

# A new regulator for rapidity divergence and pT resummation for Higgs production at N3LL

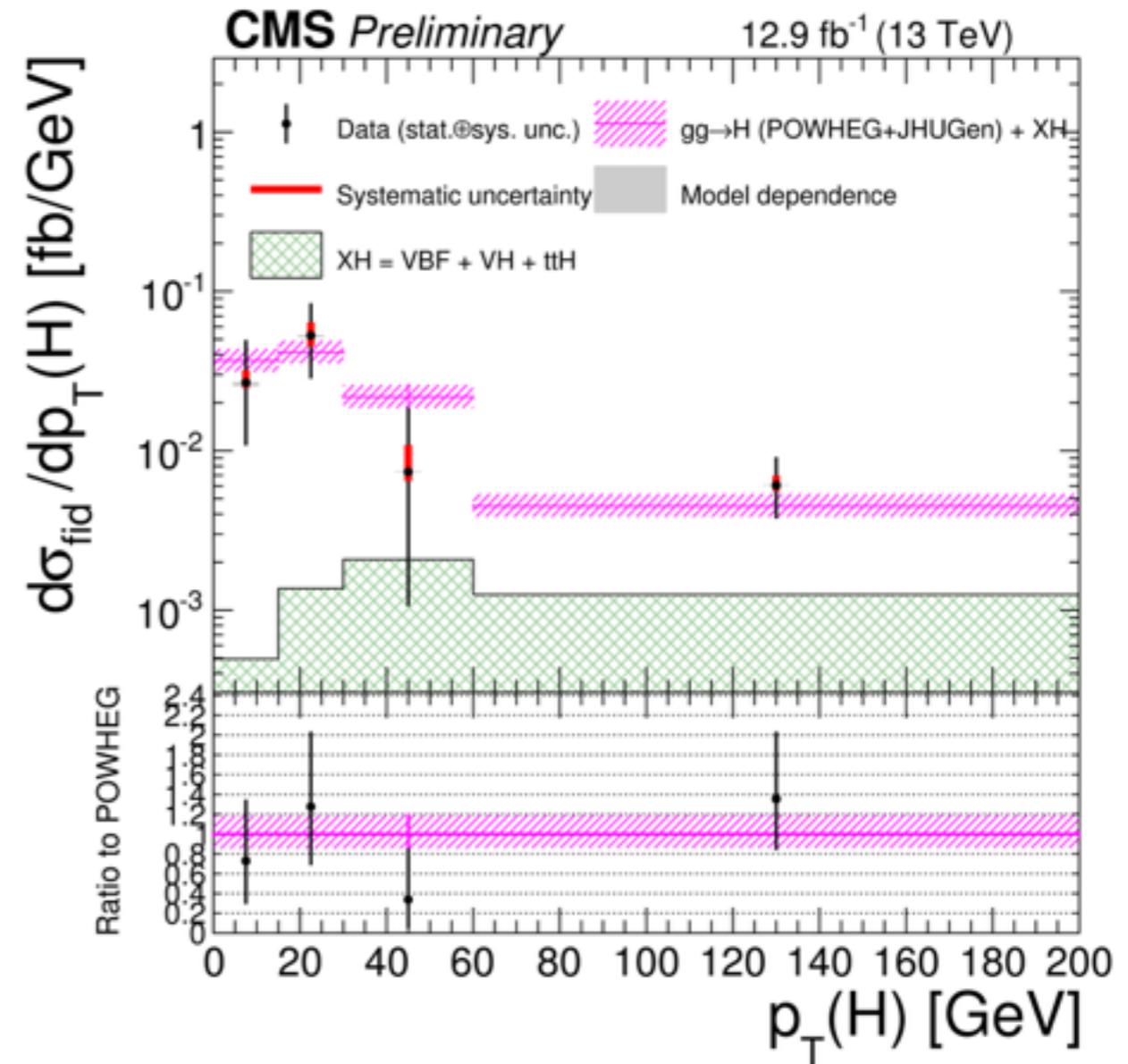
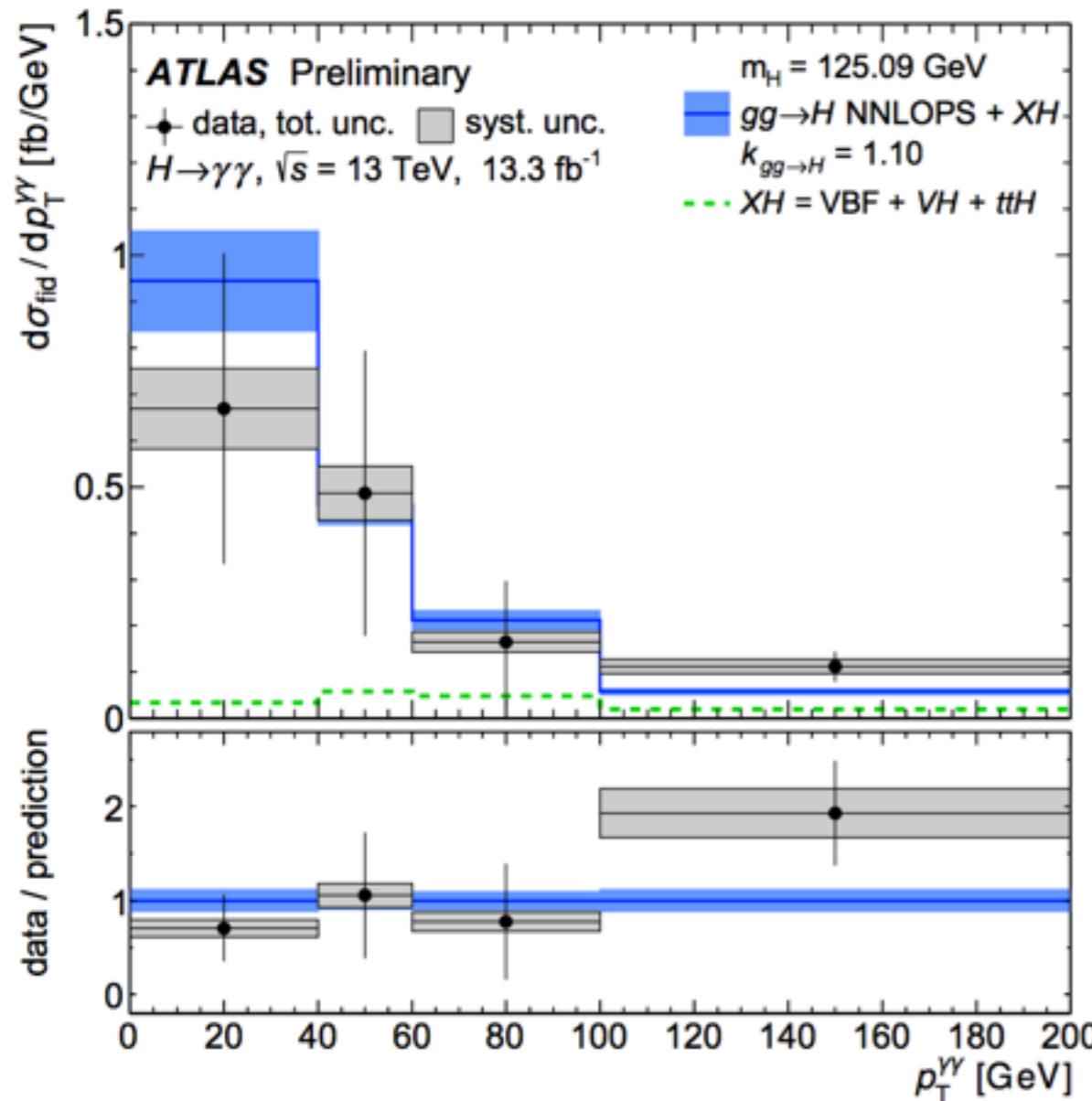
**Hua Xing Zhu**

with Ye Li, Duff Neill, 1604.00392, 1604.01404;  
and work in progress

**LoopFest XV, Buffalo**



# Entering the data rich era for Higgs physics



- ❖ Many interesting talks on Higgs pT distribution in this workshop
  - ❖ top-quark mass effects Neumann;
  - ❖ Fully-differential distribution: Mistlberger; Light-quark mass effects: Penin
- ❖ High energy resummation: Forte;

# Sudakov small pT resummation

Collins-Soper-Sterman, 1985

- ❖ At small pT differential distribution contains large logarithms:

$$\alpha_s^n \frac{1}{q_T^2} \ln^m \frac{M_H^2}{q_T^2} \xrightarrow{\text{Fourier transform}} \int d^2 \vec{q}_T \exp [i \vec{b} \cdot \vec{q}_T] \alpha_s^n \ln^{m+1} (M_H^2 b^2)$$

$$\begin{aligned} \ln \sigma(b) &\sim - \int_{1/b^2}^{m_H^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[ \ln \left( \frac{m_H^2}{\bar{\mu}^2} \right) A[\alpha_s(\bar{\mu})] + B[\alpha_s(\bar{\mu})] \right] \\ &= \alpha_s \left[ \ln^2(b^2 m_H^2) + \ln(b^2 m_H^2) \right] \\ &\quad \alpha_s^2 \left[ \ln^3(b^2 m_H^2) + \ln^2(b^2 m_H^2) + \ln(b^2 m_H^2) \right] \\ &\quad \alpha_s^3 \left[ \ln^4(b^2 m_H^2) + \ln^3(b^2 m_H^2) + \ln^2(b^2 m_H^2) + \ln(b^2 m_H^2) \right] \\ &\quad + \dots \end{aligned}$$

**LL**

$A_1$

**NLL**

$A_2$

**NNLL**

$A_3$

**N3LL**

$A_4$

See von Manteuffel's talk

$B_3$

**This talk!**

**Current states of the art**

# Factorization of pT distribution in SCET

- ❖ No operator definition for A and B. Going to higher order in logarithmic accuracy highly non-trivial and difficult
- ❖ Soft-Collinear Effective Theory can help!

