Soft Gluon Resummation for associated $t\bar{t}H$ Production at the LHC

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arXiv:1509.02780 and in preperation

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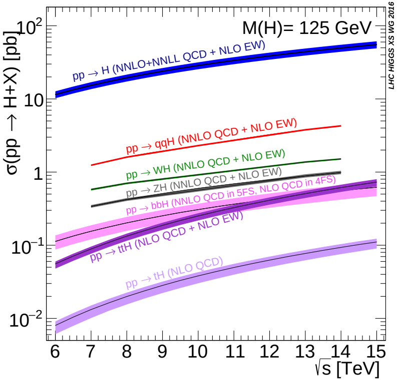
Importance of $pp \to t\bar{t}H$

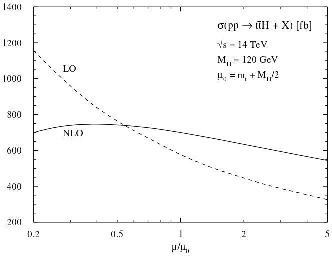
- A Higgs boson with a mass close to 125 GeV
- Precision study needed to determine if it is SM Higgs
- $t\bar{t}H$ direct way to access Yukawa coupling



[LHCHXSWG]

Current status of $pp \to t\bar{t}H$

- QCD Corrections up to NLO *[Beenakker et al. , '02] [Dawson et al. , '02]*
- Matched to parton showers by: aMC@NLO *[Frederix et al. , '11]*, PowHel *[Garzelli et al. , '11]*, Sherpa *[Hoeche et al., '12]*, POWHEG-BOX *[Hartanto et al. , '14]*
- Electroweak correction *[Frixione et al. , '14,'15][Zhang, '14]*
- Including top decays *[Denner, Feger, '15]*
- Absolute threshold at NLL *[Kulesza, Motyka, Stebel, VT, '15]*
- Expansion of NNLL in SCET *[Broggio et al., '15]*



[Beenakker et al. , '02]

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Why resummation for $t\bar{t}H$?

Gains

- NNLO corrections out of reach
- Resummation can help reduce scale uncertainty
- Good process to start:
  - Simple color structure
  - Massive particles $\to$ no final state collinear divergences

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Pitfalls

- $2 \to 3$ phase space supressed near threshold ($\sigma \propto \beta^4$)
- Small corrections from near absolute threshold

Definition of Threshold

Q-approach

Threshold variable $\hat{\tau}_Q = \frac{Q^2}{\hat{s}}$

Q^2: the invariant mass final state particles

$$1 - \hat{\tau}_Q = 1 - \frac{Q^2}{\hat{s}}$$

$$\sim \frac{\text{energy of the emitted gluons}}{\text{total available energy}}$$

M-approach (absolute threshold)

Threshold variable $\hat{\tau}_M = \frac{M^2}{\hat{s}}$

M: the sum of final state masses

$$1 - \hat{\tau}_M = 1 - \frac{M^2}{\hat{s}}$$

$$\sim \frac{\text{maximum energy of the emitted gluons}}{\text{total available energy}}$$

$\sqrt{\hat{s}}$: the partonic center of mass energy

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Logarithms

Q-approach

The IR divergences lead to logarithms:

$$(1-\hat{\tau}_Q)^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta\,(1-\hat{\tau}_Q) + \left(\frac{1}{1-\hat{\tau}_Q}\right)_+ - 2\epsilon\left(\frac{\log(1-\hat{\tau}_Q)}{1-\hat{\tau}_Q}\right)_+$$

$$\boxed{\alpha_s^n\left(\frac{\log^m(1-\hat{\tau}_Q)}{1-\hat{\tau}_Q}\right)_+}$$

