Fully Differential predictions for top-quark pair production at NNLO using STRIPPER

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NNLO – Cross section

\[ \sigma_{h_1h_2}(P_1, P_2) = \sum_{ab} \int_0^1 dx_1 dx_2 \, f_{a/h_1}(x_1, \mu_F^2) \, f_{b/h_2}(x_2, \mu_F^2) \, \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \, \alpha_s(\mu_R^2), \, \mu_R^2, \, \mu_F^2) \]

- Partonic cross section expansion in $\alpha_s$ at NNLO
  \[ \hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)} \]

- NNLO contribution
  \[ \hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{RR} + \hat{\sigma}_{ab}^{RV} + \hat{\sigma}_{ab}^{VV} + \hat{\sigma}_{ab}^{C1} + \hat{\sigma}_{ab}^{C2} \]
  \[ \text{Sum is Finite} \]

- STRIPPER
  - Subtraction scheme to consistently cancel IR - singularities

R: Real radiation
V: Virtual
C: Collinear Factorization

Real IR - poles  Virtual IR - poles

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Subtraction/slicing schemes at NNLO

- **Antenna subtraction**
  - $e^+e^- \rightarrow 3$ jets, $pp \rightarrow 2$ jets, $qq \rightarrow tt, H + jet$

- **Colorful subtraction**
  - $H \rightarrow bb$, $e^+e^- \rightarrow 3$ jets

- **qT – slicing**
  - $pp \rightarrow H$, $pp \rightarrow V$, $pp \rightarrow H + V$, $pp \rightarrow VV$, $qq \rightarrow tt$ (flavour off-diagonal)

- **N-jettiness slicing**
  - $pp \rightarrow H + jet$, $pp \rightarrow W + jet$, $pp \rightarrow Z + jet$, $pp \rightarrow H + V$, $pp \rightarrow \gamma\gamma$
Sector improved residue subtraction (STRIPPER)

**SecToR Improved Phase sPacE for real Radiation**

- Local subtraction scheme for NNLO (no approximations)
  - First formulation  
    - $pp \rightarrow tt$  
      (total cross section, $A_{FB}$ at Tevatron, distributions at Tevatron)
  - $pp \rightarrow H + \text{jet}$, $Z \rightarrow e^+ e^-$, Muon – decay, $b$ – decay, $top$ – decay, single top production
  - Generalization to 4 dimensions  
    - $pp \rightarrow \tau^+ \tau^-$

- References:
  - [Czakon, '10, '11]
  - [Czakon, Fiedler, Mitov; '13, '15]
  - [Czakon, Fiedler, DH, Mitov; '16]
  - [Boughezal, Caola, Melnikov, Petriello, Schulze; '13 '14]
  - [Boughezal, Melnikov, Petriello, '11]
  - [Caola, Czernecki, Liang, Melnikov, Szafron, '14]
  - [Brucherseifer, Caola, Melnikov, '13, '13, '14]
STRIPPER – Main Idea

- Numerical cancellation of IR – poles between NNLO contributions

- Example: Double real radiation (most complicated)

\[
\hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2s} \frac{1}{N_{ab}} \int d\Phi_{n+2} \langle M_{n+2}^{(0)} M_{n+2}^{(0)} \rangle F_{n+2}
\]

1) Use selector functions to split phase space into triple and double collinear sectors

2) Use physical parametrization (angles, energies)  

3) Physical sector decomposition: Factorization of non-commuting singularities  

4) Generate subtraction terms  

5) Laurent series in \( \epsilon \) \( \rightarrow \) numerical integration of all coefficients  

[Frixione, Kunszt, Signer (FKS); ’95 ]

[Binoth, Heinrich; ’00 ]
STRIPPER – General formulation

\[ \hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{RR} + \hat{\sigma}_{ab}^{RV} + \hat{\sigma}_{ab}^{VV} + \hat{\sigma}_{ab}^{C1} + \hat{\sigma}_{ab}^{C2} \]

