(B)SM Higgs Boson Production via Gluon Fusion

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Introduction

Gluon Fusion (Fixed-order calculations)
  LO and NLO QCD
  NNLO and \( N^3LO \) QCD / NLO elw. corrections
  Dim-6 Operators

Soft and collinear gluon resummation in Gluon Fusion
  Previous and current work
  Mass effects and collinear effects
  Numerical implementation
  Numerical Results

Conclusions
Higgs boson production

- At the LHC there are mainly 4 relevant production mechanism for a single SM Higgs $h$ and pseudoscalar Higgs $A$
Gluon Fusion: Fixed order calculations

- **Leading order (LO)**
  - Due to large Yukawa coupling and large gluon luminosities gluon fusion dominant production mechanism in the SM [Georgi et. al. (1978)]

- **NLO-calculations (next-to-leading order)**
  - Effective theory in the limit of a heavy top quark [Dawson (1991), Spira et al. (1991)]
  - Massive calculation [Spira et al. (1993,1995), Harlander, Kant (2005), Anastasiou et al. (2007), Aglietti et al. (2007)]

  - Increase of the hadronic cross section by about $50 - 90\% \Rightarrow K$-factor $K_\infty = \frac{\sigma^{NLO}}{\sigma^{LO}}\mid_{M_t \to \infty}$ huge!
  - Effective calculation is in accordance with the full massive calculation of $K$-factor within $\mathcal{O}(5\%)$ for $M_H = 125$ GeV
  - NLO cross section can be expressed in good approximation by $K_\infty$-factor rescaled by massive Born term

- **Effective NNLO calculation in the limit** $M_t^2 \gg M_H^2$ [Harlander, Kilgore (2001), Anastasiou, Melnikov (2002), Ravindran et al. (2003)]

  - Further increase of the cross section by about $\approx 30\%$
  - Scale dependence at NNLO reduces by a factor of 2 with respect to NLO
NNLO and $N^3$LO QCD corrections / NLO electroweak corrections

- Massive NNLO calculation only partly available [Harlander et al., Steinhauser et al. (2009)] in asymptotic mass expansion. Mass effects below $\mathcal{O}(1\%)$ in K-factor.
- State-of-the-art calculation at $N^3$LO [Anastasiou et al. (2014, 2015, 2016), Li et al. (2014)]
  - Soft + virtual approximation or threshold expansion (singular terms in the limit $z \to \infty$)
  - Terms originating from collinear region $\sim \ln^m(1-z)$, $0 \leq m \leq 5$
  - Quite recently: Full three loop result
  - $N^3$LO results lead to a further increase of the cross section by $+3.2\%$ for $\mu_R = \mu_F = M_H/2$ in the effective theory approach,
- NLO electroweak corrections $\mathcal{O}(\alpha_s^2 \alpha)$ in the completely factorized scheme $\sigma_{\text{tot}} = \sigma_{\text{QCD}}(1 + \delta_{\text{elw}})$, [Degrassi et al. (2004), Aglietti et al. (2006), Actis et al. (2008, 2009)]
- approximate mixed QCD and elw NNLO corrections $\mathcal{O}(\alpha_s^3 \alpha)$ [Anastasiou et al. (2009)]
Dim-6 Operators

- Higher-dimension operators of weakly interacting theories up to certain scale $\Lambda$ generate deviation of the effective Higgs coupling to gluons

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{\pi} \left\{ \frac{c_t}{12} (1 + \delta) + c_g \right\} G^{a,\mu\nu} G_{\mu\nu}^a \frac{h}{\nu}$$

- Novel coupling $c_g$ does not receive QCD corrections but develops a RGE as of the trace anomaly $\Theta_{\mu}^\mu = [1 + \gamma_m(\alpha_s)] m_t \bar{t}t + \frac{\beta(\alpha_s)}{2\alpha_s} G^{a,\mu\nu} G_{\mu\nu}^a \frac{h}{\nu}$ [Adler et al. (1979)]

$$c_g(\mu^2) = c_g(\mu_0^2) \frac{\beta_0 + \beta_1 \frac{\alpha_s(\mu^2)}{\pi} + \beta_2 \left( \frac{\alpha_s(\mu^2)}{\pi} \right)^2}{\beta_0 + \beta_1 \frac{\alpha_s(\mu_0^2)}{\pi} + \beta_2 \left( \frac{\alpha_s(\mu_0^2)}{\pi} \right)^2}$$

