NLO corrections to Higgs boson pair production in gluon fusion with full top quark mass dependence

Matthias Kerner

LoopFest XV

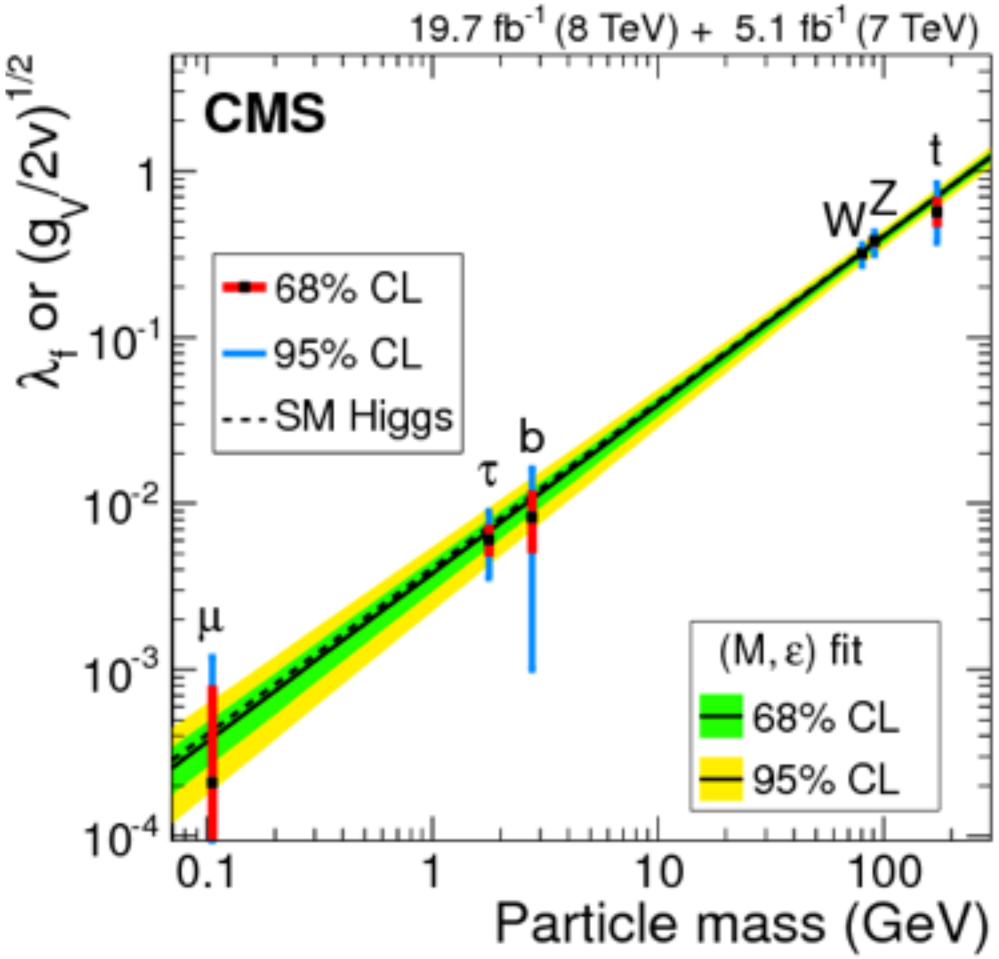
Buffalo — August 17, 2016

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

In collaboration with:
S. Borowka, N. Greiner, G. Heinrich, S. Jones, J. Schlenk, U. Schubert, T. Zirke

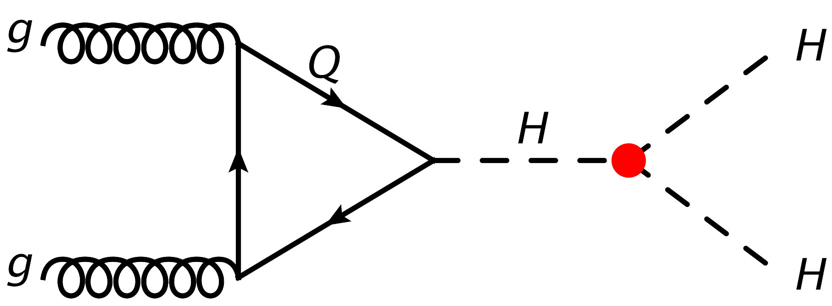
Motivation



Measurements of Higgs couplings agree with SM predictions, but

triple-Higgs coupling not established yet

→ Higgs pair production



$$V(\Phi) = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4$$

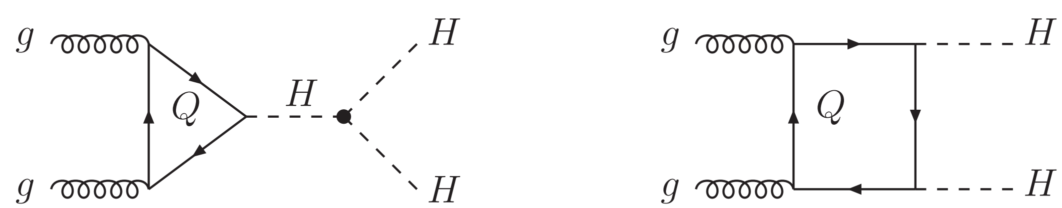
EW symmetry breaking

$$\frac{m_H^2}{2}H^2 + \frac{m_H^2}{2v}H^3 + \frac{m_H^2}{8v^2}H^4$$

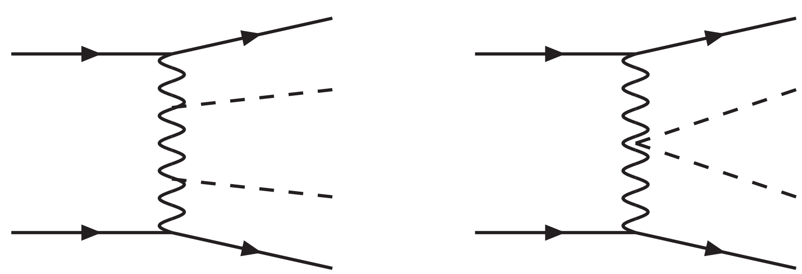
Test of Higgs potential & EW symmetry breaking

Higgs Pair Production channels

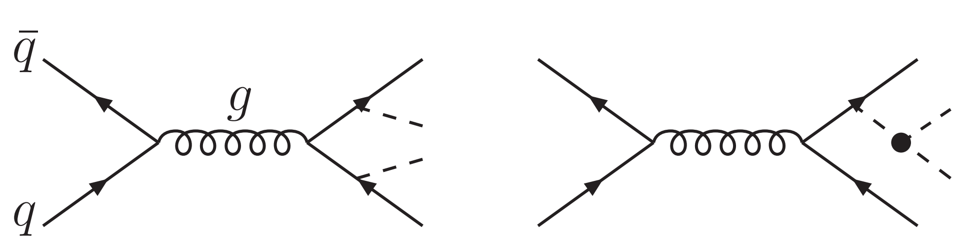
- gluon fusion



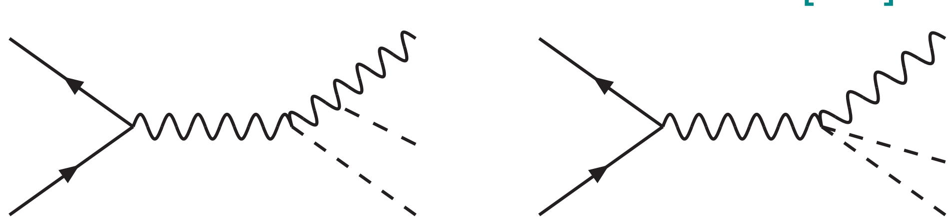
- vector boson fusion NLO: [1,2] NNLO: [3]

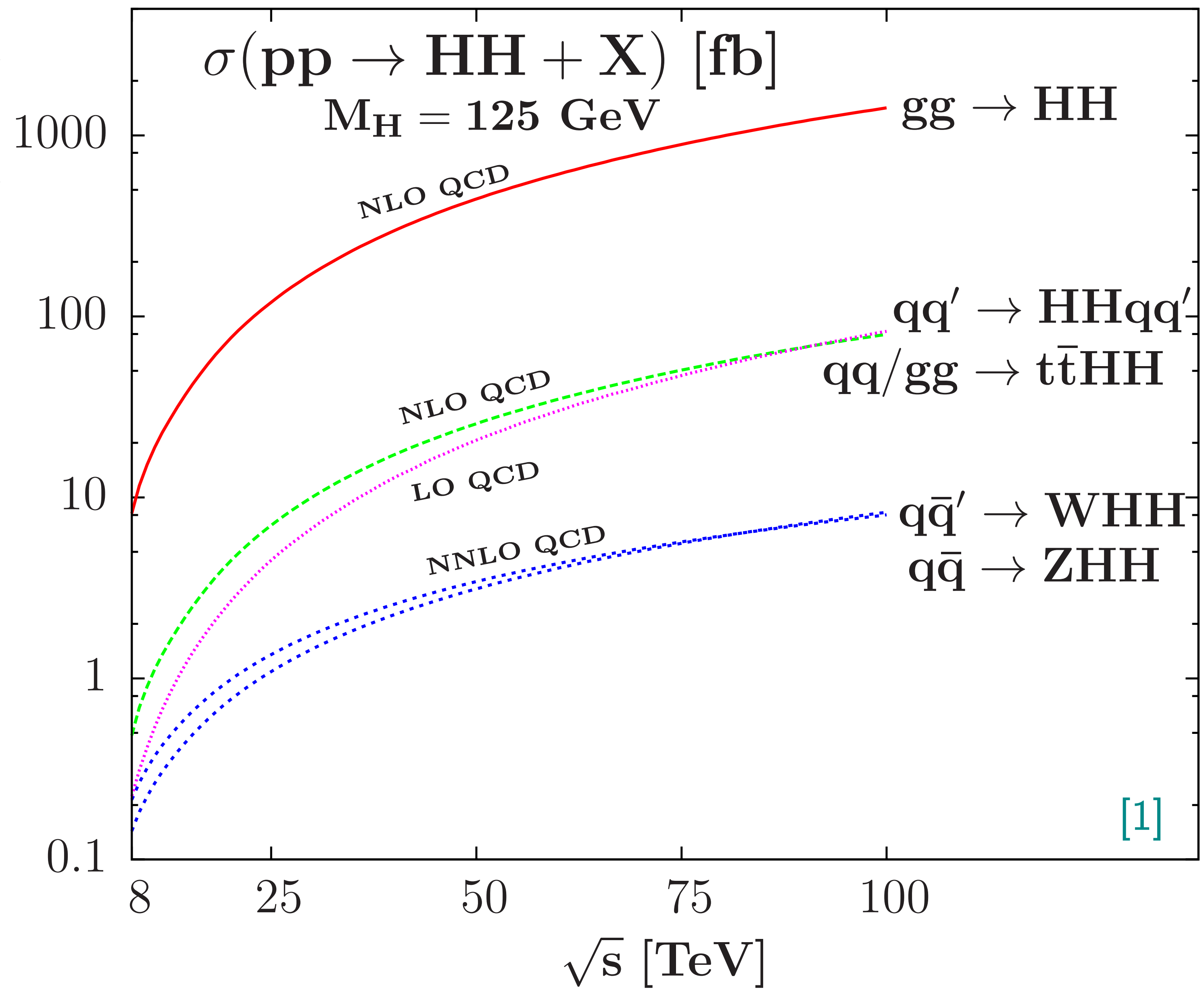


- top-quark associated NLO: [2]



- Higgs strahlung NLO: [1,2] NNLO: [1,4]





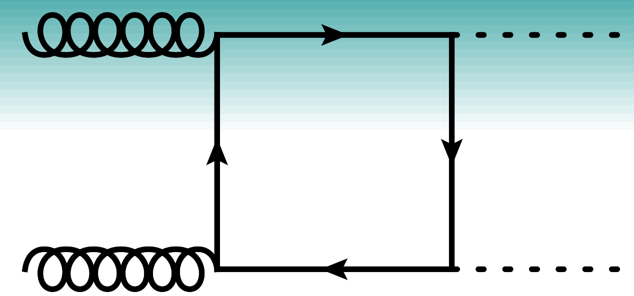
[1] Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira `12

[2] Frederix, Frixione, Hirschi, Maltoni, Mattelaer Torrielli Vryonidou, Zaro `14

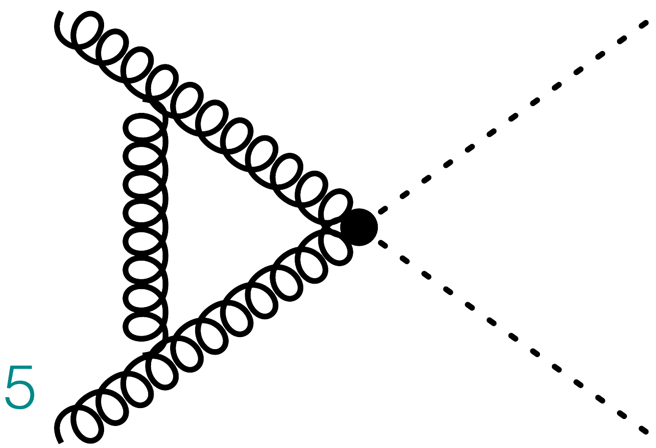
[3] Ling, Zhang, Ma, Guo, Li, Li `14

[4] Li, Wang `16

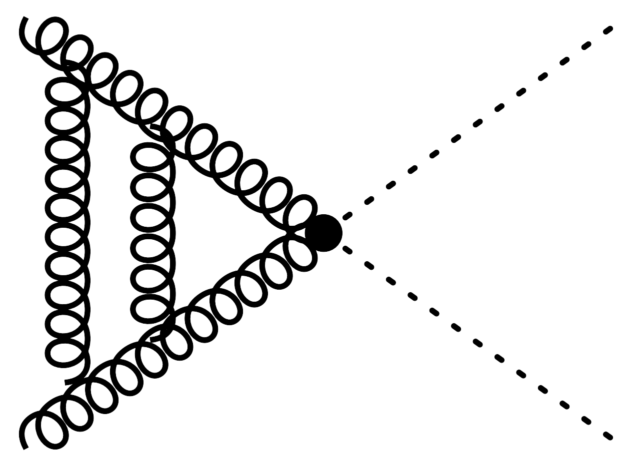
gg→HH known results



1. LO, including full m_T Glover, van der Bij `88
2. NLO ($m_t \rightarrow \infty$ limit) **K ≈ 2**
   Dawson, Dittmaier, Spira `98
   - including full m_T dependence in real radiation
     Maltoni, Vryonidou, Zaro `14
   - including $1/m_T$ expansion
     Grigo, Hoff, Melnikov, Steinhauser `13; Grigo, Hoff, Steinhauser `15
     Degrassi, Giardino, Gröber `16



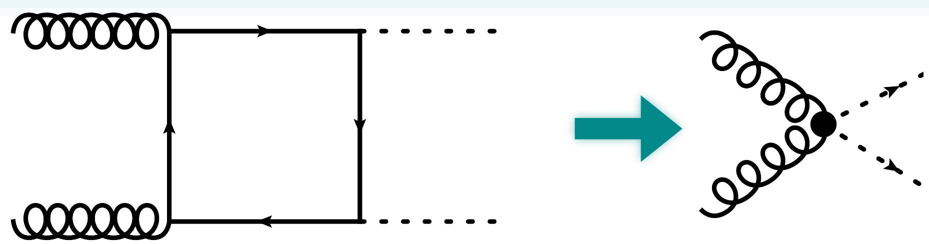
3. NNLO ($m_t \rightarrow \infty$ limit) **+20%**
   de Florian, Mazzitelli `13
   - including all matching coefficients
     Grigo, Melnikov, Steinhauser `14
   - including $1/m_T$ expansion
     Grigo, Hoff, Steinhauser `15
   - NNLL soft gluon resummation
     Shao, Li, Li, Wang `13
   - NNLL + NNLO matching
     de Florian, Mazzitelli `15
   - fully differential
     de Florian, Grazzini, Hanga, Kallweit,
     Lindert, Maierhöfer, Mazzitelli, Rathlev `16