- Each contribution is a Laurent series in \( \epsilon \)
- Separation of independently finite contributions (check number of unresolved particles)
  - Finite contribution (all particles resolved)
  - Single unresolved \( |\mathcal{M}_{n+1}^{(0)}\rangle \)
  - Double unresolved \( |\mathcal{M}_{n}^{(0)}\rangle \)
  - Finite Remainder \( |\mathcal{F}_{n}^{(1)}\rangle = |\mathcal{M}_{n}^{(1)}\rangle - Z^{(1)} |\mathcal{M}_{n}^{(0)}\rangle \)

\[
\hat{\sigma}_{F}^{RR}, \hat{\sigma}_{F}^{RV}, \hat{\sigma}_{F}^{VV}, \hat{\sigma}_{FR} = \hat{\sigma}_{FR}^{RV} + \hat{\sigma}_{FR}^{VV} + \hat{\sigma}_{FR}^{C2},
\]

\[
\hat{\sigma}_{SU} = \hat{\sigma}_{SU}^{RR} + \hat{\sigma}_{SU}^{RV} + \hat{\sigma}_{SU}^{C1}, \quad \hat{\sigma}_{DU} = \hat{\sigma}_{DU}^{RR} + \hat{\sigma}_{DU}^{RV} + \hat{\sigma}_{DU}^{VV} + \hat{\sigma}_{DU}^{C1} + \hat{\sigma}_{DU}^{C2}
\]

- Make sure that SU and DU cancel independently (\( \rightarrow \) resolved particles in 4 dimensions)
Differential top-quark pair production at NNLO

[Czakon, DH, Mitov; 2015, 2016]
Top-quark pairs at the LHC

- Total cross section measured (~ 7 %)
  \[ \sigma_{t\bar{t}}(13 \text{ TeV}) \approx (800 \pm 50) \text{ pb} \]

- Uncertainty of the prediction
  - LO (30 %) → NLO (15%) → NNLO + NNLL (5%)

→ Percent-level precision required

- Precision tests of the Standard Model
- Background for many searches and processes (Higgs, New Physics, ...)
- Constrain gluon PDF at high x
- ...

...
Scale dependence and best scale choice

[Čakon, DH, Mitov; 2016]

\[ \sigma_{h_1 h_2}(P_1, P_2) = \sum_{ab} \int_0^1 dx_1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2) \]

- What is the “best” (dynamical) scale?
  - Difference between different dynamical scales could be as large as difference between dynamical scale and fixed scale
  - Precision predictions only possible for deliberately chosen scale
  - Comparative study of perturbative convergence based on different scales

- Selection of the “correct” scale is based on the following criteria:
  - Perturbative convergence for both total and differential cross section
  - Limiting behavior: Low \( p_T (m_t) \): ~ \( m_{\text{top}} \) ↔ High \( p_T (m_t) \): ~ \( p_T \)
  - Restriction to simple functional forms studied in the past (\( H_T, m_t, \ldots \) )
Scale dependence – Total cross section

- Look for convergence
  - Scale value which minimizes difference
    - NLO → NNLO → (NNLO + NNLL)
  - Best convergence: $\mu_0 < m_{\text{top}}$
  - Little dependence on PDF set at NNLO

- Value of NNLO cross section at point of best convergence equals the NNLO+NNLL at the usual canonical scale $\mu_0 = m_{\text{top}}$

→ Therefore: Resummation has negligible impact on the total cross section at the point of fastest convergence

[Czakon, DH, Mitov; 2016]
Scale dependence – Differential Distributions

• Main guidance is perturbative convergence to discriminate between scales
  
  • Invariant mass distribution
    \[ \mu_0 = \frac{H_T}{4} \checkmark \]
    \[ H_T = \sqrt{m_t^2 + p_T^2} + \sqrt{m_{\bar{t}}^2 + p_{\bar{T}}^2} \]
    
