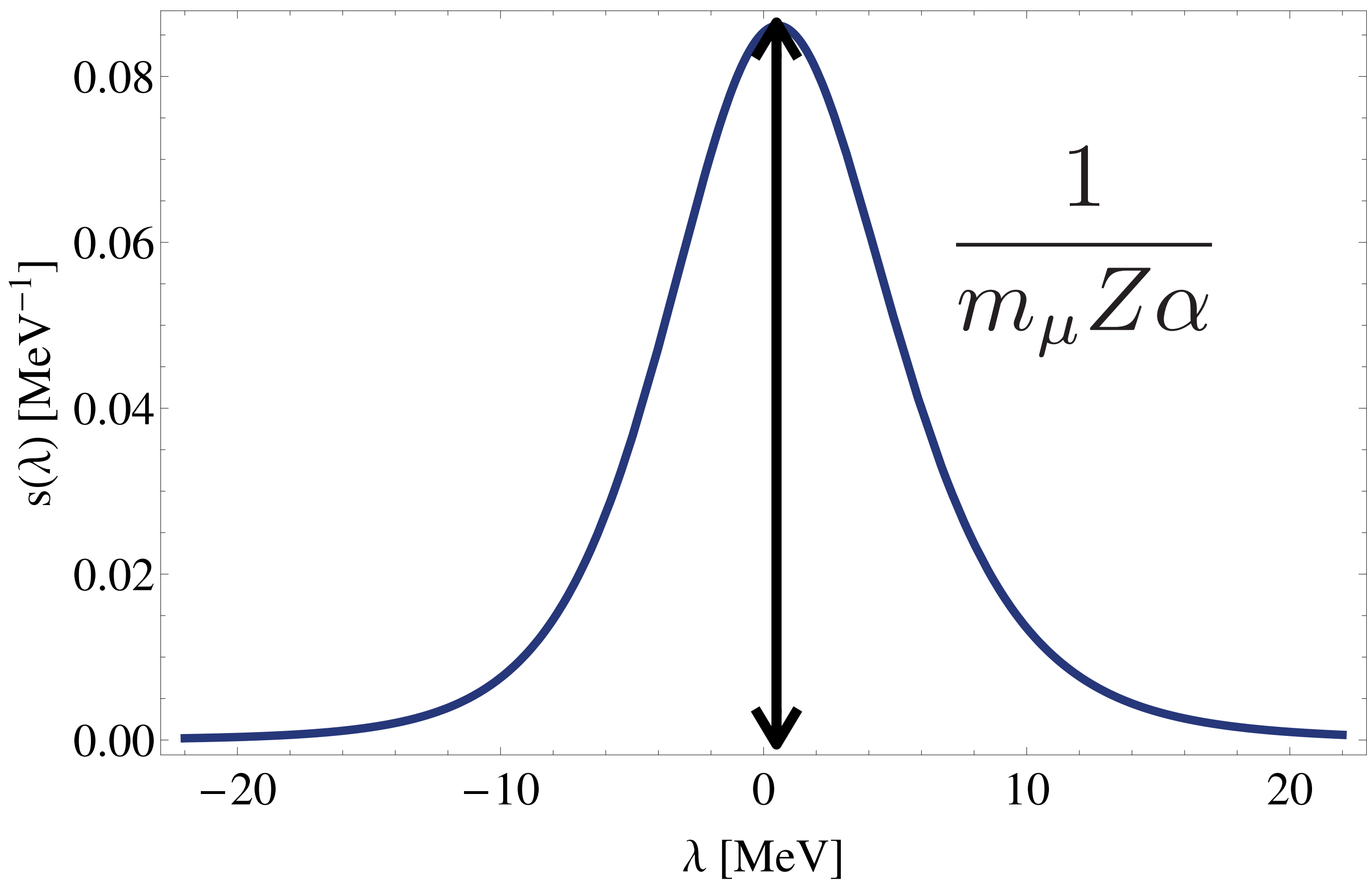

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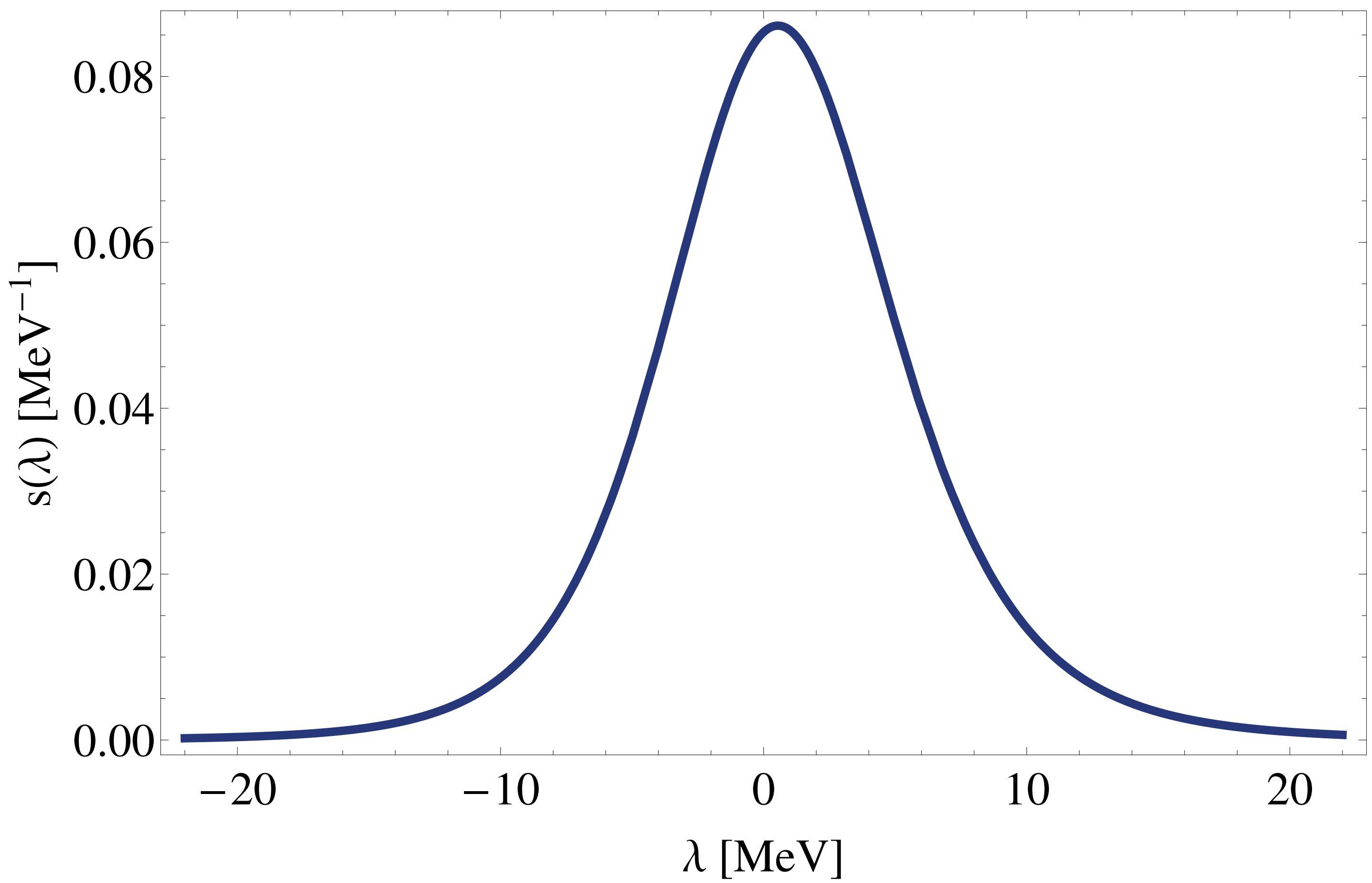

