Heavy Dark Matter Annihilation at One loop

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Based on soon to appear work w/ Grigory Ovanesyan, Tracy Slatyer and Iain Stewart

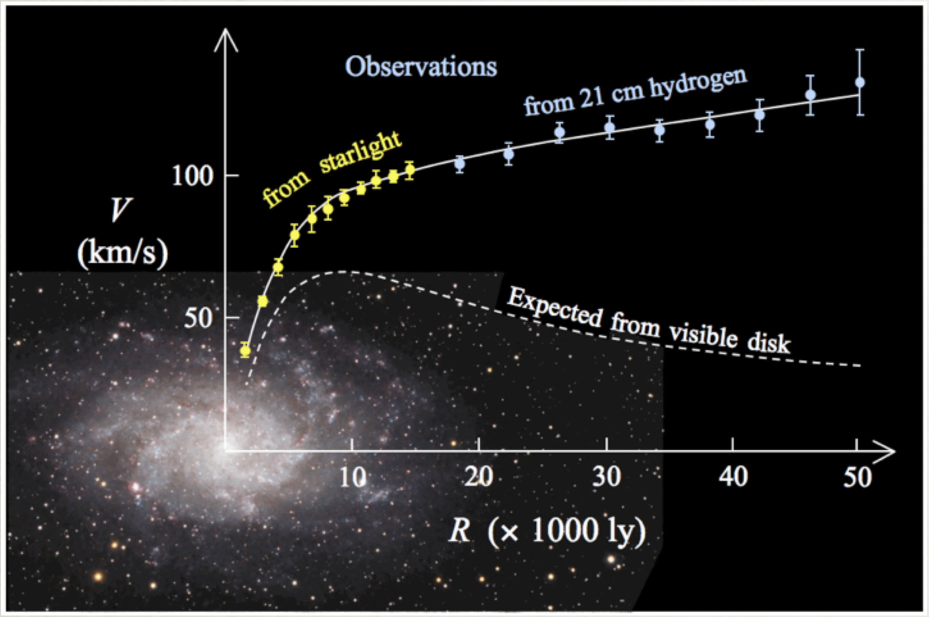
LoopFest XV
15 August 2016

Outline

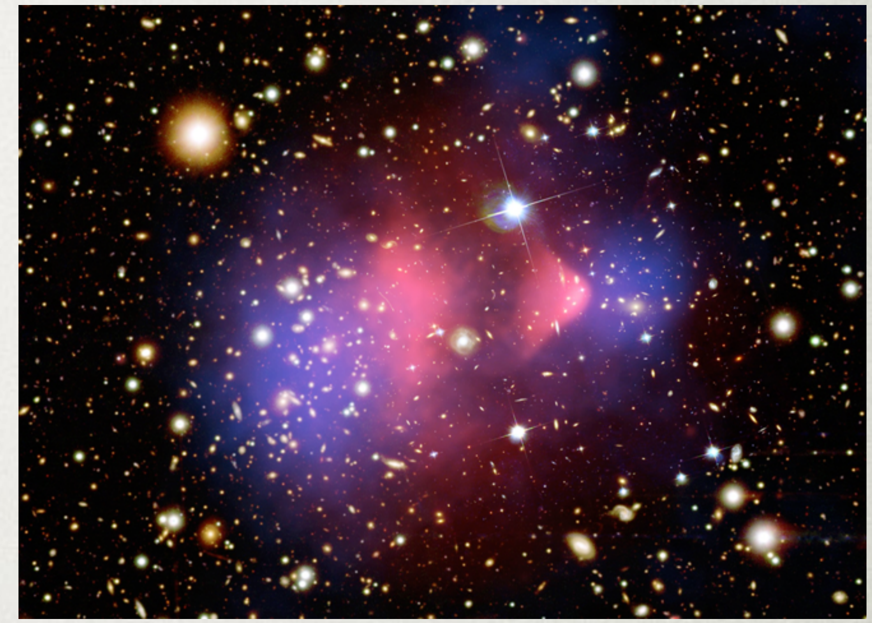
- **Motivation:** for precision/loop calculations for dark matter
- **Setup:** the DM model and EFT framework
- **Selected Highlights:** of the calculation
- **Results**

