Light Quark Mass Effects in Higgs Transverse Momentum Distribution

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Topics discussed

- High-order b-quark effect in Higgs p_t-distribution
- Double-logarithmic approximation
  - *kinematics and helicity amplitudes*
  - *Non-Sudakov double logarithms*
- High-order Abelian terms
  - *two-loop result*
  - *all-order resummation*
- Numerics for $d\sigma_{pp\to H+j}/dp_\perp^2$
- Based on: *K. Melnikov, A. Penin, JHEP 05 (2016) 172*

Higgs production at LHC

- Experiment

  Discovery mode ⇨ Precision measurements

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- Theory (heavy top)

  - *Total cross section at NNLO*

    Anastasiou, Duhr, Dulat, Herzog, Mistlberger, Phys.Rev.Lett. **114** (2015) 212001

  - *Higgs plus jet at NNLO*

    Boughezal, Caola, Melnikov, Petriello, Schulze, Phys.Rev.Lett. **115** (2015) 082003

  ↪ *percent accuracy*

Higgs production at LHC

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  - *Total cross section at NNNLO*

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  ↪ *percent accuracy*

- Fine effects become relevant
  - *bottom-quark loop contribution beyond NLO/LO*

  ↪ *important for differential distributions*

Higgs production in gluon fusion

- Quark loop mediated amplitude

  $\boxed{M(gg \to H) \propto m_H^2}$ *for* $m_q \gg m_H$

  $\boxed{M(gg \to H) \propto m_q^2 \ln^2(m_H^2/m_q^2)}$ *for* $m_q \ll m_H$

- NLO bottom-loop contribution to $gg \to Hg$
  - $\frac{\alpha_s}{4\pi}\frac{m_b^2}{m_h^2}\ln^4(m_h^2/m_q^2) \approx 2\%$ *of leading top-loop result*
  - *new kind of double logs* $\ln^2(p_\perp^2/m_b^2)$

  ➥ *changes the shape of* $p_\perp$*-distribution*

Bottom-loop contribution beyond NLO/LO

Some recent development

D. de Florian, G. Ferrera, M. Grazzini and D. Tommasini, JHEP **1111** (2011) 064

M. Grazzini and H. Sargsyan, JHEP **1309** (2013) 129

H. Mantler and M. Wiesemann, Eur. Phys. J. C **73** (2013) 2467

A. Banfi, P. F. Monni and G. Zanderighi, JHEP **1401** (2014) 097

E. Bagnaschi and A. Vicini, JHEP **1601** (2016) 056

R. Mueller and D. G. Oeztuerk, JHEP **1608** (2016) 055

R. Frederix, S. Frixione, E. Vryonidou and M. Wiesemann, JHEP **1608** (2016) 006

F. Caola, S. Forte, S. Marzani, C. Muselli and G. Vita, arXiv:1606.04100 [hep-ph]

N. Greiner, S. Hoeche, G. Luisoni, M. Schonherr, J. C. Winter, arXiv:1608.01195 [hep-ph]

...

Higgs production with transverse momentum

- Kinematics of soft emission $g_1 g_2 \to H g_3$

$$m_b \ll E_3 \ll E_{1,2} \sim m_H \qquad \longrightarrow \qquad m_b^2 \ll p_\perp^2 \ll m_H^2$$

- Double logarithmic approximation
  - *small parameter*

$$\frac{m_b}{E_3} \sim \frac{E_3}{E_{1,2}} \sim \lambda \ll 1,$$

  - *series for the amplitude*

$$\mathcal{M}_{gg\to Hg} = \frac{g_s}{\lambda} \sum_{n=1}^{\infty} C_n \alpha_s^n \ln^{2n}(\lambda) + \ldots .$$

➥ *maximal Abelian part of* C_n *for all* n

$gg \to Hg$ helicity amplitudes

- Two independent helicity amplitudes

$$M^{\rm soft}_{++\pm} \propto A_{++\pm}(x,\tau,\zeta) = \sum_{n=0}^{\infty} x^n A^{(n)}_{++\pm}(\tau,\zeta)$$