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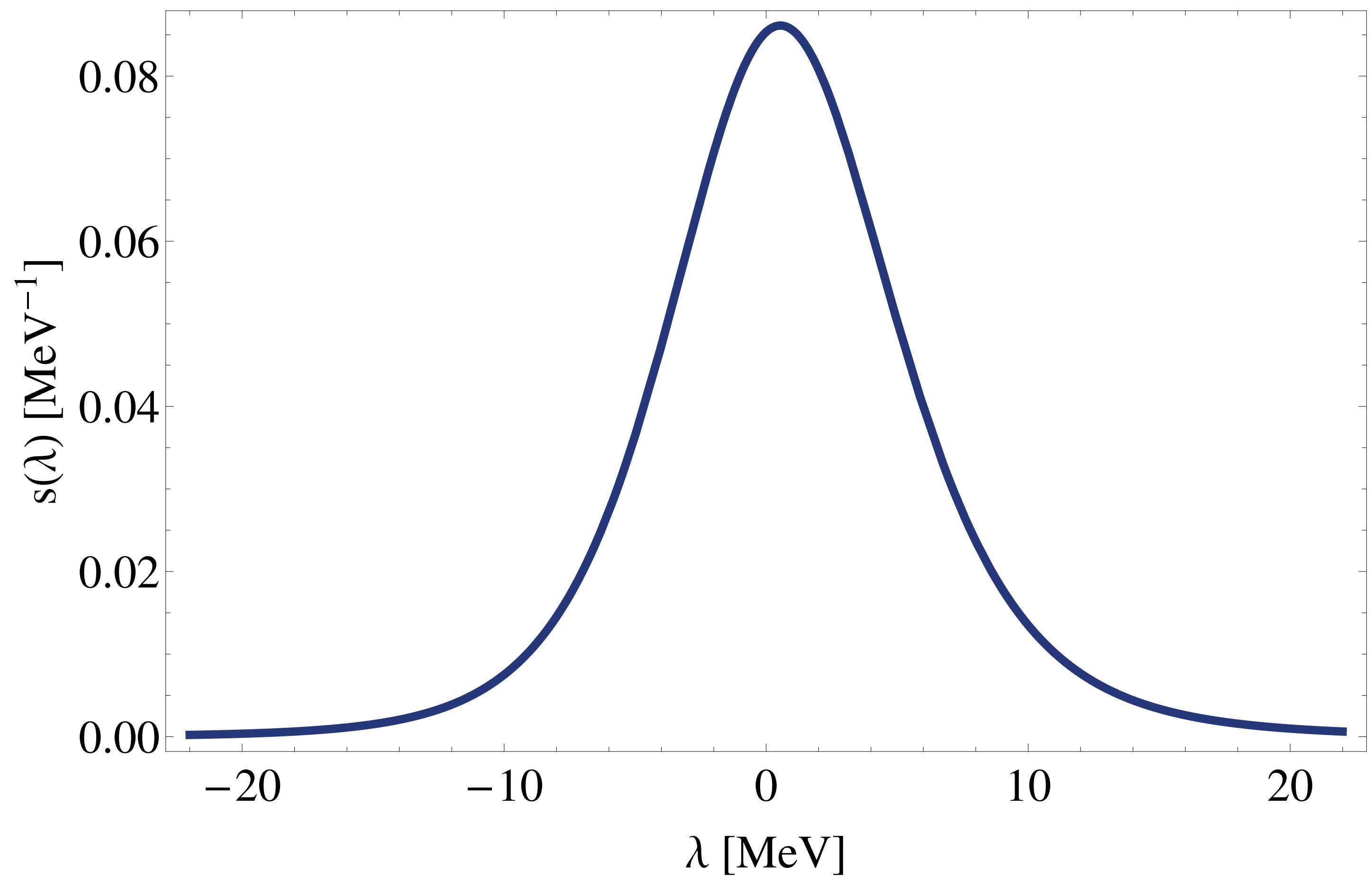
Scaling $\lambda \sim m_\mu Z\alpha$