We know a lot about Dark Matter

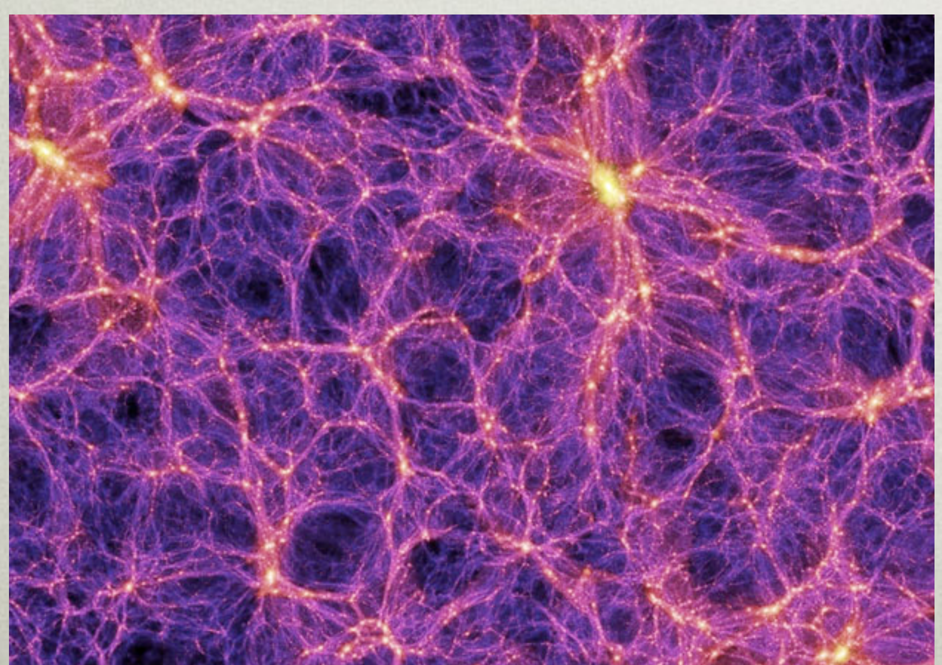
Galaxy Rotation Curves



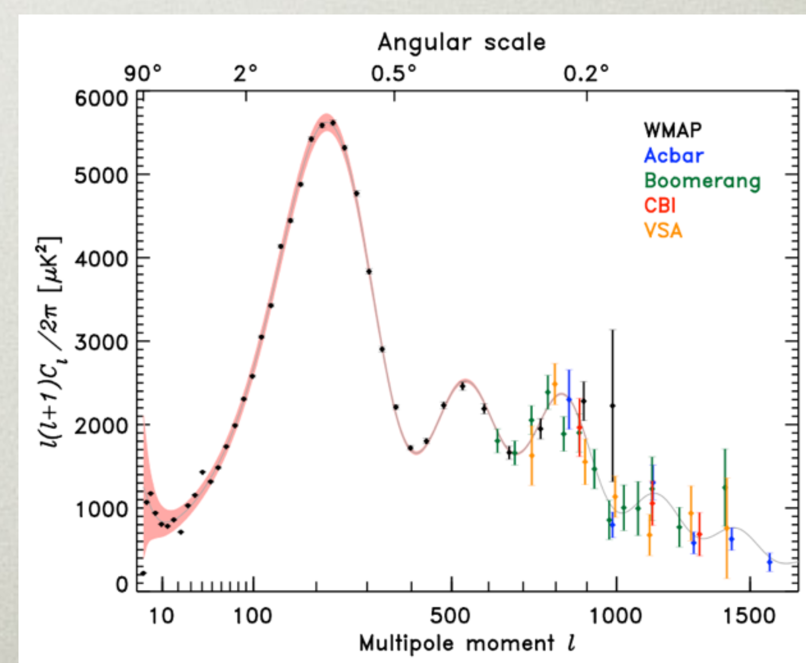
Bullet Cluster



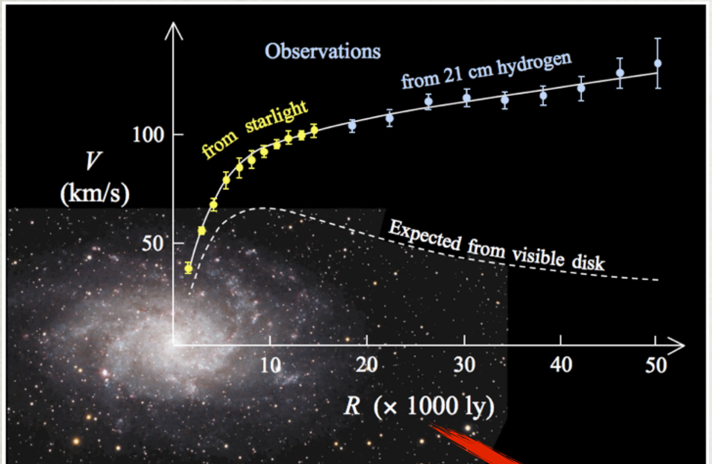
Structure Formation

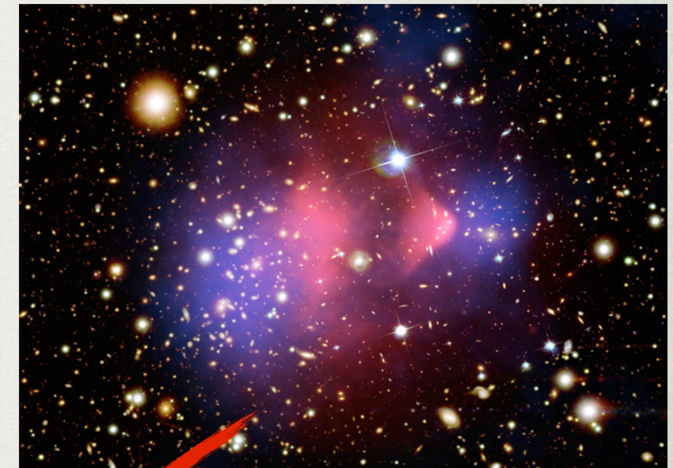


CMB Power Spectrum

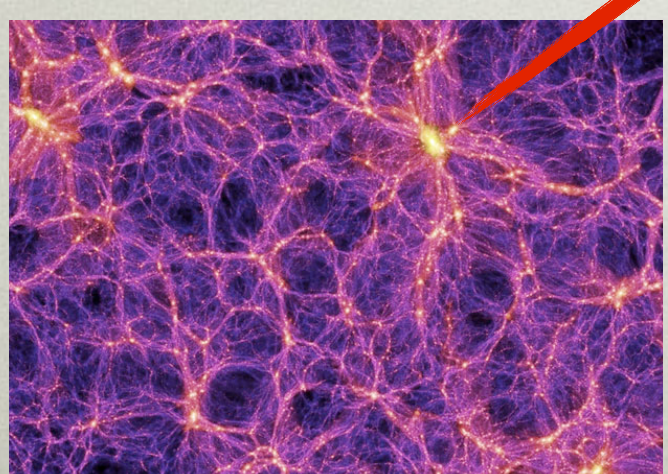


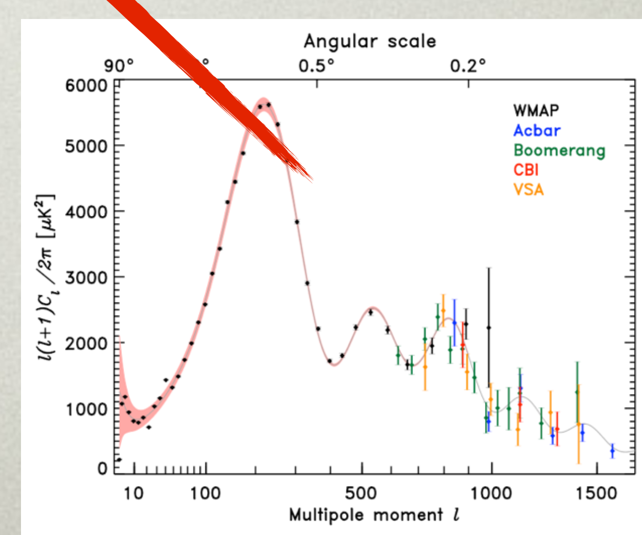
We know a lot about Dark Matter



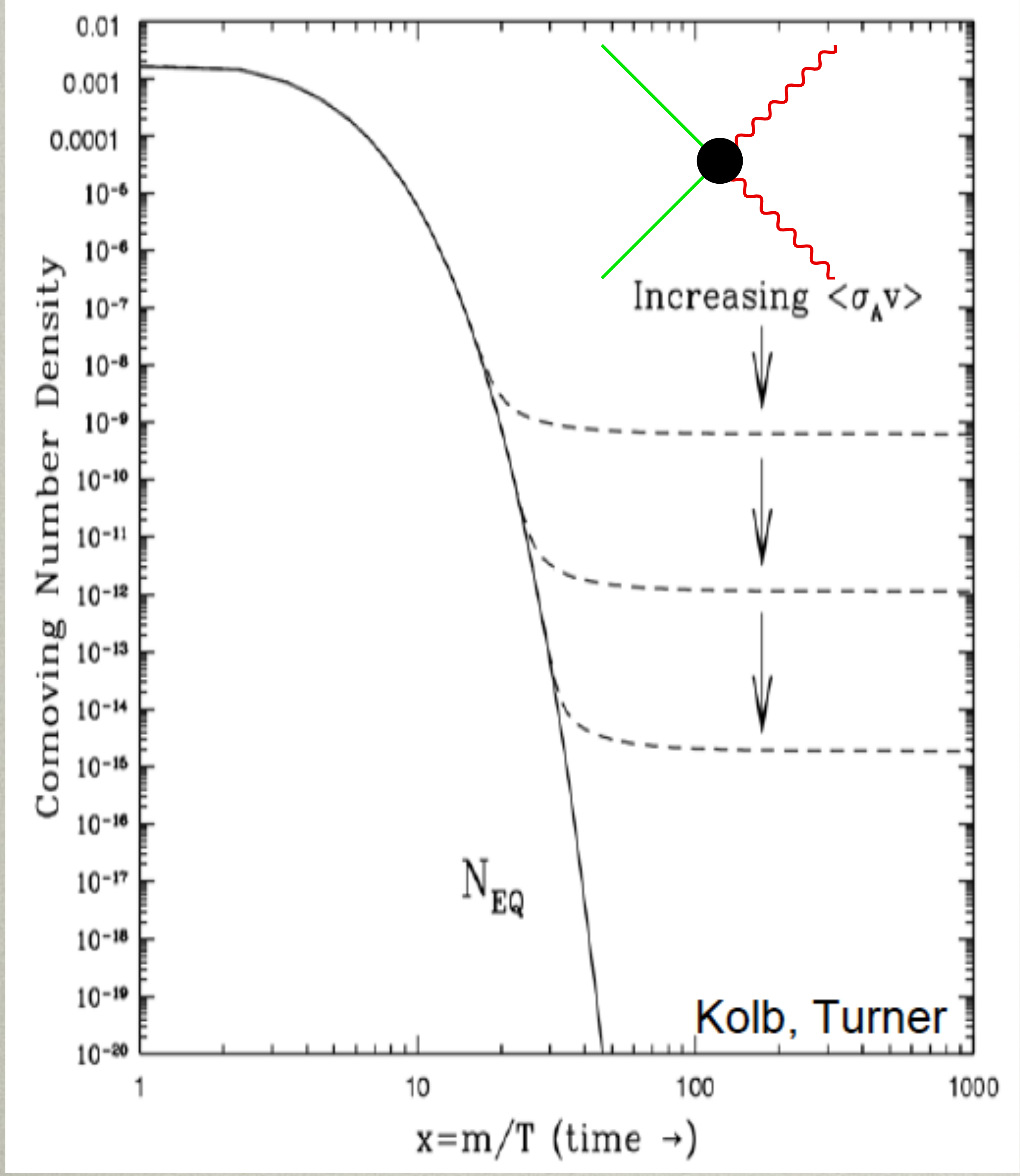


Particle Dark Matter





We know a lot about Dark Matter



Wimp Miracle

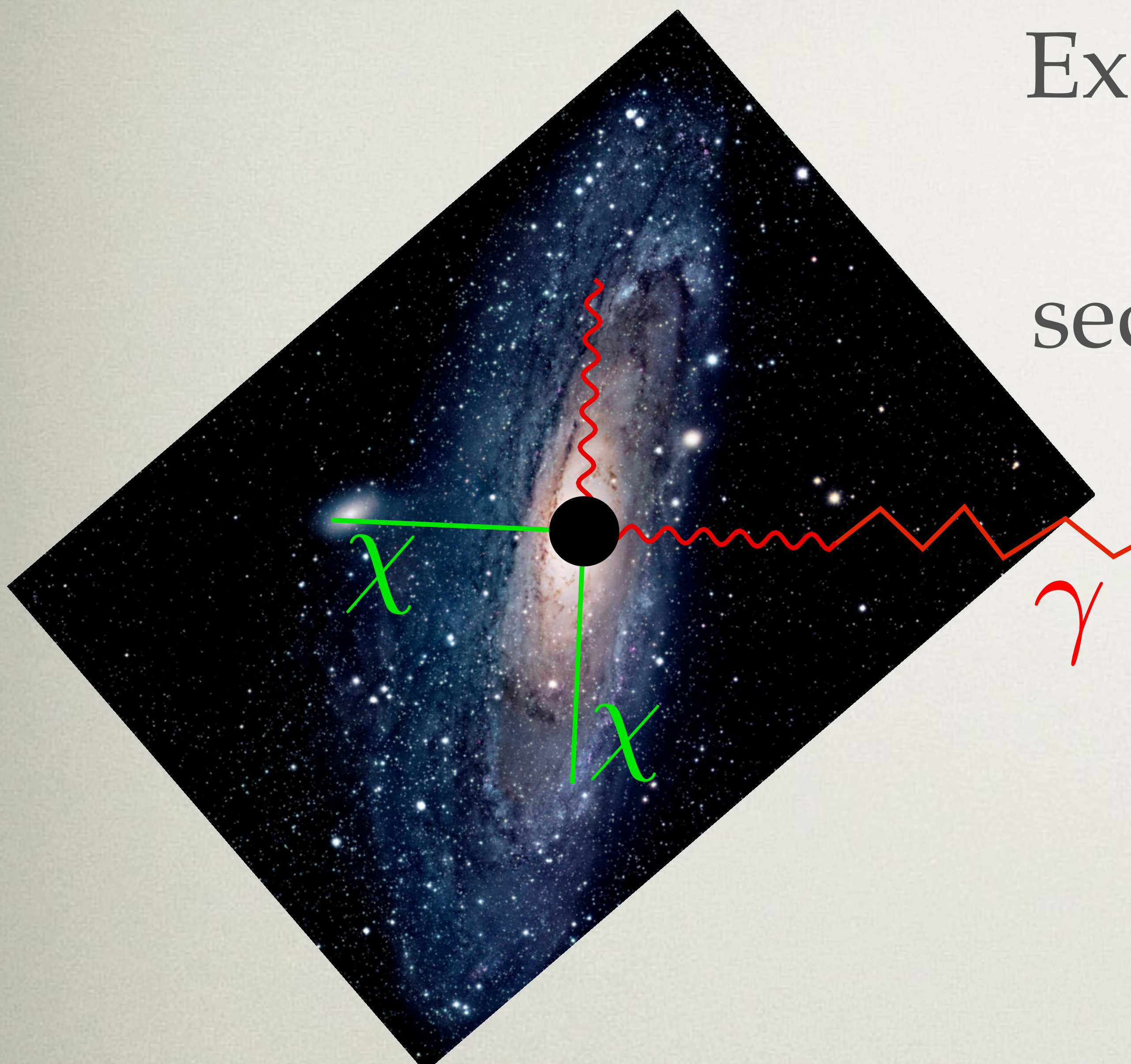
- If DM is a new heavy state initially in thermal equilibrium, its relic density scales as

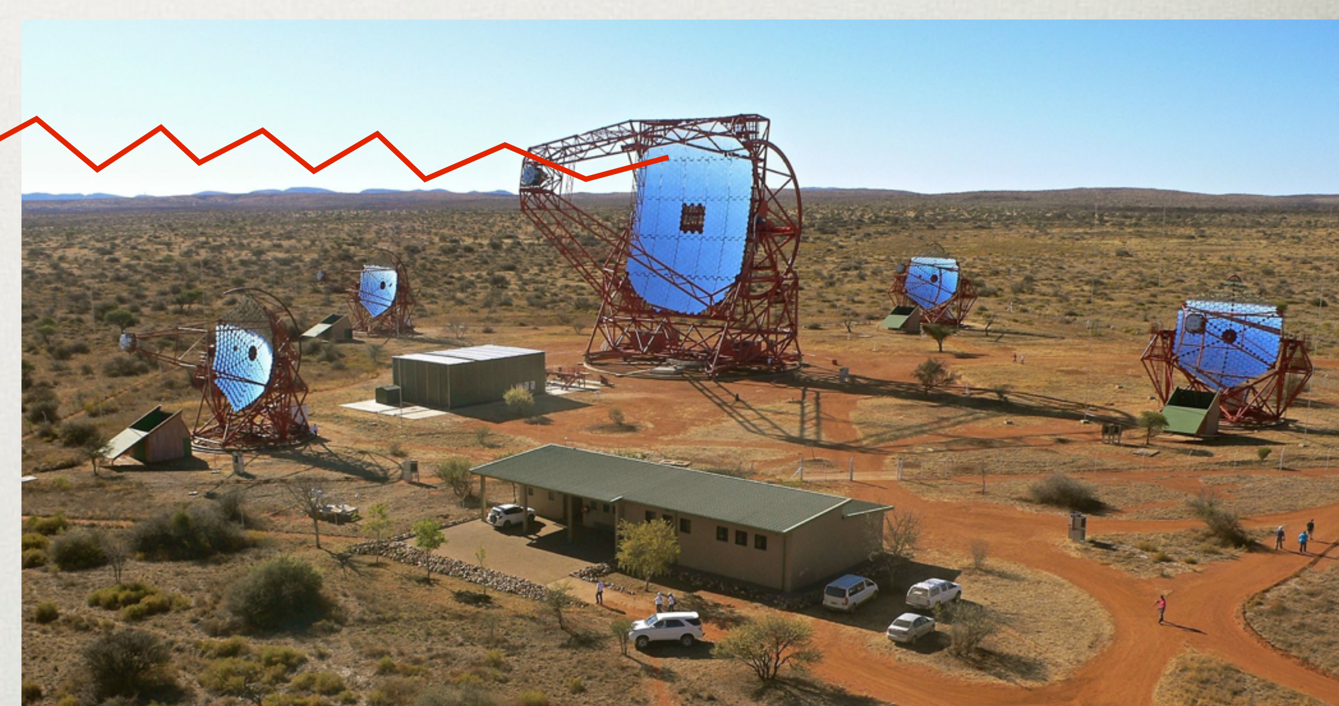
$$\Omega \propto \frac{1}{\langle \sigma v \rangle} \propto \frac{m^2}{g^4}$$