In general logarithms of $1-\hat{\tau}_Q$

M-approach

After integration over $\hat{\tau}_Q$: logarithms of $1-\hat{\tau}_M$:

$$\boxed{\alpha_s^n\log^m(1-\hat{\tau}_M)}$$

Logarithms become large in threshold limit: $\hat{\tau} \to 1$

Mellin Transform

Mellin transform is used with respect to τ (needed for factorization of phase space):

$$\boxed{\begin{aligned} \tilde{\sigma}_{pp\to t\bar{t}H}(N) &\equiv \int_0^1 d\tau\, \tau^{N-1} \sigma_{pp\to t\bar{t}H}(\tau, \mu_R, \mu_F) \\ &= \sum_{i,j} \tilde{f}_{i/p}(N+1, \mu_F) \tilde{f}_{j/p}(N+1, \mu_F) \tilde{\hat{\sigma}}_{ij\to t\bar{t}H}(N, \mu_R, \mu_F) \end{aligned}}$$

- $\tilde{f}_{i/p}(N+1, \mu_F)$: Mellin transform with respect to x
- $\tilde{\hat{\sigma}}_{ij\to t\bar{t}H}(N, \mu_R, \mu_F)$: Mellin transform with respect to $\hat{\tau}$

$\log^n(1-\hat{\tau}) \Rightarrow \log^n N$ and threshold $\hat{\tau} \to 1 \sim N \to \infty$

First application for $2 \to 3$ in Mellin space

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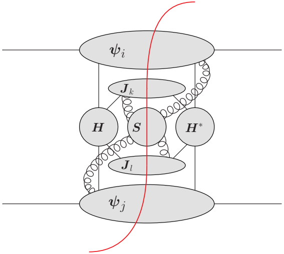
General factorization

In general cross section factorizes into:

$$\hat{\sigma}_{ij\to kl\ldots} = H_{ij\to kl\ldots,IJ} \otimes \psi_i \otimes \psi_j \otimes S_{JI} \otimes J_k \otimes J_l \ldots$$

- $H_{ij\to kl,IJ}$ Hard function
- $\psi_{i,j}$ Initial state collinear emission
- $J_{k,l,\ldots}$ Final state collinear emission
- S_{JI} Soft emission



Each of these functions is computed through renormalization group equations

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Differential formalism

General formalism developed for $2 \to 2$. *[Kidonakis,Oderda,Sterman'98]*, *[Laenen,Oderda,Sterman'98]*

Using a infra-red safe weight (ω) to describe the soft limit of the emission.
For $2 \to 2$:

- Pair invariant mass $\omega_{\rm PIM} = z = 1 - Q^2/\hat{s}$
- One particle inclusive $\omega_{\rm 1PI} = s_4 = ((p_1 + p_2 - p_3)^2 - m_4^2)/\hat{s}$

Resummation for differential distributions

$$\boxed{W = \omega_1 c_1 + \omega_2 c_2 + \omega_s + \omega_k + \omega_l + \dots}$$

$$\int dW e^{-NW} \frac{d\sigma_{ij\to kl\dots}}{d\hat{\Pi}_n}$$

$$= \int dW e^{-NW} H_{IJ}\left(\hat{\Pi}_n\right) \int d\omega_1 d\omega_2 d\omega_s d\omega_k d\omega_l \dots$$

$$\delta\left(W - \omega_1 c_1 - \omega_2 c_2 - \omega_s - \omega_k - \omega_l - \dots\right)$$

$$\psi_{i/i}\left(\omega_1\right) \psi_{j/j}\left(\omega_2\right) S_{JI}\left(\omega_s\right) J_k\left(\omega_k\right) J_l\left(\omega_l\right) \dots$$

$$= H_{IJ}\left(\hat{\Pi}_n\right) \tilde{\psi}_{i/i}\left(Nc_1\right) \tilde{\psi}_{j/j}\left(Nc_2\right) \tilde{S}_{JI}\left(N\right) \tilde{J}_k\left(N\right) \tilde{J}_l\left(N\right) \dots$$

- $\omega_{1/2}$ Initial state collinear weights
- ω_s Soft-wide angle weight
- $\omega_{k/l\dots}$ Final state collinear weights

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$2 \to 3$ weights for differential notation

$pp \to Q\bar{Q}B + X \qquad \Rightarrow Q^2$

$$z_5 = \frac{\hat{s} - s_{345}}{\hat{s}}$$

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$2 \to 3$ weights for differential notation