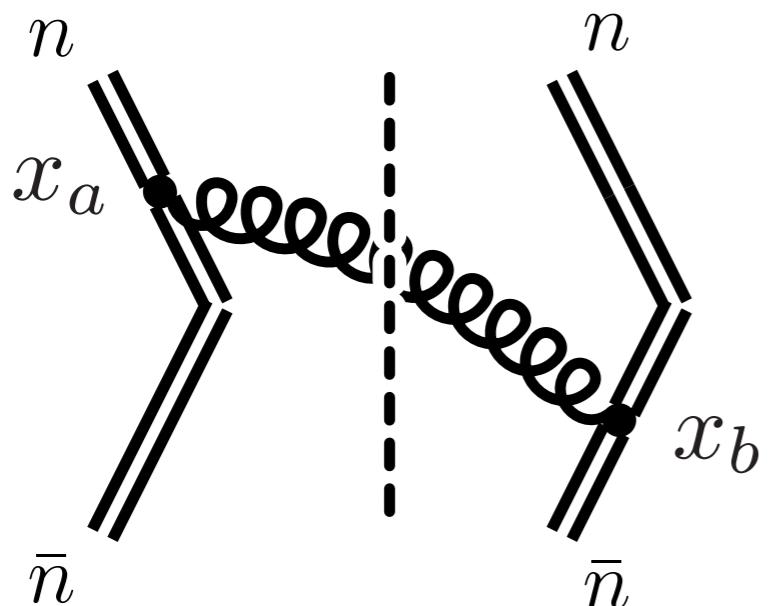
<b>hard function</b>	<b>Beam function</b>	<b>soft function</b>
$\frac{1}{\sigma} \frac{d\sigma}{d^2\vec{Q}_T dY dQ^2} \sim H(\mu) \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{Q}_T}$	$[B \otimes B](\vec{b}_\perp, \mu, \nu)$	$\cdot S_\perp(\vec{b}_\perp, \mu, \nu)$

- ❖ Cross section in SCET factorize into Wilson coefficients from integrating hard off-shell mode (hard function), matrix element of collinear fields (the beam function), and matrix element of soft Wilson line (soft function).
- ❖ Individual function contain UV and rapidity divergence. After regularization and renormalization: **μ** and **v** dependence

# Origin of rapidity divergence

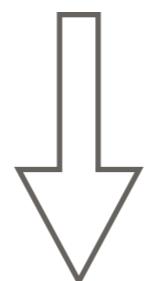
**unregulated soft function**

$$S_{\perp}(b) = \text{Tr} \langle 0 | T[S_{\bar{n}}^\dagger S_n(0)] \bar{T}[S_n^\dagger S_{\bar{n}}(\vec{b})] | 0 \rangle$$



$$\sim \int dx_a dx_b D_+(x_{ab}^2)$$

$$\sim \int_0^\infty dt_1 \int_0^\infty dt_2 \frac{1}{(t_1 t_2 + \vec{b}_\perp^2)^{1-\epsilon}}$$



**change of variable**

$$r = \frac{t_1}{t_2} \quad v = t_1 t_2$$

**unregulated rapidity divergence**

$$\sim \int_0^\infty \frac{dr}{r} \int_0^\infty \frac{dv}{(v^2 + \vec{b}_\perp^2)^{1-\epsilon}}$$

- ❖ There are many different proposals for rapidity regulator in the market:
  - ❖ Ji, Ma, Yuan '05: Tilting the Wilson line off light-cone
  - ❖ Mantry, Petriello '10: fully unintegrated collinear matrix element
  - ❖ Becher, Neubert '11; Becher, Bell, '12: asymmetric analytic regulator
  - ❖ Echevarria, Idilbi and Scimemi '11, Delta regulator
  - ❖ Collins '11: Tilting the Wilson line off light-cone with square-root soft subtraction
  - ❖ Chiu, Jain, Neill, Rothstein '12: CMU Rapidity Regulator
  - ❖ .....
- ❖ Our original goal was trying to use one of these regulator to compute anomalous dimension associated with three-loop rapidity divergence (which can then be related to  $B_3$ ). We end up finding yet a new regulator for rapidity divergence which worths exploring.

# A new regulator for rapidity divergence

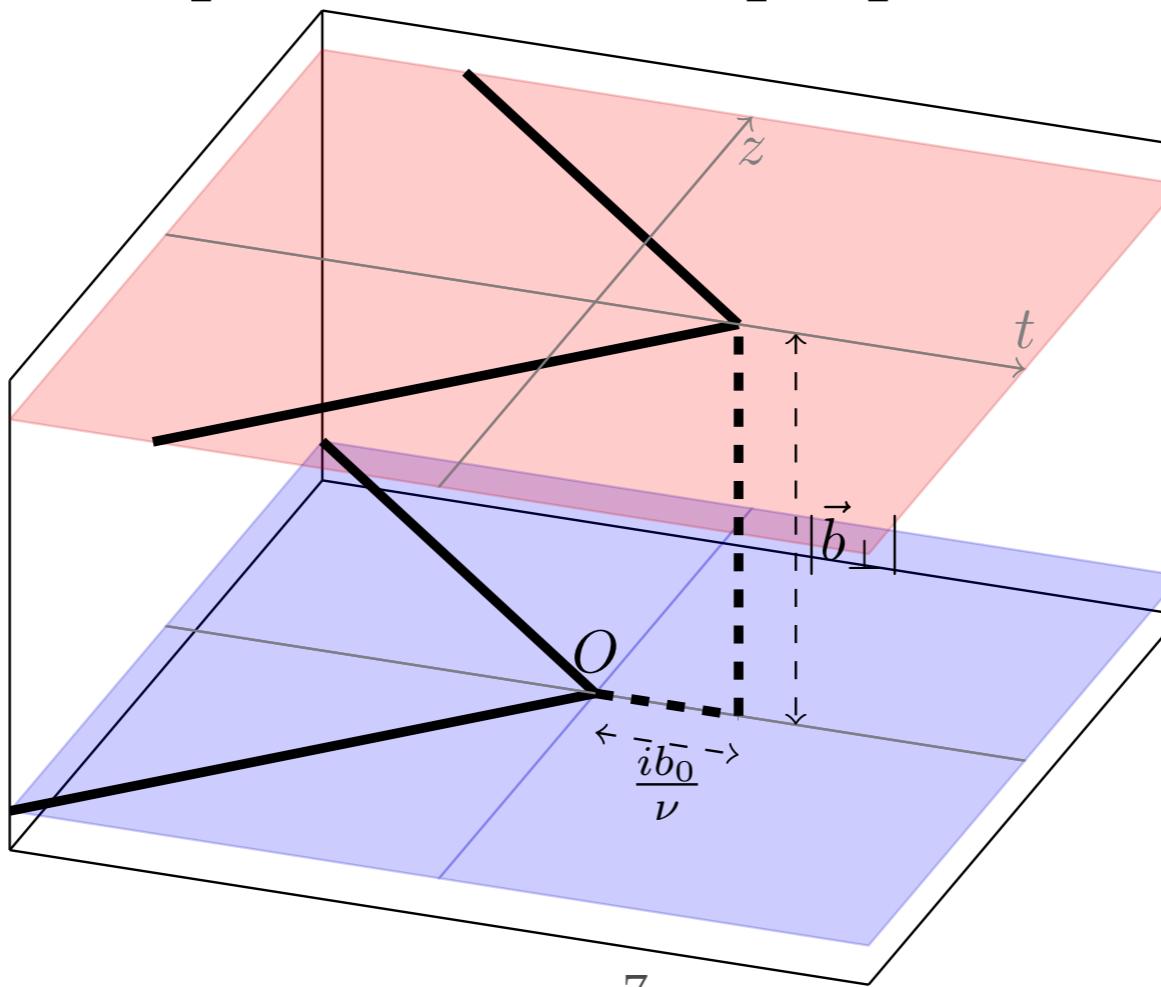
**unregulated soft function**  $S_{\perp}(\vec{b}_{\perp}) = \text{Tr} \langle 0 | T \left[ S_{\bar{n}}^{\dagger} S_n(0, 0, \vec{0}_{\perp}) \right] \overline{T} \left[ S_n^{\dagger} S_{\bar{n}}(0, 0, \vec{b}_{\perp}) \right] | 0 \rangle$