$A_{++\pm}$ *are dimensionless functions of logarithmic variables:*

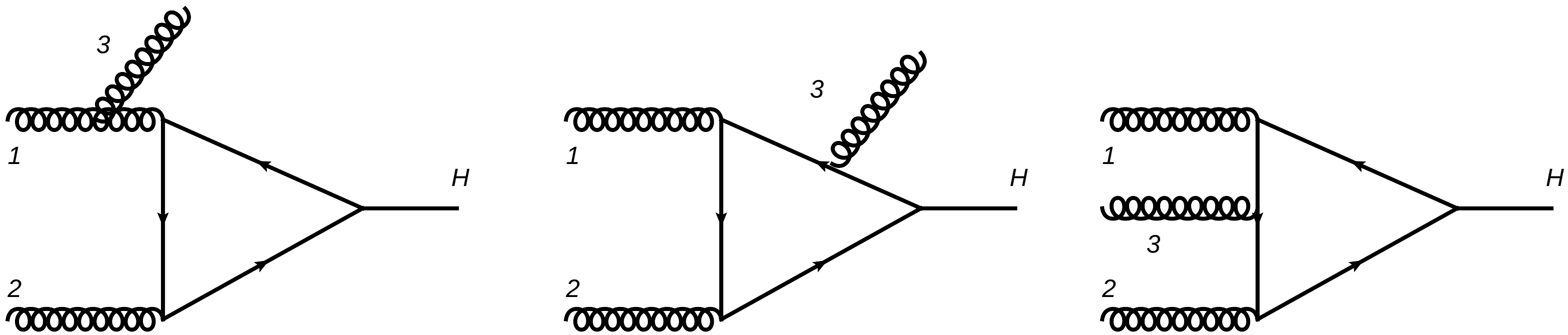
$$\tau = \ln(m_b^2/p_\perp^2)/L, \qquad \zeta = \ln(u/t)/L, \qquad x = \frac{C_F\alpha_s}{2\pi}L^2$$

where $\quad L = \ln(m_b^2/s), \qquad 0 < \tau, |\zeta| < 1, \qquad x \sim 1$

- Leading contribution to the cross section

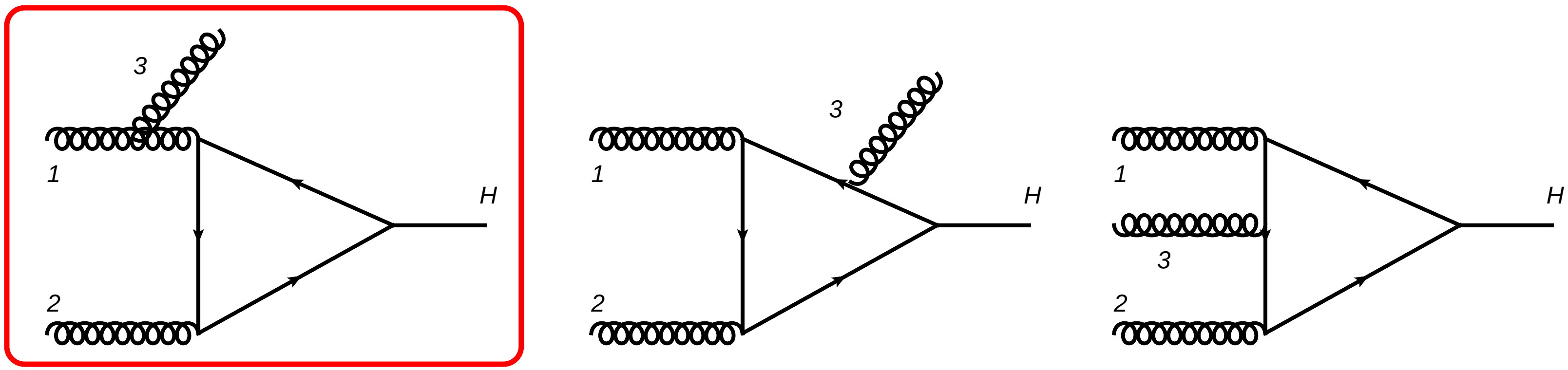
$$d\sigma_{gg\to Hg} = d\sigma^{(0)}_{gg\to Hg} \times \left[1 - \frac{3}{2}\frac{m_b^2}{m_H^2}L^2\left(A_{+++} - A_{++-}\right) + \mathcal{O}(m_b^4)\right],$$