- Weak scale coupling and TeV masses give the observed relic density
- Weakly coupled TeV scale DM is theoretically well motivated

We know a lot about Dark Matter

Experiments probing TeV scale DM with weak scale cross sections are running right now





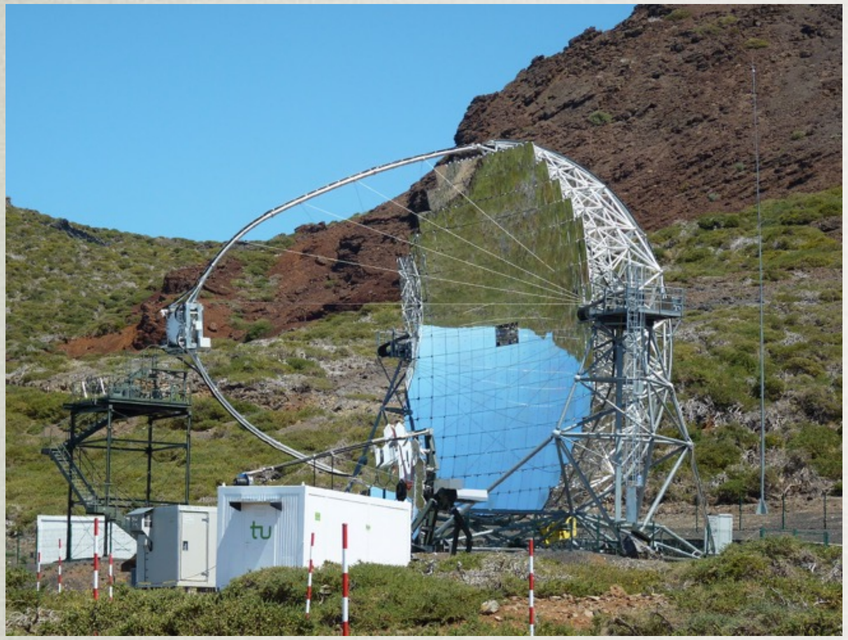
Order of magnitude calculations for the cross sections no longer suffice - **need precision**

High Energy Stereoscopic System (H.E.S.S.) Telescope

Taking data since 2012

Located in Namibia 

We know a lot about Dark Matter



MAGIC (2004)

Canary Islands 

HAWC (2015)

Mexico 

VERITAS (2007)

Arizona, USA 

CTA (2018?)

TBD

... Yet We know almost nothing about Dark Matter

What is the DM mass scale?

10^{-10} eV — TeV — 10^{25} eV → m_{DM}

Dark Photon Condensate
1105.2812

Wimpzilla
hep-ph/9810361

- Even so, many of the lessons we learn from the TeV scale could apply to other models
- Lots of space to explore!

A Simple Model

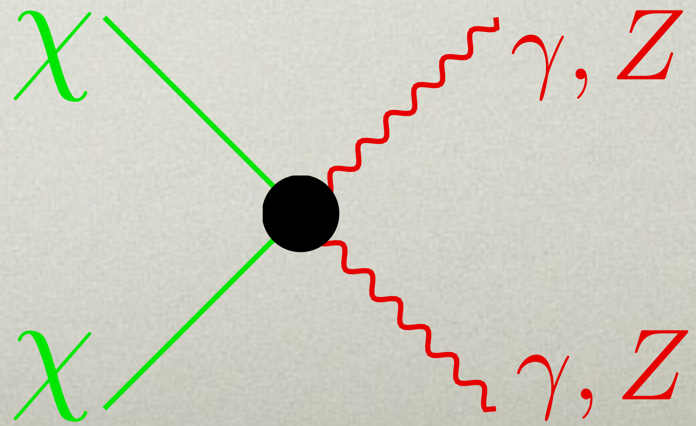
- **Model:** pure wino dark matter
  - A simple model where loop effects are known to be important
- Add SU(2) triplet Majorana fermion

$$\chi = \begin{pmatrix} \chi^0/\sqrt{2} & \chi^+ \\ \chi^- & -\chi^0/\sqrt{2} \end{pmatrix}$$

- with Lagrangian (mass is the only new parameter):

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}{\rm Tr}\bar{\chi}\left(i\not{D} - M_\chi\right)\chi$$

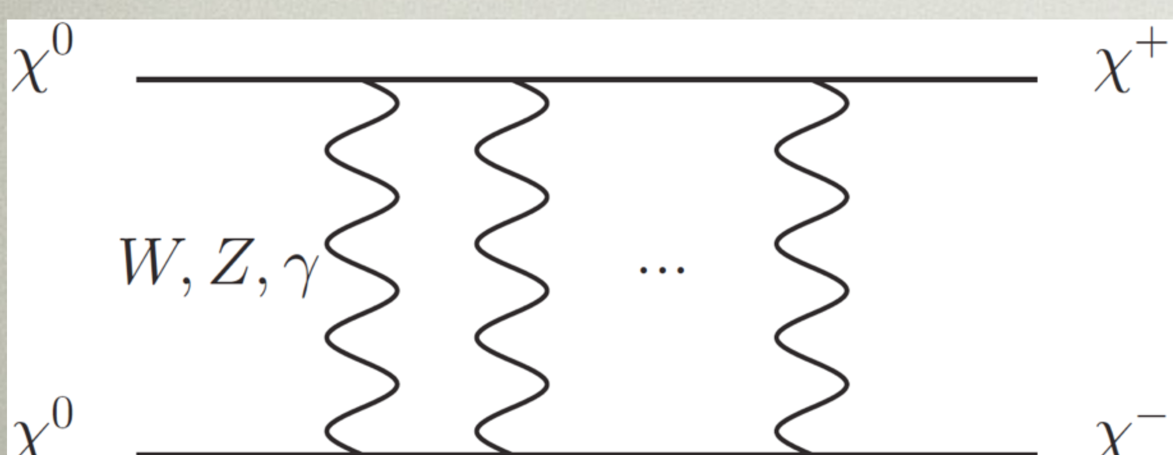
- Study annihilation into photons within the model



A Simple Model where Loops Matter

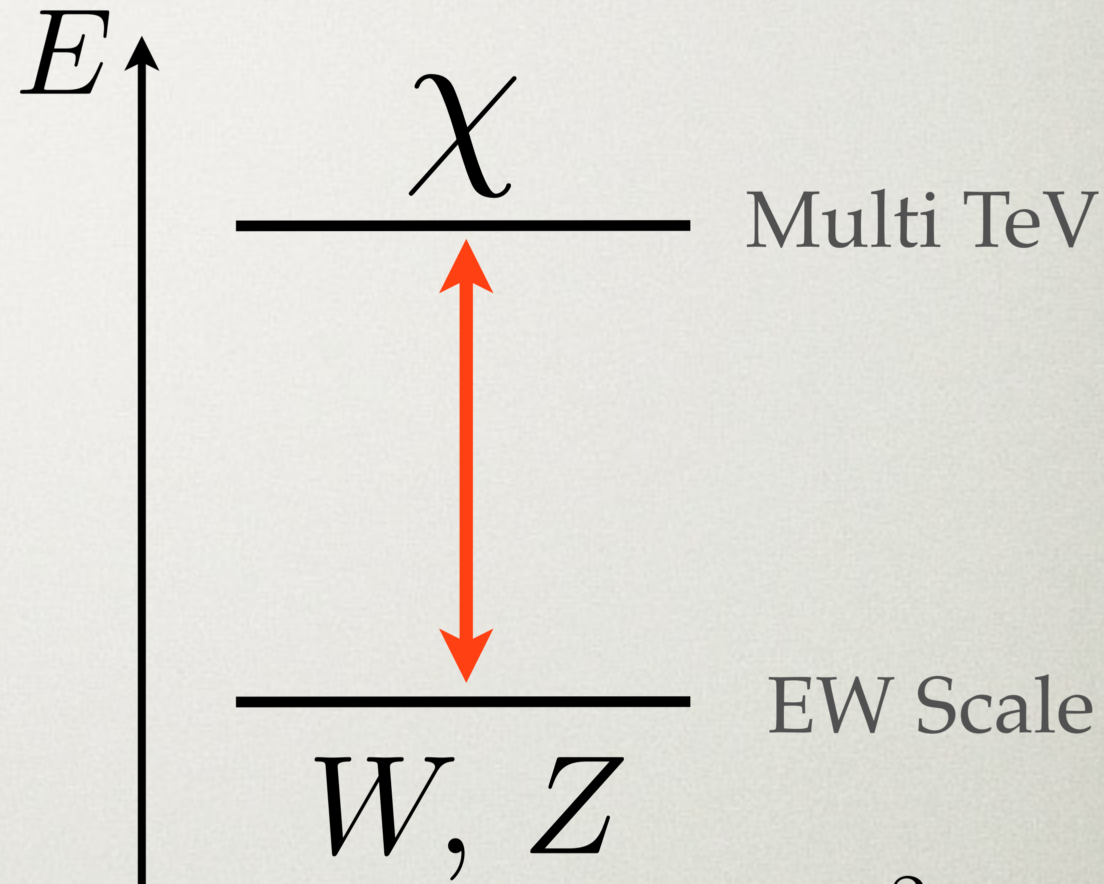
Sommerfeld Enhancement

- **Fundamentally:** an attractive potential can enhance the cross section by orders of magnitude (scales as 1/v)
- Determine numerically by solving the Schrodinger equation
- Even if start with neutral particles, the charged ones can initiate the hard annihilation - and dominate due to tree-level photon coupling



See e.g 0810.0713, 0910.5713, 1307.4082, 1603.01383

Sudakov Double Logs



1 TeV DM: $\alpha_2 \ln^2 \dfrac{4m_\chi^2}{m_Z^2} \simeq 1.3$

- Need to resum the large logs - use EFT techniques
- Also important for transverse WW production at the LHC (0909.0012)