  • Limiting behaviour
    \[ \mu_0(p_T \to 0) \to \frac{m_t}{2} \]
    \[ \mu_0(p_T \to \infty) \to \frac{(p_{T,t} + p_{T,\bar{t}})}{4} \]
    
  • Scales based on the invariant mass itself
    \[ \mu \propto m_{t\bar{t}} \ \times \]

[Czakon, DH, Mitov; 2016]
Scale dependence – Differential Distributions

• Main guidance is perturbative convergence to discriminate between scales

• Choose individual scales for top and antitop $p_T$

• Transverse mass scale

$$\mu_0 = \frac{1}{2} m_T(t/\bar{t}) = \frac{1}{2} \sqrt{m_t^2 + p_{T,t/\bar{t}}^2}$$

• Average distributions afterwards

Different scale choices for different observables
Differential Distributions @ 13 TeV

Dynamical scales → extended kinematical regime

- Comparison with data → Good agreement
Bump hunting in top-pair events

(Example: 750 GeV)

[Czakon, DH, Mitov; 2016]
Bumps in top-pair invariant mass distribution

- Minimize theory uncertainty → choose appropriate normalization

- Trade off between experimental uncertainty and theory uncertainty to choose N

- Minimize dependence on the top-mass $\ll 1\%$, checked at NLO

- Analytic fit of the distribution allows flexible rebinning

[NNLO scale + (approx.) PDF uncertainty added in quadrature]
Bumps in top-pair invariant mass distribution

- Discriminate Signal from Background

- Signal Model (BSM) from [Hespel, Maltoni, Vryonidou 2016] → 1.1 pb

- Significance depends on the bin-width
Bumps in top-pair invariant mass distribution

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Bumps in top-pair invariant mass distribution

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- Significance depends on the bin-width
Bumps in top-pair invariant mass

- Discriminate Signal from Background
- Significance depends on the position of the bin as well

\[
significance = \frac{(\text{SM} + \text{BSM})_{\text{central}} - (\text{pure SM})_{\text{central}}}{(\text{pure SM})_{\text{error}}}
\]

\[N = 1\]

\[N = \sigma(600 \text{ GeV} < m_{t\bar{t}} < 700 \text{ GeV})\]
Summary and Outlook

- Implementation of the Sector improved residue subtraction (STRIPPER)
- Applied to differential top-quark pair productions
  - High quality predictions for LHC at 8 TeV and 13 TeV
  - Precision could be used for new physics searches (Example at 750 GeV)

Outlook

- Combine NNLO-QCD with NLO-EW (published soon, in collaboration with Pagani, Tsinikos and Zaro)
- Include top-quark decays at NNLO
Back Up Slides
STRIPPER - Implementation

- General purpose event generator for NNLO computation
- Based on four-dimensional formulation of the subtraction scheme
- Complete independent implementation
- SM tree-level matrix elements are included [vanHameren, Bury; ’09, ’15]
- Process independent: User has to interface the one-loop and two-loop finite contributions
- Speed: Monte Carlo over processes and polarizations
- Simultaneous computation of:
  - Different PDF sets (LHAPDF interface)
  - Different renormalization and factorizations scales
  - Different observables
Differential Distributions @ 8 TeV

- **$P_T$ ot the top**
- **Rapidity of the top-pair**
- **Invariant mass**

- **NNLO has important impact (Good perturbative convergence)**
- **Good agreement with data → [CMS 2015, ATLAS 2015]**
- **However: Results with fixed scales applicable only to limited kinematical range**
Scale dependence – Differential Distributions

- Comparison between different scale choices
  - Difference within uncertainty
  - Main impact on scale dependence at high values and the K-factor
- Independence of PDF sets has been checked

[Czakon, DH, Mitov; 2016]
Differential Distributions @ 13 TeV

- PDF - dependence

- Useful to constrain PDF sets

[Czakon, DH, Mitov; 2016]