One-loop amplitudes



exact result: U. Baur, E. W. N. Glover, Nucl. Phys. **B339** (1990) 38

One-loop amplitudes



Evaluation of double logs

- Soft “scalar” quark exchange

$$\frac{\hat{l}+m_b}{l^2-m_b^2+i0} \to \frac{m_b}{l^2-m_b^2+i0}$$

➥ *non-Sudakov double logarithms*

- Sudakov method
  - *Sudakov parameters:* $l=\alpha p_1+\beta p_2+l_\perp$

$$\frac{1}{l^2-m_b^2+i0} \to -i\pi\delta(l^2-m_b^2) = -i\pi\delta(s\alpha\beta-l_\perp^2-m_b^2).$$

$$\frac{1}{(p_2-l)^2-m_b^2} \to \frac{1}{s\alpha}, \qquad \frac{1}{(p_1-p_3-l)^2-m_b^2} \to \frac{1}{t-\beta s}.$$

  - *double log region:* $|t|/s<\beta<1, \quad m_b^2/s<\alpha<1, \quad m_b^2/s<\alpha\beta$

Evaluation of double logs

- New variables $\eta = \ln\alpha/L, \quad \xi = \ln\beta/L$
  - *double log region:* $\eta < 1-\tau_t, \quad \eta+\xi < 1$, *where* $\tau_t = \ln\left(m_b^2/|t|\right)/L$
- *Double log coefficient*

$$A^{(0)}_{++\pm} = \pm 2\int_0^{1-\tau_t} \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi = \pm(1-\tau_t^2)$$