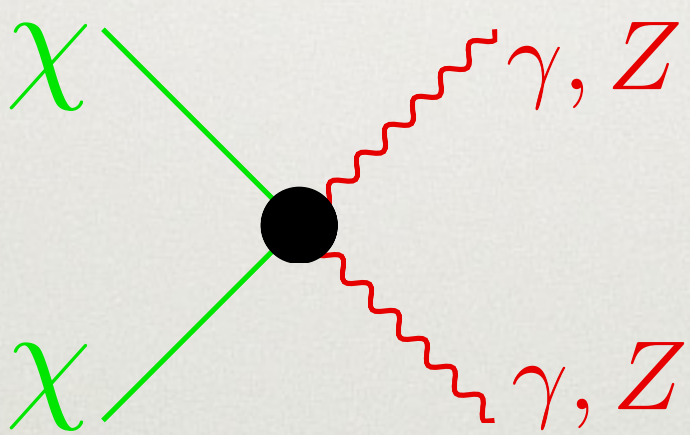
See e.g 1409.8294, 1409.7392, 1409.4415

A Simple Model mapped to an EFT

- We follow 1409.8294, who did the NLL calculation, and map to NRDM-SCET

Non-Relativistic Dark Matter

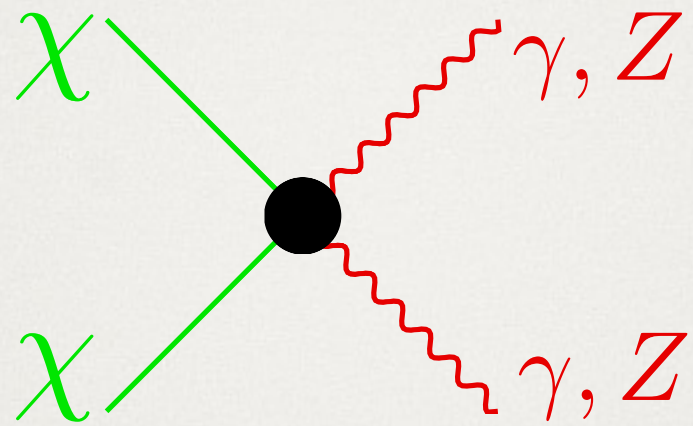
- DM is heavy and non-relativistic
- Treat like an HQET Field
- Influenced by potential sourced by EW bosons



Soft-Collinear Effective Theory

- DM is much heavier than EW scale
- Outgoing bosons are highly boosted/collinear
- Naturally described by SCET - specifically $\mathrm{SCET_{II}}$

A Simple Model mapped to an EFT

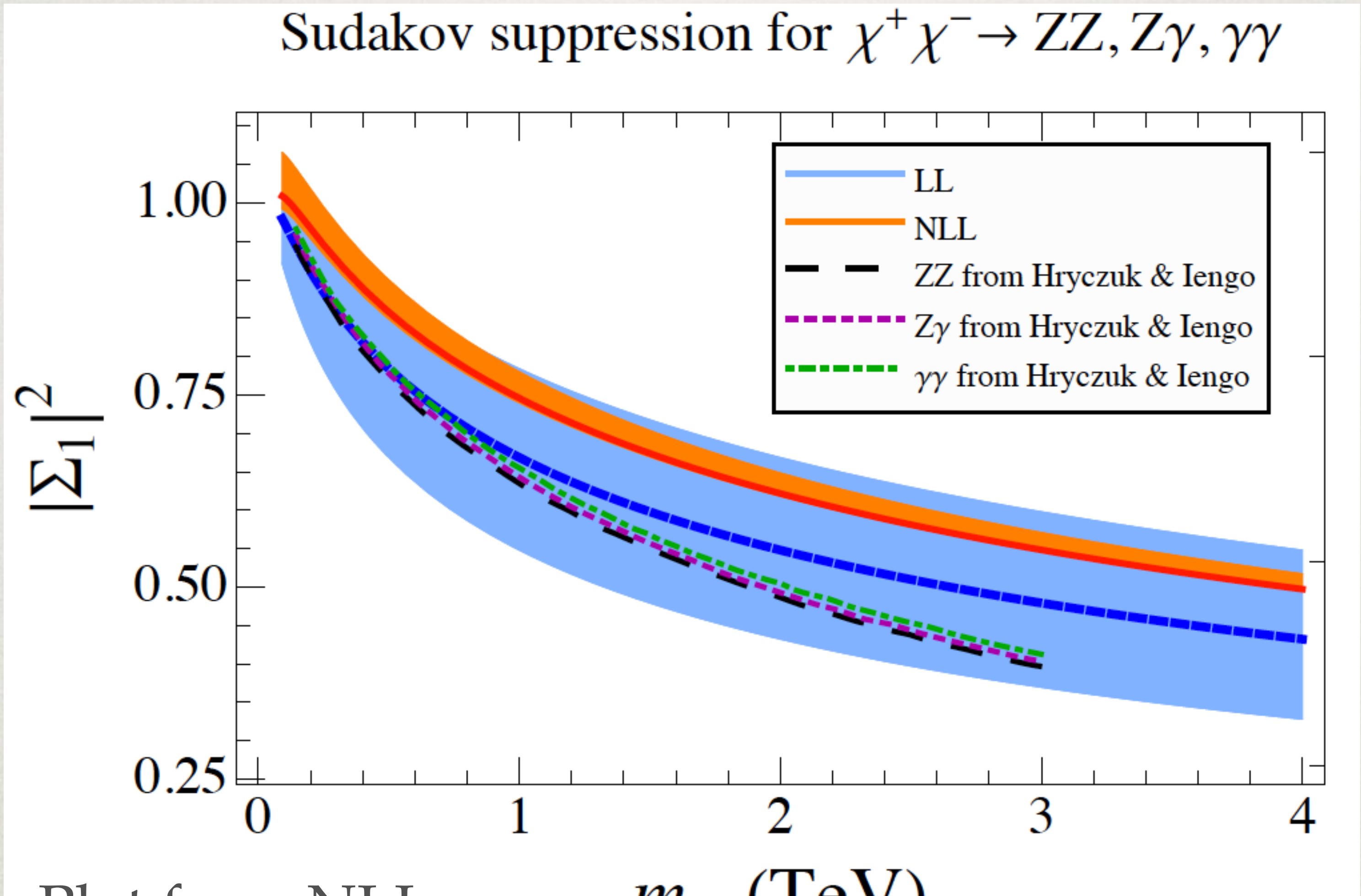


- **Power of the EFT:** at leading power, NRDM has no dependence on soft/collinear gauge bosons, whilst SCET has no dependence on the DM
- Thus Sommerfeld and Sudakov factorise to the order we are working(X can be $\gamma\gamma$, γZ or ZZ):

$$\mathcal{M}_{\chi^0\chi^0\to X} = 4\sqrt{2}m_\chi P_X \left[s_{00} \left(\Sigma_1 - \Sigma_2\right) + \sqrt{2}s_{0\pm}\Sigma_1 \right]$$

Sudakov Contribution

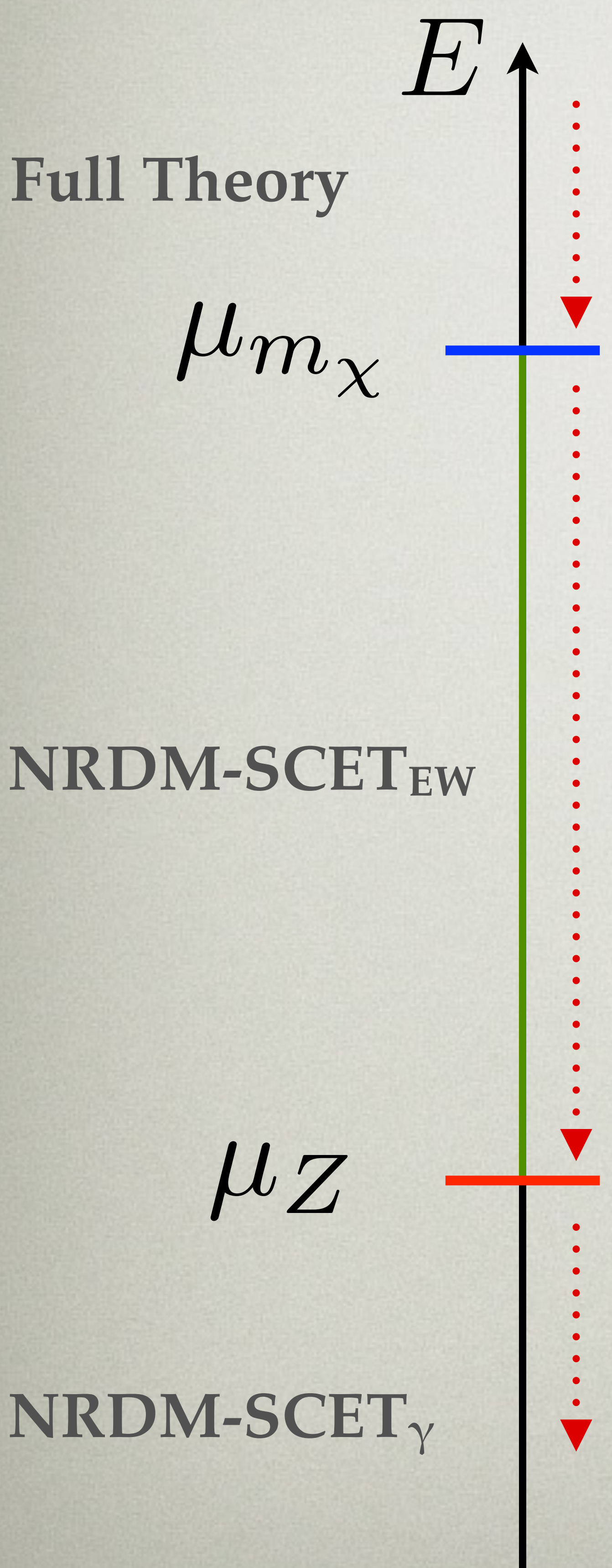
A Simple Model: Sudakov Logs to NLL



Plot from NLL

- Sudakov correction is clearly important
- Hryczuk & Iengo (1111.2916) did a one-loop fixed order numerical calculation of this process
  - Seems inconsistent with NLL calculation - might be an error
  - Warranted rethinking about this in an EFT setup

One-Loop Matching: Going beyond NLL



High Scale Matching

- At the high scale match the NRDM-SCET onto the full theory
- Wilson coefficients only sensitive to UV physics: calculate in the unbroken full theory without gauge bosons masses
- NLL only needed tree level matching, **we extend this to one loop**

Running by Anomalous Dimension

- Run from the natural scale of the hard annihilation (~2m$_\chi$), to the scale of the Sommerfeld annihilation (~m$_Z$)
- Resums Sudakov double logs
- We will take the NLL result