HEFT and approximated NLO results

- $m_T \to \infty$ limit (Higgs EFT)

  (valid for $\sqrt{s} \ll 2\, m_T$)



- Born-improved NLO HEFT

$$d\sigma_{NLO} \approx d\sigma_{NLO}^{HEFT} = \frac{d\sigma_{NLO}(m_t \to \infty)}{d\sigma_{LO}(m_t \to \infty)}\, d\sigma_{LO}(m_t)$$

K ≈ 2

Spira et al. (HPAIR)

- further improvements:

Maltoni Vryonidou, Zaro `14

$$d\sigma_{NLO}^{V,HEFT}$$

-10%

$$d\sigma_{NLO}^{R}(m_t)$$

Grigo, Hoff, Melnikov, Steinhauser `13

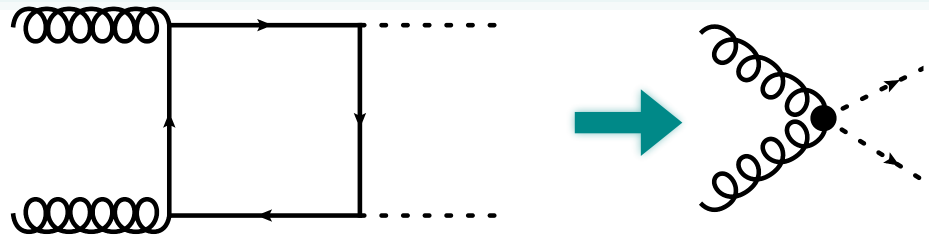
$$\sigma_{exp} = \sum_n^6 c_n \rho^n, \quad \rho = \frac{m_H^2}{m_t^2}$$

$$\sigma^{NLO} = \sigma_{exp}^{NLO} \cdot \frac{\sigma^{LO}}{\sigma_{exp}^{LO}}$$

+10%

HEFT and approximated NLO results

- $m_T \to \infty$ limit (Higgs EFT)
  (valid for $\sqrt{s} \ll 2\,m_T$)



- Born-improved NLO HEFT

$$d\sigma_{NLO} \approx d\sigma_{NLO}^{HEFT} = \frac{d\sigma_{NLO}(m_t \to \infty)}{d\sigma_{LO}(m_t \to \infty)}\, d\sigma_{LO}(m_t)$$

K ≈ 2

Spira et al. (HPAIR)

- further improvements:

Maltoni Vryonidou, Zaro `14

$$d\sigma_{NLO}^{V,HEFT}$$

-10%

$$d\sigma_{NLO}^{R}(m_t)$$

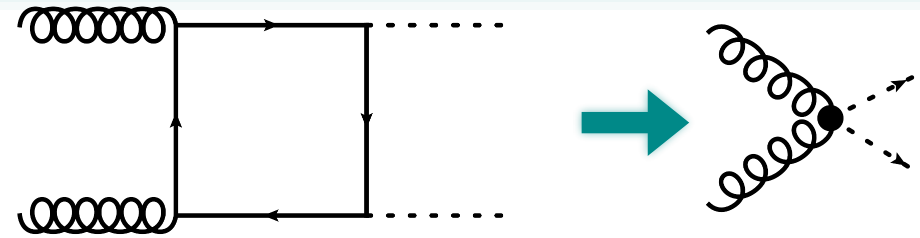
Grigo, Hoff, Steinhauser `15

$$\sigma_{exp} = \sum_n^6 c_n \rho^n, \quad \rho = \frac{m_H^2}{m_t^2}$$

±10%

$$\sigma^{NLO} = \int \mathrm{d}Q^2 \frac{\mathrm{d}\sigma_{exp}^{NLO}}{\mathrm{d}Q^2} \cdot \frac{\mathrm{d}\sigma^{LO}/\mathrm{d}Q^2}{\mathrm{d}\sigma_{exp}^{LO}/\mathrm{d}Q^2}$$

HEFT and approximated NLO results



- $m_T \to \infty$ limit (Higgs EFT)
  (valid for $\sqrt{s} \ll 2\, m_T$)

- Born-improved NLO HEFT

$$d\sigma_{NLO} \approx d\sigma_{NLO}^{HEFT} = \frac{d\sigma_{NLO}(m_t \to \infty)}{d\sigma_{LO}(m_t \to \infty)}\, d\sigma_{LO}(m_t)$$

K ≈ 2

Spira et al. (HPAIR)

- further improvements:

Maltoni Vryonidou, Zaro `14

$$d\sigma_{NLO}^{V,HEFT}$$

-10%

$$d\sigma_{NLO}^{R}(m_t)$$

Grigo, Hoff, Steinhauser `15

$$\sigma_{exp} = \sum_{n}^{6} c_n \rho^n, \quad \rho = \frac{m_H^2}{m_t^2}$$

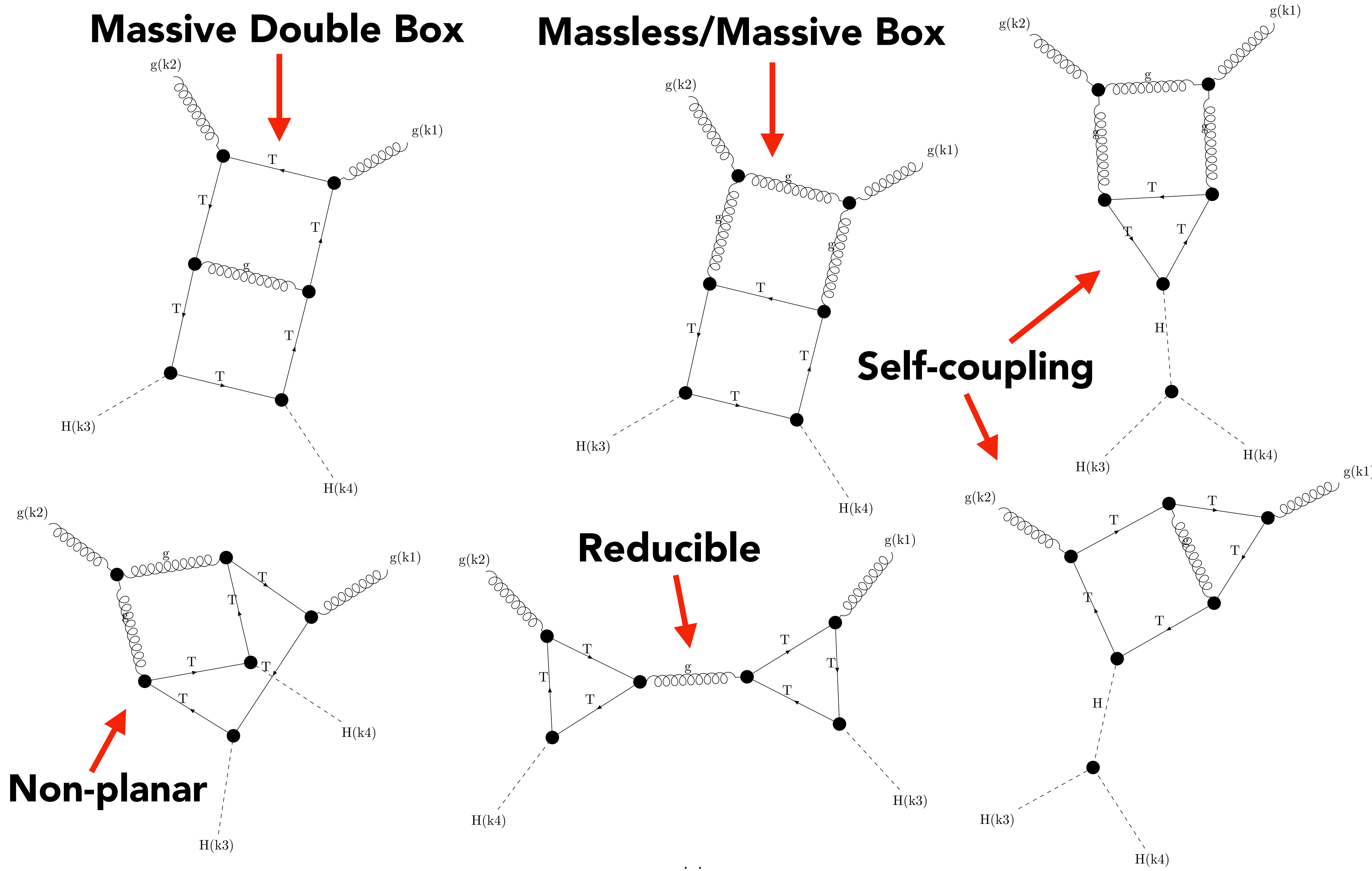
±10%

$$\sigma^{NLO} = \int \mathrm{d}Q^2 \frac{\mathrm{d}\sigma_{exp}^{NLO}}{\mathrm{d}Q^2} \cdot \frac{\mathrm{d}\sigma^{LO}/\mathrm{d}Q^2}{\mathrm{d}\sigma_{exp}^{LO}/\mathrm{d}Q^2}$$

mass effects largest uncertainty

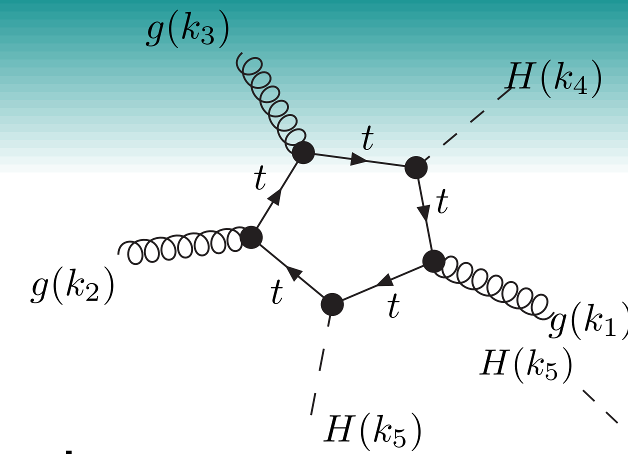
$\to$ NLO calculation with full mass dependence needed

Two Loop Diagrams

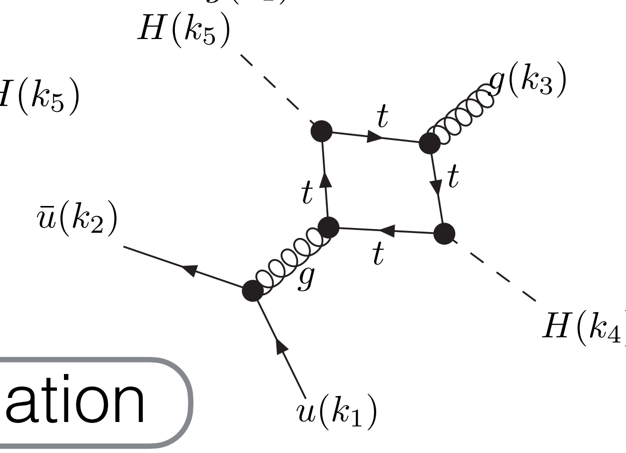


most complicated integrals not known analytically
→ numeric calculation using SecDec

Tools



- LO and Real Radiation
  - Gosam
    Cullen, van Deurzen, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Schlenk, von Soden-Fraunhofen, Tramontano
  - dipole subtraction
    Catani Seymour



this calculation

future?

- Virtual Corrections — GoSam-2Loop

reduction ⟷ amplitude generation ⟷ loop integrals

reduction

- Reduze
  von Manteuffel, Studerus`12
- FIRE
  Smirnov, Smirnov `13
- LiteRed
  Lee `13
- integrand reduction
  Mastrolia, Ossola, Peraro, Schubert

amplitude generation

GoSam-2Loop
GoSam-1L collaboration + Jahn, Jones, MK, Zirke

using QGRAF (Nogueira `93)
and FORM (Vermaseren et al. `12)

HH: 2nd implementation
using QGRAF, Reduze, Mathematica

loop integrals

- SecDec
  Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke
- analytic results
  Mastrolia, Schubert
- Loopedia
  Papara, et.al.

Two Loop Amplitude

- tensor structure Glover, van der Bij `88

$$\mathcal{M} = \epsilon_\mu(p_1, n_1)\epsilon_\nu(p_2, n_2)\, \mathcal{M}^{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = A_1(s, t, m_H^2, m_t^2, D)\, T_1^{\mu\nu} + A_2(s, t, m_H^2, m_t^2, D)\, T_2^{\mu\nu}$$

with

$$\boxed{\begin{aligned} \mathcal{M}^{++} &= \mathcal{M}^{--} = -A_1 \\ \mathcal{M}^{+-} &= \mathcal{M}^{-+} = -A_2 \end{aligned}}$$

triangle diagrams gg→H→HH only contribute to A_1

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu\, p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2\,(p_1 \cdot p_2)} \left\{ m_H^2 p_1^\nu\, p_2^\mu - 2\,(p_1 \cdot p_3)\, p_3^\nu\, p_2^\mu - 2\,(p_2 \cdot p_3)\, p_3^\nu\, p_1^\mu + 2\,(p_1 \cdot p_2)\, p_3^\nu\, p_3^\mu \right\}$$

- projectors

construct $P_i^{\mu\nu} = \sum_j c_{ij} T_j^{\mu\nu}$ such that

$$\begin{aligned} P_1^{\mu\nu}\mathcal{M}_{\mu\nu} &= A_1(s, t, m_H^2, m_t^2, D) \\ P_2^{\mu\nu}\mathcal{M}_{\mu\nu} &= A_2(s, t, m_H^2, m_t^2, D) \end{aligned}$$

Integral Reduction

Reduction to master integrals using Reduze

- integral families with 9 propagators: 5(3) planar(non-planar) families
- full dependence on $s,\ t,\ m_t^2,\ m_H^2$ challenging
  ➡ simplification: fix $m_t = 173\,\mathrm{GeV},\ m_H = 125\,\mathrm{GeV}$
- (mostly) finite basis
  von Manteuffel, Panzer, Schabinger
- non-planar sectors still unreduced

Integrals	**1-loop**	**2-loop**
Direct	63	9865
+ Symmetries	21	1601
+ IBPs	8	~260-270 currently: 327

145 (+83 crossed) planar masters
70 (+29) non-planar integrals (mostly unreduced)

Non-planar integrals

rewrite inverse prop. → scalar products

$$\int \mathrm{d}^d p_1 \mathrm{d}^d p_2 \frac{(p_1+k_1)^2}{p_1^2} f(p_i,k_i) = \int \mathrm{d}^d p_1 \mathrm{d}^d p_2 \left(1 + \frac{k_1^2}{p_1^2} + \frac{2\,p_1\cdot k_1}{p_1^2}\right) f(p_i,k_i)$$

rank-2

rank-1

up to 4 inverse propagators → up to rank-4 tensors

Amplitude — Loop Integrals

SecDec

- sector decomposition of loop integrals Binoth, Heinrich
- contour deformation Nagy, Soper