- Results into a rescaling of the $t, b, c$ Yukawa couplings and effective $Hgg$ coupling [S., Spira (2016), Liebler et al. (2016), Anastasiou et al. (2016)]
Threshold-resummation, part 1

- Partonic cross sections contain singular plus distributions

\[ D_i = \left[ \ln^i(1 - z) \right] / (1 - z) \]

at every perturbative order

- These logarithmically enhanced terms spoil the convergence of the perturbative expansion in the kinematical region \( z \to 1 \)

- Physical explanation: Near partonic threshold the phase space only permits the emission of soft gluons.

- First observation: Leading Plus distributions show a recurrent pattern \([Parisi (1980)]\) ⇒ Possibility to resum these large contributions
Threshold-resummation, part 2

- Transformation into Laplace- or Mellin-space $N$.

\[ \sigma_N(m_h^2) = \int_0^1 d\tau \tau^{N-1} \sigma(s, m_h^2) \]

- Limit $z \to 1$ corresponds to limit $N \to \infty$

- $D_i \to c_i \ln^{i+1} N + \mathcal{O}(\ln^i N)$

- Renormalization group method:
  - Factorization of divergent hard scattering cross section in the soft region into a soft, soft-collinear and hard part
  - Solution of the RG equations leads to the Sudakov exponentiation [Sterman et al. (1986, 1997), Catani et al. (1989)]

\[
\hat{\sigma}_{gg \to h} = \alpha_s^2(\mu_R) C_{gg} \left( \frac{\alpha_s^2(\mu_R)}{\mu_R^2}, \frac{m_h^2}{\mu_R^2}, \frac{m_h^2}{\mu_F^2} \right) \\
\times \exp \left[ G_h \left( \alpha_s^2(\mu_R), \ln N, \frac{m_h^2}{\mu_R^2}, \frac{m_h^2}{\mu_F^2} \right) \right]
\]
Threshold resummation in inclusive Higgs production via Gluon-Fusion

- Conventional QCD resummation
  - Threshold resummation at NLO+NLL $m_t^2 \gg M_H^2$ [Krämer, Laenen, Spira (1997)]
  - Soft-gluon resummation at NNLO+NNLL in the limit of a heavy top-quark [Catani et al. (2003)], [de Florian, Grazzini (2009)]
  - Inclusion of finite mass effects in the resummation [de Florian, Grazzini (2012)]
  - Resummation large-$x$ + small-$x$ + approximate N$^3$LO [Ball et al., Bonvini et al. (2014)]
  - Approximate N$^3$LO [deFlorian et al. (2014)]
  - Approx. N$^3$LO [Catani et al. (2014)]
  - N$^3$LO+N$^3$LL [Bonvini et al. (2015,2016), Anastasiou et al. (2016)]

- SCET
  - SCET resummation at NNLO+NNLL [Ahrens et al. (2009)]
  - SCET resummation at N$^3$LO+N$^3$LL [Anastasiou et al. (2016)]
Inclusion of mass effects into resummed kernel

- Soft+virtual $gg$-channel contains mass dependent NLO contribution $c(\tau_q)$ [de Florian, Grazzini (2012)]

$$ C_{gg}^{(1)}(\tau_q^\phi) = \pi^2 + c_\phi(\tau_q^\phi) + \frac{33 - 2N_F}{6} \log \frac{\mu_R^2}{\mu_F^2} + 6\gamma_E^2 + \frac{\zeta_2}{6} - 6\gamma_E \ln \frac{M_H^2}{\mu_F^2} $$

$$ c_H(\tau_t^H) \xrightarrow{\tau_t^H \to \infty} \frac{11}{2}, \quad \frac{\tau_t^\phi}{m_t^2} = \frac{4m_t^2}{m_\phi^2} $$