Low Scale Matching

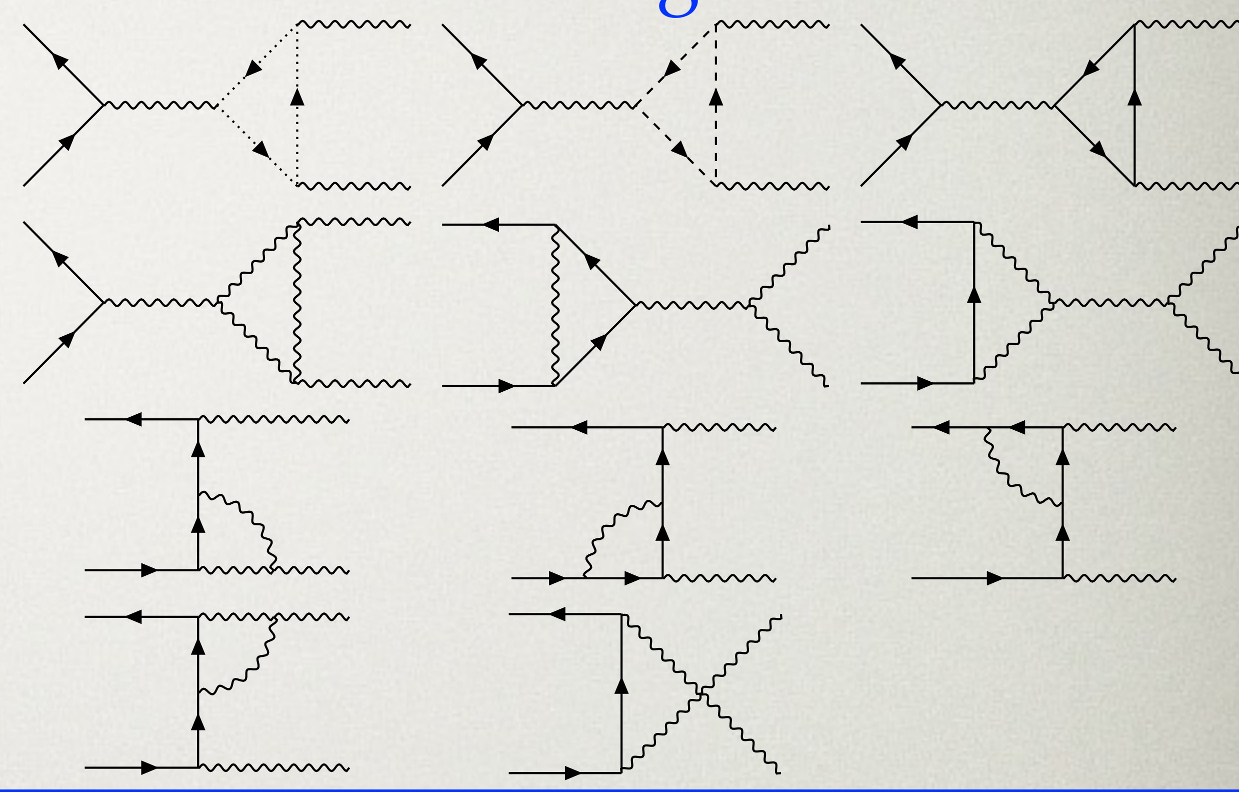
- Match from a theory with EW degrees of freedom onto one without
- Accounts for gauge boson mass effects
- Use SCET$_{EW}$ formalism (e.g. 0709.2377) and extend for NR particles
- **We do this matching fully at one loop**

- At this stage combine results with Sommerfeld calculation

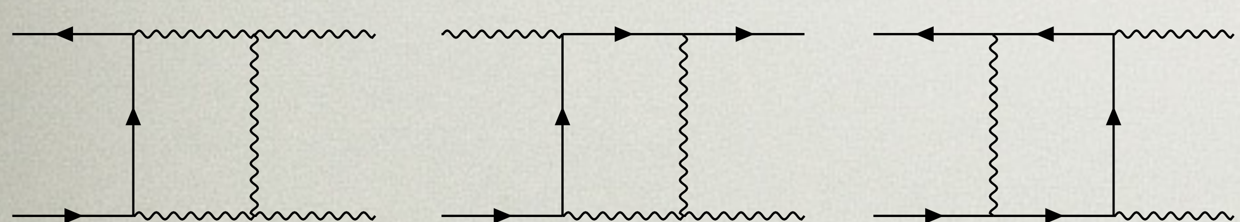
High Scale Matching: 28 Graphs at 1-loop

- Standard 1-loop matching calculation
- Use dim-reg so all EFT graphs are scaleless - we can get the matching just from the full theory graphs

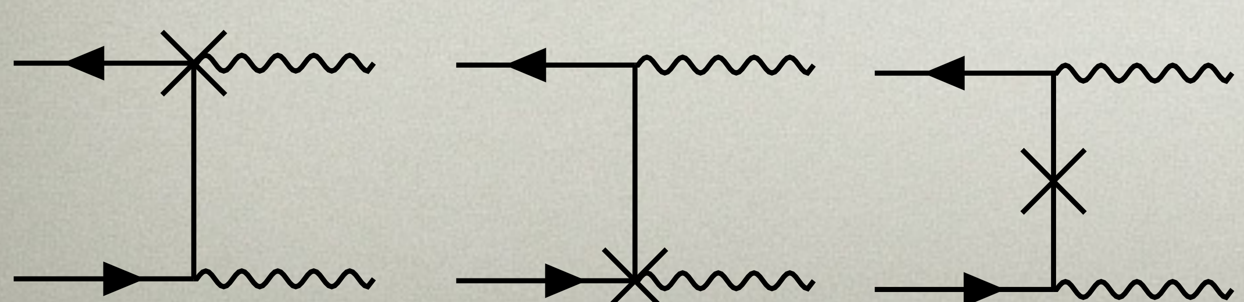
Triangles



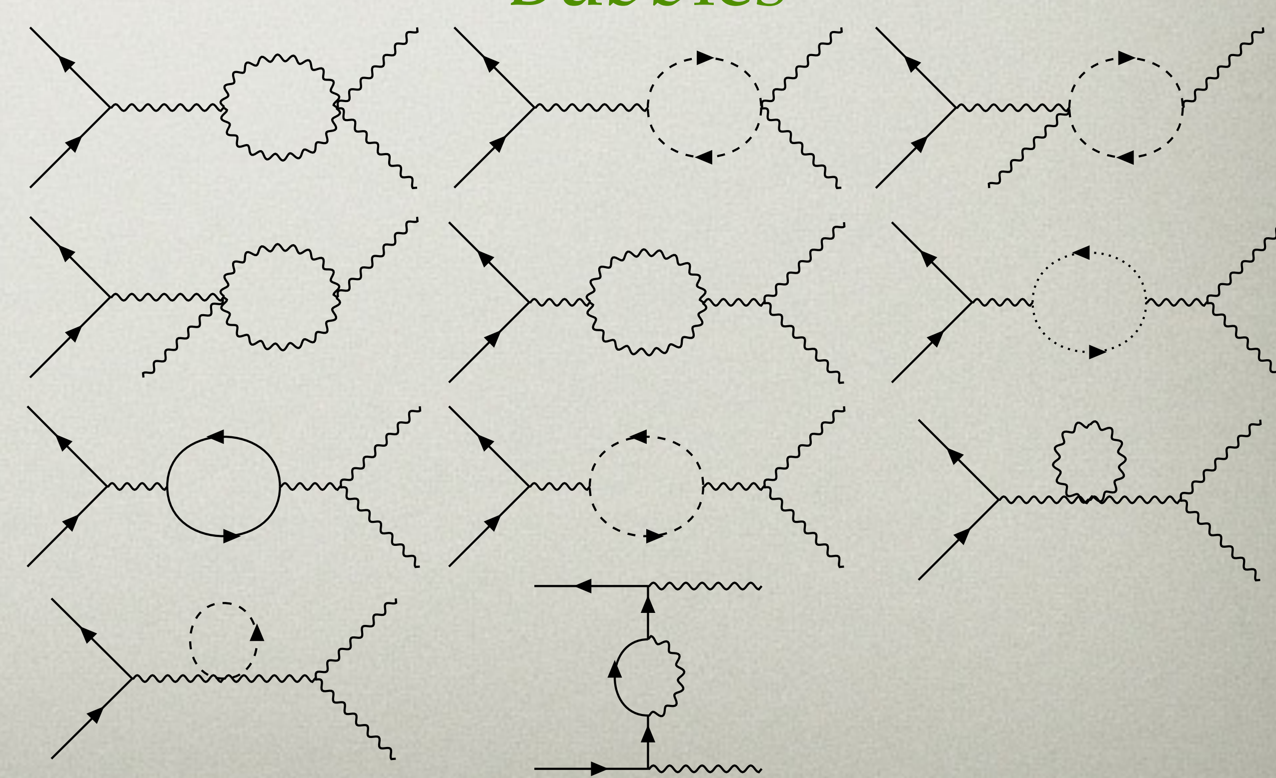
Boxes



Relevant Counter-terms

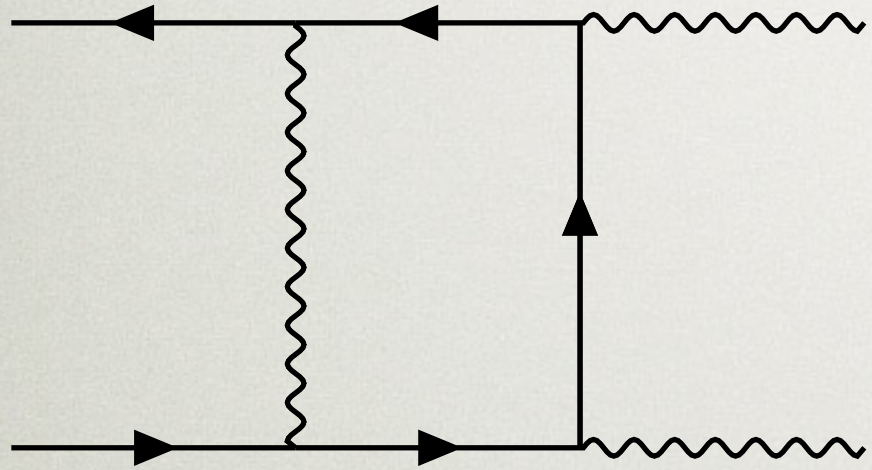


Bubbles

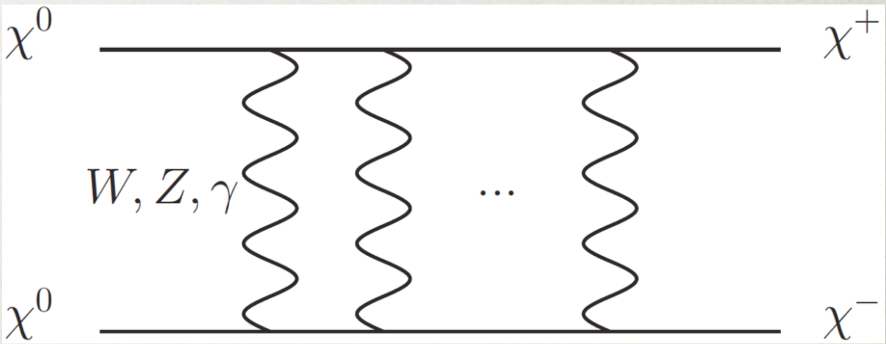


One-loop Highlights: Maintaining Factorisation

- **Aim:** extend calculation of Sudakov logs to higher order, keeping it separate from Sommerfeld contribution to avoid double counting
- But the following box graph has contributions from both:



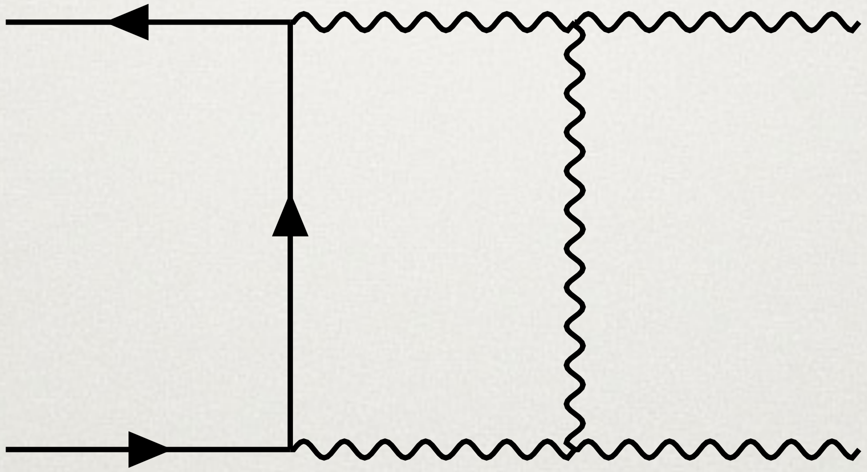
c.f.