→ numerical integration possible

interface

Amplitude & numerical integration

- using Quasi-Monte-Carlo (QMC) integration
  $\mathcal{O}(n^{-1})$ scaling of integration error
- split each integral into sectors
- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral } i} t_i + \lambda \left(\sigma^2 - \sum_i \sigma_i^2 \right) \qquad \sigma_i = c_i \cdot t_i^{-e}$$

$\sigma_i =$ error estimate (including coefficients in amplitude)

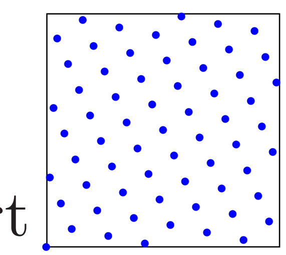
$\lambda =$ Lagrange multiplier $\qquad \sigma =$ precision goal

- avoid reevaluation of integrals for different orders in ε and form factors
- parallelization on gpu

QMC rank-1 lattice rule

$$I = \int d\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^{n} f(\vec{x}_{i,k})$$

$$\vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\}$$



$\{\ldots\} =$ fractional part

$\vec{g} =$ generating vector

$\vec{\Delta}_k =$ randomized shift

m different estimates $I_1 \ldots I_m$

$\rightarrow$ error estimate

Li, Wang, Yan, Zhao `15
Review: Dick, Kuo, Sloan

Amplitude Structure

rewrite loop integrals with r propagators and s inverse propagators as

$$I_{r,s}(s,t,m_h^2,m_t^2) = (M^2)^{-L\epsilon}(M^2)^{2L-r+s} I_{r,s}\left(\frac{s}{M^2},\frac{t}{M^2},\frac{m_h^2}{M^2},\frac{m_t^2}{M^2}\right)$$

arbitrary scale

and write renormalized form factors as

$$F^{\text{virt}} = aF^{(1)} + a^2(\frac{n_g}{2}\,\delta Z_A + \delta Z_a)F^{(1)} + a^2\delta m_t^2 F^{ct,(1)} + a^2F^{(2)} + \mathcal{O}(a^3)$$

$$F^{(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\varepsilon}\left[b_0^{(1)} + b_1^{(1)}\varepsilon + b_2^{(1)}\varepsilon^2 + \mathcal{O}(\varepsilon^3)\right], \qquad \text{(1-loop)}$$

$$F^{ct,(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\varepsilon}\left[c_0^{(1)} + c_1^{(1)}\varepsilon + \mathcal{O}(\varepsilon^2)\right], \qquad \text{(mass counter-term)}$$

$$F^{(2)} = \left(\frac{\mu_R^2}{M^2}\right)^{2\varepsilon}\left[\frac{b_{-2}^{(2)}}{\varepsilon^2} + \frac{b_{-1}^{(2)}}{\varepsilon} + b_0^{(2)} + \mathcal{O}(\varepsilon)\right], \qquad \text{(2-loop)}$$

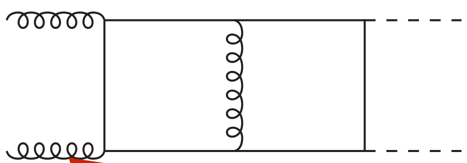
$\rightarrow$ scale variations do not require re-computation of $b_i^{(n)}$, $c_i^{(n)}$

Amplitude Evaluation — Example

$$\sqrt{s} = 327.25\,\mathrm{GeV},\ \sqrt{-t} = 170.05\,\mathrm{GeV},\ M^2 = s/4$$

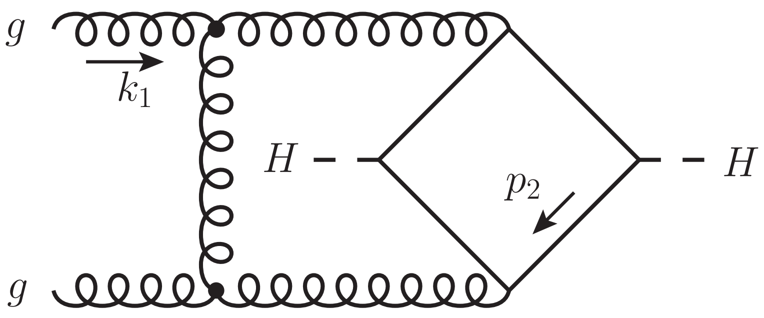
contributing integrals:

integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342



≈ 700 integrals

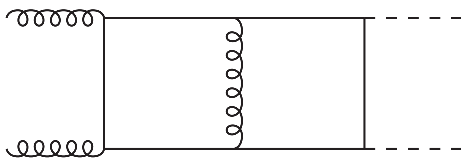
$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon}\Gamma(3+2\epsilon)M^{-4}\left(\frac{A_{-2}}{\epsilon^2}+\frac{A_{-1}}{\epsilon^1}+A_0+\mathcal{O}(\epsilon)\right)$$



Amplitude Evaluation — Example

$\sqrt{s} = 327.25\,\text{GeV},\ \sqrt{-t} = 170.05\,\text{GeV},\ M^2 = s/4$

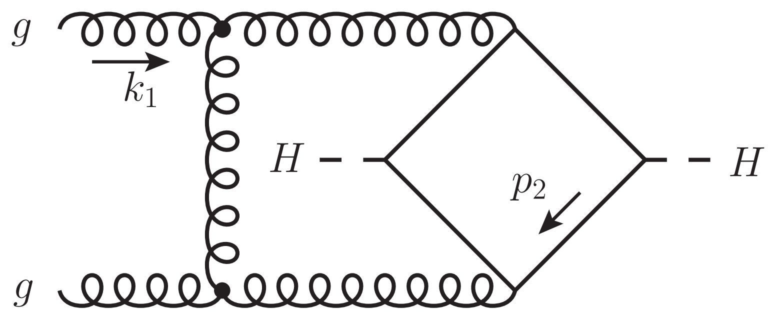
contributing integrals:



integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342

≈ 700 integrals

$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon}\Gamma(3+2\epsilon)M^{-4}\left(\frac{A_{-2}}{\epsilon^2}+\frac{A_{-1}}{\epsilon^1}+A_0+\mathcal{O}(\epsilon)\right)$$



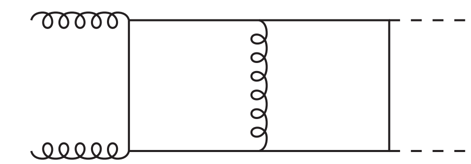
sector decomposition

sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

Amplitude Evaluation — Example

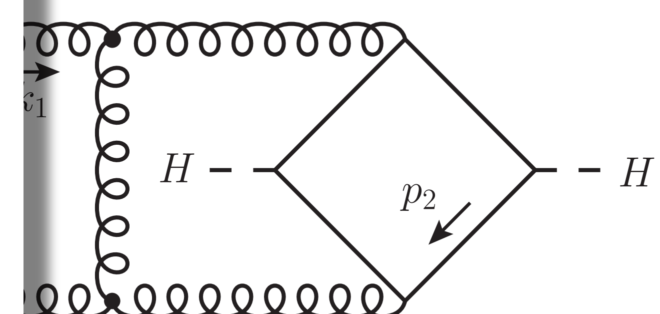
$\sqrt{s} = 327.25\,\mathrm{GeV},\ \sqrt{-t} = 170.05\,\mathrm{GeV},\ M^2 = s/4$

contributing integrals:

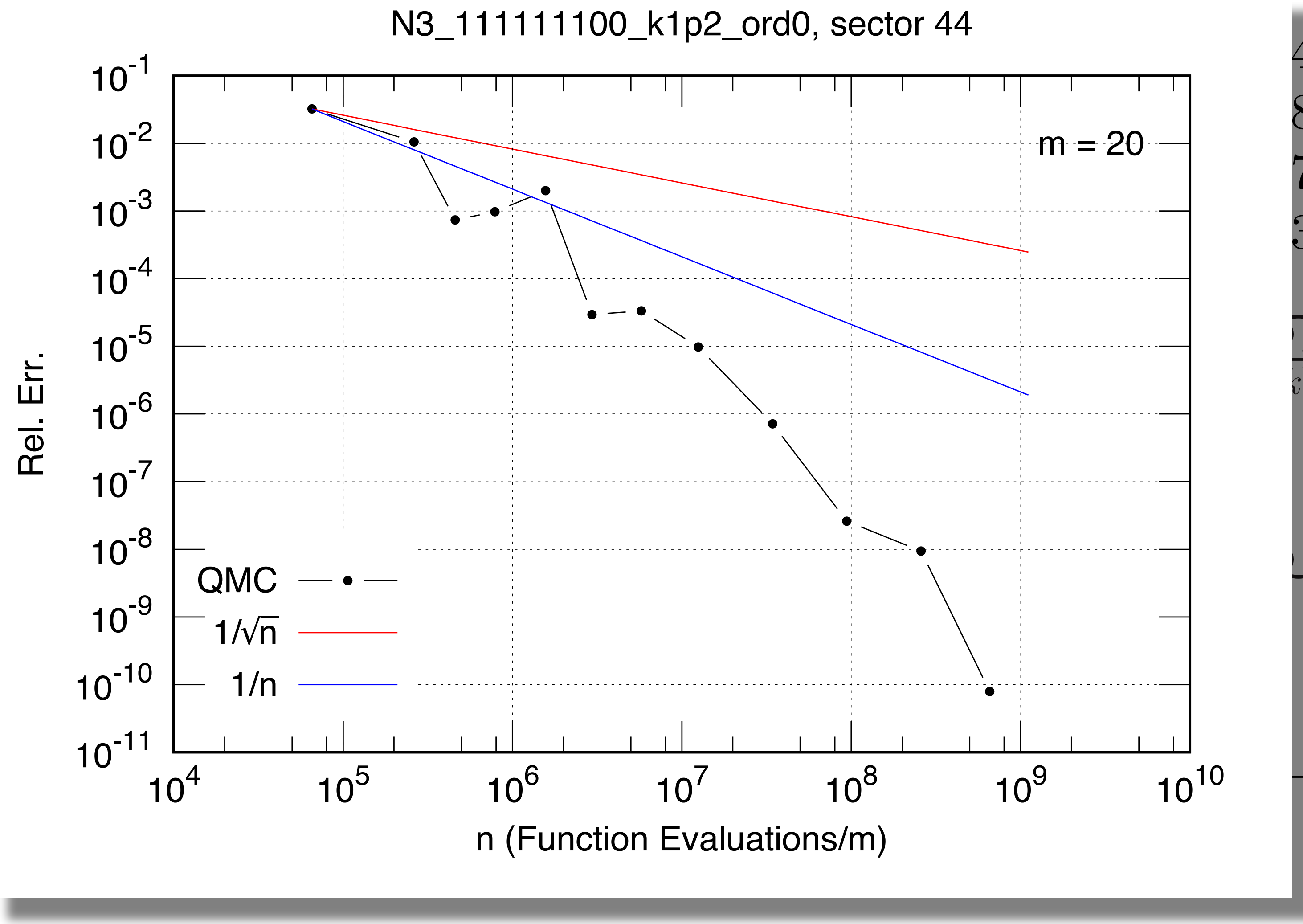


integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2			412
N3_111111100_1_orc			896
N3_111111100_k1p2			794
N3_111111100_k1p2			342

≈ 700 integrals



$I(s,t,m_t^2,m_h^2) = -($

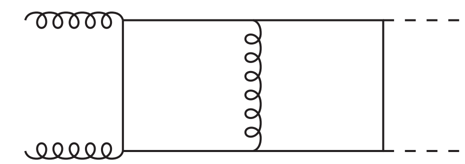


sector	in			
5	(-1.34e-0			
6	(-1.58e-03			
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

Amplitude Evaluation — Example

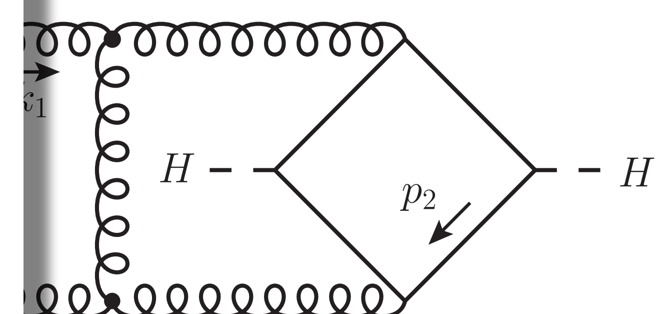
$$\sqrt{s} = 327.25\,\mathrm{GeV},\ \sqrt{-t} = 170.05\,\mathrm{GeV},\ M^2 = s/4$$

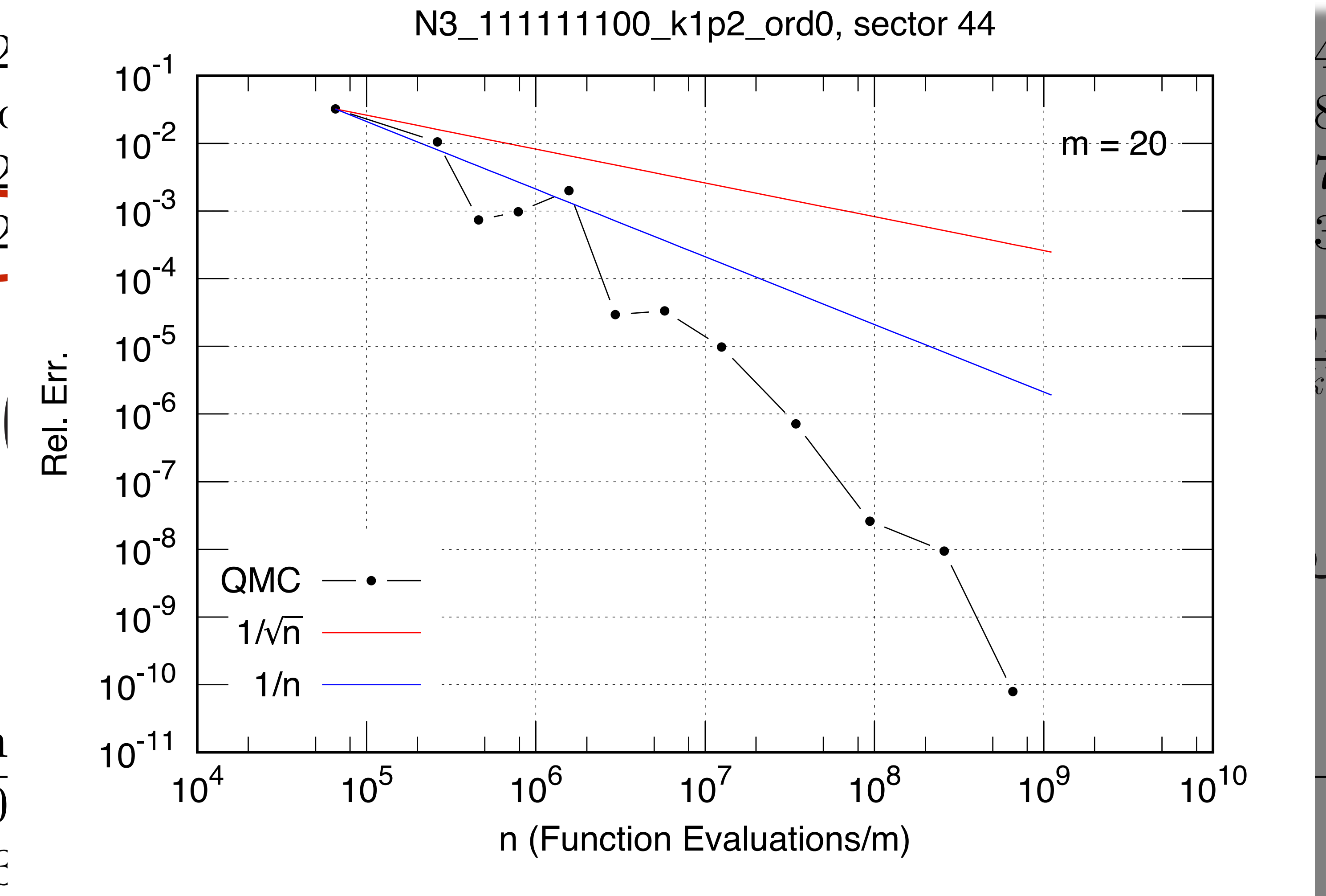
contributing integrals:



integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2			412
N3_111111100_1_or(			896
N3_111111100_k1p2			794
N3_111111100_k1p2			342

