Measuring Elliptic Apertures using Rotating Coils

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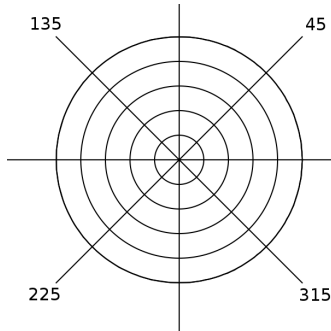
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Outline

## Motivation

- FAIR @ GSI →
    - with elliptic beam pipe: SIS 100, NESR, RESR
    - consice description of the field required
    - valid within the whole aperture
        - describe / compare the quality of the magnet(s)
        - managable data for users: e.g. beam dynanics

Theory
Test on Field Calculations for SIS 100
Field Reconstruction for Measurement

Circular Multipoles vs. Elliptic Multipoles
Converting Elliptic to Cyclic Multipoles

Coordiante systems

I/II

circles and lines

ellipse and hyperbola

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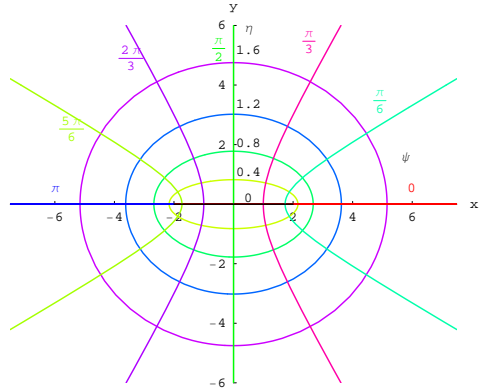
Coordiante systems

II/II

Plane polar coordinates

$$0 \leq r < \infty \quad -\pi \leq \phi \leq \pi,$$

$$x = r \cos \phi \quad y = r \sin \phi,$$

Elliptic coordiantes:

$$
\begin{aligned}
x &= e \cosh \eta \, \cos \psi \\
y &= e \sinh \eta \, \sin \psi \\
&\quad 0 \leq \eta < \infty, \\
&\quad -\pi \leq \psi \leq \pi.
\end{aligned}
$$

$$\varepsilon := e/a = \sqrt{1 - b^2/a^2}$$

Reference ellipse:

$$a = e \cosh \eta_0, \quad b = e \sinh \eta_0.$$

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Field description for potential equation $\Delta\Phi = 0$

Set of functions

$$
\begin{array}{c|ccc}
r^m e^{\mathrm{i}m\phi} & \sinh(n\eta)\, e^{\mathrm{i}n\psi} & \cosh(n\eta)\, e^{\mathrm{i}n\psi} & \text{for } n \neq 0 \\
r^m e^{-\mathrm{i}m\phi} \quad -\infty < m < \infty & 1 & \eta & \text{for } n = 0
\end{array}
$$

General solution

circular:

$$\Phi(r,\phi) = \sum_{m=-\infty}^{\infty} C_m \, (r/R_{Ref})^{|m|}\, e^{\mathrm{i}m\phi}$$

elliptic:

$$\Psi_g(\eta,\psi) = A_0 + B_0\eta + \sum_{n=-\infty}^{n=\infty}{}' \left[B_n \sinh(|n|\eta) + A_n \cosh(|n|\eta) \right] e^{in\psi}$$

The primed sum does not contain the term $n = 0$.

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Elliptic Multipoles

Required solution with real functions

$$
\Psi(\eta, \psi) = A_0 \underbrace{\frac{1}{2}}_{:=ce_0} + \sum_{n=1}^{\infty} \Big[\; A_n \underbrace{\cos(n\psi) \frac{\cosh(n\eta)}{\cosh(n\eta_0)}}_{:=ce_n} +
$$

$$
B_n \underbrace{\sin(n\psi) \frac{\sinh(n\eta)}{\sinh(n\eta_0)}}_{:=se_n} \Big]. \quad (1)
$$

$\Psi(\eta, \psi)$ can represent a scalar or e. g. $\mathbf{B} = B_y + i \, B_x$.
η_0 corresponds to R_{Ref}.

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Calculating A_n, B_n

Calculation of A_m and B_m using $\Psi(\eta_0, \psi)$ (Euler Formulas) or a FFT.

$$
\begin{aligned}
A_0 &= 2D_0; \\
A_n &= \mathrm{Re}(D_n + D_{-n}) + i\,\mathrm{Im}(D_n + D_{-n}), \\
B_n &= i\,\mathrm{Re}(D_n - D_{-n}) - \mathrm{Im}(D_n - D_{-n}); \\
&\qquad n = 1, 2, 3, ...
\end{aligned}
\tag{2}
$$

$$
D_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Psi(\eta_0, \psi)\, e^{(-in\,\psi)} \mathrm{d}\psi, \qquad -\infty < n < \infty; \tag{3}
$$

The last formulas still simplify if $\Psi(\eta_0, \psi)$ is real, since then $D_n^* = D_{-n}$.