- Real $gg$-, $gq$- and $qq$-channels have the same limit $z \to 1$ as for $m_t \to \infty$ relative to Born term (universal factorization) $\Rightarrow$ mass effects can be included in resummation

- Since no massive NNLO calculation available mass effects at NNLL unknown
Collinear Logarithms

- Universal collinear effects $\ln^k N/N \sim \ln^k (1 - z)$ are numerically relevant.
- At NLL they exponentiate together with the constant terms $\leftarrow$ conjecture [Krämer, Laenen, Spira (1997)]
- Alternative: Inclusion into constant terms $C_{gg}^{(1)} \rightarrow C_{gg}^{(1)} + 2C_A \frac{\ln N}{N}$ [Catani et al. (2001,2003)]
- Alternative approach [S., Spira (2015)]

$$C_{gg}^{(1)} \rightarrow C_{gg}^{(1)} + 2C_A \frac{\tilde{L}}{N}, \quad C_{gg}^{(2)} \rightarrow C_{gg}^{(2)} + (48 - N_F) \frac{\tilde{L}^2}{N} \text{ with } \tilde{L} = \ln \frac{Ne^{\gamma_E} \mu_F}{M_\Phi}$$

- Correctly predicts leading logarithms $((\alpha_s/\pi)^{2n-1} \ln^n N/N)$ as well as subleading logarithms $\ln^2 N/N$ at NNLO and $\ln^4 N/N$ at $N^3$LO.

- Next-to-eikonal approach [Laenen, Magnea, Stavenga (2008,2015)]
- Physical kernel evolution resums the next-to-soft terms by altering the soft function [Moch, Vogt (2014)]
Minimal prescription

- Mellin inversion

\[
\sigma^{(\text{res})} = \sigma^{(0)} \int_{C_{MP}+i\infty}^{C_{MP}-i\infty} \frac{dN}{2\pi i} \left( \frac{M_H^2}{s} \right)^{-N+1} f_g/h_1, N(\mu_F^2) f_g/h_2 N(\mu_F^2) \\
\times \hat{g}_{gg \rightarrow \phi, N}(\alpha_s(\mu_R^2), M_H^2/\mu_R^2; M_H^2/\mu_F^2)
\]

- Minimal Prescription = choosing carefully the integration contour in order to avoid non-perturbative poles

- Necessity for \( N \)-space PDF's \( \Rightarrow \) Fitting linear combinations of \( x^\alpha(1-x)^\beta \) to \( x \)-space PDF's for different \( \mu_F \) and transforming results to \( N \)-space [de Florian, Vogelsang]

- Alternative: QCD-PEGASUS. Takes PDF's at input scale \( \mu_{F,0} \) in the 9-parameter form and evolves them with DGLAP-equations in Mellin-space up to higher scales
Usage of $x$-space PDF’s

- Parton derivatives / Fake parton luminosities [Kulesza et al. (2002)]
  - Multiplication of the cross section by one:

  $$
  \sigma^{(\text{res})} = \sigma^{(0)} \int_{c_{\text{MP}}-i\infty}^{c_{\text{MP}}+i\infty} \frac{dN}{2\pi i} \rho^{-N+1} \times f_{g/h_1, N(\mu_F^2)} (N-1)^2 f_{g/h_2, N(\mu_F^2)} (N-1)^2 \hat{\sigma}_{gg \rightarrow \phi, N/(N-1)^4} \nabla \nabla \nabla \nabla$$

  $$= \sigma^{(0)} \int_{\rho}^{1} \frac{dz}{z} \int_{\rho/z}^{1} \frac{dy}{y} G^{(2)}(y, \mu_F^2)G^{(2)}(\frac{\rho}{y \cdot z}, \mu_F^2) \times \frac{1}{2\pi i} \int_{c_{\text{MP}}-i\infty}^{c_{\text{MP}}+i\infty} dz z^{-N} \hat{\sigma}_{gg \rightarrow \phi, N/(N-1)^4}$$