≈ 700 integrals





$I(s,t,m_t^2,m_h^2) = -$ (

sector	in			
5	(-1.34e-0			
6	(-1.58e-03			
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

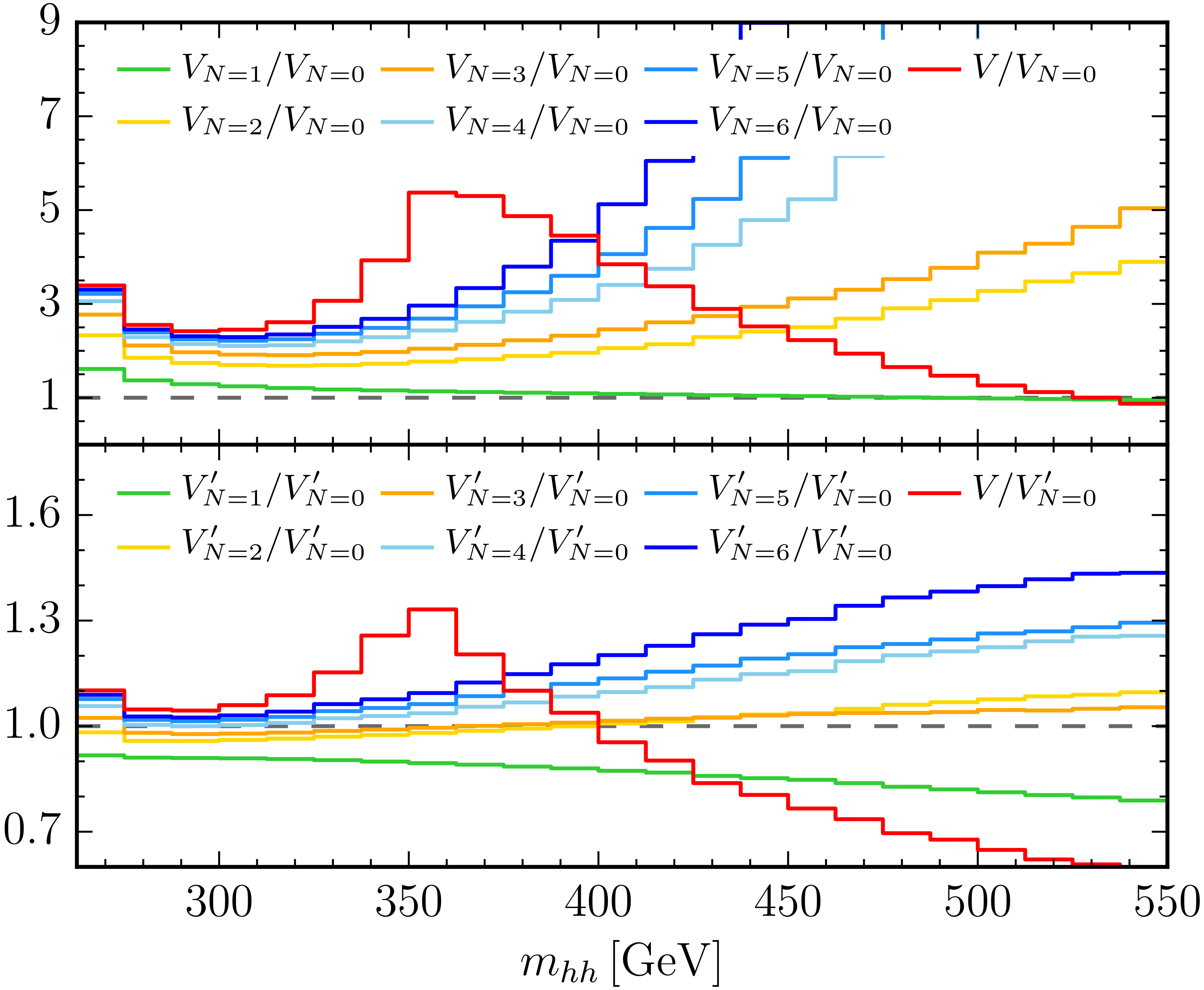
amplitude result:

gpu time: 2 h

accuracy F_1 (F_2): 0.2% (11%)

Results - Amplitude

comparison to HEFT and expansion in $1/m_t$



$$V_N = \left(d\hat{\sigma}^{\text{virt}}_{\text{exp},N} + d\hat{\sigma}^{\text{LO}}_{\text{exp},N}(\epsilon) \otimes I\right) \frac{d\hat{\sigma}^{\text{LO}}(\epsilon)}{d\hat{\sigma}^{\text{LO}}_{\text{exp},N}(\epsilon)}$$

$$d\hat{\sigma}_{\text{exp},N} = \sum_{\rho=0}^{N} d\hat{\sigma}^{(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho}$$

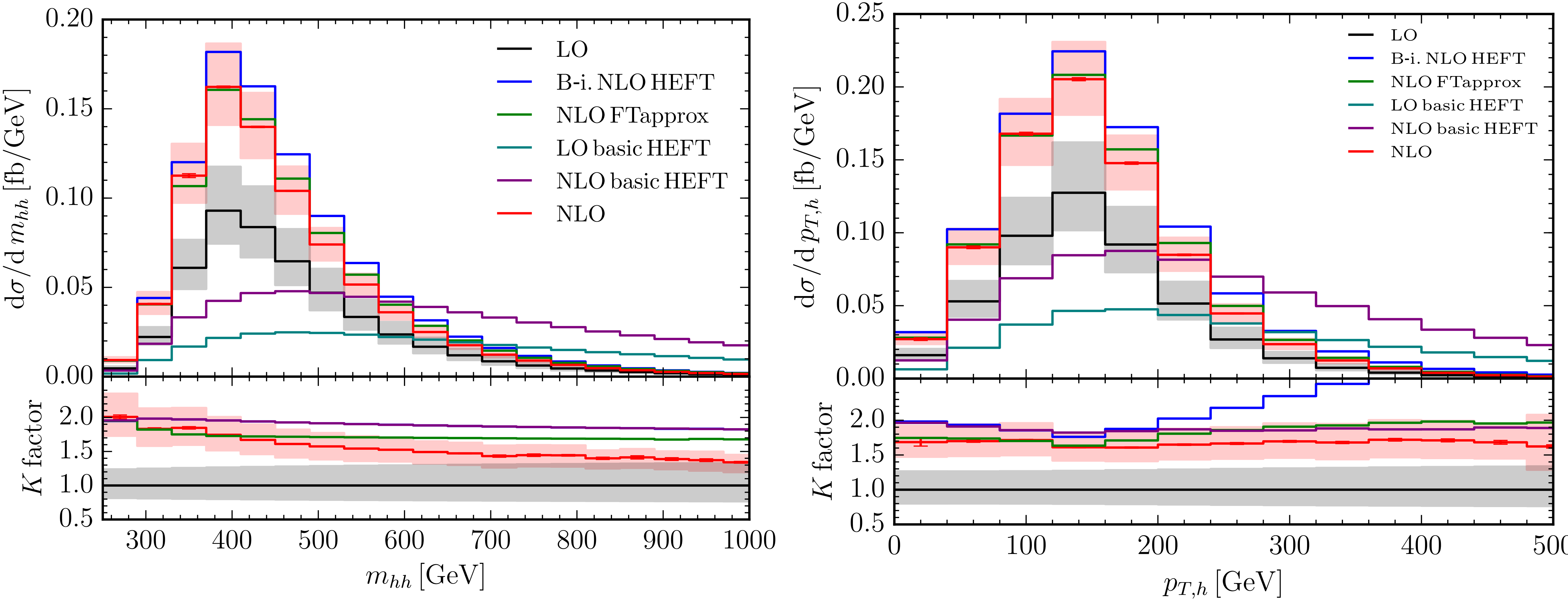
$$\Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_h\right\}$$

$$V'_N = V_N \cdot \frac{B}{B_N}$$

$V_{N\geq4}$: thanks to J. Hoff

Results - Cross Section

LHC@14TeV



$$\sigma^{NLO} = 32.91^{+14\%}_{-13\%}\,\text{fb} \pm 0.3\%\,(\text{stat.}) \pm 0.1\%\,(\text{int.})$$

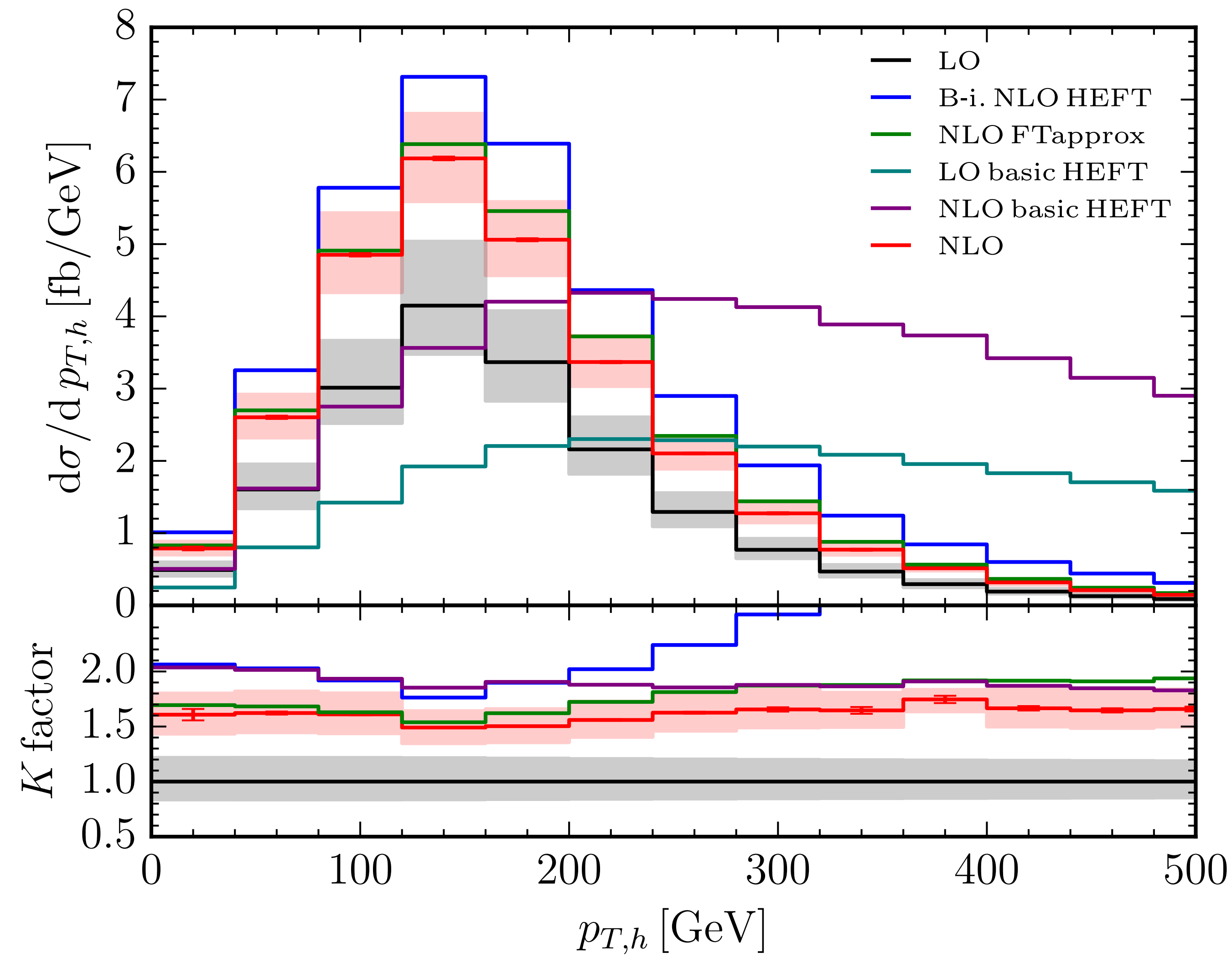
Born-improved HEFT: $\sigma^{NLO}_{HEFT} = 38.32^{+18\%}_{-15\%}\,\text{fb}$

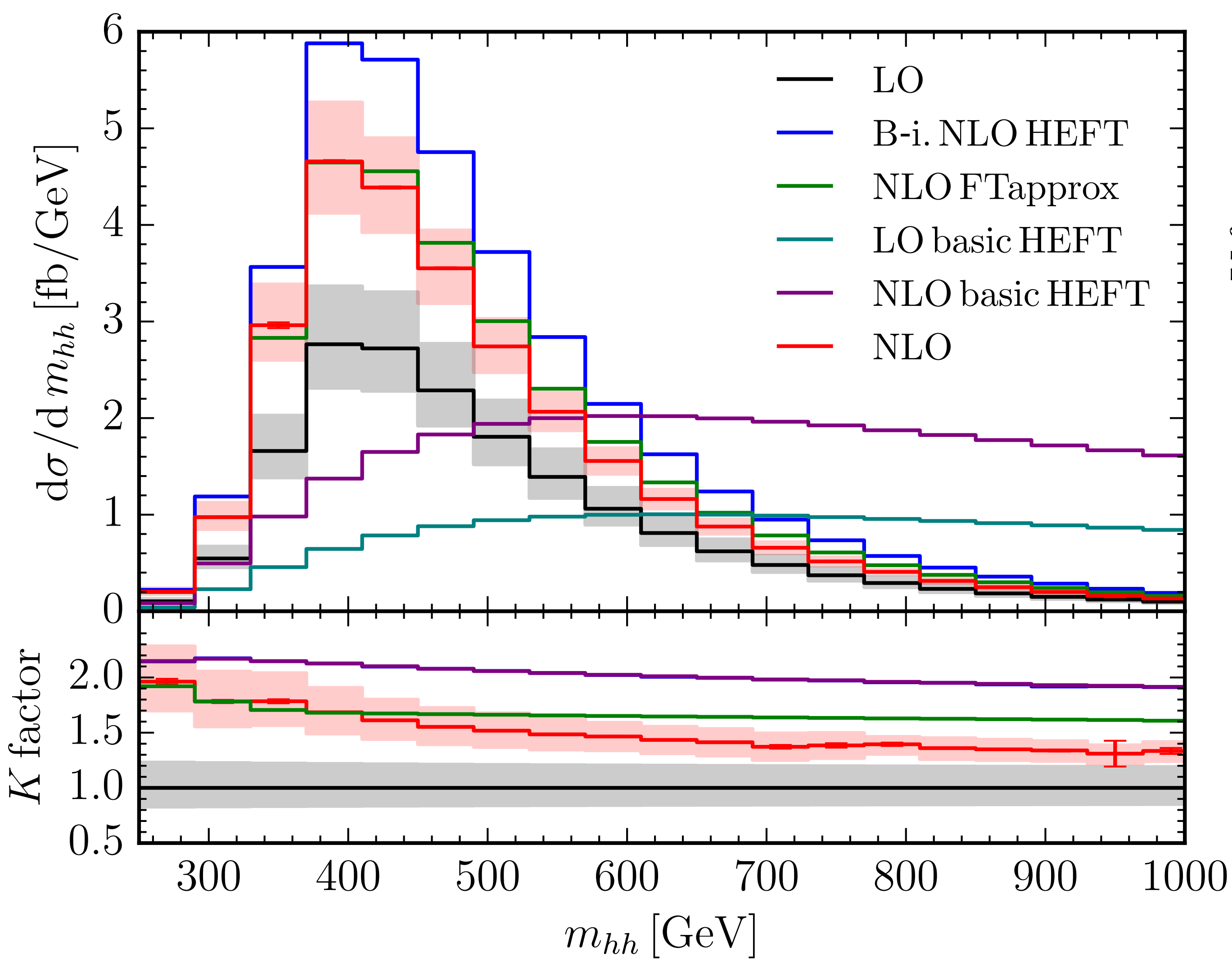
LO: $\sigma^{LO} = 19.85^{+28\%}_{-21\%}\,\text{fb}$