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From Elliptic to Cyclic Multipoles

- Circular multipoles widespread (e.g beam dynamics)
- calculation of the multipoles:
  - elliptic multipoles A_n, B_n
  - the inverse transformation matrix $\hat{S}$ elliptic $\rightarrow$ circular
- calculating $\hat{S}$
  - insert elliptic coordinates (7) and (8) into $(z/R_0)^m$ and $(z^*/R_0)^m$; $m \geq 0$.
  - expand resulting expression in harmonics of hyperbolic and trignometric functions $\rightarrow$ $ce_n(\eta, \psi)$, $se_n(\eta, \psi)$
  - invert the matrix $\hat{T} \rightarrow \hat{S}$

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Theory Summary

- Circular Multipoles: Fourier Series on Circles
- Elliptic Multipoles: Fourier Series on Ellipse
- harmonics for both can be calculated using FFT
- but
  - only the circular ones correspond to the Carthesian ones
  - for the elliptic: only if $B_y + iB_x$ is imposed on the ellipse ones gets a straightforward transform to circular ones
  - SSW measurement on ellipse measures B_η!

Test on SIS 100 magnet options

dipole 8 turn single layer dipole 0.13 T at $873A$

quadrupole 5 turn quadrupole providing 35 T/m at $6600A$

high current in the cable $\rightarrow$ field quality

Field quality in units

$$\mathbf{\Delta B}(z) = \left[\mathbf{B}(z) - \mathbf{C_m} \left(\frac{z}{R_{Ref}} \right)^{m-1} \right] \frac{10^4}{\mathbf{C_m}}$$

with $\mathbf{C_m}$ the strength of the main multipole

Dipole: Original Data

B_y:
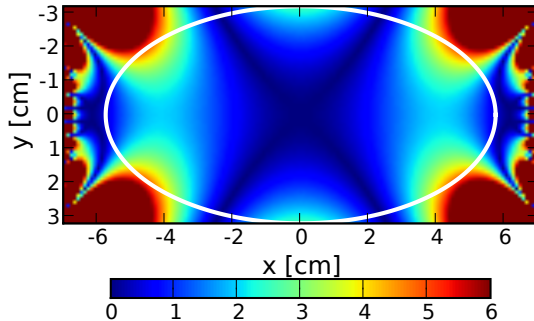
Dipole: Interpolation
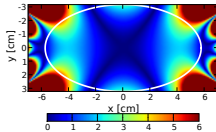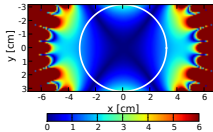
elliptic $\mathcal{C}_e$
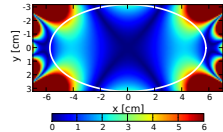
cyclic $\mathcal{C}$
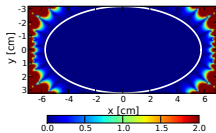
ell. $\rightarrow$ cyclic in $\mathcal{C}_e$
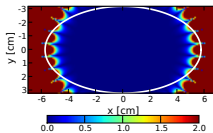
Δ elliptic $\mathcal{C}_e$
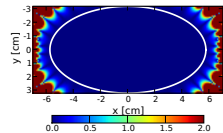
Δ cyclic $\mathcal{C}$

Δ ell. $\rightarrow$ cyclic in $\mathcal{C}_e$
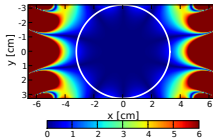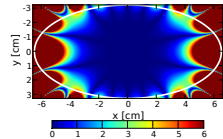
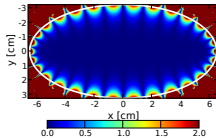# Quadrupole: Original Data

B_y:
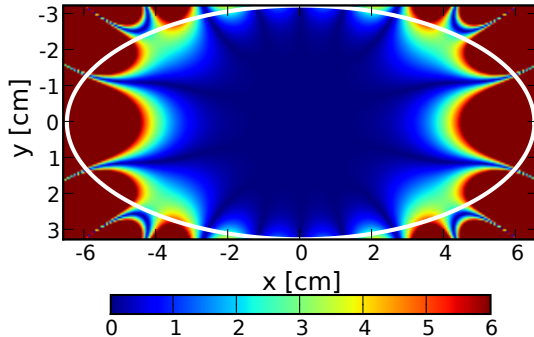
Quadrupole: Interpolation
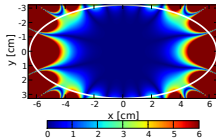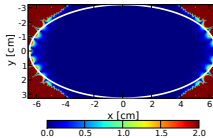

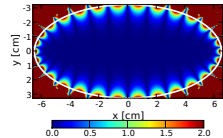elliptic $\mathcal{C}_e$ cyclic $\mathcal{C}$ ell. $\rightarrow$ cyclic in $\mathcal{C}_e$

Δ elliptic $\mathcal{C}_e$ Δ cyclic $\mathcal{C}$ Δ ell. $\rightarrow$ cyclic in $\mathcal{C}_e$

Motiviation for rotating coils measurements

- rectangular apertures $\rightarrow$ search coils
    - require a precise reference surface $\rightarrow$ pol
    - do not provide angle of the magnet
- SIS 100 $\rightarrow$ superconducting $\rightarrow$ anticryostat (circular, moveable)
    - rotating coils $\rightarrow$ circular multipoles
    - not covering whole area of interest
    - thus measurements at different locations
    - how many locations are enough?
    - how to combine the measurements?

Postion of measurement
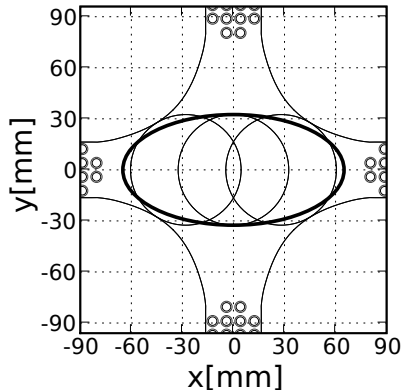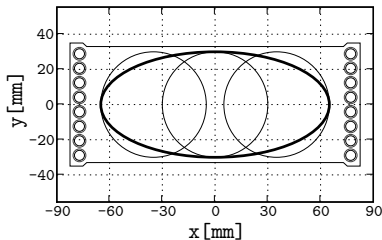
Not the whole ellipse is covered!

Connecting measurement data   I/II

- Interpolation between the circles

$$\mathbf{B_i}(\mathbf{z}) = (1 - \lambda) \sum_{n=1}^{N} \mathbf{C_n^c} \left(\frac{\mathbf{z}}{R_{Ref}} \right)^{(n-1)} + \lambda \sum_{n=1}^{N} \mathbf{C_n^{l,r}} \left(\frac{\mathbf{z} \pm a}{R_{Ref}} \right)^{(n-1)}$$

- How calculate lambda?

- validity of extrapolation: eg. distance from centre

$$w^l = \frac{R_{Ref}}{|\mathbf{z} - a|} \qquad w^c = \frac{R_{Ref}}{|\mathbf{z}|} \qquad w^r = \frac{R_{Ref}}{|\mathbf{z} + a|}$$

$$\lambda^{cl} = w^c / \left(w^c + w^l \right) \qquad \lambda^{cr} = w^c / \left(w^c + w^r \right)$$

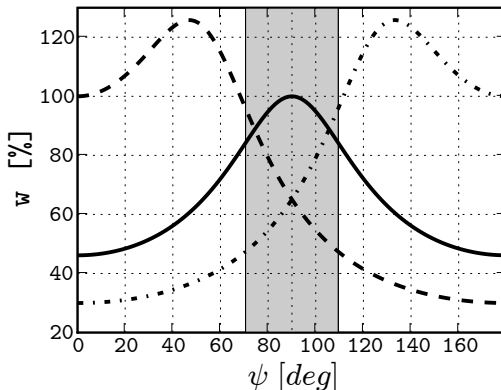# Connecting measurement data   II/II

$\lambda(p0) = 0 \ \lambda(p1) = 1$
$\lambda'(p_0) = 0 \ \lambda'(p_0)$
$\lambda(p) = 3p^2 - 2p^3$

p_0 intersection ellipse, measurment at left/right, p_1 at $\psi = 90(270)$ (centre)

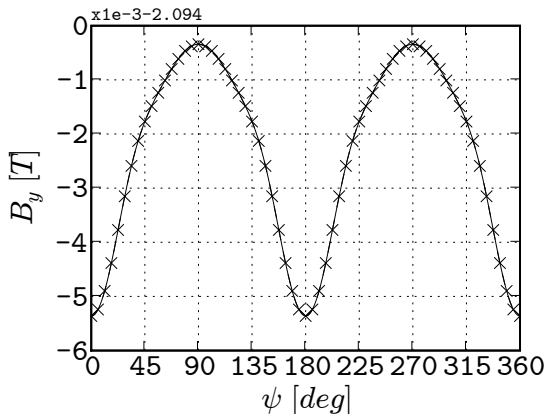## Field on the ellipse

Field on the ellipse $\mathbf{B(z)} =$
$\begin{cases} \sum_{n=1}^{N} \mathbf{C_n^{l,r}} \left(\frac{\mathbf{z} \pm a}{R_{Ref}} \right)^{(n\cdot}} \\ \mathbf{B_i(z)} \end{cases}$
$\psi = cosh^{-1}(z/e)$
Multipoles on the interpolation

Archieved precision

- for dipole: difference reconstructed $\leftrightarrow$ field 1 ppm
- for quadrupole 1 per mille $\rightarrow$ improvement required

Conclusion

- elliptic multipoles solution of $\Delta\Phi = 0$
- describe field within reference ellipse
- elliptic multipole $\rightarrow$ cicular multipoles
- field reconstruction from measurement
  - demonstrated for dipole
  - refinement required for quadrupole