- Second derivative

  $$G^{(2)}(x, \mu_F^2) = \frac{d}{dx} \left\{ x \frac{d}{dx} \left( x \cdot g(x, \mu_F^2) \right) \right\}$$

  stabilizes numerical integration over the phase space. Good agreement with QCD-PEGASUS.
Matching by including mass effects

- Improved matching by only incorporating top mass effects in the resummed kernel

  - Large double logarithms (DL) $\ln^2 \frac{M_H^2}{m_q^2}$ in the case of bottom and charm quarks $\Rightarrow$ Numerically relevant, no soft gluon dominance
  - For MSSM Higgs DL’s of bottom quarks scale with $\tan \beta$ $\Rightarrow$ Resummation only relevant for moderate $\tan \beta \lesssim 10 - 15$

\[
\sigma_{tt}^{(NNLO+N^3LL)} = \left[ \sigma_{tt}^{(0)} K_{tt,\infty}^{(NNLO)} \right]^{x-space} + \left[ \sigma_{tt}^{(0)} K_{tt,\infty}^{(N3LL)} - \sigma_{tt}^{(0)} K_{tt,\infty}^{(NNLO)} \right]^{N-space} \\
+ \left[ \sigma_{t+b+c}^{(NLO)} - \sigma_{tt}^{(0)} K_{tt,\infty}^{(NLO)} \right]^{x-space} \\
+ \left[ \sigma_{tt}^{(0)} K_{tt}^{(NLL)} - \sigma_{tt}^{(0)} K_{tt}^{(NLO)} \right]^{N-space} \\
- \left[ \sigma_{tt}^{(0)} K_{tt,\infty}^{(NLL)} - \sigma_{tt}^{(0)} K_{tt,\infty}^{(NLO)} \right]^{N-space}
\]
Scale variation: SM Higgs

(a) Scale variation with respect to the factorization scale $\chi_F = \mu_F / M_H$

(b) Scale variation with respect to the renormalization scale $\chi_R = \mu_R / M_H$

(c) Scale variation with identified scales $\chi = \mu / M_H$
Total cross section: SM Higgs

\[ \sigma(pp \rightarrow H + X) \ [pb] \]

\[ M_H \ [\text{GeV}] \]

**Figure**: Total hadronic cross section with uncertainty band due to 7-point scale variation and PDF+\(\alpha_S\) uncertainties according to the PDF4LHC15 recommendations
Pseudoscalar Higgs: Total hadronic cross section and $K$-Factor

(a) Total hadr. cxn in the $m_h^{mod+}$ scenario

(b) $K$-factor in the $m_h^{mod+}$ scenario

- Resummation effects amount to about 5% for $\tan \beta = 3$ and are small for large $\tan \beta = 30$.
- Bumbs and spikes at $M_A \sim 2M_t$ related to $t\bar{t}$ threshold that generates Coulomb singularity
- Squark loops [Anastasiou et al. (2007), Aglietti et al. (2007)], SUSY-QCD corrections [Anastasiou et al. (2008), Mühlleitner et al. (2010)] and $N^3$LO threshold effects [Ahmed et al. (2015,2016)] not yet included
Dim-6 Operator: Total hadronic cross section and $K$-Factor

- Novel coupling $c_g$ consistently included at NNLO in HIGLU [Spira et al. (1995)], resummation effects not yet examined.
- SM value recovered for $c_g (\mu^2_R) = 0$.
- Large constructive and destructive effects depending on the value of $c_g$ due to Born term interference.
- Hadronic cross section becomes minimal where $c_g$ cancels the quark-loop contributions.

(a) Total hadr. cxn by variation of the novel coupling $c_g$

(b) $K$-factor by variation of the novel coupling $c_g$
Conclusions

- Gluon fusion dominant production mechanism over the entire energy spectrum at the LHC
- Higher order corrections in pQCD and elw. theory are sizeable
- Threshold resummation proves to permit insight into higher orders in QCD
- Inclusion of mass effects in resummation turns out to be small
- Collinear effects not negligible
- Matched result at NNLO+N^3LL agrees with full N^3LO within O(2%) for μ_R = μ_F = M_H/2 for MS-masses (no inclusion of missing mass effects)
- Dim-6 Operator included at NNLO
Thank you for your attention!