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$$z_5 = \frac{\hat{s} - s_{345}}{\hat{s}}$$

$pp \to B + X[Q\bar{Q}] \qquad \Rightarrow P_{T,5}, y_5$

$$v_5 = \frac{(p_1 + p_2 - p_5)^2 - s_{34}}{\hat{s}} = \frac{\hat{s} + \tilde{t}_{15} + \tilde{t}_{25} + m_5^2 - s_{34}}{\hat{s}}$$

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$pp \to Q\bar{Q} + X[B] \qquad \Rightarrow P_{T,34}, y_{34}, Q_{34}^2$

$$s_5 = \frac{(p_1 + p_2 - p_3 - p_4)^2 - m_5^2}{\hat{s}} = \frac{\hat{s} + \tilde{t}_{13} + \tilde{t}_{14} + \tilde{t}_{23} + \tilde{t}_{24} + s_{34} - m_5^2}{\hat{s}}$$

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Orders of Resummation

Large logarithms $\log N \equiv L$ for $N \to \infty$

Perturbation needs to be reordered in α_{s} and L:

$$\boxed{\tilde{\sigma} \sim \tilde{\sigma}_{LO} \times \mathcal{C}(\alpha_{\mathrm{s}}) \exp\left[L g_1(\alpha_{\mathrm{s}} L) + g_2(\alpha_{\mathrm{s}} L) + \alpha_{\mathrm{s}} g_3(\alpha_{\mathrm{s}} L) + \cdots\right]}$$

	$\Downarrow$	$\Downarrow$	$\Downarrow$
With orders of precision:	LL	NLL	NNLL
	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$\alpha_{\mathrm{s}}^n \log^{n+1}(N)$	$\alpha_{\mathrm{s}}^n \log^n(N)$	$\alpha_{\mathrm{s}}^{n+1} \log^n(N)$

Exponential functions are universal for initial state emission

[Kodaira, Trentadue, '82][Sterman, '87][Catani, d'Emilio, Trentadue, '88][Catani, Trentadue, '89]

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Color space

Need to project the matrix element onto a color basis.
Use the s-channel color basis:

$q\bar{q}$

$$\mathbf{1}: \quad \delta_{a_2 a_1}\delta_{a_3 a_4}$$

$$\mathbf{8}: \quad T^D_{a_2 a_1} T^D_{a_3 a_4}$$

gg

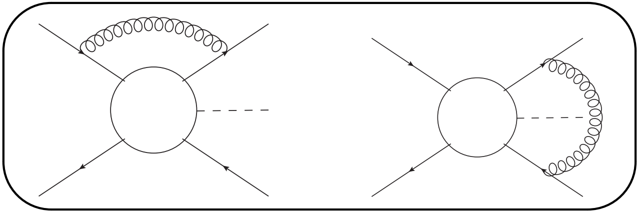
$$\mathbf{1}: \quad \delta^{A_1 A_2}\delta_{a_3 a_4}$$

$$\mathbf{8_S}: \quad T^D_{a_3 a_4} d^{D A_1 A_2}$$

$$\mathbf{8_A}: \quad i T^D_{a_3 a_4} f^{D A_1 A_2}$$

Basis for diagonalization of soft anomalous dimension in absolute threshold limit.
Same as for $t\bar{t}$ production

Soft wide-angle



[Kidonakis et al., '97-'01]

$$\tilde{S}_{ij\to kl}\left(\frac{Q}{\mu N}\right) = \bar{\mathrm{P}}\exp\left[\int_{\mu}^{Q/N}\frac{dq}{q}\Gamma^{\dagger}_{ij\to kl}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right]\tilde{S}_{ij\to kl}$$

$$\times\mathrm{P}\exp\left[\int_{\mu}^{Q/N}\frac{dq}{q}\Gamma_{ij\to kl}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right]$$

Soft anomalous dimension matrix

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,11} = -\frac{\alpha_s}{2\pi} 2C_F \left(L_{\beta_{34}} + 1\right)$$