**regulated soft function**

$$S_{\perp}^{\text{reg}}(\vec{b}_{\perp}) = \lim_{\tau \rightarrow 0} \text{Tr} \langle 0 | T \left[ S_{\bar{n}}^{\dagger} S_n(0, 0, \vec{0}_{\perp}) \right] \overline{T} \left[ S_n^{\dagger} S_{\bar{n}}(ib_0\tau/2, ib_0\tau/2, \vec{b}_{\perp}) \right] | 0 \rangle$$

$$b_0 = 2e^{-\gamma_E}$$

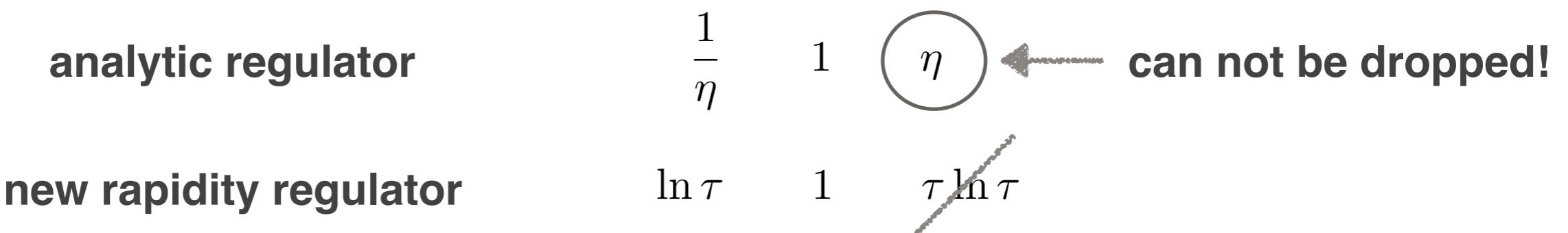
$$\nu = \frac{1}{\tau}$$



# Properties of the new rapidity regulator

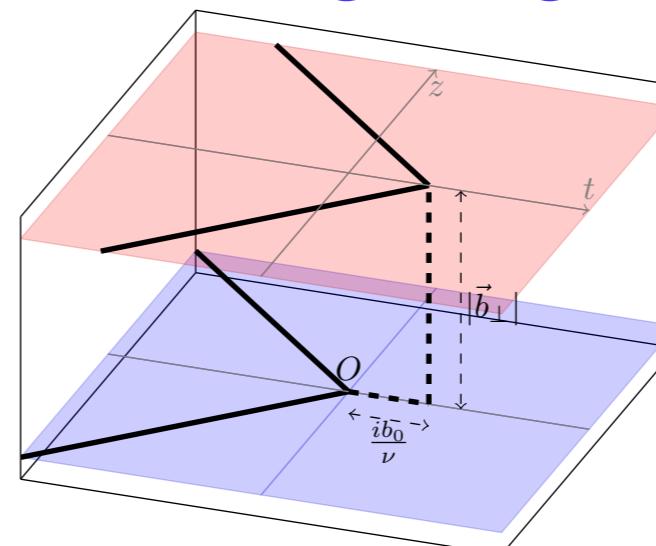
- ❖ Admit operator definition. Can be used to defined transverse-momentum dependent PDF non-perturbatively

- ❖ mass-like regulator v.s. analytic regulator



- ❖ Manifestly gauge invariant for non-singular gauge at infinite

Semi-infinite Wilson lines



- ❖ Preserve non-Abelian exponentiation theorem for soft Wilson loops

# Computing the three-loop soft function with the new regulator

$$\frac{d \ln S_\perp(\vec{b}_\perp, \mu, \nu)}{d \ln \nu^2} = \int_{\mu^2}^{b_0^2/\vec{b}_\perp^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \Gamma_{\text{cusp}} \left[ \alpha_s(\bar{\mu}) \right] + \gamma_r \left[ \alpha_s(b_0/|\vec{b}_\perp|) \right]$$

 $\gamma_r^0$ 

- ❖ Gehrmann, Lubbert, Yang (2012,2014)
- ❖ Echevarria, Scimemi, Vladimirov (2015)
- ❖ Luebbert, Oredsson, Stahlhofen (2016)

 $\gamma_r^1$  $B_1$ 

- ❖ Davies, Webber, Stirling (1985)
- ❖ Grazzini, de Florian (2000)

 $B_2$  $\gamma_r^2$  $B_3$ **Previously unknown**

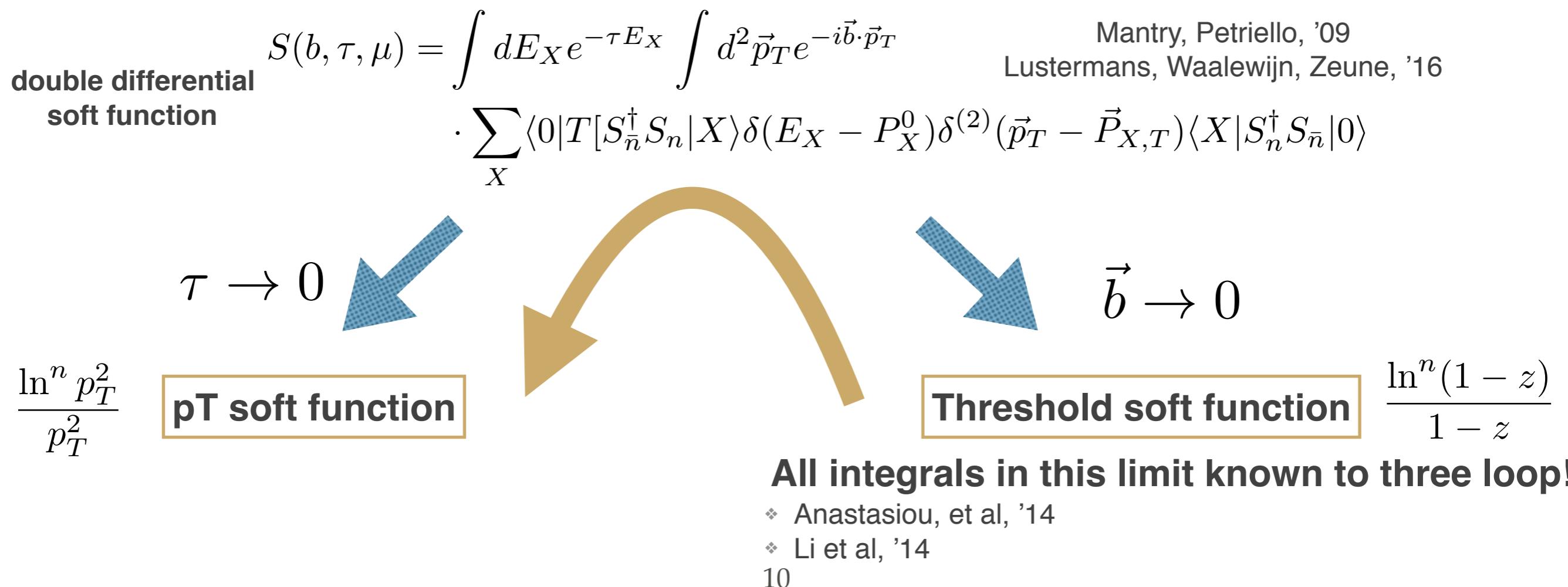
$$B_1 = \gamma_0^V - \gamma_0^r$$

$$B_2 = \gamma_1^V - \gamma_1^r + \beta_0 c_1^H$$

$$B_3 = \gamma_2^V - \gamma_2^r + \beta_1 c_1^H + 2\beta_0 \left( c_2^H - \frac{1}{2} (c_1^H)^2 \right)$$

# Double differential soft function

- ❖ To get the three-loop pT soft function, we take a detour
  - ❖ Lifting  $\tau$  as a dynamical variable, the soft function become double differential soft function
  - ❖ Taking the  $\tau \rightarrow 0$  limit afterwards to recover the pT soft function



# Two-loop double differential soft function

- ❖ Two-loop result can be extracted from 1105.5171 (Ye Li, Mantry, Petriello)
- ❖ Original expression written in terms of classical polylogarithms (Li2, Li3, Li4, Nielsen's polylogarithms). Can easily converted to HPL representation

$$S^{(1)}(b, \mu = 1/\tau) = 4C_F H_{0,1}(x) + \frac{\pi^2 C_F}{3}$$

$$S^{(2)}(b, \mu = 1/\tau) = C_A C_F \left[ -\frac{4}{3}\pi^2 H_{0,1}(x) + \frac{268}{9} H_{0,1}(x) + \frac{44}{3} H_{0,0,1}(x) \right.$$

$$\left. - \frac{44}{3} H_{0,1,1}(x) - 8 H_{0,0,0,1}(x) - 16 H_{0,0,1,1}(x) - 8 H_{0,1,0,1}(x) - 16 H_{0,1,1,1}(x) \right]$$

$$+ C_F n_f \left[ -\frac{40}{9} H_{0,1}(x) - \frac{8}{3} H_{0,0,1}(x) + \frac{8}{3} H_{0,1,1}(x) \right] + \frac{1}{2} \left[ 4C_F H_{0,1}(x) + \frac{\pi^2 C_F}{3} \right]^2$$

$$+ \left[ -\frac{22\zeta(3)}{9} + \frac{2428}{81} + \frac{67\pi^2}{54} - \frac{\pi^4}{3} \right] C_A C_F + \left[ \frac{4\zeta(3)}{9} - \frac{328}{81} - \frac{5\pi^2}{27} \right] C_F n_f$$

one-loop squared

$$x = -\frac{b^2}{b_0^2 \tau^2}$$

$$S^{(3)}(b, \tau) = ?$$

threshold constant

- ❖ We will do the calculation in N=4 Supersymmetric Yang-Mills theory first. Due to the maximal supersymmetry, the ansatz will be much simpler than QCD: uniform transcendentally
- ❖ N=4 SYM will capture the most complicated part of QCD
- ❖ QCD can be reconstructed from N=4 SYM by appropriate combination of color and matter content

transcendental weight	N=4 SYM	QCD	pure gluon	fermion	scalar
6		=			
5	$\emptyset$				
4	$\emptyset$				
3	$\emptyset$				
2	$\emptyset$				