First moment is zero

$$\int d\lambda \lambda S(\lambda) = 0$$

$$\Downarrow$$

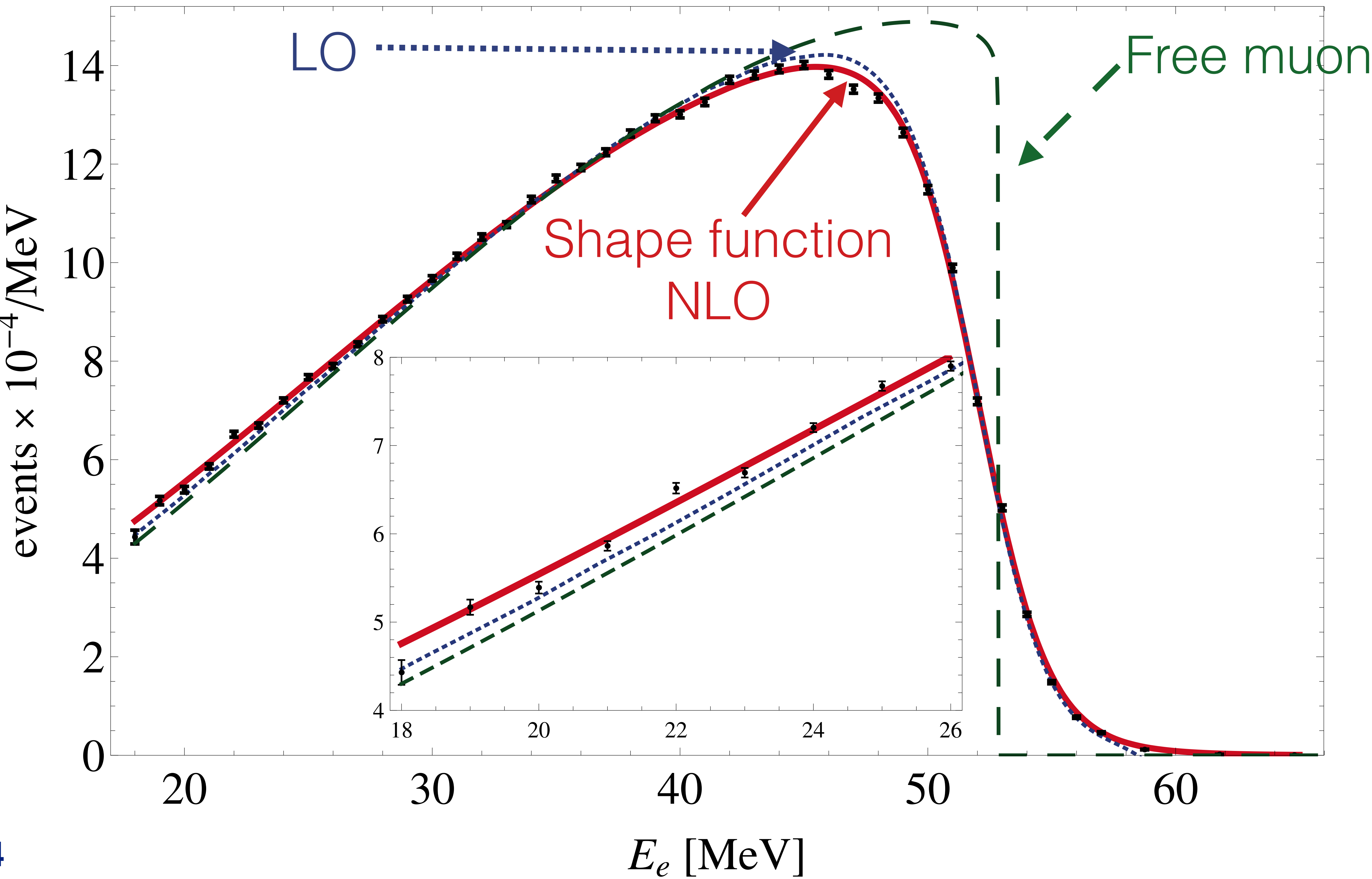
$$\Gamma_{\rm DIO} = \Gamma_0 + \mathcal{O}(Z^2\alpha^2)$$