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,12} = \frac{\alpha_s}{2\pi} \frac{2C_F}{N_C} \Omega_3 = \frac{C_F}{2N_C} \Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,21}$$

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,22} = \frac{\alpha_s}{2\pi} \left[(N_C - 2C_F)\left(L_{\beta_{34}} + 1\right) + N_C \Lambda_3 + (8C_F - 3N_C)\,\Omega_3\right]$$

$$\Omega_3 = \left(T_{13} + T_{24} - T_{14} - T_{23}\right)/2$$

$$\Lambda_3 = \left(T_{13} + T_{24} + T_{14} + T_{23}\right)/2$$

$$T_{ij} = \log\left(\frac{m_j^2 - t_{ij}}{m_j\sqrt{s}}\right) + \frac{i\pi - 1}{2}$$

with $t_{ij} = (p_i - p_j)^2$ and $\beta_{34}^2 = 1 - (m_3 + m_4)^2/s_{34}$

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Agrees with $q\bar{q} \to Q\bar{Q}$

for $p_5 \to 0$ and $m_5 \to 0$

$$\Omega_3 = (T_{13} + T_{24} - T_{14} - T_{23})/2$$

$$\Lambda_3 = (T_{13} + T_{24} + T_{14} + T_{23})/2$$

$$T_{ij} = \log\left(\frac{m_j^2 - t_{ij}}{m_j \sqrt{s}}\right) + \frac{i\pi - 1}{2}$$

with $t_{ij} = (p_i - p_j)^2$ and $\beta_{34}^2 = 1 - (m_3 + m_4)^2 / s_{34}$

$\Downarrow$ $\Downarrow$

For $2 \to 2$: $t_{13} = t_{24}$, $t_{23} = t_{14}$ $\quad$ $s_{34} = \hat{s}$, $\beta_{34} = \beta$

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Soft wide-angle emission absolute threshold

Absolute threshold limit for soft anomalous dimension:

$$2\mathcal{R}e\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}B,IJ,\mathrm{thr.}} = \frac{\alpha_{\mathrm{s}}}{\pi}\mathrm{diag}\left(0, -N_C\right)$$

$$2\mathcal{R}e\Gamma^{(1)}_{gg\to Q\bar{Q}B,IJ,\mathrm{thr.}} = \frac{\alpha_{\mathrm{s}}}{\pi}\mathrm{diag}\left(0, -N_C, -N_C\right)$$

results in soft wide-angle contribution:

$$\Delta_I^{\mathrm{NLL}} = \exp\left[h_2(\alpha_{\mathrm{s}}L, -C_I)\right]$$

With C_I the quadratic Casimir invariant

Hard Matching Coefficient

$$\mathcal{C}(\alpha_{\rm s}) = 1 + \frac{\alpha_{\rm s}}{\pi}\mathcal{C}^{(1)} + \ldots$$

- Massive final state dipoles *[Catani,Dittmaier,Seymour,Trócsányi,'02]*
- Virtual contribution from PowHeg-Box *[Hartanto,Jäger,Reina,Wackeroth,'15]* confirmed by aMC@NLO *[Hirschi,Frederix,Frixione,Garzelli,Maltoni,Pittau,'11]*
- Include Coulomb correction $\frac{1}{\beta_{34}}$

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Matching to Fixed Order

Resummed Cross Section

$$\sigma^{(\mathrm{NLO+NLL})}(\tau) = \sigma^{(\mathrm{NLO})}(\tau) + \int_{\mathrm{CT}} \frac{dN}{2\pi i} \tau^{-N} \tilde{f}_{g/p}(N+1) \tilde{f}_{g/p}(N+1) \times \left[\tilde{\hat{\sigma}}^{(\mathrm{NLL})}(N) - \tilde{\hat{\sigma}}^{(\mathrm{NLL})}(N)\big|_{(\mathrm{NLO})} \right]$$

Matching to fixed order required to avoid double counting.