- *Sum of all diagrams*

$$\boxed{A^{(0)}_{++\pm} = \pm 1 - \frac{\tau^2}{2}}$$

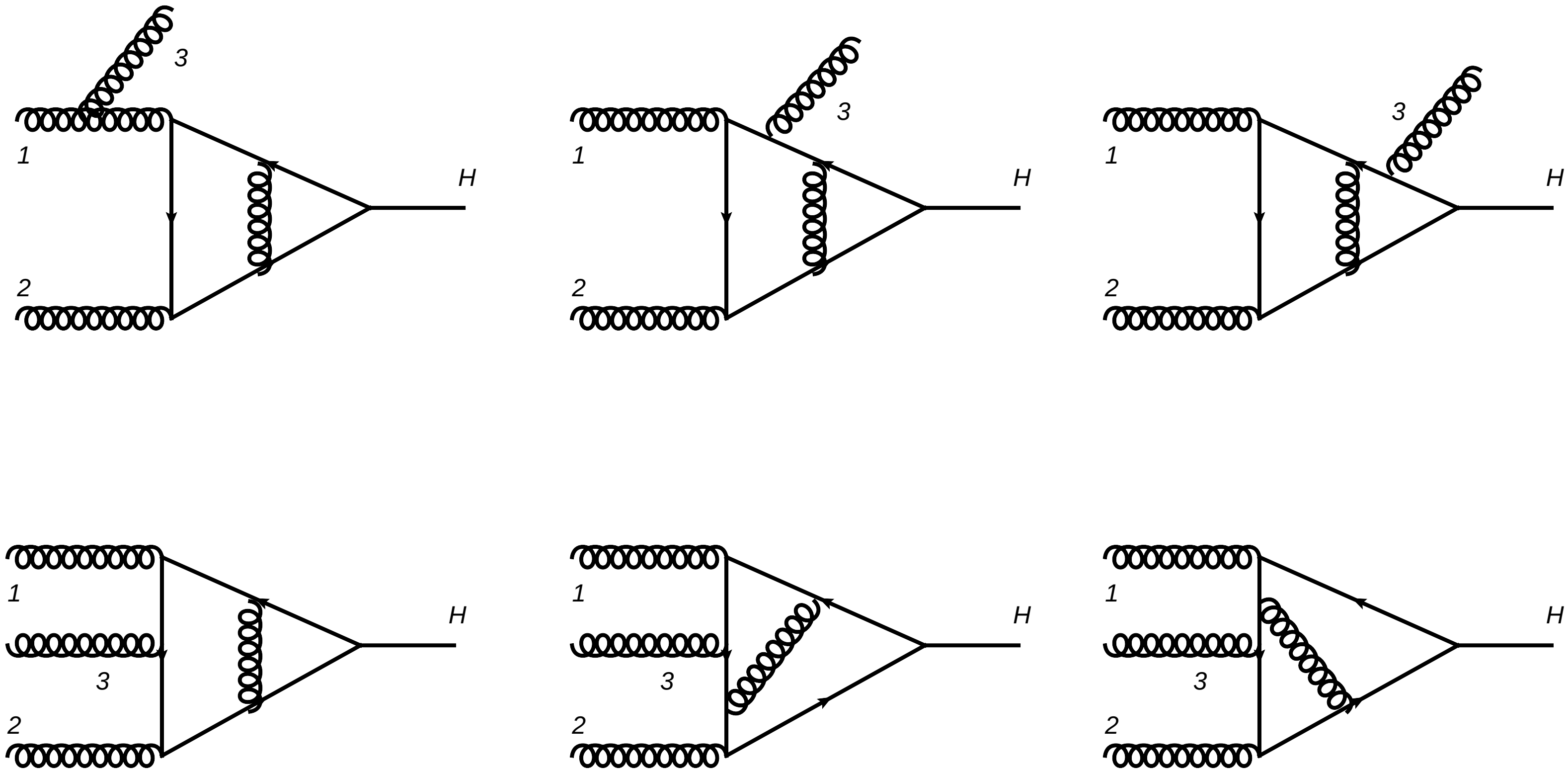
agrees with Baur and Glover

Non-standard soft emission

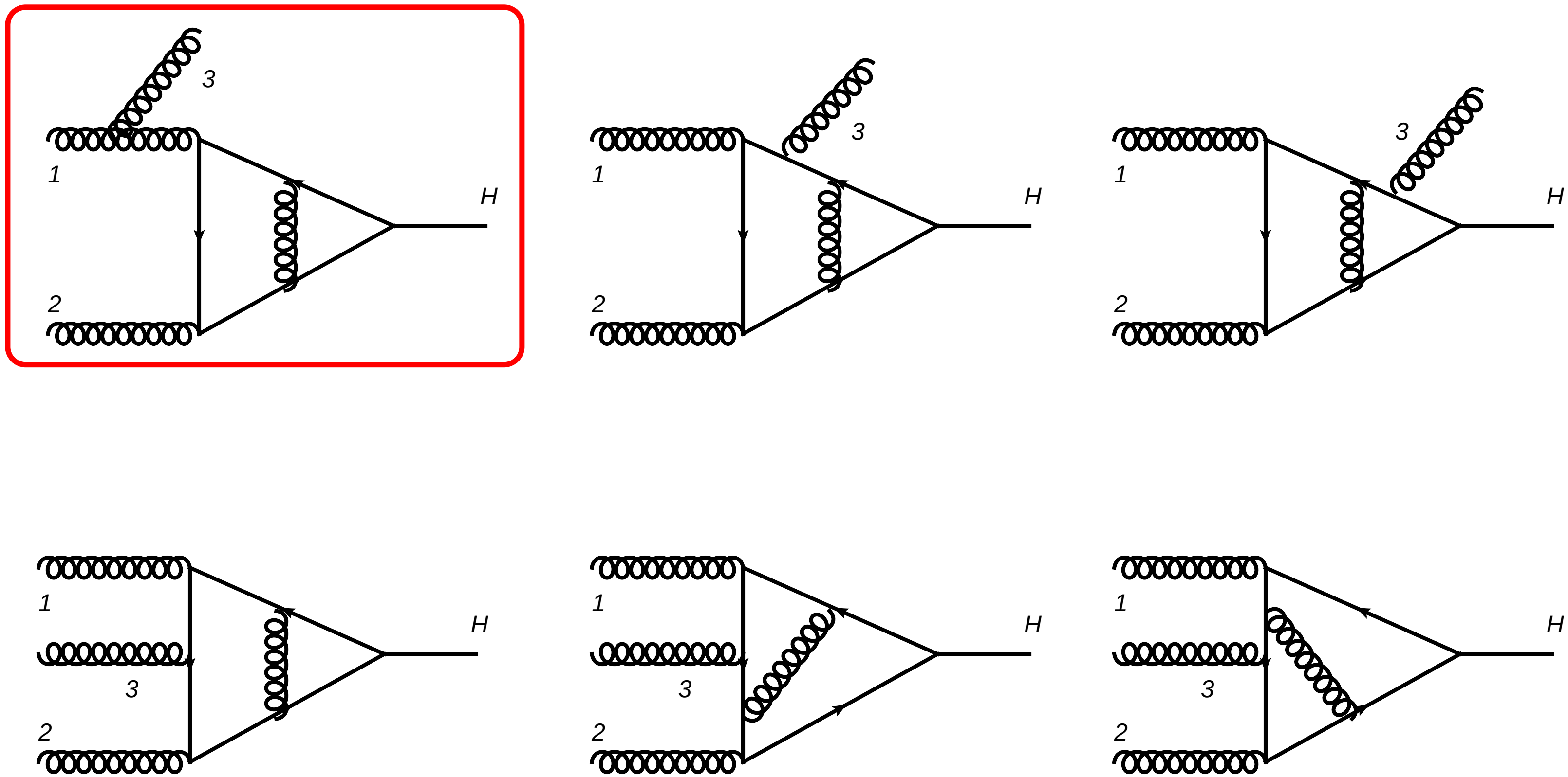
- Color dipole emission
  - *factorization*
  - $\boxed{A_{++-} = -A_{+++}}$
- Emission from soft quark line
  - $\boxed{G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu} f^{abc}}$

  ➥ *no factorization!*
  - $\boxed{A_{+++} = 0}$

Two-loop amplitudes



Two-loop amplitudes



Evaluation of double logs

- Selected diagram

$$A^{(1)}_{++\pm} = \mp 2\int_0^{1-\tau_t} \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi \int_0^{\eta} \mathrm{d}\eta' \int_0^{\xi} \mathrm{d}\xi' = \mp \frac{1-4\tau_t^3+3\tau_t^4}{12}$$

- Sum of all diagrams

$$\boxed{A^{(1)}_{+++} = -\frac{2-3\tau^2+2\tau^3+3\tau^2\zeta^2}{24}}$$

$$\boxed{A^{(1)}_{++-} = \frac{2+3\tau^2-6\tau^3+4\tau^4-3\tau^2\zeta^2}{24}},$$

Resummation of double logs

- Only three examples of non-Sudakov resummation
  - *electron-muon backward scattering*

    V. G. Gorshkov, V. N. Gribov, L. N. Lipatov and G. V. Frolov, Yad. Fiz. **6** (1967) 129
  - *Higgs two-photon width*

    M. I. Kotsky and O. I. Yakovlev, Phys. Lett. **B418** (1998) 335
  - *Dirac form factor in QED (power suppressed contribution)*

    A. A. Penin, Phys. Lett. **B745** (2015) 69
- Higher order double logs are due to soft gluons

  ➥ *Abelian part* ⇨ *factorization/exponentiation of soft photons*
  - *e.g. for the selected topology*

$$A_{++\pm} = \pm 2 \int_0^{1-\tau_t} \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi \, e^{-x\xi\eta} = \pm 2 \int_0^{1-\tau_t} \frac{1 - e^{-x\eta(1-\eta)}}{x\eta} \mathrm{d}\eta.$$