Results - Cross Section

New in arXiv 1608.xxxx

- results @ 100 TeV



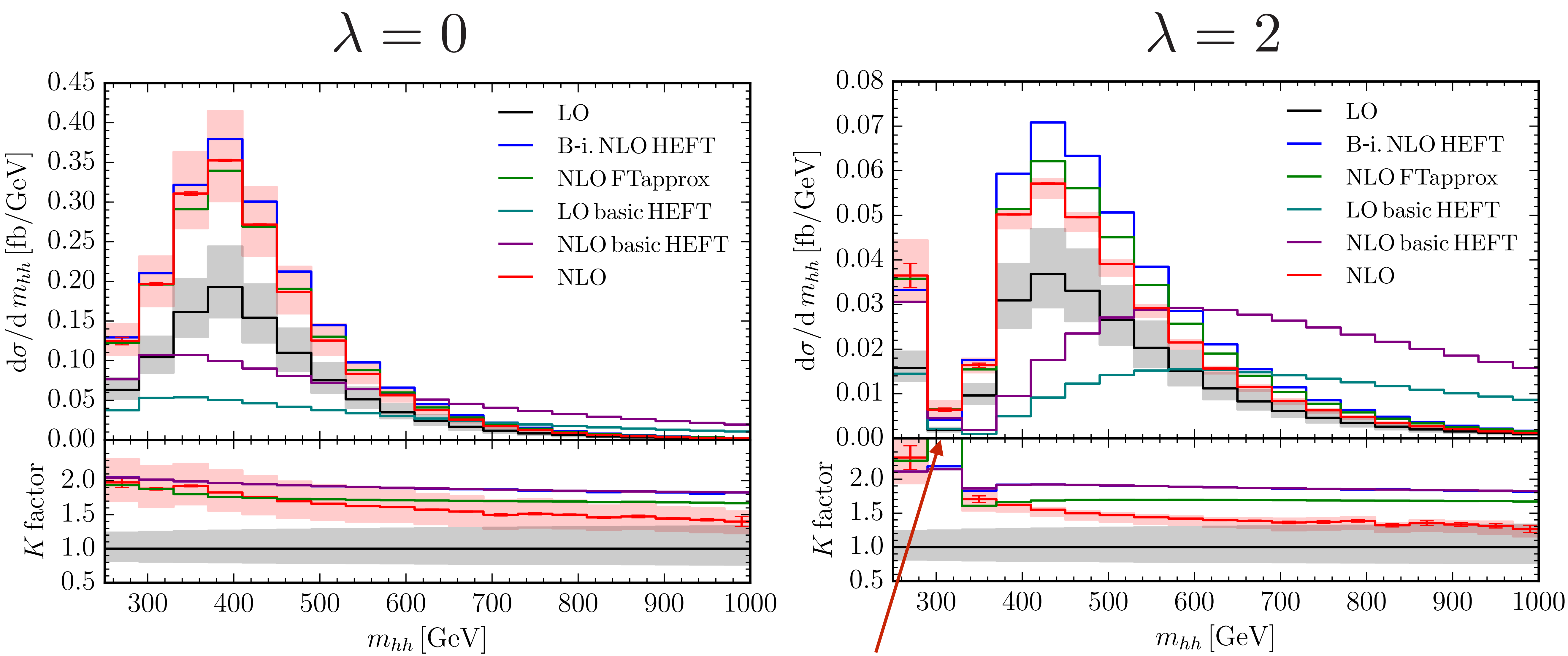


finite m_t effects pronounced compared to 14TeV results

Results - Cross Section

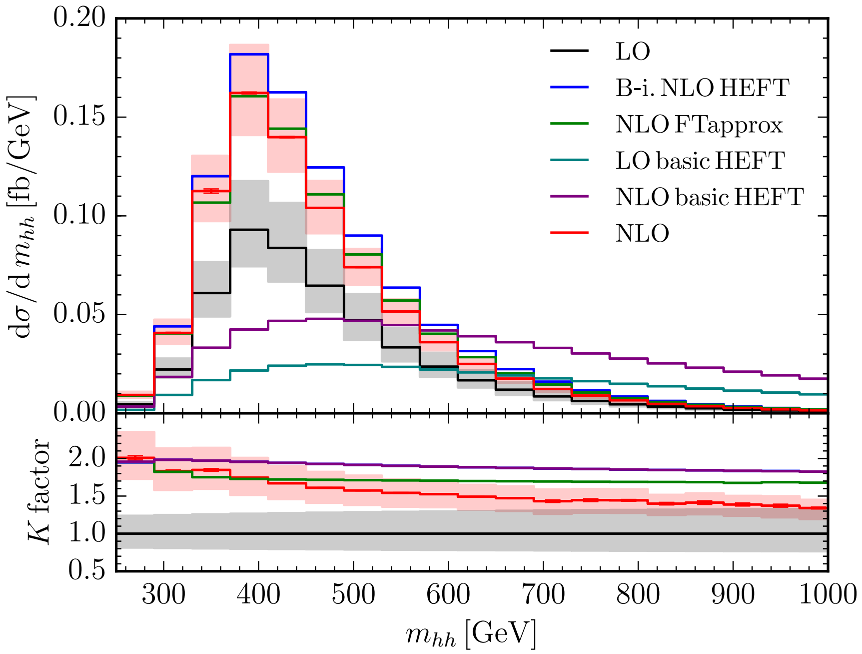
New in arXiv 1608.xxxx

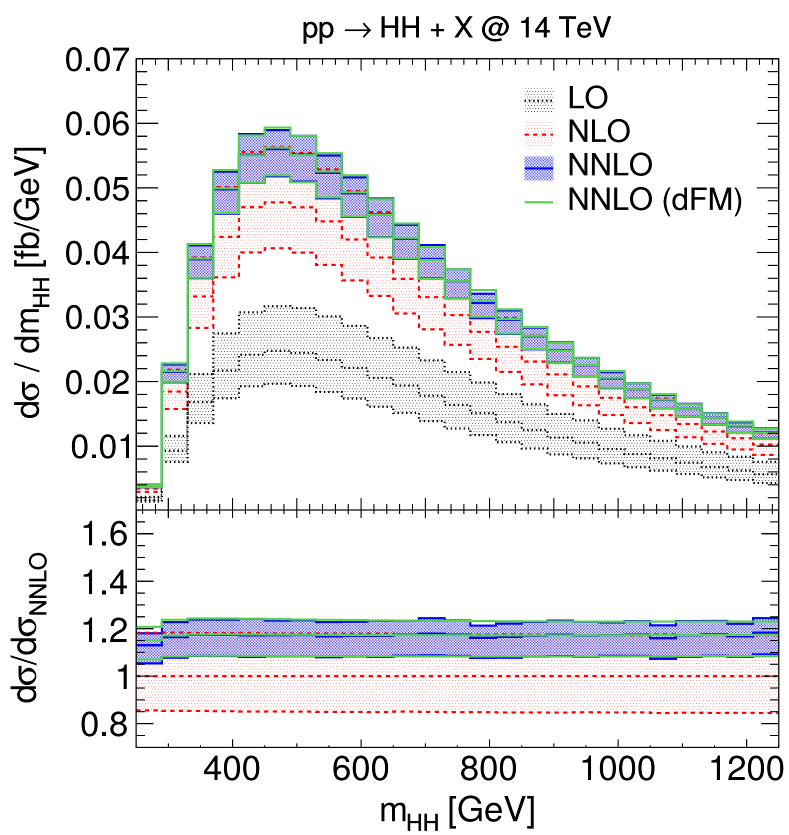
- modified Higgs self-interaction: $g_{hhh} = \lambda \cdot g_{hhh}^{SM}$



destructive interference of Δ and $\square$ contributions for $m_{hh} = m_h\sqrt{1+3\lambda}$

Results - Combination with NNLO$_{\text{HEFT}}$



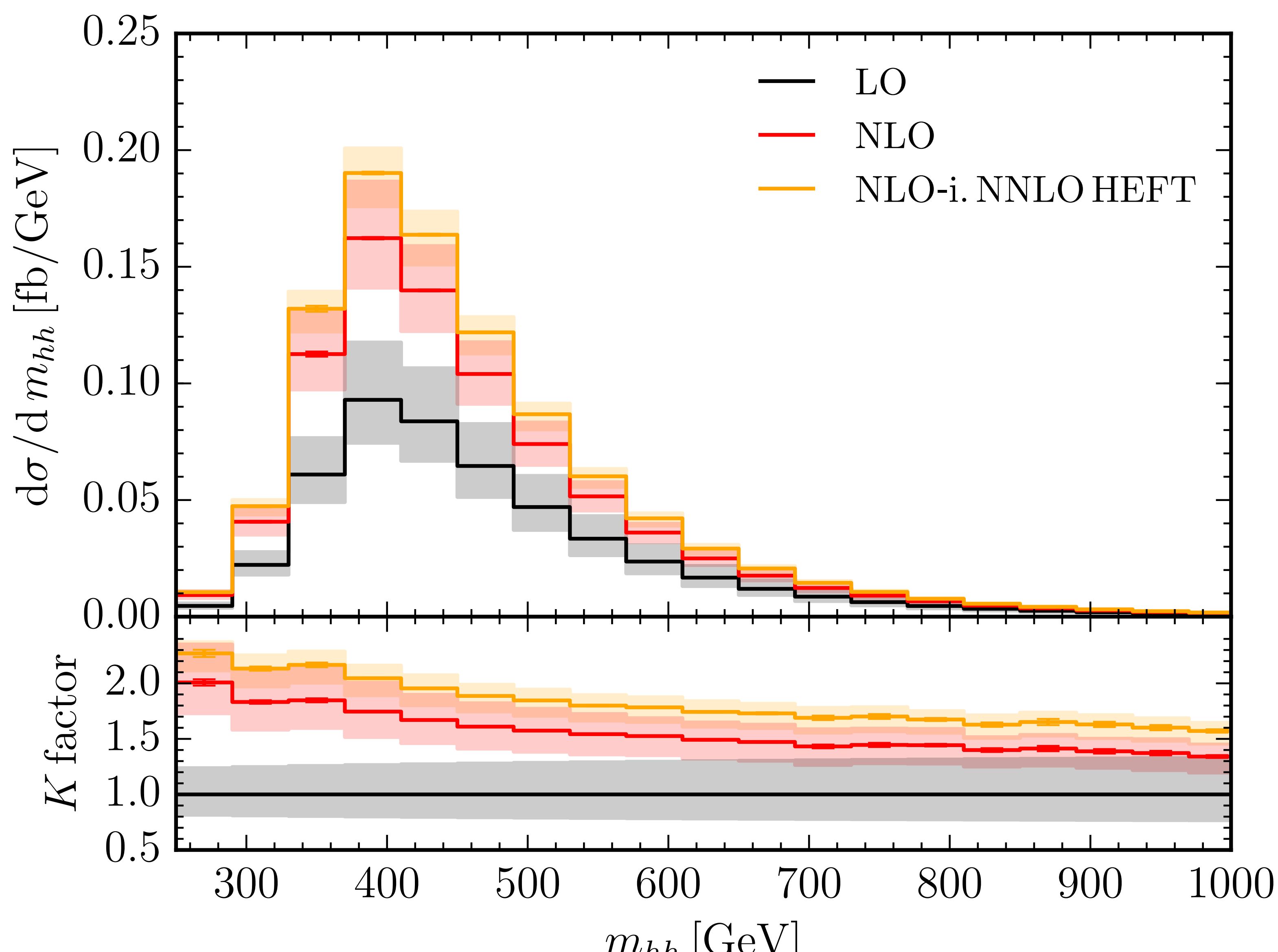


de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev `16

first attempt to combine NLO$_{\text{full}}$ with NNLO$_{\text{HEFT}}$

NLO-improved NNLO HEFT:

$$\frac{d\sigma^{\text{full}}_{\text{NLO}}}{dm_{hh}} \cdot \frac{d\sigma^{\text{HEFT}}_{\text{NNLO}}/dm_{hh}}{d\sigma^{\text{HEFT}}_{\text{NLO}}/dm_{hh}}$$



Summary

Higgs pair production at NLO

- full m_t dependence
- significant deviations from Born-improved HEFT
- reduce cross section by 14% relative to Born-improved HEFT

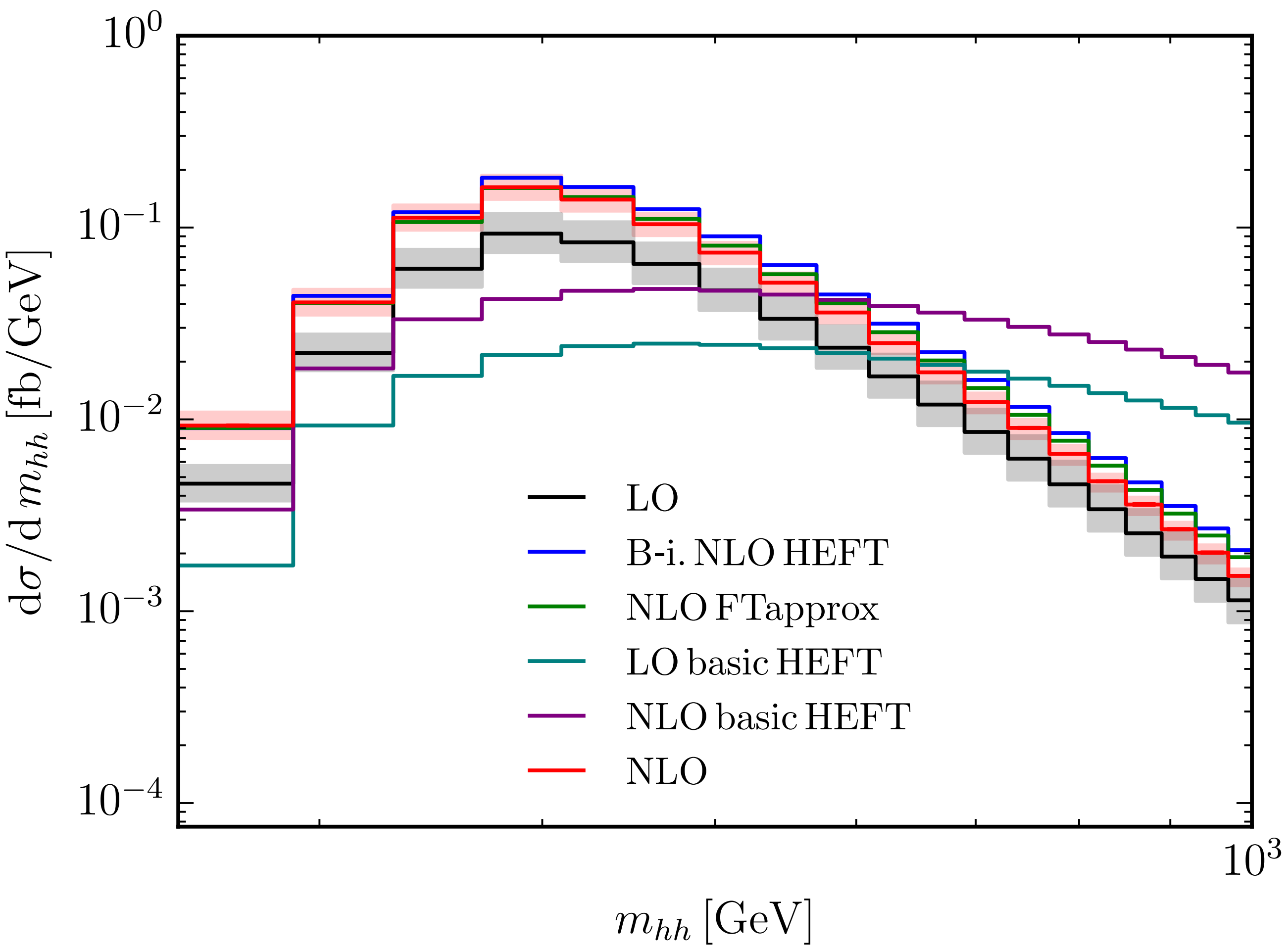
$\rightarrow$ relevant contribution to cross section