- We exploit a central feature of the Sommerfeld enhancement to remove it entirely: its scaling as 1/v
- Setting v=0 at the start of our calculation (justified as DM is non-relativistic), means 1/v contributions now scale as power divergences
- As we calculate in dim-reg, power divergences are sent to 0!
- This rigorously ensures we are not double counting

One-loop Highlights: Maintaining Factorisation

- There is one catch to this solution though. Consider the following diagram:



- When doing Passarino-Veltman reduction on the integrals that appear, we find terms proportional to:

$$[s - 4m_\chi^2]^{-1}$$

- which diverge if we set v=0…
- Origin is actually very simple, related to a basic assumption of PVR

One-loop Highlights: Maintaining Factorisation

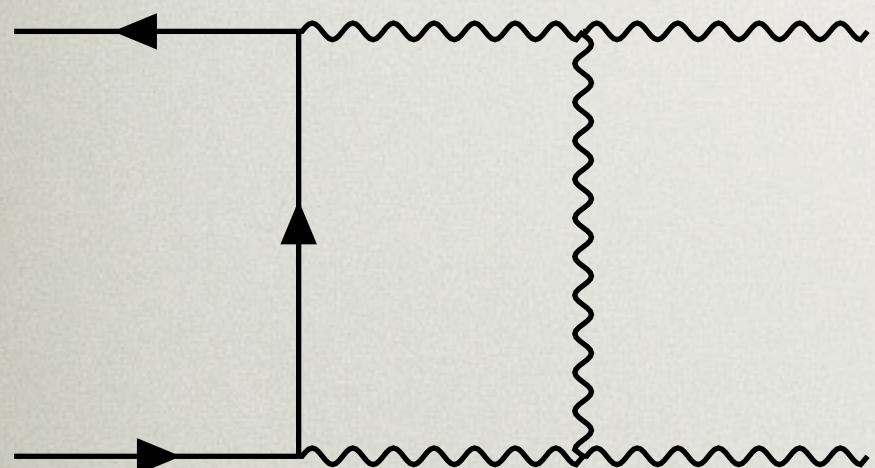
- Basic idea of Passarino-Veltman reduction is to reduce tensor integrals to scalar integrals, exploiting Lorentz invariance to see e.g.

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^\mu}{[k^2 - m_1^2][(k+p_1)^2 - m_2^2][(k+p_2)^2 - m_3^2]} = p_1^\mu C_1 + p_2^\mu C_2$$

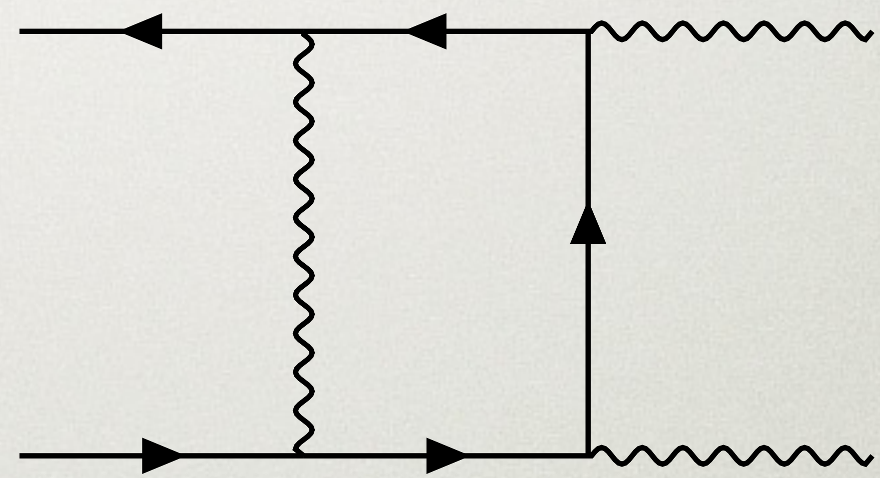
- and then solve for C_1 and C_2
- But this assume p_1 and p_2 are linearly independent. If v=0, then for us p_1=p_2 and this assumption breaks down
- The result is exactly these additional spurious divergences

One-loop Highlights: Maintaining Factorisation

- Most straightforward regulator is to reintroduce a finite velocity, which ensures p_1 and p_2 are not identical, and then remove the regulator by setting v=0 at the end of the calculation. We can only do this because the problem appears in the left graph below but not the right



$\sim v^0$

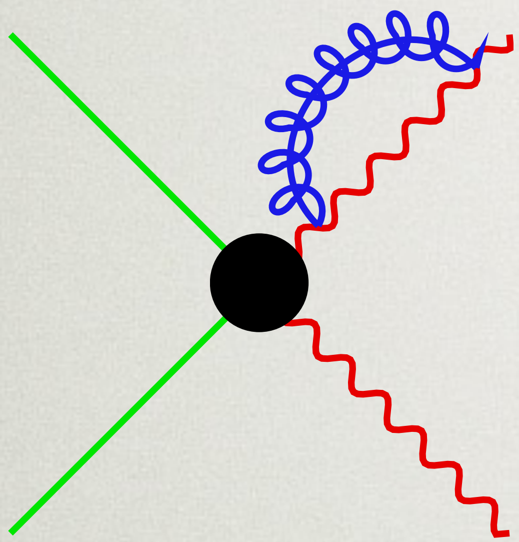


$\sim v^{-1}$

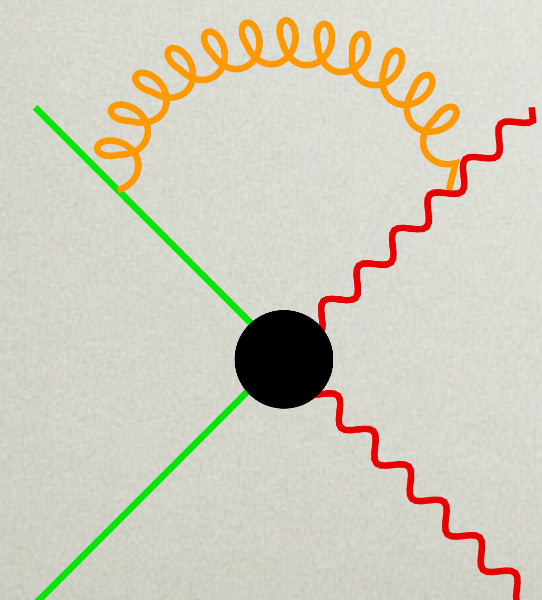
- If this appeared on the right, we would need to also introduce a velocity for the EFT graphs and they would no longer be scaleless
- Then we would need to account for the equivalent 1/v terms appearing in the EFT graphs and subtract them off to maintain factorisation

One-loop Highlights: Low-Scale Organisation

Collinear Graphs



Soft Graphs



Color Dipole

$$\mathbf{T}_i \cdot \mathbf{T}_j$$

- Use color operator notation (hep-ph/9602277)
- Allows color structure to be greatly simplified

Color Diagonal

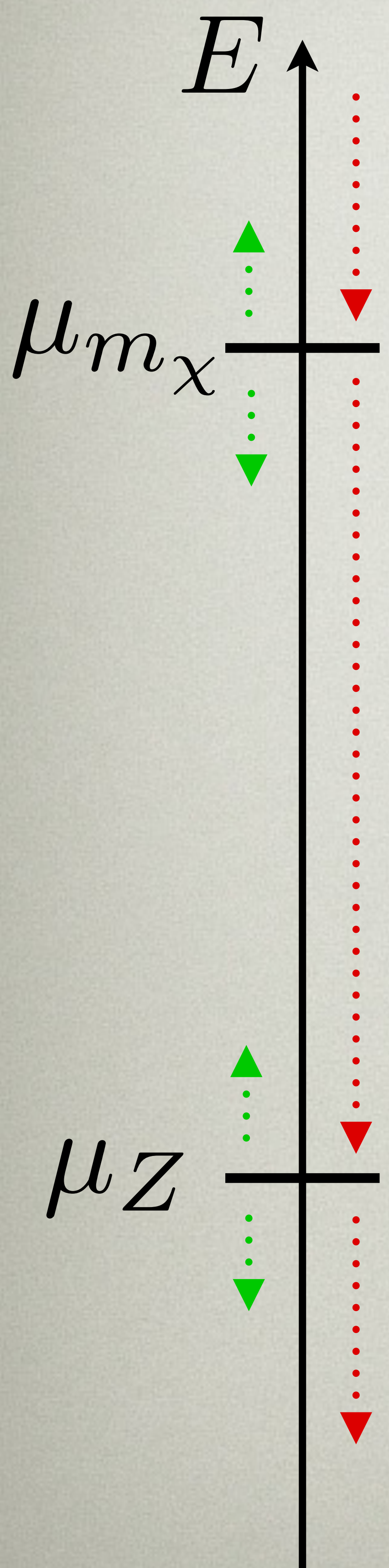
- Does not mix operators
- Process independent
- Depends on details of external particle