[N=4 SYM] = 1 gluon + 4 majorana fermion + 3 complex scalar

# Ansatz for N=4 SYM

$$\begin{aligned} & \text{NC}^3 \left( \frac{1}{90} \pi^4 c_{23} H_{0,1}[x] + c_{21} \text{Zeta}[3] H_{0,0,1}[x] + c_{22} \text{Zeta}[3] H_{0,1,1}[x] + \right. \\ & \frac{1}{6} \pi^2 c_{17} H_{0,0,0,1}[x] + \frac{1}{6} \pi^2 c_{18} H_{0,0,1,1}[x] + \frac{1}{6} \pi^2 c_{19} H_{0,1,0,1}[x] + \\ & \frac{1}{6} \pi^2 c_{20} H_{0,1,1,1}[x] + c_1 H_{0,0,0,0,0,1}[x] + c_2 H_{0,0,0,0,1,1}[x] + \\ & c_3 H_{0,0,0,1,0,1}[x] + c_4 H_{0,0,0,1,1,1}[x] + c_5 H_{0,0,1,0,0,1}[x] + c_6 H_{0,0,1,0,1,1}[x] + \\ & c_7 H_{0,0,1,1,0,1}[x] + c_8 H_{0,0,1,1,1,1}[x] + c_9 H_{0,1,0,0,0,1}[x] + c_{10} H_{0,1,0,0,1,1}[x] + \\ & c_{11} H_{0,1,0,1,0,1}[x] + c_{12} H_{0,1,0,1,1,1}[x] + c_{13} H_{0,1,1,0,0,1}[x] + \\ & \left. c_{14} H_{0,1,1,0,1,1}[x] + c_{15} H_{0,1,1,1,0,1}[x] + c_{16} H_{0,1,1,1,1,1}[x] \right) \end{aligned}$$

- ❖ The ansatz has uniform degree of transcendentality
- ❖ Ci are rational coefficients need to be fixed

# Fixing the coefficients by expanding in small impact parameter

- The ansatz admits a simple Taylor series expansion around  $b=0$ .

$$\begin{aligned}
 & Nc^3 \left( c_1 + \frac{\pi^2 c_{17}}{6} + \frac{\pi^4 c_{23}}{90} + c_{21} \text{Zeta}[3] \right) x + \\
 & \frac{1}{64} Nc^3 \left( c_1 + 2 c_2 + 4 c_3 + 8 c_5 + 16 c_9 + \frac{2 \pi^2 c_{17}}{3} + \frac{4 \pi^2 c_{18}}{3} + \right. \\
 & \left. \frac{8 \pi^2 c_{19}}{3} + \frac{8 \pi^4 c_{23}}{45} + 8 c_{21} \text{Zeta}[3] + 16 c_{22} \text{Zeta}[3] \right) x^2 + \frac{1}{11664} \\
 & Nc^3 \left( 16 c_1 + 72 c_2 + 180 c_3 + 72 c_4 + 486 c_5 + 108 c_6 + 216 c_7 + 1377 c_9 + \right. \\
 & 162 c_{10} + 324 c_{11} + 648 c_{13} + 24 \pi^2 c_{17} + 108 \pi^2 c_{18} + 270 \pi^2 c_{19} + \\
 & \left. 108 \pi^2 c_{20} + \frac{72 \pi^4 c_{23}}{5} + 432 c_{21} \text{Zeta}[3] + 1944 c_{22} \text{Zeta}[3] \right) x^3 + O[x]^4
 \end{aligned}$$

$$x = -\frac{b^2}{b_0^2 \tau^2}$$

# Fixing the coefficients by expanding in small impact parameter

- ❖ On the other hand, the coefficients can be obtained from direct calculation

$$S(b, \tau, \mu) = \int dE_X e^{-\tau E_X} \int d^2 \vec{p}_T e^{-i \vec{b} \cdot \vec{p}_T}$$

$$\cdot \sum_X \langle 0 | T[S_{\bar{n}}^\dagger S_n | X \rangle \delta(E_X - P_X^0) \delta^{(2)}(\vec{p}_T - \vec{P}_{X,T}) \langle X | S_n^\dagger S_{\bar{n}} | 0 \rangle$$

$$e^{-i \vec{b} \cdot \vec{p}_T} = 1 + (-i \vec{b} \cdot \vec{p}_T) + \frac{1}{2!} (-i \vec{b} \cdot \vec{p}_T)^2 + \frac{1}{3!} (-i \vec{b} \cdot \vec{p}_T)^3 + \frac{1}{4!} (-i \vec{b} \cdot \vec{p}_T)^4 + \dots$$

$$x = -\frac{b^2}{b_0^2 \tau^2}$$

- ❖ Obtained a system of linear equation
- ❖ The system is overdetermined. If there is a solution, it is unique

$$\begin{aligned} & \{-48 + c_1 = 0, -1296 + c_1 + 2c_2 + 4c_3 + 8c_5 + 16c_9 = 0, \\ & 165\,864 + 632c_1 - 72c_2 - 828c_3 - 72c_4 - 1134c_5 - \\ & 108c_6 + 432c_7 - 2025c_9 - 162c_{10} + 324c_{11} - 648c_{15} = 0, \\ & 3\,853\,584 + 14\,607c_1 + 2574c_2 - 23\,364c_3 - 2448c_4 - 24\,168c_5 - 3888c_6 + \\ & 18\,864c_7 - 864c_8 - 36\,976c_9 - 6912c_{10} + 13\,536c_{11} - 27\,072c_{15} = 0, \\ & 3\,256\,166\,328 + 8\,874\,264c_1 + 4\,801\,500c_2 - 21\,783\,000c_3 - 2\,286\,000c_4 - \\ & 18\,157\,500c_5 - 3\,807\,000c_6 + 21\,409\,500c_7 - 1\,620\,000c_8 - \\ & 22\,875\,625c_9 - 7\,728\,750c_{10} + 14\,902\,500c_{11} - 29\,797\,500c_{15} = 0, \\ & -4\,171\,466\,664 - 6\,429\,500c_1 - 8\,552\,500c_2 + 29\,538\,400c_3 + 2\,961\,000c_4 + \\ & 20\,718\,530c_5 + 5\,134\,500c_6 - 32\,517\,000c_7 + 3\,150\,000c_8 + \\ & 20\,440\,861c_9 + 11\,616\,750c_{10} - 22\,131\,000c_{11} + 44\,235\,000c_{15} = 0, \\ & 12\,326\,351\,229\,944 + 5\,714\,697\,530c_1 + 30\,446\,309\,250c_2 - 90\,474\,451\,130c_3 - \\ & 8\,541\,729\,000c_4 - 55\,036\,370\,270c_5 - 15\,341\,875\,500c_6 + \\ & 107\,266\,753\,580c_7 - 12\,101\,040\,000c_8 - 39\,368\,935\,291c_9 - \\ & 38\,025\,837\,500c_{10} + 71\,742\,192\,130c_{11} - 143\,339\,171\,780c_{15} = 0, \\ & -235\,454\,875\,529\,744 + 110\,392\,538\,305c_1 - 655\,804\,080\,750c_2 + \\ & + \dots \end{aligned}$$

# The N=4 SYM solution

**one loop**  $S_1^{\mathcal{N}=4}(\tau, \vec{b}_\perp, \mu = \tau^{-1}) = c_1^{s, \mathcal{N}=4} + 4N_c H_2$

**two loop**  $S_2^{\mathcal{N}=4}(\tau, \vec{b}_\perp, \mu = \tau^{-1}) = c_2^{s, \mathcal{N}=4} + N_c^2 \left( -8\zeta_2 H_2 - 8H_4 - 8H_{2,2} - 16H_{3,1} - 16H_{2,1,1} \right)$

**three loop**  $S_3^{\mathcal{N}=4}(\vec{b}_\perp, \tau, \mu = \tau^{-1}) = c_3^{s, \mathcal{N}=4} + N_c^3 \left( 16\zeta_2 H_4 + 48\zeta_2 H_{2,2} + 64\zeta_2 H_{3,1} + 96\zeta_2 H_{2,1,1} + 120\zeta_4 H_2 + 48H_6 + 24H_{2,4} + 40H_{3,3} + 72H_{4,2} + 128H_{5,1} + 16H_{2,1,3} + 56H_{2,2,2} + 80H_{2,3,1} + 80H_{3,1,2} + 144H_{3,2,1} + 224H_{4,1,1} + 64H_{2,1,1,2} + 96H_{2,1,2,1} + 160H_{2,2,1,1} + 256H_{3,1,1,1} + 192H_{2,1,1,1,1} \right)$

- ❖ All terms at given loop are integers with uniform sign
- ❖ Alternating sign between different loop order
- ❖ These are highly non-trivial check of the correctness of the result!

- ❖ We are ultimately interested in QCD. Two different approaches
  - ❖ Direct Feynman diagram calculation of the fermionic contribution (method of differential equation, many integrals, known, 12 new integrals)

**[N=4 SYM] = 1 gluon + 4 majorana fermion + 3 complex scalar**

- ❖ Similar to N=4 SYM case, we made an ansatz and try to fix the coefficient by expanding in small impact parameter
  - ❖ most complicated terms (highest weight terms) given by N=4 SYM
  - ❖ new complication: need new terms in the ansatz

$$, \left( H_{1,1}[\mathbf{x}] - \frac{H_{1,1}[\mathbf{x}]}{\mathbf{x}} \right)$$