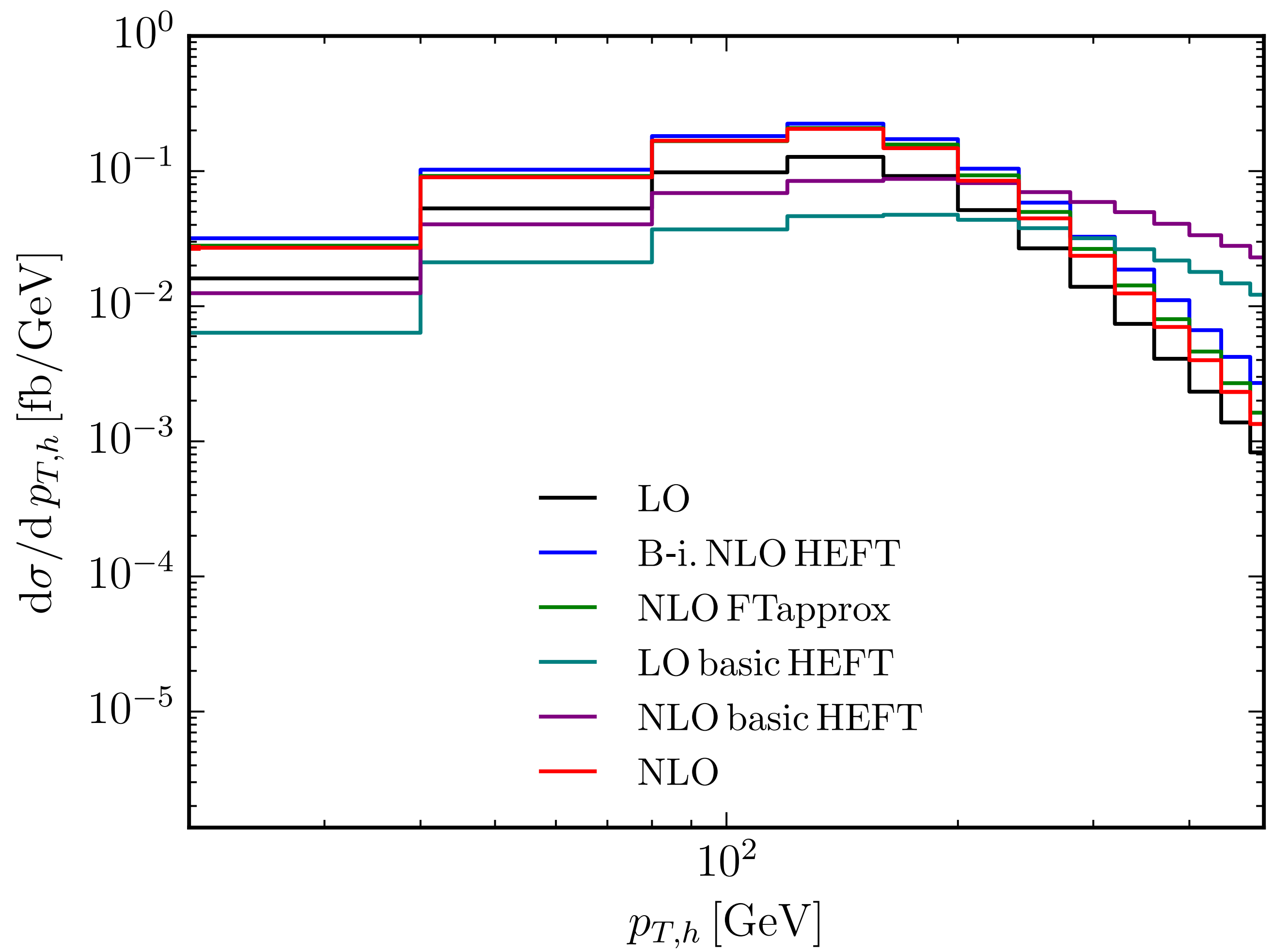
- numerical integration of loop integrals using SecDec
  - new interface to amplitude code
  - dynamically adjust #sampling points
  - Quasi Monte Carlo