Results for real atom

Czarnecki, Dowling, Garcia i Tormo, Marciano, Szafron; 2014

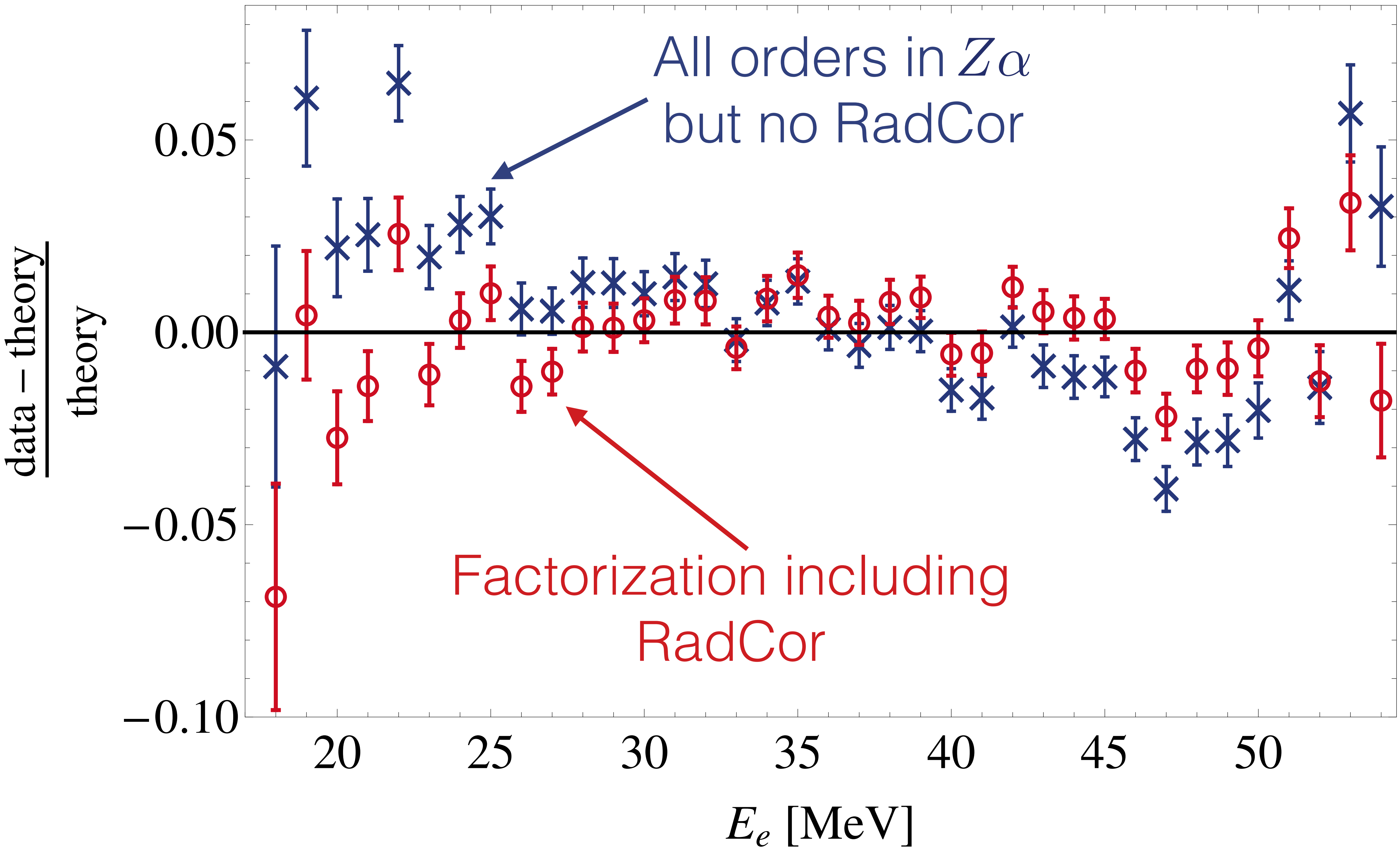
and their relation to the TWIST data



Leading Corrections

and their relation to the TWIST data

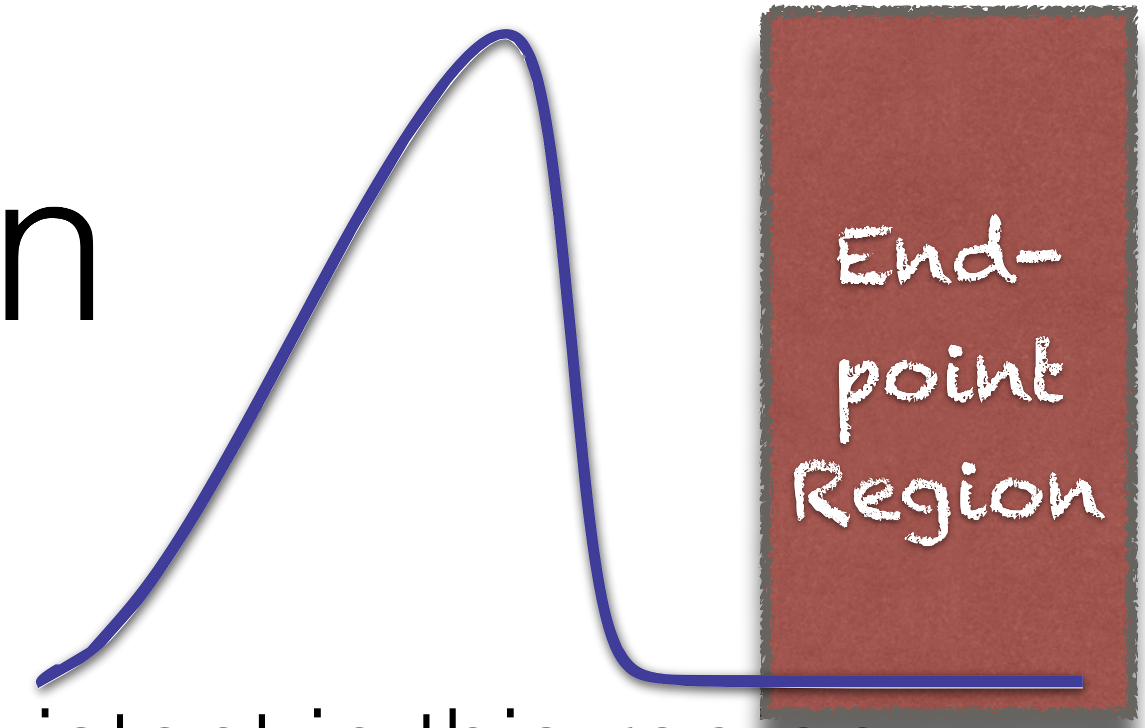
Czarnecki, Dowling,
Garcia i Tormo,
Marciano, Szafron 2014



Endpoint Region

(conversion background)

$E_e \sim m_\mu$



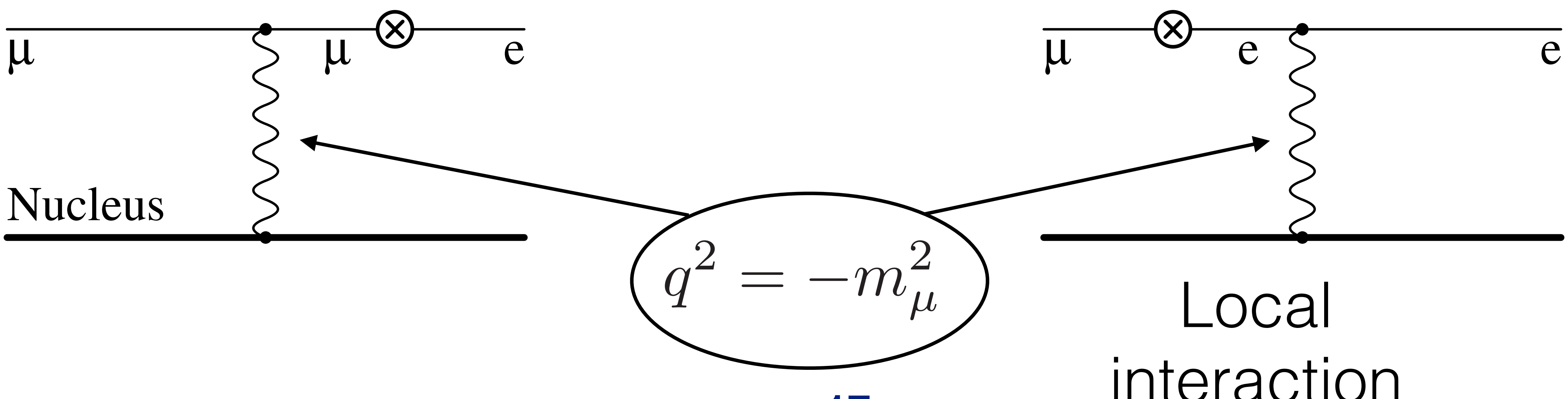
- Free muon spectrum is nonexistent in this region
- Binding effects constitute the LO terms
- Typical momentum transfer between the nucleus and the muon is large ($q^2 \sim m_\mu^2$)
- Both wave functions and propagators can be expanded in powers of $Z\alpha$

Endpoint expansion

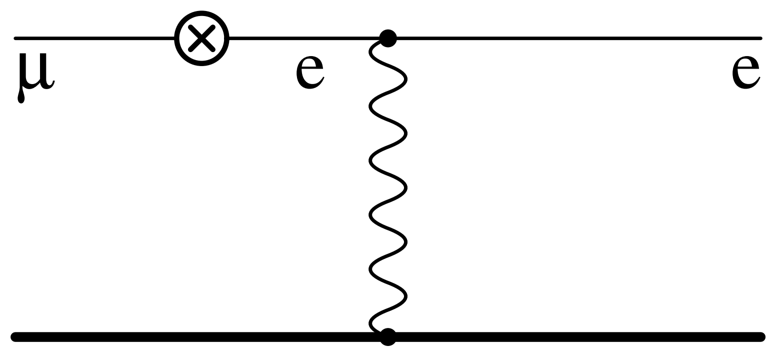
Near the endpoint, the dominant contribution comes from the exchange of hard virtual photons. Szafron, Czarnecki; 2015

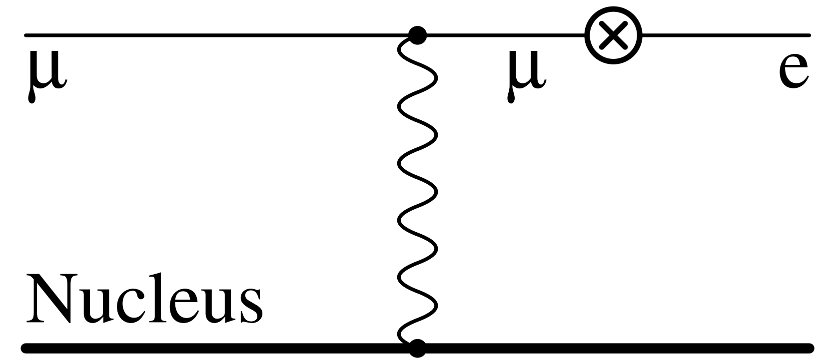
$$\frac{m_\mu}{\Gamma_{Free}} \frac{d\Gamma}{dE_e} \approx \frac{1024}{5\pi} (Z\alpha)^5 \left(\frac{\Delta}{m_\mu} \right)^5$$

$$\Delta = E_{max} - E_e$$



Binding suppression

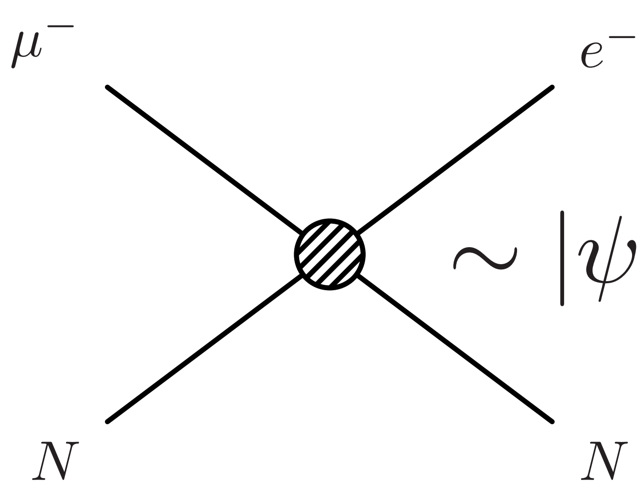




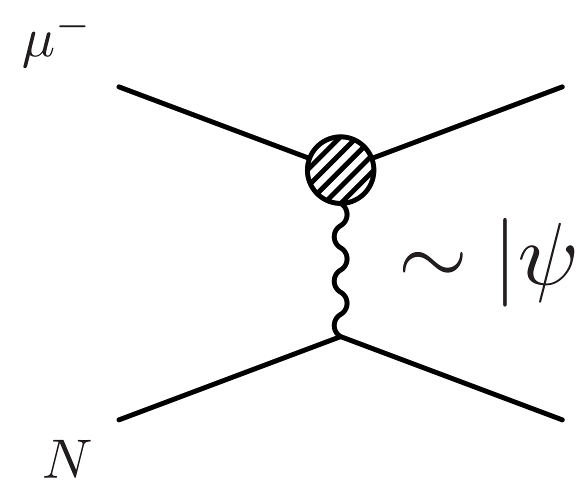
$$|\mathcal{M}|^2 \sim |\psi(0)|^2 \times \left|V(m_\mu^2)\right|^2 \sim (Z\alpha)^3 \times (Z\alpha)^2$$

$$|\psi(0)|^2 \sim (Z\alpha)^3$$

$$V(k^2) \sim -\frac{Z\alpha}{k^2}$$



$$\sim |\psi(0)|^2$$



$$\sim |\psi(0)|^2 \times \left|V(m_\mu^2)\right|^2$$

Endpoint Radiative Correction

- Soft vacuum polarization correction to the muon wave-function at the origin

$$\Psi(0) \to \Psi(0)\left(1 + \frac{\alpha}{\pi}\delta_0\right)$$

$$\delta_0 = 3.27$$

- Hard vacuum polarization

$$\delta_{VP} = \frac{\alpha}{\pi}\left(\frac{4}{3}\ln\frac{m_\mu}{m_e} - \frac{10}{9} + 0.12\right)$$

- Soft photon emission

$$\delta_S = \frac{\alpha}{\pi}\left(2\ln\frac{2m_\mu}{m_e} - 2\right)$$

- Hard correction

$$\delta_H = \frac{\alpha}{\pi}\left(6.31 - \frac{26}{15}\ln\frac{m_\mu}{m_e}\right)$$

$$\frac{1}{\Gamma_{\mathrm{free}}}\frac{d\Gamma}{dE_e} = \Delta^5 \frac{1024}{5\pi m_\mu^6}(Z\alpha)^5\left(\frac{\Delta}{m_\mu}\right)^{\delta_S}(1 + \delta_0 + \delta_{VP} + \delta_H)$$

Interpolating between regions

- We also need to know the spectrum for intermediate electron energies
- We have identified the leading corrections and it is possible to calculate them!

1. Real radiation can be approximated by taking into account collinear photon emission
2. Vacuum polarization can be included when we solve the Dirac equation numerically

Vacuum polarization

$$V(r) = -\frac{Z\alpha}{r} + Z\alpha\frac{\alpha}{\pi}V_U(r, m_e)$$

Electron loop generates long distance potential and this leads to large logarithmic corrections

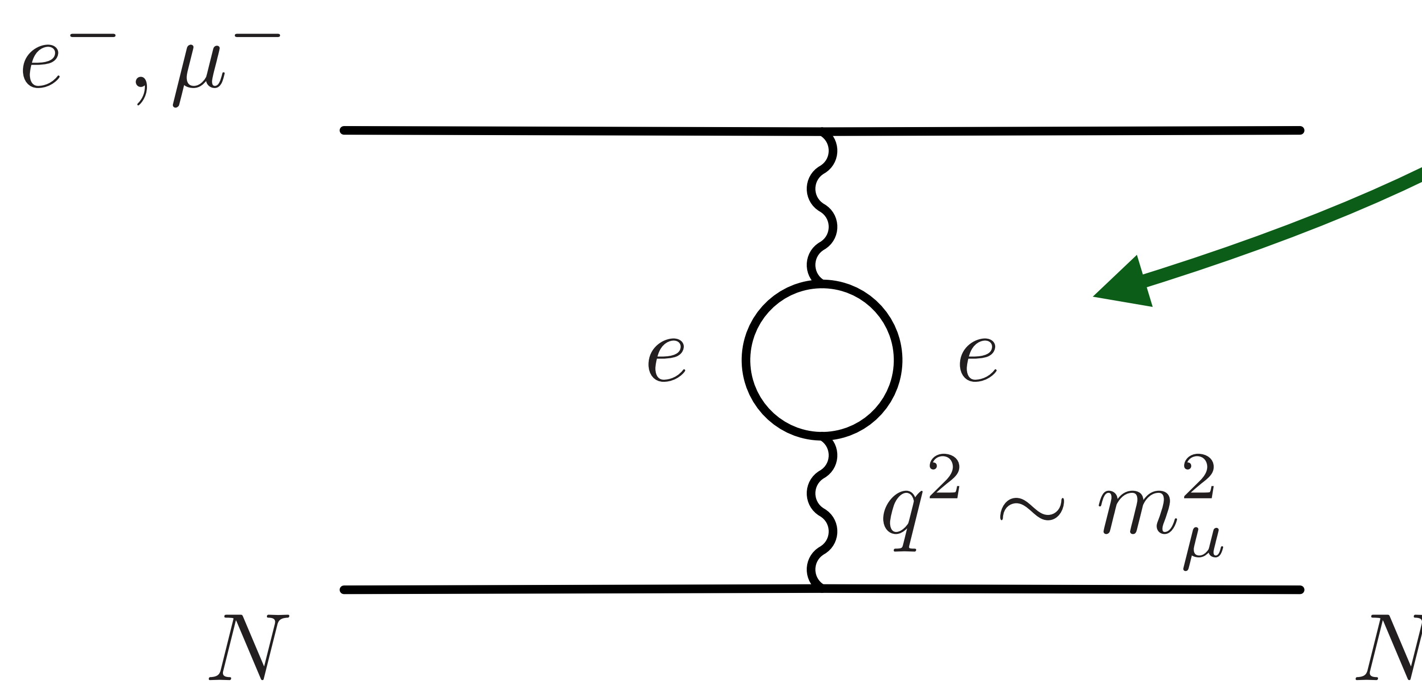
$$r \sim \frac{1}{m_e} \gg \frac{1}{m_\mu Z\alpha}$$

Correction range

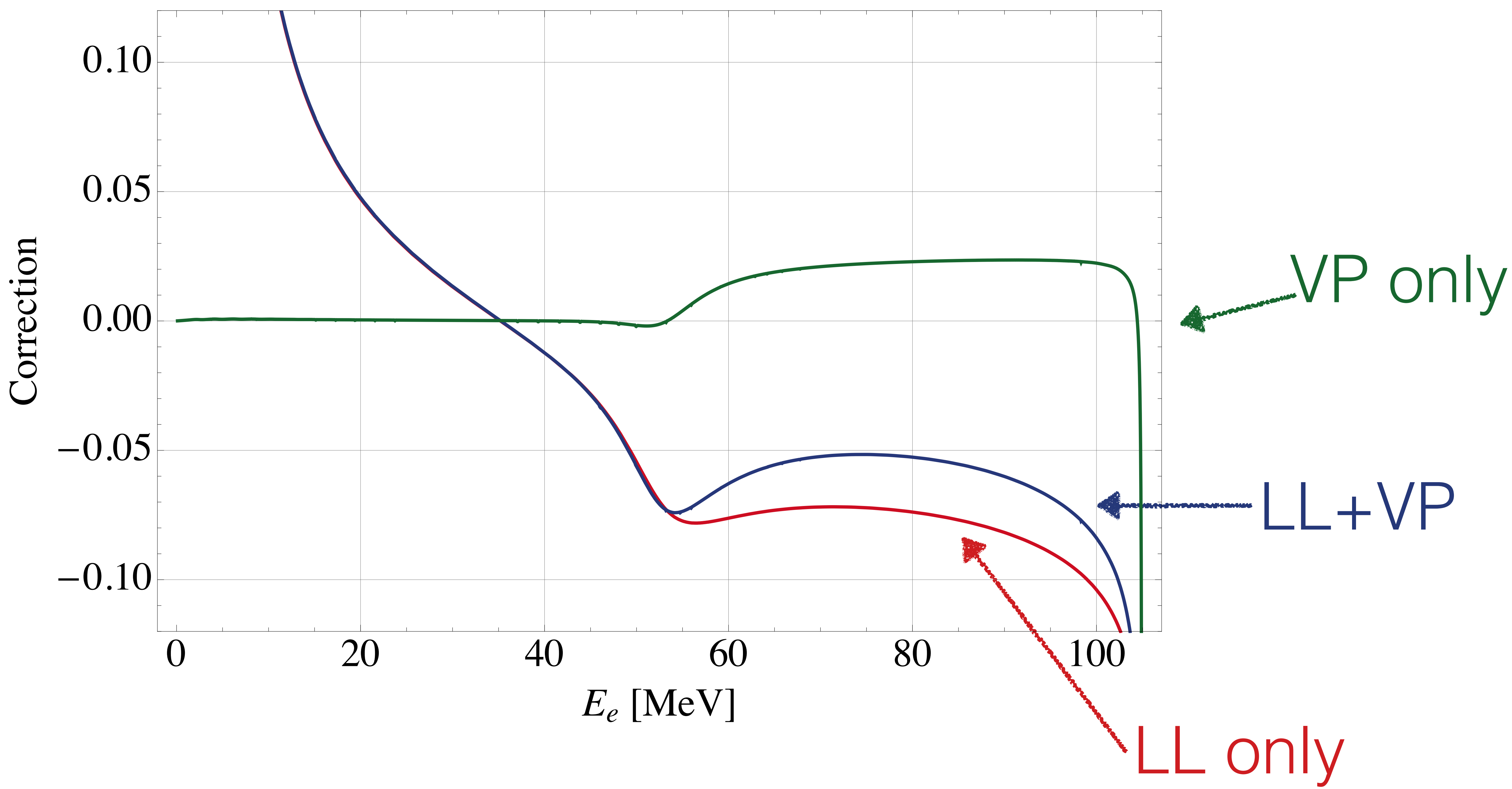
Atom size

$$\ln\frac{Z\alpha m_\mu}{m_e}$$

$$\ln\frac{m_\mu}{m_e}$$



Correction to the DIO spectrum



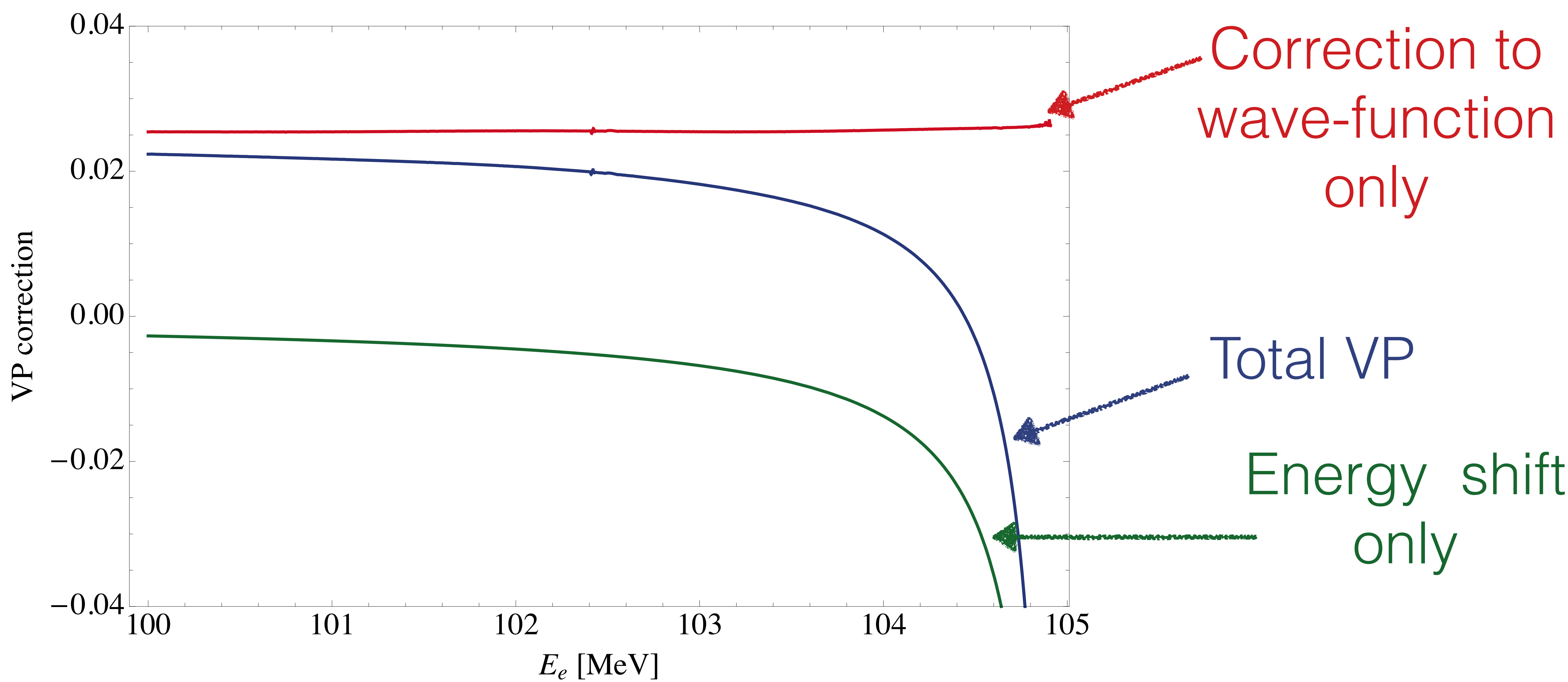
Vacuum polarization correction

$$E_b \to E_b + \frac{\alpha}{\pi}\delta E_b$$

Correction to the endpoint energy

$$\psi(p) \to \psi(p) + \frac{\alpha}{\pi}\delta\psi(p)$$