Color Non-Diagonal

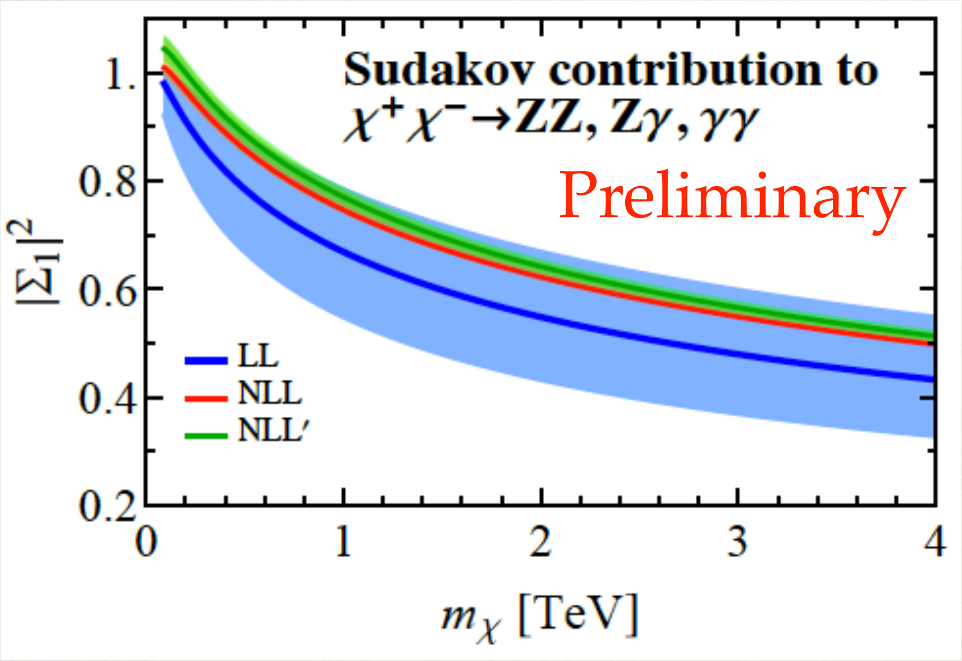
- Mixes operators
- Hardest parts can be calculated in the unbroken theory

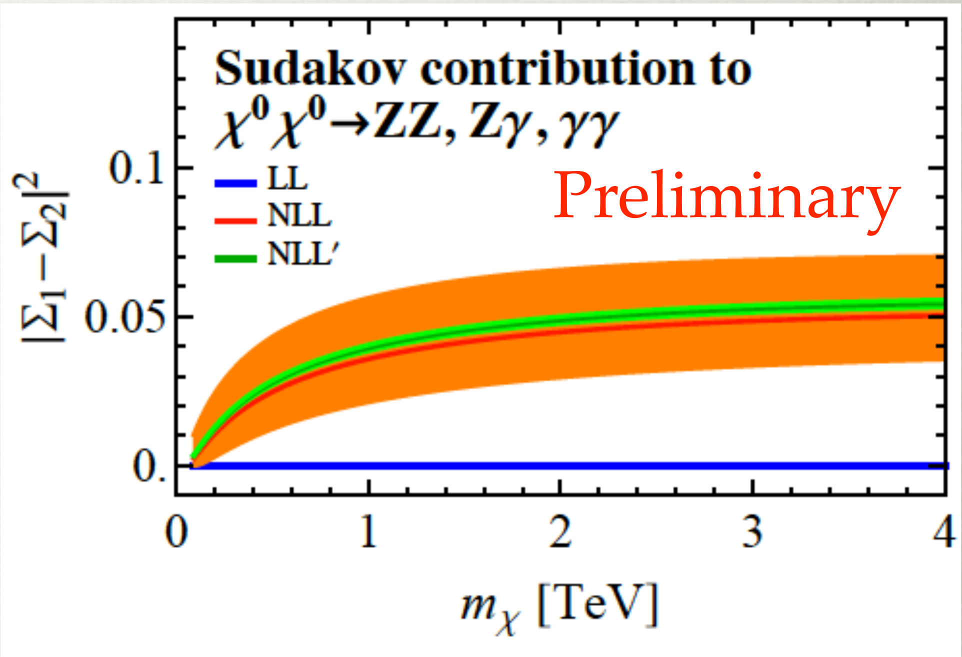
Use $\mathrm{SCET}_{\mathrm{EW}}$ formalism (e.g. 0709.2377, 0909.0012, 0909.0947) extended for NR particles

Results: NLL'=NLL+O(α)