First step towards automated 2-loop calculations using GoSam-2L

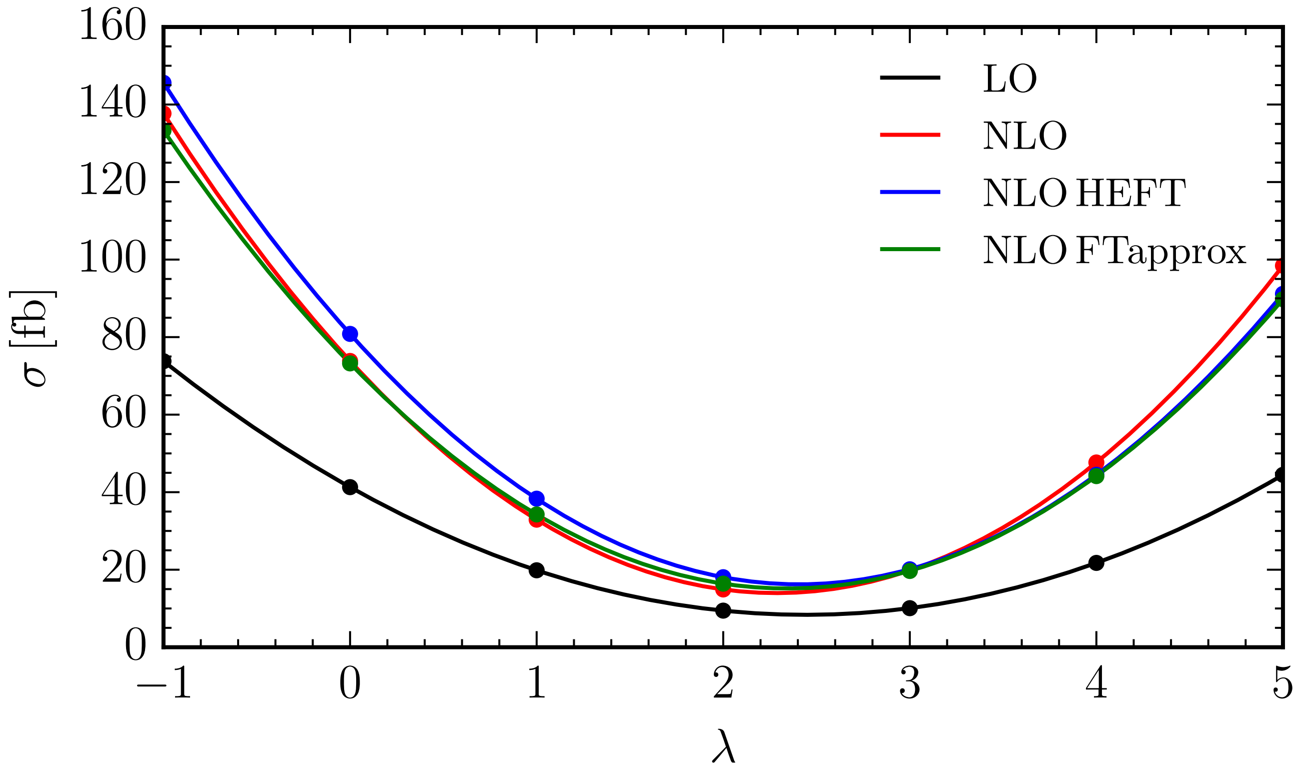
Backup

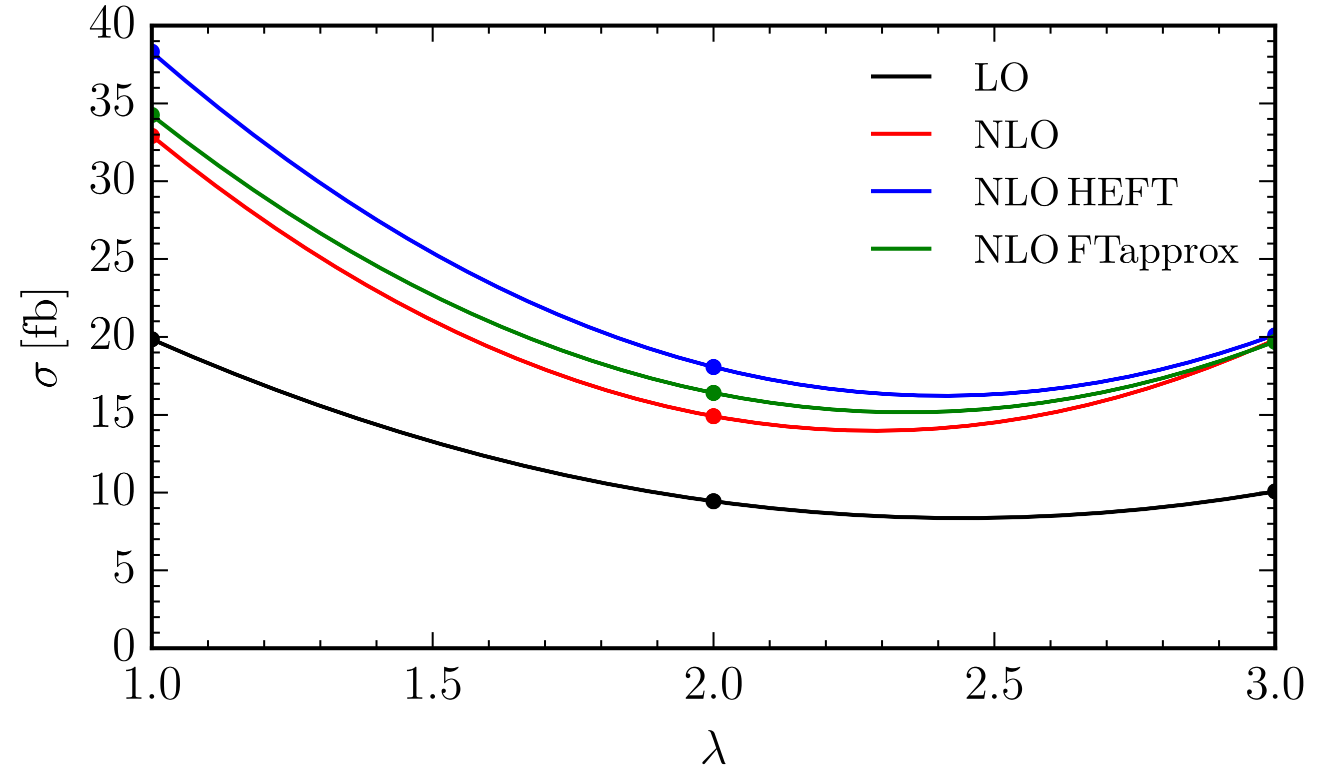
scaling behavior



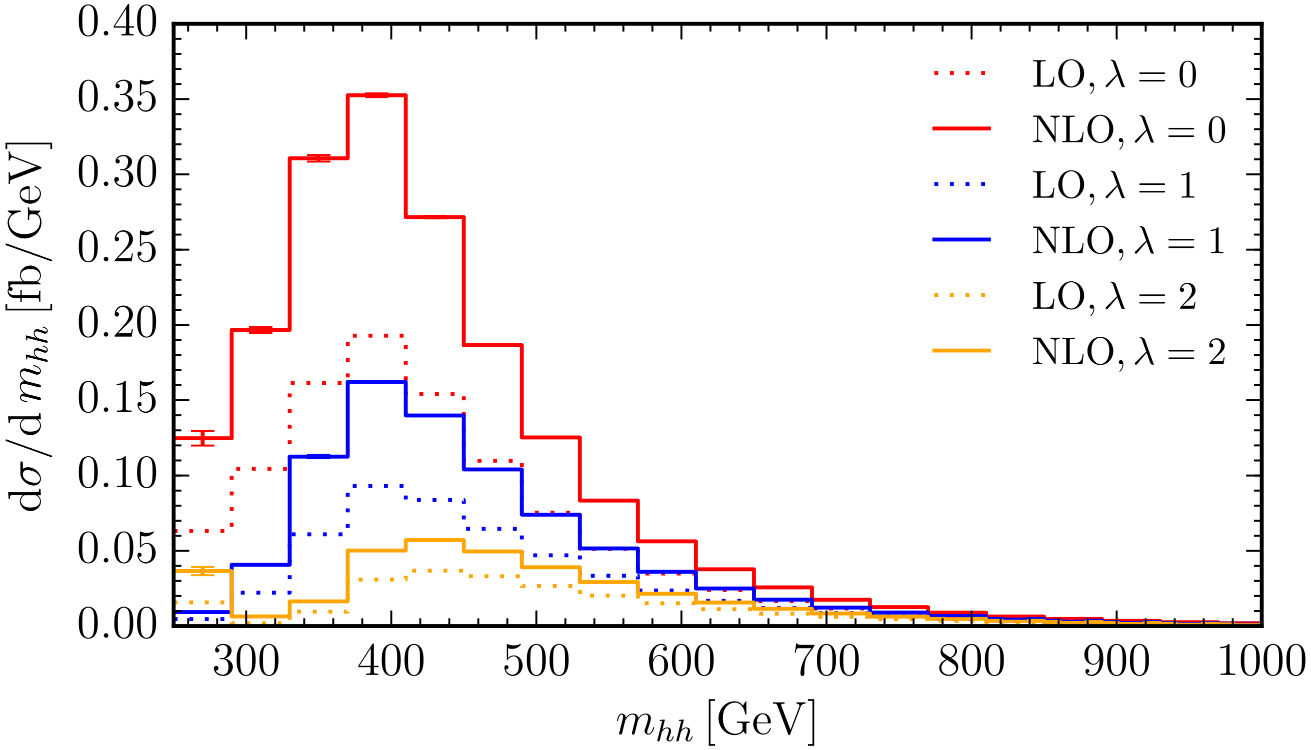


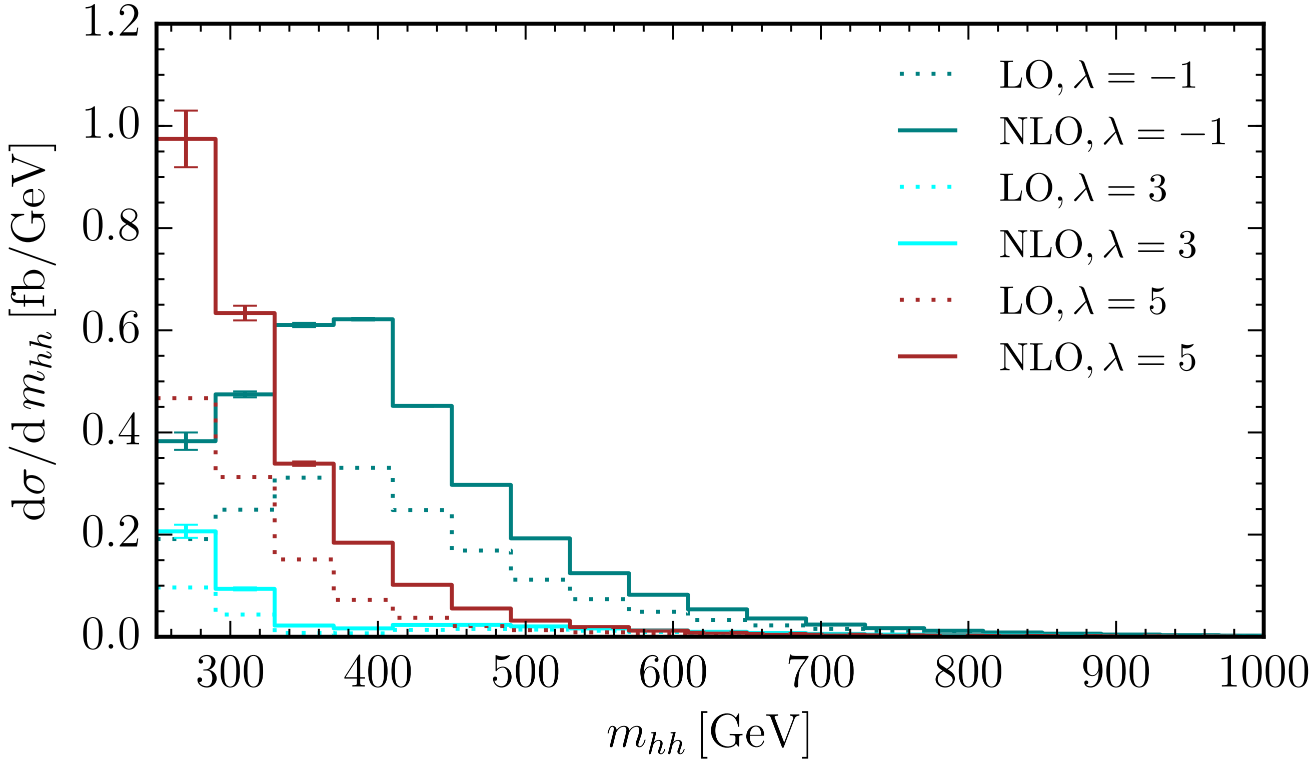
modified Higgs self-interactions





modified Higgs self-interactions





Checks

Real Emission / Subtraction Terms

- Independence of dipole-cut α parameter Nagy `03
- Agreement with Maltoni, Vryonidou, Zaro `14

Virtual Corrections

- Two calculations of amplitude up to reduction
- Amplitude result invariant under $t \leftrightarrow u$
- Pole cancellation
- Mass renormalization using two methods:
  counter-term insertion vs. calculating $\mathrm{d}\mathcal{M}^{\mathrm{LO}}/\mathrm{d}m_t^2$ numerically

- Agreement of contributions gg $\rightarrow$ H $\rightarrow$ HH with SusHi Harlander,Liebler, Mantler
- Convergence of $1/m_T$ expansion to full result
  where agreement is expected

Calculation of σ^V

- Importance sampling:

  LO calculation
  ↓
  unweighted events
  ↓
  sampling points virtual amplitude

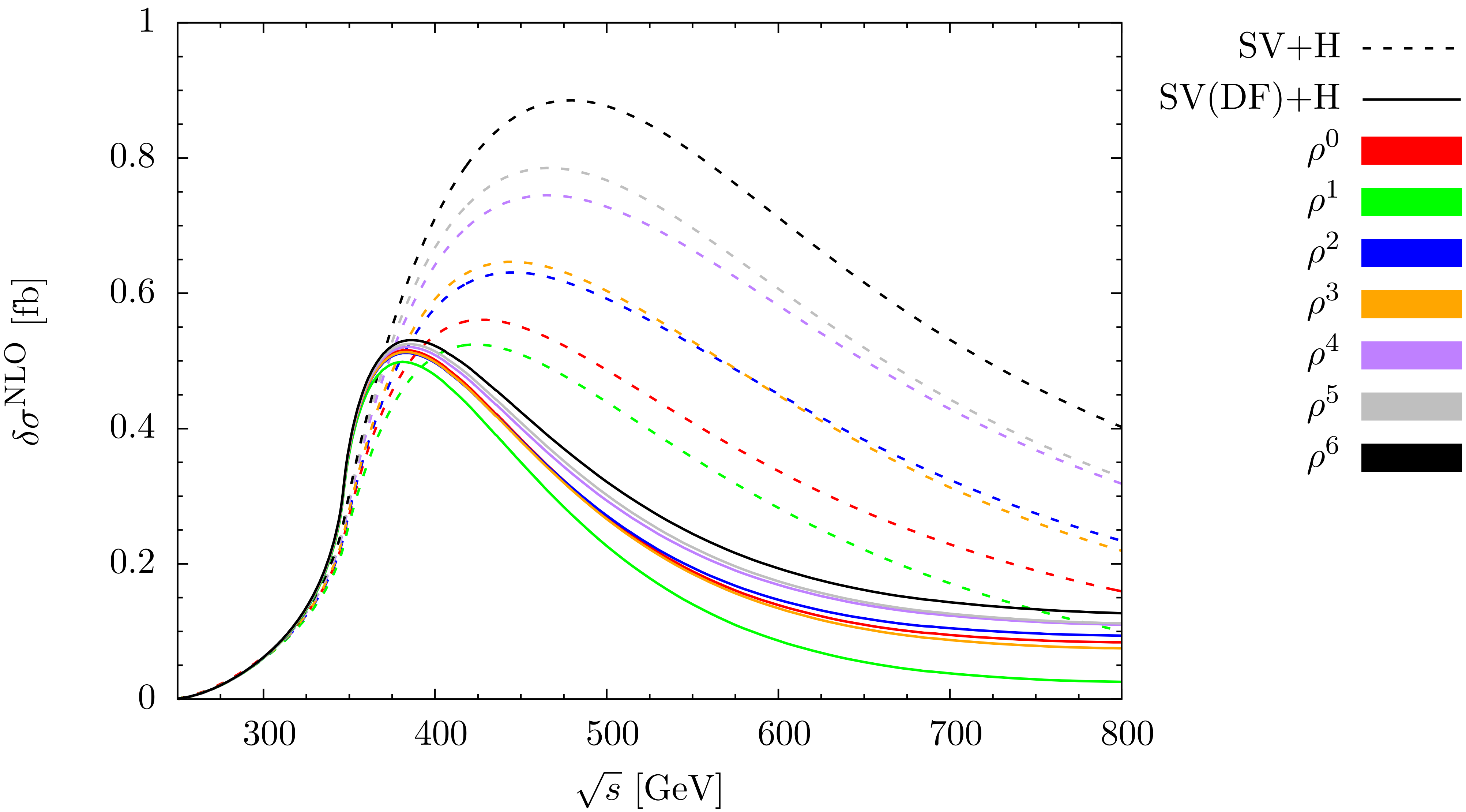
  σ^V with 2.5% accuracy

  using

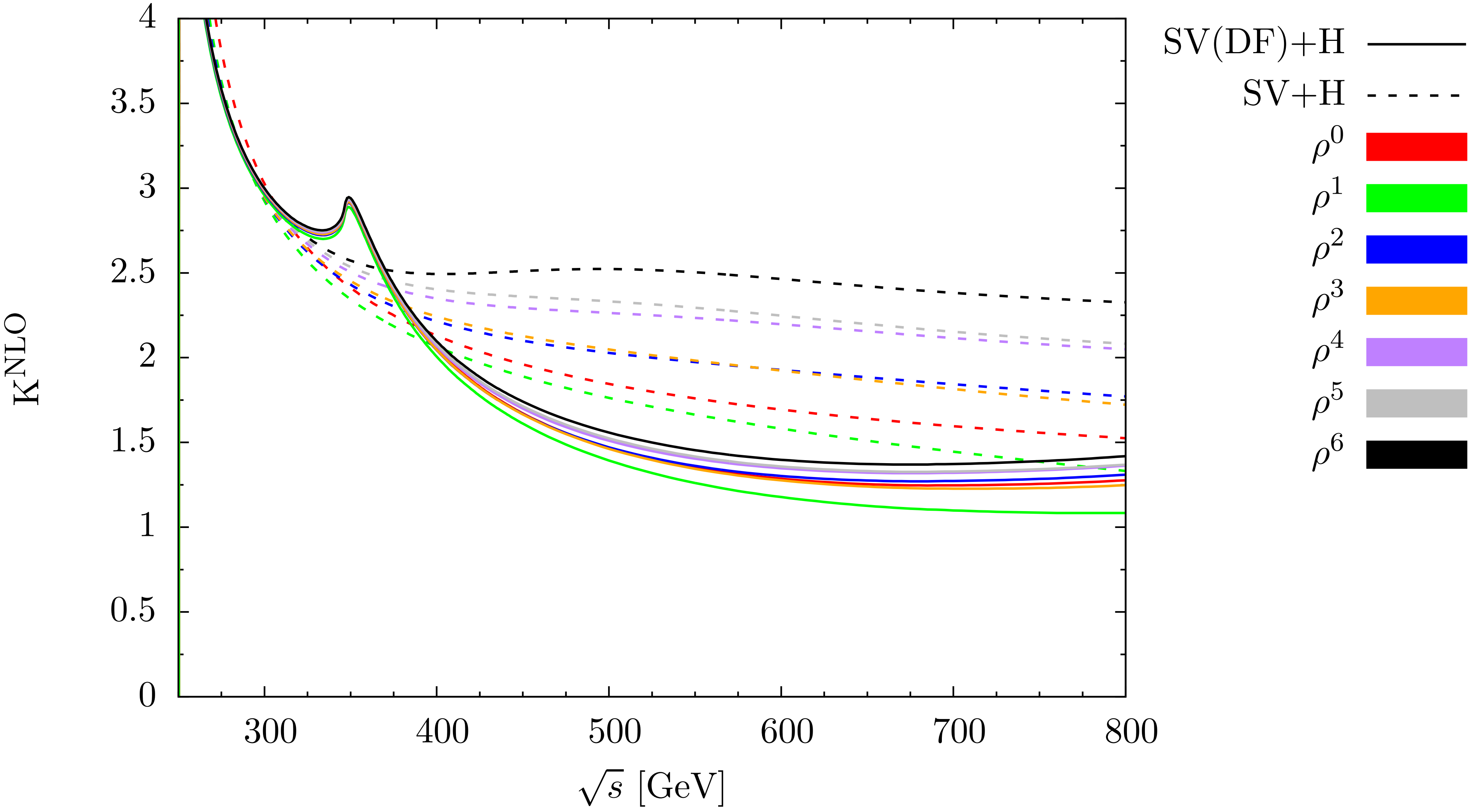
  ~1000 phase-space points

- Accuracy goal:
  - 3% for form factor F_1
  - 5-20% for form factor F_2 (depending on F_2/F_1)