# Full three-loop double differential soft function in QCD

$$\begin{aligned}
& -\frac{8}{3} C_A^2 C_F \left( H_{1,1}[x] - \frac{H_{1,1}[x]}{x} \right) + \frac{8}{3} C_A C_F n_f \left( H_{1,1}[x] - \frac{H_{1,1}[x]}{x} \right) + \xleftarrow{\text{Cancel in N=1 SYM}} \\
& C_F^2 n_f \left( -\frac{110}{3} H_{0,1}[x] + 32 \text{Zeta}[3] H_{0,1}[x] - 8 H_{0,0,1}[x] + 8 H_{0,1,1}[x] \right) + \\
& C_F n_f^2 \left( \frac{400}{81} H_{0,1}[x] + \frac{160}{27} H_{0,0,1}[x] - \frac{160}{27} H_{0,1,1}[x] + \frac{32}{9} H_{0,0,0,1}[x] - \frac{32}{9} H_{0,0,1,1}[x] + \frac{32}{9} H_{0,1,1,1}[x] \right) + \\
& C_A C_F n_f \left( -\frac{7988}{81} H_{0,1}[x] + \frac{160}{9} \zeta_2 H_{0,1}[x] - \frac{2312}{27} H_{0,0,1}[x] + \frac{16}{3} \zeta_2 H_{0,0,1}[x] + \right. \\
& \left. \frac{2312}{27} H_{0,1,1}[x] - 16 \zeta_2 H_{0,1,1}[x] - \frac{64}{3} H_{0,0,0,1}[x] + \frac{224}{3} H_{0,0,1,1}[x] + \frac{160}{9} H_{0,1,0,1}[x] - \right. \\
& \left. \frac{32}{9} H_{0,1,1,1}[x] + \frac{80}{3} H_{0,0,0,0,1}[x] + \frac{64}{3} H_{0,0,0,1,1}[x] + 16 H_{0,0,1,0,1}[x] - \frac{64}{3} H_{0,0,1,1,1}[x] + \right. \\
& \left. \frac{16}{3} H_{0,1,0,0,1}[x] - \frac{64}{3} H_{0,1,0,1,1}[x] - \frac{16}{3} H_{0,1,1,0,1}[x] - 64 H_{0,1,1,1,1}[x] \right) + \\
& C_A^2 C_F \left( \frac{30790}{81} H_{0,1}[x] - \frac{1072}{9} \zeta_2 H_{0,1}[x] + 120 \zeta_4 H_{0,1}[x] - 176 \text{Zeta}[3] H_{0,1}[x] + \frac{7120}{27} H_{0,0,1}[x] - \right. \\
& \left. \frac{88}{3} \zeta_2 H_{0,0,1}[x] - \frac{7120}{27} H_{0,1,1}[x] + 88 \zeta_2 H_{0,1,1}[x] - \frac{104}{9} H_{0,0,0,1}[x] + 16 \zeta_2 H_{0,0,0,1}[x] - \right. \\
& \left. \frac{3112}{9} H_{0,0,1,1}[x] + 64 \zeta_2 H_{0,0,1,1}[x] - \frac{1072}{9} H_{0,1,0,1}[x] + 48 \zeta_2 H_{0,1,0,1}[x] - \frac{392}{3} H_{0,1,1,1}[x] + \right. \\
& \left. 96 \zeta_2 H_{0,1,1,1}[x] - \frac{440}{3} H_{0,0,0,0,1}[x] - \frac{352}{3} H_{0,0,0,1,1}[x] - 88 H_{0,0,1,0,1}[x] + \frac{352}{3} H_{0,0,1,1,1}[x] - \right. \\
& \left. \frac{88}{3} H_{0,1,0,0,1}[x] + \frac{352}{3} H_{0,1,0,1,1}[x] + \frac{88}{3} H_{0,1,1,0,1}[x] + 352 H_{0,1,1,1,1}[x] + 48 H_{0,0,0,0,0,1}[x] + \right. \\
& \left. 128 H_{0,0,0,0,1,1}[x] + 72 H_{0,0,0,1,0,1}[x] + 224 H_{0,0,0,1,1,1}[x] + 40 H_{0,0,1,0,0,1}[x] + 144 H_{0,0,1,0,1,1}[x] + \right. \\
& \left. 80 H_{0,0,1,1,0,1}[x] + 256 H_{0,0,1,1,1,1}[x] + 24 H_{0,1,0,0,0,1}[x] + 80 H_{0,1,0,0,1,1}[x] + 56 H_{0,1,0,1,0,1}[x] + \right. \\
& \left. 160 H_{0,1,0,1,1,1}[x] + 16 H_{0,1,1,0,0,1}[x] + 96 H_{0,1,1,0,1,1}[x] + 64 H_{0,1,1,1,0,1}[x] + 192 H_{0,1,1,1,1,1}[x] \right)
\end{aligned}$$

- ❖ Taking the  $\tau \rightarrow 0$ , rapidity divergence manifest as  $\text{Log}(\tau)$

$$\gamma_0^r = 0$$

$$\gamma_1^r = C_a C_A \left( 28\zeta_3 - \frac{808}{27} \right) + \frac{112C_a n_f}{27}$$

$$\begin{aligned} \gamma_2^r = & C_a C_A^2 \left( -\frac{176}{3}\zeta_3\zeta_2 + \frac{6392\zeta_2}{81} + \frac{12328\zeta_3}{27} + \frac{154\zeta_4}{3} \right. \\ & \left. - 192\zeta_5 - \frac{297029}{729} \right) + C_a C_A n_f \left( -\frac{824\zeta_2}{81} - \frac{904\zeta_3}{27} \right. \\ & \left. + \frac{20\zeta_4}{3} + \frac{62626}{729} \right) + C_a n_f^2 \left( -\frac{32\zeta_3}{9} - \frac{1856}{729} \right) \\ & + C_a C_F N_f \left( -\frac{304\zeta_3}{9} - 16\zeta_4 + \frac{1711}{27} \right) \end{aligned}$$

# Intriguing relation between rapidity anomalous dimension and threshold anomalous dimension

Control  $\left[ \frac{1}{1-z} \right]_+$  of threshold logarithms

$$\begin{aligned}\gamma_0^r &= \gamma_0^s \\ \gamma_1^r &= \gamma_1^s - \beta_0 c_1^s \\ \gamma_2^r &= \gamma_2^s - 2\beta_0 c_2^s - \beta_1 c_1^s + 2\beta_0 \zeta_4\end{aligned}$$

constant term in threshold soft function

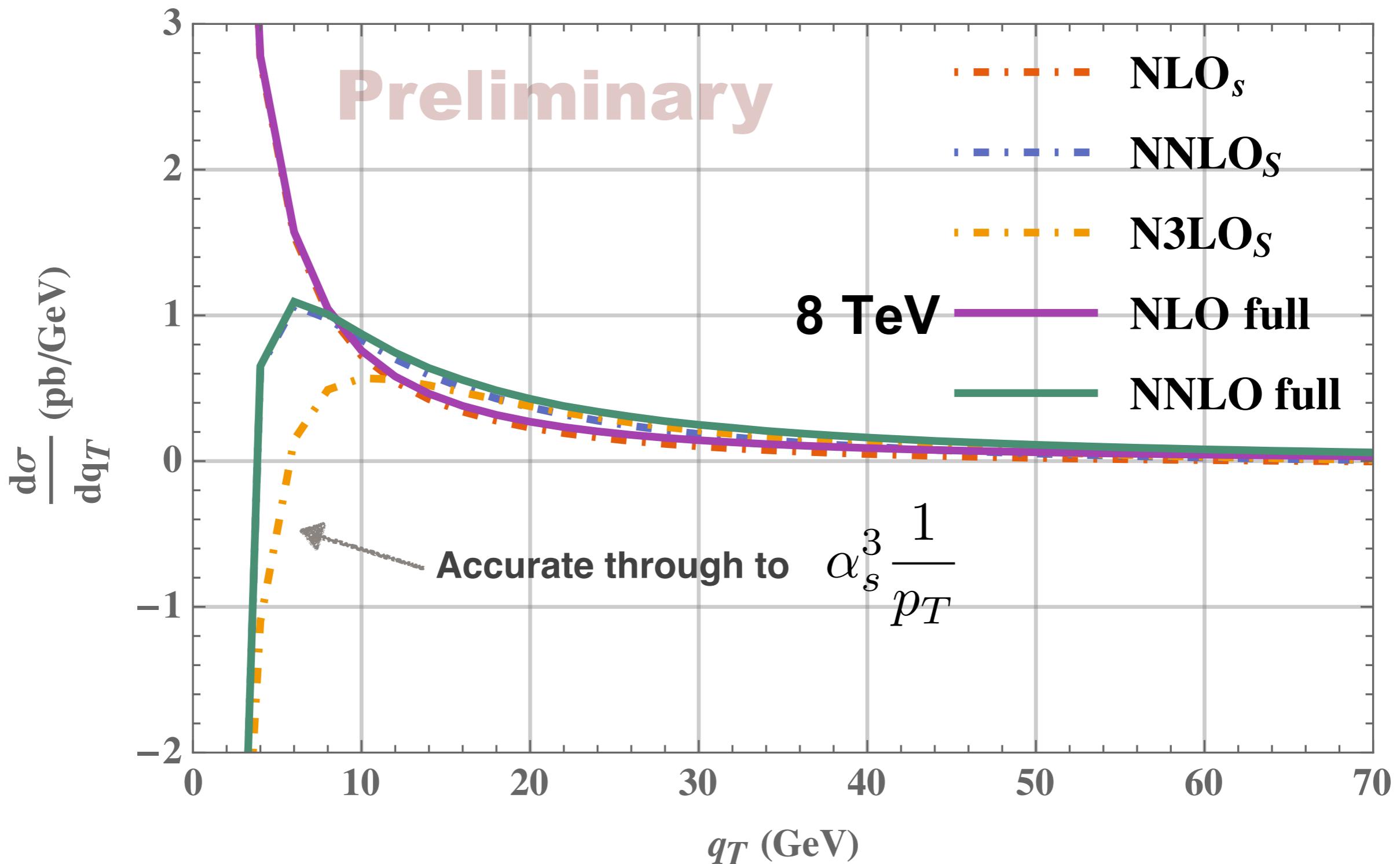
Control  $\left[ \frac{1}{P_T^2} \right]_*$  of pT distribution

# pT resummation for Higgs production at N3LL

- ❖ **Resummation performed in b space**
- ❖ **Perturbative order of various ingredients:**
  - ❖ Two-loop hard function, beam function, soft function
  - ❖ Three-loop normal anomalous dimension
  - ❖ Three-loop splitting function
  - ❖ Three-loop rapidity anomalous dimension (new)
  - ❖ Four-loop cusp anomalous dimension (Pade approximation)
- ❖ **Scale uncertainties estimated by varying hard scale, beam and soft  $\mu$  scale, soft v scale.**
- ❖ **Simple  $b^*$  scheme for non-perturbative effects**
- ❖ **Light quark mass effects included at fixed order**