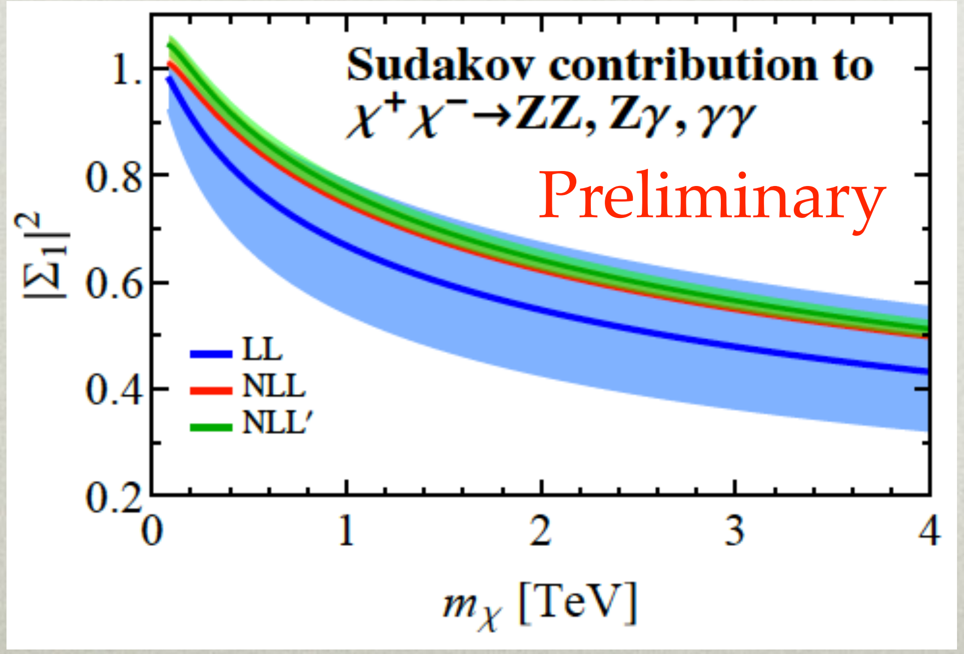


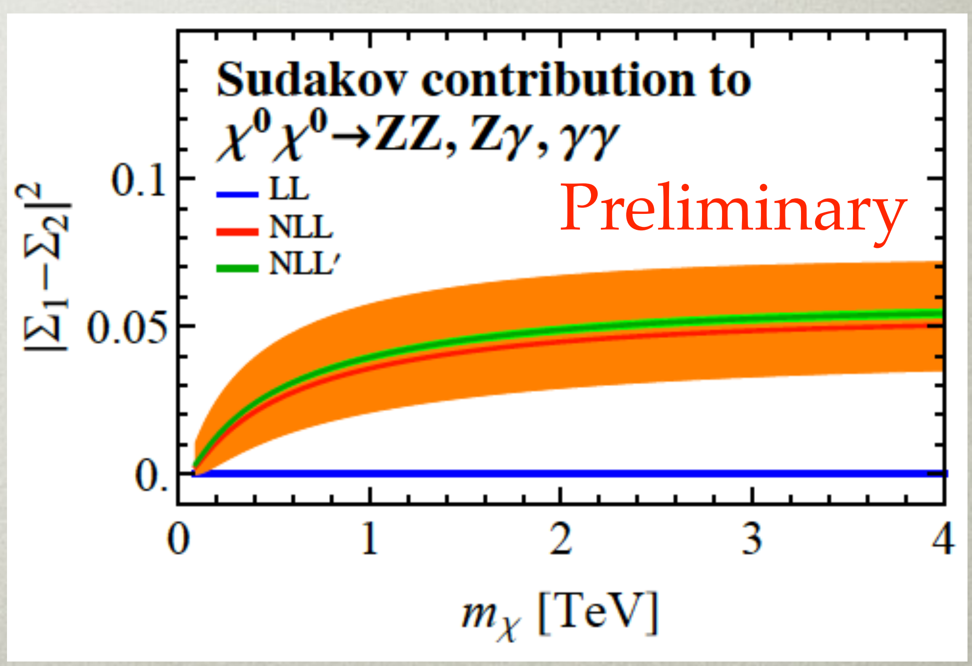
High Scale Variation



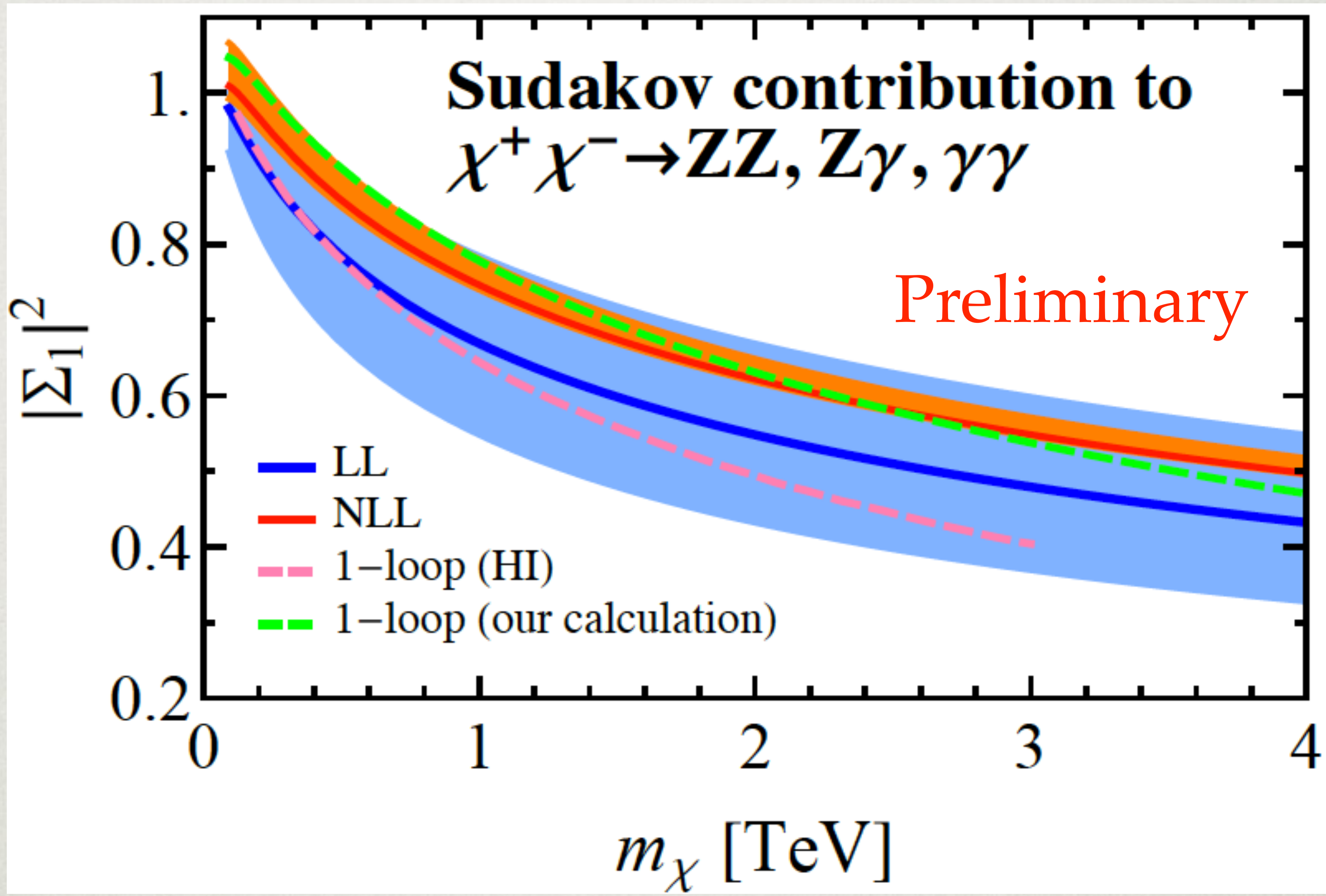


Low Scale Variation



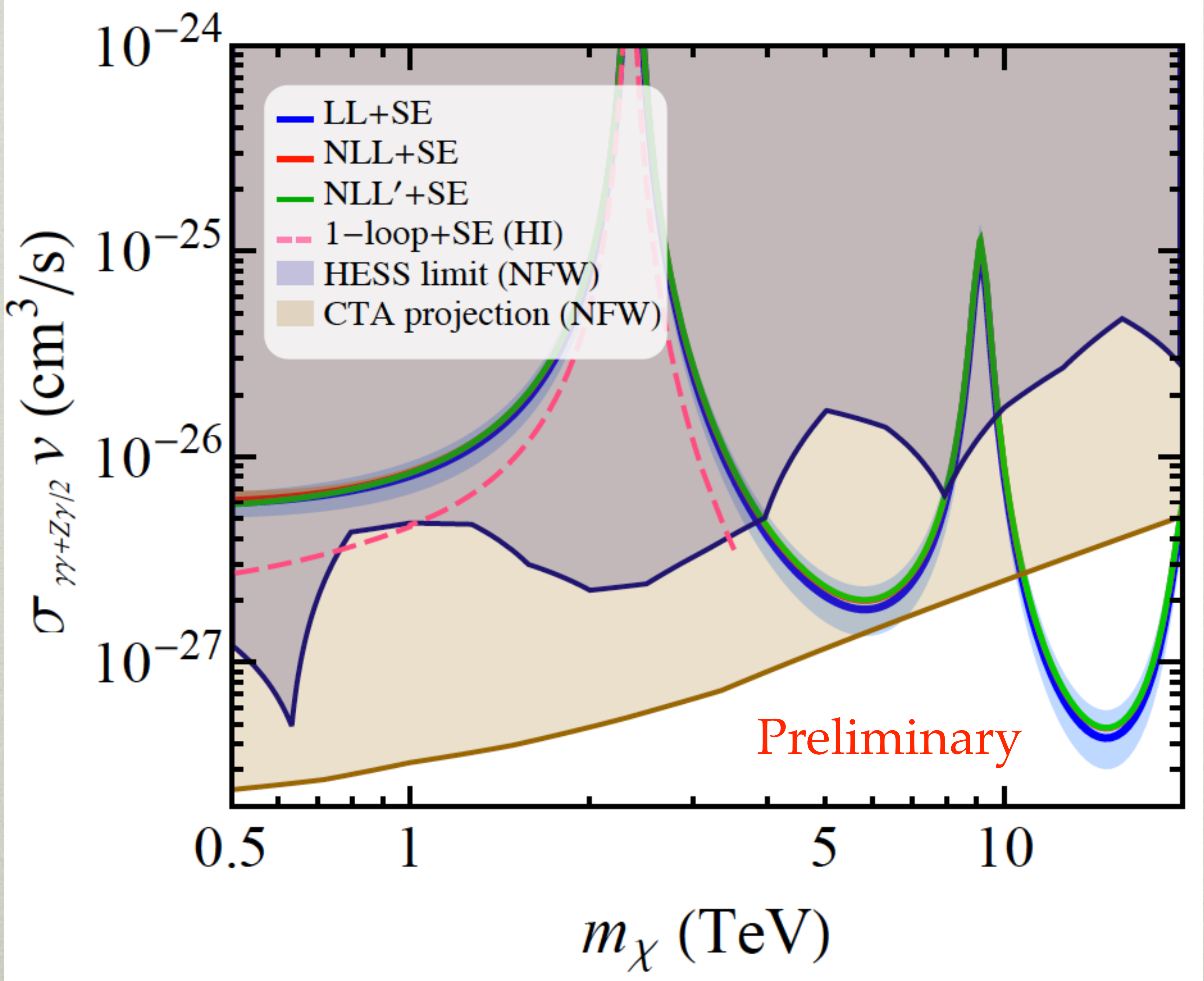


Results: Pure One-Loop Correction



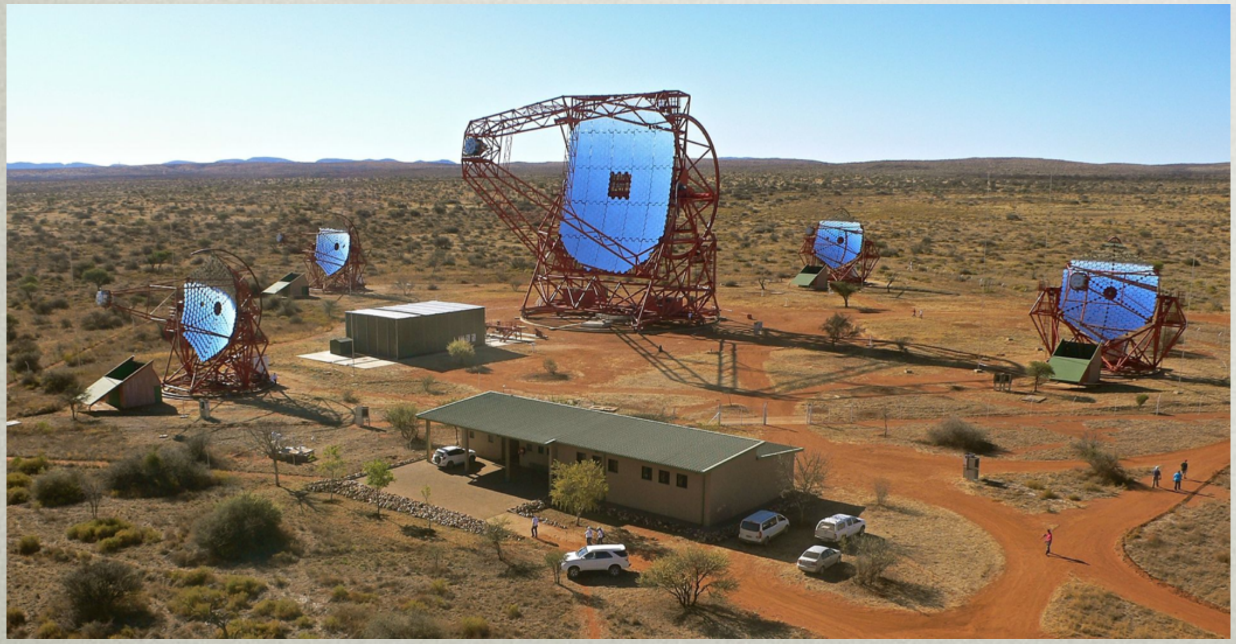
- Turning off the running can take our pure loop results and compare to HI - our result is in good agreement with NLL
- Similar shape to HI, disagreement largely a constant offset that could be due to a difference in finite terms
- Regardless our full analytic result is in a form that can be used to extend the calculation to NNLL

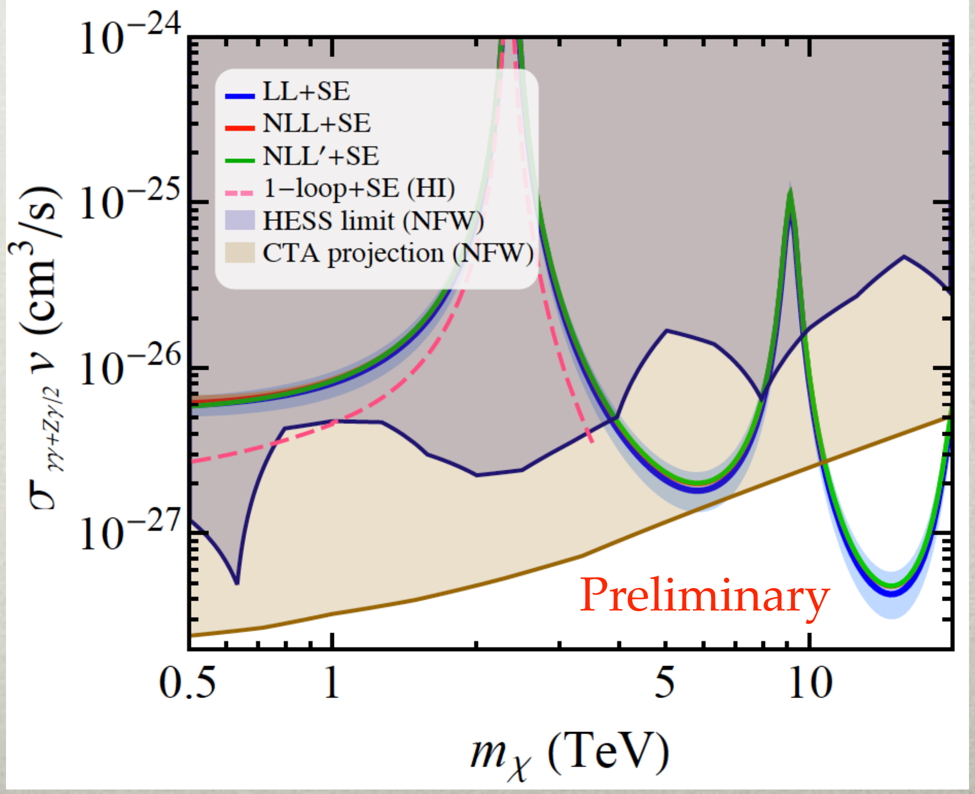
Results: Full Combination



Conclusion

- Accurate calculations of the expected indirect detection signal for TeV scale dark matter requires a careful treatment of two different loop effects:
  - Sommerfeld Enhancement - order of magnitude corrections
  - Sudakov Logarithms - order 50% corrections
- Our NLL′ calculation brings total uncertainty to 1% level





Backup Slides

Order Counting for Sudakov Double Logs

$$1 + \alpha L^2 + (\alpha L^2)^2 + (\alpha L^2)^3 + \ldots \qquad \text{LL}$$

$$\alpha L \left[1 + \alpha L^2 + (\alpha L^2)^2 + (\alpha L^2)^3 + \ldots\right] \qquad \text{NLL}$$

$$\alpha \left[\underline{1} + \alpha L^2 + (\alpha L^2)^2 + (\alpha L^2)^3 + \ldots\right] \qquad \text{NNLL}$$

NLO contribution

Large Electroweak Sudakovs at the LHC

- Although new in a DM setting - this is hardly unprecedented, and well known in an LHC context
- E.g. Chiu, Golf, Kelley, Manohar (0909.0012) have shown the electroweak Sudakov corrections to transverse WW production can suppress cross section by 40% at 2 TeV
  - Percent correction to transverse WW production in red: