Four-loop Electronic contributions to the anomalous magnetic moment of muon

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in collaboration with

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Overview

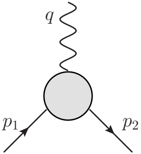
1. Motivation

2. Calculation

3. Results

Anomalous magnetic moment

$$V_{int} = -\vec{\mu}\cdot\vec{B}, \quad \vec{\mu} = g\left(\frac{e}{2m}\right)\vec{s}.$$



$$= \quad -ie\ \bar{\psi}(p_2)\left(\gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right)\psi(p_1)$$

$$F_1(0) = 1 \quad F_2(0) = \frac{g-2}{2} \equiv a_l$$

$$a_\mu^{exp} = 116592089(63)\times 10^{-11}$$ [PDG]

$$a_\mu^{th} = 116591803(49)\times 10^{-11}$$

Note that $a_\mu^{4\ell}(e) \sim 386\times 10^{-11}$ [Aoyama, Hayakawa, Kinoshita, Nio 2012]

Experiment: Fermilab E989 and J-PARC

Calculation

- `QGRAF`: generate Feynman diagrams [Nogueira]
- `q2e`: bridge between QGRAF and expansion [Harlander, Seidensticker, Steinhauser]
- `asy/in hause`: asymptotic expansion with mass hierarchy [Pak, Smirnov; Jantzen, Smirnov, Smirnov]
- `FORM`: calculate diagrams [Vermaseren]
- `FIRE/Crusher`: reduction to master integrals [Smirnov]/[Marquard, Seidel]
- `FIESTA`: MIs evaluations [Smirnov]

Leptonic contributions

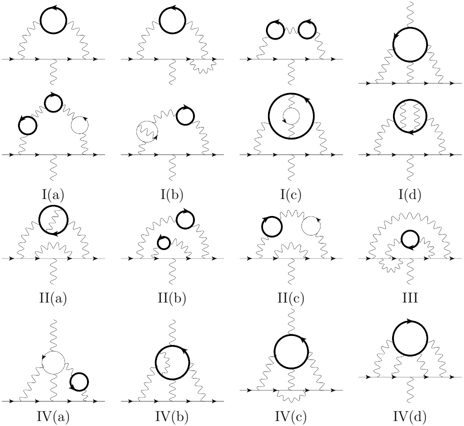
$$a_\mu = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

2ℓ [Elend 1966]

3ℓ [Laporta, Remiddi 1993; Laporta 1993; ...]

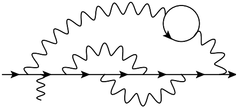
4ℓ [Kinoshita, Nio 2005; Lee et al 2013]

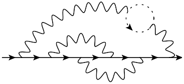
5ℓ [Aoyama, Hayakawa, Kinoshita, Nio 2012]



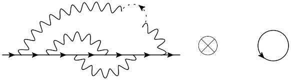
asymptotic expansion $m_\tau^2 \gg m_\mu^2 \gg m_e^2$

Graphical example

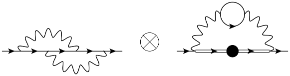




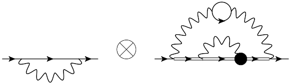
all hard



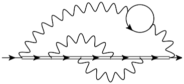
three hard



two hard



one hard

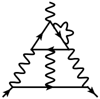


all soft

Linear propagator appears: $\frac{1}{(\ell+q)^2-m_\mu^2} = \frac{1}{2\ell\cdot q}\sum_{n=0}^{\infty}\left(\frac{-\ell^2}{2\ell\cdot q}\right)^n$

- all the 3-loop integrals are known analytically
  $\longrightarrow$ all analytical CTs
- 4-loop OS-shell integrals:
  $\simeq 70 = 40_{\texttt{ana/high prec.}} + 30_{\texttt{num}}$ [Marquard,Smirnov,Smirnov,Steinhauser 2015]
- 4-loop linear integrals:
  $\simeq 70 = 20_{\texttt{ana/high prec.}} + 50_{\texttt{num}}$
- Two different sets of auxiliary propagators used for each topology. Several different basis of MIs to get stable numerical results.

One example for IV(b)



$A_2^{(8),\mathrm{IV(b)}}$ $x = m_e/m_\mu \simeq 1/206.7682843$

$$
\begin{aligned}
&=27.395 \pm 0.014 + (4.93482 \pm 0.00003)\ell_x \\
&\quad + x\left[-0.81 \pm 1.22 + 59.0235\ell_x\right] \\
&\quad + x^2\left[142.5 \pm 7.6 + 40.6546\ell_x + 20.5582\ell_x^2 - 9.6167\ell_x^3 + 0.8333\ell_x^4\right]
\end{aligned}
$$

$$
\begin{aligned}
=&27.395 \pm 0.014 + (-26.3105 \pm 0.0002) \\
&+ [-0.0039 \pm 0.0059 - 1.5219] \\
&+ [0.003334 \pm 0.0001769 - 0.005070 + 0.01367 + 0.03409 + 0.01575] \\
=&[1.084 \pm 0.014] + [-1.5259 \pm 0.0059] + [0.06177 \pm 0.00018] \\
=&-0.380 \pm 0.016
\end{aligned}
$$