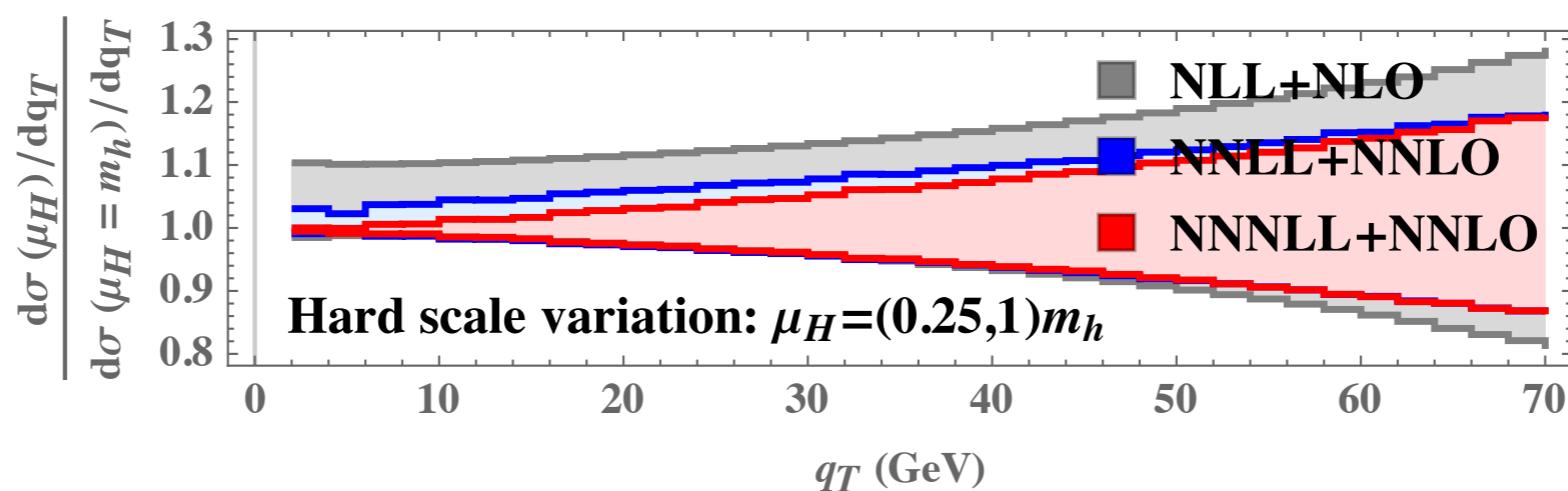
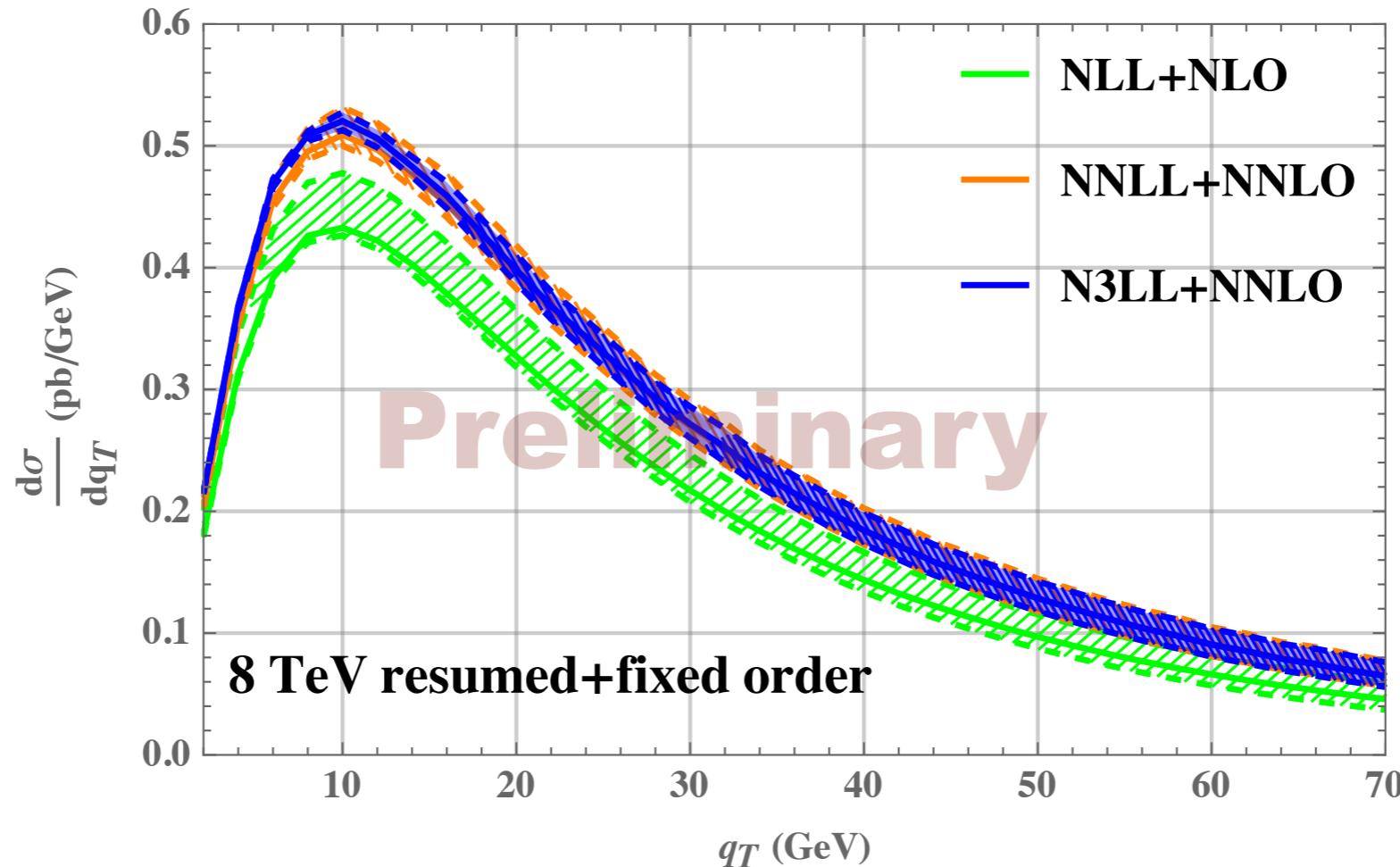
$$b^* = \frac{b}{\sqrt{1 + b^2/b_{\max}^2}}$$

# Singular distribution and fixed order

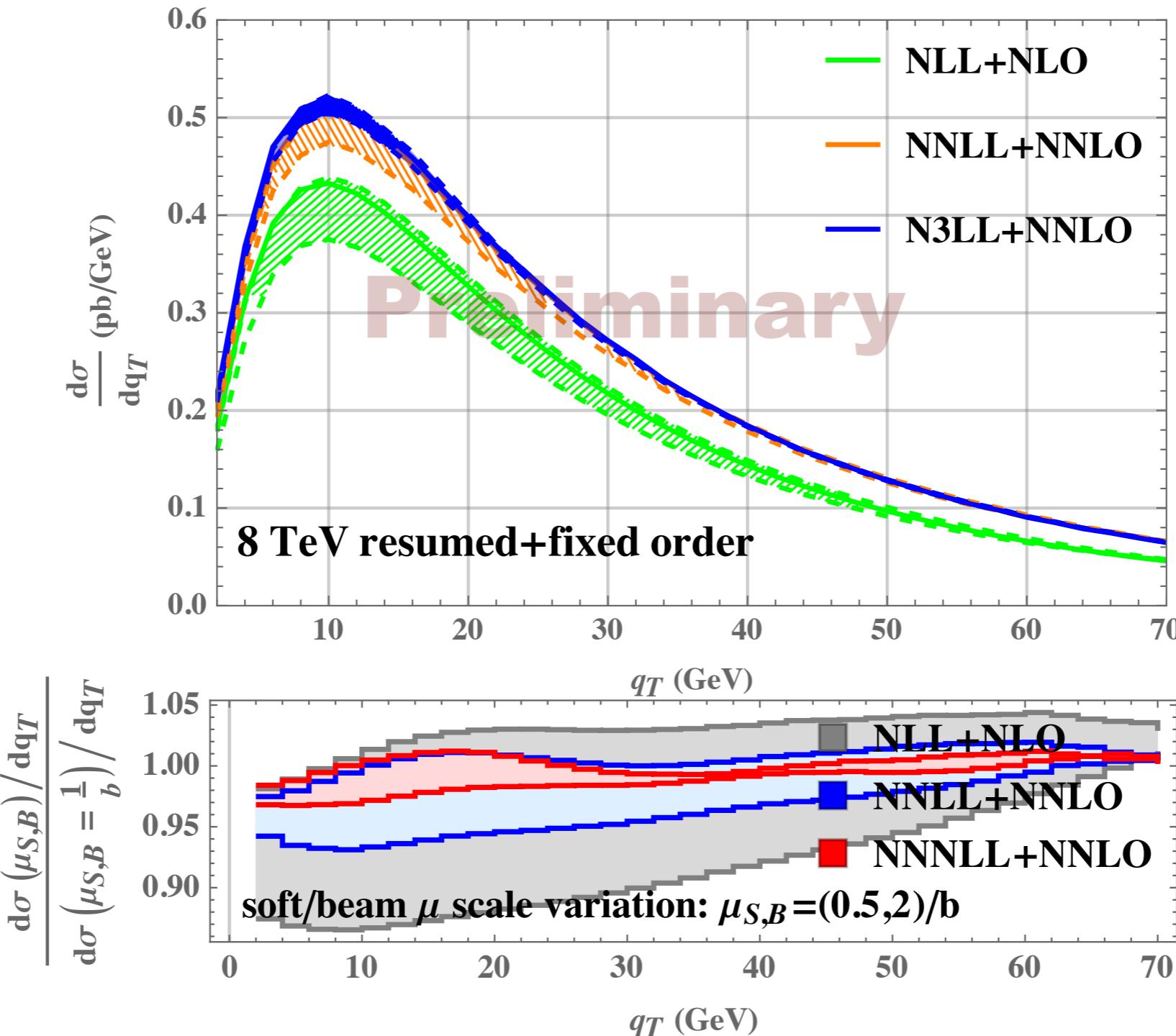


- ❖ NLO full: LO H+j production; NNLO full: NLO H+j production

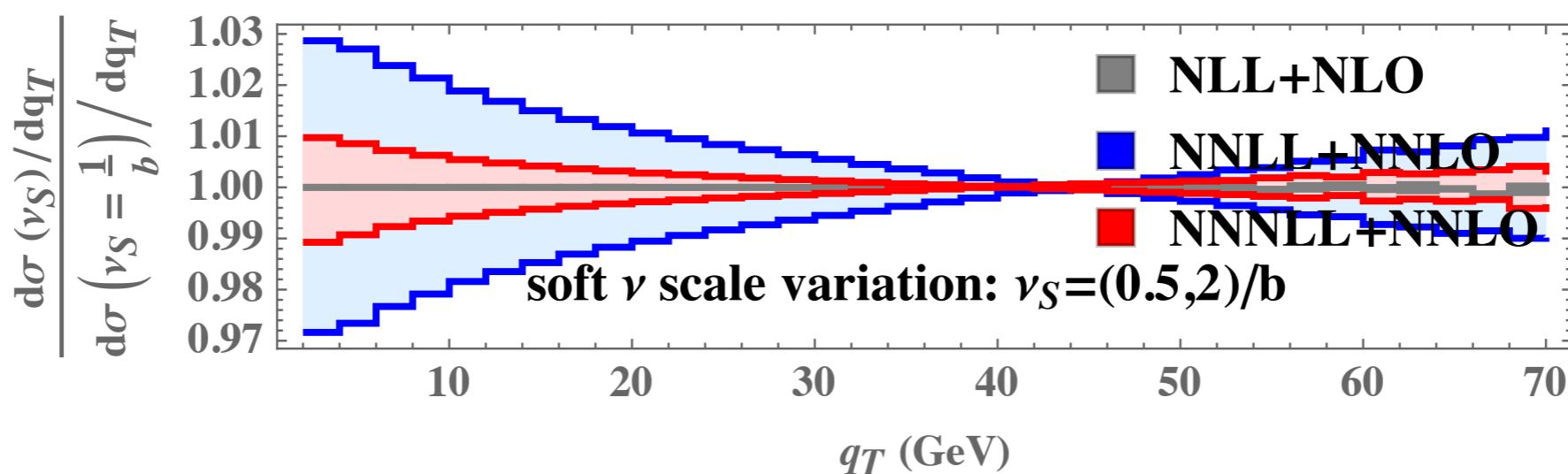
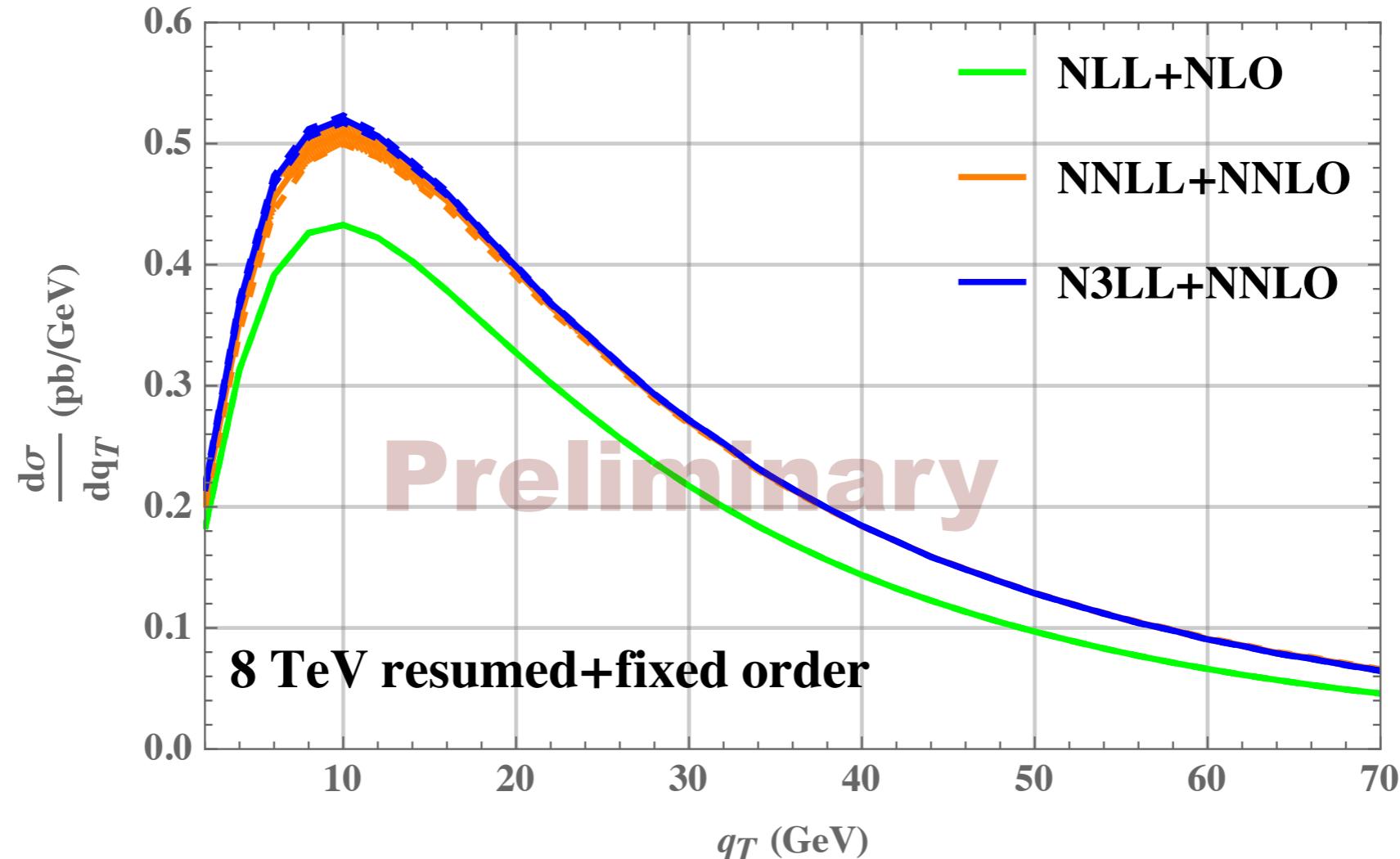
# Hard scale variation