All-order cross section

- Partonic cross section

$$d\sigma_{gg\to Hg} = d\sigma^{(0)}_{gg\to Hg} \times \left[1 - \frac{3}{2}\frac{m_b^2}{M_H^2} L^2 f(x,\tau,\zeta) + \mathcal{O}(m_b^4)\right]$$

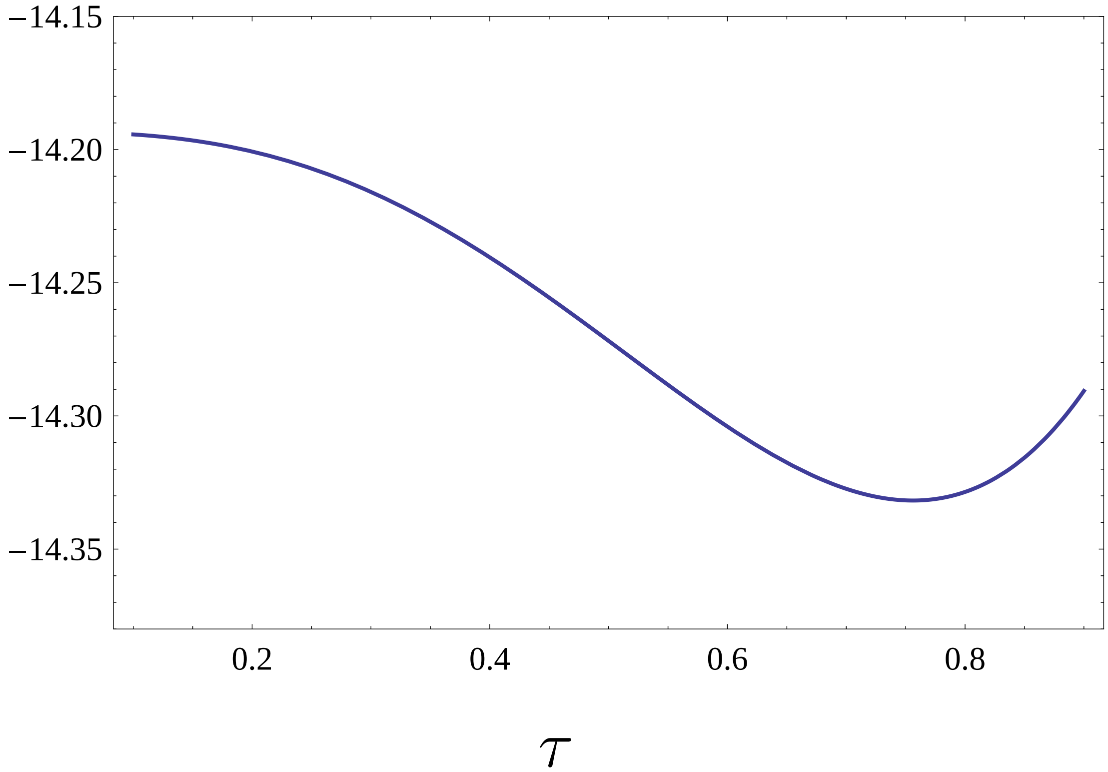
$$f = 2 - \frac{x}{6}\left(1 - \tau^3 + \tau^4\right) + \frac{x^2}{24}\left(\frac{4}{15} - \tau^3 + 2\tau^4 - \frac{7\tau^5}{5} + \frac{2\tau^6}{5} + \zeta^2\left(\tau^3 - \tau^4\right)\right) + \ldots .$$

- Hadronic cross section

$$\frac{d\sigma_{pp\to H+j}}{dp_\perp^2} = \frac{d\sigma^{(0)}_{pp\to H+j}}{dp_\perp^2}\left\{1 - \frac{3m_b^2}{m_H^2} L^2 \left[1 - \frac{x}{12}\left(1 - \tau^3 + \tau^4\right) + \frac{x^2}{48}\left(\frac{4}{15} - \tau^3 + 2\tau^4 - \frac{7\tau^5}{5} + \frac{2\tau^6}{5}\right) + \mathcal{O}(x^3)\right] + \mathcal{O}(m_b^4)\right\}$$