Corrections to the wave-functions

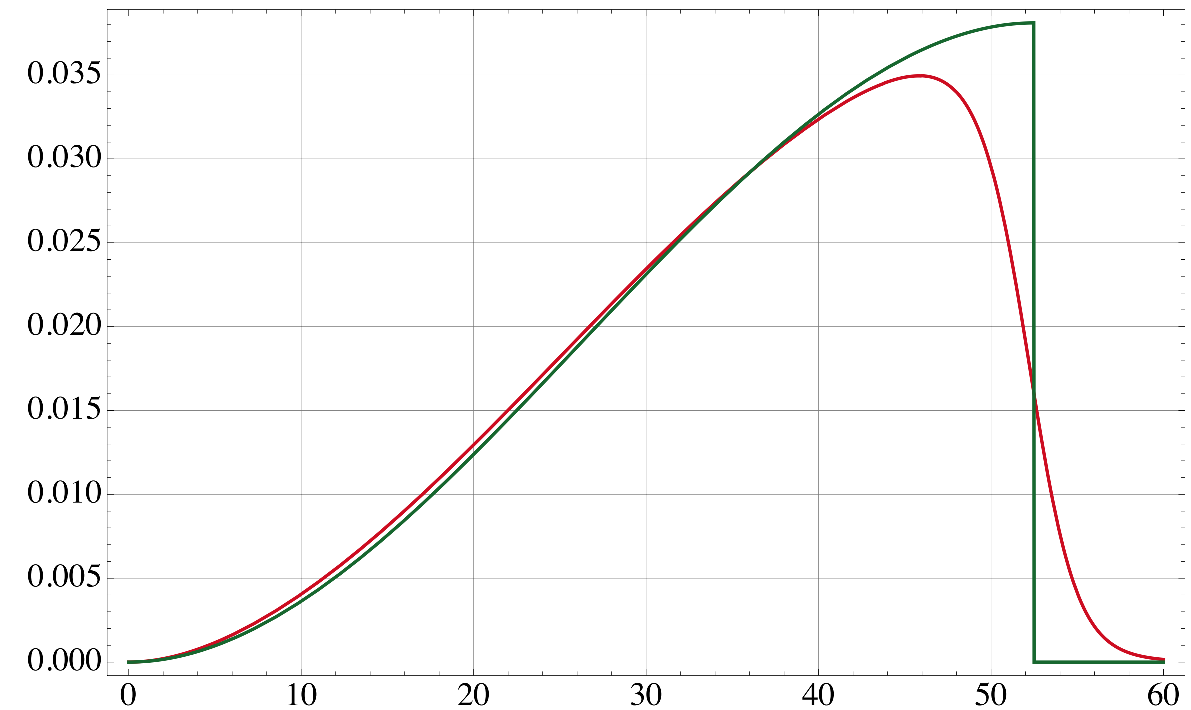


Summary

- We can correctly reproduce TWIST measurement
- Vacuum polarization gives large corrections to the DIO spectrum
- Endpoint spectrum is very sensitive to the binding energy
- Large finite nucleus size effects

Backup

Free / Bound



Ratio
Free/Bound