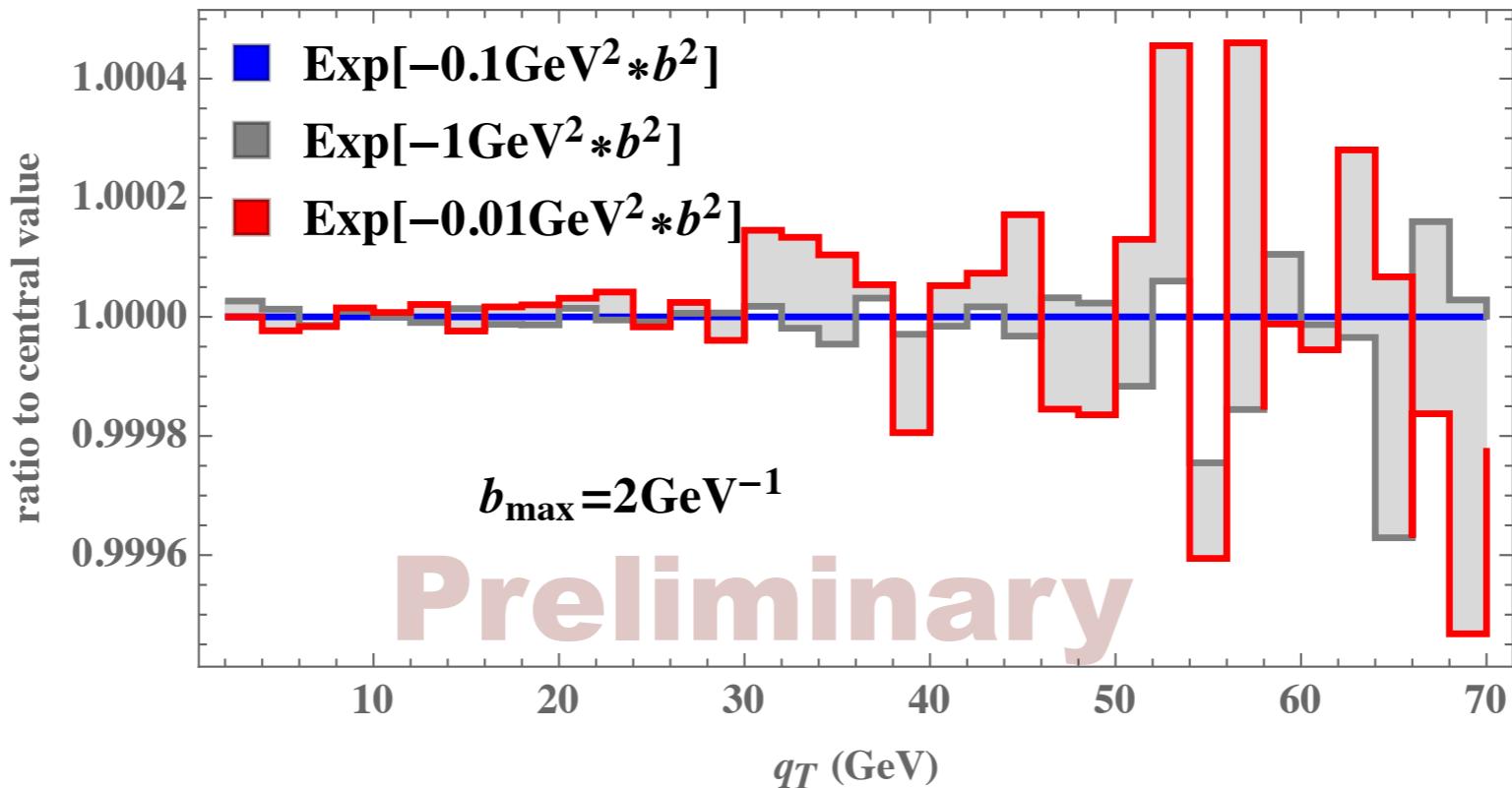
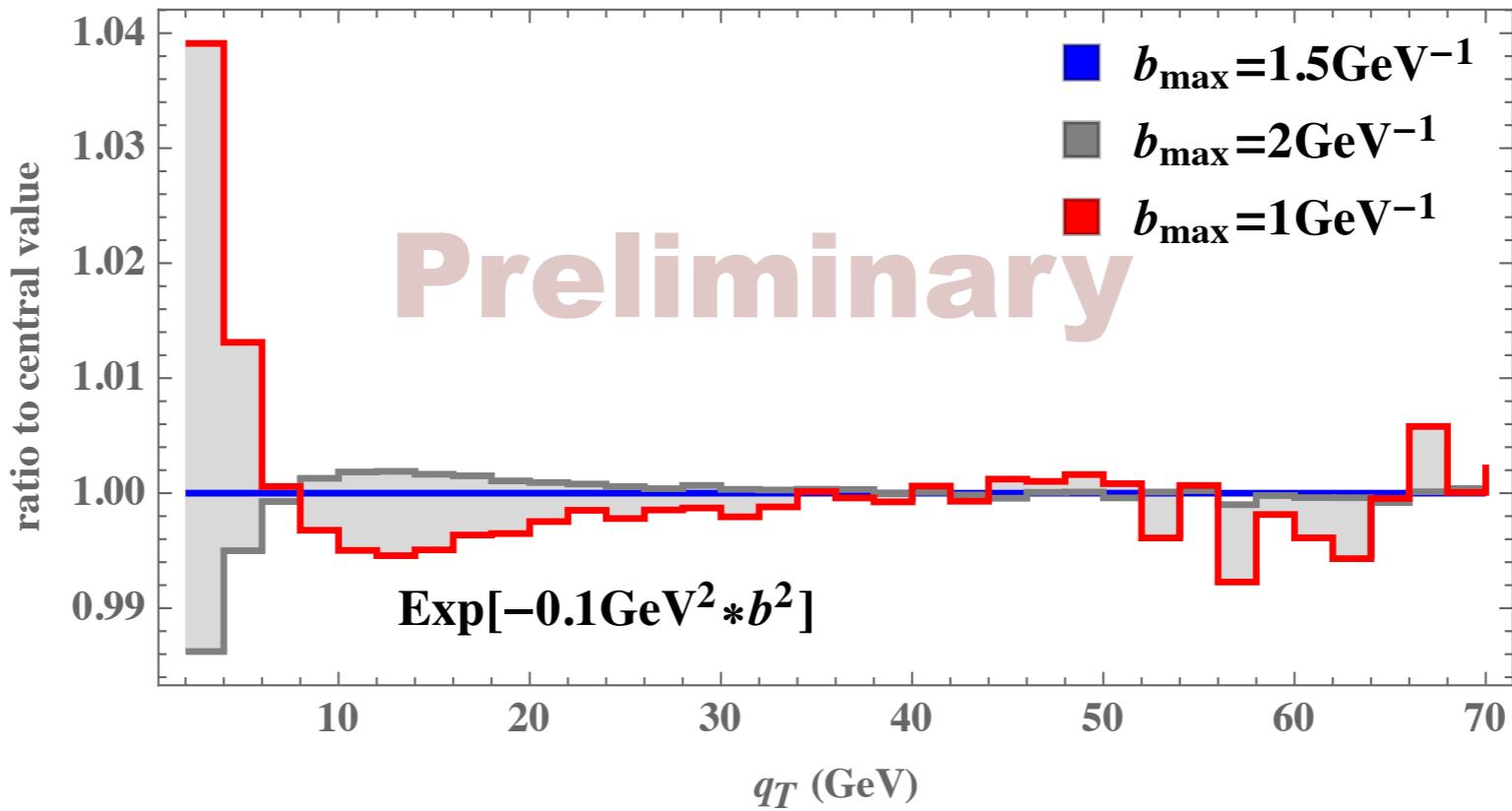
# soft/beam $\mu$ scale variation



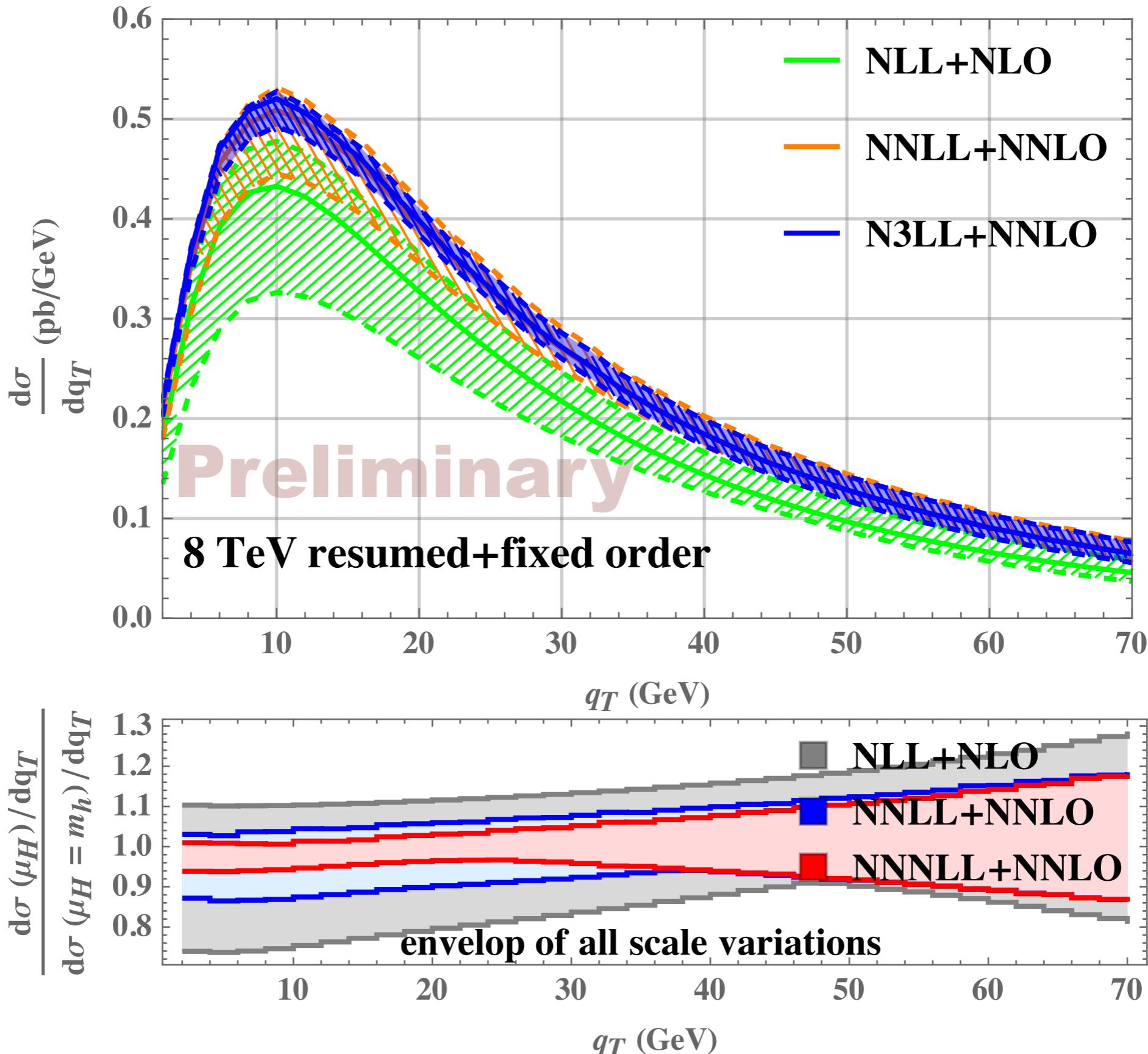
# v scale variation



# Non-Perturbative uncertainties



# Total scale uncertainties



# Summary

- ❖ **Introduce a new regulator for rapidity divergence in SCET description of transverse-momentum distribution.**
- ❖ **Analytic calculation of the resulting three-loop soft function through three-loops for the first time, extracting the rapidity anomalous dimension (also known as collinear anomaly  $d_2$ )**
  - ❖ Lifting the rapidity regulator as an dynamical variable: double differential soft function
  - ❖ Compute the double differential soft function (the  $N=4$  part) by making an ansatz, and then fixing the coefficient using expansion around  $b=0$ . Two different method for the remaining QCD part.
  - ❖ Intriguing relation between rapidity anomalous dimension and soft anomalous dimension.
- ❖ **N3LL pT resummation for Higgs production (except for four-loop cusp)**
  - ❖ Significant reduction of uncertainties. About 10% total uncertainties in the resummed region.

**Thank you for your attention!**