Numerics

The bottom-loop correction in percent to the top-loop contribution



- *NLO Abelian correction:* $+1.5\%$ *and* 0.2% *variation*
- *Full NLO correction:* $\sim 3\%$ *and* $\sim 0.4\%$ *variation*

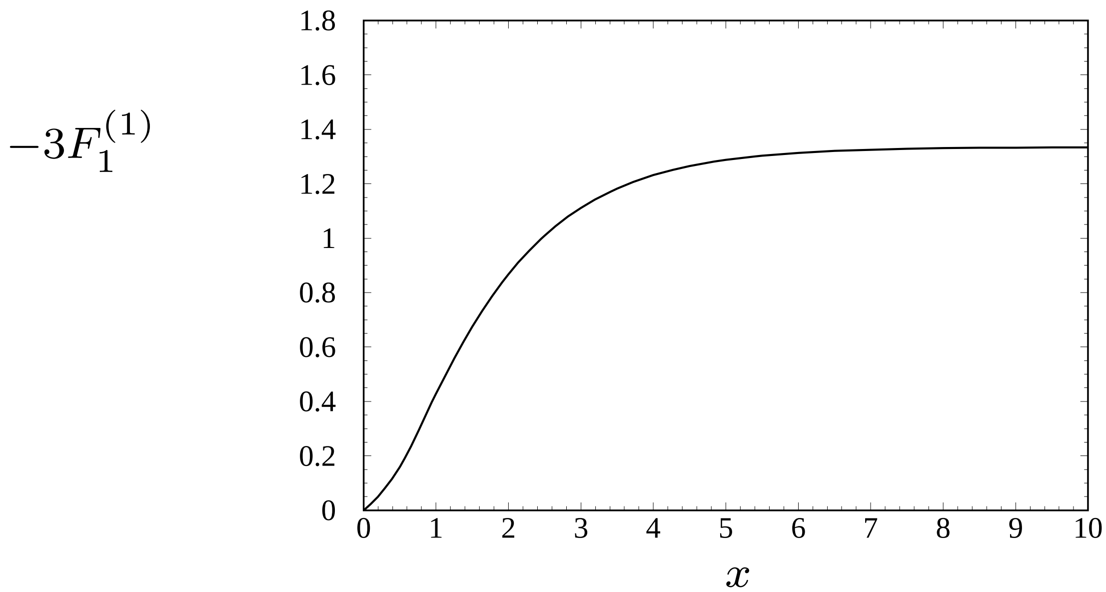
Dirac form factor at high energy

$$F_1 \propto \sum_{n=0}^{\infty} \left(m_e^2/s\right)^n F_1^{(n)}(x)$$

- Sudakov form factor: $\boxed{F_1^{(0)} = e^{-x}}$ Sudakov (1956)

- Leading power correction

Penin (2015)



↪ *double logarithmic enhancement*

Two-loop Bhabha scattering

A. Penin and N. Zerf, Phys.Lett. **B760** (2016) 816

- Two-loop power suppressed terms
  - *double logarithmic approximation*

$$\delta\sigma = -\frac{\alpha^2}{s}\left[\frac{m_e^2}{s}\left(\frac{\alpha\ln^2(m_e^2/s)}{4\pi}\right)^2\frac{4+166x-323x^2+266x^3-323x^4+166x^5+4x^6}{3(1-x)x^3}\right]$$

$x = t/s$

Summary

- Bottom quark effects in Higgs $p_\perp$-distribution
  - *Method of calculation for NLO double logs*
  - *All-order resummation of Abelian terms*
    - *Corrections saturated by two-loop term*
    - *Strong cancellation of $p_\perp$-dependence*
  - *non-Abelian NLO term to be computed*

Summary

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- Power suppressed IR logs
  - *first results (electron form-factor, Bhabha scattering)*

    A.Penin, Phys.Lett. **B745** (2015) 69; A.Penin, N.Zerf, Phys.Lett. **B760** (2016) 816
  - *interesting features very different from Sudakov logs*
  - *effective field theory RG is still missing*