- Run time: (gpu time)
  - 80 min - 2 d (≙wall-clock limit)
  - median: 2h

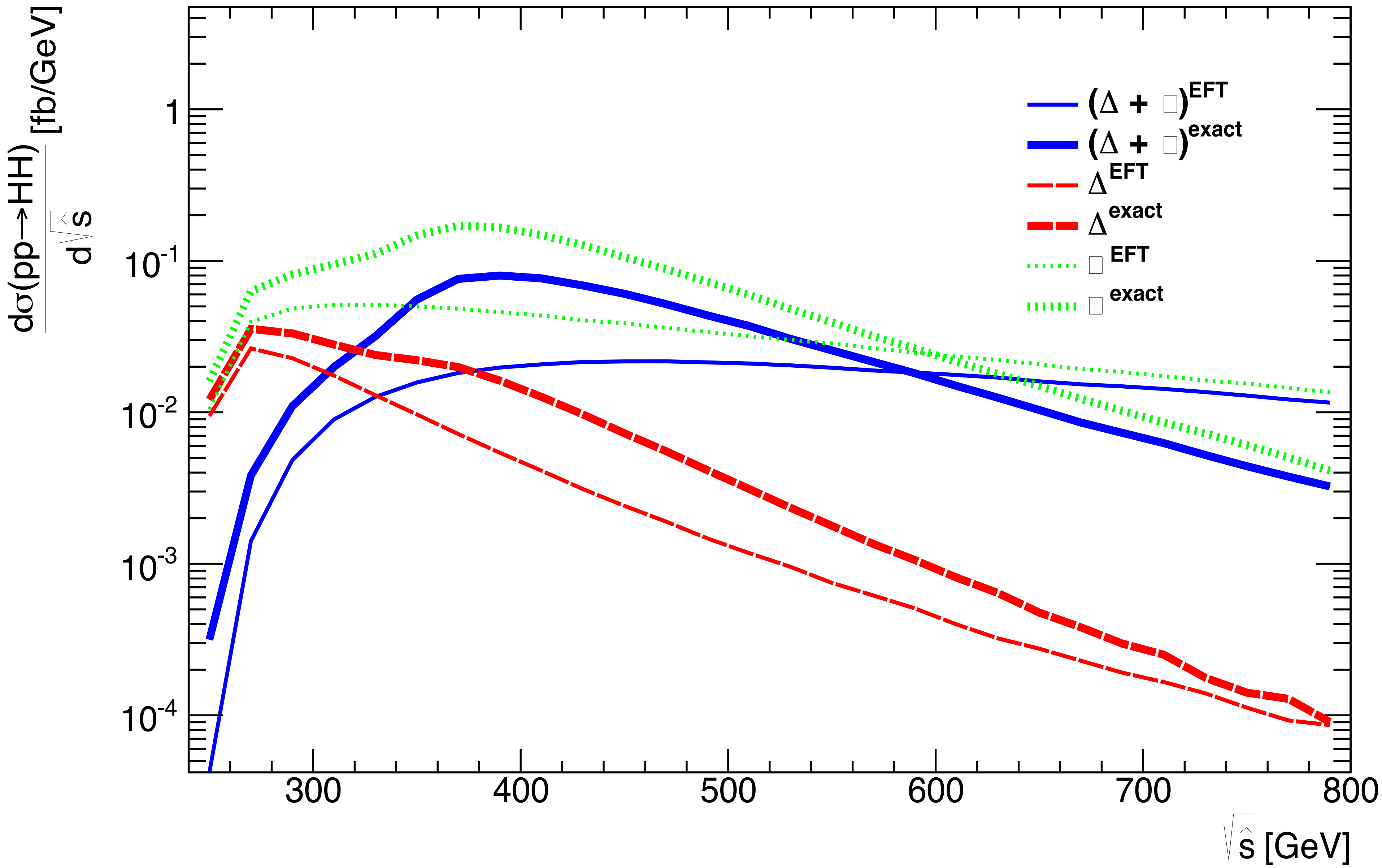


Grigo, Hoff, Steinhauser `15



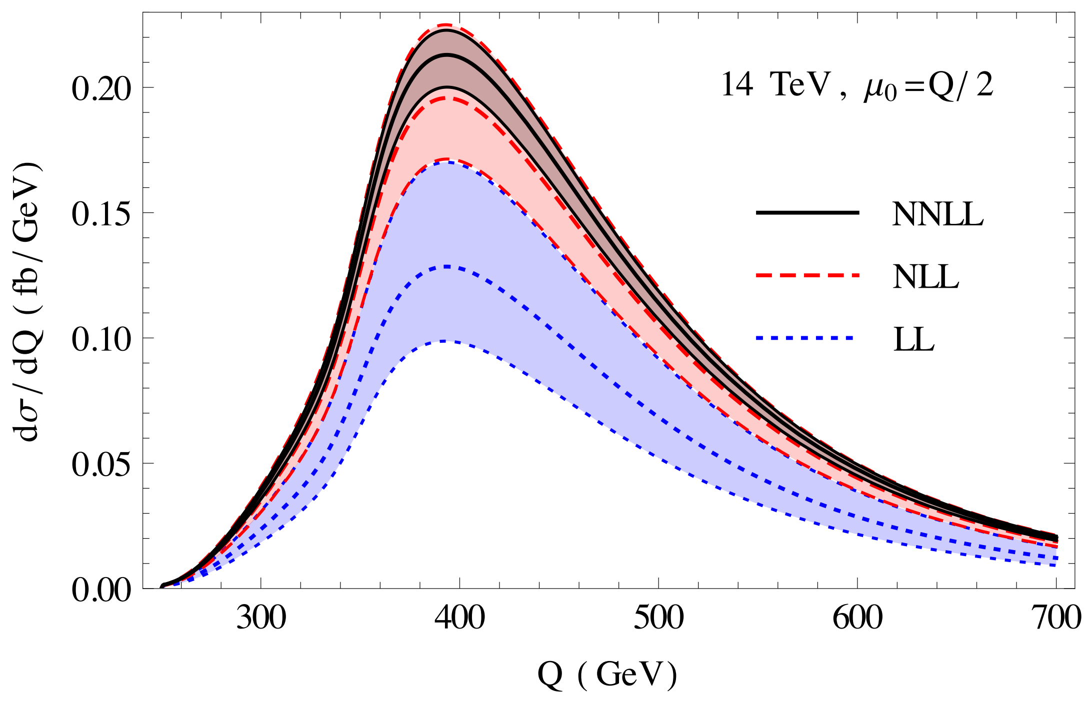
Grigo, Hoff, Steinhauser `15

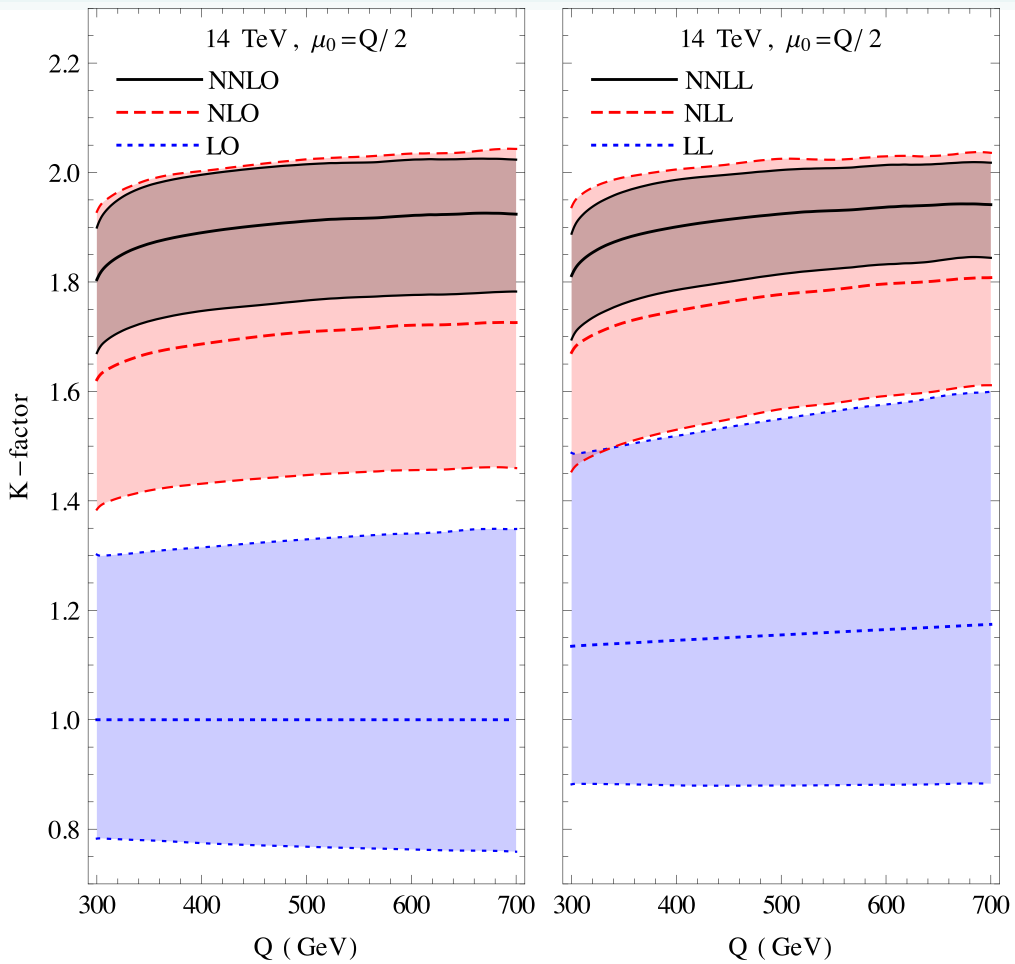
Differential Cross Section



Slawinska, van den Wollenberg,
van Eijk, Bentvelsen `14

NNLO and NNLL results





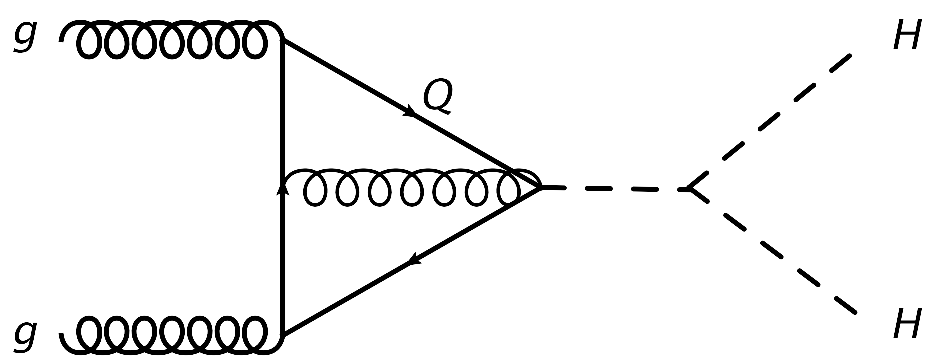
de Florian, Mazzitelli `15

$\mu_0 = Q$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+α_S unc. (%)
8 TeV	9.92	$+9.3-10$	10.8	$+5.4-5.9$	$+5.6-6.0$	$+9.3-9.2$
13 TeV	34.3	$+8.3-8.9$	36.8	$+5.1-6.0$	$+4.0-4.3$	$+7.7-7.5$
14 TeV	40.9	$+8.2-8.8$	43.7	$+5.1-6.0$	$+3.8-4.0$	$+7.5-7.3$
33 TeV	247	$+7.1-7.4$	259	$+5.0-6.1$	$+2.2-2.8$	$+6.1-6.1$
100 TeV	1660	$+6.8-7.1$	1723	$+5.2-6.1$	$+2.1-3.0$	$+5.7-5.8$

$\mu_0 = Q/2$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+α_S unc. (%)
8 TeV	10.8	$+5.7-8.5$	11.0	$+4.0-5.6$	$+5.8-6.1$	$+9.6-9.3$
13 TeV	37.2	$+5.5-7.6$	37.4	$+4.2-5.8$	$+4.1-4.3$	$+7.8-7.6$
14 TeV	44.2	$+5.5-7.6$	44.5	$+4.2-5.9$	$+3.9-4.1$	$+7.6-7.4$
33 TeV	264	$+5.3-6.6$	265	$+4.6-6.1$	$+2.4-2.7$	$+6.3-6.1$
100 TeV	1760	$+5.3-6.7$	1762	$+4.9-6.4$	$+2.2-3.1$	$+6.2-7.0$

Analytically known integrals

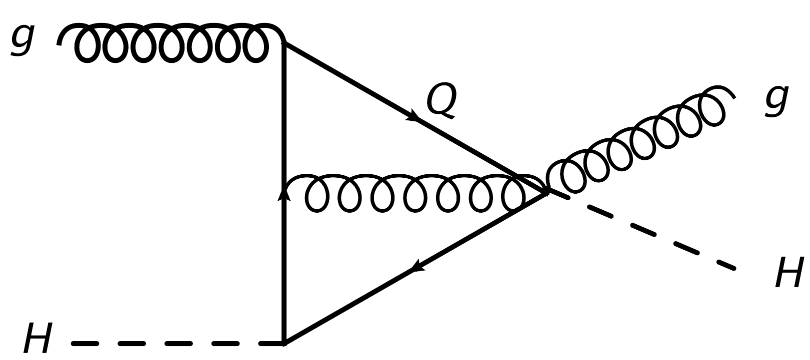
3-point, 1 off-shell leg



Spira, Djouadi et al. `93, `95
Bonciani, Mastrolia `03, `04
Anastasiou, Beerli et al. `06

$\rightarrow$HPLs

3-point, 2 off-shell leg



Gehrmann, Guns, Kara `15

$\rightarrow$ generalized HPLs, 12 letters

Amplitude Structure

Form factors are sums of rational functions multiplied by integrals that depend on ratios of the scales s, t, m_h^2, m_t^2 and the arbitrary scale M^2

$$F^{(L)} = \sum_i \left[\left(\sum_j C_{i,j}^{(L)} \epsilon^j \right) \cdot \left(\sum_k I_{i,k}^{(L)} \epsilon^k \right) \right]$$
$$= \quad \epsilon^{-2} \left[C_{1,-2}^{(L)} \cdot I_{1,0}^{(L)} + C_{1,-1}^{(L)} \cdot I_{1,-1}^{(L)} + \ldots \right]$$
$$+ \epsilon^{-1} \left[C_{1,-1}^{(L)} \cdot I_{1,0}^{(L)} + \ldots \right] + \ldots$$

compute only once

Additionally, all L-loop form factors are computed simultaneously without re-evaluating common integrals

Note: $gg \to HH$ is a loop induced process, real subtraction and mass factorisation contained in $\mathbf{I}, \mathbf{P}, \mathbf{K}$ operators (not discussed here)

Catani, Seymour 96

Slide: Stephen Jones — L&L 2016

Phase-Space Sampling

Phase-space implemented by hand

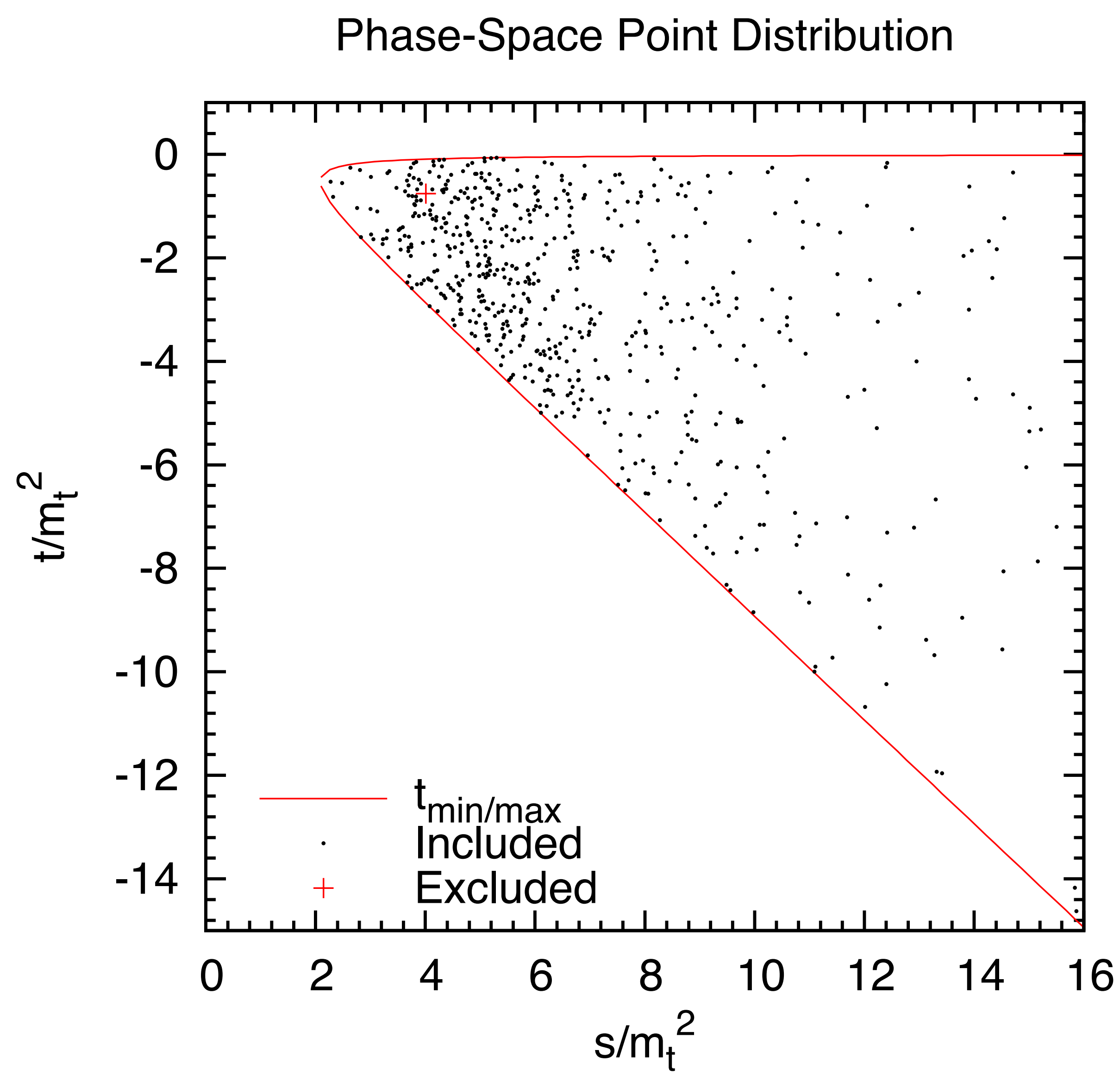
limited to 2-3 w/ 2 massive particles

Events for virtual:

1) VEGAS algorithm applied to LO matrix element $\mathcal{O}(100k)$ events computed

2) Using LO events unweighted events generated using accept/reject method $\mathcal{O}(30k)$ events remain

3) Randomly select 666 Events (woops), compute at NLO, exclude 1



Note: No grids used either for integrals or phase-space

Slide: Stephen Jones — L&L 2016