τ-loop contributions

- the hardest region to be tadpoles which known analytically
- no 4-loop OS or linear integrals

$$
\begin{aligned}
A_3^{(8)}\left(\frac{m_\mu}{m_e},\frac{m_\mu}{m_\tau}\right) = & \frac{m_\mu^2}{m_\tau^2}\Bigg(\frac{1}{135}\ln^2\frac{m_e^2}{m_\mu^2}+\frac{89}{810}\ln^2\frac{m_\mu^2}{m_\tau^2}+\ln\frac{m_\mu^2}{m_\tau^2}\left(\frac{22493}{291600}-\frac{3\zeta_3}{2}\right)\\
& +\ln\frac{m_e^2}{m_\mu^2}\left(-\frac{23}{270}\ln\frac{m_\mu^2}{m_\tau^2}-\frac{3\zeta_3}{2}+\frac{2\pi^2}{45}+\frac{74597}{97200}\right)\\
& +\frac{17\zeta_3}{135}+\frac{2\pi^4}{75}+\frac{193\pi^2}{810}-\frac{984587}{486000}-\frac{8}{135}\pi^2\log(2)\Bigg)\\
& +\frac{m_e m_\mu}{m_\tau^2}\left(\frac{4\pi^2}{15}\ln\frac{m_\mu^2}{m_\tau^2}-\frac{821\pi^2}{900}\right)+\ldots.
\end{aligned}
$$

Results I

$A_2^{(8)}(m_\mu/m_e)$	our work	literature	
I(a0)	7.223076	7.223077 ± 0.000029	[Kinoshita et al. 2004]
		7.223076	[Laporta 1993]
I(a1)	0.494072	0.494075 ± 0.000006	[Kinoshita et al. 2004]
		0.494072	[Laporta 1993]
I(a2)	0.027988	0.027988 ± 0.000001	[Kinoshita et al. 2004]
		0.027988	[Laporta 1993]
I(a)	7.745136	7.74547 ± 0.00042	[Aoyama et al. 2012]
I(bc0)	8.56876 ± 0.00001	8.56874 ± 0.00005	[Kinoshita et al. 2004]
I(bc1)	0.1411 ± 0.0060	0.141184 ± 0.000003	[Kinoshita et al. 2004]
I(bc2)	0.4956 ± 0.0004	0.49565 ± 0.00001	[Kinoshita et al. 2004]
I(bc)	9.2054 ± 0.0060	9.20632 ± 0.00071	[Aoyama et al. 2012]
I(d)	− 0.2303 ± 0.0024	− 0.22982 ± 0.00037	[Aoyama et al. 2012]
		− 0.230362 ± 0.000005	[Baikov et al. 1995]
II(a)	− 2.77885	− 2.77888 ± 0.00038	[Aoyama et al. 2012]
		− 2.77885	[Laporta 1993]
II(bc0)	−12.212631	−12.21247 ± 0.00045	[Kinoshita et al. 2004]
II(bc1)	− 1.683165 ± 0.000013	− 1.68319 ± 0.00014	[Kinoshita et al. 2004]
II(bc)	−13.895796 ± 0.000013	−13.89457 ± 0.00088	[Aoyama et al. 2012]
III	10.800 ± 0.022	10.7934 ± 0.0027	[Aoyama et al. 2012]
IV(a0)	116.76 ± 0.02	116.759183 ± 0.000292	[Kinoshita et al. 2004]
		111.1 ± 8.1	[Calmet et al. 1975]
		117.4 ± 0.5	[Chlouber et al. 1975]
IV(a1)	2.69 ± 0.14	2.697443 ± 0.000142	[Kinoshita et al. 2004]
IV(a2)	4.33 ± 0.17	4.328885 ± 0.000293	[Kinoshita et al. 2004]
IV(a)	123.78 ± 0.22	123.78551 ± 0.00044	[Aoyama et al. 2012]
IV(b)	− 0.38 ± 0.08	− 0.4170 ± 0.0037	[Aoyama et al. 2012]
IV(c)	2.94 ± 0.30	2.9072 ± 0.0044	[Aoyama et al. 2012]
IV(d)	− 4.32 ± 0.30	− 4.43243 ± 0.00058	[Aoyama et al. 2012]

Uncertainties

- The uncertainties in the second column are multiplied by a factor five.
- $A_2^{(8)} = 126.34(38) + 6.53(30) = 132.86(48)$
- $0.5 \times (\alpha/\pi)^4 \approx 1.5 \times 10^{-11}$
- $a_\mu^{exp} = 116592089(63) \times 10^{-11}$ [PDG]

Results II

	$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau)$	
group	our work	[Aoyama et al. 2012]
I(a)	0.00320905(1)	0.003209(0)
I(b) + I(c)	0.00442289(2)	0.004422(0)
II(b) + II(c)	−0.02865753(1)	−0.028650(2)
IV(a)	0.08374757(9)	0.083739(36)

The discrepancy comes from the mass ratio of m_μ/m_τ.

Summary

- $A_\mu^{(8)} = A_\mu^{(8)}|_{\mathrm{univ.}} + 132.86(48) + 0.0424941(53) + 0.062722(10)$
- agreement with Kinoshita's results.
- systematic improvement possible.

Thanks for your attention!