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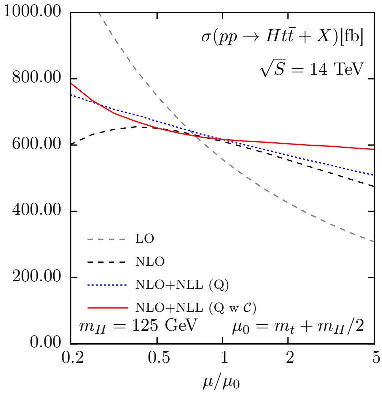
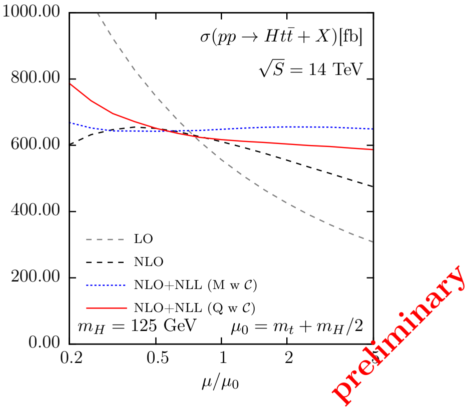
Results

[Kulesza, Motyka, Stebel, VT, '15 and in preperation]

PDFs used: MMHT2014NLO





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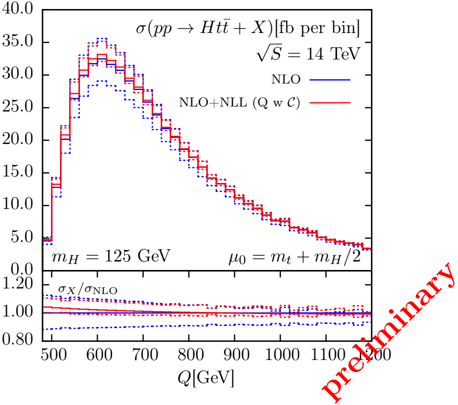

Results

[Kulesza, Motyka, Stebel, VT, '15 and in preperation]

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$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL M with $\mathcal{C}$		NLO+NLL Q with $\mathcal{C}$	
		Value [fb]	K-factor	Value [fb]	K-factor
13	$506^{+5.9\%}_{-9.4\%}$	$537^{+8.2\%}_{-5.5\%}$	1.06	$512^{+5.1\%}_{-6.2\%}$	1.01
14	$613^{+6.2\%}_{-9.4\%}$	$650^{+7.9\%}_{-5.7\%}$	1.06	$619^{+5.2\%}_{-6.4\%}$	1.01

preliminary

Using 7-point method:

$(\mu_F/\mu_0, \mu_R/\mu_0) = \{(0.5, 0.5), (0.5, 1), (1, 0.5), (1, 1), (1, 2), (2, 1), (2, 2)\}$

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Thank you for your attention

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Backup

[Kulesza, Motyka, Stebel, VT, 'in preperation]

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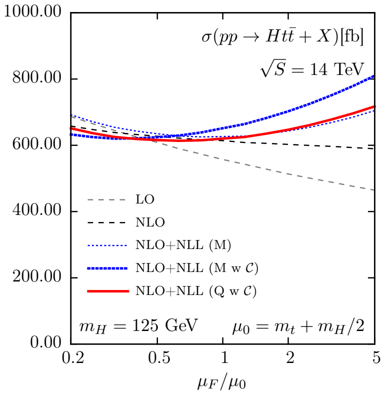

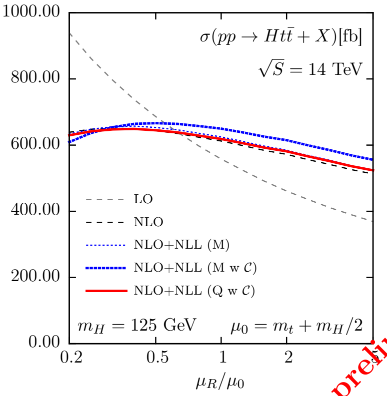

preliminary

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[Kulesza, Motyka, Stebel, VT, in preperation]

PDFs used: MSTW2008NNLO